

Knowledge Disclosure As Intellectual Property Rights Protection

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Abstract

This paper addresses in a duopoly framework the incentives to disclose information about an innovation (a trade secret) that a firm has elected not to patent. The following features are crucial to the disclosure decision: The trade secret holder might be excluded from using her innovation if a second inventor were to obtain a valid patent for a similar technology. Because patent applications are reviewed in light of the prior art, rival patents are more difficult to obtain the more the trade secret holder discloses. But disclosures to invalidate a subsequent patent must convey technical knowledge that the rival can freely use in her rediscovery activities. My analysis leads to following conclusions: First, a more profitable trade secret may result in a higher equilibrium level of disclosures. Second, a more permissive patent policy has an ambiguous effect on the disclosure decision. Third, the more complex the innovation is the smaller are the incentives to disclose useful information. Finally, by comparing the equilibrium level of disclosure with the level that maximizes (expected) total surplus, I show that there may be too much or too little disclosure in equilibrium.

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1 Introduction

Secrets and patents are two of the most common means inventors employ to protect their intellectual assets. Empirical evidence - see Cohen, Nelson and Walsh [2] for instance - strongly suggests that secrecy has become one of the most heavily employed mechanisms in several industries in the US. Secrecy is widely used, for instance, in semiconductors, machine tools, aerospace and chemical equipments. Lerner [4], examining the importance of various methods of intellectual property protection for a sample of manufacturing firms, reports that for smaller firms trade secret protection is critical for appropriating the returns of their R&D programs.¹ Although patenting is viewed as less important than secrecy, patents among large firms have the highest effectiveness score as appropriation mechanisms. The bottom line is that for some type of firms patents are effective as a means to capture innovative rents, while for others secrecy is the best option to protect their intellectual endeavors.

In spite of their increasing importance, the major drawback of secrets is that innovators cannot prevent subsequent firms from independently rediscovering the invention, patenting it and knocking the first inventor out of the market.

In this paper I focus on the defensive maneuvers that owners of trade secrets employ to mitigate the risks of patenting activities conducted by subsequent inventors. In particular, I examine the decision of a trade secret owner concerning disclosures of an innovation to invalidate potential future patents for competing technologies. Thus by analyzing how the rights of patentees to exclude “prior” - rather than “subsequent” - inventors affect the decision to disclose an innovation, I adopt a different perspective from the innovation literature².

¹Lanjouw and Schankerman’s [3] found that the burden of enforcing patent rights is more severe for certain type of patentees. They suggest that the benefits of litigating one patent spill over to the protection of other patents through reputation effects. Therefore, high enforcement costs appear to be a prevalent feature for small startup firms with insufficient experience in dealing with patent disputes.

²To the best of my knowledge, only Denicolo and Franzoni [6] have studied the relationship between patents and secrecy. Nevertheless, their focus is on optimal patent design when innovators can rely on secrecy to protect their innovations.

Three features of the economic and legal environment are crucial to understand the disclosure decision. First, trade secret users might be excluded from practicing their innovations if second inventors obtain a valid patent for similar technologies. Second, the fact that patent applications are reviewed in light of the prior art provide incentives to first inventors to strategically disclose some details of their findings in order to enlarge prior art.³ Third, disclosures to invalidate subsequent patents must also convey technical knowledge that competitors can freely use in their rediscovery activities. The simultaneous problem of disclosing to enlarge prior art at the cost of enhancing the duplication capabilities of a rival is the heart of this paper.⁴

I frame my analysis in a duopoly model where one firm, the innovator, has elected to keep her innovation a secret and a second firm invests in duplication activities to obtain a similar invention. If the rediscovery process is a success, the second firm will attempt to patent the invention. This built-in asymmetry between the firms' choice of intellectual property protection might seem, at first, a little strange because it is natural to think that the first inventor should also rely on patents. Nevertheless, this asymmetry is based on the stylized facts about the remarkable heterogeneity of intellectual property protection in most industries: secrets are widely used by firms whose costs of detecting misappropriation and enforcing their patent rights are significant. Besides, it is not the purpose of this paper to study the choice of intellectual property protection. And from a formal point of view, this assumption allows me to isolate the major drawback of using secrets to capture innovative rents.

Given my simple duopoly framework, I show that a pure strategy equilibrium exists and establish its fundamental properties. In particular, my analysis leads to five main

³Prior art can be considered as all the public knowledge that existed prior to the filing of a patent application

⁴It is important to notice that the model and results below are valid for cases in which knowledge is protected either through trade secrets - legal rights - or informal secrecy - de facto rights -. If knowledge were protected by trade secret law, in the US, subsequent inventors' patents would be considered valid by courts in most cases and first inventors do not have prior user rights. Even if first inventors had prior user rights, they should also demonstrate that they had discovered the innovation earlier. In most cases, a printed publication may be enough for this purpose.

results. First, a higher premium for technological leadership - that is a higher difference in the profits obtained by being the exclusive user of the innovation compared to a duopolistic exploitation of it - may result in a higher equilibrium level of disclosures. The intuition behind this somewhat paradoxical result is easy to understand. A higher premium weakens the incentives to disclose by increasing the forgone profits when the rival duplicates. Nevertheless, a greater market premium, by making replication efforts more profitable, increases the rediscovery probability. This improves the benefits of disclosing, because disclosures only benefit the first inventor when her rival duplicates the invention.⁵ Under these circumstances an increase in the market premium may lead to a higher disclosure level.

Second, the greater the additional profits the innovator reaps by successfully avoiding a rival's patent, the higher the equilibrium level of disclosures. This result confirms the intuition that in environments where competition is less intense the room for information transmission is enlarged.

Third and counterintuitive, a more permissive patent policy - that is the chances of the second inventor of obtaining a valid patent are greater for any given disclosure level - has an ambiguous effect on the equilibrium level of disclosures. The intuition behind this is as follows. On one hand, disclosures become more costly because their effect on the second firm's patenting possibilities is diminished. But on the other hand, a "pro-patent" shift in public policy induces higher duplication efforts. This acts as an opposing force to the first one, by increasing the attractiveness of disclosures.

Fourth, an increase in the technical complexity of the innovation causes the equilibrium level of disclosures to diminish. The reason for this is twofold. First, the second inventor will react devoting fewer resources to duplication activities because - for more complex innovations - her rediscovery cost increases. Therefore, by diminishing the threat of duplication, a more technically complex innovation weakens the importance of disclosures

⁵This is obviously true because if the second inventor duplicates the innovation, he will try to obtain a patent.

to dissuade patenting activities. Second, the marginal cost of disclosing is also higher because, for a more difficult technical problem, the contribution of the knowledge contained in the disclosure becomes more productive for duplication activities.

Finally, I compare the equilibrium level of disclosure with the level that maximizes (expected) total surplus. I show that in equilibrium there may be too much or too little disclosure. This can be easily explained for the particular case of a drastic innovation.⁶ In this case, the natural intuition is that given the total surplus does not change whether duplication is a success or not, disclosures have no “social” benefits but only effects on the distribution of property rights between the firms. Thus an appealing conjecture is that in equilibrium there is too much disclosure. Nevertheless this intuition is incorrect. Disclosures have also positive external effects because they decrease the costs associated with duplication activities and as a result the equilibrium may involve too little disclosure. Also, notice that the result that, under some circumstances, there is too much disclosure contradicts the natural conjecture that knowledge, as a pure public good, should be completely shared between the firms.

The model and results below rest on the basic notion that the validity of the second inventor’s patent is strongly affected by prior art and judiciary discretion. In general, establishing that a patent is invalid amounts to showing that the invention that this protects is not novel. Courts have ruled that, in some circumstances, a printed publication accessible to the public is enough to invalidate a patent. Allison and Lemley [1] found that once a patent has been issued, the chance that a court will hold it valid is only slightly better than even. Also, the majority of the grounds for invalidity are rooted in prior art. At the other extreme, in *Gillman v. Stern*, see Merger [5], the court held that the inventor did not take any steps to make his invention publicly known. The court concluded the invention could not be considered prior art and did not invalidate the second inventor’s patent. Similarly, in *Gore v. Garlok* the judiciary decision was that the

⁶In this context, an innovation is “drastic” if when only one firm uses it, then market competition results in monopoly.

secret commercial exploitation of a new process did not invalidate a subsequent patent. Nevertheless, in *Dunlop v Ram*, see Denicolo and Franzoni [6], the court invalidated a second inventor’s patent based on the fact that the product manufactured by the prior user had been distributed earlier to the public.

The extent of the enabling knowledge, contained in the disclosure, is the other key ingredient to understand the revelation decision. A recent series of newspaper articles anecdotally describes the widespread use of this tactic to protect innovations.⁷ In particular, it is underscored that to dilute the transfer of enabling knowledge to competitors “*firms often publish anonymously, and they sometimes use vague language to describe an invention*”. Nevertheless, the risks associated with this practice are that “*...if competitors are unable to understand an idea, there is a good chance that patent examiners will not either*”. Another example of the disclosure - duplication game firms play in innovative markets is offered by Milgrim [7]. A French company developed and kept secret a process for producing cellophane. Du Pont spent many years and millions of dollars trying to replicate or develop a similar technology. Finally, Du Pont gave up and obtained a license from the French company. As this illustrates, the technical complexity of innovations is an obstacle that might result in the most important barrier to duplicate new techniques.

