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PATENTABILITY, INDUSTRY STRUCTURE, AND INNOVATION

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Abstract

To qualify for a patent, an invention must be new, useful, and nonobvious. This paper presents a model of sequential innovation in which industry structure is endogenous and a standard of patentability determines the proportion of all inventions that qualify for protection. There is a unique patentability standard, or inventive step, that maximizes the rate of innovation by maximizing the number of firms engaged in R&D. Surprisingly, this standard is more stringent for industries disposed to innovate rapidly. If a single standard is applied to heterogeneous industries, it will encourage entry, and therefore innovation, in some industries while discouraging it in others. The model suggests a number of important implications for patent policy.

1. Introduction

Recently, economists have investigated, in the context of cumulative innovation, the relationship between the availability of patent protection and the rate of innovation (Bessen and Maskin 2002, Hunt 1999a, O'Donoghue 1998, and Scotchmer 1996). The general conclusion is that an industry's rate of innovation is maximized by protecting some inventions, but not others.

This paper presents a model in which industry structure (the number of firms engaged in R&D) depends in part on the share of all discoveries that qualify for protection, that is, by the stringency of the criteria used to examine applications for a patent. In the model, the number of firms actively engaged in R&D is the primary determinant of an industry's rate of progress. This in turn depends on the fixed cost of establishing a research facility, the productivity of R&D, and the resulting profits generated in the output market. Patentability criteria affect expected profits because they determine the likelihood that a firm's invention will lead to a competitive advantage and the speed with which that advantage will be eroded. When we speak of a standard of patentability in this paper, we focus on patent law's requirement of *nonobviousness*, or what is called the *inventive step* in Europe. To qualify for a patent it is not sufficient for an invention to be new; it must also represent a sufficiently large advance from the prior art. One can think intuitively of the nonobviousness requirement as specifying the minimal advance—the 'height' of the inventive step—necessary to qualify for protection (for the remainder of the article, we will use the term inventive step).

In the model, industry structure is characterized by a single firm in the output market that is eventually replaced by a firm that develops a patentable innovation. We show that the arrival rate of these innovations is a non-monotonic function of the inventive step required for an invention to be patentable. There is a unique inventive step that maximizes the rate of innovation in a given industry by maximizing the number of firms that choose to engage in R&D. This "optimal" inventive step is a function of exogenous parameters that influence an industry's propensity to innovate. We show that the optimal inventive step is more stringent (taller) for an industry that is otherwise pre-disposed to innovate rapidly and less stringent (shorter) for an industry that is predisposed to innovate more slowly.

Consider the following two-stage game. In the first stage, a social planner seeking to maximize the rate of innovation (via patents) would set an optimal inventive step for each industry.[1] In the second stage, firms make their entry decisions and engage in an unending sequence of patent races. The social planner would set a higher inventive step for industries where R&D is more productive, where the fixed or variable costs of doing R&D are lower, or where the industry-specific discount rate is lower. We would then observe that in the industries that innovate most rapidly, the share of inventions that qualify for protection would be lower than the share of inventions protected in other industries.

In practice, patent law generally applies one set of criteria to evaluate inventions across all industries. Subject to this constraint, adopting a higher inventive step will more likely encourage entry into rapidly innovating industries than it will entry into industries that innovate more slowly. Thus, so long as a single standard is employed, the choice of patentability criteria is also a choice about industrial

[1] Throughout this paper, we assume that the central policy problem is an under-provision of innovation by private firms, which follows from the fact that each innovation leads to a permanent increase in social benefits while, at best, firms are only able to appropriate the resulting total surplus for an indeterminate, but strictly finite period of time. An explicit welfare analysis can be found in section 3.4.

policy. But, as the model makes clear, the intuition about which industries are favored over others is the opposite of what was assumed by policymakers in the U.S. nearly two decades ago.[2]

The results lead to two additional implications. First, so long as intellectual property rights are applied in the same way to all industries, the optimal patentability standard will vary across countries, depending on their mix of industries and the relative sizes of those industries. That suggests that a policy of harmonizing patentability criteria across countries with disparate economies, as has recently been proposed, is likely to result in disparate effects across those economies.

Second, if the "optimal" standard depends on industry characteristics that influence the rate of innovation, it will change as conditions in the industry change. In other words, optimal patentability criteria seem more like a common law balancing test rather than a rigid standard set out in law. Such a test can be characterized in the following way: Does granting exclusive rights to the marginal invention generate more rents than are destroyed by shortening the expected duration of rents earned on future inventions?

The remainder of the paper is organized as follows. Section 2 introduces the model and compares it to the existing literature. Section 3 presents the equilibrium, describes its properties, and compares it to the solution to the social planner's problem. Section 4 describes the relationship between the inventive step and the rate of innovation and derives the optimal standard as a function of an industry's propensity to innovate. Section 5 concludes. The Appendix contains the proofs of all the propositions. Figures are found at the end of the paper.

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[2] This point is explored in the conclusion.

2. The Model

2.1 An Infinite Sequence of Stochastic Patent Races

Time is continuous and the horizon is infinite. Let $r > 0$ denote the discount rate. Discoveries occur at different points in time. It is convenient to divide time into the intervals between these discoveries and call them patent races. Because there is randomness in the process that generates discoveries, the actual duration of patent races will vary.

At any point in time there are n+1 firms in the industry, where $n \geq 0$ is determined according to a free entry constraint that reflects the expected return to innovation and the fixed cost $k > 0$ of setting up an R&D lab. This cost is sunk on commencement of the firm's first patent race, and new fixed investments are not required thereafter. Firms are indexed by the superscript i. At the beginning of a race, firms simultaneously choose their R&D intensity, denoted $h^i \in [0, \overline{h}]$, where $\overline{h}$ is a very large, but finite, point of saturation. Firms maintain their research intensity until a discovery occurs and the current race ends. The flow cost of conducting R&D, denoted $C(h^i)$, is strictly increasing and twice continuously differentiable in R&D intensity.

All firms share the same R&D technology. A firm's discoveries arrive through time according to a Poisson process, where the arrival rate is determined by its R&D intensity. Thus the arrival rate of ideas for firm i in patent race q is $l\, h_q^i$, where ? is an industry-specific productivity parameter. The probability that firm i discovers an invention before date t in the patent race q is $1 - e^{-l\, h_q^i \cdot t}$. The firm faces a constant *rival hazard rate* $l\,a_q^i \equiv l\sum_{j\neq i} h_q^j$. The probability that firm i wins patent race q is $h_q^i/[h_q^i + a_q^i]$, the ratio of firm i's hazard rate to the hazard rate for the entire industry.

2.1.1 A Passive Incumbent. A firm that owns a patented invention will be called an *incumbent*. The other firms will be called *challengers*. The model contains an additional assumption about the nature of technological competition: A firm that makes a patentable discovery does not compete in the subsequent patent race. This ad hoc restriction considerably simplifies the model and subsequent analysis, but it does not affect the qualitative properties of a model of patent races where the only difference between the incumbent and other firms is the rents it earns. In models of this type, being successful in a given patent race does not convey a natural advantage over rivals in subsequent races. It can be shown that the incumbent will race less aggressively than other firms, because it takes into account the fact that its R&D may replace profits it already earns (Reinganum 1985). In other models (Grossman and Helpman 1991), incumbents do not race at all.[3]

2.2 The Nature of Inventions and a System of Property Rights

A discovery is an improvement in product quality. The extent of an improvement is denoted $u_q \in [0, \bar{u}]$, where $\bar{u} < \infty$.[4] The magnitude of improvements is random, unknown until the time of invention, and common knowledge thereafter. For each invention, u is drawn from the continuous

[3] In that model firms borrow to finance their R&D investments, and the arrival rate of innovations is linear in firms' investments. In that case, the incumbent is at such a disadvantage vis-a-vis its rivals that it cannot finance subsequent innovations. It should be noted that in models that contain more asymmetry between firms, the assumption made here would significantly affect the properties of the resulting equilibria.

[4] Alternatively, we can express innovations as some percent reduction in the cost of producing the final good. The analysis would yield the same results so long as we assume cost reductions are perfectly compatible, so that a cost

density $f(u)$ with corresponding cumulative density $F(u)$. This distribution is constant through time and unaffected by the level of a firm's R&D spending.

Once a discovery has been made, it can be reverse-engineered at zero cost by all other firms. If a patent is granted, the inventor receives an exclusive right to produce and sell that invention. The statutory life of the patent is infinite. Not all inventions will be protected, however. Let $s \in [0, \overline{u}]$ denote the minimum extent of improvement for which the patent office is willing to grant a patent. In the model, this is the inventive step or standard of nonobviousness. An invention whose extent is less than s is not protected and becomes available to all firms.[5] In other words, it is added to the public domain of product improvements. Let $q(s) = 1 - F(s)$ denote the ex-ante probability of obtaining patent protection, given the patentability standard s. The expected quality improvement of a patentable invention will be denoted $\tilde{u} = \int_s^{\overline{u}} u dF(u) \big/ [1 - F(s)]$.

2.2.1 Reverse Engineering. Patent claims are defined as the improvement itself, so each improvement does not infringe a patent on another improvement. But when, and under what conditions, will an inventor be able to use prior generations of improvements in her product? For example, the firm might be required to license all prior improvements from their inventors. At the other extreme, an inventor could use all prior discoveries without obtaining a license. In this paper, we adopt an intermediate case: If an invention satisfies the standard of patentability, the inventor may use all prior discoveries without licensing them. However, if the standard is not satisfied, the prior discoveries

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reduction applied to different vintages of technology achieves the same percent reduction in cost.

[5] In the typology of O'Donoghue, Scotchmer, and Thisse (1998), we assume that lagging breadth is equivalent to

remain proprietary. One implication of this specification is that there is always, at most, one protected invention. Thus while the statutory length of patent protection is infinite, the economic life of a patent is the amount of time until the next patentable invention.[6]

Lach and Rob (1996) adopt an alternative approach, where firms embody new technology in vintage-specific capital goods. In a model of Cournot competition, the introduction of new technologies leads to a more gradual erosion of profits until the older firms exit altogether. In the model of O'Donoghue (1998), owners of patented inventions must cross license with each other if they are to produce a final good using the best available technology. To reduce complexity, O'Donoghue assumes such licenses are achieved but at the expense of an exogenous transactions cost. In his model, a social planner would respond to a higher transactions cost by raising the standard of patentability. If cross licensing were required in the model presented here, the same intuition would apply.

