

Reason, Emotion and Information Processing in the Brain *

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Abstract

Building on evidence from neurobiology and neuroscience, we model the physiological limitations faced by individuals in the process of decision-making that starts with sensory perception and ends in action selection. The brain sets a neuronal threshold, observes whether the neuronal cell firing activity reaches the threshold or not, and takes the optimal action conditional on that (limited) information. We show that the optimal threshold is set in a way that existing beliefs are most likely to be confirmed and least likely to be refuted. The conclusion holds in static and dynamic settings, and with linear and quadratic loss functions. We then relate our result to the somatic marker theory, and argue that it provides support for the hypothesis that emotions help decision-making. Last, we discuss the implications for choices in concrete vs. abstract situations, for interactions in cooperative vs. competitive activities, for reactions to expected vs. unexpected events, and for the choice of cognitive vs. affective encoding channels.

Keywords: Neuroeconomic theory, neurobiology, neuroscience, reason, emotion, information processing, Bayesian learning.

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1 Introduction

Economic theory has traditionally been interested in the analysis of choices. In particular and with some exceptions (such as Bernheim and Rangel (2004)), the *processes* by which individuals reach decisions have been overlooked, mainly because we had very little knowledge of the pathways going from perception to action. With the development of increasingly sophisticated designs and techniques to measure brain activity, the neurobiology and neuroscience literatures have dramatically improved our understanding of the *biological mechanisms that transform sensory perceptions into voluntary actions*. These results can now be incorporated into formal economic models of decision-making.

Building theoretical models of brain processes is an important step both for economics and neuroscience. For economics, incorporating *physiological costs and constraints* in the capacity of individuals to evaluate situations, process information and reach conclusions will help provide microfoundations for some well-documented heuristics, systematic errors and biases in choices, without having to rely on ad-hoc explanations.¹ For neuroscience, developing formal models of the brain as a modular entity can provide testable implications about the functionality of different brain systems and their interactions in decision-making. Since our theory builds on literatures that, in principle, are quite distant from economics, we start with a brief overview of the recent research that is relevant for our study.

1.1 Some background from the brain sciences

The basic premises for our theory come from two overlapping literatures.

1. *Neurobiology*. Researchers in neurobiology have studied the neural mechanisms underlying the transformation of sensory signals into decisions. One of the early theories, the “Efficient Coding Hypothesis” and its mathematical formulation in terms of bayesian estimation, postulates that neurons encode information as compactly as possible, so as to use resources efficiently (see Barlow (2001) and Simoncelli (2003) for a synthesis). This theory has recently led to a myriad of sophisticated statistical models that describe bayesian stochastic processing of information by neurons in visual, auditory and haptic perception tasks (see e.g. Schwartz and Simoncelli (2001), Ernst and Banks (2002), Körding and Wolpert (2004) and Ma et al. (2006)).

¹One could draw a parallel with the theory of organizations, where a more accurate modelling of organizational constraints (agency problems, restricted information channels, limited resources) has helped understanding the tendency of organizations to take certain decisions and avoid some others.

In another classical study, Hanes and Schall (1996) use single cell recording to analyze the neural processes responsible for the duration and variability of reaction times in monkeys. The authors find that movements are initiated when neural activity reaches a certain threshold activation level, in a winner-takes-all type of contest.² Also, stochastic variability in cell firing rates is responsible for the observed differences in reaction times. Building on this work, Shadlen et al. (1996) and Gold and Shadlen (2001) study a motion discrimination task, where monkeys must decide whether the net direction of dots that appear on a monitor is upward or downward. The studies show that, first, neurons in the extrastriate visual cortex respond to the motion stimuli, and then, neurons in the parietal and frontal association cortex carry these signals. The authors argue that information is processed in a specific way. Neurons “compute” approximately the log-likelihood ratio of the alternatives in order to determine which hypothesis should be supported by the evidence. Thus, according to this result, neurons incorporate the two major ingredients of bayesian theory: prior probabilities, and stochastic information processing (see also Deneve et al. (1999) for a numerical simulation model of cortical circuitry that can implement a close approximation to maximum likelihood).

The work by Shadlen et al. (1996) has led other researchers to study whether neural circuits have a similar way to encode information in more complex situations. Ditterich et al. (2003) show that when the task is more difficult (fewer dots move in synchrony), monkeys require more time to take a decision and make more mistakes, two results also consistent with a stochastic information accumulation theory. Also, using fMRI studies, Heekeren et al. (2004) conclude that the mechanism by which the brain of a monkey computes perceptual decisions is also at work for humans and for more sophisticated choices, such as image recognition. Last, in the work that is possibly most closely linked to economics, Platt and Glimcher (1999) demonstrate that neurons react not only to probabilities of gains (as already discussed) but also to magnitudes of gains, hence computing approximately the “expected value” associated to each alternative (see also Glimcher et al. (2005) and the review by Glimcher and Rustichini (2004)).

2. *Affective neuroscience.* The increased interest during the 1990s in understanding the neural basis of emotions led to the development of a new subdiscipline called “affective neuroscience” (Davidson and Sutton, 1995). Within this area, fear is arguably the emotion that has been studied in more detail. It has been shown, for example, that the amygdala is

²Nichols and Newsome (2002) provide a further analysis of the type of situations where information processing is likely to follow a winner-takes-all vs. a vector averaging pattern.

a key neural pathway to fear. An emotional deficit results in anxiety disorders that prevent the normal functioning of the individual (LeDoux (1996, 1998)). Numerous studies using both PET scan and fMRI techniques have been conducted on other emotions and reached qualitatively similar conclusions. First, emotions are crucial for decision-making as they guide actions towards salient goals (Davidson and Irwin, 1999). Second, abnormalities in the functioning of specific brain regions are responsible for emotional disorders, including social phobias (Reiman, 1997) and depression (Drevets et al., 1997). These pathologies result in poor decision-making.³

In his review of the brain areas involved in reward processing, Schultz (2000) concludes that neurons adapt their activity according to ongoing experiences. Of special importance are neurons in the orbitofrontal cortex (Tremblay and Schultz, 2000), an area that plays a major function in the experience of emotions. Inspired by this research, and in many cases preceding it, Damasio (1994) developed the “somatic marker hypothesis”, a thought-provoking but also controversial neural theory that rests on two key ingredients. First, the way emotions affect decisions is by modifying neuronal thresholds. The direction of the change in thresholds is very precise: the individual becomes more receptive to information that supports current beliefs and less receptive to information that contradicts current beliefs. As summarized by Bechara and Damasio (2005):

[P]re-existing somatic states influence the threshold of neuronal cell firing in trigger structures (e.g., VM cortex) so that subsequent somatic states from thoughts (secondary inducers) are triggered more or less easily. [...] While pre-existing negative somatic states reinforce subsequent negative states, they may impede the effectiveness of positive ones. Similarly, pre-existing positive states reinforce positive states, but they may impede negative ones (p. 363-4).

Second, this threshold modification induced by somatic dispositions improves decision-making. Again, in Bechara and Damasio’s (2005) words:

These somatic states are indeed beneficial, because they consciously or non-consciously bias the decision in an *advantageous* manner (p. 351, italics added).

Rustichini et al. (2005) provide further evidence that the emotional circuitry is active in the task of processing information. In our view, one weakness of the somatic marker

³The primary goal in most of these studies is not so much to discuss these well-established facts but, instead, to identify the brain circuitry behind each particular emotion or emotion disorder. These details, however, are less crucial for the purpose of our study.

theory is that it does not explain why a modification in that specific direction is beneficial. As discussed below, one objective of our paper is to build a formal framework that can address this issue.

1.2 Implications for decision-making and overview of the results

From the neurobiology literature reviewed above, the physiological information processing mechanism can be summarized in three basic principles. *First*, neurons carry information from the sensory to the decision-making system. However, the encoding technology is imperfect: the level of neuronal cell firing depends stochastically on the information obtained. *Second*, the motor cortex triggers an action whenever the cell firing activity in favor of one alternative reaches a certain threshold. *Third*, the individual has the ability to modify the triggering threshold. By acting on the threshold, the individual affects the likelihood of interpreting evidence for and against each option.

Assume now that the brain has been developed (say, through evolution) in a way that it maximizes the acquisition of information, in order to take the best possible action. However, due to the constraints described above, it is only able to choose a neuronal threshold and learn whether that cell firing level is reached or not. The first objective of our study is to find the optimal threshold. That is, we determine what is the best way to process information given the documented physiological limitations, and then discuss its implications for decision-making. We also study how the thresholds vary over time, across activities, under different priors, and for different objectives. Finally, we analyze the effect on choices of an impairment in the capacity of an individual to set thresholds optimally. The second and complementary objective of the paper is to relate our findings to somatic dispositions. According to the affective neuroscience literature also reviewed above, emotions play an important role in the regulation of neuronal activity. The somatic marker theory goes one step beyond in arguing that: (i) emotions operate on neuronal thresholds, (ii) they modify thresholds in a way that existing beliefs are likely to be confirmed, and (iii) this modulation of thresholds improves decision-making. However, it does not provide a compelling argument for the desirability of this threshold modulation. Since our model determines the optimal threshold, it is equipped to discuss whether and why the threshold modifications postulated by the somatic marker theory are beneficial. In other words, if we accept (i) and (ii) as a premise, our model can determine whether (iii) is indeed its logical consequence.⁴

⁴For the reader skeptical of the somatic marker hypothesis that emotions operate on neuronal thresholds,

To formalize the neurobiological principles described above, we consider a simple model with two states, A and B . We assume that the level of cell firing stochastically depends on the state, with high cell firing being more likely in state A and low cell firing in state B . The brain sets a threshold and determines only whether the cell firing surpasses it or not. Given the information obtained, an action is undertaken. Payoffs depend on the combination of action and state. The first contribution of the paper is to show that the threshold is optimally set in a way that beliefs are likely to be supported. That is, if the agent becomes more confident that the state is A , the threshold is decreased. Thus, the new threshold is more likely to be surpassed *whether the state is indeed A or not* and, as a result, the agent is more likely to take the action which is optimal in that state but suboptimal in the other. Conversely, if the individual becomes more confident that the state is B , the threshold is increased and therefore less likely to be reached (Proposition 1). The logic for this property of the optimal threshold is simple. As the likelihood of A increases, stronger contradictory information is required to reverse that belief. From simple Bayesian theory, we know that stronger information towards state B can only be obtained if a *lower* threshold is not reached, hence the result. We then check the robustness of this conclusion. We show that it generalizes to a continuum of states and to different payoff formulations (e.g., linear and quadratic loss functions). Under some conditions, it also holds when multiple thresholds can be set sequentially, which implies that the ability to modify neuronal thresholds has a snowball effect on decision-making: a stronger belief towards one state implies a greater threshold variation in its favor, therefore a higher probability that new information supports it, and so on (Propositions 2 to 6).

The second contribution is to relate this result to the somatic marker theory. According to Bechara and Damasio (2005), the role of emotions is to act on neuronal thresholds by rendering the individual more receptive to information that supports current beliefs relative to information that contradicts current beliefs. If we accept this as a premise, then our previous results demonstrate that the somatic marker’s claim –namely that emotions improve decisions– is indeed correct. In other words, a person with an emotional deficit will not modulate thresholds in this direction and, as a result, will take suboptimal actions more frequently. Furthermore, the effect of emotions on thresholds is all the more important in dynamic settings: as time passes, emotionally balanced individuals are most likely to maintain the beliefs initially favored (Corollaries 1 and 2).

our model has nothing to say about the effect of somatic dispositions on choices. Even in that case, it is still a useful model of the neurobiological mechanisms in the brain, as it determines the (constrained) optimal way to process information and discusses the implications for decision-making in different contexts.