Related Theoretical Studies

The role of disclosures in innovative settings has also been analyzed by Anton and Yao [1]. In their model, using a strategic substitute setting, knowledge is disclosed in a patent of uncertain validity. The main forces that shape strategic disclosures are the extent of the knowledge potentially transferred to the competitor and the signal that the patent sends about the total private knowledge relevant for downstream competition. My model has several different features. First, disclosures are made by a trade secret owner only to vitiate the validity of future patents. Second, in my setting information is complete

⁷ “Suddenly, ‘Idea Wars’ Take On a New Global Urgency” The New York Times.....and “Protecting Intellectual Property” The New York Times 02/18/2002.

and symmetric and the incentives to disclose are not crucially affected by the nature of downstream competition.

Also, Anton and Yao [2] examine the choice between patents and secrecy to protect a process innovation. The amount of the information disclosed in the patent is one of the crucial ingredients to understand the property right choice. One of the main conclusions is that large or economically important innovations are protected through secrecy when property rights are weak. Although my focus is not on the choice of intellectual property rights, my model suggest that under some circumstances the incentives to disclose are enhanced for important innovations. This result is due to the fact that, in my model, subsequent firms can rediscover the invention, patent it and knock the first inventor out of the market.

Battacharya and Ritter [3] study an environment in which partial disclosure of technical information reduces the cost of capital to a firm competing in a R&D race, but generates additional entry into that race. My model differs from Battacharya and Ritter by focusing on disclosures of technical information about an innovation that has already been discovered but that might be subsequently duplicated.

Green and Scotchmer [7] focus on the impact that the requirements of novelty and nonobviousness have on both the incentives to innovate and to disclose intermediate discoveries in a multistage race. In their case, partial disclosure is not allowed, and knowledge can be interpreted as an indivisible commodity. Furthermore, they conclude that firms may be reluctant to disclose intermediate discoveries because they can not appropriate the cost advantages that disclosure provides to competitors in subsequent stages. Although, I do not analyze a multistage race, in my model the firm's incentive to disclose are rooted in the possibility of invalidating future patents by strategically creating prior art, an aspect that is absent in their analysis.

Denicolo and Franzoni [6] analyze optimal patent design when innovators can rely on secrecy to protect their innovations. And even though they do not consider the possibility of disclosures by trade secret owners, the heart of their paper and mine is the rights granted

by patents to exclude “prior” rather than “subsequent” inventors. Finally, Severinov [8] also investigates the issue of information sharing in R&D context through information exchange between employees. He focuses on the incentives of firms, in a duopoly game, to regulate communication flows through incentive contracts. My problem differs from Severinov in several aspects. First, I do not consider issues related to agency problems and in my model only one firm - the one that has superior knowledge - has the option to disclose. Second and important, my results are significantly different from his findings. To illustrate, he finds that a higher premium for technological leadership always provide incentives to withhold information transmission. Nevertheless, I find that under some cases a higher technological premium may result in a higher equilibrium level of disclosures

The rest of the paper is organized as follows. Section 2 sets forth the basics of my model. In section 3 the existence of equilibrium is established. In section 4, I characterize the equilibrium and discuss the economic properties of the disclosure strategies. Section 5 characterizes the level of disclosures that maximize expected total surplus. Finally, section 6 contains concluding remarks.

2 The Model

A firm, denoted by A , has obtained a certain innovation. It might be either a new technique that lowers the unit cost of production or an innovation that improves the quality of a given commodity.⁸ I start by assuming that A has elected trade secrets to protect her innovation. A second firm, denoted by B , could duplicate or rediscover the innovation by using a stochastic invention technology. To capture the fundamental trade-off discussed in the Introduction, I suppose that if the innovation were successfully duplicated, B would attempt to obtain a patent. This basic asymmetry between the players’ choice of intellectual property protection allows me to understand the basic interactions between patents, secrets and disclosures.

⁸Secrecy is often used for process rather than for product innovations. In this sense, the model describes more closely a process innovation case.

Firm A must choose a disclosure level, d , from her feasible disclosure set, $[0, 1]$. Disclosures may be interpreted, for instance, as technological information regarding the newly innovated technique. Thus $d = 0$, indicates that A has chosen to keep her innovation entirely hidden. At the other extreme, when $d = 1$, the understanding is that the best possible description of the innovation has been made, given the available publication technologies - i.e. printed publications, photographs, drawings, etc.-. Partial knowledge transfers, $d \in (0, 1)$, are similarly interpreted.

In the duplication phase, B has a unique opportunity to use a stochastic technology to successfully duplicate the innovation. Naturally, the probability of duplication depends on the innovative efforts taken by firm B . Nevertheless, the following reparametrization is a useful shortcut for both simplifying the model and directly emphasizing the role of the duplication probability in the determination of equilibrium disclosures. To that purpose, I consider that, instead of efforts, B directly chooses the duplication probability, $f \in [0, 1]$. If B 's duplication activities are successful, she will try to obtain an exclusive right to practice the technology, by relying on patents.

Patents are granted or alternatively they are found valid in courts if the inventions protected by them are novel and exhibit a sufficient "inventive step" - i.e. are non-obvious -. An invention is considered new if it is not anticipated by prior art, that is if it is not anticipated by all the knowledge that existed prior to the filing of a patent application whether it existed by way of written or oral disclosure. The requirement of "inventive step" conveys the idea that, for owning a valid patent, it is not enough that the claimed invention is different from what existed in prior art - be novel - but that this difference must also be noticeable. Although, both novelty and obviousness are difficult to prove, the validity of a patent depends on the set of all relevant disclosures prior to the filing of a patent application. Therefore, by disclosing some elements of her innovation, A can strategically affect the possibilities to B of obtaining a valid patent.

In this paper, I assume that prior to market competition a governmental decision maker - i.e. (PO) - determines the validity of patents. After that decision, every residual

uncertainty about patent validity vanishes: if the patent is found valid, B will exclude A from using her innovated technology in the market competition game.⁹

Timing of the Disclosure - Duplication (DD) Game

The order of play is as follow,

Stage 1

A chooses a disclosure level, $d \in [0, 1]$

Stage 2

B , after observing d , chooses a duplication probability f .

Stage 3

If B rediscovers the innovation, an expert - PO - observes a signal of the true disclosures made by A . Then a resolution of property rights is made by using a commonly known property right rule.

Stage 4

A and B engage in duopoly competition.

Equilibrium. The natural equilibrium concept for the DD game is subgame perfect Nash equilibrium (SPNE). I concentrate on pure strategy SPNE. Hence, strategic options are as follows. A strategy for firm A is a choice of disclosure, d , from the feasible disclosure set $[0, 1]$. A strategy for B is a real-valued function $f : [0, 1] \rightarrow [0, 1]$. The pair of strategies (d, f) is a SPNE for the game if it induces a Nash Equilibrium in every subgame of DD game

2.1 Market Competition Stage

The payoffs firms obtain in the market competition game depend on the duplication and patenting outcomes. In general, each possible event is represented by a pair (D, P) , with

⁹Notice that although I do not make any distinction between the process of awarding patents and the challenges to their validity frequently made by competitors, the modeling assumption that the decision about a patent's validity is considered before market competition corresponds, for instance, to a patent reexamination request. This type of procedure deals mainly with issues of patentability related to prior patents and printed publications.

the first element in the pair standing for the result of the duplication activities and the second one for the property right resolution. For example, the pair (S, F) represents the event in which duplication is a success, but B fails to obtain a valid patent. The following table represents the firms' payoffs for each possible event:

Events	Profits to A	Profits to B
(S, S)	π_{12}	π_{21}
(S, F)	π_{22}	π_{22}
(F, F)	π_{21}	π_{12}

Notice that for disclosures to be possible in equilibrium, the profits for firm A when both competitors use the technology, π_{22} , must be strictly greater than the profits the firm obtains when she is excluded from exploiting the innovation, π_{12} . Therefore, I assume that the payoffs satisfy the following ordering structure: $\pi_{21} > \pi_{22} > \pi_{12} \geq 0$. This payoff structure is satisfied, for instance, for the cases of Cournot and Stackelberg leadership competition. However, these conditions are not met for the case of price competition.

2.2 Patent Decisions Stage

Faced with a property right resolution, I assume that PO observes a signal of the true disclosures made by A . This observation is given by

$$s = d + v$$

where $v \in [a, b]$ and hence $s \in \Sigma = [d + a, d + b]$.

The assumption that PO observes a signal of the true disclosures captures the idea that the resolution of a patent's validity is based not only on disclosures but also on other important legal features, such as previous ruling in similar disputes and judiciary discretion¹⁰.

¹⁰ Alternatively, it is easy to imagine that some intrinsic measurement error is involved in the observation of disclosures made by the patent office and courts.

A patent decision is chosen from the set of possible property rights resolutions $\Omega = \{V, NV\}$, where V - i.e. validity - must be understood as a resolution in favor of a valid patent for B . An element of Ω is chosen using a property right rule, $P : \Sigma \rightarrow \Omega$ with the following features

$$\begin{aligned} P(s) &= V \text{ if } s \leq \bar{s} \\ &= NV \end{aligned}$$

Notice that the decisions about a patent's validity are crucially influenced by both the “measurement error” v , and the exogenously given cutoff value, $\bar{s} \geq 0$. Bigger values for v might be interpreted, for example, as positive opinions about disclosures and hence as “good” news for A . On the other hand, a bigger \bar{s} reduces the strategic effects of disclosures on property rights decisions.