2.3 The Output Market and Flow Profits

From the preceding section, it is clear that during patent race q, the current holder of a patent can offer a product with the best available technology, i.e., one that embodies all the quality improvements invented prior to this race. The best any competing firm may offer is a product embodying all the improvements except for this last patented invention. Let $\hat{u}_q$ denote the extent of the innovation protected during race q. Note this is not necessarily the invention that ended the previous race.

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the magnitude of any patentable invention, while there is no leading breadth.

[6] This definition is consistent with the "reverse engineering" defense Congress established for *mask rights*, a sui generis form of intellectual property protecting the physical layout of computer chips (Hunt 1999a).

All consumers are identical and aggregate demand is normalized to one. Consumers only care about the quality of the good they are consuming. The reservation value of the final product to consumers, then, is simply the level of its quality, multiplied by p, the price of the final good relative to the R&D inputs (we'll suppress p until it becomes important in the comparative statics).[7] Firms compete in prices and the cost of production is zero. Thus the equilibrium price of the final good during the qth race is $\hat{u}_q$ and the incumbent earns flow profit $\hat{u}_q$. All other firms earn zero flow profits.

Flow Profits Earned During Patent Race *q*+1

	Innovation q was	
The firm is	Patentable	Unpatentable
The leader from race (q-1)	0	$\hat{u}_{q+1} = \hat{u}_q$
The winning challenger i	$\hat{u}_{q+1} = u_q$	0
All other challengers	0	0

We are interested in the flow profits earned by firms in the next (q+1) race. Several things might happen during the current race. Suppose that challenger i invents first in the qth race, but the invention is too small to qualify for protection. Because all firms can use that invention, the competitive position of firms in the output market is unchanged. In that case, the leader during race q continues to earn flow profits of $\hat{u}_q$ in the next race while all other firms earn nothing (see the last column of the table). Alternatively, the magnitude of i's invention is sufficiently large that it qualifies for protection. According to the property rights defined in section 2.2.1, firm i can also use all previously patented

[7] If we characterize innovations as cost reductions, we get the same behavior by assuming a constant elasticity of demand function with an elasticity of one.

inventions. In that case, during race q+1, firm i will earn flow profit $\hat{u}_{q+1} = u_q$, while the previous leader and all other firms earn nothing (see the first column of the table).

2.4 The Existing Literature

The theoretical literature on innovation and intellectual property design is voluminous. This section reviews only the work most closely related to the model presented here.

2.4.1 Patent Races and Endogenous Growth. The model builds on an extensive literature on stochastic patent races (Loury 1979, Dasgupta and Stiglitz 1980, Lee and Wilde 1980, and Reinganum 1985). The resulting equilibrium is similar to ones analyzed in certain models of endogenous growth (Aghion and Howitt 1992 and Grossman and Helpman 1991). One can interpret these models as an extreme case of the model constructed here, when all innovations satisfy the standard of patentability and every discovery eliminates the rents associated with the prior one.

2.4.2 Optimal Patent Design. The early literature on optimal patent design focused on the tradeoff between providing an incentive to innovate and the distortions that result from monopolistic pricing. The initial work (Nordhaus 1969) focused exclusively on patent length. More recent work considers the optimal combination of patent length and *breadth* (Gilbert and Shapiro 1990 and Klemperer 1990). Breadth is the degree to which a product or process must differ from a patented one to avoid infringement of the patent.

Patent breadth and obviousness are distinct concepts. Patent breadth affects the likelihood that a new invention will infringe the patent on a prior one. The inventive step or nonobviousness requirement distinguishes between proprietary and non-proprietary discoveries. An invention may be obvious and yet may not infringe an existing patent. Conversely, an invention may be not be obvious and yet still infringe the claims of a prior patent.

2.4.3 Patent Design with Cumulative Innovation. Another line of research (Green and Scotchmer 1995, Scotchmer 1996, and Denicolò 2000) examines the role of patents in the context of cumulative innovation, i.e., where inventions build on each other. These papers examine, in a two-period model, how patents should be designed to achieve an optimal allocation of rents between an initial and subsequent innovators.

There are a number of papers that evaluate the effects of intellectual property in dynamic models of sequential innovation in which firms compete over an unending sequence of races. For example, Bessen and Maskin (2002) use a simple model to show that an environment without any patent protection may generate more R&D investment and more innovation than an environment with patents. The key to this result in their model is that inventions are both complementary and essential, so that firms benefit from their rivals' R&D even if they must also share rents with them.

The finding that the rate of innovation is a non-monotonic function of the extent (or availability) of patent protection is found in this and a number of other papers.[8] For example, in O'Donoghue (1998), firms choose how much to invest in R&D and may choose a deterministic invention size. O'Donoghue shows that by specifying a minimum invention size, a social planner can induce more rapid innovation. The mechanism is essentially the same as the one described in this paper – lengthening the

[8] See also the papers by Cadot and Lippman (1997) and Chou and Haller (1995). In these models, the incentive to innovate is a non-monotonic function of rivals' exogenously specified capacity to imitate.

duration of incumbency can increase the rents associated with an innovation and consequently stimulate R&D investments. O'Donoghue also shows that if the social planner is limited to choosing a standard of nonobviousness, it will be larger than the first best.

In Horowitz and Lai (1996), firms choose how fast to race and the extent of the innovation they are targeting. They find the market leader will innovate just before its existing patent expires and that the extent of its innovation is an increasing function of the patent term. The overall rate of innovation is the product of the innovation rate and the extent of innovations. Horowitz and Lai show that this is maximized with a patent of finite duration but that social welfare is maximized with an even shorter patent term. In their model, patent length (measured in time) plays a role comparable to nonobviousness in this model, where effective patent length is endogenous and stochastic.

The primary differences between the model presented here and the models in the literature are that the magnitude of innovations is stochastic and industry structure is endogenous. It is then easy to show that the availability and extent of patent protection influence industry structure. In this environment, the relevant policy parameter is not patent life, which is also endogenous, but the minimum invention size that qualifies for protection. The inventive step that maximizes the rate of innovation in an industry is the one that maximizes the number of firms engaged in R&D.[9] Using the model, it is easy to show how the optimal inventive step varies across industries, depending on the exogenous parameters that affect their relative propensity to innovate.

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[9] Bernheim (1984) shows that in industries subject to sequential entry, excessively vigorous antitrust enforcement results in more concentration, not less. The underlying mechanism is similar to the one explored in this paper – if government policies reduce the likelihood of earning significant rents, only a few firms are able to amortize their cost of entry. I am grateful to an anonymous referee for pointing out this parallel.

2.4.4 Innovation with a Fixed Industry Structure. The properties of this model can be compared to those of a model in which the number of firms is assumed to be fixed (Hunt 1999a). In that model, changes in exogenous parameters (productivity of R&D, relative prices of R&D inputs, or the discount rate) affect the rate of innovation at the industry level entirely through changes in R&D investments at the firm level. In the model presented here, only changes in the discount rate and the fixed cost of setting up an R&D lab affect the amount of R&D at the firm level (see Proposition 3). All other effects operate through changes in the number of firms actively engaged in R&D (see Proposition 2). The effect of changes in the inventive step on innovation at the industry level is qualitatively the same, but it operates through the entry decision in this model rather than through changes in firms' R&D intensity as in Hunt (1999a).[10]

3. Equilibrium

3.1 The Stage Games

In this model, the leading firm is a passive recipient of rents earned on its previous patentable discovery. Eventually an innovation will occur, ending the current race and possibly the incumbent's rents. During the current race, challengers select the R&D intensity that maximizes expected current cash flow plus the expected present value of competing optimally in future races. The exact magnitude of flow profits associated with a patentable discovery is not known until the discovery has actually occurred. Firms take into account the expected invention magnitude of patentable discoveries $(\tilde{u})$ when choosing their

[10] One can posit an alternative, more complicated model that captures elements of both. In that model there is free entry, but firms must sink fixed R&D costs in each patent race. The comparative statics results of such a model lie somewhere between those reported here and in Hunt (1999a). The results for the optimal inventive step are

R&D intensity. The challengers move simultaneously, taking the number of their rivals as given (the participation constraint is addressed explicitly in the next section).

Let $V^i(h^i, a^i)$ denote the value function for the challenger i. Let V^w and V^l denote, respectively, the continuation values associated with playing optimally in all future races when the firm wins or loses the current one (the time subscripts have been suppressed in the text). Firms incur R&D expenses until the first discovery occurs. The value of competing actively in the current race, after sinking the cost of establishing an R&D lab, is then

$$[1] \qquad V^i(h^i, a^i) = \int_0^{\infty} \left\{ l\,h^i V^w + l\,a^i V^l - C(h^i) \right\} e^{-l[h^i + a^i]t - rt} dt = \frac{l\,h^i V^w + l\,a^i V^l - C(h^i)}{l\,h^i + l\,a^i + r}.$$

The first-order condition of the firm problem identifies the level of R&D where the marginal cost of additional effort is just equal to the marginal benefit of winning, rather than losing, the current race, that is $C'(h^i) = l[V^w - V^i(h^i, a^i)]$.

3.2 The Stationary Symmetric Equilibrium of the Game

A strategy of a firm in the game is a specification of a feasible R&D intensity to be played in each race, for each possible history of the game preceding that race. At the beginning of each race, each firm knows the play of all firms in the prior races and the outcomes of those races. When the firm is the incumbent, its only feasible R&D intensity is zero. Whenever the firm is a challenger, the set of feasible R&D intensities is always the same subset of $\mathbb{R}$. There are likely to be many equilibria of the game, but

qualitatively the same as those presented here.

we focus on stationary equilibria where firms choose identical strategies. In the Appendix, we prove the following:

Proposition 1 - Suppose the R&D cost function satisfies the following assumptions:

(i) $C(h) > 0,\ C'(h) > 0\ \forall h > 0;$

(ii) $C''(h) > 0\ \forall h > \hat{h} \in [0, \infty);$

(iii) $\text{Lim}_{h \to 0}\ C(h)/h = \text{Lim}_{h \to 0}\ C'(h) = 0;$

(iv) $\text{Lim}_{h \to \infty}\ C'(h) = \infty;$

(v) $C(\bar{h}) < \infty,\ \forall \bar{h} \in [0, \infty).$

Then, there exists a unique, stationary, symmetric equilibrium of the game in which every challenger chooses a flow R&D intensity $s \in (0, \bar{h}]$.

