Financial Contagion

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Abstract

Financial contagion is modeled as an equilibrium phenomenon. Because liquidity preference shocks are imperfectly correlated across regions, banks hold inter-regional claims on other banks to provide insurance against liquidity preference shocks. When there is no aggregate uncertainty, the first-best allocation of risk sharing can be achieved. However, this arrangement is financially fragile. A small liquidity preference shock in one region can spread by contagion throughout the economy. The possibility of contagion depends strongly on the completeness of the structure of interregional claims. Complete claims structures are shown to be more robust than incomplete structures.

1 Introduction

There is a long tradition of regarding dislocation in the financial sector as a cause of economic fluctuations (Friedman and Schwartz (1963), Bernanke (1983), Bernanke and Gertler (1989)). According to this view, financial crises are important because they raise the costs of intermediation and restrict credit, which in turn restrain the level of activity in the real sector and ultimately can lead to periods of low growth and recession.

The prevalence of financial crises has led many to conclude that the financial sector is unusually susceptible to shocks. One theory is that small shocks, which initially only affect a few institutions or a particular region of the economy, spread by contagion to the rest of the financial sector and then infect the larger economy. In this paper, we focus on one channel of contagion, the overlapping claims that different regions or sectors of the banking system have on one another. When one region suffers a bank crisis, the other regions suffer a loss because their claims on the troubled region fall in value. If this spillover effect is strong enough, it can cause a crisis in the adjacent regions. In extreme cases, the crisis passes from region to region and becomes a contagion.

In order to focus on the role of one particular channel for financial contagion, we exclude other propagation mechanisms that may be important for a fuller understanding of financial contagion. In particular, we assume that agents have complete information about their environment. Incomplete information may create another channel for contagion. If a shock in one region serves as a signal predicting a shock in another region, then a crisis in one region may create a self-fulfilling expectation of a crisis in another region.

We also exclude the effect of international currency markets in the propagation of financial crises from one country to another. Currency crises have been extensively studied and Calvo (1995) and Chang and Velasco (1998), among others, have studied the interaction of the banking system and currency markets in a crisis; but the role of currency markets in financial contagion is left as a subject for future research. For a survey of recent work on crises, see Calomiris (1995).

The central aim of this paper is to provide some microeoconomic foundations for financial contagion. Although the analysis may have some relevance to the recent Asian financial crisis, the model developed in this paper is not intended to be a description of any particular episode. If it has a family resemblance to any historical episode, it would be to the banking crises in the United States in the late nineteenth and early twentieth centuries (Hicks (1989)).

We take as our starting point the model presented in Allen and Gale (1998a). The basic assumptions about technology and preferences have become the standard in the literature since the appearance of the Diamond and Dybvig (1983) model. There are three dates $t = 0, 1, 2$ and a large number of identical consumers, each of whom is endowed with one unit of a homogeneous consumption good. At date 1, the consumers learn whether they are early consumers, who only value consumption at date 1, or late consumers, who only value consumption at date 2. Uncertainty about their preferences creates a demand for liquidity.

Banks have a comparative advantage in providing liquidity. At the first date, consumers deposit their endowments in the banks, which invest them on behalf of the depositors. In exchange, depositors are promised a fixed amount of consumption at each subsequent date, depending on when they choose to withdraw. The bank can invest in two assets. There is a short-term asset that pays a return of one unit after one period and there is a long-term asset that pays a return $r < 1$ after one period or $R > 1$ after two periods. The long asset has a higher return if held to maturity, but liquidating it in the middle period is costly, so it is not very useful for providing consumption to early consumers. The banking sector is perfectly competitive, so banks offer risk-sharing contracts that maximize depositors' ex ante expected utility, subject to a zero-profit constraint.

Using this framework, we are interested in constructing a model in which small shocks lead to large effects by means of contagion, more precisely, in which a shock within a single sector can spread to other sectors and lead to an economy-wide financial crisis. One view is that financial crises are purely random events, unrelated to changes in the real economy (Kindleberger (1978)). The modern version of this view, developed by Diamond and Dybvig (1983) and others, is that bank runs are self-fulfilling prophecies. An alternative view is that financial crises are an inherent part of the business cycle (Mitchell (1941), Gorton (1988), Allen and Gale (1998a)). The disadvantage of treating contagion as a "sunspot" phenomenon is that, without some real connection between different regions, any pattern of correlations is possible. So sunspot theories are equally consistent with contagion and the absence of contagion. We are interested in establishing a stronger result, that under certain circumstances any equilibrium of the model must be characterized by contagion. This form of contagion must be driven by real shocks and real linkages between regions.

The economy consists of a number of regions. The number of early and late consumers in each region fluctuates randomly, but the aggregate demand for liquidity is constant. This allows for inter-regional insurance as regions with liquidity surpluses provide liquidity for regions with liquidity shortages. One way to organize the provision of insurance is through an interbank market in deposits. Suppose that region A has a large number of early consumers when region B has a low number of early consumers, and vice versa. Since regions A and B are otherwise identical, their deposits are perfect substitutes. The banks exchange deposits at the first date before they observe the liquidity shocks. If region A has a higher than average number of early consumers at date 1 then banks in region A can meet their obligations by liquidating some of their deposits in the banks of region B. Region B is happy to oblige, because it has an excess supply of liquidity in the form of the short asset. At the final date the process is reversed, as banks in region B liquidate the deposits they hold in region A to meet the above-average demand from late consumers in region B.

Inter-regional cross holdings of deposits work well as long as there is enough liquidity in the banking system as a whole. If there is an excess demand for liquidity, however, the financial linkages caused by these cross holdings can turn out to be a disaster. While cross holdings of deposits are useful for reallocating liquidity within the banking system, they cannot increase the total amount of liquidity. If the economy-wide demand from consumers is greater than the stock of the short asset, the only way to provide more consumption is to liquidate the long asset. This is very costly, however, so banks try to avoid doing this whenever possible (see Shleifer and Vishny (1992) and Allen and Gale (1998a) for a discussion of the costs of premature liquidation). As a result, banks in regions that are not immediately affected by an excess demand for liquidity will avoid providing liquidity to regions with an excess demand for liquidity. The result may be bank runs and bankruptcy in the immediately affected region. What begins as a financial crisis in one region can then spread by contagion to other regions because of the cross-holdings of deposits.

Whether the financial crisis does spread in this way depends crucially on the pattern of inter-connectedness generated by the cross holdings of deposits. If the interbank market is *complete* and each region is connected to all the other regions, the initial impact of a financial crisis in one region may be attenuated. On the other hand, if the interbank market is *incomplete* and, as a result, each region is connected with a small number of other regions, the initial impact of the financial crisis may be felt very strongly in those neighboring regions, with the result that they too succumb to a crisis. As each region is affected by the crisis, it prompts premature liquidation of long assets, with a consequent loss of value, so that previously unaffected regions find that they too are affected because their claims on the region in crisis have fallen in value.

It is important to note the role of the free rider problem in explaining the difference between a complete and incomplete interbank market. There is a natural pecking order among different sources for liquidity. A bank will meet withdrawals first from the short asset, then from deposits held in other regions, and only in the last resort will it choose to liquidate the long asset. Cross holdings of deposits are useful for redistributing liquidity, but they do not create liquidity; so when there is a global shortage of liquidity (withdrawals exceed short assets), the only solution is to liquidate long assets. If every region takes a small hit (liquidates a small amount of the long asset) there may be no need for a global crisis. This is what happens with complete markets: banks in the troubled region have direct claims on banks in every other region and there is no way to avoid paying one's share. With incomplete markets, banks in the troubled region have a direct claim only on the banks in adjacent regions. The banks in other regions maximize their own interests and refuse to liquidate the long asset until they find themselves on the front line of the contagion.