When A and B must execute their plans, their information about the measurement error, v , is incorporated in the following common prior distribution, $G_V(v)$ for $v \in [a, b]$ with associated density $\frac{\partial G_V(v)}{\partial v} = g_V(v)$.

The following lemma formalizes, in terms of stochastic dominance, the intuition that higher disclosures lower the chances to B of obtaining a patent.

Lemma 1. a) For a fixed $d \in [0, 1]$, the probability firms attach to the event $P(s) = V$ is given by the cdf $\gamma(d; \bar{s}) = G_V(\bar{s} - d)$.

b) The location family of cdfs $\{G_V(s - d), d \in [0, 1]\}$ is stochastically increasing in d .

Proof: See the Appendix.

I suppose that $a < \bar{s} < 1 + b$. This assumption is made to avoid trivial cases. For example, if $\bar{s} \leq a$, then no matter the level of disclosures, B will never be able to get a patent.

More important, I assume that

Assumption 1. $\gamma(d; \bar{s})$ is a C^2 convex function of d and $\frac{\partial \gamma(d; \bar{s})}{\partial \bar{s}} > 0$.

Additionally, I impose on $\gamma(d; \bar{s})$ the following two properties,

Property 1. $\gamma(d = 0; \bar{s}) \leq 1$, and

Property 2. $\gamma(d = 1; \bar{s}) \geq 0$

Property 1 says that even when there are no disclosures, the probability of obtaining a patent might be less than one. Similarly, property 2 maintains that full disclosures are not enough to completely prevent property rights resolutions in favor of B . Finally, notice that together, properties 1 and 2, determine that $\bar{s} \in [1 + a, b]$.

Example 1. Suppose that beliefs about the random component, v , are uniformly distributed between a and b and let $\beta \equiv b - a$. Then, it is easy to check that properties 1 and 2 are satisfied if and only if $\beta \geq 1$. In this simple case, the probability of patenting is given by $\gamma(d; a, b, \bar{s}) = 1 - (\beta)^{-1}(d + b - \bar{s})$ for $1 + a \leq \bar{s} \leq b$. And in the special case of $\beta = 1$, we have the simple linear probability function $\gamma(d) = 1 - d$.

2.3 Duplication Stage

The process of duplication involves a choice of R&D efforts by firm B . Equivalently, as I discussed before, it can be considered that instead of efforts, B chooses directly the probability of rediscovery, $f \in [0, 1]$. Greater knowledge, emerging from information disclosures by A is valuable in that it reduces the private cost to B of achieving a given probability of duplication. Let $C(f, d)$ denote the private cost to B of achieving a duplication probability f , when the operating disclosure is d . $C(f, d)$ is assumed to be a differentiable, non-negative, increasing and strictly convex function¹¹, i.e. $\forall (f, d) \in [0, 1] \times [0, 1]$ $C(f, d) \geq 0, C_f(f, d) \geq 0, C_{ff}(f, d) > 0$ and also that $\forall d \in [0, 1]$ $C(0, d) = 0, C_f(0, d) = 0$.

With a higher disclosure level, it is assumed that both the total and marginal cost to B of achieving a given f are lower, i.e. $\forall f \in [0, 1]$ $C_d(f, d) < 0, C_{fd}(f, d) < 0$.

After observing any $d \in [0, 1]$, B chooses the duplication probability, f , to maximize her expected payoff. Notice that with probability $(1 - f)$ duplication is a failure and the profits B obtains are equal to π_{12} . With complementary probability, f , B duplicates

¹¹ All derivatives are denoted by subscripts.

the innovation. In this case, two different situations are possible. First, with probability $\gamma(d; \bar{s})$, she gets a patent and appropriates π_{21} . And second, with probability $(1 - \gamma(d; \bar{s}))$, both firms use the innovation. In this case, both of them get π_{22} . Thus, if B duplicates the innovation, her expected payoff is $\gamma(d; \bar{s})\pi_{21} + (1 - \gamma(d; \bar{s}))\pi_{22}$. Now, notice that $R^B(d, \pi, \bar{s}) = \gamma(d; \bar{s})\pi_{21} + (1 - \gamma(d; \bar{s}))\pi_{22} - \pi_{12}$ - where $\pi = (\pi_{21}, \pi_{22}, \pi_{12})$ - represents the (expected) extra profits the second inventor obtains in the case of successful duplication. Given that the firm gets π_{12} and has to pay the positive duplication costs, $C(f, d)$, whether duplication is a success or not, the expected payoff for firm B can be written as,

$$EU^B = \pi_{12} + fR^B(d, \pi, \bar{s}) - C(f, d)$$

To avoid corner solutions when $d = 0$, I assume both that the maximum value function $W(d, \pi) = \max_{f \in [0,1]} \{R^B f - C(f, d) \mid d = 0\} > 0$ and $C_f(1, 0) = \infty$.

More important is the following assumption,

Assumption 2. $C_f(1, 1) > R^B(d, \pi, \bar{s})$ for all $d \in [0, 1]$

This assumption captures the idea that even with full disclosure, B would optimally choose an $f < 1$. In other words, this assumption guarantees that selecting to rediscover the innovation with probability one is not profitable from an economic point of view, because expected returns are relatively small with respect to the duplication costs¹².

Under this assumption, the optimal duplication probability by B is determined by the following necessary and sufficient first order condition (FOC),

$$\gamma(d; \bar{s})(\pi_{21} - \pi_{22}) + (\pi_{22} - \pi_{12}) - C_f(f, d) = 0 \quad (1)$$

Notice that $(\pi_{21} - \pi_{22})$ are the extra profits firm B attains by excluding her competitor in the use of the innovation. In other words, this is the most B would pay to avoid sharing

¹² Assumption 1 can easily be relaxed without altering any of our results. If there were a $\bar{d} \in (0, 1)$ such that for $\forall d \geq \bar{d}$, $f^* = 1$, then A would not choose any $d \in [\bar{d}, 1)$. Hence optimal disclosures, $d^* \in [0, \bar{d}) \cup \{1\}$ and the same analysis done in sections 4 and 5 completely holds.

the new technology with A . From B 's point of view, this is the marginal contribution of the patent, i.e. $(\pi_{21} - \pi_{22}) = \Delta_P$. Hence $\gamma(d; \bar{s})(\pi_{21} - \pi_{22}) = E\Delta_P$ is the private expected marginal contribution of the patent. On the other hand, $(\pi_{22} - \pi_{12})$ are the extra profits B appropriates by using the innovation when A is also exploiting it. From B 's perspective, $(\pi_{22} - \pi_{12})$ is the marginal contribution of the invention, i.e. $\pi_{22} - \pi_{12} = \Delta_I$. Therefore, the (FOC) is equivalent to

$$E\Delta_P + \Delta_I - C_f(f, d) = 0 \quad (2)$$

Equation (2) shows that B will increase the probability of duplication up to the point at which the joint (expected) marginal contribution of the patent and the innovation equals the marginal cost.

But the (FOC) also demonstrates that disclosures weaken the probability of obtaining a patent and therefore, they decrease $E\Delta_P$. Consequently, by releasing innovative knowledge, A changes both B 's private cost and benefits of investing in duplication activities. Lemma 2 formalizes these basic effects of disclosures.

Lemma 2. a) B 's best response exists and it is a C^1 function $f(d, \bar{s}, \pi)$ where $\pi = (\pi_{21}, \pi_{22}, \pi_{12})$.

b) It can be verified that

$$\frac{\partial f}{\partial d} = \frac{\gamma_d(\pi_{21} - \pi_{22}) - C_{fd}}{C_{ff}} = \frac{\frac{\partial E\Delta_P}{\partial d} - C_{fd}}{C_{ff}} \leq 0$$

and that for a given disclosure level,

$$\frac{\partial f}{\partial \Delta_P} > 0, \frac{\partial f}{\partial \Delta_I} > 0, \frac{\partial f}{\partial \bar{s}} > 0.$$

Proof: See the Appendix.

Part b) of Lemma 2 shows that the duplication probability could either be lower or higher as a result of increasing knowledge transfers. What yields this result is the

combination of two opposing forces: a positive technological spillover effect and a negative pecuniary effect. The first effect is captured by the lower marginal cost to B of obtaining any given f , i.e. by $-C_{fd}$, and the second one, $\frac{\partial E\Delta_P}{\partial d}$, correspond to a lower expected return to B because her chances of obtaining a patent are reduced when disclosures become higher.

In this paper, I concentrate on the case in which higher disclosures unambiguously improve the duplication probability. Hence it is assumed that,

Assumption 3. $\forall (f, d) \in [0, 1] \times [0, 1] \quad \frac{\partial E\Delta_P}{\partial d} - C_{fd} > 0$

For technical convenience, I also assume that,

Assumption 4. $C_{ffd} = 0$ and $C_{fdd} < 0$.¹³

This last assumption says, that the rate at which the marginal cost increases is independent of the level of disclosures and the rate at which the marginal cost decreases with disclosures is itself a decreasing function of innovative knowledge. The best response probability function for firm B is completely characterized in the following lemma.

Lemma 3. a) Under assumption 3, B 's best response, f , is a monotonically increasing function of disclosures.

b) Under assumption 4, f is a C^2 strictly convex function of disclosures.