The first two assumptions tell us that R&D is costly and is eventually subject to diminishing returns. Together with the third and fourth assumptions, this assures us there will be an interior equilibrium of the stage games. To ensure the existence of a Markov Perfect Equilibrium, we need only verify that per period payoffs are bounded. This is assured by the fifth assumption and the fact that the largest invention magnitude is finite.

The R&D technology, the distribution of invention magnitudes, and the relationship between patented technology and expected profits do not vary across races.[11] The expected outcome of the races, then, varies only if firms choose different R&D intensities over time. If all challengers choose the same R&D intensity in all patent races, the probabilities of winning and losing, together with the

[11] The evolution of industries suggests that the distribution of invention magnitudes could vary over time. A more general model would allow for exhaustion of technological opportunities or spillovers from advances in other fields. The resulting dynamics would be both complicated and interesting.

expected length of races, will be the same in each race. Similarly, the continuation values associated with being the incumbent or a challenger, denoted $V^I(h)$ and $V^C(h)$, are the same across patent races.

We examine an equilibrium where firms respond to the same conditions in the same way through time. In this case, using [1], the expected values of being an incumbent or a challenger, respectively, are

$$[2] \qquad V^I(s)=\frac{\tilde{u}+lns\,V^l(s)}{r+lns} \quad \text{and} \quad V^C(s)=\frac{ls\,V^w(s)+l(n-1)s\,V^l(s)-C(s)}{r+lns}.$$

For a challenger, the continuation value associated with losing the current race is the expected value of being a challenger in the subsequent race, i.e., $V^l=V^C(s)$. For the winner of the current race, the expected value of making the first discovery is a weighted average of the continuation values associated with starting the next race as the incumbent or as a challenger. The weights depend on the probability that the invention is patentable:

$$V^w=qV^I(s)+(1\text{-}q)V^C(s).$$

Substituting these expressions into the equations in [2] and solving for $V^I(s)$ and $V^C(s)$, we find

$$V^I(s)=\frac{\tilde{u}+qlns\,V^l(s)}{r+qlns} \quad \text{and} \quad V^C(s)=\frac{qls\,V^I(s)-C(s)}{r+qls}.$$

Using these two expressions to solve for $V^I(s)$ and $V^C(s)$, we find

$$V^I(s)=\frac{[r+qls]\tilde{u}\text{-}qlns\,C(s)}{r[r+ql(n+1)s]} \qquad \text{and} \qquad V^C(s)=\frac{qls\,\tilde{u}\text{-}[r+qlns]C(s)}{r[r+ql(n+1)s]}.$$

Thus when firms choose the same R&D intensity in each patent race, the continuation values are simply a weighted average of the expected flow profit enjoyed by incumbents and the R&D

expenditures of challengers, weighted by the shares of time firms expect to be in each state of the world.

Finally, substituting these continuation values into the first-order condition yields

[3] $$C'(s) = ql[V^I(s) - V^C(s)] = ql\left(\frac{\tilde{u} + C(s)}{r + ql(n+1)s}\right).$$

Challengers select the R&D intensity that equates marginal cost and marginal benefits—the difference in the expected value of the cash flows associated with starting the next race as the incumbent rather than as a challenger. The denominator on the right-hand side of [3] is a measure of the *economic* life of patents. As $ql(n+1)s$ becomes larger, patentable discoveries occur more frequently. The incumbent enjoys her rents for less time, on average, so the present value of the rents is smaller.

3.2.1 The Participation Constraint. The first-order condition specifies a challenger's R&D intensity as a function of exogenous parameters and the number of rivals, which is endogenous. The number of rivals is determined by a participation constraint—firms enter a patent race so long as $V^C(s,(n-1)s) - k \geq 0$. After some simple algebraic manipulation, this constraint can be expressed as

[4] $$qls[V^I(s) - V^C(s)] \geq C(s) + rk.$$

We will assume the participation constraint binds, so the average cost of entering into R&D competition, including fixed costs, is just equal to the marginal benefit associated with winning the current patent race.[12] But this also implies that average cost and marginal cost are the same:

[5] $$C'(s)s = C(s) + rk.$$

[12] It should be noted we are ignoring integer constraints.

Using [3] and [4] the first-order condition can also be expressed as

[6] $$C'(s) = \frac{q\,l\,[\tilde{u} - rk]}{r + q\,l\,ns}.$$

Thus there will be no active patent races unless the revenues generated in the output market can amortize the fixed R&D costs. In the next section [3] and [5] will be used to explain some important results.

3.3 Properties of the Equilibrium

In a stationary equilibrium where there are no random shocks, firms that wish to compete will sink their fixed R&D investments at the beginning of the first patent race. Thereafter, if we consider marginal changes in certain parameters, the number of firms engaged in R&D would not decline because the expected value of actively competing in subsequent races is strictly positive (so long as $k > 0$). [13]

Consider two games involving industries with a different value for a single exogenous parameter. The industries are otherwise identical. We compare the resulting symmetric stationary equilibria. Firms take into account the exogenous parameters when deciding whether to incur the fixed cost of an R&D lab. In the Appendix we show the following:

> **Proposition 2** - The R&D intensity of individual firms does not vary with differences in output prices (p) and the productivity of R&D (l). But more firms will engage in R&D in the industry with either the higher output price or more productive R&D. Consequently, the industrywide rate of innovation will be higher.

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[13] More precisely, in this model, once firms decide to enter the industry in the first race, there are no shocks to firms' research productivity or costs that would imply any subsequent entry or exit.

Proposition 3 - The R&D intensity of individual firms varies with differences in the discount rate (r) and the cost of setting up an R&D lab (k). Higher discount rates or higher fixed R&D costs are associated with more R&D at the level of individual firms. But fewer firms will engage in R&D and the resulting industrywide rate of innovation will be lower.

Propositions 2 and 3 tell us that the industrywide rate of innovation is higher when the output price or the productivity of R&D is higher, but lower when the discount rate or the fixed cost of establishing an R&D facility is higher. In each of these cases, the determining factor is the number of firms engaged in R&D.

3.3.1 Implications of Propositions 2 and 3. Assuming the same cost function in each industry, there can be a difference in R&D intensity at the firm level only if these industries have a different discount rate or fixed cost.[14] This follows from equation [5]. But then [3] tells us that when r and k are the same in both industries, the expected value of making a patentable discovery must also be the same. If there are differences in the relative price (p) or productivity (λ) of R&D between these industries, there must be a corresponding difference in the number of firms such that the right-hand side of [3] is the same for both industries. This explains the result in Proposition 2. If R&D is cheaper, or more productive, in an industry, there are also more firms in that industry, which implies the industry innovates more rapidly. This more rapid innovation dissipates any differential in rents earned in the two industries.

[14] We have also implicitly assumed the same distribution of invention magnitudes and the patentability standard

Now suppose that p and λ are the same in both industries, but there is a difference in either r or k. From [5], we know that R&D intensity at the firm level will be higher in the industry where r or k is higher. From [3], we also know that the expected value of making a patentable discovery will be higher in the industry with the higher R&D intensity.

Given that p and λ are the same, this difference must result from a difference in the number of firms doing R&D and the difference in R&D per firm across industries.

3.3.1 Market Structure and Innovation. The parameters described in propositions 2 and 3 are sufficient to describe a wide variety of industry structures and innovation rates. These propositions also demonstrate that industry structure, in itself, cannot explain variations in the rate of innovation across industries. Consider the following example.

In the environment most closely analogous to perfect competition, fixed costs are close to zero; so there are many firms, each engaged in a little R&D. Whether we consider this industry to be highly innovative will depend on the productivity of R&D and its relative cost. Greater concentration, in the sense of relatively few firms competing to innovate, occurs when fixed costs are relatively high. Individual firms will do more R&D than we would see in the previous case. But whether we would consider this industry highly innovative again depends on the productivity of R&D and its relative cost.

Other things equal, we would expect the industry with lower fixed costs to be more innovative (proposition 3). But when other things are not equal, it is possible that a more concentrated industry would innovate more rapidly. This might occur, for example, when an industry has a relatively high fixed

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for both industries.

cost to establishing research facilities but relatively low marginal costs of using them more intensively. Thus the model may explain why empirical research on the question of which market structures are most conducive to innovation has largely been inconclusive (Cohen and Levin 1989, Scherer 1992).

3.4 Comparing the Equilibrium to the Social Planner's Problem

It is useful to compare the private equilibrium R&D intensity and number of firms to the social planner's solution.[15] Unlike firms, society enjoys a permanent benefit from every innovation, however small. The expected social value of an innovation, before taking into account the cost of R&D, is simply $\tilde{v} \equiv \frac{p}{r}\int_{\underline{u}}^{\overline{u}} u dF(u)$. Expected social welfare at the beginning of the game is

$$W = \underset{h,n}{Max}\left\{\frac{n}{r}\left(l h\tilde{v} - C(h) - rk\right)\right\}. \quad [7]$$

The first derivatives are

$$\frac{\partial W}{\partial h} = \frac{n}{r}[\, l\,\tilde{v} - C'(h)\,], \qquad \frac{\partial W}{\partial n} = \frac{1}{r}[\, l\, h\tilde{v} - C(h) - rk\,].$$

Thus the social planner would specify a firm level R&D intensity that equalizes marginal cost and the marginal social benefit of the next discovery. This R&D intensity will be greater than the level attained in the private equilibrium whenever

$$\frac{l\, p}{r}\int_{\underline{u}}^{\overline{u}} u dF(u) \geq \frac{q\, l\,[\, p\tilde{u} + C(s\,)]}{r + q\, l\,(n+1)s} = \frac{q\, l\,[\, p\tilde{u} - rk]}{r + q\, l\, ns}. \quad [8]$$

The middle term in [8] is the marginal private benefit of an increase in R&D intensity, evaluated at the private equilibrium and taking into account the standard of patentability. The equality with the last term

[15] In this model maximizing the rate of innovation, by adopting appropriate patentability standards will usually

in [8] follows from the binding entry constraint. To see why the inequality in [8] is always satisfied, recall that the expected value of patentable discoveries is simply

$$\tilde{u}(s) = \frac{\int_s^{\overline{u}} udF(u)}{1 - F(u)} = \frac{1}{q}\int_s^{\overline{u}} udF(u).$$

The inequality is strict whenever there are positive fixed costs or more than one firm is engaged in R&D.