This paper is related to a diverse literature. The closest to our own approach is the interesting paper by Lagunoff and Schreft (1998), which studies the spread of crises in a probabilistic model. Financial linkages are modeled by assuming that each project requires two participants and each participant requires two projects. When the probability that one's partner will withdraw becomes too large, all participants simultaneously withdraw and this is interpreted as a financial crisis. Financial multipliers are modeled by Kiyotaki and Moore (1998). In their model, the impact of illiquidity at one link in the credit chain travels down the chain. Edison, Pongsak and Miller (1998) use the Kiyotaki and Moore (1997) model of credit cycles as the basis for a model of financial crises: shortage of liquidity leads to collateral sales which put pressure on other firms' ability to borrow. A similar "meltdown" phenomenon in asset markets occurs in Allen and Gale (1998a). Chan-Lau and Chen (1998) present a model of financial crises based on costly monitoring. Models of crises based on multiple equilibria are Chari and

Kehoe (1997), Cole and Kehoe (1996) and Cooper and Corbae (1997). Fujiki, Green and Yamazaki (1997) study settlement risk. Rochet and Tirole (1996) use monitoring as a means of triggering correlated crises: if one bank fails, it is assumed that other banks have not been properly monitored and a general collapse occurs. Holmstrom and Tirole (1998) have studied the efficiency of the provision of liquidity in a model where firms have several methods of providing future liquidity. In equilibrium, the provision of liquidity may be inefficient because of the use of non-contingent debt instruments.

The notion of contagion appears in game theory in a number of guises (see Morris (1997) and Morris, Rob and Shin (1995)). The work of Ellison (1993) emphasized the importance of local interactions for the diffusion of cooperative behavior through a network. Chwe (1998) examines how the structure of information networks influences the possibility of cooperation in the context of a coordination problem. There is also a large literature on local interactions in macroeconomics (e.g., Durlauf (1993), Scheinkman and Woodford (1994)).

The rest of the paper is organized as follows. In Section 2 we present a model of liquidity preference based on Diamond and Dybvig (1983) and Allen and Gale (1998a). In Section 3 we characterize optimal risk sharing in terms of a planning problem subject to incentive constraints and show that the incentive-efficient allocation is in fact the same as the first best. Section 4 shows how the first-best allocation can be decentralized through a competitive banking system with an interbank market in deposits. Several different market structures are described and each turns out to be consistent with the first best. The robustness of this schema is tested in the next three sections. We do this by perturbing the model to allow for an (aggregate) excess demand for liquidity in some states of nature. In Section 5 it is shown that with an incomplete interbank market and a high degree of interconnectedness, a liquidity shock that causes a crisis in one region will spread by contagion to others. In Section 6, we consider a complete interbank market and an even higher degree of connectedness. It is shown that, with the same size shock and the same model parameters, there is no contagion. In Section 7, it is shown that with an incomplete interbank market and a low degree of connectedness, there is again no contagion. So the interaction of connectedness and incompleteness appear to be conducive to contagion.

2 Liquidity Preference

In this section we describe a simple model in which stochastic liquidity preference provides a motive for risk-sharing. The framework is based on Diamond and Dybvig (1983) and Allen and Gale (1998a), with some significant differences.

There are three dates $t = 0, 1, 2$. There is a single consumption good which serves as the numeraire. This good can also be invested in assets to produce future consumption. There are two types of assets, a liquid asset and an illiquid asset. The liquid asset is represented by a storage technology. One unit of the consumption good invested in the storage technology at date t produces one unit of the consumption good at date $t+1$. Because the returns to this asset are available one period later, we refer to it as the *short asset*. The illiquid asset has a higher return but requires more time to mature. For this reason we call it the *long asset*. Investment in the long asset can only take place in the first period and one unit of the consumption good invested in the long asset at the first date produces $R > 1$ units of output at the final date.

The long asset is not completely illiquid. Each unit of the long asset can be prematurely liquidated to produce $0 < r < 1$ units of the consumption good at the middle date. Here we assume that liquidation takes the form of physical depreciation of the asset and the liquidation value is treated as a technological constant, the "scrap value". In practice, it is more likely that assets are liquidated by being sold, in which case the liquidation value is determined by the market price. Introducing a secondary market on which assets can be sold would complicate the analysis without changing the qualitative features of the model. Allen and Gale (1998a) incorporates an asset market and endogenizes the liquidation values of assets. The analysis in Allen and Gale (1998a) confirms that the liquidation value will be low for appropriate parameter values.

The economy is divided into four ex ante identical regions, labeled A, B, C, and D. The regional structure is a spatial metaphor that can be interpreted in a variety of ways. The important thing for the analysis is that different regions receive different liquidity shocks. Any story that motivates different shocks for different (groups of) banks is a possible interpretation of the regional structure. So a region can correspond to a single bank, a geographical region within a country, or an entire country; it can also correspond to a specialized sector within the banking industry.

Each region contains a continuum of ex ante identical consumers (depositors). A consumer has an endowment equal to one unit of the consumption good at date 0 and nothing at dates 1 and 2. Consumers are assumed to have the usual Diamond-Dybvig preferences: with probability ω they are early consumers and only value consumption at date 1; with probability $(1-\omega)$ they are late consumers and only value consumption at date 2. Then the preferences of the individual consumer are given by

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \omega \\ u(c_2) & \text{with probability } 1-\omega \end{cases}$$

where c_t denotes consumption at date $t = 1, 2$. The period utility functions $u(\cdot)$ are assumed to be twice continuously differentiable, increasing and strictly concave.

The probability ω varies from region to region. Let ω^i denote the probability of being an early consumer in region i. There are two possible values of ω^i, a high value and a low value, denoted ω_H and ω_L, where $0 < \omega_L < \omega_H < 1$. The realization of these random variables depends on the state of nature. There are two equally likely states S_1 and S_2 and the corresponding realizations of the liquidity preference shocks are given in the table below:

Table 1: Regional Liquidity Shocks

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H

Note that ex ante each region has the same probability of having a high liquidity preference shock. Also, the aggregate demand for liquidity is the same in each state: half the regions have high liquidity preference and half have low liquidity preference.

All uncertainty is resolved at date 1 when the state of nature S_1 or S_2 is revealed and each consumer learns whether he is an early or late consumer. A consumer's type is not observable, so late consumers can always imitate early consumers.

Before introducing the banking sector into our story, it will be convenient to characterize the optimal allocation of risk.

3 Optimal Risk-Sharing

In this section we characterize optimal risk sharing as the solution to a planning problem. Since consumers are ex ante identical, it is natural to treat consumers symmetrically. For this reason, the planner is assumed to make all the investment and consumption decisions to maximize the unweighted sum of consumers' expected utility.

We begin by describing the planner's problem under the assumption that the planner can identify early and late consumers. The symmetry and concavity of the objective function and the convexity of the constraints simplifies the problem considerably.

- Since there is no aggregate uncertainty, the optimal consumption allocation will be independent of the state.

- Since the consumers in one region are ex ante identical to consumers in another region, all consumers will be treated alike.

Without loss of generality, then, we can assume that every early consumer receives consumption c_1 and every late consumer receives c_2, independently of the region and state of nature. At the first date, the planner chooses a portfolio $(x, y) \geq 0$ subject to the feasibility constraint

$$x + y \leq 1, \tag{1}$$

where x and y are the per capita amounts invested in the long and short assets respectively.

- Since the total amount of consumption provided in each period is a constant, it is optimal to provide for consumption at date 1 by holding the short asset and to provide for consumption at date 2 by holding the long asset.

