Competitive Equilibria with Asymmetric Information*

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Abstract

This paper studies competitive equilibria in economies where agents trade in markets for standardized, non-exclusive financial contracts, under conditions of asymmetric information (both of the moral hazard and the adverse selection type). The problems for the existence of competitive equilibria in this framework are identified, and shown to be essentially the same under different forms of asymmetric information. We then show that a 'minimal' form of non-linearity of prices (a bid-ask spread, requiring only the possibility to separate buyers and sellers), and the condition that the aggregate return on the individual positions in each contract can be perfectly hedged in the existing markets, ensure the existence of competitive

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equilibria both in the case of adverse selection and moral hazard.

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1. Introduction

This paper studies competitive equilibria in economic environments characterized by the presence of asymmetric information; both situations of moral hazard and adverse selection are allowed for.

Agents are assumed to trade in spot markets and markets for financial contracts. These contracts are (i) standardized and (ii) non-exclusive, in the sense that the terms of each contract (its price and its payoff) are, respectively, common across many relationships involving different agents, and independent of the level of transactions made by an agent in other markets. Under these conditions the partners of a contractual relationship have a very limited power over the terms of the contract. Also, at an equilibrium each agent will typically trade in different markets, and enter different contracts at the same time. We argue that in such a situation a general equilibrium approach is useful to analyze the interaction among trades in different markets; also that we may analyze contracts, as well as commodities, as traded in competitive markets. Our analysis builds on the earlier work by Dubey, Geanakoplos and Shubik (1995), who explicitly addressed this issue in a model where asymmetric information is generated by the possibility of default.

Hence we depart from the analysis of exclusive contractual relationships, where an agent can only choose one out of a menu of contracts, or equivalently the terms of a contract depend on the position of the agent in all markets. The implementation of these contracts imposes a very strong informational requirement as all the transactions of an agent need to be observed. The non-exclusivity of contracts matches then the observation that, for instance, agents often hold various insurance policies, and get loans both from banks and from credit card companies[1]. Also, the terms of contracts very rarely depend on the agents' transactions in other markets[2]. Moreover, well-defined markets operate where standardized

[1] Petersen and Rajan (1994) provide some evidence on the composition of credit sources for small businesses in the U.S.

[2] See Smith and Warner (1979) for an analysis of the terms of debt contracts in the U.S..

contracts are traded: markets for credit and insurance contracts, or mortgages, are important examples, as well as markets for any financial security which allows for a default clause.

The main objective of this paper is to analyze the conditions for the 'viability' of markets for contracts in the presence of asymmetric information. In this context, allowing for non-exclusive contracts has important implications for the 'viability' of markets. For standardized contracts, as the ones considered in this paper, the payoff typically depends on the characteristics of the agents trading the contract (as, for instance, in insurance contracts against an agent's individual risk), so that the same contract, when entered by different agents is effectively a different contract. However, when the agents' characteristics are only privately observed these different contracts cannot be separated and are traded together in a single market.

If the quantities traded by an agent in all markets are a fully revealing signal of the agent's type, exclusive contracts, where the terms of a contract depend on the trades of the agent in the various markets, allow to solve this problem and to clear markets. This was shown by Prescott and Townsend (1984a) to be possible, under general conditions, when asymmetric information is of the moral hazard type, though not in the case of adverse selection. In adverse selection economies in fact quantities traded may not fully reveal the agents' type, agents' types cannot be separated, so that what are effectively different contracts are traded in a single market at the same price. This obviously generates a problem of ensuring feasibility in such markets: there are not 'enough prices' to clear these markets. Moreover, as we already argued, the possibility to implement exclusive contracts is a very demanding requirement. When this does not hold, the feasibility problem we identified above is relevant and arises also for moral hazard economies.

We show that two prices (a bid and an ask price) for each contract are enough to guarantee the existence of a competitive equilibrium in economies where trade takes place under asymmetric information. The result requires the additional condition that the aggregate return on the individual positions in a given contract can be perfectly hedged on the existing markets (i.e. that markets are complete in some sense), or is also marketed as a distinct claim (a 'pool' security[3]). The importance of the role of 'pool' securities for the existence of competitive equilibria in economies with asymmetric information was first stressed by Dubey, Geanakoplos

[3] The securitization which is frequently observed of aggregates of standardized contracts (as in the case of mortgage-backed securities, securitized loans, ...) can be viewed as an instance of the creation of 'pool' securities.

and Shubik (1995).

Under this condition the simple ability to differentiate prices for buying and selling positions is sufficient for the existence of competitive equilibria. We should stress that this is the case both under moral hazard and adverse selection, and no matter what is the 'dimension' of the sources of asymmetric information in the economy (i.e. of the cardinality of the set of unobservable possible types or actions of the agents trading the contract). This form of non-linearity of prices (i.e. of dependence of the price of a contract on the quantity traded of that contract) is 'minimal' in the sense that it is only at one point and, more importantly, can be implemented by observing only the level (in fact the sign) of trades in each particular transaction, without the knowledge of the other transactions of the agent in the same contract. But it is also 'minimal' in the sense that without it, i.e. if prices of contracts are linear over the whole domain, competitive equilibria may fail to exist in economies with asymmetric information. Evidently, our result also implies the existence of competitive equilibria for the case in which stronger forms of non-linearity of prices can be implemented (i.e. when all trades in the same market can be monitored), as long as the price is non-linear at the level of zero trades.

In this paper we clearly show that, at the root of the difficulties for the 'viability' of markets in economies with asymmetric information, is the fact that whenever agents' types cannot be separated (e.g. via exclusivity clauses), what are effectively different contracts are restricted to trade at the same price. As a consequence, our results apply, at a more general (and abstract) level, to economies where several different goods are restricted to trade at the same price. Such a situation may also arise in various other circumstances: for instance electricity prices are often restricted not to vary at different times of the day, and many commodity prices are 'sticky' over time (because, e.g., of menu costs; see Akerlof and Yellen (1985))[4].

The Economy. The analysis is developed in the framework of a two-period, pure exchange economy. A large economy is considered, with finitely many types of agents, and infinitely many agents of each type. All agents of the same type are ex ante identical but will typically be different ex-post, after the realization of the uncertainty. The uncertainty has the following structure: there is an aggregate shock, which affects all agents in the economy, and idiosyncratic shocks each of which affects only finitely many agents (which we can metaphorically say consti-

[4]See Balasko (1997) for a related analysis.

tute a 'village'). Idiosyncratic shocks are un-correlated among them (i.e. across 'villages'). Agents have access to a set of spot markets, where the commodities are traded, and to contingent contracts, whose payoff depends on the realization of the aggregate and the specific idiosyncratic shock affecting them. These are standardized contracts (or securities)[5], in the sense that their specification (their payoff structure) is common across agents. All contracts of the same type are ex-ante identical and sell at the same price, though the payoff of each of them depends on the realization of a specific idiosyncratic risk.

Aggregate shocks are commonly observed. On the other hand agents may have some private information over the realization of their own idiosyncratic shock. When this happens, markets for contingent contracts will be characterized by the presence of asymmetric information. Different information structures will be considered, which generate different types of situations with asymmetric information, and are contrasted with the case of symmetric information.

All markets are competitive. In this economy agents are small in terms of the level of their trades but not informationally small, and hence we can have competitive markets where traders retain some private information (over the payoff of the contract). To this effect, the consideration of a large economy, and the structure of the uncertainty (in particular the fact that private information concerns 'small' risks, the idiosyncratic component of the uncertainty) play an important role. On the other hand, many other aspects of the specification, as the two period horizon, or the pure exchange nature of the economy are not essential.

Results. While in the case of symmetric information competitive equilibria always exist under standard assumptions, this is no longer true when there is asymmetric information. We present a robust example of an economy with adverse selection, where no competitive equilibrium exist, when contracts trade at linear prices.

The identification of the 'problems' for existence of competitive equilibria with linear prices in economies with asymmetric information is one of the main contributions of this paper. There are two main kinds of 'problems' for existence. First, private information over the payoff of the contracts being traded, in particular over its support, gives the agents additional arbitrage opportunities (with respect to the symmetric information case), so that the set of no-arbitrage prices may be empty. Secondly, with asymmetric information the level of agents' trades depends on their private information over the payoff of the contracts; this implies that the

[5]In the paper we will use interchangeably the terms contract and security.

total return on the positions held by agents in a given contract is no longer a linear function of the total quantity traded. As a consequence, market clearing in financial markets may not ensure feasibility of transactions in contracts (as there are not enough prices to clear both commodities' and securities' markets). Consider, e.g. the case in which 'bad risk' wish to buy and 'good risk' wish to sell the same insurance contract by the same amount: demand will equal supply but the total payment to agents trading this contract will be positive.

We also show that not only these problems are common both to moral hazard and adverse selection economies but that in our framework adverse selection can be seen, at an abstract level, as a reduced form of moral hazard.

Additional conditions with respect to the case of symmetric information are then needed to ensure the existence of competitive equilibria: in particular some restrictions have to be imposed on trades (and/or on the payoff structure) in markets characterized by the presence of asymmetric information. We show that the existence of competitive equilibria is ensured, both for economies with adverse selection and with moral hazard, under the following set of conditions:

1. an upper bound on the level of trades in contracts for which there is asymmetric information is imposed (or, alternatively restrictions on the contracts' payoff are introduced, ensuring there are no unbounded arbitrage opportunities);

2. for each of these contracts a 'pool' security is also marketed , whose payoff is the average total net amount due to agents who traded the contract (or, alternatively, markets are complete with respect to aggregate risk, so that the total return on individual positions in each contract can be perfectly hedged);

3. A positive spread is allowed between bid and ask prices for contracts whose payoffs are contingent on private information (or, alternatively one-side constraints are imposed, which let agents be only on one side - and all on the same side - of the market for these contracts).

The crucial requirements characterizing these conditions are the separation of buyers and sellers (achieved via bid-ask spreads, whose level is endogenously determined at equilibrium, or one-side constraints), and the presence of 'pool' securities. The above conditions are tight, in the sense that they cannot be relaxed without generating problems for existence. However, one can also find other sets of conditions which guarantee existence of competitive equilibria, without involving separation of buyers and sellers (e.g. the introduction of entry fees); we intend to look more closely at these issues in another paper.

The structure of the paper is as follows. Section 2 describes the class of economies we consider. In section 3 the case of symmetric information is examined; this provides a natural benchmark for the rest of the analysis. In section 4 asymmetric information is introduced and the problems for the existence of competitive equilibria are identified and are shown to be essentially the same for economies with different kinds of asymmetric information. Section 5 provides a robust example of non-existence of competitive equilibrium for a simple adverse selection economy. In section 6 the existence of competitive equilibria is established.

Related Literature. Our analysis was significantly inspired by the work of Dubey, Geanakoplos and Shubik (1995) (also Geanakoplos, Zame and Dubey (1996)). These authors study the competitive equilibria of economies where standardized securities are traded and agents have the choice to default on their contractual obligations; thus a situation of asymmetric information originates and is captured by the possibility of default. The fact that both in their and our set-up agents have some control over the payoff of the securities they trade generates important similarities in the analysis. Moreover, we also use Dubey, Geanakoplos and Shubik (1995)'s construction of 'pool' securities to aggregate the payoff of securities traded by agents under asymmetric information.[6] In the model considered by Dubey et al. the possibility of default is the only source of an agent's ability to affect the payoff of the contracts he trades; since agents may only default on their short positions, a one-side constraint naturally originates in such framework. Consistently with our existence result then, no feasibility problem arises in this model and existence can be proved under standard conditions.

Extending this work, Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (1998) show that competitive equilibria for several types of asymmetric information economies (including for instance insurance economies with adverse selection and/or moral hazard, Akerlof's 'lemons' economies, economies with default, monitoring, tournaments, and others) share a common structure and can be analyzed within the set-up of a common abstract model.

[6] This construction is also used by Minelli and Polemarchakis (1996) in an analysis of Akerlof's model of the used car market.

The pioneering work of Prescott and Townsend (1984a,b) constitutes an important reference for any analysis of economies with asymmetric information in the framework of general equilibrium, competitive models. These authors consider an economy where competitive markets for all state-contingent commodities are open at the beginning of time and agents' consumption plans are subject to the additional restriction that they have to be incentive compatible. Prescott and Townsend show that, for economies with moral hazard, competitive equilibria always exist, and are constrained efficient, while the extension of their approach to economies with adverse selection is problematic.[7] It is easy to see that the contractual relationships which generate the equilibria considered in their analysis satisfy a very strong exclusivity condition (indirectly induced by the fact that contingent consumption plans are restricted by an overall incentive compatibility constraint).[8]

Helpman and Laffont (1975) (see also (Laffont (1976)) present an example of an economy with moral hazard where, with linear prices, no competitive equilibrium exists. As we argue more in detail in another paper (Bisin and Gottardi (1997)), the lack of existence in that example can be imputed to the same kinds of problems as the ones discussed here.

Competitive equilibria for adverse selection economies in a general equilibrium framework are also studied by Gale (1992), (1996). Though the structure of markets, and in particular the role of prices and the specification of the market clearing conditions are rather different from the ones considered here, it is important to notice that Gale looks at economies where agents can be ex-ante partitioned into buyers and sellers (which again introduces what is effectively a form of one-side constraint).

The importance and the consequences of asymmetric information in large economies are examined by Gul and Postlewaite (1992), Postlewaite and McLean (1996). They study a class of economies with adverse selection for which they show that, as the economy becomes large, the consequences of the presence of asymmetric information tend to disappear (the set of constrained efficient allocations tends to coincide with the set of fully Pareto efficient allocations). This is not the case in our set-up: as we already argued, even though the economy is large, agents still retain some exclusive information over the payoff of the securities they

[7] See however, Hammond (1989).

[8] See Lisboa (1996), Bennardo (1997), Citanna and Villanacci (1997) for existence results under an explicit exclusivity condition when, in addition, re-trading in future spot markets is allowed.

trade.