The third and last contribution of the paper is to investigate in more detail the costs of having a coarse information partition and the implications of our theory. We show that the limited ability of the individual to process information does not prevent efficient decision-making if the environment is static and there are only two relevant alternatives, which we label as “basic” situations. We also argue that cognitive encoding channels with multiple thresholds are relatively more valuable in complex environments whereas affective encoding channels with one or few thresholds are more appropriate in simple environments (Propositions 7 and 8). As for the implications, we prove that optimal thresholds are more sensitive to initial beliefs the weaker the correlation between state and cell firing. This suggests that threshold modulation is most important in activities where information is subjective and difficult to interpret. We also show that thresholds are more sensitive to beliefs when agents benefit if their peers also take the correct action (positive externalities) than when they benefit if they alone take the correct action (negative externalities). Thus, optimal threshold modulation is more important in cooperative environments than in competitive ones. Last, we highlight the importance of the correlation between action and payoff when discussing the reaction of individuals to expected and unexpected events. If the correlation is high, an agent who sets thresholds optimally will exhibit a low (positive) reaction to success and high (negative) reaction to failures, as emphasized in the neuroscience literature. However, if the correlation is weak, that agent understands the importance of luck on outcomes and has a low reaction to both successes and failures. (Propositions 9 to 11).

1.3 Related literature

The existing neuroeconomic literature that models the interplay between reason and emotion assumes competition between a (rational) cognitive system and an (impulsive) affective system as well as a specific cost-benefit tradeoff between the two (see e.g. Bernheim and Rangel (2004), Benhabib and Bisin (2005), Loewenstein and O’Donoghue (2005); see also Camerer et al. (2005) for a brief review).⁵ Our work departs quite substantially from

⁵Brocas and Carrillo (2005) and Fudenberg and Levine (2006, 2007) adopt the model by Thaler and Shefrin (1981), where the two competing systems have different temporal horizons. A difference, however, is that the myopic system is not necessarily irrational; it simply has a different (in this case, more immediate) objective. Therefore, these papers do not directly address the role of emotions in decision-making. There is also a literature where the individual makes choices anticipating that emotions –such fear, anxiety or rejoicing– affect utility (Caplin and Leahy (2001), Palacios-Huerta (2004)) and some other recent works where the individual has imperfect self-knowledge resulting in other types of intrapersonal conflicts (Carrillo and Mariotti (2000), Benabou and Tirole (2002, 2004), Bodner and Prelec (2003)).

this literature in that we do not presuppose any tradeoff between reason and emotion. In fact, we do not even assume separation and competition between cognitive and affective systems. Instead, we propose a more primitive model that incorporates the *physiological constraints faced by the brain in the decision-making process*, and discuss the role of emotions in this constrained optimization problem.⁶ Since we focus on the physiological mechanisms behind the choice process, our paper is closer to the “physiological expected utility” theory developed by Glimcher et al. (2005). It is worth noting that the dichotomy between rational/cognitive and automatic/affective systems has been very successful in neuroeconomic circles, mainly because it captures in a parsimonious way the tension between reason and passion. However, it should also be acknowledged that most neuroscientists strongly disagree with a literal interpretation of this dichotomy. According to Glimcher et al. (2005, p. 251-2): “[T]here is no evidence that hidden inside the brain are two fully independent systems, one rational and one irrational. [...] What we cannot stress strongly enough is that the vast majority of evolutionary biologists and neurobiologists reject this view.” Similarly, Phelps (2006, p. 27) argues “that the classic division between emotion and cognition may be unrealistic and that an understanding of human cognition requires the consideration of emotion.”⁷ There is no doubt that the metaphor is adopted mainly for modelling purposes, and therefore can be a reasonable first approximation in many contexts. Nonetheless, the objections raised by neuroscientists suggest that it can be complementary and interesting to explore other ways (maybe more directly connected to evidence from the brain sciences) to model and understand the role of emotions on decision-making. The present paper offers one alternative approach.

Finally, there are several areas outside economics that study a related problem, although from a different angle. First, there is a literature on information processing that adopts a purely statistical approach. Neurobiologists have worked on complex statistical algorithms that mimic what neurons in the brain do (see the references in section 1.1). Theoretical neuroscientists have constructed mathematical and computational models of the brain based on the underlying biological mechanisms (see Dayan and Abbott (2005) for an introduction). Psychophysicists have developed a “signal-detection theory” to study

⁶From a modelling viewpoint, the paper is in fact a generalized version of the research conducted in the theory of organizations that studies optimal partition of information when evidence is scarce: advisors who can only convey binary information (Calvert, 1985), agents who can only announce whether the organization’s standard is met (Sah and Stiglitz, 1986), or supervisors who can only declare who is the winner of a contest (Meyer, 1991).

⁷See also LeDoux (1996, ch. 4) for a non-technical, historical perspective of the misconceptions about the physical location of functions in the brain.

the likelihood of finding a weak signal in a noisy environment, depending on the statistical properties of the noise and signal random variables (see McNicol (1972) or Wickens (2002) for an introduction). Second, there is also a literature on neural networks and artificial intelligence which builds models inspired by the architecture of the brain in order to solve specific tasks like data processing or filtering (see Abdi (1996) for a review).

2 A model of neuronal cell firing activity

2.1 The brain: a constrained processor of information

An individual (he) has to take an action with uncertain payoff. Prior to that, he can obtain some information about the relative desirability of the alternatives. His objective is to process the information as efficiently as possible, given the physiological limitations of his brain. As reviewed above, the neurobiology literature highlights three key aspects of signal processing in the brain. First, information is scarce and imperfect: neuronal cell firing is stochastically correlated with the state. Second, the brain has a limited capacity to process information. It can only determine whether the neuronal cell firing activity surpasses a given threshold or not. Third, the brain (and, more specifically, the soma according to the somatic marker theory) has the ability to choose the neuronal threshold.

To understand the effects of this limited capacity to process information, we consider a concrete and very basic choice situation. The individual must decide how far from the cave to go hunting. There are two states, $S \in \{A, B\}$: leaving the cave is either dangerous (A) or safe (B), and the individual assigns probability $p \in (0, 1)$ to state A . We denote by $\gamma \in [0, 1]$ the action, where lower values of γ denote going farther away from the cave to hunt. The payoff of the individual is $\pi_A l(\gamma - 1)$ if $S = A$ and $\pi_B l(\gamma - 0)$ if $S = B$, where $l(z) = l(-z)$ for all z and $l'(z) < 0$ for all $z > 0$. Thus, if hunting is dangerous ($S = A$), the individual should stay as close as possible to the cave ($\gamma = 1$), since each mile traveled increases the probability of encountering a predator. If hunting is safe ($S = B$), then he should go as far away as possible to find the best preys ($\gamma = 0$).⁸ Note that π_S captures the marginal cost of taking a wrong action given that the state is S . So, for example, $\pi_A > \pi_B$ would reflect the idea that hunting on a dangerous day is more costly than staying in the cave on a safe day. Given a belief p , the expected payoff function that

⁸We deliberately chose an example encountered by primitive individuals because it illustrates reasonably well evolutionary relevant situations. For current applications, it makes more sense to think of the states as investment A or B being most profitable, relationship A or B being more gratifying, etc.

the individual maximizes is:

$$L(\gamma; p) = p \left[\pi_A l(\gamma - 1) \right] + (1 - p) \left[\pi_B l(\gamma) \right] \quad (1)$$

Before making his choice, the individual takes one step out of the cave and looks around. The information transmitted by the sensory system is modeled in a way to incorporate the three premises reviewed above. First, we assume that there is some stochastic level of neuronal cell firing c that depends on the state. Formally, the likelihood of a cell firing level $c \in [0, 1]$ is $g(c)$ if the state is A and $f(c)$ if the state is B , with $G(c) = \int_0^c g(y)dy$ and $F(c) = \int_0^c f(y)dy$ representing the probability of a cell firing activity not greater than c when the state is A and B , respectively. Furthermore, high cell firing is relatively more likely when $S = A$ and low cell firing is relatively more likely when $S = B$. Said differently, on dangerous days, there are *stochastically* more neurons in the amygdala carrying a message of fear to the brain than on safe days. Formally, the functions $g(\cdot)$ and $f(\cdot)$ satisfy the standard Monotone Likelihood Ratio Property (MLRP):⁹

Assumption 1 (MLRP) $\left(\frac{f(c)}{g(c)} \right)' < 0$ for all c . (A1)

This assumption states that the probability of state B (hunting is safe) rather than A (hunting is dangerous) monotonically decreases with the level of cell firing. In other words, an increase in the number of neurotransmitters carrying the signal “fear” indicates a higher likelihood that a predator is present. Figure 1 provides a graphical representation of cell firing probabilities for the case of symmetric functions $f(\cdot)$ and $g(\cdot)$.

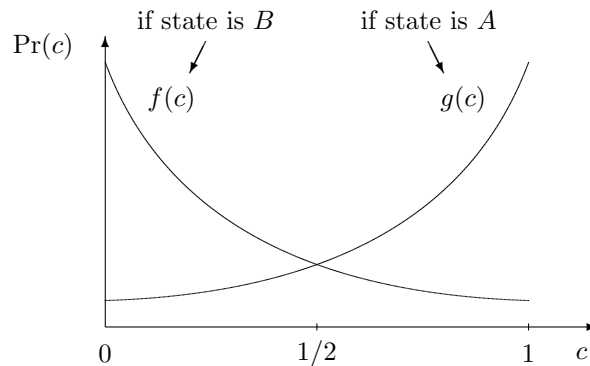


Figure 1. Stochastic cell firing level.

⁹MLRP implies: (i) $\frac{1-G(c)}{g(c)} > \frac{1-F(c)}{f(c)}$, (ii) $\frac{G(c)}{g(c)} < \frac{F(c)}{f(c)}$, and (iii) $G(c) < F(c) \forall c \in (0, 1)$.

Second, the neuronal activity is an indicator of the state, and therefore of the optimal action to be taken. However, physiological constraints prevent the brain from learning the exact level of cell firing c . Following the second premise, we assume that the brain only determines whether the neuronal activity is above or below a threshold x . This means that, for *any* given threshold x , a cell firing activity above it will suggest (more or less strongly) that $S = A$, whereas a cell firing activity below it will indicate (more or less strongly) that $S = B$. Last, according to the third premise, we allow the brain to modify the threshold, that is, to select x . In the next section, we determine how the threshold should be set in order to optimize learning.

The timing of our game can thus be summarized as follows. In stage 0, nature picks a state $S \in \{A, B\}$. In stage 1, the brain sets a threshold $x \in (0, 1)$, the individual looks around, there is neuronal activity $c \in [0, 1]$, and the brain finds out whether the cutoff is reached or not ($c \geq x$). In stage 2, the individual updates his beliefs and chooses an action $\gamma \in [0, 1]$. Final payoffs depend on the action γ and the state S . Figure 2 depicts this sequence of events.

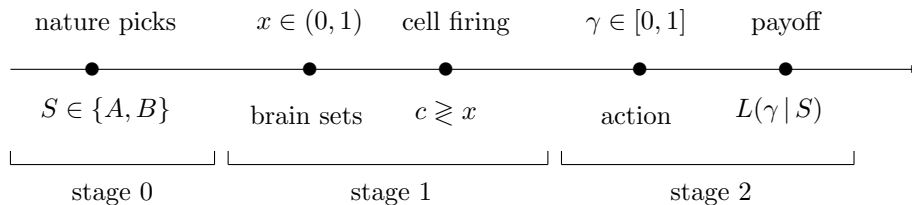


Figure 2. Timing.

2.2 Optimal cell firing threshold

Our first objective is to determine the optimal threshold. We solve this game by backward induction. In stage 2, the optimal action depends on the shape of the payoff $l(\cdot)$. We first assume that $l(z)$ is weakly convex on both sides of its bliss point $z = 0$: $l''(z) \geq 0$ for all z , so that departures from the optimal action are decreasingly costly. A special case of this class of functions is the linear loss function $l(z) = -|z|$.¹⁰ Note that when $l(z)$ is weakly convex, then the expected payoff function, $L(\gamma; p)$, is weakly convex in γ and differentiable in $(0, 1)$, so corner solutions are optimal. If we denote by $\gamma^*(p) = \arg \max_{\gamma} L(\gamma; p)$, necessary and sufficient conditions for $\gamma^*(p) = 1$ and $\gamma^*(p) = 0$ are,

¹⁰In section 3.1, we extend the analysis to a strictly concave loss function.

respectively, $L(1; p) > L(0; p)$ and $L(0; p) > L(1; p)$. Using (1) and given that $l(0) > l(1)$, we have:

$$\gamma^*(p) = 1 \text{ if } p > p^* \equiv \frac{\pi_B}{\pi_A + \pi_B} \quad \text{and} \quad \gamma^*(p) = 0 \text{ if } p < p^* \equiv \frac{\pi_B}{\pi_A + \pi_B}$$

Not surprisingly, if the marginal cost of an incorrect action in a given state increases, then the individual is more willing to take the action optimal in that state even at the increased risk of erring in the other state. Formally, $dp^*/d\pi_A < 0$ and $dp^*/d\pi_B > 0$. In our example, as predators become smarter and more dangerous, the individual is more likely to decide to stay in the cave, even on days that are apparently safe.