As for the optimal number of firms, there is a razor's edge result. Generically, the marginal social benefit of innovation will be either greater or less than a firm's average cost at the first best R&D intensity. Consequently, the social planner will desire either an infinite number of firms or no firms to establish R&D facilities.[16] If we assume the average cost is less than the marginal social benefit to innovation, it is apparent the social planner would wish to subsidize both entry and R&D effort.

To put it another way, the social planner could implement the optimal R&D intensity by allowing only one firm to innovate and granting infinitely lived patents on all the resulting innovations (recall that deadweight losses have been assumed away). But if the social planner would desire one such firm, she would in fact desire an infinite number of them. That would dissipate firms' profits so that the optimal R&D intensity would not be obtained. So if the social planner is limited to using patents of the sort described in this model, the best she can do is to maximize the rate of innovation in the industry.

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maximize welfare.

[16] This is an artifact of the inexhaustibility of discoveries in the model. Such a result would not hold if we relaxed the assumption that firms' inventions arrive independently of the activities of their rivals, or if there were congestion costs—i.e., R&D labs become more expensive when more firms wish to build them.

4. Patentability Standards and the Rate of Innovation

4.1 General Results

We typically think of the U.S. patent system as applying a common set of criteria to inventions in all technology fields and industries. In this section, however, we construct a hypothetical in which two otherwise identical industries are subject to different standards of patentability. Firms take this standard into account when deciding whether to sink the fixed cost of an R&D lab. In this way we allow for the possibility that patentability criteria affect the number of firms engaged in R&D.

In equilibrium, firms equate the marginal cost of additional R&D effort to the expected gain associated with inventing first. This gain is affected by patentability criteria in two ways. First, there is the likelihood that any given invention by a firm qualifies for protection. Second, there is a relationship between that probability and the number of rivals a firm competes with. In the Appendix we show the following:

> **Proposition 4** - In the stationary symmetric equilibrium, differences in the standard of patentability (the inventive step) do not affect the R&D intensity of individual firms, but they do affect the number of firms actively engaged in R&D and, therefore, the industrywide rate of innovation.

The intuition behind propositions 2 and 4 is very similar. Differences in patentability criteria do not affect the left-hand side of equations [3] and [6], so marginal and average cost in the two industries will be the same.[17] This implies the marginal benefit to innovating first is also the same. This equality is the result of a difference in the number of firms in these industries. But which industry has more firms and therefore innovates faster? In the Appendix, we show

> **Proposition 5** - There exists a unique standard (inventive step), denoted s^*, such that in the interval $[0, s^*)$, industry-wide R&D activity increases as the standard is made more strict. In the interval $(s^*, \bar{u}]$, industrywide R&D activity decreases as the standard is made more strict.

Proposition 5 tells us that differences in the rate of innovation between two industries will depend on the industry-specific patentability standards relative to each other and relative to s^*. Consider two otherwise identical industries with standards s_1 and s_2. When $s^* \le s_1 < s_2$, the first industry will innovate more rapidly than the second. But when $s_1 < s_2 \le s^*$, the industry with the more stringent standard innovates faster. Thus, without knowing s^* and the actual patentability standard relative to it, we cannot say a priori whether a change in the standard will increase or decrease the rate of innovation.

4.1.1 Deriving the Optimal Patentability Standard. In the Appendix, we show that s^* is implicitly defined by the equation $\Psi(s) = 0$, where

[17] As noted in the discussion of Proposition 2, under different modeling assumptions the amount of R&D performed by each firm would likely change, but the direction of change in R&D performed by an entire industry would be the same.

[9] $$\Psi(s) \equiv \left(\frac{ql(n+1)s}{r + ql(n+1)s} \right) [p\tilde{u} + C(s)] - [ps + C(s)].$$

As the standard of patentability is made more strict (requiring a larger inventive step), firms encounter the following tradeoff. On the one hand, a firm that makes a marginal discovery would not obtain a patent. The cost of this is the forgone value of the marginal patent plus the R&D expended in the subsequent patent race. That is reflected in the second term of [9]. This is the static effect of an increase in the patentability standard.

But raising the standard also has a dynamic effect because firms are able to earn flow profits for a longer period of time. The expected gain is the average value of patentable inventions, plus the R&D that would otherwise be expended in the next patent race. The sign of $\Psi(s)$ depends crucially on the relative weight placed on these gains and losses (the ratio in the first term of [9]). That ratio depends on the arrival rate of patentable discoveries relative to the discount rate. If patentable discoveries arrive very often, the first term in [9] receives nearly the same weight as the second and $\Psi(s) \geq 0$. If the arrival rate of patentable discoveries is very small relative to the discount rate, it is likely that $\Psi(s) \leq 0$.

At first, it may seem counter-intuitive that the benefit to preserving an incumbent's rents is larger when patentable inventions are more frequent. Given that the expected duration of those rents is small one might expect the present value of the rents preserved would also be small. But when we consider changes in the standard of patentability, we are considering marginal changes in those rents. In a rapidly innovating industry, the rents that are affected are earned relatively soon and therefore are not discounted very much. In an industry that innovates less rapidly, increasing the standard of patentability contributes additional rents, but they are earned far in the future and are discounted accordingly.

Now consider how $\Psi(s)$ changes as we vary the patentability standard from a very low to a very high value. When the standard is very weak (s = 0), the static effect is irrelevant because rents earned on the marginal invention cannot affect the participation decision (the static effect is important only when $ps \geq rk$). In this range, adopting a more strict standard would increase the number of firms actively engaged in R&D. But eventually, as the standard is made increasingly more strict, the dynamic effect becomes smaller ($q\,l\,ns$ eventually declines as s increases) while the static effect becomes larger. When the patentability standard is very strict, the static effect dominates. There is only one standard, or height of the inventive step, where the two effects are exactly equal.

4.2.1 Implications. Clearly the optimal standard (optimal in the sense that it maximizes the rate of innovation) implied by [9] depends on the characteristics of the industry that determine its underlying propensity to innovate. If those parameters change, so would s^*. If those parameters vary across industries, there would be a unique, but different, standard that maximizes the rate of innovation in each of those industries. Industry structure, which is determined by r and k in this model, is relevant in the determination of s^* only to the extent that it influences the industry's rate of innovation. In the Appendix, we prove

> **Proposition 6** - The critical standard, s^*, is increasing in the productivity of R&D and decreasing in the discount rate and fixed and variable costs of doing R&D.

This proposition has two very natural interpretations:

> **Corollary 1 –** Suppose there are two industries, *a* and *b*, and in each industry the standard of nonobviousness is set to maximize the rate of innovation in that industry. Without loss of generality, suppose that industry *a* innovates more rapidly than industry *b*. Then, a smaller share of innovations will qualify for protection in industry *a* than in industry *b* ($s_a^* > s_b^*$).

> **Corollary 2 –** Suppose a single patentability standard is applied to industries *a* and *b* and that we consider relaxing this standard. It is more likely that doing so will raise the rate of innovation in industry *b*, than it will in industry *a*.

In order to maximize the rate of innovation in industries already pre-disposed to innovate rapidly, the minimum inventive step required for patent protection should be set higher than for other industries. If industries are all drawing from the same distribution of invention magnitudes, that means that a smaller share of inventions will qualify for protection in industries that innovate the most rapidly.

Now suppose we arbitrarily choose a single patentability requirement *s* that is applied to both industries. By definition, it is more likely that $s \leq s_a^*$ than $s \leq s_b^*$. It is therefore more likely that raising *s* would increase the rate of innovation in industry *a* than in industry *b*. Conversely, as stated in Corollary 2, it is more likely that lowering *s* will increase the rate of innovation in industry *b* than in industry *a*. Thus, in the absence of changes in exogenous parameters of the model, a policy of relaxing patentability criteria would appear to favor innovation in more traditional industries at the expense of innovation in high technology industries. We return to this intuition in the discussion.

4.2.2 Examples. These results are illustrated in Figures 1-4, which are generated under the additional assumptions that R&D costs are quadratic and that invention magnitudes are drawn from a normal distribution. Each figure plots out the industrywide level of R&D, as a function of the minimum inventive step, for two industries that differ from each other in only one parameter. As described in Proposition 2, the industry with more productive R&D, a higher output price (e.g., a lower relative price of R&D), lower fixed R&D costs, or a lower discount rate engages in more R&D and consequently innovates more rapidly. We call that industry industry a.

The figures also depict the change in industrywide R&D as a function of the minimum inventive step. As described in Proposition 6, the rate of innovation is maximized with a larger inventive step in industry a than in industry b. That is consistent with Corollary 1, which tells us the R&D maximizing inventive step is larger for the industry that is pre-disposed to innovate more rapidly than another one. It is also the case that the rate of innovation in industry a is more sensitive to the specification of the minimum inventive step than it is for industry b. In other words, unless patentability standards were initially very strict, any increase in inventive activity in industry b would not offset a decline in such activity in industry a.

5. Discussion and Conclusions

This paper develops a model of cumulative innovation where the profitability of inventions is eroded by the introduction of new, competing technologies through time. When firms can readily duplicate each other's discoveries, patentability criteria, in particular the requirement of nonobviousness (the inventive step), play an important role in determining the share of future discoveries that will affect the expected profits earned on patented inventions discovered today.

In such an environment, there exists a unique inventive step that maximizes the rate of innovation in an industry, by maximizing the number of firms that enter into R&D competition. The effect of changes in the inventive step on the industrywide rate of innovation depends on whether the initial standard is more or less stringent than this optimal value. This optimal standard will be more stringent for industries pre-disposed to innovate rapidly than for industries pre-disposed to innovate slowly. In other words, under the optimal patentability standard, a smaller share of inventions qualifies for protection in rapidly innovating industries than in other industries.

When a common inventive step is applied to all industries in an economy, the number of firms engaged in R&D in each of those industries will depend on the stringency of the standard. Generally speaking, when the standard is more stringent, there will be more firms in industries disposed to innovate more rapidly, and fewer firms in industries disposed to innovate less rapidly. In setting patentability criteria, then, we are also setting industrial policy. A social planner would take these effects into account when setting the optimal standard.