The efficiency of competitive equilibria for economies with moral hazard and exclusive contracts, after the decentralization result obtained by Prescott and Townsend, has been recently examined by Lisboa (1996), Bennardo (1997), Citanna and Villanacci (1997), Magill and Quinzii (1997). On the other hand, the consequences for efficiency of abandoning exclusivity have been discussed (again only for the moral hazard case, and for simple economies with a single market and ex-ante identical agents) by Hellwig (1983), Arnott and Stiglitz (1993), Bisin and Guaitoli (1996), Kahn and Mookerjee (1997). The efficiency properties of competitive equilibria in our framework will be analyzed in another paper.

Finally the literature on General Equilibrium models with Incomplete Markets should also be mentioned.[9] This has developed a general framework which extends the methodology of the Arrow-Debreu model to the analysis of competitive equilibria under uncertainty (with symmetric information), when agents are not able to fully insure against all sources of risk. In such framework the set of markets in which agents are allowed to trade (in particular of contingent markets) is taken as exogenously given. We show in this paper that the presence of asymmetric information generates some restrictions on the set of the agents' insurance opportunities arising endogenously from the agents' incentive compatibility constraints and the conditions for the viability of markets.[10]

2. The Structure of The Economy

We consider a two-period pure exchange economy. There are L commodities, labelled by $l \in L = \{1, ..., L\}$, available for consumption both at date 0 and at date 1; commodity 1 is the designated numeraire in every spot. The agents in the economy are of finitely many types, indexed by $h \in H = \{1, ..., H\}$, and there are countably many agents of each type. An agent is then identified by a pair (h, n), where $n \in N$ (N is the set of natural numbers); λ^h denotes the fraction of the total population made of agents of type h.

Uncertainty. Uncertainty is described by the collection of random variables $\tilde{\sigma}$, $(\tilde{s}^{h,n})_{h \in H, n \in N}$, with support, respectively Σ and $(S^h)_{h \in H}$ (the same for all n).

[9] See Geanakoplos (1990), and Magill and Shafer (1991) for surveys of this literature.

[10] See Duffie and Rahi (1995) for a survey of other approaches to endogenizing market incompleteness.

Both Σ and S^h are assumed to be finite sets, $\Sigma = \{1, ..., \Sigma\}$ and $S^h = \{1, ..., S^h\}$, with generic element σ and s^h.

The random variables $\tilde{s}^n \equiv (\tilde{s}^{h,n})_{h\in H}$ describe the idiosyncratic uncertainty affecting the (H) agents of index n, while $\tilde{\sigma}$ represents the economy's aggregate shock. We assume that the variables $(\tilde{s}^n)_{n\in N}$ are, conditionally on σ, identically and independently distributed across n. Let π denote the common probability distribution of $(\tilde{\sigma}, \tilde{s}^{h,n})$, and $\pi(s/\sigma) \equiv \pi((\tilde{s}^{1,n} = s^1, .., \tilde{s}^{H,n} = s^H)/\sigma)$. We have so:

Assumption 1

- $\pi(\tilde{s}^{h,n} = s^h) = \pi(\tilde{s}^{h,n'} = s^h)\ \forall n, n' \in N, s^h \in S^h$.
- $\pi(\tilde{s}^{h,n} = s^h, \tilde{s}^{h',n'} = s^{h'} \mid \sigma) = \pi(\tilde{s}^{h,n} = s^h \mid \sigma)\pi(\tilde{s}^{h',n'} = s^{h'} \mid \sigma)\ \forall n \neq n' \in N; h, h' \in H; s^h, s^{h'} \in S^h$.

On the other hand we allow $\tilde{s}^{h,n}$ to be correlated, conditionally on σ, with $\tilde{s}^{h',n}$, for $h' \neq h$. We also allow for the correlation of $\tilde{s}^n$ with $\tilde{\sigma}$.[11]

A metaphor may be useful to clarify the structure of the uncertainty: we may think of n as indexing different villages, while h indexes different professional types inside each village. The idiosyncratic shocks affecting the different professional types inside each village may be correlated, and may also be correlated with the aggregate shocks affecting the whole economy. On the other hand, idiosyncratic shocks are independent across villages, conditionally on the aggregate shocks.

Endowments. We will consider, with no loss of generality, the case in which uncertainty enters the economy via the level of the agents' date 1 endowment. Each agent $(h, n) \in H \times N$ has an endowment w_0^h at date 0, and his date 1 endowment, $w_1^h(\tilde{\sigma}, \tilde{s}^n)$, depends upon the realization of $\tilde{\sigma}$ and $\tilde{s}^n$. Let $S \equiv \Pi_{h\in H} S^h$ and $s \equiv (s^h)_{h\in H}$. We assume:

Assumption 2

$w_0^h \in \Re^L_{++}, w_1^h \equiv (w_1^h(\sigma, s); \sigma \in \Sigma, s \in S) \in \Re^{L(\Sigma S)}_{++}$.

[11] In the following analysis the variable $\tilde{s}^{h,n}$ will in turn describe the component of the idiosyncratic shock $\tilde{s}^n$ which is only affecting agent (h, n) or a signal he receives over the realization of his idiosyncratic uncertainty. The presence of aggregate uncertainty, $\tilde{\sigma}$, not only allows for greater generality but also will play an important role in the case of one of the uncertainty structures we consider; see Remark 5 below.

Preferences. A consumption plan for an arbitrary agent (h, n) specifies the level of consumption of the L commodities at date 0 and 1 in every state. The consumption set is the non-negative orthant of the Euclidean space. Agents are assumed to have Von Neumann - Morgenstern preferences over consumption plans. The utility index of agent (h, n) is given by a function $u^h : \Re_+^{2L} \to R$ satisfying:

Assumption 3
u^h is continuous, strictly increasing and strictly concave.

Information Structure. Throughout the analysis we will maintain the assumption that the aggregate shock $\tilde{\sigma}$ is realized at date 1, its distribution and realization are known to all agents. On the other hand, different cases will be considered with respect to the information agents have over their idiosyncratic shocks.

In our framework *asymmetric information economies* are characterized by the fact that either the realization of the idiosyncratic shock component $\tilde{s}^{h,n}$ or its distribution are private information of agent (h, n). In particular, we will examine the case of *adverse selection* economies, where the agents have private information, at the beginning of date 0, over the realization of their idiosyncratic shock, and of *hidden information* economies, where it is the realization of the shock, at date 1, to be private information. We intend to argue that the latter have essentially the same properties as economies with the more standard form of moral hazard, with hidden action, and allow, at the same time, to maintain the source of the informational asymmetry in information over an exogenous state component[12].

3. Symmetric Information Economy

It is convenient to introduce the structure of markets and the nature of the market clearing conditions in our framework by considering first the case of symmetric information.

In this section we suppose that all idiosyncratic shocks $(\tilde{s}^{h,n})_{h \in H, n \in N}$, as well as aggregate shocks $\tilde{\sigma}$, are realized at date 1, are commonly observed, and their distribution is known by all agents at date 0. Agents' information is then perfectly symmetric. We can have therefore at date 0 markets for securities (contracts) with

[12] Both in hidden information and moral hazard economies, as we will see in the next sections (also Bisin and Gottardi (1997)), agents have the possibility to affect the distribution of the payoff of existing securities.

payoff contingent on every relevant source of uncertainty.

Market Structure. Spot markets for the L commodities open both at date 0 and in every possible state at date 1.

At date 0 a set of markets for contingent securities also open. More precisely, for every pair $(\tilde{s}^n, \tilde{\sigma})$ there are J (linearly independent) securities: each security $j \in J$ pays $r_j(s, \sigma)$ units of numeraire if and only if the realization of $(\tilde{s}^n, \tilde{\sigma})$ equals (s, σ); and there is one of these securities for every n. These are standardized securities in the sense that ex-ante all securities of a given type j are identical, i.e. their payoff has the same distribution for all n; ex-post however, their payoff will be different, as it will vary with the specific realization of $(\tilde{s}^n)_{n \in N}$ across n. This is natural in insurance markets, where insurance policies are standardized, but payments depend on individual realization of shocks; similar considerations hold for standard credit contracts, mortgages,...

Altogether there are then countably many of these securities in the economy, but we will consider the case in which each agent (h, n) can only trade the J securities with payoff contingent on his idiosyncratic shock $\tilde{s}^n$ (as well as $\tilde{\sigma}$). We will show that this is not restrictive provided all agents can also trade J 'pool' securities. The payoff of 'pool' security j is defined so as to equal the opposite of the average net amount (of the numeraire commodity) due to - or owed by - all agents who traded securities of type j. Hence each 'pool' security can be viewed as a claim against the (net) position of all agents in 'individual' securities of a given type. By the Law of Large Numbers the payoff of 'pool' security j will only depend on σ and be given by:

$$r_j^p\ (\sigma) = -\frac{\sum_h \lambda^h \sum_s \pi(s/\sigma) r_j\ (s, \sigma) \theta_j^h}{\sum_h \lambda^h \theta_j^h},\ \sigma \in \Sigma$$

where θ_j^h denotes the amount of security j held by agents of type h (independent of n as we will show). This expression clearly simplifies to $r_j^p\ (\sigma) = -\sum_s \pi(s/\sigma) r_j\ (s, \sigma)$ (and we can take this as the obvious specification of $r_j^p(\sigma)$ also when $\sum_h \lambda^h \theta_j^h = 0$). All this has a very natural interpretation: the payoff of each 'pool' security is obtained from the payoff of the underlying security by averaging out the idiosyncratic component of its return (which could be diversified away by holding a position in infinitely many ex-ante identical securities).

Thus the role of 'pool' securities here is to summarize all agents could do by trading in the existing securities of index n different from their own.

Perfect Competition. Markets are perfectly competitive. By this we mean not only that agents act as price-takers, but also that all securities that are ex-ante identical (only differ for the value of n) sell at the same price. Securities offering insurance against the same kind of idiosyncratic shock in different villages are in fact equivalent to the eyes of the outside investors, and hence should sell at the same price. This embodies the idea that there is a unified, large, competitive market where all (standardized) securities of a given type are traded.[13]

Linear Prices. Prices in both financial and spot markets are independent from agents' observable characteristics (e.g. of their type h), and from the level of their trades. The unit price of security j is then denoted by q_j (by the above perfect competition assumption, independent of n); $q \equiv (q_j)_{j \in J}$. The (normalized) vector of spot prices of the L commodities at date 0 and at date 1 when the aggregate shock is σ, are denoted respectively by p_0 and $p_1(\sigma)$.

We also impose the condition that each 'pool' security j sells at the opposite price of the underlying 'individual' security; $-q_j$ is then the price of 'pool' security j. This can be viewed as a no arbitrage condition whenever agents are free to trade on securities with payoff contingent on other agents' (other 'villages') idiosyncratic shocks or, as we will argue later, as a zero profit condition if intermediaries are explicitly modelled.

In the present framework all agents of a given type h face the same choice problem, and this problem is convex. All these agents make then the same choice, so that this will be independent of n, and will be described by a portfolio respectively of 'individual' and 'pool' securities, $\theta^h = (.., \theta^h_j, ..) \in R^J$, $\theta^h_p = (.., \theta^h_{p,j}, ..) \in R^J$, and a consumption plan $c^h = (c^h_0; c^h_1 = c^h_1(s, \sigma), s \in S, \sigma \in \Sigma)) \in \Re^{L(1+S\Sigma)}_+$. The consumption plan specifies the level of consumption at date 0 and at date 1, for every possible realization of the aggregate uncertainty and the idiosyncratic uncertainty affecting the agent.

A *competitive equilibrium with symmetric information* is then a collection of prices $< p_0, (p_1(\sigma)_{\sigma \in \Sigma}), (q_j)_{j \in J} >$, consumption and portfolio plans for every agent's type $< (c^h, (\theta^h, \theta^h_p))_{h \in H} >$, and a specification of the payoff of 'pool' securities $[r^p_j(\sigma)]_{\sigma, j}$ such that:

[13] Since each village is populated by a finite number of agents, price taking would not be justified in an economy with securities' prices indexed by the name of the village.

- agents optimize: for all $h \in H$ the plan $(c^h, (\theta^h, \theta_p^h))$ solves the problem

$$\max_{[(c_0^h, c_1^h) \in \Re_+^{L(1+S\Sigma)}, (\theta^h, \theta_p^h) \in \Re^{2J}]} \sum_{s,\sigma} \pi(\sigma)\pi(s/\sigma)u^h(c_0^h, c_1^h(s,\sigma)) \qquad (P_{SI}^h)$$

s.t.

$$p_0 \cdot (c_0^h - w_0^h) + q \cdot (\theta^h - \theta_p^h) \leq 0$$

$$p_1(\sigma) \cdot (c_1^h(s,\sigma) - w_1^h(s,\sigma)) \leq \sum_j r_j(s,\sigma)\theta_j^h + r_j^p(\sigma)\theta_{j,p}^h, \ (s,\sigma) \in S \times \Sigma$$

- markets clear:

$$\sum_h \lambda^h (c_0^h - w_0^h) \leq 0 \qquad (3.1)$$

$$\sum_h \lambda^h \sum_s \pi(s/\sigma)(c_1^h(s,\sigma) - w_1^h(s,\sigma)) \leq 0, \sigma \in \Sigma \qquad (3.2)$$

$$\sum_h \lambda^h (\theta_j^h - \theta_{p,j}^h) = 0, \ j \in J \qquad (3.3)$$

- the payoff $r_j^p(\sigma)$ of each 'pool' security satisfies:

$$r_j^p(\sigma) = -\sum_s \pi(s/\sigma) r_j(s,\sigma), \ j \in J, \sigma \in \Sigma \qquad (3.4)$$

Under assumption 1, we have been able to exploit the Law of Large Numbers to write the feasibility condition for date 1 in (3.2) in terms of conditional expectations. The market clearing condition for securities (3.3) is then stated as the equality of the total position in 'individual' securities of a given type and the total position in the associated 'pool' security. It is easy to show, by using again the Law of Large Numbers and the above specification of the payoff of 'pool' securities that this ensures that the aggregate payoff of the portfolios held by agents equals 0, for all possible realizations of the uncertainty at date 1, i.e. ensures feasibility.

