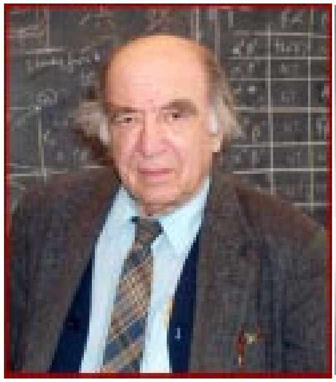
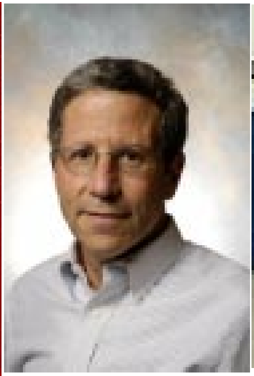
Introductory Lecture

What this class is about

- economic science as it exists today
- intrinsically a mathematical subject
- the heart of economics is mechanism design theory
- the goal of the class is to give you a limited working knowledge of mechanism design theory


The Problem of the Consumer

$\max u(x_1, x_2)$

subject to $p_1 x_1 + p_2 x_2 \le I$ (budget set)

x_1, x_2 quantity of good 1,2 consumed respectively

p_1, p_2 money prices of goods 1 and 2

I money income

Utility Theory

- utility we take as given in this class
- utility is not primitive, preferences are
- under mild assumptions preferences can be *represented* by a utility function
- the cocktail party approach to science
  - all things are relative
  - economists assume people are lightning calculators of pleasure and pain

Demand Theory

$u(x_1, x_2) = \bar{u}$

solution of the consumer problem

$\max u(x_1, x_2)$

subject to $p_1 x_1 + p_2 x_2 \leq I$ (budget set)

indifference curve $u(x_1, x_2) = \bar{u}$

find the slope: $\frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = \underbrace{d\bar{u} = 0}$

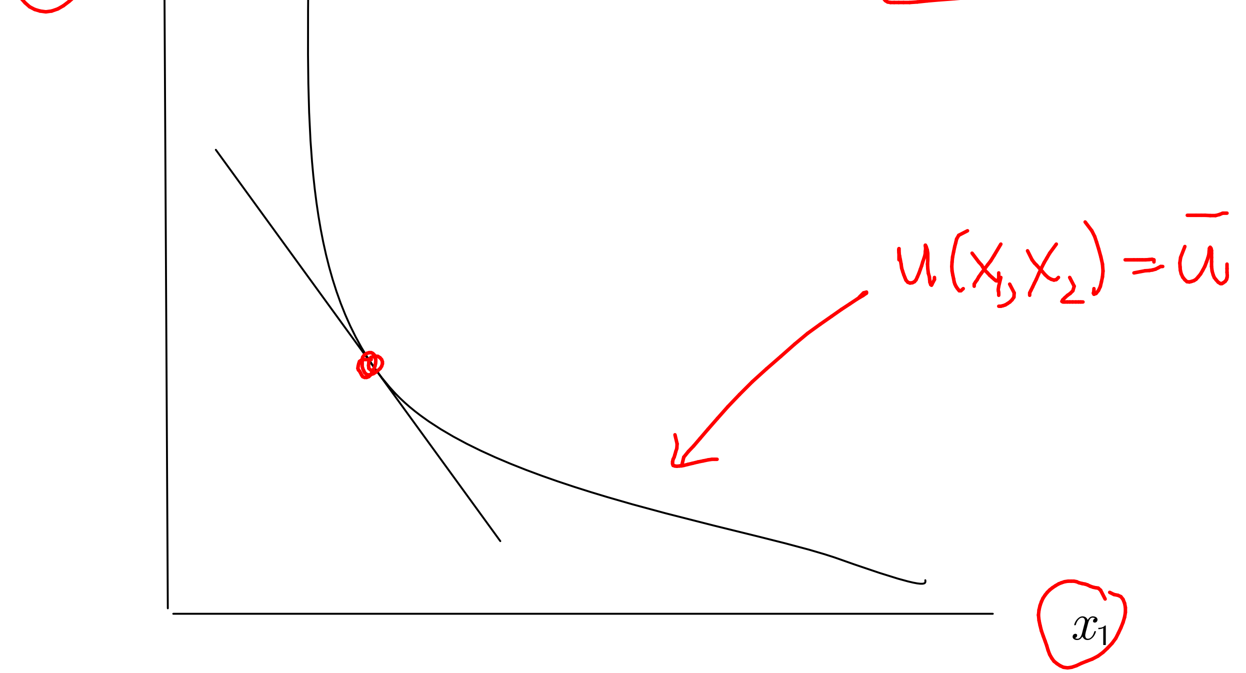
solution: $\frac{dx_2}{dx_1} = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2}$