Proof: See the Appendix

2.4 Disclosure Stage

Firm A must choose a disclosure level that belongs to the feasible disclosure set, anticipating B 's best response given by equation (1). In other words, A 's problem is

$$\begin{aligned} \max_K EU^A(f, d) &= \max_K [(1 - f)\pi_{21} + fR^A(d, \pi_{12}, \pi_{22}, \bar{s})] \\ K &= \left\{ (f, d) \in [0, 1] \times [0, 1] : \gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d) = 0 \right\} \end{aligned} \quad (3)$$

¹³I also assume to simplify calculations that $C_{fff} = 0$.

where $R^A(d, \pi_{12}, \pi_{22}, \bar{s}) = \gamma(d; \bar{s})\pi_{12} + (1 - \gamma(d; \bar{s}))\pi_{22}$. A 's problem reflects the basic trade-off of disclosing innovative knowledge: by releasing valuable information, the probability of exclusively exploiting the innovation, $(1 - f)$, is reduced. But if the discovery were independently obtained the chances of obtaining a bigger payoff, R^A , - i.e. not being excluded of using the improved technology in the market competition game - would be higher.

3 Equilibrium

In this section, I establish the existence of equilibrium for the DD game and analyze sufficient conditions under which the equilibrium involves partial information transmission. Although much of the characterization of the equilibrium is provided in the next section, here I discuss some basic insights about the economic principles that determine equilibrium disclosures.

The existence of an equilibrium for the DD is equivalent to establish that the problem for the disclosing firm - i.e. the problem defined in the disclosure stage - has a solution. The following is the basic existence theorem.

Theorem 1. **a)** A pure strategy subgame perfect Nash equilibrium exists.
b) Suppose that the payoff function of the disclosing firm, $EU^A(f, d)$, is quasiconcave, i.e. for all $d \in [0, 1]$ $\gamma_{dd}[\frac{(\pi_{21} - \pi_{22})}{(\pi_{22} - \pi_{12})} + \gamma(d, \bar{s})] \geq 2(\gamma_d(d, \bar{s}))^2$. Then the equilibrium is unique.¹⁴

Proof: See the Appendix

When does the equilibrium entails partial disclosures? Proposition 1 formalizes the basic economic principles under which null disclosures are impossible in equilibrium.

¹⁴Showing that $EU^A(f, d)$ is quasiconcave if and only if for all $d \in [0, 1]$ $\gamma_{dd}[\frac{(\pi_{21} - \pi_{22})}{(\pi_{22} - \pi_{12})} + \gamma(d, \bar{s})] \geq 2(\gamma_d(d, \bar{s}))^2$ is a straightforward proof that I omit.

Proposition 1. Let $\alpha \equiv \frac{(\pi_{21}-\pi_{22})}{(\pi_{22}-\pi_{12})}$. If at $d = 0$,

$$\begin{aligned} f &> \frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s})) \\ 0 &= \gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d) \end{aligned}$$

Then, absence of disclosures is impossible in equilibrium.

Proof: See the Appendix.

The second equation of Proposition 1 states that the f used in the first condition is B 's optimal response to a zero disclosure strategy played by A . More relevant, the first condition allows us to consider some of the factors determining whether the innovator will completely prevent disclosures or not.

What Proposition 1 confirms is the intuitive idea that a high probability of rediscovery, in the absence of disclosures, is a sufficient condition to avoid full secrets. In other words, owners of “easy” innovations - i.e. a sufficiently high f - are tempted to disclose, at least, partially their findings. Obviously, a high impact of disclosures over patenting possibilities, - i.e. a high $-\gamma_d$ - and a small marginal effect of leakages on the rediscovery probability - i.e. a low f_d - are factors that contribute to avoid complete secrets.

Also, this Proposition shows that a high probability of patenting, when there are no disclosures - i.e. a high γ - lowers the profitability of disclosures. This finding clearly illustrates the trade-off involved in the revelation decision: in the case of a sufficiently high γ , information revelation would transfer valuable commercial knowledge that with high probability could be later privatized by a competitor.

Finally, whether α deters or not information revelation should be carefully considered. A detailed explanation must await the next section.

Similar to Proposition 1, in the next proposition sufficient conditions are provided to avoid full information transmission.

Proposition 2. Let $\alpha \equiv \frac{(\pi_{21}-\pi_{22})}{(\pi_{22}-\pi_{12})}$. If at $d = 1$,

$$\begin{aligned}
f &< \frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s})) \\
0 &= \gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d)
\end{aligned}$$

Then, full disclosures are impossible in equilibrium.

Proof: See the Appendix.

The intuition behind Proposition 2 follows similar lines of reasoning to those of Proposition 1 and I do not provide any additional explanations.

In the rest of the paper, I concentrate on the case of partial disclosures. Notice that an interior equilibrium exist if and only if Propositions 1 and 2 hold simultaneously. In other words, for an interior equilibrium is necessary that $[\frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))]_{d=0} < f]_{d=0} < f]_{d=1} < [\frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))]_{d=1}$.¹⁵ When do these inequalities are satisfied? On one hand, notice that $f]_{d=0} < f]_{d=1}$ because by Lemma 3, the duplication probability, f , is a monotonically increasing function of disclosures. On the other hand, it is necessary that $[\frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))]_{d=1} > [\frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))]_{d=0}$. By simple algebraic manipulations, it can be shown that this last inequality is satisfied if, for instance, the ratio $\alpha/\gamma]_{d=0}$ is sufficiently high.¹⁶ For the rest of the paper, I assume that the functions f and γ and the payoff ratio α are such that the above inequalities are satisfied.

To study the interior equilibria is useful to introduce the following notation. The marginal benefit of disclosures will be denoted by MB_d and the marginal cost of disclosing by MC_d . The next Corollary, using Propositions 1 and 2, provides sufficient conditions for existence of an interior equilibrium and offers a full characterization of it.

¹⁵Obviously, the pair of constraints defining the best response of firm B must hold at both $d = 0$ and $d = 1$.

¹⁶Notice that $z_0 \equiv [\frac{f_d}{-\gamma_d}]_{d=0} < [\frac{f_d}{-\gamma_d}]_{d=1} \equiv z_1$ because (i) f is a monotonically increasing and strictly convex function of disclosures and (ii) γ is a decreasing convex function of disclosures. Then $[\frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))]_{d=0} < [\frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))]_{d=1}$ if and only if $(\frac{z_1}{z_0} - 1)\frac{\alpha}{\gamma]_{d=0}} + \frac{z_1}{z_0}\frac{\gamma]_{d=1}}{\gamma]_{d=0}} > 1$. This is satisfied, for example, if $\frac{\alpha}{\gamma]_{d=0}}$ is sufficiently high. Although this last condition is not necessary condition.

Corollary 1

Let $\Pi(d, \bar{s}, \pi) \equiv (\pi_{21} - \pi_{22}) + \gamma(d, \bar{s})(\pi_{22} - \pi_{12})$ and assume that Propositions 1 and 2 hold simultaneously. Then the necessary conditions for an interior equilibrium are

$$\begin{aligned} MB_d &\equiv -\gamma_d(d; \bar{s})f(\pi_{22} - \pi_{12}) = MC_d \equiv \frac{\partial f}{\partial d}\Pi(d, \bar{s}, \pi) \\ 0 &= \gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d) \end{aligned} \quad (4)$$

The SOC is provided in the appendix.

Proof: See the Appendix.

The second equation implicitly defines the best response of firm B to disclosures made by A and the first one defines the optimal disclosures chosen by A given the optimal duplication probability function chosen by B . Hence these two equations completely describe the properties of any interior SPNE.

An alternative way of characterizing an interior equilibrium is by using level sets or iso-profits curves for firm A . They describe all the combinations of f and d that keep (expected) profits for A fixed at some given level. Because an increase in f decreases the expected payoff to A , d must be increased to keep expected profits at the same level. In other words, the marginal rate of substitution between f and d is strictly positive and given by,

$$MRS_{fd} = - \left[\frac{f\gamma_d(d; \bar{s})}{\alpha + \gamma(d; \bar{s})} \right] > 0$$

In Figure 1, a typical iso-profit curve - for a payoff function of firm A strictly quasi-concave - is drawn. It is also shown the best response of firm B . At the equilibrium, A chooses the highest iso-profit curve subject to the constraint given by B 's best response. It is not difficult to imagine that the equilibrium is characterized by a tangency point of

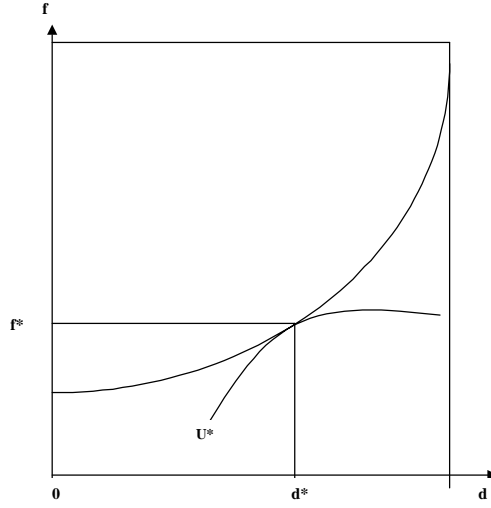


Figure 1:

the form,

$$MRS_{fd} = - \left[\frac{f \gamma_d(d; \bar{s})}{\alpha + \gamma(d; \bar{s})} \right] = \frac{\partial f}{\partial d}$$

And in this case, in which $EU^A(f, d)$ is quasiconcave, the equilibrium is unique.