It is immediate to see that the set of securities' prices precluding arbitrage opportunities is a non-empty, open set. Moreover, both the agents' choice problem P_{SI}^h and the equilibrium problem are finite-dimensional and well-behaved problems. The following result then follows by an application of standard arguments:

Theorem 1. *Under assumptions 1-3, a competitive equilibrium with symmetric information exists, such that the price of every security $j \in J$ is 'fair', conditionally on σ :*

$$q_j = \sum_{\sigma \in \Sigma} \rho_\sigma \sum_{s \in S} \pi(s/\sigma) r_j(s, \sigma) = - \sum_{\sigma \in \Sigma} \rho_\sigma r_j^p(\sigma)$$

for some $\rho \equiv (.., \rho_\sigma, ..) \gg 0$.

Let R denote the $S\Sigma \times J$ payoff matrix, with generic element $r_j(s, \sigma)$, and $Sp[R]$ the linear space generated by the columns of R.. We also have:

Corollary 1. *If, in addition, $Sp[R] = \Re^{S\Sigma}$, competitive equilibria with symmetric information and fair prices are Pareto optimal and such that consumption allocations only depend on the aggregate shock σ (i.e. all idiosyncratic shocks are fully insured).*

When $Sp[R] = \Re^{S\Sigma}$ we can say therefore that markets are complete and that the above market structure allows to decentralize Pareto optimal allocations via securities with exogenously given payoff. Our result complements the results of Magill and Shafer (1992), Cass, Chichilniski and Wu (1996) where, building on the original analysis of Malinvaud (1972), Pareto optima are decentralized via a set of mutual insurance contracts. It also confirms the fact that the restriction we imposed on agents' behavior, by preventing them from trading in securities of different index n, is not binding.

Remark 1. *This specification of the equilibrium condition for securities has important implication for the pattern of contingent trades across agents. Condition (3.3) implies that trades among agents of different index n (across different villages) take place both by compensating positions in the same type of security in different villages (i.e. by aggregating their positions in this security) as well as by compensating them with positions in the associated 'pool' securities.*
Both elements are important as can be seen by the fact that both are required for the validity of the decentralization result in Corollary 1. Without 'pool' securities, market clearing would require long positions in each security j to be matched by short positions in the same security, and this is unduly restrictive. The role of 'pool' securities will be even more important in the analysis of economies with asymmetric information.

Remark 2. *Though the set of available securities is taken as given and financial intermediaries are not explicitly modelled, competitive intermediaries, who design* *and market these securities, could be introduced with no substantial change in the structure of the model or the definition of an equilibrium. In particular, the economies we study are equivalent to economies in which intermediaries take positions in individual securities, compensate them across 'villages', and issue, on that basis, 'pool' securities. Intermediaries maximize profits and act on the basis of competitive conjectures. The condition we imposed on the price of 'pool' securities together with the specification of the market clearing condition for securities imply then that a zero-profit condition holds, at equilibrium, for all intermediaries.*

This equivalence between the specification of the model with an exogenously given set of financial markets and the one with competitive, profit-maximizing intermediaries extends to all the following analysis of asymmetric information economies.

4. Asymmetric Information Economies

In this section asymmetric information is introduced: different information structures, leading to different types of economies with asymmetric information are presented. We show that in these economies the existence of competitive equilibria cannot be proved under the same set of assumptions as with symmetric information (i.e. assumptions 1-3 are no longer enough to ensure that competitive equilibria exist). The nature of the existence problems is identified, and is shown to be common to economies with various kinds of informational asymmetries. This will provide the basis for the determination of additional conditions under which general existence results will be proved in section 6.

4.1. Adverse Selection Economy

Consider the case in which:

- the idiosyncratic shocks $(\tilde{s}^{h,n})_{h\in H,n\in N}$ are realized at the beginning of date 0, but the realization of $\tilde{s}^{h,n}$ is privately observed by agent (h,n) and becomes commonly known only at date 1.

Let the structure of markets be the same as in the previous section. At date 0, markets for the L commodities and securities open. For every n there are J securities with payoff contingent on $((\tilde{s}^n),\tilde{\sigma})$;[14] in addition, there are J 'pool'

[14] Payoffs conditional on $\tilde{s}^n$ are made possible by the fact that, while private information at date 0, the idiosyncratic shocks $\tilde{s}^n$ are publicly observed in period $1, \forall n$.

securities. At date 1, after the realization of $((\tilde{s}^n)_{n\in N}, \tilde{\sigma})$ becomes known to all agents, securities liquidate their payoff and the commodities are again traded on spot markets. All markets are perfectly competitive. We begin by looking at the case in which prices are restricted to be linear, to examine the possible problems which may arise under this condition.

With the above information structure the economy will be characterized by the presence of adverse selection: at date 0 agents trade contingent securities having different information over their payoff. In particular each agent (h, n), before choosing the level of trade in 'individual' securities, knows the realization of $\tilde{s}^{h,n}$, which acts as a signal over the payoff of these securities.[15]

We will also impose the following condition on the structure of the uncertainty:

- the idiosyncratic shocks $(\tilde{s}^{h,n})_{h\in H, n\in N}$ are independent of $\tilde{\sigma}$: $\pi(s/\sigma) = \pi(s)$ $\forall s, \sigma$..

This condition says that the idiosyncratic uncertainty and the agents' signals are un-correlated with the aggregate uncertainty. It implies that no information is revealed at a competitive equilibrium.[16] In the present framework in fact, date 0 prices could reveal at most the aggregate component of the uncertainty (differently from Radner (1979)), since the economy is large and hence the private information of an agent over an idiosyncratic source of uncertainty will have a negligible impact on aggregate trades.

Remark 3. *In the economy we described agents are 'small' as far as the level of their trades is concerned, but retain some monopolistic power with regard to their information, i.e. they are not 'informationally small'. More precisely, their level of trade in security j is negligible with respect to the aggregate level of trade in securities of type j, and this is what is relevant for the equilibrium price q_j, thus justifying the price-taking behavior. On the other hand agents have some specific (and exclusive) information over the payoff of the security they are trading, and*

[15] The 'individual' securities, with payoff contingent on $(\tilde{s}^n, \tilde{\sigma})$ are traded, as in the case of symmetric information, by agents of index n to insure against their own idiosyncratic shock. However there is now an additional factor determining their trades: agents are also willing to trade these securities to 'speculate' on the basis of the private information they have over their payoff. On the other hand, the role of 'pool' securities from the agents' point of view is unchanged: they offer them the possibility to trade, indirectly, contracts contingent on the uncertainty affecting other 'villages'.

[16] This is just for clarity, and all our results also hold when we allow for correlation between aggregate and idiosyncratic uncertainty.

so the effects of the informational asymmetries remain even though the economy is large (unlike in the models considered by Gul and Postlewaite (1990), McLean and Postlewaite (1996)).

A formal description of the agents' problem and a definition of competitive equilibrium for the adverse selection economy is now presented.

Let q_j be the price of securities of type j (again, by the assumption of perfect competition, the same for all n), and $-q_j$ be the price of the associated 'pool' security; $q \equiv ((q_j)_{j \in J})$; p_0 and $p_1(\sigma)$ are commodity spot prices. We will still consider the case in which agent (h, n) is restricted to trade only the J securities contingent on his own idiosyncratic shock $\tilde{s}^n$ as well as the J 'pool' securities.[17] Given the assumed information structure the agent will choose the level of trades at date 0, in securities and consumption goods, after learning the realization s^h of $\tilde{s}^{h,n}$. His portfolio and consumption plans are then contingent on s^h. At the same time his date 1 consumption plan will specify now the level of consumption for every possible realization of the remaining uncertainty, i.e. for every possible value $s^{-h} \equiv ((s^{h'})_{h' \neq h})$ of the shocks affecting the other agents, and for every σ. See Figure 1.

We will see below that all agents of the same type face the same optimization problem, that the feasible set is convex, and their objective function is strictly concave; their optimal choice therefore will be, as in the case of symmetric information, identical for all n (and the index n can then be omitted here).

Let $S^{-h} \equiv \Pi_{h' \neq h} S^h$ and $\pi(s^{-h}/s^h) \equiv \pi((\tilde{s}^{h',n} = s^{h'})_{h' \neq h}/s^h)$. The consumption and portfolio plans of agents of type h are then described by the vectors $(\theta^h(s^h); \theta^h_p(s^h)) = (.., \theta^h_j(s^h), ..; .., \theta^h_{p,j}(s^h), ..) \in R^J \times R^J$, and $c^h(s^h) = (c^h_0(s^h);$ $c^h_1(s^h) = c^h_1(s^{-h}, \sigma; s^h), s^{-h} \in S^{-h}, \sigma \in \Sigma)) \in \Re_+^{L(1+S^{-h}\Sigma)}$, $s^h \in S^h$, and are obtained as solutions of the following program:

$$\max_{[(c^h(s^h)) \in \Re_+^{L(1+\Sigma S^{-h})}; (\theta^h(s^h), \theta^h_p(s^h)) \in \Re^{2J}]} \sum_{s^{-h}, \sigma} \pi(\sigma)\pi(s^{-h}/s^h) u^h(c^h(.; s^h)) \qquad (P^h_{AS})$$

s.t.

$$p_0 \cdot (c^h_0(s^h) - w^h_0) + q \cdot (\theta^h(s^h) - \theta^h_p(s^h)) \leq 0$$

[17] However, this may not be now the optimal choice of agents, at the above prices, if free to trade in all existing securities' markets. In the presence of asymmetric information 'pool' securities are not only, as with symmetric information, a synthesis of what agents can do by trading in other villages but also play, as we will argue more extensively later, an important role in ensuring feasibility. Hence the trading restriction we imposed here could be dropped, but at the cost of greater complications in the description of the structure of trades.

$$p_1(\sigma) \cdot (c_1^h(s^{-h}, \sigma; s^h) - w_1^h(s, \sigma)) \leq \sum_j \theta_j^h(s^h) r_j\ (s, \sigma) + \sum_j \theta_{p,j}^h(s^h) r_j^p\ (\sigma);$$
$$(s, \sigma) \in S \times \Sigma$$

The unit payoff of 'pool' security $j \in J$ is again defined by the opposite of the average total net amount (of the numeraire commodity) due to - or owed by - all agents who traded securities of type j, for all n; this is when the average is well defined, and it takes an arbitrary value otherwise:

$$r_j^p\ (\sigma) = \left\{ \begin{array}{l} -\frac{\sum_h \lambda^h \sum_s \pi(s) r_j\ (s,\sigma)\theta_j^h(s^h)}{\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h)},\ \text{if } \sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h) \neq 0 \\ \textit{arbitrary},\ \text{if } \sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h) = 0 \end{array} \right\},\ \sigma \in \Sigma \tag{4.1}$$

Let R^p be the $\Sigma \times J$ matrix with generic element $r_j^p(\sigma)$.

A *competitive equilibrium with adverse selection* is defined by a specification of the 'pool' securities' payoff R^p, a collection of prices $(p_0, (p_1(\sigma))_{\sigma \in \Sigma}), q)$, and of contingent consumption and portfolio plans for every agents' type $(c^h(s^h), \theta^h(s^h), \theta_p^h(s^h);$ $s^h \in S^h)_{h \in H}$ such that:

- for all h, the plan $(c^h(s^h), \theta^h(s^h), \theta_p^h(s^h); s^h \in S^h)$ solves (P_{AS}^h) at the prices $(p_0, (p_1(\sigma))_{\sigma \in \Sigma}), q)$ and 'pool' securities' payoff R^p;
- commodity markets clear:

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)(c_0^h(s^h) - w_0^h) \leq 0 \tag{4.2}$$

$$\sum_h \lambda^h \sum_s \pi(s) \left((c_1^h(s^{-h}, \sigma; s^h) - w_1^h(s, \sigma))\right) \leq 0,\ \sigma \in \Sigma \tag{4.3}$$

- security markets clear: for all $j \in J$:

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h) \left(\theta_j^h(s^h) - \theta_{p,j}^h(s^h)\right) = 0 \tag{4.4}$$

- the payoff $r_j^p(\sigma)$ of each 'pool' security j satisfies (4.1), for all σ.

4.1.1. Why Existence is a Problem with Adverse Selection

The presence of adverse selection, specifically the fact that each agent (h, n) trades securities (of index n) by having some private information over its payoff, poses two main problems for the analysis of this economy with respect to the case of symmetric information considered in section 3.

1. *Arbitrage.* Agents have additional arbitrage opportunities.
   With symmetric information the set of securities' prices precluding arbitrage is always non-empty and open. On the other hand, when agents have private information over the support of the payoff of securities this set may well be empty.
   More precisely, the set:

   $$K(s^h) \equiv \{q \in R^J : \exists \rho \in \Re_{++}^{S^{-h}\Sigma}, q = \sum_{s^{-h},\sigma} \rho_{s^{-h},\sigma} r(s^h, s^{-h}, \sigma)\}$$

   denotes the set of prices of the J individual securities precluding arbitrage opportunities to agents of type h when they observed state s^h. Therefore, for no agent to have any arbitrage opportunity we need:

   $$\underset{h \in H, s^h \in S^h}{\cap} K(s^h) \neq \emptyset \tag{NA}$$

   The greater the set of securities with payoff contingent on $(\tilde{s}^{h,n})_{h \in H, n \in N}$, i.e. the larger the insurance offered against the states over which some agents have private information, the less likely is that condition (NA) will be satisfied. In particular it will always be empty if R has full rank, so that un-restricted trade in a complete set of markets is not feasible in the present situation.
   The nature of the problem can be clearly seen by considering the following extreme case. Agents receive different signals over the future realization of the idiosyncratic uncertainty, so it may happen that agent (h, n) knows that some shock realization s is not possible, while some other agent (h', n) gives it positive probability. Suppose there is a security paying one unit in state s and 0 in all other states. No-arbitrage for agent (h', n) requires that this security sells at a positive price, while no-arbitrage for agent (h, n) requires that the security's price is 0. Hence the no-arbitrage set is empty in this case.