Let the average fraction of early consumers be denoted by $\gamma = (\omega_H + \omega_L)/2$. Then the feasibility constraint at date 1 is

$$\gamma c_1 \leq y \tag{2}$$

and the feasibility constraint at date 2 is

$$(1 - \gamma) c_2 \leq Rx. \tag{3}$$

At date 0 each consumer has an equal probability of being an early or a late consumer, so the ex ante expected utility is

$$\frac{1}{2}u(c_1) + \frac{1}{2}u(c_2) \tag{4}$$

and this is what the planner seeks to maximize, subject to the constraints (1), (2) and (3). The unique solution to this unconstrained problem is called the *first-best allocation*.

The first-best allocation satisfies the first-order condition

$$u'(c_1) \geq u'(c_2).$$

Otherwise, the objective function could be increased by using the short asset to shift some consumption from early to late consumers. Thus, the first-best allocation automatically satisfies the *incentive constraint*

$$c_1 \leq c_2, \tag{5}$$

which says that late consumers find it weakly optimal to reveal their true type, rather than pretend to be early consumers. The *incentive-efficient allocation* maximizes the objective function (4) subject to the feasibility constraints (1), (2), and (3), and the incentive-constraint (5). What we have shown is that the incentive-efficient allocation is the same as the first-best allocation.

Proposition 1 *The first-best allocation (x, y, c_1, c_2) is equivalent to the incentive-efficient allocation, so the first best can be achieved even if the planner cannot observe the consumers' types.*

In order to achieve the first best, the planner has to transfer resources among the different regions. In state S_1, for example, there are ω_H early consumers in regions A and C and ω_L early consumers in regions B and D. Each region has γc_1 units of the short asset, which provide γc_1 units of consumption. So regions A and C each have an excess demand for $(\omega_H - \gamma)c_1$ units of consumption and regions B and D each have an excess supply of $(\gamma - \omega_L)c_1 = (\omega_H - \gamma)c_2$ units of consumption. By re-allocating this consumption, the planner can satisfy every region's needs. At date 2, the transfers flow in the opposite direction, because regions B and D have an excess demand of $(\omega_H - \gamma)c_2$ units each and regions A and C have an excess supply of $(\omega_h - \gamma)c_2$ units each.

4 Decentralization

In this section we describe how the first-best allocation can be decentralized by a competitive banking sector. There are two reasons for focusing on the first best. One is technical: it turns out that it is much easier to characterize the equilibrium conditions when the allocation is the first best. The second reason is that, as usual, we are interested in knowing under what conditions the market "works". For the moment, we are only concerned with the feasibility of decentralization. The optimality of the banks' behavior is discussed in Section 8.1.

The role of banks is to make investments on behalf of consumers and to insure them against liquidity shocks. We assume that only banks invest in the long asset. This gives the bank two advantages over consumers. First, the banks can hold a portfolio consisting of both types of assets, which will typically be preferred to a portfolio consisting of the short asset alone. Secondly, by pooling the assets of a large number of consumers, the bank can offer insurance to consumers against their uncertain liquidity demands, giving the early consumers some of the benefits of the high-yielding long asset without subjecting them to the high costs of liquidating the long asset prematurely at the second date.

In each region there is a continuum of identical banks. We focus on a symmetric equilibrium in which all banks adopt the same behavior. Thus, we can describe the decentralized allocation in terms of the behavior of a representative bank in each region.

Without loss of generality, we can assume that each consumer deposits his endowment of one unit of the consumption good in the representative bank in his region. The bank invests the deposit in a portfolio $(x^i, y^i) \geq 0$ and, in exchange, offers a deposit contract (c_1^i, c_2^i) that allows the depositor to withdraw either c_1^i units of consumption at date 1 or c_2^i units of consumption at date 2. Note that the deposit contract is not contingent on the liquidity shock in region i. In order to achieve the first best through a decentralized banking sector, we need to put $(x^i, y^i) = (x, y)$ and $(c_1^i, c_2^i) = (c_1, c_2)$, where (x, y, c_1, c_2) is the first-best allocation.

The problem with this approach is that, while the investment portfolio satisfies the bank's budget constraint $x + y \leq 1$ at the first date, it will not satisfy the budget constraint at the second date. The planner can move consumption between regions, so he only needs to satisfy the average constraint $\gamma c_1 \leq y$. The representative bank, on the other hand, has to face the possibility that the fraction of early consumers in its region may be above average, $\omega_H > \gamma$, in which case it will need more than y to satisfy the demands of the early consumers. It can meet this excess demand by liquidating some of the long asset, but then it will not have enough consumption to meet the demands of the late consumers at date 2. In fact, if r is small enough, the bank may not be able to pay the late consumers even c_1. Then the late consumers will prefer to withdraw at date 1 and store the consumption good until date 2, thus causing a bank run.

There is no overall shortage of liquidity, it is just badly distributed. One way to allow the banks to overcome the maldistribution of the liquidity is by introducing an interbank market in deposits.

4.1 The Interbank Deposit Market

Suppose that banks are allowed to exchange deposits at the first date. This case of complete markets is illustrated in Figure 1. Each region is negatively correlated with two other regions. We therefore assume that every bank in region i holds $z^i = (\omega_H - \gamma)/2 > 0$ deposits in each of the regions $j \neq i$. Since bank deposits are identical and worth one unit each at the first date, the representative bank's budget constraint will still be satisfied at date 0. At the beginning of the second period the state of nature S is observed and the banks have to adjust their portfolios to satisfy their budget constraints. If the region has a high demand for liquidity, $\omega^i = \omega_H$, it liquidates all of its deposits in other regions. On the other hand, if it has a low demand for liquidity, $\omega^i = \omega_L$, it retains the deposits it holds in the other regions until the final date.

Consider the budget constraint of a bank in a region with a high demand for liquidity. It must pay c_1 to the fraction ω_H of early consumers in its own region and also redeem the $z^j = (\omega_H - \gamma)/2$ deposits of the other high-demand region. So the total demand for repayment is $[\omega_H + (\omega_H - \gamma)/2]c_1$. On the other side of the ledger, it has y units of the short asset and claims to $3z^i = 3(\omega_H - \gamma)/2$ deposits in the other three regions. Thus, the budget constraint that must be satisfied is

$$[\omega_H + (\omega_H - \gamma)/2]c_1 = y + 3(\omega_H - \gamma)/2$$

which simplifies to the planner's constraint

$$\gamma c_1 = y.$$

Regions with low liquidity demand must pay c_1 to a fraction ω_L of their own depositors and redeem $2z^i = (\omega_H - \gamma)$ deposits from the banks in the regions with high liquidity demand. It has y units of the short asset to meet these demands, so the budget constraint that must be satisfied is

$$[\omega_L + (\omega_H - \gamma)]c_1 = y.$$

Since $\omega_H - \gamma = \gamma - \omega_L$, this equation simplifies to the planner's constraint $\gamma c_1 = y$. In both cases, the cross holdings of deposits allow the banks to meet the demands of their depositors without liquidating the long asset.

At the last date, all the banks liquidate their remaining assets and it is easy to show that if the budget constraints at the second date are satisfied, the budget constraints at the third date are automatically satisfied too. For example, the budget constraint at date 2 for a region that had high liquidity preference at date 1 will be

$$[(1 - \omega_H) + (\omega_H - \gamma)]c_2 = Rx$$

where the left hand side is the demand for withdrawals, comprising the demand of the late consumers in the region $1 - \omega_H$, plus the demand from the two regions with low liquidity preference $2z^j = (\omega_H - \gamma)$. On the right hand side, we have the liquidation value of the long asset Rx. This simplifies to the planner's constraint $(1 - \gamma)c_2 = Rx$. The same is true of the budget constraint for the regions with a low liquidity shock:

$$[(1 - \omega_L) + (\omega_H - \gamma)/2]c_2 \leq Rx + 3[(\omega_H - \gamma)/2]c_2.$$

Thus, by shuffling deposits among the different regions, it is possible for banks to satisfy their budget constraints in each state S and at each date $t = 0, 1, 2$ while providing their depositors with the first-best consumption allocation through a standard deposit contract.