4 Comparative Statics

In this section, I further study the economic properties of the equilibrium and I also underscore the economic motivations that lead to disclosures in the DD game. In particular, I focus in the following two important factors. First, I analyze how market competition shapes the optimal disclosure strategy. Second, the effect of patent policy on the optimal disclosure strategy is carefully addressed. For that purpose, it is useful to start by considering the economic principles that govern the optimal disclosure policy.

On the benefit side, Corollary 1 shows that the higher the optimal probability of

duplication, f , the bigger the impact of leakages on patenting possibilities, $-\gamma_d$, and the more important the profit incentives $(\pi_{22} - \pi_{12})$, the higher the innovator's gains from intentional leakages.

The following two observations underscore the intuition behind the above factors. First, disclosures are employed by firm A only with the purpose of dissuading patenting of similar competing innovations by B . But B can only apply for a patent if she replicates the innovation. Therefore a higher probability of duplication - i.e. a higher f - improves the returns to disclosing. Second, $(\pi_{22} - \pi_{12})$ constitutes the market incentives to release innovative knowledge by capturing the additional profits A obtains by successfully avoiding patenting activities. This is called the *preemptive effect*.

On the cost side, disclosures facilitate duplication by transferring enabling knowledge to B . This effect is formally captured by $\frac{\partial f}{\partial d}$. And given that duplication is an event with positive probability, the forgone (expected) profits for firm A are $\Pi(d, \bar{s}, \pi) \equiv (\pi_{22} - \pi_{12}) + \gamma(d; \bar{s})(\pi_{21} - \pi_{22})$.

This last expression shows that the *preemptive effect* also increases the disclosing costs. The explanation for this is rather simple: when the *preemptive effect* becomes higher, $(\pi_{21} - \pi_{12})$ also increases, making disclosures a less attractive alternative. But $\Pi(d, \bar{s}, \pi)$ is in addition determined by $(\pi_{21} - \pi_{22})$. This last expression represents the market rewards for being the technological leader and it is the most A would be willing to spend in protecting her trade secrets. This is called the *premium effect*. When the premium effect is high, the cost of releasing innovating knowledge is also significant; and the intuition is simple: the scenario in which A appropriates the benefits of having the technological leadership is less likely to happen when disclosures are high.

Now let me interpret $\alpha \equiv \frac{(\pi_{21} - \pi_{22})}{(\pi_{22} - \pi_{12})}$ as a market payoff ratio measuring the relative importance of being at the cutting-edge of technological advance compared to the benefits of preempting the rival's patenting activities. How does α shape the disclosure strategy? Does a more intense competition - i.e. a higher α - reduce the incentives to disclose?

Finally, let me emphasize that all the following comparative static results are obtained

by using the second order condition that must be satisfied close to any regular maximum.

Changes in the premium effect

How does a change in the premium for technological leadership affect disclosures? Intuitively, one expects that when α increases, due to a bigger market premium, disclosures should diminish: the innovator would be more conservative in her disclosure strategy under the threat of missing the chance of capturing higher rents for being the technological leader. Even though this logic seems appealing, Proposition 3 shows that changes in the premium effect have inconclusive consequences on disclosures.

Proposition 3. An increase in the *technological premium* has an ambiguous effect on the equilibrium level of disclosures d^*

Proof: See the Appendix.

The intuition for this proposition is as follows. On one hand, a higher *premium effect* weakens the incentives to disclose by increasing the cost of releasing knowledge in two different ways: by rising the forgone profits in case B duplicates - i.e. by increasing $\Pi(d, \bar{s}, \pi)$ - and by augmenting the marginal effect of disclosures on the duplication probability. This last effect is just captured by $(-\frac{\partial}{\partial \Delta_P} \frac{\partial f}{\partial d})$, and it is always negative. These two effects by reinforcing to each other make disclosing a less attractive alternative.

However, on the benefit side, increases in the *premium effect* make duplication more attractive to B , thereby rising the duplication probability. This last channel, represented by $\frac{\partial f}{\partial \Delta_P} > 0$, unambiguously improve the benefits of intentional leakages, because the state of the world in which disclosures are useful - i.e. duplication - becomes more likely.

Under these countervailing effects is impossible to rule out the case in which increases in the *market premium* - i.e. a more virulent market competition in the sense of a higher α - leads to higher knowledge transmission. Conditions that contribute to this provoking result include a relatively moderate effect of disclosures over the duplication probability compared to the reaction of the duplication efforts to stronger market incentives.

Changes in the preemptive effect

When the benefits of preempting the competitor's patent increases - i.e. a lower α - one should expect an environment more favorable to disclosures. Although this logic is confirmed in the following proposition, the intuition is more complicated than it appears to be and involves several contradicting forces.

Proposition 4. An increase in the *preemption rewards* causes the equilibrium level of disclosures d^* to rise.

Proof: See the Appendix.

This result can easily be explained using our taxonomy of benefits and cost of disclosing. On the benefit side, two important economic forces encourage disclosures. On one hand, an increase in $(\pi_{22} - \pi_{12})$ simply represents the extra profit the firm appropriates by obstructing her rival's patent. On the other hand, and more subtle, an increase in the *preemptive effect* tightens the incentives to invest in duplication activities and - as it was remarked before - this effect encourages leakages by increasing the probability of the state of the world in which disclosures are advantageous.

However, increases in $(\pi_{22} - \pi_{12})$ make disclosures more costly by raising the forgone profits for A in case B duplicates, that is by rising $\Pi(d, \bar{s}, \pi)$.

The proposition shows that ultimately the benefit effects outweigh the cost consequences of increased disclosures and confirms, at least partially, the intuition that in environments where competition is less intense the room for information transmission is enlarged.

Changes in patent policy

The attitudes of the PO towards patents and disclosures are crucially influenced by the commonly known parameter \bar{s} . A greater value for \bar{s} might be interpreted as a shift in favor of patentees to exclude prior users. Under these circumstances, one may be tempted to conclude that disclosures should diminish. Contrary to this appealing intuition, the following proposition highlights that an increase in \bar{s} , under some conditions, encourages disclosures.

Proposition 5. An increase in the rights of patentees to exclude first inventors has an ambiguous effect on the equilibrium level of disclosures d^*

Proof: See the Appendix.

The intuition behind this result is relatively easy to understand. When \bar{s} increases, disclosures becomes more costly because their effect on others' patenting possibilities is diminished. This effect is captured by the fact that $\frac{\partial \gamma}{\partial \bar{s}} > 0$.¹⁷ On the other hand, a bigger value for \bar{s} improves the perspectives to B of obtaining a patent and induces higher duplication efforts, i.e. $\frac{\partial f}{\partial \bar{s}} > 0$. As I discussed before, this influence acts as a countervailing force to the first one, by increasing the attractiveness of disclosures.

The result that, under some conditions, a public policy “biased” in favor of patents induces -for those who use secrecy - a more aggressive disclosure strategy is supported, at least partially, by casual evidence: recall the articles cited in the Introduction highlighting the importance of carefully managed disclosures for strategically protecting intellectual property. Even more important, they also suggest the increasing relevance of this practice under the new and more favorable environment to patents installed in 1982 since the creation, in the US, of the Courts of Appeals for the Federal Circuit.¹⁸

Changes in the technical complexity of innovations

How are disclosures affected by the technical complexity of inventions?. Do innovators use a more aggressive disclosure strategy for relatively “easy” innovations? To formally answer these questions, one needs to incorporate a measure of technical complexity.

To capture the issue of technical complexity - a measure of how “easy” or “difficult”

¹⁷In this case, I assume that the cross partial derivate $\frac{\partial}{\partial \bar{s}} \frac{\partial \gamma}{\partial d} = 0$. This assumption highly simplifies the calculations involved and the same results are obtained if it is assumed that $\frac{\partial}{\partial \bar{s}} \frac{\partial \gamma}{\partial d} \leq 0$.

¹⁸The Courts of Appeals for the Federal Circuit, a specialized appellate court to solve patent cases, was established in 1982 by Congress. The Court's decisions has been regarded as being widely “pro-patent”. See for instance, Kortum and Lerner [10]

innovations are - let me introduce the family of cost functions,

$$C_\lambda = \left\{ \lambda C(f, d) : \lambda \in [\underline{\lambda}, \bar{\lambda}] \right\}$$

For any λ , I assume that $C(f, d)$ has the same properties as those discussed earlier. The idea is that for a more difficult innovation, achieving a given duplication probability is more costly. Thus more complex innovations can be associated with higher values for λ . Following this interpretation, λ can be considered the degree of technical complexity. Assuming that for all $\lambda \in [\underline{\lambda}, \bar{\lambda}]$, the solution to B 's problem is interior, now the (FOC) that characterize the SPNE are:

$$\begin{aligned} 0 &= \gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d) \\ MB_d &= -\gamma_d(d, \bar{s})f(\pi_{22} - \pi_{12}) = MC_d = \frac{\partial f}{\partial d}\Pi(d, \bar{s}, \pi) \end{aligned}$$

Using these equations, it is easy to show the following proposition.

Proposition 6. An increase in the technical complexity of the innovation causes the equilibrium level of disclosures d^* to diminish.

Proof: See the Appendix.