2. *Feasibility.* Market clearing for the aggregate positions on 'individual' and the associated 'pool' securities (as in condition (4.4)) is no longer enough to ensure feasibility of trades in securities.
   The problem is that now security holdings, unlike in the case of symmetric information, are not the same for all agents of the same type as the portfolio choice of each agent (h, n) depends on the observed realization s^h of $\tilde{s}^{h,n}$ and the payoff of the securities purchased also depends on this realization. As a consequence, condition (4.4), which is the direct analogue of the market clearing condition (3.3) for the symmetric information case) does not ensure that the aggregate payoff on securities is 0 (which is evidently required for feasibility):

$$\sum_h \lambda^h \sum_s \pi(s)(r_j\ (s,\sigma)\theta_j^h(s^h) + r_j^p(\sigma)\theta_{p,j}^h(s^h)) = 0 \tag{4.5}$$

   To see this, suppose (4.4) holds and, furthermore, $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h) = \sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_{p,j}^h(s^h) = 0$ (i.e. 'individual' and 'pool' securities' markets clear separately).
   Then, while $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_{p,j}^h(s^h) = 0$ implies $\sum_h \lambda^h \sum_s \pi(s) r_j^p(\sigma)\theta_{p,j}^h(s^h) = 0$, $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h) = 0$ does not imply $\sum_h \lambda^h \sum_s \pi(s) r_j(s,\sigma)\theta_j^h(s^h) = 0$, since the term $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h)$ cannot be factored out of this sum (when $\theta_j^h(s^h)$ depends non trivially on s^h, i.e. when adverse selection matters).[18]
   Again the nature of the problem can be clearly seen for an extreme case. Suppose signal s implies that the return on buying the security (say, a credit contract) will be high, while s' on the contrary implies that the return will be low. Then it may happen that agents who received signal s will buy this credit contract, while agents who received s' will sell. In this case, even if the aggregate position on this type of contract is 0, still in period 1 the agents who bought credit (the 'bad' risks) cannot be paid out of the proceeds from agents who sold it (the 'good' risks), so that feasibility is not satisfied.

Remark 4. *At a more general level we can view the feasibility problem as arising from the fact that each of the various contracts of the same type is now a different*

[18] Even though the average payoff defining the payoff of the 'pool' security is not defined when $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta_j^h(s^h) = 0$, the statement in the text is true no matter what is the specification of the payoff of the 'pool' in this case.

object not only ex-post (as the realization of the payoff depends on the village) but also ex-ante, as the level of trades by an agent depends on the specific realization of signal received over its payoff. On the other hand, with linear prices, only one price exists to clear the market for all these contracts.

4.2. Hidden Information Economy

Suppose:

- the idiosyncratic shocks $(\tilde{s}^{h,n})_{h\in H,n\in N}$ are realized at date 1 (and may now be correlated with $\tilde{\sigma}$);
- the realization of $\tilde{s}^{h,n}$ is privately observed by agent (h,n) - before the realization of $\tilde{\sigma}$ is commonly observed - and never becomes known to the other agents, for all h,n.

To ensure that agents are able to observe their own endowment we will also assume here that:

- the endowment of agent (h,n) depends upon the realization of $\tilde{\sigma}$ and $\tilde{s}^{h,n}$ only.

Under these conditions contracts with payoff directly contingent on $\tilde{s}^{h,n}$ can no longer be written, as $\tilde{s}^{h,n}$ is private information and never publicly observable. Thus agents will only be able to get some insurance against their idiosyncratic shocks as long as this is compatible with their incentives. We will model this by considering securities whose payoff is contingent on what the agents say about the state, on the messages they send after learning the realization of their idiosyncratic shock.

More precisely, let M^h be the space of messages which an agent of type h can send. We will assume that M^h is finite; let M^h also be its cardinality and denote by m^h its generic element (we can have, for instance, $M^h = S^h$, i.e. each agent simply announces one of the possible states he has privately observed). For every n there are so J securities whose payoff depends on the realization of $\tilde{\sigma}$, commonly observed, and on the messages sent by agents of index n over the realization of $(\tilde{s}^{h,n})_{h\in H}$. One unit of security j, $j = 1,..,J$ pays $r_j(m,\sigma)$ units of the numeraire commodity at date 1 when state σ is realized, and $m \equiv (m^1,...,m^H) \in M = \Pi_{h\in H} M^h$ is the collection of messages sent by the H agents of index n. In addition, there are again J 'pool' securities, associated with each type of 'individual' security, and the $\Sigma \times J$ matrix R^p, with generic element $r_j^p(\sigma)$, describes their payoffs.

Except for this important difference, the structure of markets is unchanged. Spot markets for the commodities' and securities' markets open at date 0, before any realization of the uncertainty. At date 1, after the agents sent their message and the realization of $\tilde{\sigma}$ has been commonly observed, the payoff of contracts is determined and liquidated. Spot markets then open where the commodities are traded. Markets are competitive. As in the case of adverse selection we examine the case in which prices are restricted to be linear; in particular, q_j is again the unit price of securities of type j (and $-q_j$ the price of the associated 'pool' security).

In this economy 'individual' securities are traded by agents to attain some insurance against their idiosyncratic shocks but also to exploit the possibility agents have, as a result of their private information, to affect their payoff. The market for these securities is then characterized by the presence of hidden information[19].

Remark 5. *The non-triviality of the choice of the message sent by agents (and hence the fact that indeed some amount of insurance against the agents' privately observed states can be achieved) is ensured by the following features of the information structure:*
(a) the correlation of $(\tilde{s}^{h,n})_{h\in H}$ *across* h;
(b) the correlation of $\tilde{s}^{h,n}$ *with* $\tilde{\sigma}$.
In the presence of (a) the message agent (h, n) *will choose to send will not typically be a constant message (the same for all* s^h*) if the securities' payoff depends jointly on the messages sent by all agents with the same index* n *(i.e. by agents whose private information is correlated). The same is true under (b) if securities' payoffs depend on the commonly observed state* σ *as well as on the agents' messages and if these are sent by agents before learning the realization of* $\tilde{\sigma}$.
The fact that correlation may induce some discipline on the agents' opportunistic behavior and hence enhance incentives is well-known in the moral hazard literature: (a) can be viewed as an abstract representation of a situation of relative performance evaluation (see e.g. Lazear and Rosen (1981)), while an application

[19]The strong similarity with the case of moral hazard, where agents can affect the securities' payoff distribution via some unobservable action, should be now more evident. The crucial distinction between adverse selection economies on one side, and hidden information as well as moral hazard economies on the other, lies in the fact that in the first case the informational asymmetry arises before the contract is signed, while in the latter agents have no private information when their trades in securities are decided, this only arises at a later date (see also Hart and Holmstrom (1987)).

of the idea behind (b) can be found in Townsend (1982). As it will appear more clear later, in the present framework, where the contracts traded are standardized contracts and strong exclusivity conditions may not be (are not) enforceable, incentive compatibility becomes trivial if neither (a) nor (b) hold (equivalently, the agents' optimal message will be constant if securities' payoff only depends on $\tilde{s}^{h,n}$*).*

Let us describe now formally the agents' choice problem and define competitive equilibria for economies with hidden information.

In the present framework the optimization problem of each agent is a non-convex problem. The agent's choice set is clearly not convex since M^h is a discrete set. But even if the agent were allowed to randomize in his choice of which message to send for every realization s^h, his problem would still be not convex (as in that case the objective function is not concave). The fact that agents can choose both the unit payoff (via their message) and the quantity traded of each 'individual' security generates in fact an inherent non-convexity in their choice problem. As a consequence, in this economy ex-ante identical agents may end up choosing, at equilibrium, different portfolio levels and different messages, so that the index n can no longer be omitted here.[20]

Each agent (h,n) faces here the following optimization problem. He has to choose (i) his date 0 consumption level $c_0^{h,n} \in \Re_+^L$ and portfolio holdings $(\theta^{h,n}, \theta_p^{h,n}) \in \Re^{2J}$; (ii) the message plan $m^{h,n} \equiv (m^{h,n}(s^h)_{s^h \in S^S}) \in (M^h)^{S^h}$, specifying the message to send at the beginning of date 1, for every possible realization of $\tilde{s}^{h,n}$; (iii) his date 1 consumption plan $c_1^{h,n} = (c_1^{h,n}(\sigma, m^{-h}, s^h), \sigma \in \Sigma, m^{-h} \in M^{-h}, s^h \in S^h) \in \Re_+^{L\Sigma M^{-h} S^h}$, specifying the level of consumption for every possible realization s^h of his idiosyncratic uncertainty, σ of the aggregate shock, and every possible collection of messages $m^{-h} \equiv ((m^{h'})_{h' \neq h})$ sent by agents of other types.

The joint dependence of security payoffs on σ and the message m sent by all agents with the same index n is important to have non trivial message choices, as we argued in Remark 5; for the same reason we assume that agents' messages have to be sent before the realization of $\tilde{\sigma}$ is commonly observed and that messages are commonly observed.

The timing of an agent's choices is illustrated in Figure 1.

[20] The presence of non-convexities has also been often associated with agents' problems with moral hazard (see Hart and Holmstrom (1987); also Prescott and Townsend (1984a)).

Formally agent (h, n) has then to solve the following program:[21]

$$\max_{[c^{h,n} \in \Re_+^{L(1+\Sigma M^{-h} S^h)}, (\theta^{h,n}, \theta_p^{h,n}) \in \Re^{2J}, m^{h,n} \in (M^h)^{S^h}]} \sum_{\sigma,m,s} \pi(\sigma, m^{-h}, s^h) u^h(c_0^{h,n}, c_1^{h,n}(\sigma, m^{-h}, s^h)) \qquad (P_{HI}^h)$$

subject to:

$$p_0 \cdot (c_0^{h,n} - w_0^h) + q \cdot (\theta^{h,n} - \theta_p^{h,n}) \leq 0$$

$$p_1(\sigma) \cdot (c_1^{h,n}(\sigma, m^{-h}, s^h) - w_1^h(\sigma, s^h)) \leq \sum_j \theta_j^{h,n} r_j(m^{h,n}(s^h), m^{-h}, \sigma) + \sum_j \theta_{p,j}^{h,n} r_j^p(\sigma);$$
$$\sigma \in \Sigma, m^{-h} \in M^{-h}, s^h \in S^h$$

where $\pi(\sigma, m^{-h}, s^h) \equiv \sum_{s^{-h}} \pi(s, \sigma)\pi(m^{-h}/s^{-h})$, and $\pi(m^{-h}/s^{-h})$ denotes the probability distribution[22] describing the 'average' strategy of agents of type $h' \neq h$ over the message they will send for every possible realization of their idiosyncratic state.

Remark 6. *Since the payoff of the 'individual' securities traded by agent (h, n) may depend on the message sent by agents of different type, but same index n, a strategic component is introduced in the agent's problem. As a consequence the equilibrium notion, as we will see below, will have a Nash component, arising from the agents' message game. The fact then that the 'average' strategy of agents of other type is considered follows from the anonymity property of this game (agents do not know the precise identity of the agents characterized by their same index n).*
On the other hand, here as well as in the previous sections the payoff of 'pool' securities is also endogenously determined by agents' portfolio choices, but is taken as given by agents as the effects of each of them on the payoff of the 'pool' is negligible.

We will show that the economy can be 'convexified' by exploiting the large number of agents. This requires that ex-ante identical agents behave differently

[21] By requiring $c^{h,n} \in \Re_+^{L(1+\Sigma M^{-h} S^h)}$ we are implicitly imposing the condition that the level of consumption of agent (h, n) has to lie in the consumption set for every possible message of agents of type $h' \neq h$, even though some of these messages may be given zero probability by $\pi(m^{-h}/s^{-h})$. This may appear unduly restrictive, in particular when markets are incomplete, and is mainly motivated by reasons of technical convenience.

[22] Even though each individual agent chooses to send - as in problem P_{HI}^h - a single message as a function of the state, we have to allow as we said for the possibility that identical agents make different choices at equilibrium, so that the 'average' strategy of agents of a given type may be described by a non-degenerate probability distribution over messages.

at equilibrium: we will consider the case in which agents of the same type will make at most an arbitrarily large but finite[23] number V of different choices at equilibrium, denoting by $c^{h,\nu}, \theta^{h,\nu}, \theta_p^{h,\nu}, m^{h,\nu}$ the ν-th choice of agents of type h and by $\gamma^{h,\nu}$ the fraction of agents of this type making such choice, $\nu = 1,..,V$.

A *competitive equilibrium with hidden information* is defined by a specification of the payoff of 'pool' securities R^p, an array of prices $(p_0, (p_1(\sigma))_{\sigma\in\Sigma}, q)$, a collection of consumption, portfolio, and message plans for agents of type h with their relative frequency $(c^{h,\nu}, \theta^{h,\nu}, \theta_p^{h,\nu}, m^{h,\nu}; \gamma^{h,\nu})_{\nu\in V}$, for all h, $((\pi(m^{-h}/s^{-h}))_{s^{-h}\in S^{-h}})_{h\in H}$, such that:

- for every h all plans $(c^{h,\nu}, \theta^{h,\nu}, \theta_p^{h,\nu}, m^{h,\nu})_{\nu\in V}$ are solutions of (P_{HI}^h) at the prices $(p_0, (p_1(\sigma))_{\sigma\in\Sigma}, q)$, 'pool' securities ' payoff R^p, and other agents' 'average' strategy $(\pi(m^{-h}/s^{-h}))_{s^{-h}\in S^{-h}}$;

- for all h $(\pi(m^{-h}/s^{-h}))_{s^{-h}\in S^{-h}}$ is consistent with the strategies chosen by individual agents of type $h' \neq h$:

$$\pi(m^{-h}/s^{-h}) = \Pi_{h'\neq h}\left(\sum_{\nu:m^{h',\nu}(s^{h'})=m^{h'}} \gamma^{h',\nu}\right) \tag{4.6}$$

- commodity markets clear:

$$\sum_h \lambda^h \left(\sum_\nu \gamma^{h,\nu} c_0^{h,\nu} - w_0^h\right) \leq 0 \tag{4.7}$$

$$\sum_h \lambda^h \sum_s \pi(s/\sigma)\left(\sum_{\nu,m^{-h}} \gamma^{h,\nu}\pi(m^{-h}/s^{-h})c_1^{h,\nu}(\sigma, m^{-h}, s^h) - w_1^h(s^h,\sigma)\right) \leq 0,\ \forall\sigma \tag{4.8}$$

- security markets clear: for all $j \in J$,

$$\sum_h \lambda^h \sum_\nu \gamma^{h,\nu}\left(\theta_j^{h,\nu} - \theta_{p,j}^{h,\nu}\right) = 0 \tag{4.9}$$

[23]Under this condition the Law of Large Numbers can still be exploited in the feasibility conditions. We will show that equilibria satisfying such condition always exist. However, there may also be other equilibria which violate it.

