Modeling Altruism in Experiments

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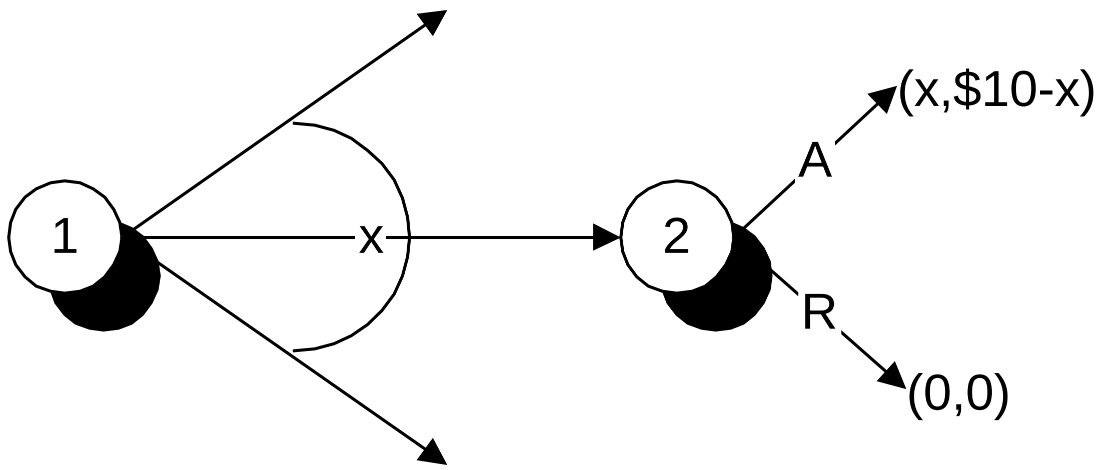
Ultimatum

Roth et al [1991]: ultimatum bargaining in four countries

extensive form



usual selfish case with $a_i = 0$ player 2 accepts any demand less than $10

subgame perfection requires player 1 demand at least $9.95

Table 1 below pools results of the final (of 10) periods of play in the 5 experiments with payoffs normalized to $10

Demand	Observations	Frequency of Observations	Accepted Demands	Probability of Acceptance
$5.00	37	28%	37	1.00
$6.00	67	52%	55	0.82
$7.00	26	20%	17	0.65

Table 1

Altruistic Preferences

- players $i = 1,\dots,n$
- at terminal nodes *direct utility* of u_i
- coefficient of altruism $-1 < a_i < 1$
- *adjusted utility*

$$v_i = u_i + \sum_{j \neq i} a_i u_j$$

$$v_i = u_i + \sum_{j \neq i} \frac{a_i + \lambda a_j}{1+\lambda} u_j \,.$$

- $0 \leq \lambda \leq 1$
- objective is to maximize adjusted utility

- since the stakes are small, ignore risk aversion, and identify direct utility with monetary payoffs
- prior to start of play, players drawn independently from population with a distribution of altruism coefficients represented by a common cumulative distribution function. $F(a_i)$
- each player's altruism coefficient a_i is privately known
- the distribution *F* is common knowledge
- we model a particular game as a Bayesian game, augmented by the private information about types
- marginal utility of money returned to experimenter is assumed zero

Related Work

$v_i = u_i + \sum_{j \neq i} \beta_{ij} u_j$,

β_{ij} determined from players types or other details about the game

- Ledyard [1995] $\beta_{ij} = \gamma_i (u_j^f - u_j)$, u_j^f is undefined “fair amount”
- Rabin [1993] $\beta_{ij} = \gamma_i (u_i - u_i^f)$ player cares about fair for himself, rather than fair for the other player; “fair amount” is a fixed weighted average of the maximum and minimum Pareto efficient payoff given player *i*’s own choice of strategy; coefficient γ_i endogenous in complicated way

Andreoni and Miller [1996]

Palfrey and Prisbrey [1997] warm glow effect

value of contributions to other players not so important as the cost of the donation

there is a "warm glow": players wish to incur a particular cost of contribution, regardless of the benefit.