A higher λ triggers two different and reinforcing effects. On the benefit side, B will react by devoting fewer resources to duplication activities because her rediscovery cost increases. Therefore, by diminishing the threat of duplication, an increase in the technical complexity of innovations weakens the importance of disclosures to dissuade patenting activities. On the other hand, the marginal cost of disclosing is also higher, and the reason is simply that the enabling effect of disclosures becomes more important for “difficult” innovations. In other words, facing a more difficult technical problem, the marginal contribution to B of a given amount of knowledge, contained in the disclosure, is now enhanced.

5 Total Surplus Maximizing Disclosures

If the disclosing firm's goal were to maximize (expected) total surplus, what would be the optimal disclosure level? Would this “social” disclosure level be greater than the equilibrium one? A natural conjecture is that the private level of disclosure is smaller than the “social” one: knowledge as a pure public good should be shared completely between the firms. Although the intuition seems appealing, the argument is not correct. In general, it is impossible to obtain a definite answer to the above questions and in equilibrium there may be too much or too little disclosure.

To obtain a better understanding of the divergences between private and “social” disclosures is useful to introduce the following concepts. I denote the situation in which both firms use the innovation as a duopoly (D) and the case in which only one firm exploits the new technology as a monopoly (M). Notice that the monopoly case is perfectly compatible with both firms competing in the downstream market: the word monopoly is used here to emphasize that only one firm uses the innovation.

Let TS^i and CS^i for $i \in \{D, M\}$ be total surplus and consumer surplus respectively. Thus $TS^M = CS^M + \pi_{21} + \pi_{12}$, $TS^D = CS^D + 2\pi_{22}$. Then “social” disclosures, d^W , solve the following problem

$$d^W(\pi, \bar{s}) = \arg \max_K [(1-f)TS^M + f\{\gamma(d; \bar{s})TS^M + (1-\gamma(d; \bar{s}))TS^D\} - C(f, d)]$$

$$K = \left\{ (f, d) \in [0, 1] \times [0, 1] : \gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d) = 0 \right\}$$

Two observations are important to understand the problem. First, in this formulation, the disclosing firm maximizes total surplus net of duplication costs. This is done to acknowledge that duplication activities involves a positive additional development cost. Second, notice that in the “social” problem, the “benevolent” firm still considers that duplication activities are performed by a profit maximizing firm. Therefore, now the firm chooses a disclosure level to maximize (expected) total surplus taking into account the same constraints as those considered in the private case. Thus, this formulation is best

interpreted as a kind of constrained optimum.

It is not difficult to show that “social” marginal benefits (SMB_d) and cost (SMC_d) of disclosing are

$$\begin{aligned} SMB_d &= -f\gamma_d(d; \bar{s})(TS^D - TS^M) - C_d \\ SMC_d &= \frac{\partial f}{\partial d}\{C_f(f, d) - (1 - \gamma(d; \bar{s}))(TS^D - TS^M)\} \end{aligned} \quad (5)$$

The following proposition is a direct consequence of manipulating equations (4) and (5).

Proposition 7. The “social” benefit and cost of disclosing are related to the private ones as follows:

- a) $SMB_d = MB_d - C_d - f\gamma_d(d; \bar{s})\{(CS^D - CS^M) - (\pi_{21} - \pi_{22})\} \leq MB_d$
- b) $SMC_d = MC_d - \frac{\partial f}{\partial d}(1 - \gamma(d; \bar{s}))(CS^D - CS^M) < MC_d$.¹⁹

Proposition 7 shows that the “social” marginal cost of disclosing is always smaller than the private one. The difference between the two comes from the fact that disclosures increase the duplication probability and when B does not get a patent - an event that occurs with probability $1 - \gamma(d; \bar{s})$ - both firms use the innovation in the market competition game. This results in a higher consumer surplus, because $CS^M < CS^D$. The disclosing firm does not consider this effect when deciding her optimal disclosure strategy because it does not capture the (expected) positive spillovers that disclosures have on consumers.

On the benefit side, there are two sources of divergences between private and “social” values. First, the disclosing firm imposes a positive externality by reducing the cost to B of obtaining a given duplication probability, $-C_d$. And second, for a given f , higher disclosures diminish the probability of patenting and the “social” solution requires to consider the (expected) changes in consumer surplus and profits resulting from the modification in the market structure. In equilibrium, the disclosing firm does not take into account neither

¹⁹These results are obtained by following simple algebraic manipulations and I omit the formal proof.

the modification in the consumer surplus nor the change in the profits of her competitor due to the alteration in the market structure.

If $(CS^D - CS^M) - (\pi_{21} - \pi_{22}) > 0$, then $SMB_d > MB_d$ and therefore $d^W > d^*$. However, as the following example illustrates, it is entirely possible that $(CS^D - CS^M) - (\pi_{21} - \pi_{22}) < 0$.

Example 2

Consider a Cournot duopoly game with linear inverse demand function $P = A - Q$. The innovation reduces the cost of producing the good from c to $c - \sigma$ for $\sigma > 0$ and $A > c$. Then $\pi_{21} = (9)^{-1}(A + 2\sigma - c)^2$, $\pi_{22} = (9)^{-1}(A + \sigma - c)^2$, $CS^M = \frac{2}{9}(A + \frac{\sigma}{2} - c)^2$ and $CS^D = \frac{2}{9}(A + \sigma - c)^2$. Therefore $CS^D - CS^M = \frac{2}{9}\{\frac{3}{4}\sigma^2 + (A - c)\sigma\}$ and $(\pi_{21} - \pi_{22}) = \frac{2}{9}\{\frac{3}{2}\sigma^2 + (A - c)\sigma\}$. Finally, $(CS^D - CS^M) - (\pi_{21} - \pi_{22}) = -(6)^{-1}\sigma^2 < 0$.

Therefore, in general, the equilibrium may involve too much or too little disclosure as the following proposition summarizes.

Proposition 8. Suppose that in equilibrium disclosures are partial, i.e. $d^* \in (0, 1)$. Then at $d^* : -C_d + \frac{\partial f}{\partial d}(1 - \gamma(d; \bar{s}))(CS^D - CS^M) - f\gamma_d(d; \bar{s})\{(CS^D - CS^M) - (\pi_{21} - \pi_{22})\} = SMB_d - SMC_d \leq 0$.²⁰

A particular and interesting case to examine is that of a “drastic” innovation, that is an innovation such that $\pi_{12} = 0$ and therefore $TS^M = TS^D$. Are “social” disclosures in this situation smaller than the private ones?. A natural intuition is that given the total surplus does not change whether duplication is a success or not, disclosures have no “social” benefits but only effects on the distribution of property rights between the firms. Following this reasoning, an appealing conjecture is that in equilibrium there is too much disclosure. Nevertheless this intuition is incorrect because given that B will invest in duplication activities, disclosures reduce the private cost to B of achieving any duplication probability.

²⁰ Also at d^* , the following constrain must hold $\gamma(d; \bar{s})\Delta_P + \Delta_I - C_f(f, d) = 0$

Using Proposition 7, at d^* , $SMB_d - SMC_d = -C_d + f\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) \leq 0$. Thus if the negative pecuniary external effect that the disclosing firm imposes on her competitor, $f\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22})$, is smaller - in absolute value - than the technological positive externality, $-C_d$, then the equilibrium involves too little disclosure. Finally, notice that if at d^* , $-C_d = -f\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22})$ then the two opposing external effects cancel out and the private level of disclosure equals the social one, that is $d^* = d^W$.

6 Conclusions

In this paper I examined the defensive maneuvers that owners of trade secrets employ to mitigate the risks of patenting activities conducted by subsequent inventors. In particular, my focus was on the decision of a trade secret owner concerning disclosures of an innovation to invalidate potential future patents for similar competing technologies.

In a simple duopoly framework I studied the links between several innovation characteristics and the properties of equilibrium disclosures. One important economic dimension of innovations is their profitability. My model shows that, under some circumstances, more profitable innovations are more aggressively disclosed. Innovations are also distinguished by their technical complexity. In this case, I found that, for more complex innovations, the strategic use of disclosures is considerably debilitated. In addition, disclosures are affected by patent law and in particular by the rights of potential patentees to exclude “prior” inventors. My results suggest that an strengthening of these rights does not necessarily lead to less disclosure: a “pro-patent” policy may increase the importance of this practice to protect innovations.

I also showed, by comparing the equilibrium level of disclosure with the level that maximizes (expected) total surplus, that in equilibrium there may be too much or too little disclosure. The result that, under some circumstances, there is too much disclosure contradicts the natural conjecture that knowledge, as a pure public good, should be completely shared between the firms.

Several interesting extensions and connected ideas to this work remain to be studied

in future research. First, in the context of sequential innovations, disclosure - through patents - of interim findings is a key ingredient to decrease the social cost of future innovations. Without proper incentives, firms, that also pursue future discoveries, may be reluctant to disclose “intermediate” innovations. The optimal design of a patent law that potentially allows first inventors to appropriate a share of other’s subsequent innovators deserves additional research efforts. Second, the link between the economic profitability of innovations - or innovation “size” - and the choice of intellectual property protection when first inventors can be excluded by subsequent innovators is also a topic to be explored in future research.

7 Appendix

Proof of Lemma 1. (a) Define a new random variable $S = V + d$. Then, $G_S(\bar{s}) = \text{prob} \{S \leq \bar{s}\} = \text{prob} \{d + V \leq \bar{s}\} = \text{prob} \{V \leq \bar{s} - d\} = G_V(\bar{s} - d)$.