- the payoff of each 'pool' security $j \in J$ is given by:

$$r_j^p(\sigma) = \left\{ \begin{array}{l} -\frac{\sum_h \lambda^h \sum_s \pi(s/\sigma)\left(\sum_{\nu,m^{-h}} \pi(m^{-h}/s^{-h})\gamma^{h,\nu} r_j\left(m^{h,\nu}(s^h), m^{-h}, \sigma\right)\theta_j^{h,\nu}\right)}{\sum_h \lambda^h \sum_\nu \gamma^{h,\nu}\theta_j^{h,\nu}}, \\ \text{if } \sum_h \lambda^h \sum_\nu \gamma^{h,\nu}\theta_j^{h,\nu} \neq 0 \\ \\ arbitrary, \text{ if } \sum_h \lambda^h \sum_\nu \gamma^{h,\nu}\theta_j^{h,\nu} = 0 \end{array} \right\}, \ \sigma \in \Sigma \tag{4.10}$$

- $\gamma^{h,\nu} \geq 0, \sum_\nu \gamma^{h,\nu} = 1$[24]

Condition (4.6) ensures the consistency of what agent (h,n) considers in (P_{HI}^h) to be the ('average') strategy over messages of agents of type $h' \neq h$ and the actual message plans chosen by these agents, i.e. ensures that the equilibrium in the message game characterized by the above equilibrium notion is a Nash equilibrium. The payoff of 'pool' securities is then endogenously determined at equilibrium, in condition (4.10), and is set equal, as in the models of the previous sections, to the opposite of the average total payoff to agents holding positions in each 'individual' security.

4.2.1. Why Existence is a Problem with Hidden Information

The main problems posed by the presence of hidden information for the viability of markets for contingent contracts are the same as the ones we found under adverse selection:

1. *Arbitrage.* The fact that the agents can affect the payoff of the securities via their message (can choose, to some extent, the support of securities' payoffs) gives them additional arbitrage opportunities.
   More precisely, the set of prices of the J 'individual' securities precluding arbitrage opportunities to agents of type h is given by:

$$K^h \equiv \{q \in R^J : \exists \rho \in \Re_{++}^{M^{-h}\Sigma}, q = \sum_{m^{-h},\sigma} \rho_{m^{-h},\sigma} r(m^h, m^{-h}, \sigma) \ \forall m^h\}$$

[24] We allow $\gamma^h = (\gamma^{h,v})_{v \in V}$ to be any real vector in the simplex Δ^{V-1} even though, with countably many agents we could limit our attention to rational numbers. Since rational numbers are dense in the reals, the equilibrium we obtain is, strictly speaking, an approximate equilibrium. To overcome this problem we could have considered, without any change in the nature of the results, the case of a continuum of agents, as in Aumann (1966), and made appeal to the results by Al-Najjar (1995) on the Law of Large Numbers in such framework.

Therefore, for no agent to have any arbitrage opportunity we need:

$$\bigcap_{h\in H} K^h \neq \emptyset \tag{NA'}$$

It is easy to see that, as for adverse selection economies, there is a trade-off between the variability of the securities' payoff with respect to agents' messages, needed to ensure larger insurance opportunities, and the absence of arbitrage opportunities.

2. *Feasibility.* The non-convexity in the agents' choice problem implies, as we saw, that ex-ante identical agents may end up choosing, at equilibrium, different portfolio levels and hence different messages. As a consequence the securities' payoff will vary non-linearly with the quantity traded by agents. We face so again the problem that the fact that the market clearing condition for securities is satisfied does not imply that the aggregate payoff will also be 0.

   Formally, if equation (4.9) holds and, in addition

   $$\sum_h \lambda^h \sum_\nu \gamma^{h,\nu}\theta_j^{h,\nu} = 0,$$

   it does not necessarily follow that

   $$\sum_h \lambda^h \sum_s \pi(s/\sigma)\left(\sum_{\nu,m^{-h}} \pi(m^{-h}/s^{-h})\gamma^{h,\nu} r_j\ (m^{h,\nu}(s^h), m^{-h}, \sigma)\theta_j^{h,\nu}\right) = 0, \forall\sigma$$

   and hence the total payoff may not be 0 (and this no matter what is in this case the payoff level of the 'pool' security). We see in fact from the above expression that the message, and hence the payoff of 'individual' securities depends on the portfolio chosen, so that again security holdings and payoffs cannot be separated.

   The argument parallels exactly the one of the previous section for adverse selection economies. The intuition is also essentially the same: we can have agents who bought a security and send a message implying a high payoff, while agents who sold the same security send a message inducing a low payoff, so that on the whole total payoff is not 0, and feasibility is not satisfied.

On the other hand, no specific, additional problems are caused by the non-convexity of the agents' choice problem. In economies with hidden information non-convexities are then a source of difficulties for existence, but only in the sense that they induce the same correlation of portfolios and returns which was at the root of the problems we have identified for adverse selection economies. We will show in the next section that existence for economies with hidden information can be established then under essentially the same conditions as for adverse selection economies.

Remark 7. *At a more abstract level we can view the main consequence of the presence of asymmetric information in markets for contingent contracts as the (endogenously determined) correlation of portfolios and returns, i.e. the fact that the actual return on a contract of a given type is not constant throughout the economy and will typically vary with the quantity traded. This feature is indeed common both to adverse selection and hidden information economies (as well as moral hazard economies) and is the source of the existence problems we discussed. In particular, while with adverse selection only the probability structure of portfolios is endogenously determined (i.e.* θ *as a function of* s*), and returns are exogenously given, with hidden information both the probability structure of portfolios and returns (via the message choice) are endogenously chosen. This explains the sense in which adverse selection economies can be viewed in our framework as a reduced form of hidden information economies.*

5. A Non-Existence Example

In the previous section we identified two classes of problems for the existence of competitive equilibria, concerning the presence of arbitrage opportunities and feasibility of trades in contracts. We present here a robust example of an adverse selection economy of the kind described above, for which no competitive equilibrium exists, because of the feasibility problem[25].

Consider an economy with countably many agents of only one type ($H = 1$) and one commodity ($L = 1$); consumption only takes place at date 1. There is no aggregate uncertainty ($\Sigma = 1$). The idiosyncratic shocks have two possible realizations, $1, 2$, and each agent receives one out of two equiprobable signals at

[25] It is then easy to find examples where equilibria do not exist because of the presence of unbounded arbitrage opportunities arising from the agents' private information.

date 0: g or b. Let $\pi_g \equiv \pi(1/g)$ and $\pi_b \equiv \pi(1/b)$ be the probability of (idiosyncratic) state 1 conditional respectively on signal g and signal b.[26] We assume that $w(1) > w(2)$ and $\pi_g > \pi_b$; hence agents who receive signal g are the 'good risks' (i.e. have a higher probability of the good realization of their future income) and agents with signal b the 'bad risks'.

Agents has Von Neumann-Morgernstern preferences over consumption of the following form: $ln(c)$.

After learning his signal but before knowing the realization of his idiosyncratic uncertainty, each agent can trade two securities, $1, 2$. Security 1 pays one unit of the commodity when the agent's idiosyncratic state is 1. Similarly security 2 pays one unit of the commodity in idiosyncratic state 2.[27] Let q and $1 - q$ denote the (normalized) prices of, respectively, security 1 and 2.

The budget constraint of an agent who received signal g is then:

$$\theta_1(g)q + \theta_2(g)(1 - q) = 0.$$

Similarly for agents with signal b.

The agents' utility maximization problem subject to the above constraint can be easily solved in this case and yields an explicit expression of the demand for consumption in the two idiosyncratic states (respectively for agents receiving signal g and b):

$$\begin{aligned} c(1; g) &= \pi_g \left(\tfrac{qw(1)+(1-q)w(2)}{q}\right) \\ c(2; g) &= (1 - \pi_g) \left(\tfrac{qw(1)+(1-q)w(2)}{1-q}\right) \\ c(1; b) &= \pi_b \left(\tfrac{qw(1)+(1-q)w(2)}{q}\right) \\ c(2; b) &= (1 - \pi_b) \left(\tfrac{qw(1)+(1-q)w(2)}{1-q}\right) \end{aligned} \tag{5.1}$$

The market-clearing condition (or equivalently the overall zero-profit condition for the two contracts) is:

$$c(1; g)\pi_g + c(1; b)\pi_b + (1 - \pi_g)c(2; g) + (1 - \pi_b)c(2; b) \tag{5.2}$$

[26]For the simplicity of the exposition we depart here slightly from the formalization of idiosyncratic uncertainty and the structure of agents' signals described in the previous section.

[27]'Pool' securities need not be explicitly modelled in this set-up since their payoff, in the absence of aggregate uncertainty, will be deterministic and agents are always able, by trading the two 'individual' securities, to generate deterministic payoffs (or to perfectly hedge the returns of 'pool' securities; see also the next section). On the other hand, 'pool' securities also play an implicit role in generating a form of the equilibrium conditions (as in (**??**) below), where only market clearing in commodities appears.

$$= \quad w(1)\pi_g + w(1)\pi_b + (1-\pi_g)w(2) + (1-\pi_b)w(2)$$

For this economy the set of no-arbitrage prices is non-empty, and is given by all prices $q \in (0,1)$. We will show that, nonetheless, for an open set of parameters, a competitive equilibrium for the economy described above does not exist.

The excess demand function (equivalently the overall profit function) we obtain from (5.1) is continuous, for all $q \in (0,1)$. However, when $\frac{w(2)\pi_g}{w(1)(1-\pi_g)} > \frac{\pi_b}{(1-\pi_b)}$ this function has a negative value both when $\frac{q}{1-q} > \frac{\pi_g}{(1-\pi_g)}$ and when $\frac{q}{1-q} < \frac{w(2)\pi_b}{w(1)(1-\pi_b)}$; it is easy to see in fact, from the expressions of the agents' demand, that in the first case agents will be buying insurance, no matter what is the signal received, and will do this at more than fair terms, while the reverse happens in the second case, so that profits will be negative in both situations. For intermediate values of the relative price ($\frac{w(2)\pi_b}{w(1)(1-\pi_b)} < \frac{q}{1-q} < \frac{\pi_g}{(1-\pi_g)}$) the sign of aggregate demand cannot be unambiguously determined without further information on the parameter values of the economy. This already suggests that, for some parameter values, aggregate excess demand (profits) may be negative for all prices. Notice that this fact is a clear manifestation of the feasibility problem we discussed in the previous section.

We will now show that we can find an open set of parameter values (in the region $\frac{w(2)\pi_g}{w(1)(1-\pi_g)} > \frac{\pi_b}{(1-\pi_b)}$) for which the above indeed happens and no equilibrium exists.

Let $w(1) = 0.8, w(2) = 0.2, \pi_b = .2$, and $\pi_g = 0.2 + \epsilon$, $\epsilon > 0$.. Solving the equations (5.1) and (5.2) for equilibrium prices and allocations we find:

$$\begin{aligned}
&c(1;g) = -.0\,4\frac{-550\epsilon^2-48-355\epsilon+125\epsilon^3+480\epsilon\rho+128\rho+500\rho\epsilon^3-700\rho\epsilon^2}{2+10\epsilon+25\epsilon^2} \\
&c(2;g) = .\,64\rho + .\,16 - .\,8\epsilon\rho - .\,2\epsilon \\
&c(1;b) = -.0\,4\frac{-115\epsilon+25\epsilon^2-48+128\rho-160\epsilon\rho+100\rho\epsilon^2}{2+10\epsilon+25\epsilon^2} \\
&c(2;b) = .\,64\rho + .\,16 \\
&\tfrac{q}{1-q} = \rho
\end{aligned}$$

where ρ takes one of the two following values:

$$\frac{\left(40+75\epsilon-125\epsilon^2+\sqrt{(576+2160\epsilon-11575\epsilon^2-6750\epsilon^3+5625\epsilon^4)}\right)}{2\,(128-160\epsilon+100\epsilon^2)}$$

$$\frac{\left(40+75\epsilon-125\epsilon^2-\sqrt{(576+2160\epsilon-11575\epsilon^2-6750\epsilon^3+5625\epsilon^4)}\right)}{2\,(128-160\epsilon+100\epsilon^2)}$$

Straightforward computations reveal that, when $\epsilon > 0.3$, no real-valued solution exist for equilibrium prices and allocations (as the argument of the square root appearing in the above expression determining ρ, $(576 + 2160\epsilon - 11575\epsilon^2 - 6750\epsilon^3 + 5625\epsilon^4)$), has in that case a negative value).

Notice that, with the above parameter values the condition $\frac{w(2)\pi_g}{w(1)(1-\pi_g)} > \frac{\pi_b}{(1-\pi_b)}$ reduces to $\epsilon > 0.3$; hence an equilibrium never exists in this region. It is then immediate to see that perturbing the values of the parameters does not allow to restore existence, so equilibria fail to exist for an open set of parameter values.

On the other hand, when $\frac{w(2)\pi_g}{w(1)(1-\pi_g)} < \frac{\pi_b}{(1-\pi_b)}$ we find that at the prices $\frac{q}{1-q} = \frac{\pi_b}{(1-\pi_b)}$ overall profits are positive (agents receiving signal b buy insurance at fair terms, while agents with signal g also buy insurance but at less than fair terms). By the continuity of the profit function we conclude that an equilibrium always exist in this region.

In particular, for the above parameter specification as we already saw two admissible equilibrium solutions exist when $\epsilon < 0.3$. Moreover, it can be easily seen that the two competitive equilibria we obtain in this region are always Pareto ranked.