implicit function theorem

example: Cobb-Douglas utility $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, $\alpha, \beta > 0$

$$\frac{dx_2}{dx_1} = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}} = -\frac{\alpha x_2}{\beta x_1}$$

$$\boxed{x_1^{1/2} \; x_2^{1/2}}$$



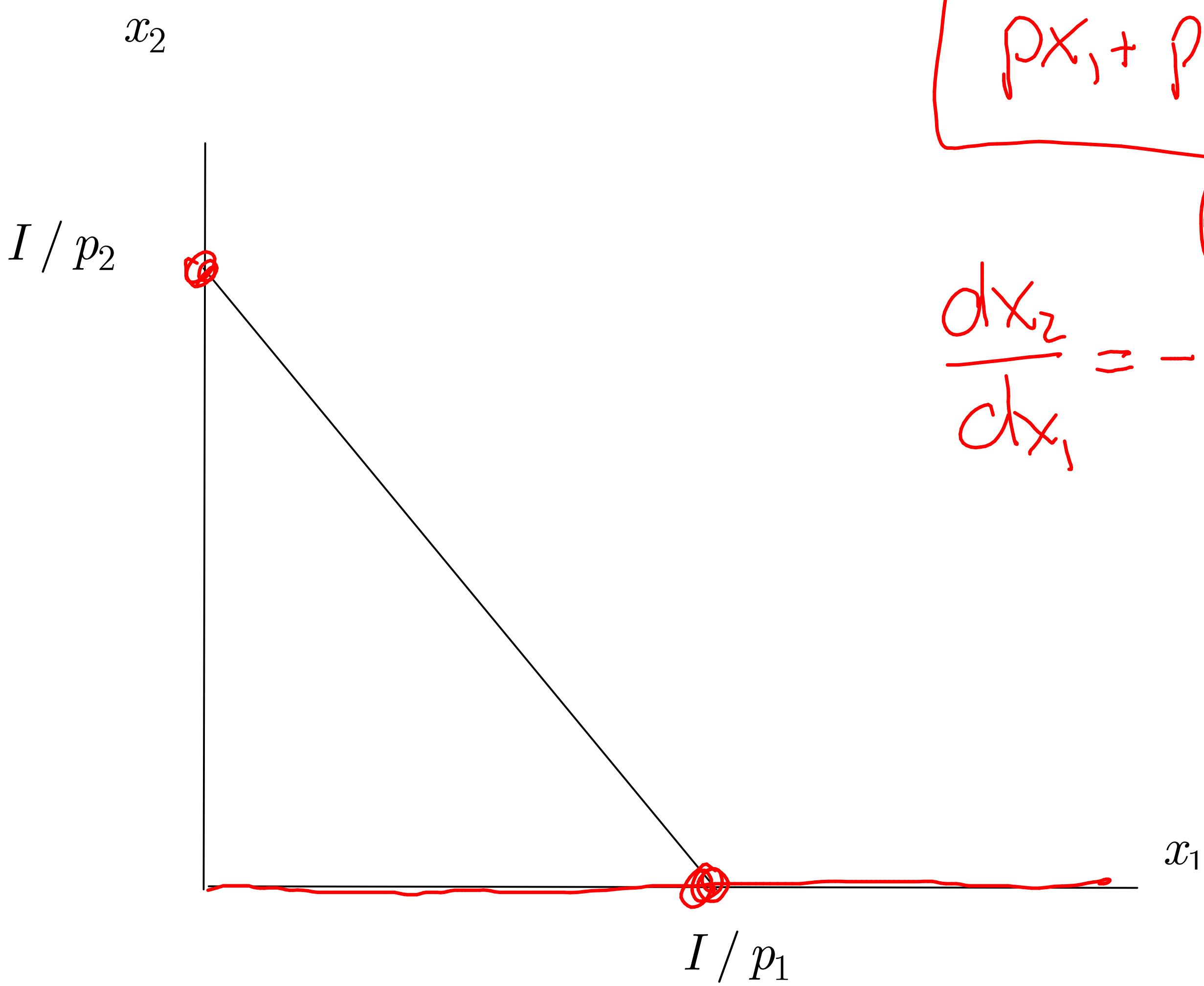
slope of the budget line: $-p_1 \,/\, p_2$