(b) The location family of cdfs is stochastically increasing in d if $d_1 > d_2 \Rightarrow G_V(s - d_1)$ is stochastically greater than $G_V(s - d_2)$. Given any $d \in [0, 1]$, we know that

$$\begin{aligned} &= 0 \text{ for } x < d + a \\ G_S(x|d) &= G_V(x - d) \text{ for } d + a \leq x \leq d + b \\ &= 1 \text{ for } d + b < x \end{aligned}$$

Consider the following cases: (1) $x \leq d_2 + a \Rightarrow x \leq d_1 + a \Rightarrow G_S(x|d_1) = G_S(x|d_2) = 0$; (2) $x \geq d_1 + b \Rightarrow x \geq d_2 + b \Rightarrow G_S(x|d_1) = G_S(x|d_2) = 1$; (3) $d_2 + a < x < d_1 + a \Rightarrow G_S(x|d_1) = 0$ and $G_S(x|d_2) > 0 \Rightarrow G_S(x|d_1) < G_S(x|d_2)$; (4) $d_2 + b < x < d_1 + b \Rightarrow G_S(x|d_1) < 1$ and $G_S(x|d_2) = 1 \Rightarrow G_S(x|d_1) < G_S(x|d_2)$; (5) $d_1 + a < x < d_2 + b \Rightarrow G_S(x|d_2) = G_S(x|d_1) + [G_S(d_1 + a|d_2) - G_S(d_2 + a|d_2)] \Rightarrow G_S(x|d_1) < G_S(x|d_2)$ ■

Proof of Lemma 2. Part (a) follows from the satisfaction of the condition for the implicit function theorem, namely that $C_{ff} \neq 0$. Part (b) follows from the characterization of the first order comparative static effects of the exogenous variables, i.e. d, π and Δ , on f given by

$$\begin{aligned} \frac{\partial f}{\partial \Delta_P} &= (C_{ff})^{-1} \gamma(d; \bar{s}) > 0 \\ \frac{\partial f}{\partial \Delta_I} &= (C_{ff})^{-1} = \frac{1}{\gamma(d; \bar{s})} \frac{\partial f}{\partial MC_P} > 0 \\ \frac{\partial f}{\partial \bar{s}} &= (C_{ff})^{-1} \frac{\partial \gamma(d; \bar{s})}{\partial \bar{s}} (\pi_{21} - \pi_{22}) > 0 \blacksquare \end{aligned}$$

Proof of Lemma 3. Part (a) follows from the comparative statics of d on f and assumption 3. Part (b) follows from the fact that under assumption 4, f is C^2 function of d and

$$\frac{\partial}{\partial d} \frac{\partial f}{\partial d} = (C_{ff})^{-1} [\gamma_{dd}(\pi_{21} - \pi_{22}) - C_{fdd}] > 0$$

Because $C_{fdd} < 0$ and $\gamma_{dd} \geq 0$ ■

Proof of Theorem 1. (a) Notice that the constrained set K is non-empty, closed and bounded. That K is non-empty follows from the fact that for all $d \in [0, 1]$ there exists an (unique) f that solves firm B 's problem. K is closed because $\gamma(d; \bar{s})$ and $C_f(f, d)$

are continuous functions of d and f . And it is also bounded because both f and $d \in [0, 1]$. Therefore K is a non-empty compact set. Then, given that $EU^A(f, d) = [(1 - f)\pi_{21} + fR^A(d, \pi_{12}, \pi_{22}, \bar{s})] : [0, 1] \times [0, 1] \rightarrow R_{++}$ is a continuous function of d and f , by Weierstrass's theorem, the problem of the disclosing firm has a solution.

(b) Given Lemma 2, the constrained set can be written as

$$K = \left\{ (d, f) \in [0, 1] \times [0, 1] : f = h(d; \Delta_P, \Delta_I, \bar{s}) \right\}$$

By part a) of the theorem there exists a pair (d_0, f_0) such that

$$EU^A((d_0, f_0)) = \text{Max}\{EU^A(d, f) : (d, f) \in K\}$$

Now define $u^* = EU^A((d_0, f_0))$. Assume that there exists a $(d_1, f_1) \in K$ such that $d_1 \neq d_0$ and $u^* = EU^A((d_1, f_1))$. Given a $0 < \theta < 1$, define $\theta(d_0, f_0) + (1 - \theta)(d_1, f_1)$. Then by quasiconcavity, $EU^A(\theta d_0 + (1 - \theta)d_1, \theta f_0 + (1 - \theta)f_1) \geq u^*$. However, notice that corresponding to $\theta d_0 + (1 - \theta)d_1$, there is a unique f given by $f = h(\theta d_0 + (1 - \theta)d_1; \cdot)$ such that $(\theta d_0 + (1 - \theta)d_1, h(\theta d_0 + (1 - \theta)d_1; \cdot)) \in K$. Then $h(\theta d_0 + (1 - \theta)d_1; \cdot) < \theta f_0 + (1 - \theta)f_1$ because by Lemma 3 $h(d; \cdot)$ is a strictly convex function of d . Finally, because $EU^A(f, d)$ is a monotonically decreasing function of f ,

$$\begin{aligned} EU^A(\theta d_0 + (1 - \theta)d_1, h(\theta d_0 + (1 - \theta)d_1; \cdot)) &> \\ EU^A(\theta d_0 + (1 - \theta)d_1, \theta f_0 + (1 - \theta)f_1) &\geq u^* \end{aligned}$$

This last result, by contradicting the initial assumption that the pair (d_0, f_0) is an equilibrium, implies uniqueness ■

Proof of Proposition 1. The Lagrangian function corresponding to the problem of the disclosing firm is:

$$\begin{aligned} L = & (1 - f)\pi_{21} + f\{\gamma(d; \bar{s})\pi_{12} + (1 - \gamma(d; \bar{s}))\pi_{22}\} - \eta\{\gamma(d; \bar{s})(\pi_{21} - \pi_{22}) + \\ & (\pi_{22} - \pi_{12}) - C_f(f, d)\} + \mu_1 d + \mu_2(1 - d) \end{aligned}$$

The (FOC) are:

$$FOC_d : -f\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) - \eta\{\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) - C_{fd}(f, d)\} + (\mu_1 - \mu_2) = 0$$

$$FOC_f : -[(\pi_{21} - \pi_{22}) + \gamma(d; \bar{s})(\pi_{22} - \pi_{12})] + \eta C_{ff}(f, d) = 0$$

$$FOC_\eta : -\gamma(d; \bar{s})(\pi_{21} - \pi_{22}) - (\pi_{22} - \pi_{12}) + C_f(f, d) = 0$$

$$FOC_{\mu_1} : d \geq 0, \mu_1 \geq 0 \text{ with complementary slackness.}$$

$$FOC_{\mu_2} : (1 - d) \geq 0, \mu_2 \geq 0 \text{ with complementary slackness.}$$

Then $d^* = 0 \Rightarrow \mu_1 > 0, \mu_2 = 0$. From FOC_d and FOC_f it must be that at $d = 0$,

$$\begin{aligned} -f\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) + \mu_1 &= \frac{1}{C_{ff}(f, d)}[\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) - C_{fd}(f, d)] \\ &\quad [(\pi_{21} - \pi_{22}) + \gamma(d; \bar{s})(\pi_{22} - \pi_{12})] \end{aligned}$$

Using Lemma 2,

$$-f\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) + \mu_1 = f_d[(\pi_{21} - \pi_{22}) + \gamma(d; \bar{s})(\pi_{22} - \pi_{12})]$$

Dividing both sides by $-\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12})$ and considering that $\mu_1 > 0$,

$$f < \frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s}))$$

Therefore sufficient conditions to avoid null disclosures in equilibrium are that at $d = 0$,

$$\begin{aligned} f &> \frac{f_d}{-\gamma_d}(\alpha + \gamma(d; \bar{s})) \\ 0 &= \gamma(d; \bar{s})(\pi_{21} - \pi_{22}) + (\pi_{22} - \pi_{12}) - C_f(f, d) \blacksquare \end{aligned}$$

Proof of Proposition 2. Analogous to that of Proposition 1

Proof of Corollary 1. In an interior equilibrium $d^* \in (0, 1)$ and $(\mu_1, \mu_2) = (0, 0)$. From FOC_d and FOC_f it must be that at d^*

$$\begin{aligned} -f\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) &= \frac{1}{C_{ff}(f, d)}[\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) - C_{fd}(f, d)] \\ &\quad [(\pi_{21} - \pi_{22}) + \gamma(d; \bar{s})(\pi_{22} - \pi_{12})] \end{aligned}$$

Then using $\Pi(d, \bar{s}, \pi) \equiv (\pi_{21} - \pi_{22}) + \gamma(d, \bar{s})(\pi_{22} - \pi_{12})$ and Lemma 2,

$$MB_d \equiv -f\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) = f_d\Pi(d, \bar{s}, \pi) \equiv MC_d$$

The sufficient SOC that must be satisfied close to any regular maximum is

$$\det M = \det \begin{bmatrix} 0 & \frac{\partial g}{\partial d} & \frac{\partial g}{\partial f} \\ \frac{\partial g}{\partial d} & \frac{\partial^2 L}{\partial d^2} & \frac{\partial^2 L}{\partial d \partial f} \\ \frac{\partial g}{\partial f} & \frac{\partial^2 L}{\partial f \partial d} & \frac{\partial^2 L}{\partial f^2} \end{bmatrix} > 0$$

where $g = \gamma(d; \bar{s})(\pi_{21} - \pi_{22}) - C_f(f, d)$. Using the FOC, assumption 4 and Lemma 2 and