4-person public goods contribution game

players must decide whether or not to contribute a single token

each period each player randomly draws value ξ_i for token, uniformly distributed on 1 to 20

token kept, the value of token is paid

token contributed fixed amount γ paid to each player

$$u_i = \xi_i - \xi_i m_i + \gamma \sum_{j=1}^{n} m_j \,.$$

each player 20 rounds with fixed value of γ

four times with different values of γ

each round players shuffled

results from the second 10 rounds with each value of γ, so players relatively experienced

	$\gamma = 3$		$\gamma = 15$	
$\xi_i - \gamma$	Gain ratio	$\overline{m}$	Gain ratio	$\overline{m}$
5	1.8	0.00	9.0	0.60
3-4	2.7	0.18	13.1	0.67
1-2	6.8	0.27	33.7	0.79
0		0.88		0.86

Table 2

data is pooled as indicated in the table.

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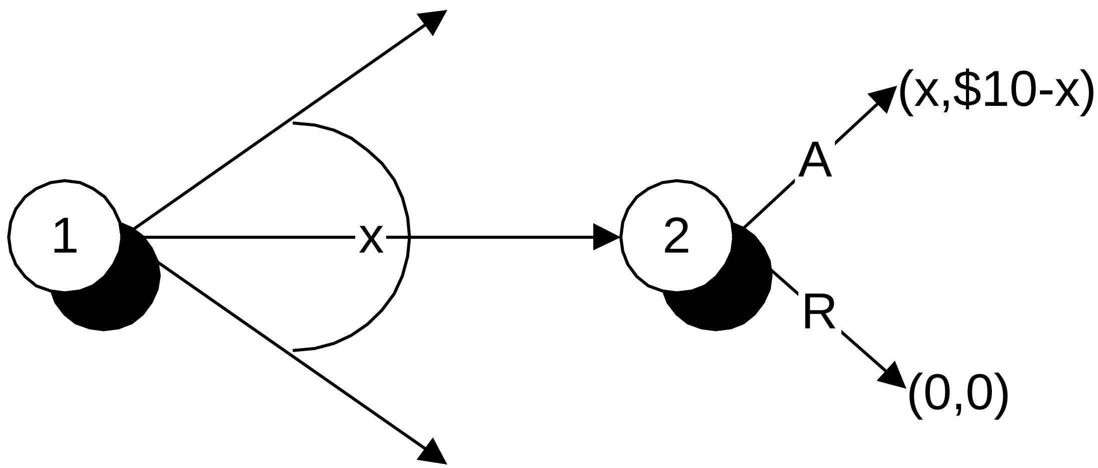

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Table 3

Proposition 1: No demand will be made for less than $5.00, and any demand of $5.00 or less will be accepted.

In fact in the data only was offer of less than $5.00 was ever made, and it was for $4.75 and was accepted, so the data are consistent with Proposition 1 assume that the distribution *F* places weight on three points $\overline{a} > a_0 > \underline{a}$

altruistic normal and spiteful types

since there are three demands made in equilibrium, and more altruistic types will prefer to make lower demands, we look for an equilibrium in which the altruistic types demand \$5.00, the normal type \$6.00 and the spiteful type \$7.00 (also require that no type wants to demand more than \$7.00)

so probabilities of the three types are 0.28, 0.52 and 0.20 respectively, as this is the frequency of demands in the sample

$5.00 demand is accepted by all three types

$6.00 demand is accepted by 82% of the population; but attribute the difference between 80% and 82% to sampling error (can’t reject at 28% level) so assume exactly spiteful types reject

$7.00 demand accepted by 65% of the population, corresponding to all the altruistic types (28%) and 71% ($0.71\times 0.52 \approx 0.37$) of the normal types

so normal types must be indifferent between accepting and rejecting a $7.00 demand consider the \$5.00 demand

all types will accept this demand, the adjusted utility received by a player demanding this amount is

$$5+\frac{a+\lambda(.28\overline{a}+.52a_0+.20\underline{a})}{1+\lambda}5$$

if the spiteful type accepts, all types will accept the demand

since offer is known to be made by the altruistic type, for spiteful type to accept we must have

$$5+\frac{\underline{a}+\lambda\overline{a}}{1+\lambda}5\geq 0$$

(this inequality is always satisfied for $\underline{a},\overline{a}>-1$)

