

1. A consumer values three good x_1, x_2, x_3 , which she may purchase at prices p_1, p_2, p_3 using money income I . Her preferences can be represented by a utility function $u(x_1, x_2, x_3) = x_1 x_2^2 x_3^3$.

a. Find the demand functions of the consumer for x_1, x_2 and x_3 .

First order conditions

$$x_2^2 x_3^3 = \lambda p_1$$

$$2x_1 x_2 x_3^3 = \lambda p_2$$

$$3x_1 x_2^2 x_3^2 = \lambda p_3$$

These equations give the relevant demand functions,

$$x_1 = \frac{I}{6p_1}$$

$$x_2 = \frac{I}{3p_2}$$

$$x_3 = \frac{I}{2p_3}$$

b. Verify that these functions are homogeneous of degree zero in prices and income.

Note that $x_1(p, I) = \frac{I}{6p_1} = \frac{cI}{c6p_1} = x_1(cp, cI)$. Similarly $x_2(p, I) = x_2(cp, cI)$ and $x_3(p, I) = x_3(cp, cI)$. Consequently, the demand functions are indeed homogeneous of degree 0.

Suppose that there is a firm that produces the good x_2 at constant marginal cost $c > 0$.

c. Find the competitive equilibrium output and price in the market for x_2 . In competitive equilibrium it must be that $p_2 = c$.

The output would therefore be, $x_2 = \frac{I}{3c}$.

d. Find the optimal output and price of a monopolist in the market for x_2 . The demand function for x_2 shows that irrespective of the price, the consumer would choose to spend a third of her income on it. Given that the monopolist faces a constant marginal cost, and a certain revenue of $\frac{I}{3}$ as long as a positive amount of output is produced. The optimal strategy would involve setting as high a price as possible along with the smallest quantity of output. Consequently the problem does not have a well defined solution.

2. A criminal must decide whether or not to accept a plea bargain of ten years in prison or pay for a fancy lawyer. If she hires the lawyer, the lawyer must decide whether or not hire a private investigator. The utility of the criminal is the negative of the expected number of years spent in jail minus the number of thousands of dollars spent on the attorney. The attorney charges a fee of five thousand dollars, and must pay three thousand dollars to hire an investigator. The utility of the attorney is the number of thousands of dollars received. If the lawyer does not hire the private investigator, the criminal will spend fifteen years in prison. If he does hire the investigator, the criminal will get off free.

In what follows (a, b) represents a strategy profile where the lawyer chooses

action (a) while the criminal chooses action (b).

a. Draw the extensive form of this game. Find the subgame perfect equilibria.

The subgame perfect equilibria involves the criminal choosing (Not Hire) and the lawyer choosing (Not Hire) if the lawyer is indeed hired.

b. Find the normal form of this game. Find the Nash equilibria.

<i>Lawyer/Criminal</i>	<i>Hire</i>	<i>Not Hire</i>
<i>Hire</i>	2, -5	0, -10
<i>Not Hire</i>	5, -20	0, -10

The Nash Equilibrium involves (Not Hire, Not Hire) resulting in a payoff of (0, -10)

c. What is the Stackelberg equilibrium if the lawyer moves first? (Hire, Hire) resulting in a payoff of (2, -5)

Now, suppose the lawyer lives forever, and discounts the future with discount factor d . The lawyer faces a sequence of different clients each of whom live one period.

d. Find a discount factor d for the surgeon and subgame perfect equilibrium strategies for both players such that the surgeon gets the same utility in the equilibrium as in the Stackelberg equilibrium of the game played once. Explain why these strategies are subgame perfect.

Lawyer: Play (Hire) in the first period. Further play (Hire) if (Hire, Hire) has always been played in the past. Otherwise, play (Not Hire) forever.

Criminal: Play (Hire) in the first period. Play (Hire) if (Hire, Hire) has always been played in the past. Otherwise play (Not Hire).

For these strategies to be subgame perfect it must be true that $2 > (1 - d)5 \Rightarrow 5d > 3$. So the above strategies are subgame perfect for $d > \frac{3}{5}$.

e. There is an equilibrium of the repeated game in which no client hires an attorney. Does this equilibrium make sense to you? Why or why not?

3. Lloyd's of London (L) has the opportunity to insure a famous golf player (G). There are two possible events - the golf player is a failure (F) or a success (S) with equal probability. In case of success, L collects a payment p . In case of failure L pays S with probability Q and $-R$ with probability $1 - Q$. Let m_L be the amount of money received by L from G (possibly negative). L simply tries to maximize m_L . Let m_G be the amount received by G from L (possibly negative). The utility of G is $2(1 + m_G)$ in case of failure. In case of success it is $6 + m_G$ for $-6 < m_G < 0$ and 6 for $m_G \geq 0$.

First suppose that L can observe whether G has success or failure. Assume

in this case that $Q = 1$. In terms of p, S ,

- a. Write the objective function of L. Write the objective function of G.

Let the objective functions for L and G be V_L and V_G .

Then,

$$V_L = \frac{1}{2}p + \frac{1}{2}(-S)$$

$$V_G = \frac{1}{2}\min\{6, 6 - p\} + 1 + S$$

- b. Write the participation constraint for G.

$$V_G = \frac{1}{2}\min\{6, 6 - p\} + 1 + S > 4$$

- c. Show that $p = 1, S = 1$ satisfies the participation constraint, yields zero profits to L, and makes G strictly better off than no contract ($p=0, S=0$)
 With $p = 1, S = 1, V_G = \frac{1}{2}(6 - 1) + 1 + 1 = \frac{9}{2} > 4$. Consequently $p = 1, S = 1$ satisfies the participation constraint (is weakly better off than no contract) and indeed makes G strictly better off than the no contract scenario. Finally, $V_L = \frac{1}{2}1 + \frac{1}{2}(-1) = 0$, thereby yielding zero profits to L.

Now suppose that L cannot observe whether G has success or failure, but must accept an announcement from G reporting whether he has succeeded or failed. G may claim failure when successful, but may not claim success when a failure. You should no longer assume that $Q = 1$.

- d. Rewrite the objective functions for L and G and the participation constraint for G for general value of Q .

$$V_L = \frac{1}{2}p + \frac{1}{2}((Q(-S)) + R(1 - Q))$$

$$V_G = \frac{1}{2}\min\{6, 6 - p\} + \frac{1}{2}Q2(1 + S) + \frac{1}{2}(1 - Q)2(1 - R)$$

The participation constraint being

$$V_G \geq 4$$

- e. Write the incentive constraint for G.

$$\frac{\min\{6, 6 - p\}}{2} + \frac{Q2(1 + S)}{2} + \frac{(1 - Q)2(1 - R)}{2} \geq \frac{Q6}{2} + \frac{(1 - Q)(6 - R)}{2} + \frac{Q2(1 + S)}{2} + \frac{(1 - Q)2(1 - R)}{2} \quad (1)$$

The right hand side for the above equation is the expected payoff to G from lying. In particular, irrespective of the true state of the world, G announces *Failure*.

f. Show that $Q = 1/2$, $S = 4$, $R = 2$, $p = 1$ satisfies the participation and incentive constraints for G and yields 0 profit to L.

$$V_L = \frac{1}{2}1 + \frac{1}{2}\left(\frac{1}{2}(-4) + 2\left(\frac{1}{2}\right)\right) = 0$$

Note also that both sides of the inequality in (1) gives $\frac{9}{2}$ which is in turn greater than the the non participation value of 4.