In stage 1, the threshold x is selected. Before proceeding, three properties must be noted. First, for any p and x , the belief about state A is revised upwards if x is surpassed and downwards if x is not reached:

$$\bar{p}(x) \equiv \Pr(A | c > x) = \frac{p(1 - G(x))}{p(1 - G(x)) + (1 - p)(1 - F(x))} > p \quad \forall p, x \in (0, 1)^2 \quad (2)$$

$$\underline{p}(x) \equiv \Pr(A | c < x) = \frac{pG(x)}{pG(x) + (1 - p)F(x)} < p \quad \forall p, x \in (0, 1)^2 \quad (3)$$

It captures the idea that low cell firing is an (imperfect) indicator of state B and high cell firing an (imperfect) indicator of state A . Second, a necessary condition for a threshold x to be optimal is that it must prescribe different actions depending on whether $c \gtrless x$. Indeed, if it prescribes the same action, stage 1 is uninformative, and the threshold does not play any role. Third, when evaluating the likelihood of A , surpassing a stringent cutoff is more unlikely than surpassing a weak cutoff, but it is also a stronger indicator that this state is correct: $\partial \Pr(A | c > x) / \partial x > 0$. Also, not reaching a stringent cutoff is more likely than not reaching a weak cutoff, but it is also a weaker indicator that this state is incorrect: $\partial \Pr(A | c < x) / \partial x > 0$. A similar argument is true for state B . This property captures the negative relation in Bayesian learning contexts between likelihood and impact of information.

Properties one and two imply that the individual will set a threshold x such that $\gamma^* = 1$ if the threshold is exceeded and $\gamma^* = 0$ if the threshold is not met. Property three highlights the trade-off between the likelihood of obtaining a given piece of information and its impact on beliefs. Taken together, it means that the optimal threshold maximizes the following value function:

$$\begin{aligned} V(x; p) &= \Pr(c > x) L(1; \bar{p}(x)) + \Pr(c < x) L(0; \underline{p}(x)) \\ &= p \pi_A \left[(1 - G(x))l(0) + G(x)l(1) \right] + (1 - p) \pi_B \left[(1 - F(x))l(1) + F(x)l(0) \right] \end{aligned} \quad (4)$$

Denote by $x^*(p) = \arg \max_x V(x; p)$ the optimal threshold as a function of the belief p , and assume that the parameters of the model are such that $\frac{f(0)}{g(0)} > \frac{p}{1-p} \frac{\pi_A}{\pi_B} > \frac{f(1)}{g(1)}$.¹¹ Our first result is the following.

Proposition 1 *When $l''(z) \geq 0$, the optimal threshold $x^*(p)$ is unique and given by:*

$$\frac{f(x^*(p))}{g(x^*(p))} = \frac{p}{1-p} \frac{\pi_A}{\pi_B} \quad (5)$$

which, in particular, implies that $dx^/dp < 0$.*

Consider two individuals who differ only in their belief about the likelihood of state A . According to Proposition 1, the individual with strongest prior in favor of A sets the lowest threshold. As a result, he is more likely to receive evidence that endorses A and less likely to receive evidence that endorses B than the other agent, *both if the true state is A and if the true state is B* . In other words, the ability to set optimal thresholds increases the likelihood that an individual be reaffirmed in his belief.

This result is due to the trade-off between the likelihood of information and its impact. To see this, suppose that the individual believes that A is more likely than B and, consequently, prefers to choose action $\gamma = 1$ given his current prior. Assume also that he sets a high threshold. If the threshold is surpassed, he will be extremely confident that the state is A , whereas if the threshold is not reached, he will be only moderately convinced about state B . In both cases, his posterior belief hinges towards A , and he takes the same action he would have taken without this information. Suppose instead that the individual sets a low threshold. If the threshold is surpassed, he will slightly increase his confidence in state A , whereas if the threshold is not reached, he will become quite convinced that the state is B . The individual ends up taking different actions and, in both cases, he is quite confident about his choice. Overall, the optimal threshold must balance the belief in favor of A conditional on the threshold being surpassed and the belief in favor of B conditional on the threshold not being reached. In order to achieve this balance, the threshold should be low whenever A is a priori more probable than B : evidence of A is weakly informative but adds to the prior in its favor, whereas evidence of B is highly informative and compensates for the prior against it. In other words, when one state is a priori more likely than the other, the individual will require strong information against it to change his beliefs.

Notice that **(A1)** ensures the uniqueness of the local (hence, global) maximum. If, instead, we assumed stochastic dominance (a substantially weaker condition than **(A1)**

¹¹This condition ensures that the optimal solution is interior.

formally defined as $F(c) > G(c)$ for all c), x^* would not necessarily be unique. Nevertheless, since the value function (4) is always submodular, the monotonic relation between threshold and likelihood of state, $dx^*/dp < 0$, would be preserved in all the local maxima.¹² Last, note also that the threshold is set in such a way that, in equilibrium, the most costly mistakes are most likely to be avoided: $dx^*/d\pi_A < 0$ and $dx^*/d\pi_B > 0$.

2.3 The role of emotions

The model presented so far relies exclusively on our three neurobiological premises. According to the somatic marker theory, there is a tight physiological link between these findings and the somatic dispositions of individuals. As discussed in section 1.1, Bechara and Damasio (2005) argue that emotions affect the way information is processed, a view consistent with the findings of Rustichini et al. (2005). More precisely, the soma modifies the neuronal threshold by increasing the likelihood of interpreting evidence in favor of the currently supported hypothesis. They also conclude that this threshold modulation is beneficial for decision-making. Our model clarifies this issue.

Corollary 1 *If we accept as a premise that somatic dispositions play the biological role described in the somatic marker theory, then emotions do improve decision-making.*

Proposition 1 states that it is optimal to set a threshold so that the likelihood of confirming a belief is greater the stronger the prior in its favor. The somatic marker theory argues that emotions affect the threshold in that precise direction. Corollary 1 simply puts these two results together. In other words, our model formally proves that the somatic marker’s claim regarding the desirability of favoring the current belief, when interpreting new information, is correct. Conversely, consider an individual with an abnormal activation of somatic signals who sets the threshold independently of his belief ($dx^*/dp = 0$) or who is most receptive to information that contradicts his current belief ($dx^*/dp > 0$). According to our theory, this person will make suboptimal choices more frequently. Last and as a matter of clarification, our model does not argue that the soma is an intelligent system that performs the sophisticated trade-off described in Proposition 1. Instead, it suggests that the development through evolution of a biological mechanism (in this case, governed by emotions) with these qualitative properties is indeed advantageous.

¹²We thank Guofu Tan for pointing this out.

3 Optimal threshold in other environments

The model presented in section 2 makes a number of assumptions. It is only natural to discuss whether the qualitative properties of Proposition 1 (and its implications for emotional dispositions presented in Corollary 1) hold in more general, or simply different, settings. In this section, we extend the basic model in a number of directions: we consider a concave payoff function (section 3.1), we increase the number of states (section 3.2), and we allow for two stages of cell firing activity (section 3.3). To avoid carrying unnecessary parameters, we will assume without loss of generality that $\pi_A = \pi_B = 1$.

3.1 Increasingly costly departures from the optimal action

Although plausible in some settings, a weakly convex payoff function may not always be the most appropriate way to represent the payoff of the individual. For example, hunting closely enough to the cave might not be dangerous, even in the presence of predators. To capture this alternative situation, we assume in this section that the payoff function is strictly concave, continuous and twice-differentiable everywhere: $l''(z) < 0$ for all z and $l'(0) = 0$. Denote by $\gamma^{**}(p) = \arg \max_{\gamma} L(\gamma; p)$. Taking the first-order condition in (1), we have:

$$\frac{l'(\gamma^{**}(p))}{l'(1 - \gamma^{**}(p))} = \frac{p}{1 - p} \quad (6)$$

Note that $\left. \frac{\partial L(\gamma; p)}{\partial \gamma} \right|_{\gamma=0} = -p l'(1) > 0$, $\left. \frac{\partial L(\gamma; p)}{\partial \gamma} \right|_{\gamma=1} = (1 - p) l'(1) < 0$, and $\frac{\partial^2 L(\gamma; p)}{\partial \gamma^2} < 0$. Thus, contrary to the previous case, extreme choices are never optimal when departures are increasingly costly: $\gamma^{**}(p) \in (0, 1)$ for all $p \in (0, 1)$. Furthermore:

$$\frac{d\gamma^{**}(p)}{dp} \propto \frac{\partial^2 L(\gamma; p)}{\partial \gamma \partial p} = -l'(1 - \gamma) - l'(\gamma) > 0$$

As the likelihood of danger increases, the individual decides to stay closer to the cave. Before making any decision, the individual looks outside the cave. The brain sets a threshold x that is used to interpret the evidence. Given (2), (3) and (6), the optimal actions when $c > x$ and $c < x$ are, respectively:

$$\frac{l'(\gamma^{**}(\bar{p}(x)))}{l'(1 - \gamma^{**}(\bar{p}(x)))} = \frac{\bar{p}(x)}{1 - \bar{p}(x)} = \frac{p}{1 - p} \frac{1 - G(x)}{1 - F(x)} \quad (7)$$

$$\frac{l'(\gamma^{**}(\underline{p}(x)))}{l'(1 - \gamma^{**}(\underline{p}(x)))} = \frac{\underline{p}(x)}{1 - \underline{p}(x)} = \frac{p}{1 - p} \frac{G(x)}{F(x)} \quad (8)$$

where $\gamma^{**}(\bar{p}(x)) > \gamma^{**}(\underline{p}(x))$. Differentiating (7) and (8) and using **(A1)**, we obtain:

$$\frac{d\gamma^{**}(\bar{p}(x))}{dx} > 0 \quad \text{and} \quad \frac{d\gamma^{**}(\underline{p}(x))}{dx} > 0 \quad (9)$$

According to (9), the individual reacts to an increase in the threshold by choosing a higher action. Indeed, if the higher threshold is surpassed, then the evidence in favor of state A is stronger. Conversely, if the higher threshold is not reached, then the evidence in favor of B is weaker. In both cases, higher actions follow. We can now define the value function of the individual. Adapting (4) to the present case, we get:

$$\begin{aligned} V(x; p) &= \Pr(c > x) L(\gamma^{**}(\bar{p}(x)); \bar{p}(x)) + \Pr(c < x) L(\gamma^{**}(\underline{p}(x)); \underline{p}(x)) \\ &= p(1 - G(x))l(1 - \gamma^{**}(\bar{p}(x))) + (1 - p)(1 - F(x))l(\gamma^{**}(\bar{p}(x))) \\ &\quad + pG(x)l(1 - \gamma^{**}(\underline{p}(x))) + (1 - p)F(x)l(\gamma^{**}(\underline{p}(x))) \end{aligned} \quad (10)$$

To solve this value function, we first need to introduce a strengthened version of MLRP:

Assumption 2 (s-MLRP) $\frac{f(c)}{1-F(c)} > \frac{g(c)}{1-G(c)}$, $\left(\frac{f(c)}{g(c)} \frac{1-G(c)}{1-F(c)}\right)' \leq 0$, $\left(\frac{f(c)}{g(c)} \frac{G(c)}{F(c)}\right)' \leq 0$ **(A2)**

Note that the first and second or the first and third parts in **(A2)** imply **(A1)**, but the converse is not true. Denote by $x^{**}(p) = \arg \max_x V(x; p)$. The optimal threshold maximizes (10) given (7) and (8). The first-order condition is:

$$\frac{\partial V(x; p)}{\partial x} \Big|_{x=x^{**}} = 0 \Rightarrow \frac{f(x^{**})}{g(x^{**})} = \frac{p}{1-p} \frac{l(1 - \gamma^{**}(\bar{p}(x^{**}))) - l(1 - \gamma^{**}(\underline{p}(x^{**})))}{l(\gamma^{**}(\underline{p}(x^{**}))) - l(\gamma^{**}(\bar{p}(x^{**})))} \quad (11)$$

and we can now state our next result.