(1) $$(6+\frac{a_0+\lambda(.35\overline{a}+.65a_0)}{1+\lambda}4).8-(5+\frac{a_0+\lambda(.28\overline{a}+.52a_0+.20\underline{a})}{1+\lambda}5)\geq 0$$

(2) $$(6+\frac{\overline{a}+\lambda(.35\overline{a}+.65a_0)}{1+\lambda}4).8-(5+\frac{\overline{a}+\lambda(.28\overline{a}+.52a_0+.20\underline{a})}{1+\lambda}5)\leq 0$$

(3) $$4+\frac{\underline{a}+\lambda a_0}{1+\lambda}6\leq 0$$

$$(7+\frac{a+\lambda(.43\overline{a}+.57a_0)}{1+\lambda}3).65$$

(4) $$(7+\frac{\underline{a}+\lambda(.43\overline{a}+.57a_0)}{1+\lambda}3).65-(6+\frac{\underline{a}+\lambda(.35\overline{a}+.65a_0)}{1+\lambda}4).8\geq 0$$

(5) $$(7+\frac{a_0+\lambda(.43\overline{a}+.57a_0)}{1+\lambda}3).65-(6+\frac{a_0+\lambda(.35\overline{a}+.65a_0)}{1+\lambda}4).8\leq 0$$

(6) $$3+\frac{a_0+\lambda\underline{a}}{1+\lambda}7=0$$

a sequential equilibrium matching the data will be given by parameters $1 > \bar{a} > a_0 > \underline{a} > -1,\ 0 \le \lambda \le 1$ such that the inequalities (1) through (5) and the equality (6) above are satisfied

Proposition 2: There is no equilibrium with $\lambda = 0$.

Proposition 3: In equilibrium $-.301 \le a_0 \le -.095$, $-1 < \underline{a} < -2/3$, $1 \ge \lambda \ge 0.222$.

Parameter's consistent with sequential equilibrium

$\overline{a}$	0.10	0.30	0.40	0.90	0.90	0.90	0.90
a_0	-0.22	-0.22	-0.22	-0.22	-0.27	-0.26	-0.20
$\underline{a}$	-0.90	-0.90	-0.90	-0.90	-0.87	-0.90	-0.90
λ	0.45	0.45	0.45	0.45	0.36	0.35	0.49

Table 4

it appears to be difficult to get $\underline{a}$ larger than -0.87 (versus the known lower bound of -2/3)

values of λ are difficult to find lower than 0.35 (against the known lower bound of 0.22)

values of λ are difficult to get higher than 0.49, although I have not been able to get an analytic upper bound on λ (other than 1)

couldn't find equilibria with values of a_0 below -0.2, although the known lower bound is only $-.301$.

Competitive Auction: Sanity Check

Roth et al report a market game experiment under similar experimental conditions

Nine identical buyers submit an offer to a single seller to buy an indivisible object worth nothing to the seller and $10.00 to the buyer. If the seller accepts he earns the highest price offered, and a buyer selected from the winning bids by lottery earns the difference between the object's value and the bid. Each player participates in 10 different market rounds with a changing population of buyers.

game has two subgame perfect equilibrium outcomes (with selfish players): either the prices is $10.00, or everyone bids $9.95

in the experiment by round 7 the price rose to $9.95 or $10.00 in every experiment, and typically this occurred much earlier let α be the coefficient of altruism adjusted for the opponent's altruism

seller accepts *x* if $x+\alpha(1-x)\geq 0$

$\alpha > -1$ so true provided that $x \geq \$5.00$

buyers: if there are multiple offers at $10.00 then no seller can have any effect on their own utility, since the seller always gets $10.00 and the buyers $0.00 regardless of how any individual seller deviates

more generally, suppose that seller offers are independent of how altruistic they are

an offer *x* accepted with probability *p* gives utility

$$p((1-x)+\alpha x)+(1-p)\alpha = \alpha + (1-\alpha)p(1-x)$$

which regardless of α are the same preferences as $1-x$

since preferences are independent of altruism, players are willing to use strategies that are independent of how altruistic they are, so every equilibrium without altruism is an equilibrium with altruism