4.2 Incompleteness in the Interbank Deposit Market

The interbank market in the preceding section is complete in the sense that a bank in region i can hold deposits in every other region $j \neq i$. In some cases, this may not be realistic. The banking sector is interconnected in a variety of ways, but transaction and information costs may prevent banks from acquiring claims on banks in remote regions. To the extent that banks specialize in particular areas of business or have closer connections with banks that operate in the same geographical or political unit, deposits may tend to be concentrated in "neighboring" banks. To capture this effect, which is crucial in the sequel, we introduce the notion of incompleteness in the interbank market by assuming that banks in region i are allowed to hold deposits in some but not all of the other regions. For concreteness we assume that banks in each region hold deposits only in one adjacent region, as shown in Figure 2. It can be seen that banks in region A can hold deposits in region B, banks in region B can hold deposits in region C and so on.

As before we suppose the representative bank in region i holds an investment portfolio $(x^i, y^i) = (x, y)$ and offers a deposit contract $(c_1^i, c_2^i) = (c_1, c_2)$. We also assume that the bank holds $z^i = (\omega_H - \gamma)$ deposits in the adjacent region at the first date, that is, the bank in region A holds $(\omega_H - \gamma)$ deposits in region B, and so on. The first-period budget constraint is satisfied as before, because the exchanges of deposits, having the same values, cancel out, leaving the budget constraint $x + y \leq 1$.

At date 1 the aggregate state is observed and banks and consumers learn the liquidity shock in each region. As before, we only need to distinguish regions according to whether they have high or low demands for liquidity. Regions with the high liquidity shock ω_H liquidate their deposits in other banks at the second date while banks with the low liquidity shock ω_L, do not. The market structure that is assumed here has the property that every region with a high liquidity shock has deposits in a region with a low liquidity shock, and vice versa. The budget constraint of a high liquidity shock region is

$$\omega_H c_1 = y + (\omega_H - \gamma)c_1$$

and the budget constraint of a low liquidity shock region is

$$[\omega_L + (\omega_H - \gamma)]c_1 = y.$$

Substituting $\omega_H - \gamma = \gamma - \omega_L$ and simplifying, it is seen that both constraints are equivalent to the planner's constraint $\gamma c_1 = y$. Likewise, at the final date the budget constraint for the high and low liquidity shock regions, respectively, are

$$[(1 - \omega_H) + (\omega_H - \gamma)]c_2 = Rx$$

and

$$(1 - \omega_L)c_2 = Rx + (\omega_H - \gamma)c_2$$

and both are equivalent to the planner's constraint $(1-\gamma)c_2 = Rx$.

So even if the interbank deposit market is incomplete, it is possible to satisfy the budget constraints by shuffling deposits through the interbank market. However, although it is possible to achieve the first best with either complete or incomplete markets, we shall see that the implications for financial fragility are very different in the two cases.

One interesting feature of the market structure in Figure 2 is that, although each region is relying on just its neighbor for liquidity, the entire economy is connected. Region A holds deposits in region B, which holds deposits in region C, and so on. In fact, this is unavoidable given the market structure assumed. Consider the alternative market structure shown in Figure 3. Region A holds deposits in region B and region B holds deposits in region A. Likewise, region C holds one unit of deposits in region D and region D holds one unit of deposits in region C. This market structure is more incomplete than the one in Figure 2 and the pattern of holdings in Figure 2 is incompatible with it. However, it is possible to achieve the first best through the pattern of holdings in Figure 3. This is true even though the economy is disconnected, since regions A and B trade with each other but not with regions C and D and regions C and D trade with each other but not with regions A and B. Again, these patterns do not seem to have any significance as far as achieving the first best is concerned; but they turn out to have striking differences for financial fragility.

5 Fragility

To illustrate the *financial fragility* of the optimal risk sharing allocation, we use the decentralization results from Section 4. Then we perturb the model to allow for the occurrence of a state $\bar{S}$ in which the aggregate demand for liquidity is greater than the system's ability to supply liquidity and show that this can lead to an economy-wide crisis.

The market structure is assumed to be given by Figure 2. The corresponding allocation requires each bank to hold an initial portfolio of investments (x, y) and offer a deposit contract (c_1, c_2), where (x, y, c_1, c_2) is the first-best allocation. In order to make this deposit contract feasible, the representative bank in each region holds $z = (\omega_H - \gamma)$ deposits in the adjacent region. Note that z is the minimal amount that is needed to satisfy the budget constraints. It will become apparent that larger cross-holdings of deposits, while consistent with the first best in Section 4, would make the contagion problem worse.

Now, let us take the allocation as given and consider what happens when we "perturb" the model. By a perturbation we mean the realization of a state $\bar{S}$ that was assigned zero probability at date 0 and has a demand for liquidity that is very close to that of the states that do occur with positive probability. Specifically, the liquidity shocks are

Table 2: Regional Liquidity Shocks with Perturbation

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H
$\bar{S}$	$\gamma + \varepsilon$	γ	γ	γ

.

In state $\bar{S}$, every region has the previous average demand for liquidity γ except for region A where the demand for liquidity is somewhat higher $\gamma + \varepsilon$. The important fact is that the average demand for liquidity across all four regions is slightly higher than in the normal states S_1 and S_2. Since the abnormal state $\bar{S}$ occurs with negligible probability (in the limit, probability zero) it will not change the allocation at date 0. In states S_1 and S_2 the continuation equilibrium will be the same as before at date 1; in state $\bar{S}$ the continuation equilibrium will be different.

In the continuation equilibrium beginning at date 1, consumers will optimally decide whether to withdraw their deposits at date 1 or date 2 and banks will liquidate their assets in an attempt to meet the demands of their depositors. Early consumers always withdraw at date 1; late consumers will withdraw at date 1 or date 2 depending on which gives them the larger amount of consumption. Because we want to focus on essential bank crises, we assume that late consumers will always withdraw their deposits at date 2 if it is (weakly) optimal for them to do so. Banks are required to meet their promise to pay c_1 units of consumption to each depositor who demands withdrawal at date 1. If they cannot do so, they must liquidate all of their assets at date 1. As in Allen and Gale (1998a) the proceeds of the liquidation are split pro rata among depositors (i.e., we do *not* assume first come first served). If the bank can meets its obligations at date 1, then the remaining assets are liquidated at date 2 and given to the depositors who have waited until date 2 to withdraw. In the rest of this section, we describe the continuation equilibrium at date 1 in state $\bar{S}$, assuming the actions consistent with the first best at date 0.

5.1 The Liquidation "Pecking Order"

At date 1 a bank can find itself in one of three conditions. A bank is said to be *solvent*, if it can meet the demands of every depositor who wants to withdraw (including banks in other regions) by using only its liquid assets, that is, the short asset and the deposits in other regions. The bank is said to be *insolvent* if it can meet the demands of its deposits but only by liquidating some of the long asset. Finally, the bank is said to be *bankrupt* if it cannot meet the demands of its depositors by liquidating all its assets.

These definitions are motivated by the assumption that banks will always find it preferable to liquidate assets in a particular order at date 1. We call this the "pecking order" for liquidating assets and it goes as follows: first, the bank liquidates the short asset, then it liquidates deposits, and finally it liquidates the long asset. To ensure that the long asset is liquidated last, we need an additional assumption,

$$\frac{R}{r} > \frac{c_2}{c_1} \tag{6}$$

which is maintained in the sequel. Since the first-best consumption allocation (c_1, c_2) is independent of r (this variable does not appear in the first-best problem in Section 3) we can always ensure that condition (6) is satisfied by choosing r sufficiently small.

