Economic 211, David K. Levine
Problems on Game Theory Fundamentals

January 12, 2005

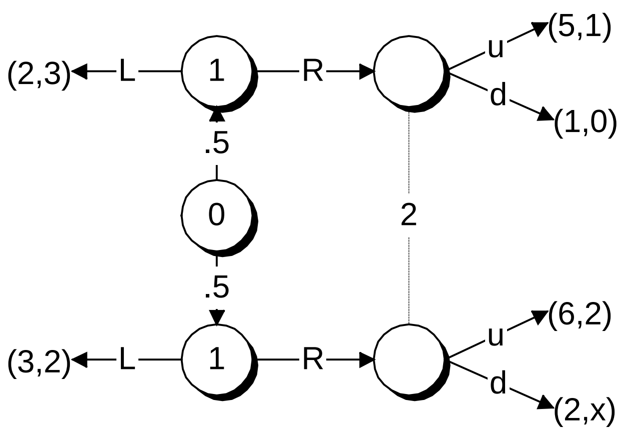
1. Risk Dominance and Pareto Efficiency

Suppose that $x \le 21$. The symmetric game below has a Nash equilibrium that Pareto dominates all other outcomes of the game, plus another pure Nash equilibrium. For what values of *x* is the Pareto dominant equilibrium also risk dominant?

	L	R
U	20,20	19,*x*
D	*x*,19	21,21

2. Refinements of Nash Equilibrium

Consider the following extensive form:



In each of the three cases $x = 1,2,3$ find the mixed and pure Nash, and pure Subgame Perfect, Sequential and Trembling Hand Perfect equilibria. Can any strategies be eliminated through iterated weak dominance?

3. The Minmax Theorem and Correlated Play

Suppose that $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ is a vector of mixed strategies in a finite game, and that $u^i(\sigma)$ are the payoffs to player *i*. Define the *maxmin* for player *i* to be the amount that a player can guarantee himself no matter how his opponents play

$$\max\min = \max_{\sigma_i} \min_{\sigma_{-i}} u^i(\sigma).$$

Define the *minmax* for player i to be the smallest amount player i's opponents can reduce his payoff to when player i knows their strategies

$$\min\max = \min_{\sigma_{-i}} \max_{\sigma_i} u^i(\sigma).$$

(a) Show that minmax$\geq$maxmin.

(b) Let ρ_{-i} be a correlated strategy for all the players other than player i. Using the fact that in two-player games minmax = maxmin, show that

$$\max\min \geq \min_{\rho_{-i}} \max_{\sigma_i} u^i(\sigma_i, \rho_{-i}).$$

(c) Construct an example of a THREE PLAYER game in which minmax>maxmin.