Proposition 2 *Under **(A2)** and when $l''(z) < 0$, a sufficient condition to have a unique optimal threshold x^{**} given by (11) and such that $dx^{**}/dp < 0$ is:*

$$\frac{d}{dx} \left[F(x)l(\gamma^{**}(\underline{p}(x))) + (1 - F(x))l(\gamma^{**}(\bar{p}(x))) \right]_{x=x^{**}} > 0 \quad (C1)$$

*Under **(A1)**, the condition guarantees $dx^{**}/dp < 0$ in every optimum but not uniqueness.*

When departures from the optimal action are increasingly costly, the quasi-concavity of the value function $V(x; p)$ is not guaranteed for generic values of the densities and payoff functions $f(\cdot)$, $g(\cdot)$ and $l(\cdot)$. In fact, there are two forces at play when choosing the optimal x . First, a higher threshold is less likely to be surpassed and therefore more likely to induce the low action ($\gamma^{**}(\underline{p}(x))$ will be taken more often than $\gamma^{**}(\bar{p}(x))$). Second,

either outcome is a weaker indicator that the state is B . Therefore, the final action will be higher both when the threshold is surpassed and when it is not reached (see (9)). In other words, conditional on being a priori more confident that the state is A , the individual can always lower the action when his ex post confidence is weakened. Therefore, and contrary to the previous section, choosing a high threshold is not necessarily always dominated. Proposition 2 provides a sufficient condition such that $V(\cdot)$ is well-behaved. The interpretation of **(C1)** is simple: starting from the optimal threshold, setting a higher x increases the payoff of the individual if and only if the state is B .¹³ In other words, as x increases, the direct effect of increasing the likelihood of choosing the low action must dominate the indirect effect of choosing relatively higher actions. It is also clear why the condition is automatically satisfied when payoffs are weakly convex: since only $\gamma^* = 0$ or $\gamma^* = 1$ are optimal, a marginal change in the threshold does not affect the choice of action (indirect effect) but it does change the likelihood of surpassing it (direct effect).

In order to provide a full characterization of the equilibrium, we will now restrict attention to quadratic payoffs, $l(z) = \alpha - \beta z^2$ with $\beta > 0$. Under this restriction, the optimal action coincides with the belief of the individual. Formally, (7) and (8) become:

$$\gamma^{**}(\bar{p}(x)) = \bar{p}(x) \quad \text{and} \quad \gamma^{**}(\underline{p}(x)) = \underline{p}(x) \quad (12)$$

Replacing (12) into (11), we obtain the following result.

Proposition 3 *Under **(A2)** and if payoffs are quadratic, the optimal threshold x^{**} is unique, it satisfies $dx^{**}/dp < 0$, and it solves:*

$$\frac{f(x^{**})}{g(x^{**})} = \frac{p}{1-p} \frac{(1 - \bar{p}(x^{**})) + (1 - \underline{p}(x^{**}))}{\bar{p}(x^{**}) + \underline{p}(x^{**})} \quad (13)$$

*Under **(A1)**, the optimal threshold satisfies $dx^{**}/dp < 0$ but it may not be unique.*

The main conclusion in Proposition 1 is that thresholds are set in such a way that existing beliefs are likely to be reaffirmed. According to Propositions 2 and 3, the result extends immediately to quadratic payoffs. It also extends to other concave payoff functions as long as **(C1)** is satisfied. The following example illustrates some other properties of the equilibrium.

¹³Given (11), **(C1)** can also be stated as: $\frac{d}{dx} \left[G(x)l(1-\gamma^{**}(\underline{p}(x))) + (1-G(x))l(1-\gamma^{**}(\bar{p}(x))) \right]_{x=x^{**}} < 0$. Therefore, a higher x decreases the payoff if the state is A .

Example 1. Suppose that the cell firing distribution functions are $G(c) = c^2$ and $F(c) = c$. From (5) and (13) and after some algebra, the optimal thresholds with linear ($l(z) = -|z|$) and quadratic ($l(z) = -z^2$) payoffs are respectively:

$$x^*(p) = \frac{1-p}{2p} \quad \text{and} \quad x^{**}(p) = \frac{\sqrt{1-p}}{\sqrt{1-p} + \sqrt{1+p}}$$

where x^* and x^{**} are interior if $p > 1/3$. In this example, the optimal threshold is always less extreme with quadratic than with linear payoffs: $x^* \gtrless x^{**} \gtrless 1/3$ for all $p \lesseqgtr 3/5$.

3.2 Enlarging the state space

In this section, we are interested in situations in which making a decision requires to assess the likelihood of many states, so that information is more complex to evaluate. To better isolate this new dimension, we consider the model presented in section 2.1 and assume that there is a continuum of states $s \in [0, 1]$, but only two possible actions $\gamma \in \{0, 1\}$. In our example, there are many predators, and s captures the proportion of predators who are currently in the neighborhood. The individual can only go hunting ($\gamma = 0$) or stay in the cave ($\gamma = 1$). We order the states by the increasing degree of danger, from safest ($s = 0$) to most dangerous ($s = 1$). The probability of a cell firing level c given state s is now $f(c|s)$. The generalization of MLRP to the continuous case is:

Assumption 1' (continuous MLRP) $\frac{d}{dc} \left(\frac{f_s(c|s)}{f(c|s)} \right) \geq 0$ for all c and s . (A1')

The individual initially believes that the state is s with probability $p(s)$, where $\int_0^1 p(s) ds = 1$. The expected payoff function described in (1) can then be generalized as:

$$L(\gamma; p(s)) = \int_0^1 p(s) l(\gamma - s) ds$$

Since we restrict the action space to $\gamma \in \{0, 1\}$, the optimal action is:

$$\hat{\gamma} = 1 \quad \text{if} \quad \int_0^1 p(s) (l(1-s) - l(s)) ds > 0 \quad \text{and} \quad \hat{\gamma} = 0 \quad \text{if} \quad \int_0^1 p(s) (l(1-s) - l(s)) ds < 0$$

Given a prior distribution $p(s)$ and a threshold x , the individual will put more weight in states closer to 1 if $c > x$ and in states closer to 0 if $c < x$. Thus, in equilibrium, the individual will choose $\hat{\gamma} = 1$ if the threshold is surpassed and $\hat{\gamma} = 0$ if the threshold is not reached (this property will be checked ex-post). The value function is then:

$$\begin{aligned} V(x; p(s)) &= \Pr(c > x) L(1; p(s|c > x)) + \Pr(c < x) L(0; p(s|c < x)) \\ &= \int_0^1 p(s) \left((1 - F(c|s)) l(1-s) + F(c|s) l(s) \right) ds \end{aligned} \tag{14}$$

Denote by $\hat{x}(p(s)) = \arg \max_x V(x; p(s))$. We have:

Proposition 4 *With a continuum of states $s \in [0, 1]$ and only two actions $\gamma \in \{0, 1\}$, the optimal threshold \hat{x} is unique and given by:*

$$-\int_0^1 p(s) f(\hat{x} | s) (l(1-s) - l(s)) ds = 0 \quad (15)$$

If $\left(\frac{q(s)}{p(s)}\right)' \geq 0$, then $\hat{x}(q(s)) < \hat{x}(p(s))$.

The main conclusions stated in Proposition 1 extend to the case of two actions and a continuum of states: (i) the optimal threshold is unique and (ii) if one individual puts more weight in higher states than another in a MLRP sense, $\left(\frac{q(s)}{p(s)}\right)' \geq 0$, then he also sets a lower threshold. This property is simply a generalization of the comparative statics on p to the case of a continuous distribution of beliefs. It thus strengthens the idea that, under an optimal threshold, existing beliefs are more likely to be supported and less likely to be refuted than opposite beliefs.

3.3 Dynamic thresholds

So far, we have assumed that the individual takes one look out of the cave before choosing whether to go hunting. Another natural extension is to consider the situation in which he can take a second look before deciding what to do. This case is interesting only if the individual can re-optimize the threshold after having looked for the first time (otherwise, it boils down to a model with one stage of cell firing and a more accurate information). We deal with this situation by adding one stage to the basic model described in section 2.1. In stage 1, the individual has a prior belief p . He sets a threshold y , learns whether $c_1 \geq y$, and updates his belief. In stage 2 and given his new belief, he sets a new threshold x , learns whether $c_2 \geq x$ and, again, updates his belief. The action (and payoff) is contingent on his posterior belief which depends on $c_1 \geq y$ and $c_2 \geq x$. We assume that c_t is independently drawn from distribution $G_t(c_t)$ if $S = A$ and distribution $F_t(c_t)$ if $S = B$, with $t \in \{1, 2\}$. Distributions may be different across stages but both $\frac{f_1(c)}{g_1(c)}$ and $\frac{f_2(c)}{g_2(c)}$ satisfy **(A1)**.

The game is solved by backward induction. The second cell firing stage is identical to that described in section 2.2. The second stage value function is then given by (4). If the first threshold y is surpassed, the posterior is $\bar{p}(y)$ and the optimal second period threshold is $x^*(\bar{p}(y))$. If the first threshold y is not reached, the posterior is $\underline{p}(y)$ and the

optimal second period threshold is $x^*(\underline{p}(y))$. Combining (2), (3) and (5), we get:

$$\frac{f_2(x^*(\bar{p}(y)))}{g_2(x^*(\bar{p}(y)))} = \frac{\bar{p}(y)}{1 - \bar{p}(y)} = \frac{p}{1 - p} \frac{1 - G_1(y)}{1 - F_1(y)} \quad (16)$$

$$\frac{f_2(x^*(\underline{p}(y)))}{g_2(x^*(\underline{p}(y)))} = \frac{\underline{p}(y)}{1 - \underline{p}(y)} = \frac{p}{1 - p} \frac{G_1(y)}{F_1(y)} \quad (17)$$

The value function that the individual maximizes in the first cell firing stage is:

$$W(y; p) = \Pr(c_1 > y) \left[V(x^*(\bar{p}(y)); \bar{p}(y)) \right] + \Pr(c_1 < y) \left[V(x^*(\underline{p}(y)); \underline{p}(y)) \right] \quad (18)$$

The first term is the likelihood of surpassing a cutoff y , in which case the posterior becomes $\bar{p}(y)$, multiplied by the second-stage value function given this posterior (see (4)), and under the anticipation of an optimal second-stage threshold $x^*(\bar{p}(y))$ (see (5)). The same logic applies to the second term. Notice that threshold y affects the utility of the individual only through its effect in the posterior belief transmitted to stage 2. Denote by $y^*(p) = \arg \max_y W(y; p)$ the optimal stage 1 threshold. It maximizes (18) under (16) and (17). Taking the first-order condition in (18) and applying the envelope theorem:

$$\left. \frac{\partial W(y; p)}{\partial y} \right|_{y=y^*} = 0 \Rightarrow \frac{f_1(y^*)}{g_1(y^*)} = \frac{p}{1 - p} \frac{G_2(x^*(\underline{p}(y^*))) - G_2(x^*(\bar{p}(y^*)))}{F_2(x^*(\underline{p}(y^*))) - F_2(x^*(\bar{p}(y^*)))} \quad (19)$$

and we get the analogue of Proposition 2 to the dynamic threshold case.