5.1 International Implications

A social planner in a country with a different industrial composition would likely adopt a different inventive step. The inventive step would likely be higher in economies that enjoy a comparative advantage in R&D. That might suggest that, under an optimal standard, patents would be easier to obtain in less developed economies than in more developed ones. Adopting the same inventive step in all countries may increase the rate of innovation in some countries but might reduce it in others. Of course, more general statements about welfare implications require a model that allows for trade, foreign direct investment, and licensing.[18]

Efforts toward patent harmonization have thus far concentrated on issues such as establishing uniform priority, a minimum patent length, fewer subject matter exceptions, adequate remedies for infringement (damages, injunctions), and adequate administrative and judicial infrastructures. One exception was the proposed Patent Harmonization Treaty, abandoned in the mid 1990s, which included a specification of patentability standards (Moy 1993). Recently, the U.S. Patent and Trademark Office proposed to include, among other things, an American-style nonobviousness test in its agenda for future international negotiations on patent harmonization (USPTO 2001).

5.2 A Common Law Standard of Patentability?

Given that the optimal standard is a function of industry characteristics that influence the industry's rate of innovation, this standard will vary as those characteristics change. An economywide increase in the productivity of R&D, for example, might suggest the inventive step should be increased in order to obtain the maximum possible benefit of this new-found productivity. If the productivity increase occurred in a single industry, a social planner would likely adopt a more stringent standard, but doing so

[18] See the surveys by Maskus (2000) and Saggi (2001). A general conclusion is that the adoption of stronger patent rights around the world could lead to significant wealth transfers from users of patented technologies in developing economies to patent holders in the most developed economies. There is some evidence of dynamic gains for developing economies, such as increased imports of high technology goods, greater foreign direct investment, and increased opportunities to license advanced technologies. Whether the benefits of these changes exceed the cost depends on the degree to which innovation is increased in developed economies, developing economies, or both.

would reduce the rate of innovation in the other industries.

The optimal inventive step derived from the model presented in this paper follows from an explicit balancing of the gains and losses generated by marginal changes in the patent standard. A social planner would reduce the inventive step until the value of granting exclusive rights to the marginal invention is just equal to the expected value of rents that are lost as the economic life of patents is reduced. This has the flavor of a common law balancing test rather than a standard specified by law.

One can argue that, for a very long time, that is how the requirement of nonobviousness functioned in the U.S. patent system. The requirement existed in court precedents about a century before it appeared in the 1952 Patent Act, which largely adopted the test used by the courts. The classic articulation of the test appeared in the 1966 decision *Graham v. John Deere*: At the time it was made, would the invention have been obvious to a practitioner of ordinary skill in the relevant field? If such a determination is influenced by factors such as research productivity or costs, the judicial test and the one described in this paper do not seem incompatible.

Recently, some legal scholars have argued that that patent standards should be influenced by a balancing of costs and benefits (Barton 2001, Rai 2002). But most patent practitioners and scholars support relatively stable patentability criteria and an equal treatment of all patentable technologies. They argue that the patent system is already costly and additional complexity would only increase these costs while also increasing uncertainty about future returns.

5.3 The American Policy Experiment of the 1980s

But patent standards have been changed before. During the 1980s, the U.S. adopted a new form of intellectual property (mask rights) to protect the physical layout of semiconductor chips, and a series of court decisions reduced the inventive step for patents (Hunt 1999a, 1999b). At the time, it was argued that these changes would stimulate innovation in America's high technology industries. The results of this paper suggest the opposite might well be true — weaker patentability standards are more likely to increase R&D in industries that innovate slowly and to reduce R&D in industries that would otherwise innovate rapidly.

The final assessment of the changes adopted in the 1980s remains an open empirical question.[19] This model suggests at least one testable implication: Historical patterns of entry and exit from industries may have changed in some systematic way – with relatively more net entry into industries that innovate slowly and relatively less net entry into industries that innovate more rapidly. A topic of future research, then, is to establish whether this conjecture is borne out in the data.

[19] Relatively few articles address this question. Bessen and Maskin (2002) argue that granting patents on computer software may have been detrimental. Hunt (1996) presents evidence of changes in the value of R&D investments in the semiconductor industry that are consistent with the theoretical model presented here. See also Kortum and Lerner (1999) and the reviews by Jaffe (2000) and Hunt (1999b).

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APPENDIX

Proposition 1 - Suppose the R&D cost function satisfies the following assumptions:

(i) $C(h) > 0,\ C'(h) > 0\ \forall h > 0;$

(ii) $C''(h) > 0\ \forall h > \hat{h} \in [0, \infty);$

(iii) $\text{Lim}_{h\to 0}\ C(h)/h = \text{Lim}_{h\to 0}\ C'(h) = 0;$

(iv) $\text{Lim}_{h\to\infty}\ C'(h) = \infty;$

(v) $C(\overline{h}) < \infty,\ \forall \overline{h} \in [0, \infty).$

Then, there exists a unique, stationary, symmetric equilibrium of the game in which every challenger chooses a flow R&D intensity $s \in (0, \overline{h}\,]$.

Proof: The proof is constructed through the lemmas that follow.

Lemma 1 - Suppose $V^{W}_{q+1} \in (0, \infty)$ and $V^{W}_{q+1} - V^{L}_{q+1} > 0$. If rivalry and the fixed R&D costs are sufficiently small, at least one challenger will choose to enter a stage game.

Proof: Note that we are treating the continuation values as exogenous parameters. Later we show that, in equilibrium, the continuation values satisfy the requirements set out in the lemmas.

Consider the case where there is no rivalry and fixed R&D costs are zero. We need to show that $V^{i}_{q}(h^{i}, 0) \geq V^{i}_{q}(0,0)$. The inequality is satisfied when there exists some positive level of R&D intensity where $l\, h^{i}_{q} V^{W}_{q+1} \geq C(h^{i}_{q})$, which is satisfied if the minimum average cost of R&D is not too high. The third assumption ensures there is at least one R&D intensity $\tilde{h} \in (0, \overline{h}\,]$ where the inequality is strict.

Now we consider a strictly positive fixed R&D cost k. In that case, a challenger chooses to enter so long as $l\, h^{i}_{q} V^{W}_{q+1} \geq C(h^{i}_{q}) + [r + h^{i}_{q}]k$. If $l\, \hat{h}^{i}_{q} V^{W}_{q+1} > C(\hat{h}^{i}_{q})$, there is also a level of fixed R&D cost where $l\, \hat{h}^{i}_{q} V^{W}_{q+1} \geq C(\hat{h}^{i}_{q}) + [r + \hat{h}^{i}_{q}]k$. Thus for k sufficiently small, there are always at least two firms, an incumbent and at least one challenger.

Now suppose there is some small positive level of rivalry. A challenger will enter the stage game if the following inequality holds:

$$\text{[A.1]} \qquad l\, h^{i}_{q} \left\{ rV^{W}_{q+1} + l\, a^{i}_{q} \left[V^{W}_{q+1} - V^{L}_{q+1} \right] \right\} \geq \left[r + l\, a^{i}_{q} \right] C(h^{i}_{q}) \left[r + l\, h^{i}_{q} + l\, a^{i}_{q} \right] rk.$$

Applying the preceding argument to this inequality, for k and a^{i}_{q} sufficiently small, there is an R&D intensity in the interval $(0, \overline{h}\,]$ where this inequality is strict. So long as $V^{W}_{q+1} > 0$ and $V^{W}_{k+1} - V^{L}_{k+1} > 0$, the magnitude of the continuation values will always define a set of pairs $(a^{i}_{q}, k) \in \mathbb{R}^{+}$ where the participation constraint is satisfied. We can also define a level of fixed R&D cost, $\hat{k}(a^{i}_{q})$ where the participation constraint just binds.

APPENDIX (continued)

Lemma 2 - If $V^W_{q+1} \in (0,\infty)$, $V^W_{q+1} - V^L_{q+1} > 0$, and $k < \hat{k}(0)$, there exists an interior equilibrium of the stage game.

Proof: The proof of existence is a modification of the existence proof in Reinganum (1985). We continue to treat the continuation values as exogenous parameters, but take into account the effect of a firm's choice of R&D intensity on the likelihood of winning and the expected length of the patent race. Firms take their rival's research intensity as given. Fixed R&D costs must be sufficiently low so that at least one firm is willing to engage in R&D.

The derivative of the firm's objective function, $\partial V^i_q / \partial h^i_q$, is

[A.2] $$\frac{r\left[\,l\,V^W_{q+1} - C'(h^i_q)\right] + l\,a^i_q\left[\,l\,[V^W_{q+1} - V^L_{q+1}] - C'(h^i_q)\right] + l\left[C(h^i_q) - C'(h^i_q)h^i_q\right]}{\left[r + l\,(h^i_q + a^i_q)\right]^2}.$$

The sign of [A1] is the sign of the numerator, which we will call $f^i(h^i_q, a^i_q)$. Note that $f^i(h^i_q, a^i_q)$ is strictly decreasing in R&D intensity:

$$\frac{\partial f^i(h^i_q, a^i_q)}{\partial h^i_q} = -\,C''(h^i_q) \bullet \left[r + l\,(h^i_q + a^i_q)\right] < 0.$$

If the saturation point of R&D ($\bar{h}$) is sufficiently large, there will be a finite level of R&D effort where $f^i(h^i_q, a^i_q) = 0$. $V^i_q(h^i_q, a^i_q)$ is maximized by this level of R&D effort. Let $h^i_q(a^i_q)$ denote the firm's best response to the level of rivalry it encounters. The strict monotonicity of $f^i(h^i_q, a^i_q)$ implies that this best response is unique. Firms never choose R&D intensities greater than $\tilde{h}^i_k$, so we can restrict the strategy space to a convex, compact, nonempty subset of $\mathbb{R}^n$, denoted $X \equiv \Pi^n_{i=1}[0, \tilde{h}^i_q]$. The vector $[h^1_q(a^1_q), h^2_q(a^2_q), \ldots h^n_q(a^n_q)]$ maps X into itself continuously. Existence of an equilibrium then follows from Brouwer's fixed point theorem. ▪

Lemma 3 - If $l\,[V^W_{q+1} - V^L_{q+1}] - [C'(h_q) + h_q C''(h_q)] < 0$, there exists a unique, symmetric equilibrium of the stage game.