$p x_1 + p_2 x_2 \leq I$

$\boxed{p x_1 + p_2 x_2 = I}$

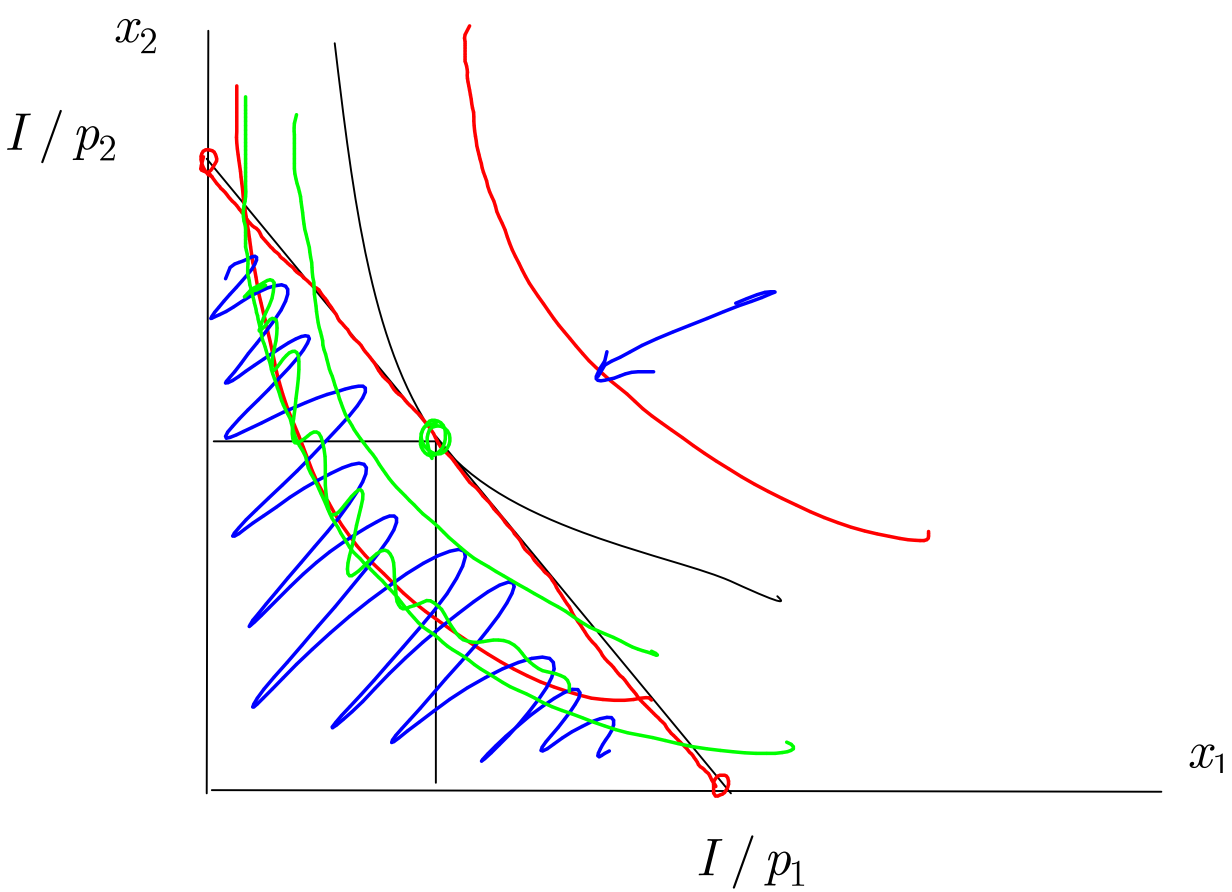
$p_1 x_1 + p_2 x_2$

$\frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$

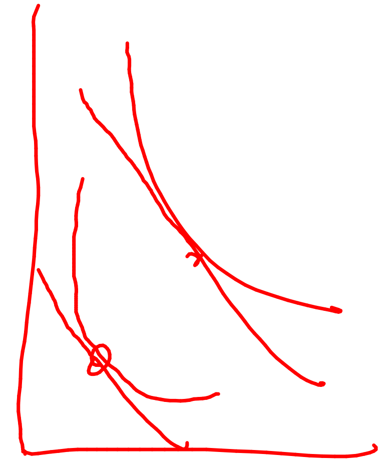


The Optimum

tangency between the budget line and indifference curve



Cobb-Douglas



$$-\frac{\alpha x_2}{\beta x_1} = -\frac{p_1}{p_2}$$ tangency condition of equal slopes

$$p_1 x_1 + p_2 x_2 = I$$ budget constraint holds with equality