Proposition 5 *Under (A2) and with two cell firing stages, a sufficient condition to have a unique optimal threshold y^* in stage 1 given by (19) and such that $dy^*/dp < 0$ is:*

$$\frac{d}{dy} \left[F_1(y) F_2(x^*(\underline{p}(y))) + (1 - F_1(y)) F_2(x^*(\bar{p}(y))) \right]_{y=y^*} > 0 \quad (\mathbf{C2})$$

Under (A1), the condition guarantees $dy^/dp < 0$ in every optimum but not uniqueness.*

It is well-known that two-stage optimization problems are easily plagued by non-convexities in the overall maximand, hence the possibility of multiple local optima. Proposition 5 determines a sufficient condition for uniqueness of the maximum. Under this condition, the familiar comparative statics with respect to p is also preserved. In fact, the two-stage model with decreasingly costly departures is technically similar to the one-stage model with increasingly costly departures. In particular, the same two effects operate when the threshold is increased. First, the direct effect: the new threshold is less likely to be surpassed. Second, the indirect effect: because surpassing a higher threshold is a

stronger indicator of state A whereas not reaching a higher threshold is a weaker indicator of state B , an increase in stage 1 threshold is always followed by a decrease in stage 2 threshold ($dx^*(\bar{p})/dy < 0$ and $dx^*(\underline{p})/dy < 0$). As before, the condition stated in the proposition ensures that the direct effect dominates the indirect one.

Following a similar procedure as in section 3.1, we impose specific functional forms to be able to characterize the equilibrium. We assume that the second stage cell firing densities are linear and symmetric, $g_2(c) = 2c$ and $f_2(c) = 2(1 - c)$, and keep a general formulation for the first stage cell firing densities. It turns out that, under this restriction, the optimal first stage threshold takes the same form as in the one-stage quadratic case.

Proposition 6 *Under (A2) and with second stage linear and symmetric cell firing densities, the optimal first stage threshold y^* is unique, it satisfies $dy^*/dp < 0$, and it solves:*

$$\frac{f_1(y^*)}{g_1(y^*)} = \frac{p}{1-p} \frac{(1 - \bar{p}(y^*)) + (1 - \underline{p}(y^*))}{\bar{p}(y^*) + \underline{p}(y^*)} \quad (20)$$

Under (A1), the optimal threshold satisfies $dy^/dp < 0$ but it may not be unique.*

Propositions 1, 3 and 4 showed the desirability of setting the threshold in a way that pre-existing beliefs are likely to be confirmed when there is one stage of cell firing. Proposition 6 shows that, under some conditions, being more receptive to information that confirms rather than contradicts existing beliefs is optimal also when further news are subsequently available. The intuition relies, just as before, on the balance between the likelihood of the information vs. its impact: the individual must sacrifice either quality or probability of obtaining information, and quality is relatively more important for the state that is a priori less favored.

As already noted, the first stage threshold y affects the belief inherited at stage 2. Then, it impacts the utility of the individual only through its effect on beliefs. By contrast, the second stage threshold x directly determines which action is chosen. With three or more stages, the thresholds at all but the last one only affect the belief inherited at the following stage. Thus, we conjecture that the main properties of the thresholds emphasized in Propositions 1 and 6 should, under reasonable conditions, be preserved in a situation involving more than two cell firing stages. Unfortunately, a formal treatment of this problem is difficult due to the increased analytical complexity of the problem.

Propositions 1 and 6 taken together have an interesting implication for the neurobiological role of emotions on decision-making.

Corollary 2 *Under the premises of the somatic marker theory, emotions improve decision-making at every stage of the information gathering process. Furthermore, an emotional individual is most likely to develop beliefs that are maintained and least likely to develop beliefs that are abandoned.*

Because beliefs are developed on the basis of relevant information, it is obvious that, other things being equal, further evidence is more likely to reaffirm the individual in his beliefs than contradict him. Corollary 2 makes a stronger argument: compared to an individual with an abnormal activation of somatic signals, an emotional individual will modulate thresholds in a way that he will stick to his beliefs more often.¹⁴ Indeed, an increase in the belief that the true state is A implies a decrease in the first period cutoff, so a higher likelihood of surpassing it. Furthermore, if it is surpassed, the belief is again updated upwards and the second stage threshold is decreased even further. Overall, somatic activations have a snowball effect on decision-making: the more information an individual obtains, the more he modulates the threshold to confirm his belief and, as a result, the more likely he is to receive further supporting evidence.¹⁵ We conclude this section with a simple example that nicely illustrates these and other effects of optimal threshold selection on the process of information acquisition.

Example 2. Consider a two-stage cell firing model and suppose that the density functions are identical in both stages, symmetric and linear: $g_t(c) = 2c$ and $f_t(c) = g(1-c) = 2(1-c)$, with $t \in \{1, 2\}$. From (5) and (20) and after some algebra, the optimal thresholds in the first and second stage are:

$$\left(\frac{1-y^*}{y^*}\right)^2 = \frac{p}{1-p} \Leftrightarrow y^*(p) = \frac{\sqrt{1-p}}{\sqrt{1-p} + \sqrt{p}} \quad \text{and} \quad \frac{1-x^*}{x^*} = \frac{p}{1-p} \Leftrightarrow x^*(p) = 1-p$$

The threshold is always less sensitive to the belief in the first stage than in the second stage, $x^*(p) \gtrless y^*(p) \gtrless 1/2$ for all $p \gtrless 1/2$, which seems quite natural. The individual anticipates that, after the first cell firing stage, there is still more information to come. Therefore, maximizing the amount of information acquired in the first stage is relatively more important than learning whether, at this point, it supports or contradicts current beliefs, and more information is on average obtained with a less extreme threshold. More generally, the result implies that for some extreme priors, the belief about which state is

¹⁴Naturally and using a basic property of martingales, if the emotional individual receives contradictory evidence, he will also revise his beliefs more strongly (see section 5.3).

¹⁵Again by the properties of martingales, if signals and threshold tend to infinity the individual always ends up learning the true state.

more probable will not be reversed after the first cell firing stage.¹⁶ This sharply contrasts with the second cell firing stage, where a necessary condition for a threshold to be optimal is that it must prescribe different actions depending on the result of cell firing.

4 The effect of physiological limitations on decision-making

The information processing mechanism by which the individual sets a threshold and learns only whether it is surpassed necessarily implies some information loss, relative to a situation where the individual could determine the exact level of cell firing. In this section, we discuss the cost of this physiological constraint as a function of the environment considered: two or more states, one or two cell firing stages, and concave or convex loss functions (section 4.1). We then propose a simple extension where the individual can choose between a low cost but coarse information partition and a high cost but precise information partition, and study in which cases one of them dominates the other (section 4.2).

4.1 Costs of a coarse information partition

Suppose that, instead of setting a threshold, the brain could determine the exact level of cell firing. Interestingly, in the environment described in section 2.2 with two states and a weakly convex utility loss, physiological limitations do not prevent efficient decision-making. The key is that $l''(z) \geq 0$ implies that only extreme choices are optimal, that is, γ^* is either 0 or 1. The threshold x^* is set in such a way that if the individual learned that $c = x^*$ (a zero-probability event), then he would be indifferent between actions $\gamma^* = 0$ and $\gamma^* = 1$. Formally and using (5):

$$\Pr(A | c = x^*) = \frac{p g(x^*)}{p g(x^*) + (1 - p) f(x^*)} = \frac{\pi_B}{\pi_A + \pi_B}$$

As a result, $\gamma^* = 1$ dominates $\gamma^* = 0$ for all $c > x^*$ and $\gamma^* = 0$ dominates $\gamma^* = 1$ for all $c < x^*$ which, in turn, means that learning whether x^* is surpassed or not is sufficient for the purpose of determining which action to take. Even though the assumptions in section 2.2 are restrictive, it is worth thinking in which circumstances they are met. Convexity of the payoff function reflects the fact that marginal departures from the ideal choice are the

¹⁶Formally, $\Pr(A|c_1 > y^*(p)) = 1 - [y^*(p)]^2 < \frac{1}{2}$ for all $p < \frac{2-\sqrt{2}}{4}$. Symmetrically, $\Pr(A|c_1 < y^*(p)) = [1 - y^*(p)]^2 > \frac{1}{2}$ for all $p > \frac{2+\sqrt{2}}{4}$. If $p \in (0, \frac{2-\sqrt{2}}{4})$, the individual will believe at the end of the first stage that B is more likely than A independently of whether $c_1 \geq y^*$. If $p \in (\frac{2+\sqrt{2}}{4}, 1)$, the individual will believe at the end of the first stage that A is more likely than B independently of whether $c_1 \geq y^*$.

most costly ones. Therefore, it is suitable to model environments where life-threatening events occur as soon as the optimal action is not taken. In our example, it can capture a high probability of both fatal injury when a predator is encountered and death by starvation if no food is collected.

The same conclusion may extend to more complex environments, like the continuum of states extension presented in section 3.2. In that setting, the individual does not suffer a utility loss from a coarse information partition either, simply because the action space is binary ($\Gamma = \{0, 1\}$). Overall, as long as (i) there is one stage of cell firing and (ii) only two actions are relevant (in equilibrium as in section 2.2 or by assumption as in section 3.2), setting one threshold will be sufficient for the purpose of decision-making. We label these environments as “basic” or “primitive” decision-making situations.

We can also see why the two conditions are necessary. Consider a situation with increasingly costly departures ($l''(z) \leq 0$) and a large action space ($\gamma \in [0, 1]$). As discussed in section 3.1, the optimal action in that environment is different for every belief even with two states (see (6)). Therefore, if the individual is only able to set a threshold, he will have only one of two posterior beliefs (depending on whether the threshold is surpassed or not), which immediately results in a strictly positive utility loss. A similar argument applies to the dynamic environment considered in section 3.3. From equations (16) and (17) we can see that every belief inherited from stage 1 implies a different optimal threshold in stage 2. Therefore, even though setting only one threshold in stage 2 is sufficient, the individual would still increase his payoff if he were able to determine the exact cell firing level in stage 1. Notice also that, with a coarse information partition, the order in which evidence is received matters. For example, under the sequence c^* ‘small’ followed by c^{**} ‘high’, $\gamma = 1$ is more likely to be chosen (i.e., stage 2 threshold is more likely to be surpassed) than under the sequence c^{**} followed by c^* . Taken together, these observations suggest that the brain is well designed to process information in static environments, but behavioral errors are inevitably generated when several pieces of information need to be processed sequentially. The results of this section are summarized as follows.

Proposition 7 *Physiological limitations in information processing do not prevent efficient decision-making only in basic situations: a static environment with a choice between two relevant alternatives.*

Finally, we provide a simple analytical example characterizing the utility loss due to a coarse information partition in a two-stage cell firing environment.

Example 3. Consider the same setting as in Example 2. Let $l(0) = 1$, $l(1) = 0$ and $p = 1/2$, which implies that $y^* = \frac{1}{2}$, $\bar{p}(\frac{1}{2}) = \frac{3}{4}$, $x^*(\bar{p}(\frac{1}{2})) = \frac{1}{4}$, $\underline{p}(\frac{1}{2}) = \frac{1}{4}$, and $x^*(\underline{p}(\frac{1}{2})) = \frac{3}{4}$. Given $p(A|c_1) = c_1$, the expected payoff if the exact level of cell firing c_1 is observed is:

$$\tilde{W} = p \int_0^1 \Pr(c_1|A) [1 - G(x^*(p(A|c_1)))] dc_1 + (1-p) \int_0^1 \Pr(c_1|B) [F(x^*(p(A|c_1)))] dc_1 = \frac{5}{6}$$

The payoff if the individual learns only whether the threshold $y^* = 1/2$ is surpassed is:

$$W = \Pr(A) \left[\Pr(c_1 > \frac{1}{2} | A) \Pr(c_1 > \frac{1}{4} | A) + \Pr(c_1 < \frac{1}{2} | A) \Pr(c_1 > \frac{3}{4} | A) \right] \\ + \Pr(B) \left[\Pr(c_1 < \frac{1}{2} | B) \Pr(c_1 < \frac{3}{4} | B) + \Pr(c_1 > \frac{1}{2} | B) \Pr(c_1 < \frac{1}{4} | B) \right] = \frac{13}{16} < \tilde{W}$$

which results in a utility loss of only 2.5%.

4.2 The affective and cognitive encoding channels

As discussed in section 1.3, the main objective of the paper is not to analyze the interplay between a rational and an emotional system in the brain. Nevertheless, understanding whether neuronal thresholds are modulated by the soma (and, if so, why) remains a challenge in neuroeconomics. In that respect, comparing the behavior of an individual supposedly using an emotional process with the behavior of an individual using a “more sophisticated” process might shed some light on this puzzle.