Proof: Existence of a symmetric equilibrium follows from the firm's objective function and first order condition, which varies only by the level of rivalry encountered. In the symmetric equilibrium, $f^i(h^i_q, a^i_q)$ becomes $f^i\left(h_q, (n-1)h_q\right)$. The corresponding first order condition is

$$r\left[\,l\,V^w_{q+1} - C'(h_q)\right] + l\,(n-1)h_q\left[\,l\,[V^w_{q+1} - V^l_{q+1}] - C'(h_q)\right] + l\left[C(h_q) - C'(h_q)h_q\right] = 0.$$

The first and third terms are strictly decreasing in R&D effort. If the second term is also strictly decreasing, then only one level of R&D intensity satisfies the equality. Hence we require that

$$l\left[V^{w}_{q+1} - V^{l}_{q+1}\right] - \left[C'(h_q) + h_q C''(h_q)\right] < 0. "$$

The symmetric equilibrium R&D intensity of the stage game with continuation values V^{W}_{q+1} and V^{L}_{q+1} is denoted $h_q(V^{W}_{q+1}, V^{L}_{q+1})$.

Lemma 4 - The game is continuous at infinity.

Proof: It is sufficient to show that total firm payoffs are a discounted sum of per period payoffs and that these per period payoffs are uniformly bounded [see Fudenberg and Tirole (1991), p. 110]. The per period payoff to firms is the present value of flow profits for the incumbent and the present value of R&D expenditures for challengers. The maximum per period return for an incumbent is $\bar{u}/r$. Per period returns for challengers are contained in the interval $[-C(\bar{h})/(r + \bar{h}), 0]$.

Lemma 5 - Lemmas 1 - 4 imply the existence of a stationary symmetric equilibrium of the game.

Proof: We return to the first order condition of the stage game, but assume that the continuation values associated with winning and losing the current race do not vary across races. Rearranging terms, we have:

[A.3] $$C'(h_q)\left[r + l\, nh_q\right] = l\left[rV^{W} + l\,(n-1)h_q[V^{W} - V^{L}] + C(h_q)\right].$$

If firms take the continuation values as given, and these values are constant across races, it is a best response for each firm to choose the same R&D intensity in each race. Lemma 3 establishes the existence of such a best response for a given specification of the continuation values. The continuation values themselves take a simple recursive form:

$$V^{I}(h) = \frac{\tilde{u} + l\, nh\left[qV^{C}(h) + (1-q)V^{I}(h)\right]}{r + l\, nh} = \frac{\tilde{u} + q\, l\, nhV^{C}(h)}{r + q\, l\, nh}, \text{ and}$$

$$V^{C}(h) = \frac{l\, h\left[qV^{I}(h) + (1-q)V^{C}(h) + (n-1)V^{C}(h)\right] - C(h)}{r + l\, nh} = \frac{q\, l\, hV^{I}(h) - C(h)}{r + q\, l\, h}$$

Solving for $V^{I}(h)$ and $V^{C}(h)$, we have,

$$V^{I}(h) = \frac{[r + q\, l\, h]\tilde{u} - q\, l\, nhC(h)}{r[r + q\, l\,(n+1)h]} \qquad \text{and} \qquad V^{C}(h) = \frac{q\, l\, h\tilde{u} - \left[r + q\, l\, nh\right]C(h)}{r[r + q\, l\,(n+1)h]}.$$

If we substitute for $V^{I}(h)$ and $V^{C}(h)$ in equation [A.3], the first-order condition reduces to

[A.4] $$C'(h) = q\, l\left(\frac{\tilde{u} + C(h)}{r + q\, l\,(n+1)h}\right).$$

APPENDIX (continued)

We use s to denote the equilibrium R&D intensity that satisfies equation [A.4]. It can be verified, using equation [A.4], that the condition required in lemma 3 for the uniqueness of the symmetric equilibrium of the stage games is satisfied.

If we substitute for $V^I(s)$ and $V^C(s)$ in equation [A.1], the participation constraint is simply $V^C(s) \geq k$. This in turn implies $C(s) + rk \leq ql\,s\left[V^I(s) - V^C(s)\right] = C'(s)s$. When the participation constraint binds, we can express it simply as

[A.5] $$C'(s) = ql\left(\frac{\tilde{u} - rk}{r + ql\,ns}\right).$$

During each race, for every challenger, the R&D intensity s is the unique best response to the continuation values $V^I(s)$ and $V^C(s)$. The strategy of playing s in every race cannot be improved upon by choosing a different R&D intensity in one race and playing s in all the others. If playing s in every race cannot be improved upon by a deviation in one stage, and the game is continuous at infinity, choosing the R&D intensity s in each race is a subgame perfect equilibrium of the game [see Fudenberg and Tirole (1991), p.110]. ∎

Lemma 6 - The symmetric stationary equilibrium is unique.

Proof: It is sufficient to show that there is only one possible intersection of the curves described by [A.4]. At $h = 0$, $C'(h) = 0$ while $ql[V^I(h) - V^C(h)] = ql\,\tilde{u}/r$. Thus at the first intersection, the marginal cost curve must be rising faster than $ql[V^I(h) - V^C(h)]$. If we can rule out an intersection where $ql[V^I(h) - V^C(h)]$ is rising faster than marginal cost is, we are done. Define $\mathrm{M}^1 = C'(h)\left[r + ql(n+1)h\right] - ql[\tilde{u} + C(h)] = 0$ and note that:

[A.6] $$\frac{\partial \mathrm{M}^1}{\partial h} = C''(h)\left[r + ql(n+1)h\right] + C'(h)ql\,n > 0.$$

This rules out an intersection where the marginal cost curve crosses $ql[V^I(h) - V^C(h)]$ from above. ∎

Proposition 2 - The R&D intensity of individual firms does not vary with differences in output prices (*p*) and the productivity of R&D (λ). But more firms will engage in R&D in the industry with either the higher output price or more productive R&D. Consequently, the industrywide rate of innovation will be higher.

Proposition 3 - The R&D intensity of individual firms varies with differences in the discount rate (r) and the cost of setting up an R&D lab (*k*). Higher discount rates or higher fixed R&D costs are associated with more R&D at the level of individual firms. But fewer firms will engage in R&D and the resulting industrywide rate of innovation will be lower.

Proof: We reintroduce the relative price of outputs in terms of inputs (*p*) and rewrite [A.4] and [A.5] in following form:

APPENDIX (continued)

$$M \equiv \begin{array}{l} \mathrm{M}^1 = C'(s)[r + ql(n+1)s] - ql[p\tilde{u} + C(s)] = 0 \\ \mathrm{M}^2 = C'(s)[r + qlns] - ql[p\tilde{u} - rk] = 0 \end{array}$$

We'll need the following derivatives:

$$\begin{array}{ll}
\mathrm{M}^1_s = C''(s)[r + qls] + [C''(s)s + C'(s)]qln & \mathrm{M}^2_s = C''(s)r + [C''(s)s + C'(s)]qln \\
\mathrm{M}^2_n = C'(s)qls & \mathrm{M}^2_n = C'(s)qls \\
\mathrm{M}^1_p = -ql\tilde{u} & \mathrm{M}^2_p = -ql\tilde{u} \\
\mathrm{M}^1_l = -\dfrac{r}{l}C'(s) & \mathrm{M}^1_l = -\dfrac{r}{l}C'(s) \\
\mathrm{M}^1_r = C'(s) & \mathrm{M}^2_r = C'(s) + qlk \\
\mathrm{M}^1_k = 0 & \mathrm{M}^2_k = rql
\end{array}$$

The Jacobian $|\mathrm{M}| = \mathrm{M}^1_s \mathrm{M}^2_n - \mathrm{M}^2_s \mathrm{M}^1_n = C''(s)C'(s)[qls]^2 > 0.$

i. Increasing the output price:

$$\frac{\partial s}{\partial p} = \frac{\mathrm{M}^2_p \mathrm{M}^1_n - \mathrm{M}^1_p \mathrm{M}^2_n}{|\mathrm{M}|} = \frac{C'(s)qls\,[ql\tilde{u} - ql\tilde{u}]}{C''(s)C'(s)[qls]^2} = 0;$$

$$\frac{\partial n}{\partial p} = \frac{\mathrm{M}^1_p \mathrm{M}^2_s - \mathrm{M}^2_p \mathrm{M}^1_s}{|\mathrm{M}|} = \frac{\tilde{u}}{C'(s)s} > 0.$$

ii. Increasing the productivity of R&D:

$$\frac{\partial s}{\partial l} = \frac{\mathrm{M}^2_l \mathrm{M}^1_n - \mathrm{M}^1_l \mathrm{M}^2_n}{|\mathrm{M}|} = \frac{C'(s)qs\,[rC'(s) - rC'(s)]}{lC''(s)C'(s)[qls]^2} = 0;$$

$$\frac{\partial n}{\partial l} = \frac{\mathrm{M}^1_l \mathrm{M}^2_s - \mathrm{M}^2_l \mathrm{M}^1_s}{|\mathrm{M}|} = \frac{r}{qsl^2} > 0.$$

iii. Increasing the fixed cost of setting up an R&D lab:

$$\frac{\partial s}{\partial k} = \frac{\mathrm{M}^2_k \mathrm{M}^1_n - \mathrm{M}^1_k \mathrm{M}^2_n}{|\mathrm{M}|} = \frac{r}{C''(s)s} > 0;$$

$$\frac{\partial n}{\partial k} = \frac{\mathrm{M}^1_k \mathrm{M}^2_s - \mathrm{M}^2_k \mathrm{M}^1_s}{|\mathrm{M}|} = \frac{-r\left[C''(s)\left[r + ql(n+1)s\right] + C'(s)qln\right]}{C''(s)C'(h)qls^2} < 0.$$

The change in industrywide R&D is therefore:

$$n\frac{\partial s}{\partial k}+s\,\frac{\partial n}{\partial k}=\frac{-r\left[r+q\,l\,(n+1)s\,\right]}{C'(h)q\,l\,s}<0.$$

iv. Increasing the discount rate:

$$\frac{\partial s}{\partial r}=\frac{\mathrm{M}_r^2\mathrm{M}_n^1-\mathrm{M}_r^1\mathrm{M}_n^2}{|\mathrm{M}|}=\frac{k}{C''(s\,)s}>0;$$

$$\frac{\partial n}{\partial k}=\frac{\mathrm{M}_r^1\mathrm{M}_s^2-\mathrm{M}_r^2\mathrm{M}_s^1}{|\mathrm{M}|}=-\left(\frac{C''(s\,)\left[C'(s\,)s\;+\left[r+q\,l\,(n+1)s\,\right]k\right]+C'(s\,)q\,l\,s\,k}{C''(s\,)C'(h)q\,l\,s^{\,2}}\right)<0.$$