Centipede

McKelvey and Palfrey [1992] 29 experiments over the last 5 of 10 rounds of play,

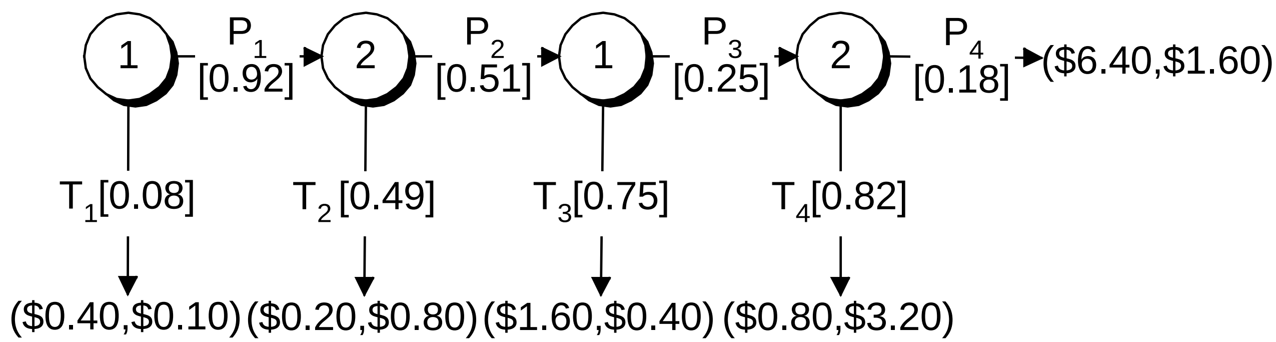

Figure 1

does not make much sense with selfish players 18% of player 2's who reach the final move choose to throw away money

with selfish preferences, the unique Nash equilibrium is for all player 1's to drop out immediately model the same model of three types we used to analyze ultimatum

assume $\lambda = 0.45$, $\underline{a} = -0.9$ and $a_0 = -0.22$, which are parameters that have been narrowed down by the data on ultimatum

probabilities of the spiteful, normal and altruistic groups are $0.20, 0.52, 0.28$ respectively virtually no player 1's drop out in the first move, so that the distribution of types the second time player 1 moves should be essentially the prior distribution

second move by player 1, 25% of the players choose to continue, which, within the margin of sampling error, is quite close to the 28% of player 1's that are altruistic. So we will assume that in player 1's final move, all the altruistic types pass, and all the other types take, and we will analyze the following modified data

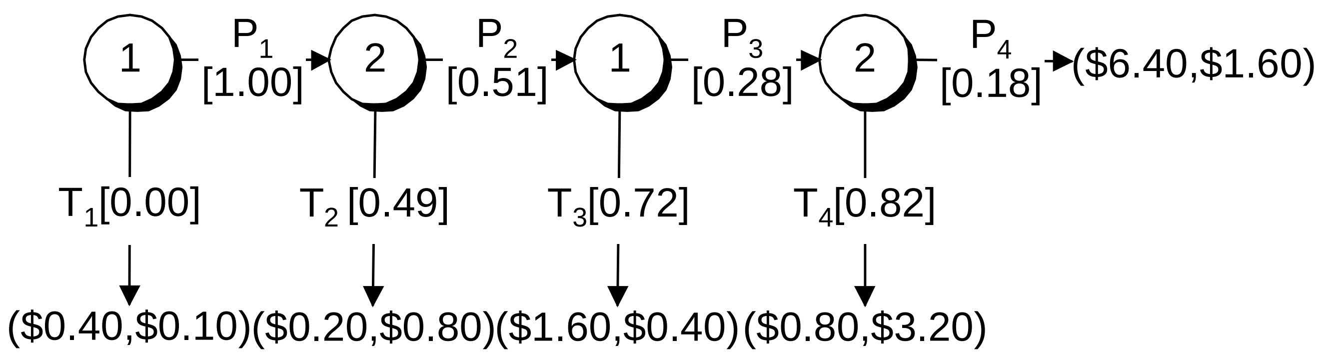

Figure 2

player 2’s at the final node first

spiteful and selfish types drop out before altruists, and fewer players pass than the 28% of the population that are altruists, we conclude that the altruistic types must be indifferent between passing and taking

all player 1’s are known to player 2 to be altruists at this point, it follows that

$$3.20+\frac{\overline{a}+\lambda\overline{a}}{1+\lambda}0.80=1.60+\frac{\overline{a}+\lambda\overline{a}}{1+\lambda}6.40.$$