In this section, we postulate that, for each decision, the brain may have to choose between different encoding channels: an affective process where the partition of information is coarse but fast vs. a cognitive process where the partition is fine but slow and/or energy demanding.¹⁷ The affective process is the process analyzed so far; it only determines whether the neuronal activity is above or below a threshold. The cognitive process is a sophisticated channel capable of interpreting neuronal activity in more detail (for example, by setting more than one threshold). It is crucial to note that this approach does not presuppose that the cognitive channel leads to correct choices whereas the affective channel makes mistakes. Rather, both channels process the information received in an optimal way. However, the latter channel faces tighter constraints than the former.

According to Proposition 7, the affective channel is sufficiently precise in basic situations. When the environment is more complex, the utility loss of a coarse partition

¹⁷Contrary to the rest of the paper (and to the general methodology advocated in it) this subsection is based more on an “intuitive trade-off” than on evidence from neurobiology. The conclusions should then be taken with extra caution.

becomes positive, and the cognitive channel becomes relatively more valuable. To study in more detail the marginal gain of using the cognitive channel, we consider the simplest extension of the basic model in which the affective channel is not fully efficient. Formally, we add a third state, $S = O$, to the two-state and a continuum of actions model of section 2. State O is intermediate and captures, for example, a case where hunting is mildly dangerous. The payoff of the individual in that state is $l(\gamma - \frac{1}{2})$ and the probability of a cell firing level c is $h(c)$, with $\left(\frac{f(c)}{h(c)}\right)' < 0$ and $\left(\frac{h(c)}{g(c)}\right)' < 0$ for all c . Given a linear loss function, $l(z) = -|z|$, a straightforward extension of the argument in Proposition 1 implies that only three actions can be optimal, $\tilde{\gamma} \in \{0, \frac{1}{2}, 1\}$. This has two implications. First, there is a utility loss whenever the affective channel is used. Second, there is no gain in utility by being able to set more than two cutoffs. Therefore, without loss of generality, we assume in what follows that the cognitive process sets exactly two thresholds.

Denote by p_S the probability of state S (with $\sum_S p_S = 1$). For the same reason as in section 2.2, a necessary condition for cutoffs x_1 and x_2 ($> x_1$) to be optimal is that $\tilde{\gamma} = 0$ if $c \in [0, x_1)$, $\tilde{\gamma} = \frac{1}{2}$ if $c \in [x_1, x_2]$ and $\tilde{\gamma} = 1$ if $c \in (x_2, 1]$. The value function that the individual maximizes is then:

$$\begin{aligned} V(x_1, x_2) &= \Pr(c < x_1)L(0; p(\cdot | c < x_1)) + \Pr(c \in [x_1, x_2])L(\frac{1}{2}; p(\cdot | c \in [x_1, x_2])) \\ &\quad + \Pr(c > x_2)L(1; p(\cdot | c > x_2)) \\ &= -p_B \left[(1 - F(x_2)) + \frac{1}{2}(F(x_2) - F(x_1)) \right] - p_O \left[\frac{1}{2}(1 - H(x_2)) + \frac{1}{2}H(x_1) \right] \\ &\quad - p_A \left[\frac{1}{2}(G(x_2) - G(x_1)) + G(x_1) \right] \end{aligned} \quad (21)$$

Taking first-order conditions in (21), we obtain that the optimal cutoffs under this cognitive information processing channel, \underline{x} and \bar{x} , solve:

$$\frac{f(\underline{x})}{g(\underline{x})} = \frac{p_A}{p_B} + \frac{p_O}{p_B} \frac{h(\underline{x})}{g(\underline{x})} \quad \text{and} \quad \frac{f(\bar{x})}{g(\bar{x})} = \frac{p_A}{p_B} - \frac{p_O}{p_B} \frac{h(\bar{x})}{g(\bar{x})}$$

Notice that $\underline{x} < \bar{x}$ for all $p_A, p_O, p_B \in (0, 1)^3$ and $\underline{x} = x^* = \bar{x}$ when $p_O = 0$. If an information partition in three regions is cognitively too demanding, the individual can activate an affective channel, characterized by one cutoff only. Information processing is then faster but partitioned in only two regions. Let \tilde{x} be the cutoff that solves:

$$\frac{f(\tilde{x})}{g(\tilde{x})} = \frac{p_A}{p_B}$$

It is immediate to see that $\tilde{x} \in (\underline{x}, \bar{x})$, and we can state the following result.

Proposition 8 *With three states, the cognitive channel sets two thresholds (\underline{x}, \bar{x}) whereas the affective channel sets one threshold which, depending on (p_A, p_O, p_B) , is \underline{x} or \tilde{x} or \bar{x} . The utility loss under the affective channel is highest when all states are equally likely and lowest when one state is highly unlikely.*

When the individual sets only one threshold, he realizes that this excessively coarse information partition results in some utility loss. In particular, the individual will be able to discriminate only between two actions, depending on whether the selected threshold is surpassed or not. The question is to decide which action is not worth pursuing. This will depend on the *relative likelihood* of the different states. When A is the most unlikely state (p_B/p_O and p_O/p_A are high), the individual sets threshold \underline{x} ; he chooses $\gamma = 0$ if $c < \underline{x}$ and $\gamma = \frac{1}{2}$ if $c > \underline{x}$. Thus, relatively to the cognitive channel, the affective channel discriminates *optimally* between the low and medium actions and fully disregards the high one. When B is most unlikely, the opposite is true: the individual sets \bar{x} and disregards the low action. Last, when O is most unlikely, the threshold is \tilde{x} and only extreme actions (0 or 1) are undertaken. Note that, just like x^* , the cutoff \tilde{x} compares the ratio of prior probabilities of A and B with the ratio of their marginal cell firing levels $g(\cdot)$ and $f(\cdot)$, ignoring any information relative to state O . When $p_O \rightarrow 0$, all three thresholds converge to the optimal threshold with two states. Overall, the affective channel sacrifices the action which is optimal in the state most unlikely to occur, and discriminates optimally between the other two. So, even though by definition the cognitive channel always outperforms the affective channel, the utility difference between the two depends crucially on the probability distribution of states. More precisely, the difference is greatest when all states are equally likely and smallest when one state occurs with very low probability.¹⁸

5 Implications of the theory

We now analyze some implications of our theory. We also discuss and evaluate some arguments developed recently in neuroscience regarding the effect of emotional deficits in decision-making. Again, emotions are viewed here as brain mechanisms that lead to optimal threshold modulation. The skeptical reader can focus on the general implications of these mechanisms and ignore the discussion about the role of the soma in these choice

¹⁸To determine the optimal choice, this utility difference should then be traded-off against the extra cost (time, energy, etc.) incurred under the cognitive channel.

processes. For tractability, we consider the basic model presented in section 2, which is simple enough to be extended in a number of directions, and we assume that $\pi_A = \pi_B = 1$.

5.1 Emotional choices in simple and complex activities

If emotions are responsible for modulating thresholds, a natural question is to determine for which activities the cost of an emotional impairment is greatest. Activities usually differ in several dimensions, such as the temporal horizon (near vs. distant), the likelihood of occurrence (common vs. exceptional) and the difficulty to mentally represent the consequences of outcomes (concrete vs. abstract). Based on studies with brain lesion patients (Anderson et al. (1999), Bechara and Damasio (2005)), the somatic marker theory argues that impairments in the emotional system have more dramatic effects in choices related to abstract, exceptional and temporally distant situations than in choices related to concrete, common and temporally close situations.

An extension of our theory can help understanding this issue. We divide activities into two categories: simple (concrete, common, temporally close) vs. complex (abstract, exceptional, temporally distant) and assume that activities in the former category are easier to evaluate than in the latter one. Formally, we assume that information obtained through cell firing is *uniformly more accurate* in simple than in complex activities. So, for example, it is easier to process news about the relative value of two goods available for immediate purchase than about the relative desirability of two future sentimental relationships. This matches the evidence presented in Ditterich et al. (2003) about the correlation between task difficulty and probability of a choice error (see section 1.1). We use subscript $k \in \{\alpha, \beta\}$ to denote the two categories of activities, where “ α ” refers to simple (concrete/common/close) activities and “ β ” refers to complex (abstract/exceptional/distant) activities. The probability of a cell firing level c in activity k is denoted by $f_k(c)$ if $S = B$ and by $g_k(c)$ if $S = A$. We assume that the complex activity satisfies **(A1)**, that is $\left(\frac{f_\beta(c)}{g_\beta(c)}\right)' < 0$. The greater accuracy of cell firing conditional on state in simple than in complex activities is also captured with a standard MLRP:

$$\left(\frac{g_\alpha(c)}{g_\beta(c)}\right)' > 0 \quad \text{and} \quad \left(\frac{f_\alpha(c)}{f_\beta(c)}\right)' < 0$$

The idea is simply that “neuronal mistakes”, defined as low cell firing when $S = A$ or high cell firing when $S = B$, are uniformly less frequent in α -activities than in β -activities. Taken together, these conditions imply that the simple activity also satisfies **(A1)**, that

is, $\left(\frac{f_\alpha(c)}{g_\alpha(c)}\right)' < 0$. Denote by $x_k^*(p)$ the optimal threshold in activity k as a function of p . We can show the existence of a belief $\hat{p} \in (0, 1)$ such that the optimal threshold coincides in both types of activities. Formally, $x_\beta^*(\hat{p}) = x_\alpha^*(\hat{p}) = \hat{x}$. Our next result is as follows.

Proposition 9 *Optimal thresholds are more sensitive to initial beliefs in complex than in simple activities, that is, $x_\beta^*(p) \leq x_\alpha^*(p) \leq \hat{x}$ for all $p \geq \hat{p}$.*

We showed in section 2 that the optimal threshold x^* is set in such a way that the ex-post confidence in the true state is balanced. When the correlation between state and cell firing is high (simple activities), small variations in the threshold are sufficient to achieve the desired balance. Conversely, when the correlation is low (complex activities), the individual is forced to resort to more extreme variations in thresholds. Said differently, the individual is more inclined to confirm his prior when neuronal mistakes are frequent, that is, when new evidence is noisy.

The implications of this simple argument are interesting. It suggests that the inability to modulate neuronal thresholds, due for example to a somatic deficit, results in relatively poorer choices and more mistakes when the situation is complex (e.g., an action that may cause a moral harm on someone) than when the situation is simple (e.g., an action that may cause a physical harm on someone). This hypothesis is largely favored in the somatic marker theory (Damasio (1994), Bechara and Damasio (2005)). Note, however, that the result is based on a purely biological reason, and not on any moral or ethical consideration.

5.2 Emotional interactions in cooperative and competitive environments

Given that the optimal threshold depends on prior beliefs and payoffs derived from ensuing decisions, we can reasonably anticipate that any element that modifies the structure of individual payoffs should impact the threshold modulation. Natural environments with this property are strategic interactions.

Consider the following extension. There are two agents, i and j with $i, j \in \{1, 2\}$, who are either in competition or cooperation for an activity. We adopt a reduced form model of the positive (cooperation) and negative (competition) externalities exerted by individuals on one another: in cooperative situations, agents benefit if their colleague also takes the correct action whereas in competitive situations, agents benefit if they alone take the correct action. This, for example, captures the cases where individuals hunt together and compete for preys, respectively.