Each of the three assets offers a different cost of obtaining current (date 1) consumption in terms of future (date 2) consumption. The cheapest is the short asset. One unit of the short asset is worth one unit of consumption today and, if reinvested in the short asset, this is worth one unit of consumption tomorrow. So the cost of obtaining liquidity by liquidating the short asset is 1. Similarly, by liquidating one unit of deposits, the bank gives up c_2 units of future consumption and obtains c_1 units of present consumption. So the cost of obtaining liquidity by liquidating deposits is c_2/c_1. From the first-order condition $u'(c_1) = Ru'(c_2)$ we know that $c_2/c_1 > 1$. Finally, by liquidating one unit of the long asset, the bank gives up R units of future consumption and obtains r units of present consumption. So the cost of obtaining liquidity by liquidating the long asset is R/r. Thus, we have derived the pecking order, short assets, deposits, long assets:

$$1 < \frac{c_2}{c_1} < \frac{R}{r}.$$

In order to maximize the interests of depositors, the bank must liquidate the short asset before it liquidates deposits in other regions before it liquidates the long asset.

The preceding argument assumes that the banks in other regions are not bankrupt. The bankruptcy rules require all assets to be liquidated immediately, so all deposit holders in a bankrupt institution will want to liquidate their deposits immediately regardless of their own condition.

5.2 Liquidation Values

The value of a deposit at date 1 is c_1 if the bank is not bankrupt and it is equal to the liquidation value of all the bank's assets if the bank is bankrupt. Let q^i denote the value of the representative bank's deposits in region i at date 1. If $q^i < c_1$ then all the depositors will withdraw as much as they can at date 1. In particular, the banks in other regions will be seeking to withdraw their claims on the bank at the same time that the bank is trying to redeem its claims on them. *All depositors must be treated equally*, that is, every depositor gets q^i from the bank for each unit invested at the first date, whether the depositor is a consumer or a bank from another region. Then the values of q^i must be determined simultaneously. Consider the representative bank in region A, for example. If all the depositors withdraw, the total demands will be $1 + z$, since the banks in region D hold z deposits and the consumers in region A hold 1 deposit. The liabilities of the bank are valued at $(1 + z)q^A$. The assets consist of y units of the short asset, x units of the long asset, and z deposits in region B. The assets are valued at $y+rx+zq^B$. The equilibrium values of q^A must equate the value of assets and liabilities

$$q^A = \frac{y + rx + zq^B}{(1 + z)}. \tag{7}$$

A similar equation must hold for any region i in which $q^i < c_1$.

If $q^B = c_1$ then we can use this equation to calculate the value of q^A; but if $q^B < c_1$ then we need another equation to determine q^B and this equation will include the value of q^C, and so on.

5.3 Buffers and Bank Runs

Suppose that a bank is insolvent and has to liquidate some of the long asset. For the moment, we assume that the late consumers wait until the last date and we ignore the role of banks in other regions. How much can the bank afford to give the consumers at the first date? The bank must give the late consumers at least c_1 at date 2, otherwise they would be better off withdrawing at date 1. So a bank with a fraction ω of early consumers must keep at least $(1-\omega)c_1/R$ units of the long asset to satisfy the late consumers at date 2. Then the amount of the long asset that can be liquidated at date 1 is $(x-(1-\omega)c_1/R)$ and the amount of consumption that can be obtained by liquidating the long asset without causing a run is

$$b(\omega) \equiv r(x-(1-\omega)c_1/R).$$

We call $b(\omega)$ the bank's buffer.

In region A, the bank has $y=\gamma c_1$ units of the short asset. The fraction of early consumers is $\gamma+\varepsilon$ in state $\bar{S}$, so in order to pay each early consumer c_1 units of consumption, the bank will have to get εc_1 units consumption by liquidating the long asset. This is feasible, without any help from the banks in other regions, if and only if the increased demand for liquidity εc_1 is less than the buffer:

$$\varepsilon c_1 \leq b(\gamma+\varepsilon). \tag{8}$$

In most of what follows, we assume that condition (8) is violated. In other words, if region A had to remain self sufficient, it would be bankrupt, because there is no way it can feasibly offer its late consumers c_1 at date 2 (to prevent a run) and meet the demands of its early consumers for c_1 at date 1.

When $\varepsilon>0$ is small enough to satisfy the inequality (8), the banks in region A are insolvent, but there are no repercussions for the banks in other regions. The late consumers in region A are worse off, because the premature liquidation of the long asset at date 1 prevents the bank from paying c_2 to depositors at date 2. (The pecking order implies that the banks in regions B, C, and D will liquidate their deposits in regions C, D, and A, respectively, rather than liquidate the long asset).

When ε is large enough to violate condition (8), banks in region A will be bankrupt. Although they have deposits in region B, these deposits are of no use as long as the value of deposits in region A is $q^A=c_1$. Other regions will liquidate their deposits in order to avoid liquidating the long asset. As long as all the deposits have the same value, the mutual withdrawals simply cancel out. Once the banks in region A are bankrupt, there will be a spillover effect to region D. A deposit in region D is worth $q^D = c_1$ and a deposit in region A is worth $q^A < c_1$, so banks in region D suffer a loss when cross holdings of deposits are liquidated. If ε is not too large, this spillover effect will make region D banks insolvent, but will not force them into bankruptcy.

If ε is larger still, so that the spillover effect exceeds region D's buffer, then region D banks will be bankrupt too. The liquidation of region D's long assets will cause a loss to banks in region C and this time the spillover effect is large enough that region C too will be bankrupt. As we go from region to region the spillover gets larger and larger, because more regions are in bankruptcy and more losses have accumulated from liquidating the long asset. So once region D goes bankrupt, all the regions go bankrupt. This result is summarized in the following proposition.

As this informal discussion suggests, two conditions must be satisfied in order for the initial shock to region A to spread to all the other regions. First, the liquidity preference shock in region A must exceed the buffer in region A:

$$\varepsilon c_1 > b(\gamma + \varepsilon). \tag{9}$$

Second, the spillover effect to region D must exceed the buffer in region D. A lower bound for the spillover effect is $z(c_1 - \bar{q}^A)$, where z is the amount of deposits held and $\bar{q}^A$ is an upper bound on the value of the deposits in region A under bankruptcy. To derive the upper bound $\bar{q}^A$ we use equation (7) and assume that $q^B = c_1$:

$$q^A \leq \bar{q}^A = \frac{y + rx + zc_1}{(1 + z)}. \tag{10}$$

Then a sufficient condition for the spillover to exceed the buffer in region D is

$$z(c_1 - \bar{q}^A) > b(\gamma). \tag{11}$$

The term zc_1 is the amount promised to the banks in Region C and $z\bar{q}^A$ is the upper bound on the value of deposits in Region A. Hence the left hand side of the condition is the difference between liabilities and the upper bound on assets in the interbank deposit market for Region D. If this exceeds Region D's buffer the spillover will force region D banks into bankruptcy.

Proposition 2 *Consider the model with market structure described in Figure* 2 *and perturb it by the addition of the zero probability state* $\bar{S}$. *Suppose that each bank chooses an investment portfolio* (x, y, z) *and offers a deposit contract* (c_1, c_2), *where* (x, y) *is the first-best investment portfolio,* (c_1, c_2) *is the first-best consumption allocation, and* $z = (\omega_H - \gamma)$. *Suppose that conditions (9) and (11) are satisfied. Then, in any continuation equilibrium, the banks in all regions must go bankrupt at date* 1 *in state* $\bar{S}$.

Proof. The proof requires several steps.