To better understand the properties of the competitive equilibria we obtain and more generally of the equilibrium structure of the economy, consider the solutions we get when $\epsilon = 0$ (in this case the signal received by the agents is totally uninformative, information is then symmetric):

$$\begin{aligned} (i)\ c(1;g) &= c(2;g) = c(1;b) = c(2;b) = .32;\ \frac{q}{1-q} = .25 \\ (ii)\ c(1;g) &= .8; c(2;g) = .2; c(1;b) = .8; c(2;b) = .2;\ \frac{q}{1-q} = .0625 \end{aligned}$$

The equilibrium in (i) is characterized by the presence of full insurance at fair prices (and is, evidently, Pareto efficient), while equilibrium (ii) has a zero level of trades for all agents.

The system of equilibrium equations (5.1) and (5.2) is clearly regular at every solution. Hence the two competitive equilibria we found with adverse selection, i.e. in the region $0.3 > \epsilon > 0$, arise by continuity (as ϵ is varied away from 0) from the two solutions obtained for the economy with symmetric information, the Pareto efficient and the no trade equilibrium.

6. Existence Results

The previous example shows that, with respect to the case in which information is symmetric, additional conditions are needed in economies with asymmetric information to overcome the problems discussed in section 4 and guarantee the existence of competitive equilibria. In particular, some restrictions have to be imposed on the agents' trades, or on the structure of payoffs, or equivalently some form of non-linearity in prices must be introduced in markets characterized by the presence of hidden information or adverse selection. This, as well as the fact that securities' payoffs are partly determined by the agents' actions (thus reflecting their incentive compatibility constraints), implies that asymmetric information generates an endogenous limit on the set of insurance possibilities which can be attained via competitive markets.

In this section we focus our attention on 'minimal' forms of non-linearity of prices of contracts (in the sense that they impose a minimal observability requirement) which are sufficient to guarantee existence of competitive equilibria in the class of economies studied; see Remark 8 below.

At this level of generality the imposition of bounds on trades (in 'individual securities') is a fairly obvious solution to the problem of unlimited arbitrage opportunities arising from the agents' private information.[28]

More precisely, the following restrictions will be imposed:[29]

$$\begin{array}{c} (i)\ \theta^h \in \Theta^h, \text{ a compact and convex subset of } \Re^J, 0 \in \Theta^h\ \forall h \\ (ii)\ Sp[(\sum_s \xi_s r_j(s,\sigma))_{\sigma,j}] = \Re^\Sigma\ \forall (..,\xi_s,..) \in \Delta^{S-1} \end{array} \qquad (C1)$$

where $(\sum_s \xi_s r_j(s,\sigma))_{\sigma,j}$ is the matrix with generic element $(\sum_s \xi_s r_j(s,\sigma))$ and Δ^{S-1} is the $(S-1)-$ dimensional simplex.

Condition $(C1(i))$ requires that agents' admissible trades in all 'individual' securities are bounded both above and below; on the other hand, no restriction is imposed on the agents' trades in 'pool' securities. Evidently, if this condition holds no agent can have unbounded arbitrage opportunities arising from his private information. Recalling that the price of each 'pool' security equals the opposite of the price of the underlying security, by a standard argument we obtain that

[28] Alternatively, we could find restrictions on the payoffs of existing securities, ensuring that condition (NA) $((NA'))$ is satisfied, that the set of no arbitrage prices is non-empty. The validity of all our results extends to such case. The difference among these two kinds of trading restrictions (on trades vs. payoffs) may, however, become important when we examine the properties of equilibrium allocations (e.g. when we address issues of security design or efficiency with asymmetric information).

[29] The condition stated here applies to adverse selection economies. In the case of economies with hidden information, the only difference is that in $(C1(ii))$ s should be replaced with m.

the non-empty open set

$$Q(R^p) = \{q \in \Re^J : \exists \rho \in \Re^{\Sigma}_{++},\ q = -(R^p)'\rho\} \tag{6.1}$$

characterizes the set of prices for which there are no arbitrage opportunities.

Condition $(C1(ii))$ is needed to ensure the continuity of the budget set in the present framework when, as in $(C1(i))$, trades in 'pool' securities are unrestricted.[30] It says that the projection of $Sp[R]$ on the $\Sigma-$dimensional space of payoffs contingent only on σ has full rank.[31] It implies, when one-side constraints are imposed (see $(C2)$ below), that whatever the level of agents' trades in 'individual' securities, we always have $Sp[R^p] = \Re^{\Sigma}$. Hence the fact that the payoff of 'pool' securities is endogenously determined at equilibrium (and that there are no bounds on trades in 'pool' securities) does not generate any discontinuity in the agents' budget set. When $(C1(ii))$ holds, agents are then able to attain all possible payoffs contingent on σ by trading in 'pool' securities (over which, under $C1(i)$ there is no restriction), so that markets are always complete with respect to the aggregate uncertainty in the economy.

We show in the Appendix that under this condition the agents' choice problem has always a solution and this is well-behaved, both with adverse selection and hidden information.

To overcome the 'feasibility' problem we will impose the condition that agents are constrained to take only long positions in 'individual' securities (e.g. that they can only buy, not sell short, insurance contracts):

$$\theta^h \in \Theta^h \subset \Re^J_+,\ \forall h \tag{C2}$$

It is immediate to see that under $(C2)$ the market clearing condition for securities ensures feasibility: in the adverse selection economy, if $(C2)$ holds

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)\theta^h_j(s^h) = 0$$

[30] The continuity of the agents' budget set could in fact be ensured also by imposing, in alternative to $(C1(ii))$, the following condition: $\theta^h_p \in \Theta^h_p \subset R^J$, compact, convex and such that $0 \in \Theta^h_p$, i.e. by imposing bounds on trades in all securities.

[31] This condition is obviously satisfied if there exists a subset of 'individual' securities with payoff only contingent on σ, spanning the whole aggregate uncertainty (in this case, as we already mentioned in the Introduction, the explicit presence of 'pool' securities is no longer needed as their payoff can be perfectly hedged in existing markets).

implies

$$\sum_h \lambda^h \sum_s \pi(s) r_j\ (s,\sigma)\theta_j^h(s^h) = 0$$

thus ensuring feasibility also when the total position in' 'individual' securities is 0 (the same argument obviously holds in the case of hidden information).

We will refer in what follows to the restrictions imposed by $(C1),(C2)$ as *One-Side Constraints.*

As an alternative to $(C2)$ we will also consider the case in which agents are allowed to go both long and short in 'individual' securities but different prices are quoted for long and short positions (i.e. bid-ask spreads are allowed). More precisely, denoting by q_j^b and q_j^s the unit buying and selling price of 'individual' security j, $j \in J$, the condition:

$$q_j^b \gtreqless q_j^s,\ \forall j \in J \tag{$C2'$}$$

describes the fact that no short-sale restriction is imposed but buying and selling prices may differ. Buyers and sellers are so clearly separated. This situation can be formally analyzed as follows: to each 'individual' security we can associate another security with opposite, but otherwise identical, payoff (so that the pair represents, respectively, long and short positions in the security), and have distinct 'pool' securities, as well as distinct prices, for each of them[32]. The model is so reduced to one with One-Side Constraints (and an expanded set of securities); by the same argument as above it follows that $(C2')$ also allows to overcome the feasibility problem.

We will refer to conditions $(C1),(C2')$ as *Bid-Ask Spreads.*

Remark 8. *The introduction of any form of trading restriction, or non-linearity of prices, requires some observability of agents' trades in financial markets. We already commented in the Introduction on the very strong informational requirements needed to implement exclusivity conditions, or also general non-linear price schedules. We intend to argue that the implementation of one-side constraints as in condition $(C2)$, or as well of bid-ask spreads as in condition $(C2')$, poses observability requirements which are, qualitatively, the least demanding. Only the level of trades in a particular transaction has in fact to be observed to implement*

[32]While this (and the fact that the total long and short positions in a given type of security need not coincide, at equilibrium) may look like a contrived structure of markets, in fact it just captures the ability of intermediaries to hedge the risk in their total position against agents, by trading other securities with payoff contingent on the aggregate uncertainty σ.

constraints or variations of prices at a zero level of trades (as in the case of one-side constraints or bid-ask spreads), while for constraints or price changes at any level different from zero the whole set of trades in same (and possibly in the other) market needs, in principle, to be observed.

On the other hand the imposition of bounds on trade requires essentially the observation of 'large' portfolios, a stronger but natural requirement.

Remark 9. *Under $(C2)$ no direct compensation of the positions in a given security in different villages is possible; hence the only contingent trades among agents of different villages take place via their trades in 'pool' securities. In each market we have on one side the buyers of a given type of 'individual' security, on the other the agents holding positions in the associated 'pool' security. With bid-ask spreads some compensation of positions is possible, though it will not be enough and, as we argued, the presence of 'pool' securities is still needed.*
More generally, we can view the 'feasibility' problem as the fact that a direct compensation of the positions in a security is not possible. Hence the need to specify how the losses arising in correspondence of the profits agents make, by trading securities on the basis of their private information, are distributed in the economy.[33] *Conditions $(C2)$ and $(C2')$ imply different mechanisms for distributing these losses.*

We will show that, with the additional restrictions imposed by One-Side Constraints (or by Bid-Ask Spreads)[34], competitive equilibria always exist, both for economies with adverse selection and with hidden information. We consider then first the case in which conditions $(C1), (C2)$ hold.

By restricting agents to be all on one side of the market for 'individual' securities condition $(C2)$ may generate a 'trivial' solution to the existence problem: we can always find in fact a level of q sufficiently high (or low according to the sign of the security's payoff) such that no agent wants to buy any 'individual' security, i.e. $\theta^h = 0 \ \forall h$. In this case no trade contingent on the agents' private information takes place, and the economy reduces therefore to a standard economy with incomplete (numeraire) security markets, where agents trade under conditions of symmetric information in all markets. To show existence of an equilibrium would then not be very informative if such trivial equilibria always belonged to the equilibrium set considered. However, it is immediate to see that the condition $q = -q^p$

[33] Or, equivalently, so as to ensure the validity of a zero-profit condition for intermediaries.

[34] Our previous analysis also shows that these conditions are tight, i.e. existence is not ensured if they are relaxed.

(and hence $q = -(R^p)'\rho$), imposed in our definition of competitive equilibrium, is not always satisfied at such 'trivial' equilibria.

Moreover, the following theorem establishes a stronger result: we will show that a competitive equilibrium always exists where the price of each 'individual' security has the property of being weakly more than fair for some agent and (weakly) less than fair for some other agent. By fair price we mean here fair with respect to the idiosyncratic shock component of the security's payoff, whose value is then equal to its expectation, conditionally on the private information of an agent.[35]

In the case of economies with adverse selection the fairness property of prices is formally stated as follows:

$$q_j \in co\{\sum_{\sigma} \rho_\sigma \sum_{s^{-h}} \pi(s^{-h}) r_j(\sigma, s^h, s^{-h}); s^h \in S^h, h \in H\} \qquad (F_{AS})$$

where $\sum_{\sigma} \rho_\sigma \sum_{s^{-h}} r_j(\sigma, s^h, s^{-h})$ constitutes a price of security j which is fair, in the above sense, for the agents of type h who observed s^h.

Similarly with hidden information:

$$q_j \in co\{\sum_{\sigma,s} \rho_\sigma \pi(s/\sigma) \sum_{m^{-h}} \pi(m^{-h}/s^{-h}) r_j(\sigma, m^h(s^h), m^{-h}); m^h \in (m^{h,v})_{\nu \in V}, h \in H\} \qquad (F_{HI})$$

where again $\sum_{\sigma,s} \rho_\sigma \pi(s/\sigma) \sum_{m^{-h}} \pi(m^{-h}/s^{-h}) r_j(\sigma, m^h(s^h), m^{-h})$ constitutes a fair price of security j for agents of type h who follow message strategy m^h. Since message strategies are endogenously chosen by agents, in (F_{HI}) prices are required to be fair only with respect to those message profiles actually chosen at equilibrium, i.e. $((m^{h,v})_{\nu \in V}$, typically a subset of $(M^h)^{S^h}$. The problem then arises that if $\theta_h = 0$ agent h will be indifferent among the possible messages he can send; we will use a condition in the spirit of 'trembling hand perfection' to restrict strategies also for agents who choose a zero level of trades[36].

We can now state the main result (the proof is in the Appendix):

Theorem 2. *Under assumptions 1-3, and conditions $(C1), (C2)$ (i.e. with One-Side Constraints), a competitive equilibrium with fair prices (satisfying, respectively, $(F_{AS}), (F_{HI})$) always exists, both for economies with adverse selection and with hidden information.*

[35] Evidently, if equilibrium prices are required to satisfy this fairness property, they cannot be set at an arbitrarily high (or low) level so as to prevent trade, as in a 'trivial' equilibrium.

[36] See Gale (1992) and Dubey, Geanakoplos and Shubik (1995) for discussions of refinements in similar environments.

As argued above, the model with Bid-Ask Spreads can always be reduced, formally, to one with One-Side Constraints, so that existence of competitive equilibria with Bid-Ask Spreads obtains as a corollary of the previous result:

Corollary 2. *Under assumptions 1-3, and conditions* $(C1), (C2')$ *(i.e. with Bid-Ask Spreads), a competitive equilibrium with fair prices exists, both for economies with adverse selection and with hidden information.*

Bid-ask spreads are endogenously determined at equilibrium, as the difference between the price for long and short positions. It is immediate to see that in the present framework the equilibrium level of the bid-ask spread will always be non-negative, and typically positive, when information is asymmetric (while it is zero under symmetric information.) The presence of a bid-ask spread is then to be imputed to the agents' private information over the payoff of securities, and the need to ensure feasibility in this case - or equivalently a zero-profit condition for intermediaries.[37]

More generally, the presence of a bid-ask spread constitutes the mechanism according to which the losses arising from the presence of asymmetric information are distributed in the economy, and hence feasibility ensured. On the other hand, with One-Side Constraints these losses are distributed to buyers of 'pool' securities, as the payoff they get is lower than the unconditional expected payoff - where the expectation is taken over the idiosyncratic uncertainty component - which was the level of the payoff under symmetric information (see (3.4)).