Agents cannot exchange information. They each set a threshold x_i , learn whether the cell firing activity is below or above the threshold, and choose an action. The state is common to both agents but, conditional on it, cell firing is independent across individuals. Denote by λ the agent's utility of taking the correct action. If, in the cooperation setting, the other agent also takes the correct action or if, in the competition setting, the other agent takes the incorrect action, the utility is increased by μ . Using the superscript “+” and “-” to denote the cooperation and competition settings respectively, the value function of agent i can be written as follows:

$$\begin{aligned} V_i^+(x_i, x_j) &= P(A)P(c > x_i|A) \left[\lambda + \mu P(c > x_j|A) \right] + P(B)P(c < x_i|B) \left[\lambda + \mu P(c < x_j|B) \right] \\ &= p(1 - G(x_i)) \left[\lambda + \mu(1 - G(x_j)) \right] + (1 - p)F(x_i) \left[\lambda + \mu F(x_j) \right] \end{aligned} \quad (22)$$

$$\begin{aligned} V_i^-(x_i, x_j) &= P(A)P(c > x_i|A) \left[\lambda + \mu P(c < x_j|A) \right] + P(B)P(c < x_i|B) \left[\lambda + \mu P(c > x_j|B) \right] \\ &= p(1 - G(x_i)) \left[\lambda + \mu G(x_j) \right] + (1 - p)F(x_i) \left[\lambda + \mu(1 - F(x_j)) \right] \end{aligned} \quad (23)$$

Note that $\mu = 0$ corresponds to the case of independent activities analyzed in section 2.2. Denote by $x_i^+ = \arg \max_{x_i} V_i^+(x_i, x_j^+)$ and by $x_i^- = \arg \max_{x_i} V_i^-(x_i, x_j^-)$. Taking first-order conditions in (22) and (23), if interior solutions exist, they will be such that:

$$\frac{f(x_i^+)}{g(x_i^+)} = \frac{p}{1 - p} \frac{\lambda + \mu(1 - G(x_j^+))}{\lambda + \mu F(x_j^+)} \quad (24)$$

$$\frac{f(x_i^-)}{g(x_i^-)} = \frac{p}{1 - p} \frac{\lambda + \mu G(x_j^-)}{\lambda + \mu(1 - F(x_j^-))} \quad (25)$$

According to this formulation, thresholds are strategic complements in cooperative activities ($dx_i^+/dx_j > 0$) and strategic substitutes in competitive activities ($dx_i^-/dx_j < 0$). The reason is simple. Suppose that agent 1 decreases his threshold. This makes him more likely to surpass it, and therefore to take action $\gamma^* = 1$. In cooperative activities, it also increases the expected benefit for agent 2 of taking action $\gamma^* = 1$ when $S = A$ relative to the expected benefit of taking action $\gamma^* = 0$ when $S = B$, since the extra cooperative payoff is more likely to be enjoyed in the first than in the second case. The cost of either wrong choice does not change. Therefore, agent 2 has an incentive to move the threshold in the same direction as agent 1. The opposite is true in competitive activities.

In what follows, we look for interior symmetric equilibria, where $x_i^+ = x_j^+ = x^+$ in the cooperative case and $x_i^- = x_j^- = x^-$ in the competitive one. We denote by $\kappa \equiv \mu/\lambda$ the

relative importance of the other agent's choice on one's payoff. In order to have interior solutions, the following technical restriction needs to be imposed.

Assumption 3 (Interior stability) *The parameter κ is such that:*

$$\frac{\kappa g(x^+)}{1 + \kappa(1 - G(x^+))} + \frac{\kappa f(x^+)}{1 + \kappa F(x^+)} < \frac{g'(x^+)}{g(x^+)} - \frac{f'(x^+)}{f(x^+)} \quad (\mathbf{A3.1})$$

$$\frac{\kappa g(x^-)}{1 + \kappa G(x^-)} + \frac{\kappa f(x^-)}{1 + \kappa(1 - F(x^-))} < \frac{g'(x^-)}{g(x^-)} - \frac{f'(x^-)}{f(x^-)} \quad (\mathbf{A3.2})$$

If assumptions **(A3.1)** and **(A3.2)** are violated, then the solutions in the cooperative and competitive settings respectively may not be interior. For some distributions, these inequalities are satisfied for all κ . Otherwise, they require κ to be small enough.¹⁹ The reason is straightforward. As κ increases, the decision of each agent becomes closer to a coordination problem. Suppose that one agent disregards information and always takes action $\gamma^* = 1$. Then, if κ is high, the other agent also has an incentive to disregard information and take action $\gamma^* = 1$ (in the cooperative setting) and action $\gamma^* = 0$ (in the competitive setting). Under these conditions, we can prove that there exists $\tilde{p} \in (0, 1)$ and \tilde{x} such that $x^+(\tilde{p}) = x^*(\tilde{p}) = x^-(\tilde{p}) = \tilde{x}$, and we have the following result.

Proposition 10 *Optimal thresholds are more sensitive to initial beliefs in cooperative than in independent activities, and less sensitive in competitive than in independent activities. Formally, $x^+(p) \leq x^*(p) \leq x^-(p) \leq \tilde{x}$ for all $p \geq \tilde{p}$.*

Proposition 10 states that, from a pure information processing perspective, threshold modulation is more decisive in cooperative than in competitive situations. Because in cooperative activities there is an extra payoff when both agents undertake the same correct action, the benefit of favoring the current belief (the state a priori most likely to be correct) is increased relative to the case of independent activities. Conversely, in competitive situations, each agent is more interested in learning that the least likely state is indeed the correct one. As a result, they both set the threshold in a way that existing beliefs are

¹⁹Technically, these assumptions ensure that the slopes of the reaction functions are smaller than 1 in absolute value at the symmetric intersection. Note that the L.H.S. in **(A3.1)** and **(A3.2)** are increasing in κ . Therefore, sufficient conditions for the inequalities to hold are:

$$\frac{g(x^+)}{1 - G(x^+)} + \frac{f(x^+)}{F(x^+)} < \frac{g'(x^+)}{g(x^+)} - \frac{f'(x^+)}{f(x^+)} \quad \text{and} \quad \frac{g(x^-)}{G(x^-)} + \frac{f(x^-)}{1 - F(x^-)} < \frac{g'(x^-)}{g(x^-)} - \frac{f'(x^-)}{f(x^-)}$$

The first inequality always holds for instance if $f(\cdot)$ and $g(\cdot)$ are linear and symmetric, as in Example 2.

less favored than under independent activities. Overall, the result of this simple extension suggests that an emotional deficit that prevents the individual from modulating neuronal thresholds (for example, keeping it always at \tilde{x}) is more costly in environments with positive externalities than in environments with negative externalities. That is, emotions are more helpful in cooperative than in competitive situations.

5.3 Emotional reactions to expected and unexpected events

Neuroscientists have been interested in the reaction of individuals to unexpected events. Schultz et al. (2000) claim that somatic states induce an overreaction to unanticipated outcomes, and determine in a series of experiments the biological mechanism behind this behavior, labeled the “Dopaminergic Reward Prediction Error” (DRPE) hypothesis. This theory argues that the response to rewards materializes in changes in dopamine release, which reflects the discrepancy between the reward itself and its prediction.²⁰ Bechara and Damasio (2005) integrate this idea in the somatic marker theory: “[T]he very quick, almost instantaneous, switch from one somatic state to another when an unexpected event occurs, can exert a disproportional impact on somatic state activation. Thus people’s decision may get reversed completely if a disappointment was encountered” (p.365).

From a Bayesian perspective, beliefs are always revised more strongly the greater the distance between prior and observation. So, without a formal benchmark for comparison, it is difficult to determine whether there is “overreaction” and a “disproportional” response, or just a natural belief reversal due to the influx of contradictory evidence.

We study this issue within the framework of our model by considering the following extension. In stage 1, the brain sets a threshold x and determines whether cell firing is above or below it. In stage 2, the individual updates his beliefs and chooses an action. In stage 3, he observes the outcome and assesses his ex post confidence in the choice he made. If the payoff is perfectly correlated with the true state, the individual learns with certainty whether he made the correct choice. That is, the ex-post probability of each state is either 0 or 1. More interestingly, under imperfect correlation, the (positive) reaction after a success and (negative) reaction after a failure may depend on the threshold. Our objective is to analyze this case in more detail. Imperfect correlation is modeled in a simple way. If $S = A$, the payoff of action γ is $l(\gamma - 1)$ with probability θ and $l(\gamma)$ with probability $1 - \theta$, with $\theta \in (1/2, 1]$. If $S = B$, the payoff is $l(\gamma)$ with probability θ and $l(\gamma - 1)$ with probability $1 - \theta$. In our example, on a dangerous day, the individual may be

²⁰Caplin and Dean (2007) provide an axiomatic approach of the DRPE hypothesis.

lucky, avoid all predators, and go back to the cave with a nice prey believing that it was a safe day. Section 2 thus corresponds to $\theta = 1$, where the true state is known ex post.

We first need to characterize the optimal strategy of the individual in this modified version of the problem. In stage 2, the expected payoff of the individual is:

$$L(\gamma; p, \theta) = \left[p\theta + (1-p)(1-\theta) \right] l(\gamma-1) + \left[p(1-\theta) + (1-p)\theta \right] l(\gamma) \quad (26)$$

Let $p' = p\theta + (1-p)(1-\theta)$ be the probability of encountering a predator. It is immediate that the strategy of the individual is the same as in section 2.2: $\gamma^* = 1$ if $p' > 1/2$ and $\gamma^* = 0$ if $p' < 1/2$, where $p' \geq 1/2$ if and only if $p \geq 1/2$. In words, the correlation between the ex post payoff and the state does not affect the optimal action.

In stage 1, the brain sets a threshold x which may or may not coincide with $x^*(p)$, the optimal one. We restrict the attention to the non-trivial case where the threshold is such that the result of the cell firing activity is informative ($\Pr(A | c < x) < 1/2 < \Pr(A | c > x)$), because otherwise this stage is ignored. The question we ask is: for any given x , how will the individual revise his beliefs after observing the outcome? Note that there are only two possible outcomes in stage 3: $l_H \equiv l(0)$ and $l_L \equiv l(1) (< l(0))$. By construction, the individual expects to be correct (otherwise he would choose the other action) and get l_H . We thus call l_H the “expected outcome” and l_L the “unexpected outcome.” The confidence of the individual is boosted when followed by a high payoff and reduced when followed by a low payoff. If the level of cell-firing is high, he takes action $\gamma^* = 1$ and believes this action is correct with probability:

$$\Pr(A | c > x) = \bar{p}(x)$$

If the outcome is the expected one, his revised belief that the action was indeed correct is:

$$\Pr(A | c > x, l_H) = \frac{p(1-G(x))\theta}{p(1-G(x))\theta + (1-p)(1-F(x))(1-\theta)} = \bar{p}(x, l_H) \quad (> \bar{p}(x))$$

Otherwise, his belief is revised downwards to:

$$\Pr(A | c > x, l_L) = \frac{p(1-G(x))(1-\theta)}{p(1-G(x))(1-\theta) + (1-p)(1-F(x))\theta} = \bar{p}(x, l_L) \quad (< \bar{p}(x))$$

Let $\bar{p}(x, l_H) - \bar{p}(x)$ be the increase in confidence after a high payoff, and $\bar{p}(x) - \bar{p}(x, l_L)$ the decrease in confidence after a low payoff. These differences measure the positive reaction to an expected event and the negative reaction to an unexpected event, respectively.

Similarly, if the level of cell firing is low, the individual takes action $\gamma^* = 0$ anticipating it is correct with probability:

$$\Pr(B | c < x) = 1 - \underline{p}(x)$$

Again, if the action is confirmed with a high payoff, his revised belief becomes:

$$\Pr(B | c < x, l_H) = \frac{(1-p)F(x)\theta}{(1-p)F(x)\theta + pG(x)(1-\theta)} = 1 - \underline{p}(x, l_H) \quad (> 1 - \underline{p}(x))$$

Otherwise, he believes the action was correct with probability:

$$\Pr(B | c < x, l_L) = \frac{(1-p)F(x)(1-\theta)}{(1-p)F(x)(1-\theta) + pG(x)\theta} = 1 - \underline{p}(x, l_L) \quad (< 1 - \underline{p}(x))$$

In that case, $\underline{p}(x) - \underline{p}(x, l_H)$ measures the reaction to the expected event, and $\underline{p}(x, l_L) - \underline{p}(x)$ measures the reaction to the unexpected event. We have the following result.

Proposition 11 *For all x , the reaction to unexpected events is always greater than the reaction to expected events, and all reactions are smaller as the environment becomes more stochastic (lower θ). More extreme cutoffs always lead to lower reactions to expected events. They lead to higher reactions to unexpected events if θ is high and to lower reactions to unexpected events if θ is low.*

When the threshold is surpassed, the individual assesses a high probability to state A and $\gamma^* = 1$ is believed to be correct. Then, by construction, the belief will be revised more in absolute value if the action is not confirmed with a high payoff than if it is. Naturally, as the environment becomes more stochastic, observing an event is less informative and beliefs are less drastically revised in either direction. These conclusions do not depend on which threshold x is selected. They occur simply because the individual integrates all information in a Bayesian way. The same reasoning holds when the threshold is not surpassed and action $\gamma^* = 0$ is taken.