two equations in two unknowns

solve the tangency $p_2 x_2 = (\beta/\alpha) p_1 x_1$

plug in the budget constraint $p_1 x_1 + (\beta/\alpha) p_1 x_1 = I$

solve for the demand function

$$\boxed{x_1 = \frac{\alpha}{\alpha + \beta} \frac{I}{p_1}}$$

bigger α the more you like x_1 and the more you demand

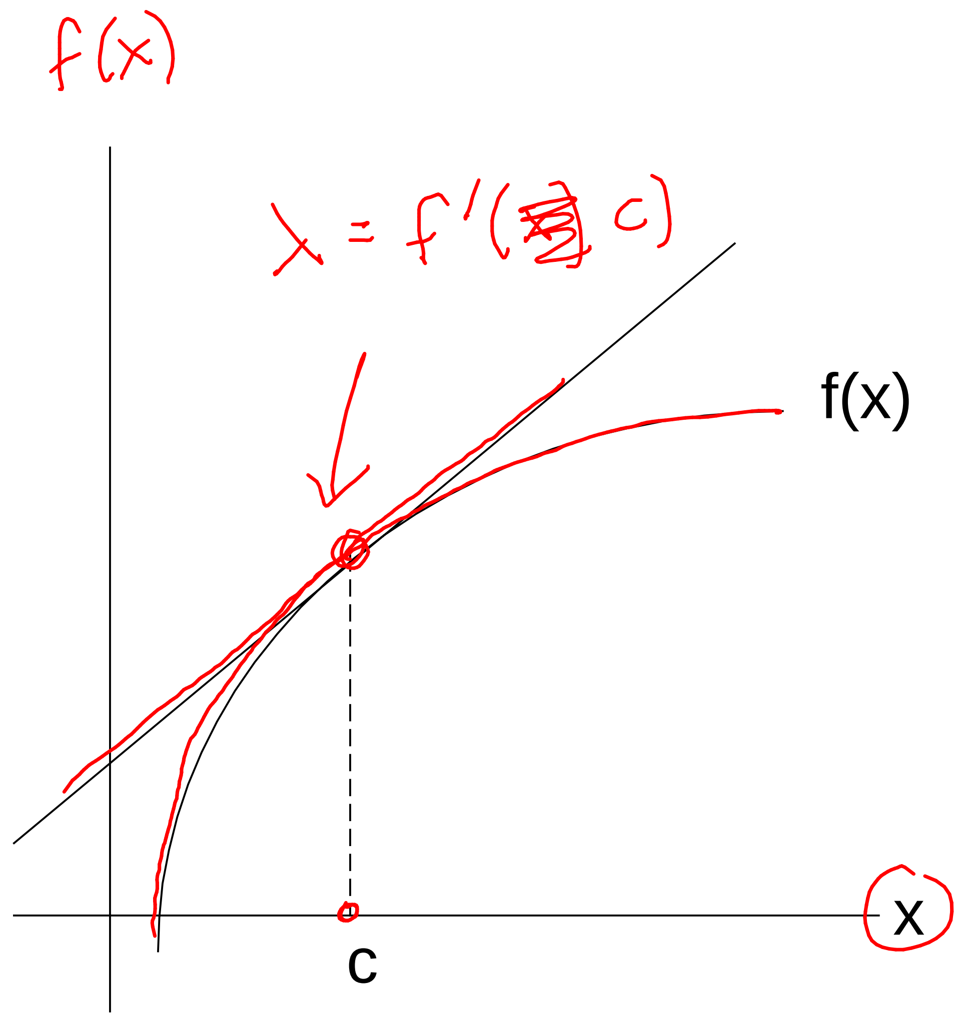
homogeneity of degree zero: depends only on the ratio I/p_1