Remark 10. *Various examples can be found of financial markets whose features resemble the ones implied by conditions* $(C1)$ *and* $(C2)$*, or* $(C2')$*. Credit markets usually have borrowers on one side and, on the other side, suppliers of funds holding 'pool' securities (depositors, or more generally holders of claims issued by intermediaries.) Similarly, in insurance markets we often have standardized contracts offering positive insurance and, on the other side, 'securitized' claims issued by insurance companies. The mortgage market is yet another example. A somewhat different situation characterizes the stock market and markets for*

[37] A similar role of bid-ask spreads has been earlier shown by Glosten and Milgrom (1985). These authors examine a specific intermediation model, with risk-neutral market-makers, and study the equilibrium of the market for one security, in the presence of adverse selection. It is interesting to notice that also in this model total long and short positions are not required to be equal, at equilibrium, and market-makers play, effectively, the same role as 'pool' securities in our framework.

derivative securities. In these cases, agents may often take both long and short positions, and market makers charge spreads to guarantee themselves zero profits in the presence of asymmetric information; also 'pool' of securities are usually held by, e.g., mutual funds.

Remark 11. *To illustrate the claim that our results also apply to more abstract economies where exogenous restrictions on prices are imposed, consider a simple economy with* 4 *commodities and* H *consumers, and suppose that the first three commodities must trade at the same price:* $p_1 = p_2 = p_3 = p$. *Clearly this economy has in general no competitive equilibrium, as there are not enough prices to clear all markets. Consider then the following conditions:*
i) in the markets for commodities $1, 2, 3$ *agents can only buy, not sell, these goods (i.e. one-side constraints are imposed);*
ii) there also exists a market where fixed bundles composed of α *units of good* 1, β *units of good* 2, *and* $(1 - \alpha - \beta)$ *units of good* 3 *can be both bought and sold, at the price* p *(i.e. a 'pool' of the commodities whose prices are restricted is also marketed);*
iii) the proportions of the various goods in this bundle (i.e. the terms α *and* β*), are determined endogenously at equilibrium by :* $\frac{\alpha}{1-\alpha-\beta} = \frac{\sum_h x_1^h}{\sum_h x_3^h}$, $\frac{\beta}{1-\alpha-\beta} = \frac{\sum_h x_2^h}{\sum_h x_3^h}$, *where* x_l^h *is the amount of good* $l, l = 1, 2, 3$, *purchased by agent* h *in the market for this good.*
By a fairly immediate reformulation of our earlier argument we can show that under the above conditions feasibility can be ensured and competitive equilibria always exist for this economy.

7. Conclusions

The analysis developed in this paper can be extended in many directions. Other conditions to overcome the existence problems we identified, and in particular the feasibility problem, should be explored. For instance the introduction of entry fees, which agents are required to pay to be able to trade in markets for 'individual' securities, and are endogenously determined at equilibrium, allows to prove the existence of competitive equilibria even with linear prices and no short-sale restrictions. In this case there is no separation between buyers and sellers and the entry fee operates as a mechanism, symmetric on the two sides of the market, to redistribute the losses arising from the presence of asymmetric information so as to ensure feasibility. In this respect, bankruptcy institutions, or sets of taxes and transfers, could also serve the same purpose and ensure existence.

An analysis of the different implications for the nature of markets under asymmetric information of these alternative conditions, as well as of the other forms of non-linearities of prices which can be implemented when information on agents' trades is easily available, constitutes an important objective of our future work. In particular we intend to look at the efficiency properties of competitive equilibria and characterize the extent to which risks are insurable when agents have private information over them.

Figure 1: Timing.

Appendix

Proof of Theorem 2

We will show first that under $(C1)$ the agents' optimization problem has a well-behaved solution. Notice that, if condition $(C1)$ holds, agents' behavior is invariant with respect to the endogenously determined value of R^p; this term can then be omitted from the specification of the determinants of demand. In addition, we can use (6.1) to replace q with ρ so that demand can be written as a function simply of (ρ, p_0, p_1) (and, with hidden information, of $\pi(m^{-h}/.)$).

Lemma A. 1. *Under assumptions 1-3 and* $(C1)$*, the individual choice problem* P^h_{AS} *has a solution for all* $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}$ *and all the values of* R^p *which can be generated by (4.1). The solution is described by the correspondence* $\left(c^h(s^h), \theta^h(s^h), \theta^h_p(s^h)\right)(\rho, p_0, p_1)$*, non-empty, upper-hemi-continuous, convex-valued, and exhibiting the following boundary behavior,* $\forall s^h, h$*: for any sequence* $\left\{\rho^{(\tau)}, p_0^{(\tau)}, p_1^{(\tau)}\right\}_\tau \in (\Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++})$*, converging to* $(\rho, p_0, p_1) \in \partial(\Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++})$, $\inf\{\|c^h, \theta^h, \theta^h_p\| : (c^h, \theta^h, \theta^h_p) \in \left(c^h(s^h), \theta^h(s^h), \theta^h_p(s^h)\right)\left(\rho^{(\tau)}, p_0^{(\tau)}, p_1^{(\tau)}\right)\} \to \infty$.
The same properties, with the only exception of convex-valuedness, hold for the solutions of P^h_{HI}*, described by* $\left(c^h, \theta^h, \theta^h_p, (m^h(s^h))_{s^h}\right)\left((\rho, p_0, p_1), \pi(m^{-h}/s^{-h})\right)$.

Proof. Consider P^h_{AS}. It is immediate to see that condition $(C1(ii))$ implies that, when agents are restricted to take only long positions in 'individual' securities the values of the payoff of 'pool' securities obtained from (4.1) are such that we always have $Sp[R^p] = \Re^{\Sigma}$. This shows, since trades in 'pool' securities are unrestricted, that agents can indeed attain any payoff which is contingent on the aggregate uncertainty and that their behavior is unaffected by changes in R^p at equilibrium (so this term can be omitted from the argument of the demand correspondence).

Under $(C1)$, using (6.1) and the date 1 budget constraints to substitute for q, $\theta^h_p(s^h)$ in the expression of the agent's constraint at date 0, the feasible set of problem P^h_{AS} can be rewritten as follows:

$$\begin{gathered} B^h_{AS}(\rho, p_0, p_1; s^h) = \{c^h(s^h) \in \Re^{L(1+\Sigma S^{-h})}_{+}, \theta^h(s^h) \in \Theta^h : \\ p_0 \cdot (c^h_0(s^h) - w^h_0) + (-(R^p)'\rho \cdot \theta^h(s^h) + \\ + \sum_\sigma \rho_\sigma [p_1(\sigma) \cdot (c^h_1(s, \sigma; s^h) - w^h_1(s, \sigma)) - \sum_j \theta^h_j(s^h) r_j\ (s, \sigma)] \leq 0;\ s \in S\} \end{gathered} \tag{A.1}$$

Hence we see that all admissible consumption and portfolio plans must satisfy the following condition:

$$[p_1(\sigma) \cdot (c_1^h(s,\sigma;s^h) - w_1^h(s,\sigma)) - \sum_j \theta_j^h(s^h) r_j(s,\sigma)] =$$
$$[p_1(\sigma) \cdot (c_1^h(s',\sigma;s^h) - w_1^h(s',\sigma)) - \sum_j \theta_j^h(s^h) r_j(s',\sigma)] \ \forall s' \neq s$$

This describes the constraints on income transfers across states arising from market incompleteness: the value of excess demand less the return on 'individual' securities has to be the same for all s.[38]

Under assumption 2 $B_{AS}^h(\rho, p_0, p_1; s^h)$ has clearly a non-empty interior, and is closed, convex and compact for all $(\rho, p_0, p_1) \in \Re_{++}^{\Sigma} \times \Re_{++}^{L(1+\Sigma)}$. Moreover, it is defined by the intersection of budget hyperplanes and the choice variables which appear (c^h, θ^h) are all, by assumption, bounded below. Therefore by a standard argument the correspondence $B_{AS}^h(\rho, p_0, p_1; s^h)$ is also continuous. Upper-hemi-continuity and convex-valuedness of demand then follow from the continuity and concavity properties of the agents' objective function (stated in assumption 1).

Under assumption 2 it is immediate to see that $B_{AS}^h(.)$ has a non-empty interior also at prices $(\rho, p_0, p_1) \in \partial(\Re_{++}^{\Sigma} \times \Re_{++}^{L(1+\Sigma)})$, so that the boundary behavior property of demand holds.

Consider next the agent's problem in the economy with hidden information. A similar expression as above can be obtained for the feasible set B_{HI}^h of problem P_{HI}^h. It is then immediate to see that B_{HI}^h has the same properties as B_{AS}^h, with the only exception of convexity. Agents have in this case an additional choice variable, the message m^h which affects the payoff of the securities they trade; as we already noticed since M^h is finite B_{HI}^h cannot be convex. The presence of other, perfectly divisible choice variables and the structure of the budget equations defining B_{HI}^h ensure that the continuity of the correspondence defined by B_{HI}^h is preserved. Hence the rest of the above argument still applies. ■

We are now ready to prove that competitive equilibria exist. We will prove first the result for economies with adverse selection.

(AS) The level of aggregate excess demand is obtained as follows from indi-

[38] Under $(C1)$ as we showed 'pool' securities allow agents to fully insure against σ, while 'individual' securities offer some partial insurance against the idiosyncratic shocks s.

vidual demands:

$$\begin{aligned} z_0(\rho,p_0,p_1) &= \sum_h \lambda^h \sum_{s^h} \pi(s^h)[c_0^h(s^h)(\rho,p_0,p_1) - w_0^h] \\ (\theta,\theta_p)(\rho,p_0,p_1) &= \sum_h \lambda^h \sum_{s^h} \pi(s^h)(\theta^h(s^h),\theta_p^h(s^h))(\rho,p_0,p_1) \\ z_1(\sigma)(\rho,p_0,p_1) &= \sum_h \lambda^h \sum_{s} \pi(s)[c_1^h(s^{-h},\sigma;s^h)(\rho,p_0,p_1) - w_1^h(s,\sigma)],\ \sigma\in\Sigma \end{aligned}$$

By Lemma 1 it follows that the above expression inherits the same properties of individual demand: it is a upper-hemi-continuous, non-empty, convex-valued correspondence for all $(\rho,p_0,p_1)\in\Re^{\Sigma}_{++}\times\Re^{L(1+\Sigma)}_{++}$, and exhibits the appropriate boundary behavior. Moreover, it satisfies the following expressions defining Walras law at date 0 and date 1 in state σ: for all (ρ,p_0,p_1)

$$p_0\cdot z_0(\rho,p_0,p_1)+\rho\cdot(R^p(\rho,p_0,p_1)(\theta^p(\rho,p_0,p_1)-\theta(\rho,p_0,p_1))=0 \qquad (A.2)$$

$$(..,p_1(\sigma)\cdot z_1(\sigma)(\rho,p_0,p_1),..)+R^p(\rho,p_0,p_1)(\theta(\rho,p_0,p_1)-\theta^p(\rho,p_0,p_1))=0 \quad (A.3)$$

where $R^p(\rho,p_0,p_1)$ denotes the map obtained by substituting agents' demand correspondences in the expression of the payoff of 'pool' securities (4.1). Equations $(A.2)$, $(A.3)$ are obtained by aggregating across agents the budget constraints, after replacing q with $-R^{p\prime}\rho$, and using the specification of $R^p(\rho,p_0,p_1)$.[39]

Normalize date 0 and date 1 prices in every aggregate state σ on the simplex. Consider then the following truncated price sets: $\Delta_\delta^{L+\Sigma-1}\equiv((\rho,p_0)\in\Re_+^{L\Sigma}:\sum_l p_{0,l}+\sum_\sigma\rho_\sigma=1;\ p_{0,l},\rho_\sigma\geqslant\delta)$, $\Delta_\delta^{L-1}\equiv((p_1(\sigma)\in\Re_+^{L}:\sum_l p_{1,l}(\sigma)=1;\ p_{1,l}(\sigma)\geqslant\delta)$, for δ sufficiently 'small'. Pick a convex, compact set $K_\delta\subset\Re^{L(1+\Sigma)}\times\Re^{2J}$ such that the image $(z_0,(..,z_1(\sigma),..),\theta,\theta_p)\left(\Delta_\delta^{L+\Sigma-1},(\Delta_\delta^{L-1})^\Sigma\right)\subset K_\delta$.

Examine next the map $R^p(\rho,p_0,p_1)$. It is immediate to see from the expression of (4.1) that $r_j^p(\sigma)\in co\{r_j(\sigma,s^h);s^h\in S^h,h\in H\}$ for all θ_j^h such that $\theta_j\neq 0$, where $co\{.\}$ denotes the convex hull of a set, and $r_j(\sigma,s^h)\equiv(\sum_{s^{-h}}r_j(\sigma,s)\pi(s^{-h}/s^h))$, i.e. is the expected payoff of security j conditionally on σ,s^h. We can then, with no loss of generality, limit our attention to the case in which $r_j^p(\sigma)\in co\{r_j(\sigma,s^h);s^h\in S^h,h\in H\}$ also when $\theta_j=0$. Hence $R^p(\rho,p_0,p_1)\in\bar{R}_{AS}\equiv\{R\in\Re^{\Sigma\times J}:r_j^p(\sigma)\in co[r_j(\sigma,s^h);s^h\in S^h,h\in H],\ j\in J,\sigma\in\Sigma\}$ for all ρ,p_0,p_1, i.e. the range of this map lies in the convex hull of a finite set

[39] Condition $(C2)$, by implying that when $\theta=0$ the aggregate payoff of 'individual' securities equals 0, is crucial for the validity of $(A.3)$ also when $\theta=0$.

of points, thus a convex, compact set. Furthermore, the range of $-(R^p)'\rho$, when $R^p \in \bar{R}_{AS}$, $(\rho, p_0) \in \Delta^{L+\Sigma-1}$, is also compact, and will be denoted by Q_{AS}.

