Consider this 2 player, 5 x 5 game. There are 5 pure equilibria, highlighted in yellow. Since there is not a single pure equilibria, theory tell the players to use mixed strategy in order to make an optimal decision.

Let’s consider player 2, who has to choose columns. Which probability is every pure strategy (C1 … C5) to be assigned, so that the expected utility for player 1 yields the same value, no matter which strategy (F1 … F5) he chooses?

Let \( p \) be the probability for player 2 to choose C1, \( q \) the probability for C2, \( r \) the probability for C3, \( s \) the probability for C4 and \( t \) the probability for C5. Player 2 has to solve the following system of equations:

\[
4 \cdot p + 0 \cdot q + 8 \cdot r + 0 \cdot s + 6 \cdot t = u
\]
\[
7 \cdot p + 9 \cdot q + 4 \cdot r + 4 \cdot s + 3 \cdot t = u
\]
\[
10 \cdot p + 8 \cdot q + 7 \cdot r + 4 \cdot s + 0 \cdot t = u
\]
\[
3 \cdot p + 3 \cdot q + 4 \cdot r + 2 \cdot s + 8 \cdot t = u
\]
\[
7 \cdot p + 3 \cdot q + 1 \cdot r + 10 \cdot s + 1 \cdot t = u
\]

Where \( u \) is the expected utility for player 1. Moreover, we have to define that \( p + q + r + s + t = 1 \), and now we have a system of 6 equations with 6 unknown. We can solve this system by means of the Gauss-Jordan method, where the matrix for this system is:

\[
\begin{array}{cccccc}
1 & 1 & 1 & 1 & 1 & 0 \\
4 & 0 & 8 & 0 & 6 & -1 \\
7 & 9 & 4 & 3 & -1 & 0 \\
10 & 8 & 7 & 0 & -1 & 0 \\
3 & 3 & 4 & 2 & 8 & -1 \\
7 & 3 & 1 & 10 & 1 & -1 \\
\end{array}
\]

The first line represents:
\[
1 \cdot p + 1 \cdot q + 1 \cdot r + 1 \cdot s + 1 \cdot t + 0 \cdot u = 1
\]

And the others, for instance the second line:
\[
4 \cdot p + 0 \cdot q + 8 \cdot r + 0 \cdot s + 6 \cdot t = 1 \cdot u
\]

Hence:
\[
4 \cdot p + 0 \cdot q + 8 \cdot r + 0 \cdot s + 6 \cdot t - 1 \cdot u = 0
\]

I’ve solved this system with three different computer programs that uses the Gauss-Jordan procedure. I wrote one of these programs, and the other two can be found here:

http://www2.unime.it/weblab/ita/Gauss/gauss_auto_es.htm
http://people.hofstra.edu/Stefan_waner/RealWorld/tutorialsf1/scriptpivot2.html
In all these three tests, the results I get are:

\[
\begin{align*}
  p &= -0.32 \\
  q &= 0.17 \\
  r &= 0.52 \\
  s &= 0.48 \\
  t &= 0.14 \\
  u &= 3.76
\end{align*}
\]

Remember that \(u\) is the expected utility for player 1.

The problem, obviously, is that negative value the system yields for the probability \(p\).

I’ve used the same procedure for other games, from 2 x 2 to 5 x 5, and most of the times the probability values are ok (that is, no negative probability).

So, what is wrong with the system that formalizes the game? I don’t think it is a calculation mistake, since the three programs work properly when solving system of equations.

I’ve also tried to formalize the game with others system of equations, without using \(u\), and the values I get are exactly the same. (Were it useful/helpful, in a next draft I can write down the whole transformations).

I’d be very grateful for any answer or clue to solve this.