Step 1. We first suppose that there is a continuation equilibrium in which $q^i = c_1$ in every region i and show that this leads to a contradiction. The demand for deposits from early consumers is $\gamma+\varepsilon$ in region A and γ in regions B, C and D. The stock of the short asset is $y = \gamma c_1$, so there is an aggregate excess demand for liquidity that can only be met by liquidating the long asset in some region. To avoid liquidating the long asset, banks must redeem at least as many deposits as are withdrawn by banks from other regions. Since no bank wants to liquidate the long asset if it can be avoided, the only equilibrium is one in which all banks simultaneously withdraw their deposits in banks in other regions at date 1. These mutual withdrawals offset each other so each region is forced to be self-sufficient, that is, no region is able to get extra liquidity from the other regions.

We have already seen that self-sufficiency implies the banks in region A are bankrupt. Thus, we have a contradiction that implies that the banks in some region must be bankrupt. In fact, the banks in region A must be bankrupt. By the earlier argument, $q^i < c_1$ for some i implies that all banks will withdraw their deposits in other regions. Then either $q^B = c_1$ and region A receives no net inflows from the interbank market or $q^B < c_1$ and the situation is even worse because Region A loses money on its deposits in region B. In either case, $q^A = c_1$ is impossible because of condition (9).

Step 2. Having established that banks in region A must be bankrupt, we next show that the financial crisis must extend to other regions. Consider region D first. For the reasons explained above, all banks will be liquidating their deposits in other regions in any continuation equilibrium in state $\bar{S}$. An upper bound on the liquidation value q^A of the deposits in region A is obtained by assuming that $q^B = c_1$, that is $q^A \leq \bar{q}^A$. If banks in region D are not bankrupt, the liabilities of the banks in region D are $(\gamma + z)c_1$, because a fraction γ of consumers withdraw early and the banks in region C withdraw z deposits and each deposit is worth c_1. The liquid assets of the bank are worth $y + zq^A$ and the buffer $b(\gamma)$ is the most that can be obtained from liquidating the long asset without violating the incentive constraint. So, to avoid bankruptcy, it must be the case that

$$\begin{aligned}(\gamma + z)c_1 &\leq y + b(\gamma) + zq^A \\ &\leq y + b(\gamma) + z\bar{q}^A.\end{aligned}$$

Since $\gamma c_1 = y$, this inequality implies that $z(c_1 - \bar{q}^A) \leq b(\gamma)$, contradicting condition (11). Thus, the banks in region D must be bankrupt also.

Step 3. The argument from Step 2 can be continued by induction. In fact, since we know that $\bar{q}^A < c_1$, we must have $q^D < \bar{q}^A$. Then it is easier to violate the non-bankruptcy condition in region C than it was in region D and this shows that $q^C < \bar{q}^A$. Then using the same argument, we have $q^i < \bar{q}^A$ for every region i. All regions are in bankruptcy and the only possible continuation equilibrium is one in which

$$q^i = y + rx < c_1$$

for $i = A, B, C, D$. ■

6 Robustness

The incompleteness of markets is essential to the contagion result in the following sense. There exist parameter values for which any equilibrium with incomplete markets involves runs in state $\bar{S}$ (this is the set of parameter values characterized in Section 5). For the same parameter values, we can find an equilibrium with complete markets that does not involve runs in state $\bar{S}$.

To see this, we go back to the complete markets equilibrium in Section 4. The values of the investment portfolio (x, y) and the deposit contract (c_1, c_2) are the same; but to make the first-best allocation feasible at dates 1 and 2, the representative bank holds $z/2 = (\omega_H - \gamma)/2$ deposits in each of the other regions. The claim on any one region is smaller than in the equilibrium in Section 5, but the total claim $3z/2$ is larger. Again, $z/2$ is the smallest amount of deposits consistent with feasibility.

Consider now what happens in a continuation equilibrium at date 1 when state $\bar{S}$ occurs, assuming that the actions at date 0 have not changed. We assume, of course, that the conditions of Proposition 2 continue to hold. Using exactly the same argument as before, we can show that banks in region A are bankrupt: there is an aggregate excess demand for liquidity, the other regions will not provide liquidity, and because condition (8) is violated the banks in region A cannot meet their depositors' demands. The question is whether this requires that the other regions should also experience bankruptcy.

To address this question, we have to calculate the liquidation value of the deposits in region A. Assuming that none of the other regions is bankrupt, we observe that the assets are valued at $y + rx + 3(z/2)c_1$ and the liabilities are valued at $(1 + 3z/2)q^A$, so

$$\overline{q}^{A*} = \frac{y + rx + 3(z/2)c_1}{(1 + 3z/2)}. \tag{12}$$

The loss to each bank in regions $j \neq A$ because of the collapse of banks in region A is $(z/2)(c_1 - \overline{q}^{A*})$ and since the bank's holding of the short asset y is just enough to satisfy its own early consumers, the bank will be insolvent, but not bankrupt, if and only if this amount is less than or equal to the buffer:

$$(z/2)(c_1 - \overline{q}^{A*}) \leq b(\gamma). \tag{13}$$

This condition (13) can be satisfied even though conditions (8) and (11) are violated because the financial interdependence, measured by $z/2$, is smaller.

7 Containment

The critical ingredient in the example of contagion analyzed in Section 5 is that any two regions are connected by a chain of overlapping bank liabilities. Banks in region A have claims on banks in region B, which in turn have claims on banks in regions C, and so on. If we could cut this chain at some point, the contagion that begins with a small shock in region A would be contained in subset of the set of regions.

Consider the incomplete market structure in Figure 3 and the allocation that implements the first best, which was described in Section 4. The allocation requires banks in regions A and B to have claims on each other and banks in regions C and D to have claims on each other, but there is no connection between the region $\{A, B\}$ and the region $\{C, D\}$. If state $\bar{S}$ occurs, the excess demand for liquidity will cause bankruptcies in region A and they may under certain conditions spread to region B, but there is no reason why they should spread any further. Banks in regions C and D are simply not connected to the troubled banks in regions A and B.

Comparing the three market structures we have considered so far, complete markets in Figure 1, incomplete markets in Figure 2, and the disconnected market structure in Figure 3, we can see that there is a non-monotonic relationship between completeness or incompleteness of markets and the extent of the financial crisis in state $\bar{S}$. With the complete markets structure of Figure 1 the crisis is restricted to region A, with the market structure in Figure 2 the crisis extends to all regions, and with the market structure in Figure 3 the crisis is restricted to regions A and B.

It could be argued that the market structures are not monotonically ordered: the complete markets network does contain the other two, but the paths in the network in Figure 3 are not a subset of the network in Figure 2. This could be changed by adding paths to Figure 2, but then the equilibrium of Figure 3 would also be an equilibrium of Figure 2. This raises an obvious but important point, that contagion depends on the *endogenous* pattern of financial claims. An incomplete market structure like the one in Figure 2 may preclude a complete pattern of financial connectedness and thus encourage financial contagion; but a complete market structure does not imply the opposite: even with complete markets there may be an endogenous choice of overlapping claims that causes contagion. In fact, the three equilibria considered so far are all consistent with the complete market structure. There are additional equilibria for the economy with the complete market structure. Like the three considered so far, they achieve the first best in states S_1 and S_2, but have different degrees of financial fragility in the unexpected state $\bar{S}$, depending on the patterns of interregional deposit holding.

What is important about the market structure in Figure 2, then, is that the pattern of interregional cross holdings of deposits that promotes the possibility of contagion is the only one consistent with this market structure. Since we are interested in contagion as an essential phenomenon, this market structure has a special role. The complete markets economy, by contrast, has equilibria with and without contagion and provides a weaker case for the likelihood of contagion.