From this we may calculate $\overline{a}=2/7\approx 0.29$. This is one of the wide range of values consistent with the ultimatum data.

consider player 1's final decision to pass or take

51% of the player 2's previously passed, including all the altruistic player 2's, so $0.28/0.51 = 0.55$ of the player 2's are altruists and the remaining 0.45 are selfish types

player 1 takes, he then places a weight on his opponents utility of

$$a_T \equiv \frac{a_0 + \lambda(0.55 \times \overline{a} + 0.45 \times a_0)}{1+\lambda} = -0.13.$$

utility if he takes is $1.60 + a_T 0.40 = 1.55$

pass, has a 0.18 chance of an altruistic opponent; gets \$6.40 for himself and \$1.60 for the opponent or \$6.31

faces a 0.82 chance of an opponent who is $0.45/0.82 = 0.55$ likely to be selfish and 0.45 likely to be altruistic yields a utility of $0.33

averaging over his opponent passing and taking in the final round, yields the expected utility to passing of $1.40

less than the utility of taking

selfish type should take; so should spiteful type. since normal type nearly indifferent altruistic type passes utility from taking and passing

Node	Type	Take Utility	Pass Utility	Difference
1's last move	a_0	$1.55	$1.40	$0.14
2's first move	a_0	$0.76	$0.85	-$0.09
1's first move	$\underline{a}$	$0.33	$0.49	-$0.16

Table 5

spiteful type 1 player willing to pass in the first period only inconsistency:

selfish type of player 2 first move should be indifferent between passing and taking, and in fact prefers to pass

Public Goods Contribution Game

public goods contribution game studied by Isaac and Walker [1988]

simultaneous move *n* person game

each individual may contribute a number of tokens to a common pool, or consume them privately

m_i is the number of tokens contributed (normalize so that the total number of available tokens per player is 1), the direct utility is given by

$$u_i = -m_i + \gamma \sum_{j=1}^{n} m_j$$

four treatments were used with different numbers of players and different values for the marginal per capital return γ

consider the final round of play only; each treatment was repeated three times

The data from the experiments

γ	n	$m_i > 0$	$m_i > 1/3$	$\overline{m}$	a^*
0.3	4	0.00	0.00	0.00	1.13
0.3	10	0.23	0.10	0.07	0.38
0.75	4	0.58	0.33	0.29	0.17
0.75	10	0.55	0.30	0.24	0.06

Table 6

vs 28% altruists w/ average coefficient of 0.29

as above assume $\lambda = 0.45$, $\underline{a} = -0.9$, $a_0 = -0.22$, $\overline{a} = 0.29$ w/ probabilities $0.20, 0.52, 0.28$

mean population altruism $\hat{a} = -0.21$

adjusted utility of contributing

$$v_i = -m_i + \gamma(\, m_i + \hat{m}_{-i}\,) + \frac{a_i + \lambda \hat{a}}{1+\lambda}(n-1)(-\hat{m}_{-i} + \gamma(m_i + (n-1)\hat{m}_{-i}\,)$$

where $\hat{m}_{-i}$ is the mean contribution by players other than player *i*.

differentiating with respect to own contribution

$$-1 + \gamma + \frac{a_i + \lambda \hat{a}}{1+\lambda}(n-1)\gamma \geq 0.$$

And calculate cutoff

$$a* = \frac{(1-\gamma)(1+\lambda)}{(n-1)\gamma} - \lambda \hat{a}.$$

Things that don't work

- dictator
- Seely and Van Huyck "Strategy Coordination and Public Goods"; gets off the boundary but have less altruism and less spite
- with more than two players does a spiteful player care about the total utility he deprives other players of, or how well he does relative to an order statistic or mean?