SECOND MIDTERM EXAM Economics 401 Fall 2008, David K. Levine

Do all three questions, each has equal weight. You have 1 hour and 23 minutes.

1. Spousal Abuse

A married woman is found dead. Suppose that 80% of married women who are murdered are murdered by their husbands. Suppose in addition that all women who are murdered by their husbands are abused by their husbands prior to their murder. On the other hand, only 10% of women who are murdered by someone other than their husband are abused by their husbands. If this woman was abused by her husband, what is the probability that her husband was her murderer? If we assign a utility of 1 to putting a murderer in jail, a utility of 0 to putting nobody in jail, and utility of –10 to putting an innocent person in jail, should we put the husband in jail for murder if we do not know whether or not he was abusive? If we find conclusive evidence that he was abusive?

Define probability
P(H) as "a dead married women is murdered by her husband "
P($\underline{H}$) as "a dead married women is not murdered by her husband"
P(A) as "a dead married women is abused"
P(A|H) as "conditional on her husband is the murder, the dead married women is abused by her husband"
P(A|$\underline{H}$) as "conditional on her husband is not the murder, the dead married women is abused by her husband"
then
P(H)=0.8
P($\underline{H}$)=0.2
P(A|H)=1
P(A|$\underline{H}$)=0.1
1 We need to show what P(H|A) is:

$$P(H \mid A) = \frac{P(A \mid H)P(H)}{P(A)} = \frac{P(A \mid H)P(H)}{P(A \mid H)P(H) + P(A \mid \underline{H})P(\underline{H})} = \frac{40}{41} \approx 0.976$$

2 If we do not know whether the husband was abusive or not, we only know that the dead women was murdered by her husband with probability 0.8
If we put the husband in jail, the utility is 0.8*1+0.2*(-10)=-1.2, smaller than 0, so we should not put him in jail.
3 If we know the husband was abusive, then we know the dead women was murdered by her husband with probability 0.976
If we put the husband in jail, the utility is 0.976*1+0.024*(-10)=0.736, larger than 0, so we should put him in jail.

2. A PD of Sorts

Consider the following symmetric game in which you can either be proud, not confess or confess. Suppose that it is infinitely repeated with discount factor $0 \leq \delta < 1$.

	proud	not confess	confess
proud	40,40	48,42	8,4
not confess	42.48	50,50	7,53

confess	4,8	53,7	10,10

a) Give an accurate sketch of the socially feasible individually rational set. What does the Folk Theorem say about this set?

10 is the minmax payoff for each player, any point that is socially feasible and not less than (10,10) for each player is in the SFIR set.
The Folk Theorem says, any payoff vector in the SFIR set could be obtained by a SPE as δ larger enough.

b) Find grim trigger strategies so that players both get 50 in equilibrium. For what discount factors are these a Nash equilibrium? If the grim trigger strategies are a Nash equilibrium, why are they also subgame perfect?

As the game is symmetric, we can construct the same grim strategy for the players: begins with playing "not confess"; goes on playing "not confess" as long as the previous outcomes are (not confess, not confess), otherwise plays "confess".
To make sure it is a Nash equilibrium, we need the condition $50 \geq 53(1-\delta)+10\delta$, which implies $1 > \delta \geq 3/43$
As the game is repeated infinitely, any subgame of the original game is the same as the original game, so any Nash equilibrium of the original game is also Nash equilibrium in any subgame, so they are subgame perfect equilibria.

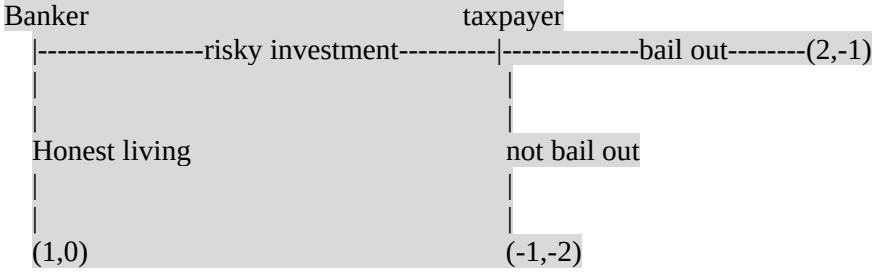
c) Find mixed strategy equilibrium of this game played once in which both players randomize between proud and confess.

Each player plays "proud" with probability 1/19, plays "confess" with probability 18/19.

3. Long Run versus Short Run

An short-lived investment banker must choose whether to earn an honest living, yielding her a payoff of 1, and to the long-run taxpayer of 0, or whether to make a risky investment. The taxpayer must decide whether or not to bail out the investment banker. If the taxpayer does not bailout, then the investment banker gets –1, but the economy collapses, so the taxpayer gets –2. If the taxpayer does bailout then the investment banker gets 2, but the taxpayer gets –1.
a) Find the extensive and normal form of this game.

Extensive form:



Normal form:

	banker		
		Honest living	Risky investment

taxpayer	Bail out	0 , 1	-1 , 2
	Not bail out	0 , 1	-2 , -1

b) What pure strategy Nash equilibria are there in the stage game; which are subgame perfect? What is the Stackelberg equilibrium of the stage game in which the taxpyaer moves first?

The pure strategy Nash equilibria are (risky investment, bail out) and (honest living, not bail out)
The subgame perfect equilibrium is (risky investment, bail out)
The Stackelberg equilibrium is: the taxpayer commits "not bail out", the banker plays "honest living"

c) If the stage game is repeated and the taxpayer is infinitely lived with discount factor equal to δ and there is a sequence of short-lived sales people propose a equilibrium strategy and a δ such that players end up playing Stackelberg every period.

Strategy: the banker begins with "honest living" and the taxpayer begins with "not bail out"; as long as the taxpayer always plays "not bail out" in the previous periods, the banker plays "honest living", otherwise she plays "risky investment"; as long as the taxpayer never played "bail out" before, he goes on playing "not bail out", otherwise he plays "bail out".
To make sure this is SPE, we need the condition $-2(1-\delta)+0\delta \geq -1$, which implies $1/2 \leq \delta < 1$.

[note: the outcome of this equilibrium is that the short-lived bankers always play "honest living". The payoff to each banker is 1 and the discounted average payoff of the taxpayer is 0. The condition $-2(1-\delta)+0\delta \geq -1$ is to make sure that if some bankers deviate, the taxpayer will still follow the equilibrium strategy. Other deviation cases could be easily checked.]