3, it can be verify that $\det M > 0$ if and only if

$$(C_{ff})^2 \{f \gamma_{dd}(\pi_{22} - \pi_{12}) + f_{dd} \Pi(d, \bar{s}, \pi) + 2\gamma_d(\pi_{22} - \pi_{12})f_d\} > 0$$

Using the definition of MB_d and MC_d , and recognizing that f is a function of d it follows that,

$$\begin{aligned} \frac{\partial MB_d}{\partial d} &= -f \gamma_{dd}(\pi_{22} - \pi_{12}) - \gamma_d(\pi_{22} - \pi_{12})f_d \\ \frac{\partial MC_d}{\partial d} &= f_{dd} \Pi(d, \bar{s}, \pi) + \gamma_d(\pi_{22} - \pi_{12})f_d \end{aligned}$$

Therefore,

$$\det M > 0 \Leftrightarrow \left(\frac{\partial MC_d}{\partial d} - \frac{\partial MB_d}{\partial d} \right) > 0 \blacksquare$$

Proof of Proposition 3. The FOC obtained in Proposition 1 define the following system of equations

$$F_1(\eta, d, f, \pi, \bar{s}) = -\gamma(d; \bar{s})(\pi_{21} - \pi_{22}) - (\pi_{22} - \pi_{12}) + C_f(f, d) = 0$$

$$F_2(\eta, d, f, \pi, \bar{s}) = -f \gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) - \eta \{ \gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) - C_{fd}(f, d) \} = 0$$

$$F_3(\eta, d, f, \pi, \bar{s}) = -[(\pi_{21} - \pi_{22}) + \gamma(d; \bar{s})(\pi_{22} - \pi_{12})] + \eta C_{ff}(f, d) = 0$$

Then if at the equilibrium $\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, d, f)} \neq 0$, the conditions for the implicit function theorem are satisfied. Notice that $\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, d, f)} = \det M$ and $\det M$ must be greater than zero around any regular maximum. Therefore there exists a C^1 function: $d^*(\pi, \bar{s})$ such that

$$\frac{\partial d^*(\pi, \bar{s})}{\partial(\pi_{21} - \pi_{22})} \equiv -\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, (\pi_{21} - \pi_{22}), f)} (\det M)^{-1}$$

Then using the FOC and Lemma 2, it can be verified that

$$\begin{aligned} -\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, (\pi_{21} - \pi_{22}), f)} &= (C_{ff})^2 \{ -\gamma_d(d; \bar{s}) \frac{\partial f}{\partial \Delta_P} (\pi_{22} - \pi_{12}) \\ -\Pi(d, \bar{s}, \pi) \left(\frac{\partial}{\Delta_P} \frac{\partial f}{\partial d} \right) - \frac{\partial f}{\partial d} \} &\leq 0 \end{aligned}$$

and therefore,

$$\begin{aligned} \frac{\partial d^*(\pi, \bar{s})}{\partial(\pi_{21} - \pi_{22})} &= \frac{1}{\left(\frac{\partial MC_d}{\partial d} - \frac{\partial MB_d}{\partial d}\right)} [-\gamma_d(d; \bar{s}) \frac{\partial f}{\partial \Delta_P} (\pi_{22} - \pi_{12}) \\ -\Pi(d, \bar{s}, \pi) \left(\frac{\partial}{\partial_P} \frac{\partial f}{\partial d}\right) - \frac{\partial f}{\partial d}] &\leq 0 \blacksquare \end{aligned}$$

Proof of Proposition 4. Using the implicit function theorem, the comparative statics of disclosures with respect to the preemptive effect can be calculated as follows

$$\frac{\partial d^*(\pi, \bar{s})}{\partial(\pi_{22} - \pi_{12})} \equiv -\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, (\pi_{22} - \pi_{12}), f)} (\det M)^{-1}$$

Then using the FOC and Lemma 2, it can be verified that

$$\begin{aligned} -\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, (\pi_{21} - \pi_{22}), f)} &= (C_{ff})^2 \{-f\gamma_d(d; \bar{s}) - \gamma(d; \bar{s}) \frac{\partial f}{\partial d} \\ -\gamma_d(d; \bar{s}) \frac{\partial f}{\partial \Delta_I} (\pi_{22} - \pi_{12})\} &\leq 0 \end{aligned}$$

Given that at an interior equilibrium $-f\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) = \frac{\partial f}{\partial d} \Pi(d, \bar{s}, \pi) \Rightarrow -f\gamma_d(d; \bar{s}) = \frac{\partial f}{\partial d} (\alpha + \gamma(d; \bar{s}))$, it follows that

$$\frac{\partial d^*(\pi, \bar{s})}{\partial(\pi_{21} - \pi_{22})} = \frac{1}{\left(\frac{\partial MC_d}{\partial d} - \frac{\partial MB_d}{\partial d}\right)} [\alpha \frac{\partial f}{\partial d} - \gamma_d(d; \bar{s}) \frac{\partial f}{\partial \Delta_I} (\pi_{22} - \pi_{12})] > 0 \blacksquare$$

Proof of Proposition 5. The sensitivity of disclosures, around a “regular” equilibrium, to changes in patent policy can be calculated as follows

$$\frac{\partial d^*(\pi, \bar{s})}{\partial \bar{s}} \equiv -\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, \bar{s}, f)} (\det M)^{-1}$$

Then using the FOC and Lemma 2, it can be verified that

$$-\det \frac{\partial(F_1, F_2, F_3)}{\partial(\eta, \bar{s}, f)} = (C_{ff})^2 \{(-\gamma_d(d; \bar{s}) \frac{\partial f}{\partial \bar{s}} - \frac{\partial f}{\partial d} \frac{\partial \gamma(d; \bar{s})}{\partial \bar{s}}) (\pi_{22} - \pi_{12})\} \leq 0$$

Therefore, it follows that

$$\frac{\partial d^*(\pi, \bar{s})}{\partial \bar{s}} = \frac{1}{\left(\frac{\partial MC_d}{\partial d} - \frac{\partial MB_d}{\partial d}\right)} \left[(-\gamma_d(d; \bar{s}) \frac{\partial f}{\partial \bar{s}} - \frac{\partial f}{\partial d} \frac{\partial \gamma(d; \bar{s})}{\partial \bar{s}}) (\pi_{22} - \pi_{12}) \right] \leq 0 \blacksquare$$

Proof of Proposition 6. The introduction of the degree of technical complexity slightly modifies the (FOC) to

$$F_1(\eta, d, f, \pi, \bar{s}, \lambda) = -\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) - (\pi_{22} - \pi_{12}) + \lambda C_f(f, d) = 0$$

$$F_2(\eta, d, f, \pi, \bar{s}, \lambda) = -f \gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) - \eta \{ \gamma_d(d; \bar{s})(\pi_{21} - \pi_{22}) - \lambda C_{fd}(f, d) \} = 0$$

$$F_3(\eta, d, f, \pi, \bar{s}, \lambda) = -[(\pi_{21} - \pi_{22}) + \gamma_d(d; \bar{s})(\pi_{22} - \pi_{12})] + \eta \lambda C_{ff}(f, d) = 0$$

The comparative statics of d^* with respect to λ can be obtained as follows

$$\frac{\partial d^*(\pi, \bar{s}, \lambda)}{\partial \lambda} \equiv -\det \frac{\partial (F_1, F_2, F_3)}{\partial (\eta, \lambda, f)} (\det M)^{-1}$$

Then using the FOC, it can be verified that

$$\begin{aligned} -\det \frac{\partial (F_1, F_2, F_3)}{\partial (\eta, \lambda, f)} &= (C_{ff} \lambda)^2 \{ \gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) \frac{C_f}{\lambda C_{ff}} \\ &\quad + \Pi(d, \bar{s}, \pi) \frac{\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22})}{\lambda^2 C_{ff}} \} \end{aligned}$$

Considering that now f is a C^1 function of λ , it is straightforward to show that for a given disclosure level

$$\begin{aligned} \frac{\partial f}{\partial \lambda} &= \frac{-C_f}{\lambda C_{ff}} < 0 \\ \frac{\partial}{\partial \lambda} \frac{\partial f}{\partial d} &= -\frac{\gamma_d(d; \bar{s})(\pi_{21} - \pi_{22})}{\lambda^2 C_{ff}} > 0 \end{aligned}$$

Hence,

$$\begin{aligned} -\det \frac{\partial (F_1, F_2, F_3)}{\partial (\eta, \lambda, f)} &= (\lambda C_{ff})^2 \{ -\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) \frac{\partial f}{\partial \lambda} \\ &\quad - \Pi(d, \bar{s}, \pi) \frac{\partial}{\partial \lambda} \frac{\partial f}{\partial d} \} < 0 \end{aligned}$$

Finally,

$$\begin{aligned} \frac{\partial d^*(\pi, \bar{s}, \lambda)}{\partial \lambda} &= \frac{1}{\left(\frac{\partial MC_d}{\partial d} - \frac{\partial MB_d}{\partial d}\right)} [-\gamma_d(d; \bar{s})(\pi_{22} - \pi_{12}) \frac{\partial f}{\partial \lambda} \\ -\Pi(d, \bar{s}, \pi) \frac{\partial}{\partial \lambda} \frac{\partial f}{\partial d}] &< 0 \blacksquare \end{aligned}$$