The change in industrywide R&D is therefore:

$$n\frac{\partial s}{\partial k}+s\,\frac{\partial n}{\partial k}=-\left(\frac{C'(s\,)s\;+\left[r+q\,l\,(n+1)s\,\right]k}{C''(s\,)C'(h)q\,l\,s^{\,2}}\right)<0.$$

Welfare - The social planner takes into account the permanent effect of an innovation, regardless of its patentability. Given that the productivity and cost of R&D do not change over time, nor does the distribution of invention magnitudes, the planner has no incentive to choose a different R&D intensity or number of firms in different patent races. The welfare function is then

$$W=\underset{h,n}{Max}\left\{\frac{l\,nh\tilde{w}-nC(h)}{r+l\,nh}-nk\right\},\text{ where }\tilde{w}=\frac{p\tilde{u}(0)}{r}+\frac{l\,nh\tilde{w}-nC(h)}{r+l\,nh}.$$

The term $\tilde{w}$ is a continuation value that reflects the expected social welfare generated in all future rounds of R&D. If we recursively substitute for $\tilde{w}$ in the preceding expression, we find that

$$W=\underset{h,n}{Max}\left\{\sum_{t=1}^{\infty}\left(\frac{l\,nh}{r+l\,nh}\right)^{t}\frac{p\tilde{u}(0)}{r}-\sum_{t=1}^{\infty}\left(\frac{l\,nh}{r+l\,nh}\right)^{t-1}\frac{nC(h)}{r+l\,nh}-nk\right\},\text{ or}$$

$$W=\underset{h,n}{Max}\left\{\frac{n}{r}\left(l\,h\frac{p\tilde{u}(0)}{r}-C(h)-rk\right)\right\}.$$

Proposition 4 - In the stationary symmetric equilibrium, differences in the standard of patentability (the inventive step) do not affect the R&D intensity of individual firms, but they do affect the number of firms actively engaged in R&D and, therefore, the industrywide rate of innovation.

Proof: We begin by calculating the derivatives**:**

APPENDIX (continued)

$$\mathrm{M}_s^1 = \mathrm{M}_s^2 = l \left\{ \frac{\partial q}{\partial s} \left[C'(s)ns + rk \right] - \frac{\partial (q\tilde{u})}{\partial s} p \right\}.$$

Recall that $q = 1 - F(s)$ and $q\tilde{u} = \int_s^{\overline{u}} u dF(u),$ which implies that $\mathrm{M}_s^1 = \mathrm{M}_s^2 = -l\, f(s)\Psi(s),$ where

[A.7] $$\Psi(s) = \left\{ \left[C'(s)ns + rk \right] - ps \right\}.$$

The expression used in the text is obtained by substituting the first order and free entry conditions for $C'(s)$ and rk respectively.

The comparative static calculations are thus:

$$\frac{\partial s}{\partial s} = \frac{\mathrm{M}_s^2 \mathrm{M}_n^1 - \mathrm{M}_s^1 \mathrm{M}_n^2}{|\mathrm{M}|} = \frac{f(s)\left[\Psi(s) - \Psi(s) \right]}{C''(s)qs} = 0;$$

$$\frac{\partial n}{\partial s} = \frac{\mathrm{M}_s^1 \mathrm{M}_s^2 - \mathrm{M}_s^2 \mathrm{M}_s^1}{|\mathrm{M}|} = \frac{f(s)\Psi(s)}{C'(h)qs}.$$

The expression for $\Psi(s)$ used in the text is obtained by substituting for $C'(s)s$ using [A.5]."

Proposition 5 - There exists a unique standard (inventive step), denoted s^*, such that in the interval $[0, s^*)$, industrywide R&D activity increases as the standard is made more strict. In the interval $(s^*, \overline{u}]$, industrywide R&D activity decreases as the standard is made more strict.

Proof: First, using [A.7] we check the slope of $\Psi(s)$:

$$\partial\Psi(s)/\partial s = \frac{\partial s}{\partial s}\left[C''(s)ns + C'(s)s \right] + C'(s)s\, \frac{\partial n}{\partial s} - p = \frac{f(s)\Psi(s)}{q} - p.$$

Thus if there is a value $s^* \in [0, \overline{u}],$ where ? (s*) = 0, we know that $\partial\Psi(s)/\partial s\big|_{s^*} = -p$. Thus there can be at most one extremum of ? (s).

Next we check the values of ? (s) as $s \to \overline{u}$ and $s \to 0$. These are evaluated most easily if we recall that in equilibrium $C'(s) = q\, l \left[p\tilde{u} - rk \right] / \left[r + q\, l\, ns \right]$. Substituting for $C'(s)$ in [A.7], we evaluate

APPENDIX (continued)

$$\Psi(s) = \left(\frac{q\, l\, ns}{r + q\, l\, ns} \right) \left[p\tilde{u}(s) - rk \right] - \left[ps - rk \right].$$

$$\text{Lim}_{s \to \bar{u}} \Psi(s) = \left(\frac{0 \cdot l\, n(\bar{u}) s\,(\bar{u})}{r + 0 \cdot l\, n(\bar{u}) s\,(\bar{u})} \right) \left[p\bar{u} - rk \right] - \left[p\bar{u} - rk \right] < 0.$$

If we assume for the moment that the participation constraint is satisfied when $s = 0$ (i.e. $p\tilde{u}(0) - rk \geq 0$),

the second limit is:

$$\text{Lim}_{s \to 0} \Psi(s) = \left(\frac{l\, n(0) s\,(0)}{r + l\, n(0) s\,(0)} \right) \left[p\tilde{u}(0) - rk \right] + rk > 0.$$

But it is possible that, depending on the distribution of invention magnitudes and the output price, under a very weak patentability standard, the participation constraint [A.5] might be violated (i.e. $p\tilde{u}(0) - rk < 0$). In that case ? (s) does not exist at $s = 0$. Instead, define $\hat{s}$ s.t. $p\tilde{u}(\hat{s}) - rk = 0$. Then $\Psi(s)$ exists for $\forall\ s \in (\hat{s}, \bar{u}]$. We also know that $\Psi(s)$ is initially positive for values of s just greater than $\hat{s}$, because $-[p\hat{s} - rk] > 0$ implies that

$$\left(\frac{[1 - F(\hat{s})]\, l\, n(\hat{s}) s\,(\hat{s})}{r + [1 - F(\hat{s})]\, l\, n(\hat{s}) s\,(\hat{s})} \right) \left[p\tilde{u}(\hat{s}) - rk \right] - \left[p\hat{s} - rk \right] > 0.$$

Existence of the extremum then follows from continuity of ? (s) over $(\hat{s}, \bar{u}]$. "

Proposition 6 - The critical standard, s^*, is increasing in the equilibrium rate of innovation. This implies that in rapidly innovating industries, a smaller proportion of inventions can be protected without causing the rate of innovation to decline.

Proof: The critical standard is defined by the equation ? (s) = 0. To examine how s^* varies with the industrywide rate of innovation, we compute comparative static derivatives with respect to the exogenous parameters explored in propositions 2 and 3:

[A.8] $$\frac{\partial s^*}{\partial z} = \frac{-\partial \Psi(s)}{\partial z} / \frac{\partial \Psi(s)}{\partial s} = \frac{\partial \Psi(s)}{\partial z} \frac{1}{p},$$

where z is either p, λ, r, or k. Note also that

APPENDIX (continued)

$$\frac{\partial \Psi(s)}{\partial z} = \left[C''(s)s + C'(s)\right]n\frac{\partial s}{\partial z} + C'(s)s\ \frac{\partial n}{\partial z} - \frac{\partial\left[ps - rk\right]}{\partial z}.$$

i. Higher output prices:

$$\frac{\partial s^*}{\partial p} = \frac{C'(s)s\,\tilde{u}(s^*)}{C'(s)s} - s^* = \tilde{u}(s^*) - s^* > 0.$$

ii. More productive R&D:

$$\frac{\partial s^*}{\partial l} = \frac{C'(s)s\,r}{qs\,l^2} - 0 = \frac{rC'(s)}{q\,l^2} > 0.$$

iii. A higher discount rate:

$$\frac{\partial s^*}{\partial r} = \frac{\left[C''(s)s + C'(s)\right]nk}{C''(s)s} - \frac{C''(s)\left[C'(s)s + \left[r + ql(n+1)s\right]k\right] + C'(s)ql\,nk}{C''(s)ql\,s} + k$$

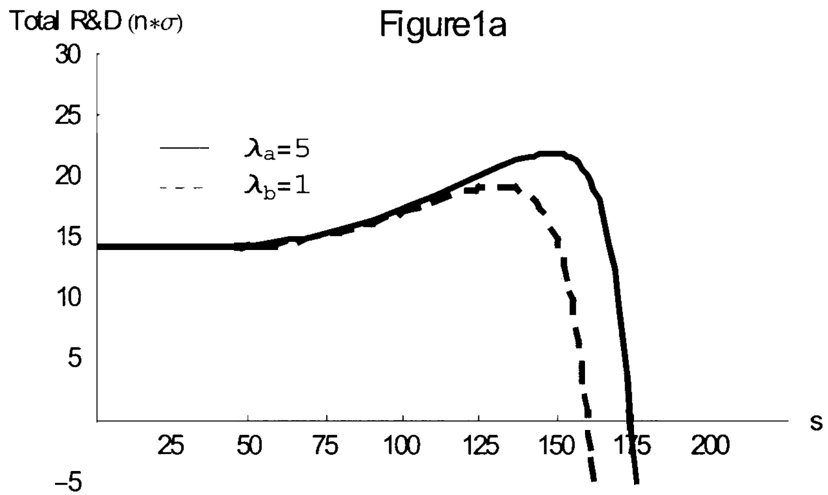
$$\frac{\partial s^*}{\partial r} = k - \frac{C''(s)\left[C'(s)s + \left[r + qls\right]k\right]}{C''(s)ql\,s} = -\left(\frac{C'(s)s + rk}{ql\,s}\right) < 0.$$