8 Discussion

8.1 Equilibrium

In the preceding sections we have discussed the continuation equilibrium at dates 1 and 2, but not said anything about the equilibrium behavior at date 0. It is clear that it is optimal for consumers at date 0 to deposit their endowment with the banks, because they could not achieve the same level of expected utility in autarky, even if they were able to invest in both the short and the long assets. What is less obvious is whether the behavior of the banks is optimal in some sense.

The bank has to choose an investment portfolio, a deposit contract, and a position in the interbank deposit market to maximize the consumers' expected utility at date 0. The tricky part is choosing how much to trade on the interbank market. In order to finance deposits in other regions, it will have to sell its own deposits to other banks. But the value of its deposits will depend on the choice of the investment portfolio and the deposit contract and the withdrawal decisions made by depositors at the second and third dates. We can finesse the complex calculation of the bank's optimal behavior by noting that since all trade is voluntary, anything the bank does to make its own consumers better off cannot make anyone else worse off (since a single bank is negligible, the rest of the economy gets zero surplus from trading with it). Then the fact that the allocation at the second and third dates is (first-best) optimal implies that there is no deviation that the bank would prefer.

The achievement of the first best depends on the assumption of no aggregate uncertainty. In general, when there is aggregate uncertainty, the first-best is not attainable and the characterization of the equilibrium actions at date 0 (and subsequent dates) will be much more difficult. This is an important topic for future research, but it lies outside the scope of this paper.

8.2 Many States and Regions

The arguments developed in this paper extend easily to the case of many regions and many states of nature. Suppose that there are n regions and suppose that the liquidity shocks ω^i are finite-valued and exchangeable. If the economy-wide fraction of early consumers is a constant, then the first best allocation (x, y, c_1, c_2) is non-stochastic and independent of the consumer's region.

A market structure is characterized by a family of neighborhoods: for each region i the neighborhood N^i is the set of regions in which a bank in region i can hold deposits. The market structure is *connected* if for any regions i and j there is a finite chain of regions, beginning with i and ending with j such that for each adjacent pair, banks in the first region can hold deposits in the second. As long as the market structure is connected the first best can be decentralized by a competitive banking sector using standard deposit contracts.

A market structure is complete if banks in region i can hold deposits in every other region $j \neq i$; otherwise it is incomplete. The degree of completeness and connectedness is crucial for determining the necessity of contagion. With complete markets and a given set of parameters, increasing the number of regions eventually eliminates the necessity of contagion, because the initial impact of a liquidity shortage in a single region becomes negligible as the number of regions becomes unboundedly large.

In the example of Section 5, a liquidity shortage in one region can lead to crises in all four regions. The same argument works with any number of regions. In fact, as contagion "spreads" from one region to another, the spillovers become larger and it is easier to keep the contagion going. In the general case, contagion can spread from a single region to an arbitrarily large number n of regions. In this sense, small shocks can have very large effects if markets are incomplete.

8.3 Alternative Market Structures

The market structure in Figure 1 results in a symmetric allocation because in either state of nature there is an alternating pattern of liquidity shocks as one passes around the circle of regions. This is not important for the achievement of the first best. In fact, all that is required for the decentralization result is that the regions be connected. However, the symmetry of the equilibrium allocation and the conditions for financial contagion do depend on the symmetry of the market structure. To see this, consider the market structure in Figure 4. In state S_1, at date 1, regions A and C are adjacent and have high shocks and regions B and D are adjacent and have low shocks. In state S_1 the shocks are reversed but the same pairing holds. This is not an obstacle to achieving the first best, but it does require a special pattern of claims. In order to satisfy its budget constraint in state S_1, region A will have to hold $(\omega_H - \gamma)$ deposits in region C. Region will not be able to spare any liquidity for region A so it will have to pass region A's demand on to region B, along with its own demand for $\omega_H - \gamma$ deposits. So region C must hold $2(\omega_H - \gamma)$ deposits in region B. Region B has an excess of $(\gamma - \omega_L)c_1 = (\omega_H - \gamma)c_1$ units of consumption that it can offer region C, so to meet the entire demand from region C it must have $(\omega - \gamma)$ deposits in region D that it can liquidate.

At the final date, the pattern is reversed. Now regions B and D are in need of liquidity and regions A and C must provide it. Region B needs $(\gamma - \omega_L) = (\omega_H - \gamma)$ deposits in region D, but since region D cannot offer any liquidity itself, it must have $2(\omega_H - \gamma)$ deposits in region A that it can liquidate in order to provide both for its own needs and those of region B. Region A needs only $(\omega_H - \gamma)$ deposits in region C to pass on part of region D's demand to region C.

In state S_2, regions B and D are in need of liquidity and, by a similar argument, region B must hold $(\omega_H - \gamma)$ deposits in D, region D must hold $2(\omega_H - \gamma)$ deposits in A, and region A must hold at least $(\omega_H - \gamma)$ in C.

So the equilibrium pattern is for regions C and D to hold $2(\omega_H - \gamma)$ in regions B and A, respectively, and for regions B and A to hold at least $(\omega_H - \gamma)$ in regions D and C, respectively. This pattern will not satisfy the budget constraint at the first date, however. The only way that banks can afford deposits in another region is to sell their own deposits. If regions C and D hold $2(\omega_H - \gamma)$ deposits in regions B and A, respectively, they must offer regions A and B, respectively, the same number of deposits in their own banks. So the cross-holdings of deposits remains symmetric in the equilibrium, even though the transfers carried out through the liquidation of those deposits is not symmetric.

What changes as a result of the bigger deposit holdings is the size of the spillover effect. Consider what happens to region A in state $\bar{S}$. If region A is bankrupt and region C is not bankrupt, the loss to region D is $2(\omega_H - \gamma)(q^A - c_1)$. There are two effects. First, region D holds more deposits in region A and this makes things worse for region D; second, region A holds more deposits in region C and this increases q^A, which makes things better for region D. Now, putting $z = (\omega_H - \gamma)$,

$$q^A = \frac{y + rx + 2zc^1}{(1 + 2z)}$$

implies that

$$\begin{aligned} 2z(q^A - c_1) &= 2z\left(\frac{y + rx + 2zc^1}{(1+2z)} - c_1\right) \\ &= 2z\left(\frac{y + rx - c_1}{1+2z}\right) \\ &> z\left(\frac{y + rx - c_1}{1+z}\right), \end{aligned}$$

so the loss to region D is greater when the size of the deposits is greater. Thus, contagion gets started more easily when the level of deposits held by banks is $2z$ rather than z.

The final possible market structure is shown in Figure 5. It can straightforwardly be seen that this is equivalent to the complete market structure shown in Figure 1 in states S_1 and S_2. The reason is that liquidity shocks in regions A and C are perfectly correlated so that these deposits offset each other. Similarly for regions B and D. The level of deposits held to achieve the efficient allocation will again be $z^i = (\omega_H - \gamma)/2$. The difference comes in state $\overline{S}$. The upper bound on the value of deposits $\overline{q}^A$ is given by (10) rather than (12) but the condition for contagion is

$$(z/2)(c_1 - \overline{q}^A) > b(\gamma).$$

It follows contagion occurs more easily than with the complete market structure in Figure 1 but less easily than with the market structure in Figure 2.

8.4 Sunspot Equilibria

The focus in this paper is on financial contagion as an essential feature of equilibrium. We do not rely on arguments involving multiple equilibria. The aim is instead to show that under certain conditions every continuation equilibrium at date 1 exhibits financial contagion. Nonetheless, there are multiple equilibria in the model and if one is so disposed one can use the multiplicity of equilibrium to tell a story about financial contagion as a sunspot phenomenon.