$x_1^{\alpha} x_2^{\beta}$

Lagrange Multipliers

$\max f(x)$ subject to $x \leq c$

obvious answer: if f is concave and $f'(c) > 0$ take $x = c$



$f(x) \le c$

$\boxed{x = c}$

$\boxed{f(x) - \cancel{\lambda(x)}}\ \boxed{\lambda \cdot x}$

$f'(x) - \lambda = 0$

Lagrange Multipliers

$$L = f(x) - \lambda(x - c)$$

first order conditions for a maximum

$$\frac{\partial L}{\partial x} = f'(x) - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = -(x - c) = 0$$

solution: $x = c, \lambda = f'(x)$

what is measured by the multiplier λ?

the increase in the objective when the constraint c is relaxed

Example

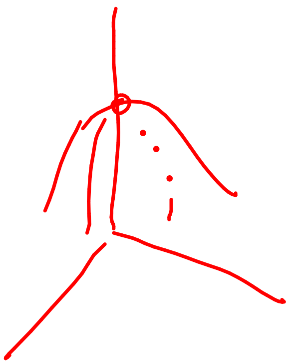
max $(x_1{}^\rho + x_2{}^\rho + x_3{}^\rho)^{1/\rho}$

subject to $p_1x_1 + p_2x_2 + p_3x_3 = I$

$$\boxed{p_1x_1 + p_2x_2 + p_3x_3 - I} \le 0$$

Lagrangean

$$L = (x_1{}^\rho + x_2{}^\rho + x_3{}^\rho)^{1/\rho} - \lambda(p_1x_1 + p_2x_2 + p_3x_3 - I)$$



solution:

$\frac{\partial u}{\partial x_i}$

how many

$$\frac{\partial L}{\partial x_i} = \frac{1}{\rho}\left({x_1}^{\rho} + {x_2}^{\rho} + {x_3}^{\rho} \right)^{(1/\rho)-1} \rho {x_i}^{\rho-1} - \lambda p_i = 0$$

$p_1 x_1 + p_2 x_2 + p_3 x_3 = I$

solve:

$$x_i = \left[\frac{\lambda p_i}{\left({x_1}^{\rho} + {x_2}^{\rho} + {x_3}^{\rho} \right)^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}}$$

$$= \left[\frac{p_i}{\left({x_1}^{\rho} + {x_2}^{\rho} + {x_3}^{\rho} \right)^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} [\lambda]^{\frac{1}{\rho-1}}$$

substitute in constraint

$$\sum_i p_i^{\,1+\frac{1}{\rho-1}} \left[\frac{\lambda}{(x_1{}^{\rho} + x_2{}^{\rho} + x_3{}^{\rho})^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} = I$$

$$[\lambda]^{\frac{1}{\rho-1}} = \frac{I}{\sum_i p_i^{\frac{\rho}{\rho-1}} (x_1{}^{\rho} + x_2{}^{\rho} + x_3{}^{\rho})^{\frac{1}{\rho}}}$$

substitute back into solution of FOC

$$x_i = \left[\frac{p_i}{(x_1{}^{\rho} + x_2{}^{\rho} + x_3{}^{\rho})^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} \frac{I}{\sum_j p_j{}^{\frac{\rho}{\rho-1}} (x_1{}^{\rho} + x_2{}^{\rho} + x_3{}^{\rho})^{\frac{1}{\rho}}}$$

$$= [\, p_i \,]^{\frac{1}{\rho-1}} \frac{I}{\sum_j p_j{}^{\frac{\rho}{\rho-1}}}$$

this is the *CES* demand function widely used in empirical research

note that it satisfies homogeneity of degree zero in prices and income