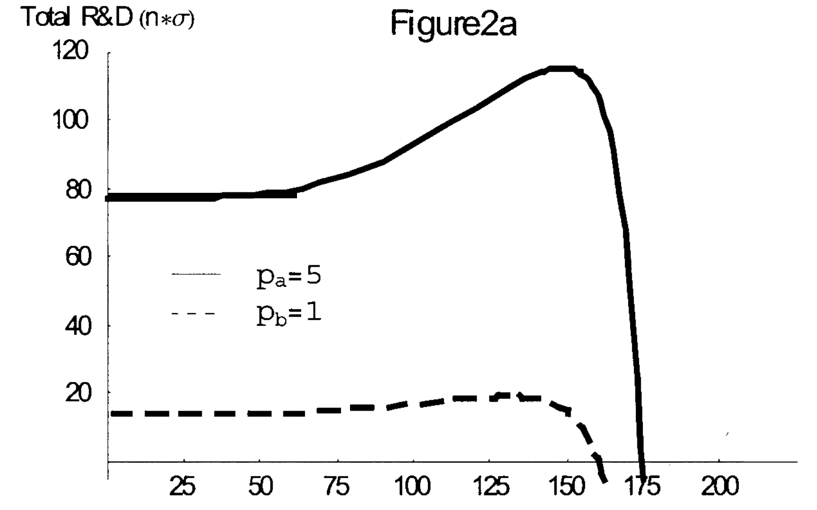
iv. Higher fixed R&D costs:

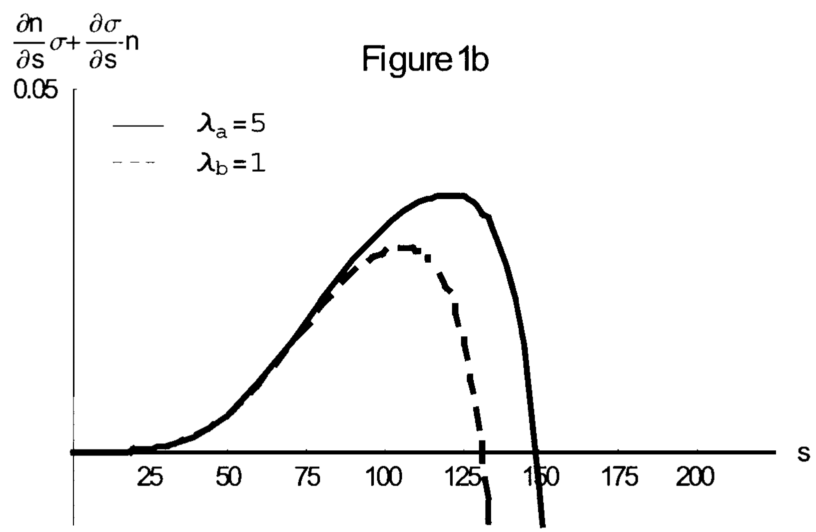
$$\frac{\partial s^*}{\partial k} = \frac{\left[C''(s)s + C'(s)\right]nr}{C''(s)s} - \frac{r\left\{C''(s)\left[r + ql(n+1)s\right] + C'(s)ql\,n\right\}}{C''(s)ql\,s} + r$$

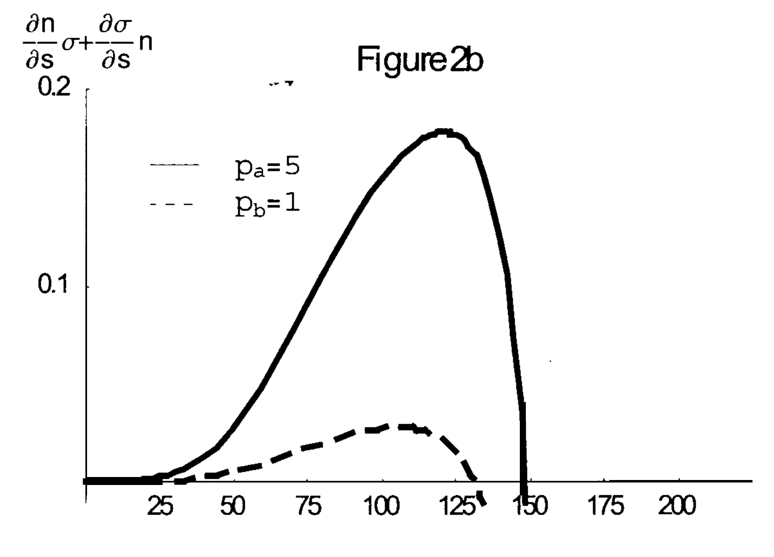
$$= r - \frac{rC''(s)\left[r + qls\right]}{C''(s)ql\,s} = \frac{-r^2}{ql\,s} < 0.$$

Illustrations of Comparative Statics*



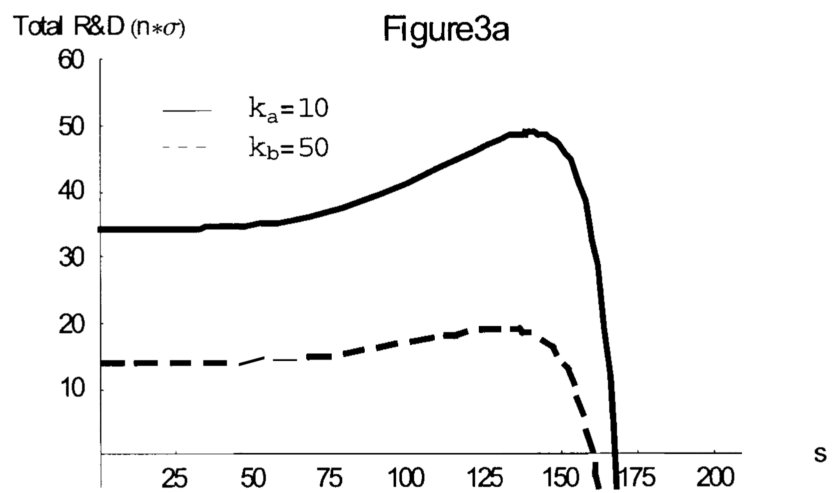


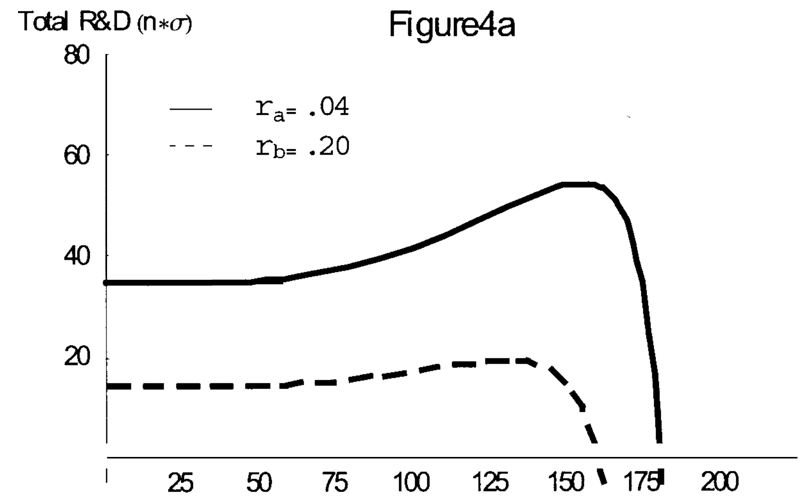


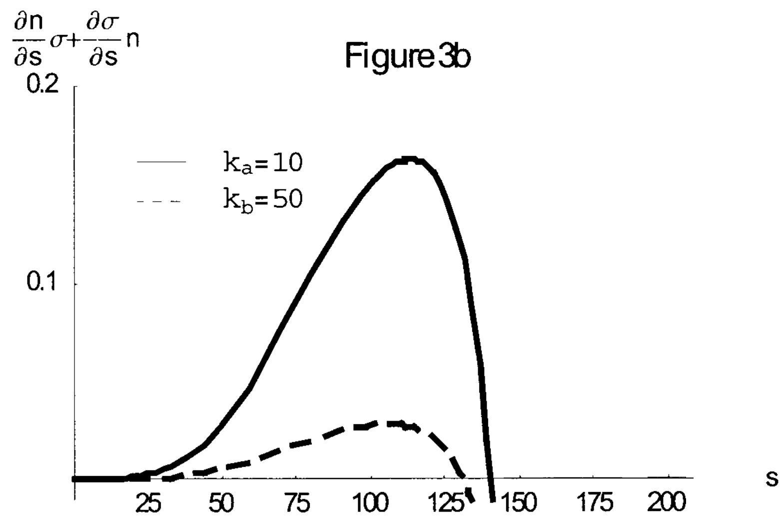


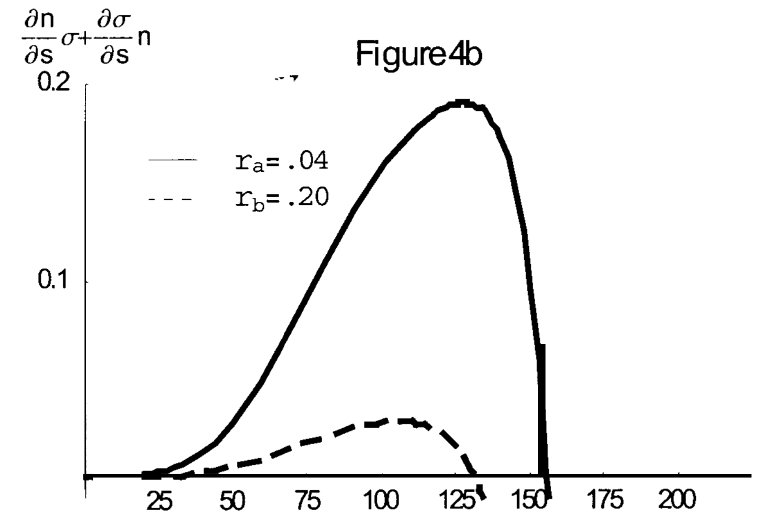
*: The figures were generated in Mathematica. Costs are assumed to be quadratic and inventions are drawn from a normal distribution with mean 100 and standard deviation of 25. Except where noted in the figures, the base parameterization is the following: $p = 1$, $\lambda = 1$, $r = 0.2$, and $k = 50$.

Illustrations of Comparative Statics*









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