To illustrate the multiplicity as simply as possible, suppose that markets are complete and the fraction of early consumers in each region is nonstochastic and equal to γ. There are no interregional cross holdings of deposits at date 0. If every consumer in every bank chooses to withdraw his deposit at date 1, regardless of the size of the liquidity shock, then the banks will all be bankrupt because $c_1 > y + rx$. This outcome is an equilibrium because it is optimal for each individual depositor to withdraw assuming that all other depositors withdraw. There is also the usual equilibrium in which late consumers choose not to withdraw until date 2 and the bank's portfolio (x, y) allows the first best to be achieved.

The low probability event $\bar{S}$ is now interpreted as a "sunspot" that does not change the demand for liquidity but simply triggers the self-fulfilling prophecy that a bank run will occur. The outcome in terms of the pattern of bank runs in state $\bar{S}$ is the same as in Section 5. Whether one wants to call this a contagion is a matter of taste.

8.5 Risky Assets

For simplicity, we have assumed that the long asset has a non-stochastic return, but it would be more realistic to assume that the long asset is risky. In Allen and Gale (1998a), the long asset is risky and negative information about future returns is what triggers bank runs. In the present framework, uncertainty about long asset returns could be used both to motivate inter-regional cross holdings of deposits and to provoke insolvency or bankruptcy. The results should be similar. What is crucial for the results is that the financial interconnectedness between the regions takes the form of claims held by banks in one region on banks in another region.

If, instead of holding claims on banks in another regions, banks were to invest directly in the long assets of that region, there would be a spillover effect, but it would be much weaker. If banks in region A hold some of the long asset in region B and it has a low return, then the depositors in region A must accept a reduction in consumption. But that is all. Banks in region B, who hold a large proportion of their assets in region B, are forced to liquidate their assets at a much greater loss since $r < R$. As long as the banks in region A are not bankrupt, they can afford to wait to get the higher return $R > r$.

On the other hand, if the banks in region A had invested in the banks in region B, then they would suffer a larger loss when the banks in region B liquidate their assets.

Another way that banks can invest indirectly in risky assets is by lending to investors. If banks cannot observe the investment portfolio chosen by the investors, the investors will engage in risk shifting. Allen and Gale

(1998b) show that this kind of behavior can lead to bubbles in asset prices and increase the probability of a banking crisis (general default).

8.6 Alternative Interbank Markets

The fact that the financial interconnectedness takes the form of pre-existing claims on other banks is crucial. If liquidity were provided on an ex post basis, there would be no possibility of contagion. Suppose, for example, that instead of an interbank market for deposits at the first date, there is an interbank for one-period loans at the second date. Then a bank that finds it has an excess demand for liquidity at date 1 has no pre-existing claim on banks in other regions, but it can borrow from banks in regions with an excess supply of liquidity. In states like S_1 and S_2, this mechanism will be enough to achieve the first best. If the lending rate is ρ then the markets will clear if

$$1 + \rho = \frac{c_2}{c_1}.$$

In state $\bar{S}$, however, there will be an economy-wide shortage of liquidity. The fraction of early consumers is $\gamma + \varepsilon$ in region A and γ in the other regions; but there is only enough of the short asset in the economy to provide for a fraction γ of early consumers. The only way that more liquidity can be provided to the banks in region A is for the long asset to be liquidated. The only way that other regions will be willing to do this is if the rate of interest on loans compensates for the cost of liquidation:

$$1 + \rho = \frac{R}{r}.$$

At this rate, the cost of borrowing is too high to be of any help to the banks in region A. It is just like liquidating more of the long asset and we already have assumed that the bank is liquidating as much as it can, subject to the incentive constraint.

So the interbank loan market turns out to be of no use in state $\bar{S}$, where there is an economy-wide shortage of liquidity. It protects other regions from contagion, but does nothing to stop the crisis in the affected region.

If the probability of state $\bar{S}$ is very small but positive, the ex post loan market is strictly preferred to the ex ante deposit market because region A is no worse off and the other regions are better off. In this very simple model, there is no advantage to negotiating liquidity contracts ex ante. In a richer model, there might well be a "hold-up" problem because banks with excess liquidity can exploit banks with insufficient liquidity. There could also be an adverse selection problem where bad banks are more likely to try to use the ex post loan market than good banks. These and other problems arising from asymmetric information could cause banks to prefer negotiating a contract ex ante. An interesting question is what kinds of arrangements banks will choose to set up, given the trade-off between the individual benefits of access to liquidity and the social costs of contagion. This is an important topic for further study.

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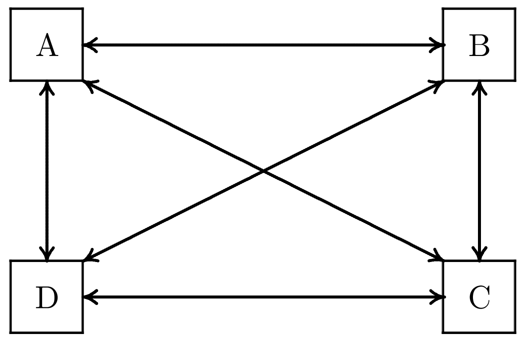

Figure 1

Complete Market Structure

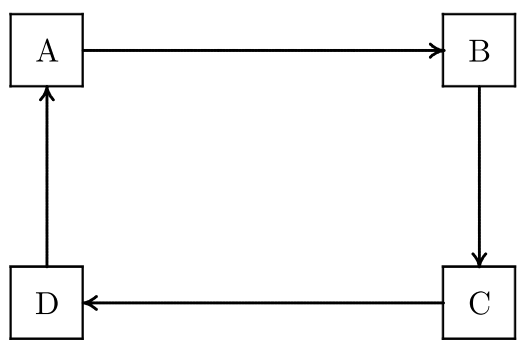

Figure 2

Incomplete Market Structure

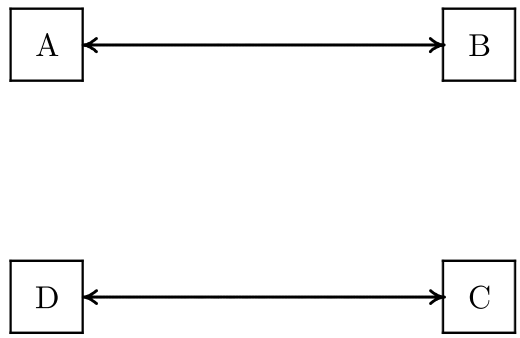

Figure 3

Disconnected Incomplete Market Structure

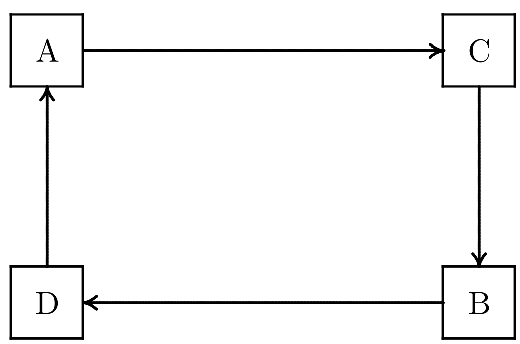

Figure 4

Alternative Incomplete Market Structure

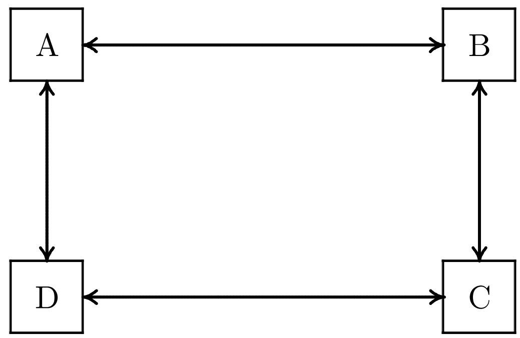

Figure 5

Partially Incomplete Market Structure