Consider then the map:

$$(z_0, (.., z_1(\sigma), ..), \theta, \theta_p, R^p, \rho, p_0, p_1, q)_\delta :$$

$$K_\delta \times \bar{R}_{AS} \times \Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma \times Q_{AS} \longrightarrow K_\delta \times \bar{R}_{AS} \times \Delta_\delta^{L+\Sigma-1}, (\Delta_\delta^{L-1})^\Sigma \times Q_{AS}$$

defined by:

$$\begin{aligned}
(z_0, (.., z_1(\sigma), ..), \theta, \theta_p) &= (z_0, (.., z_1(\sigma), ..), \theta, \theta_p)\,(\rho, p_0, p_1) \\
r_j^p\,(\sigma) &= -\frac{\sum_h \lambda^h \sum_s \pi(s) r_j\,(s, \sigma)\theta_j^h(s^h)\,(\rho, p_0, p_1)}{\theta_j},\ \forall \sigma, j \\
\rho, p_0 &\in \arg\max\{p_0 \cdot z_0 + \rho \cdot (R^p(\theta^p - \theta))\} \\
p_1(\sigma) &\in \arg\max\{p_1(\sigma) \cdot z_1(\sigma)\},\ \forall \sigma \\
q &= -(R^p)'\rho
\end{aligned}$$

Under the above assumptions this map is upper-hemicontinuous and convex-valued, and its domain is compact, convex. Therefore, by Kakutani's Theorem it has a fixed point $[z_0, (.., z_1(\sigma), ..), \theta, \theta_p, R^p, \rho, p_0, p_1, q]_\delta$.

Recalling the expression of Walras' laws derived above it is immediate to see that if, at the fixed point , $(\rho, p_0, p_1)_\delta \in int\{\Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma\}$, we have $[(z_0), (R^p(\theta^p - \theta)), (.., z_1(\sigma), ..)]_\delta = 0$, i.e. an equilibrium[40] for the perturbed economy. If not, let $\delta \to 0$ and consider the associated sequence of fixed points. By a standard argument (see, e.g., Werner (1985)) we can show that this sequence is convergent and, given the boundary behavior property of excess demand, the limit value $(\rho, p_0, p_1)^* \in int\{\Delta^{L+\Sigma-1} \times (\Delta^{L-1})^\Sigma\}$.

Furthermore, notice that at the equilibrium we obtained we have, for all j :

$$q_j^* \in co\{\sum_\sigma \rho_\sigma^* r_j(\sigma, s^h); s^h \in S^h, h \in H\} \qquad (A.4)$$

Recalling the definition of $r_j(\sigma, s^h)$ it is immediate to see that $(A.4)$ is equivalent to condition (F_{AS}). Therefore we have shown that at equilibrium the price of every security is always (weakly) more than 'fair' for some agent and (weakly)

[40] The equality $R^p(\theta^p - \theta) = 0$ implies that the values $\theta^p - \theta = 0$ also belong to the aggregate demand correspondence.

less than 'fair' for some other agent (where at least one of the two inequalities is strict).

(HI) Part of the proof for economies with hidden information is essentially the same as for economies with adverse selection. However, in this case we have also to show that the economy can be 'convexified' by exploiting the large number of agents. Furthermore, to impose a restriction, as was stated, in the spirit of 'trembling hand perfection' on the message strategies of agents who choose a zero level of trades (for whom the message choice is trivial), we will have to introduce a perturbation of the economy and proceed then by a limit argument. In what follows we will focus on the new parts of the argument, referring to the proof above for the common parts.

Let B_ε^J be an ε-ball in R^J. We will prove first the existence of competitive equilibria for the 'perturbed' economy where agents' trades in 'individual' securities are restricted to lie in the set $\tilde{\Theta}^h \equiv \Theta^h \backslash B_\varepsilon^J$, for all h and for ε sufficiently 'small'. By taking the limit as $\varepsilon \to 0$ we obtain a sequence of 'perturbed' economies which converges to the 'original' economy where agents' behavior is subject to the 'original' trading constraints $(\Theta^h)_{h \in H}$. In a 'perturbed' economy agents have to trade some nonzero amount of the 'individual' securities; therefore the payoff of 'pool' securities is always given by the 'average' payoff of individual securities and the price is fair with respect to the message strategy optimally chosen by the agents. In the limit, the same property also holds.

Let $E^h \equiv [e^i \in \Re^{M^h} : e_i^i = 1, e_j^i = 0 \ \forall j \neq i;\ i \in M^h]$ be the collection of unit vectors in $\Re^{M^h}$. Evidently, there is a one-to-one correspondence between elements of E^h and of M^h, so that we can equivalently state the agents' message choice in terms of the choice of an element of E^h. Let $\left(c^h, \theta^h, \theta_p^h, (e^h(s^h))_{s^h}\right)$ $\left((\rho, p_0, p_1), \pi(e^{-h}/s^{-h})_{s^{-h}}; \tilde{\Theta}^h\right)$ denote then the solution of P_{HI}^h when trades in 'individual' securities are restricted to lie in the set $\tilde{\Theta}^h$, and m^h has been replaced by e^h.

Define

$$\left(\hat{c}^h, \hat{\theta}^h, \hat{\theta}_p^h, (\hat{e}^h(s^h))_{s^h}, [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\theta_j^h]_{j,\sigma,s^h}\right) \left((\rho, p_0, p_1), \pi(e^{-h}/s^{-h})_{s^{-h}}; \tilde{\Theta}^h\right) \equiv$$

$$co\{\left(c^h, \theta^h, \theta_p^h, e^h\right) \left((\rho, p_0, p_1), \pi(e^{-h}/s^{-h})_{s^{-h}}; \tilde{\Theta}^h\right);$$
$$[r_j(e^h(s^h), e^{-h}, \sigma)\theta_j^h \left((\rho, p_0, p_1), \pi(e^{-h}/s^{-h})_{s^{-h}}; \tilde{\Theta}^h\right)]_{j,\sigma,s^h}\}$$

where $co\{\Phi(.)\}$ denotes, for any map $\Phi(.)$, the convex hull of the image of the map. By Lemma 1 it follows that the above expression is a upper-hemicontinuous, nonempty[41] correspondence for all $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}, \pi(e^{-h}/s^{-h})_{s^{-h} \in S^{-h}} \in \Pi_{h' \neq h}(\Delta^{M^{h'}})^{S^{h'}}$ and exhibits the appropriate boundary behavior; moreover it is also, by construction, convex-valued.

Let $\Delta_{\delta}^{L+\Sigma-1}, \Delta_{\delta}^{L-1}$ be defined as before.

The range of the map $R^p((\rho, p_0, p_1), \pi(e^h/s^h)_{s \in S}; (\tilde{\Theta}^h)_{h \in H})$ we obtain by substituting the expression of $\hat{\theta}^h\left((\rho, p_0, p_1), \pi(e^{-h}/s^{-h})_{s^{-h} \in S^{-h}}; \tilde{\Theta}^h\right), h \in H$, in the expression of (4.10) lies now in the set
$\bar{R}_{HI} \equiv \{R \in \Re^{S \times J} : r_j^p(\sigma) \in co[\sum_s \pi(s/\sigma) \sum_{m^{-h}} \pi(m^{-h}/s^{-h})\ r_j(\sigma, m^h(s^h), m^{-h});$ $m^h \in (M^h)^{S^h}, \pi(e^{-h}/s^{-h})_{s^{-h} \in S^{-h}} \in \Pi_{h' \neq h}(\Delta^{M^{h'}})^{S^{h'}}, h \in H\}$, convex, compact. Similarly, let Q_{HI} be the range of $-(R^p)'\rho$, when $R^p \in \bar{R}_{HI}$, $(\rho, p_0) \in \Delta^{L+\Sigma-1}$, also compact.

Consider then the map:

$$\begin{array}{l}\left(z_0, (.., z_1(\sigma), ..), \theta, \theta_p, R^p, \rho, p_0, p_1, q, \pi(e^h/s^h)_{s^h,h}\right)_{\delta} : \\ K_{\delta} \times \bar{R}_{HI} \times \Delta_{\delta}^{L+\Sigma-1} \times (\Delta_{\delta}^{L-1})^{\Sigma} \times Q_{HI} \times \Pi_h(\Delta^{M^h})^{S^h} \longrightarrow \\ K_{\delta} \times \bar{R}_{HI} \times \Delta_{\delta}^{L+\Sigma-1} \times (\Delta_{\delta}^{L-1})^{\Sigma} \times Q_{HI} \times \Pi_h(\Delta^{M^h})^{S^h}\end{array}$$

defined by:

$$\begin{array}{rcl}z_0 & = & \sum_h \left(\lambda^h \hat{c}_0^h(.) - w_0^h\right) \\ (.., z_1(\sigma), ..)) & = & \sum_{h,s} \lambda^h \pi(s \mid \sigma) \left[\Pi_{h' \neq h}\left(\pi^{h'}(e^{h'}/s^{h'})\right) \hat{c}_1^h(\sigma, e^{-h}, s^h)(.) - w_1^h(s^h, \sigma)\right], \ \sigma \in \Sigma \\ (\theta, \theta_p) & = & \sum_h \lambda^h(\hat{\theta}^h, \hat{\theta}_p^h)(.) \\ r_j^p(\sigma) & = & -\dfrac{\sum_{h,s} \lambda^h \pi(s/\sigma) \sum_{e^{-h}} \left(\Pi_{h' \neq h} \pi^{h'}(e^{h'}/s^{h'})\right) [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\theta_j^h](.)}{\theta_j}, \ \forall \sigma, j \\ \pi(e^h/s^h) & = & \hat{e}^h(s^h) \ \forall h, s^h \\ q & = & -(R^p)'\rho \\ \rho, p_0 & \in & \arg\max\{p_0 \cdot z_0 + \rho \cdot (R^p(\theta^p - \theta))\}\end{array}$$

[41] Under assumption 2 we can always find ε sufficiently small so that the agents' feasible set is nonempty also when they are restricted to trade nonzero amounts of the 'individual' securities.

$$p_1(\sigma) \quad \in \quad \arg\max\{p_1(\sigma) \cdot z_1(\sigma)\}, \ \forall \sigma$$

This map is upper-hemicontinuous and convex-valued, and its domain is compact, convex. Therefore Kakutani's theorem can again be applied, yielding the existence of a fixed point. Proceeding as above we can show that the same expression of Walras' laws hold, and that, by letting $\delta \to 0$ the associated sequence of fixed points converges to an equilibrium of the perturbed, 'convexified' economy $\left(z_0, (.., z_1(\sigma), ..), \theta, \theta_p, R^p, \rho, p_0, p_1, q, \pi(e^h/s^h)_{s^h,h}\right)^*_\varepsilon$. If we then let $\varepsilon \to 0$ we obtain another sequence of fixed points (each of which is an equilibrium of the associated perturbed, 'convexified' economy) which converges to an equilibrium of the unperturbed, 'convexified' economy, $\left(z_0, (.., z_1(\sigma), ..), \theta, \theta_p, R^p, \rho, p_0, p_1, q, \pi(e^h/s^h)_{s^h,h}\right)^*$. At this equilibrium demand and messages are determined by
$\left(\hat{c}^h, \hat{\theta}^h, \hat{\theta}^h_p, (\hat{e}^h(s^h))_{s^h}, \ [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\theta^h_j]_{j,\sigma,s^h}\right) \left((\rho, p_0, p_1), \ \pi(e^{-h}/s^{-h})_{s^{-h} \in S^{-h}}; \Theta^h\right)^*$,
i.e. by the 'convexified' demand map at the 'original' trading constraints $(\Theta^h)_h$, and are such that, at the prices $(\rho, p_0, p_1, q)^*$, commodity and securities' markets clear, the payoff of 'pool' securities is consistent with agents' messages, and $\pi(e^h/s^h)^*_{s^h,h}$ is consistent with $(\hat{e}^h(s^h))^*_{s^h}$ (the Nash equilibrium component).

By Caratheodory's theorem, as long as $V \geqslant [(L(1 + \Sigma M^{-h} S^h) + 2J + M^h S^h + J\Sigma M]$, we can always find a set of weights $(\gamma^{h,\nu})^*_{h \in H, \nu \in V}$ and a set of points, all belonging to the original demand map, such that

$$\left(\hat{c}^h, \hat{\theta}^h, \hat{\theta}^h_p, (\hat{e}^h(s^h))_{s^h}, \ [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\theta^h_j]_{j,\sigma,s^h}\right) \left((\rho, p_0, p_1), \ \pi(e^{-h}/s^{-h})_{s^{-h} \in S^{-h}}; \Theta^h\right)^* =$$
$$= \sum_\nu \gamma^{h,\nu} \left(c^{h,\nu}, \theta^{h,\nu}, \theta^{h,\nu}_p, (e^{h,\nu}(s^h))_{s^h}, \ [r_j(e^{h,\nu}(s^h), e^{-h}, \sigma)\theta^{h,\nu}_j]_{j,\sigma,s^h}\right)^* \forall h,$$

where $(c^{h,\nu}, \theta^{h,\nu}, \theta^{h,\nu}_p, (e^{h,\nu}(s^h))_{s^h})^* \in$
$\left(c^h, \theta^h, \theta^h_p, (e^h(s^h))_{s^h}\right) \left((\rho, p_0, p_1)^*, \ \pi(e^{-h}/s^{-h})^*_{s^{-h} \in S^{-h}}; \Theta^h\right) \forall \nu$.
Hence $\left((c^{h,\nu}, \theta^{h,\nu}, \theta^{h,\nu}_p, (e^{h,\nu}(s^h))_{s^h})^*_{\nu,h}, \ (\gamma^{h,\nu})^*_{\nu,h}, \ (\rho, p_0, p_1)^*, \ \pi(e^{-h}/s^{-h})^*_{s^{-h} \in S^{-h}}\right)$ constitutes a competitive equilibrium of the original economy.

It is then immediate from the inspection of the fixed point map and the limit argument, that at this equilibrium property (F_{HI}) holds, i.e. that the prices of every security are always (weakly) more than 'fair' with respect to the message strategy followed by some agent and (weakly) less than 'fair' for some other agent (one of the two inequalities at least is strict). Moreover, the strategies of agents who hold a zero amount of securities are restricted to be consistent with (i.e. 'close' to) their optimal strategy when they hold a small amount of securities.

This completes the proof. ■

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