Thresholds also affect the magnitude of the reaction to different events. An individual who surpasses an extreme threshold (close to 1) is more confident in his action (here $\gamma^* = 1$) than an individual who surpasses a weak threshold. Therefore, he also experiences less (positive) surprise when his decision is confirmed by a high payoff. More interesting is the case of an individual who obtains a low payoff. If the environment is deterministic (θ high), he becomes almost sure that his decision was incorrect. Given that extreme thresholds lead to stronger beliefs, they also lead to a higher (negative) surprise. If the environment

is stochastic (θ low), there is little information in the payoff and the posterior belief hinges towards the prior. Here, given that extreme thresholds lead to stronger beliefs, they also lead to a smaller surprise.

The arguments in the neuroscience literature briefly discussed above capture some but not all the effects associated with the reaction to anticipated and unanticipated events. This literature emphasizes that an individual with emotional stability exhibits a small positive surprise to expected events and a large negative surprise to unexpected events. This is partly consistent with our findings. Indeed, the ability to set extreme cutoffs always results in low reaction to expected events. Since an individual with emotional stability sets optimal thresholds and since optimal thresholds are somewhat extreme, such individuals will experience low positive reactions. However, Proposition 11 also qualifies that claim. Our model predicts a high negative reaction to unexpected events only if outcomes are sufficiently deterministic. This literature thus ignores the fact that, in highly stochastic environments, emotionally balanced individuals realize the high importance of luck on payoffs, and therefore react mildly to all outcomes, whether positive or negative.

6 Concluding remarks

We have modeled the physiological limitations faced by individuals when processing information. We have shown that, under such constraints, it is optimal to set neuronal thresholds in a way that initial beliefs are favored. The conclusion holds in a wide array of settings and has interesting implications for the somatic marker theory.

As discussed in detail, the general methodology used in the paper consists in building an economic model of decision processes based on evidence from neuroscience and neurobiology. This “neuroeconomic theory” approach to decision-making has two main advantages. First, by modeling the underlying mechanisms that lead to choices, it is possible to predict behavior more accurately. Second, the models provide testable predictions concerning the effects of pathologies on choices. We have extensively discussed the first point but only briefly mentioned how emotional deficiencies that translate into suboptimal thresholds affect decisions. We have therefore not exploited the full potential of the methodology in the second dimension. A natural alley for future research would be to test some of our implications in patients with brain lesions.

The model has also implications that extend beyond the realm of brain information processing. First, we can apply it to a standard individual learning problem, under the

assumption that news obtained is coarse. Under that interpretation, our model provides “a rationale for stubbornness”: agents are less likely to change their mind as time passes, not only because they are more confident about which alternative they prefer, but also because they modify thresholds in a way that existing beliefs are most likely to be reinforced. At the same time, when they change their mind, they do it more drastically. It also suggests that stubbornness should be more prevalent in complex issues, where information is more difficult to interpret. Second and related, two individuals with opposite beliefs will set thresholds at opposite ends and therefore may interpret the same evidence in opposite directions. In other words, in a world of different priors, common information may increase polarization of opinions, at least in the short run.²¹ Third, we have seen that the information sequence matters in dynamic settings. This suggests that behavior can be influenced by manipulating the order in which news is revealed. One could possibly develop a theory of framing based on this approach. Last, the way information is interpreted in strategic settings seems to affect behavior substantially. The multi-agent application we offered is introductory. It would be interesting to analyze interactions in more general environments.

²¹See Sobel (2007) for a different study of polarization in group decision-making.

Appendix

A1. Proof of Proposition 1

Taking the first-order condition in (4), we find that $x^*(p)$ satisfies (5). Given **(A1)**, $x^*(p)$ is unique and the local second-order condition is satisfied:

$$\begin{aligned} \frac{\partial^2 V}{\partial x^2} \Big|_{x^*} &= -p g'(x^*) \pi_A (l(0) - l(1)) + (1-p) f'(x^*) \pi_B (l(0) - l(1)) \\ &= (1-p) \pi_B g(x^*) (l(0) - l(1)) \left(\frac{f(x^*)}{g(x^*)} \right)' < 0. \end{aligned}$$

$dx^*/dp < 0$ is immediate from **(A1)**. For $V(x^*; p)$ to be the value function, we must check ex-post that it is optimal to select $\gamma^* = 1$ when $c > x^*$ and $\gamma^* = 0$ when $c < x^*$. We have:

$$\begin{aligned} \Pr(A|c > x^*) > \frac{\pi_B}{\pi_A + \pi_B} &\Leftrightarrow p \pi_A (1 - G(x^*)) > (1-p) \pi_B (1 - F(x^*)) \Leftrightarrow \frac{1-G(x^*)}{g(x^*)} > \frac{1-F(x^*)}{f(x^*)} \\ \Pr(A|c < x^*) < \frac{\pi_B}{\pi_A + \pi_B} &\Leftrightarrow p \pi_A G(x^*) > (1-p) \pi_B F(x^*) \Leftrightarrow \frac{G(x^*)}{g(x^*)} < \frac{F(x^*)}{f(x^*)} \end{aligned}$$

Both inequalities are satisfied given **(A1)**. This completes the proof.

A2. Proof of Proposition 2

From (11) and using (7) and (8), we get:

$$\begin{aligned} \frac{\partial^2 V(x; p)}{\partial x \partial p} \Big|_{x=x^{**}} &= -\frac{1}{p} \left[f(x^{**}) \left(l(\gamma^{**}(\underline{p})) - l(\gamma^{**}(\bar{p})) \right) + F(x^{**}) l'(\gamma^{**}(\underline{p})) \frac{d\gamma^{**}(\underline{p})}{dx} \Big|_{x^{**}} \right. \\ &\quad \left. + (1 - F(x^{**})) l'(\gamma^{**}(\bar{p})) \frac{d\gamma^{**}(\bar{p})}{dx} \Big|_{x^{**}} \right] \\ &= -\frac{1}{p} \times \frac{d}{dx} \left[F(x) l(\gamma^{**}(\underline{p})) + (1 - F(x)) l(\gamma^{**}(\bar{p})) \right]_{x=x^{**}} \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{\partial^2 V(x; p)}{\partial x^2} \Big|_{x=x^{**}} &= (1-p) \frac{\left(\frac{f(x)}{g(x)} \right)'}{\frac{f(x)}{g(x)}} \left[f(x) \left(l(\gamma^{**}(\underline{p})) - l(\gamma^{**}(\bar{p})) \right) + F(x) l'(\gamma^{**}(\underline{p})) \frac{d\gamma^{**}(\underline{p})}{dx} \frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)} \right)'} \frac{\left(\frac{F(x)}{G(x)} \right)'}{\frac{F(x)}{G(x)}} \right. \\ &\quad \left. + (1 - F(x)) l'(\gamma^{**}(\bar{p})) \frac{d\gamma^{**}(\bar{p})}{dx} \frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)} \right)'} \frac{\left(\frac{1-F(x)}{1-G(x)} \right)'}{\frac{1-F(x)}{1-G(x)}} \right]_{x=x^{**}} \end{aligned}$$

By **(A2)**, $\frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)} \right)'} \frac{\left(\frac{F(x)}{G(x)} \right)'}{\frac{F(x)}{G(x)}} \leq 1$ and $\frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)} \right)'} \frac{\left(\frac{1-F(x)}{1-G(x)} \right)'}{\frac{1-F(x)}{1-G(x)}} \leq 1$. Therefore, $\frac{\partial^2 V(x; p)}{\partial x \partial p} \Big|_{x=x^{**}} < 0 \Rightarrow \frac{\partial^2 V(x; p)}{\partial x^2} \Big|_{x=x^{**}} < 0$ and the proposition follows.

A3. Proof of Proposition 3

Let $P \equiv \frac{p}{1-p}$. The F.O.C. in the quadratic case can be rewritten as:

$$\frac{1-p}{p^2} \frac{\partial V(x; p)}{\partial x} \Big|_{x^{**}} = k(x^{**}, P)$$

where $k(x, P) \equiv \left(\frac{1-G(x)}{(1-G(x))P+(1-F(x))} - \frac{G(x)}{G(x)P+F(x)} \right) \left[f(x) \left(\frac{1-G(x)}{(1-G(x))P+(1-F(x))} + \frac{G(x)}{G(x)P+F(x)} \right) - g(x) \left(\frac{1-F(x)}{(1-G(x))P+(1-F(x))} + \frac{F(x)}{G(x)P+F(x)} \right) \right]$. Differentiating the F.O.C., we get

$$\frac{\partial^2 V(x; p)}{\partial x \partial p} \Big|_{x^{**}} \propto \frac{\partial k(x^{**}, P)}{\partial P}$$

After some tedious algebra, and using (13), we get:

$$\frac{\partial k(x^{**}, P)}{\partial P} = -g(x^{**}) \frac{\left(\frac{F(x^{**})}{G(x^{**})} - \frac{1-F(x^{**})}{1-G(x^{**})} \right)^3}{\left(2P + \frac{F(x^{**})}{G(x^{**})} + \frac{1-F(x^{**})}{1-G(x^{**})} \right) \left(P + \frac{F(x^{**})}{G(x^{**})} \right)^2 \left(P + \frac{1-F(x^{**})}{1-G(x^{**})} \right)^2} < 0$$

and the result follows.

A4. Proof of Proposition 4

Taking the F.O.C. in (14), we obtain (15). The local S.O.C. is:

$$\begin{aligned} \frac{\partial^2 V}{\partial x^2} \Big|_{\hat{x}} &= - \int_0^1 p(s) f_x(\hat{x} | s) (l(1-s) - l(s)) ds \\ &= \int_0^1 \left(- \frac{f_x(\hat{x} | s)}{f(\hat{x} | s)} \right) p(s) f(\hat{x} | s) (l(1-s) - l(s)) ds \end{aligned}$$

Let $h(s) \equiv - \frac{f_x(\hat{x} | s)}{f(\hat{x} | s)}$. By (A1'), $h'(s) \leq 0$. We can then rewrite the local S.O.C. as:

$$\begin{aligned} \frac{\partial^2 V}{\partial x^2} \Big|_{\hat{x}} &= \int_0^{1/2} h(s) p(s) f(\hat{x} | s) (l(1-s) - l(s)) ds + \int_{1/2}^1 h(s) p(s) f(\hat{x} | s) (l(1-s) - l(s)) ds \\ &< h(1/2) \left[\int_0^{1/2} p(s) f(\hat{x} | s) (l(1-s) - l(s)) ds + \int_{1/2}^1 p(s) f(\hat{x} | s) (l(1-s) - l(s)) ds \right] = 0 \end{aligned}$$

Regarding the comparative statics, if $\left(\frac{p(s)}{q(s)} \right)' \leq 0$, then:

$$\begin{aligned} \frac{\partial V(x; p(s))}{\partial x} \Big|_{\hat{x}(p(s))} &= - \int_0^1 \left(\frac{p(s)}{q(s)} \right) q(s) f(\hat{x}(p(s)) | s) (l(1-s) - l(s)) ds \\ &> - \left(\frac{p(1/2)}{q(1/2)} \right) \int_0^1 q(s) f(\hat{x}(p(s)) | s) (l(1-s) - l(s)) ds \end{aligned}$$

Therefore,

$$\left. \frac{\partial V(x; p(s))}{\partial x} \right|_{\hat{x}(p(s))} = 0 > \left(\frac{p(1/2)}{q(1/2)} \right) \left. \frac{\partial V(x; q(s))}{\partial x} \right|_{\hat{x}(q(s))} \Rightarrow \hat{x}(p(s)) > \hat{x}(q(s))$$

Last, we need to check that it is indeed optimal to choose $\hat{\gamma} = 1$ when $c > \hat{x}$ and $\hat{\gamma} = 0$ when $c < \hat{x}$. Let $\mathcal{L}(x) \equiv L(1; p(s|c=x)) - L(0; p(s|c=x))$, also $p(s|c=x) \equiv j(s|x) = \frac{p(s)f(x|s)}{\int_0^1 p(s)f(x|s)ds}$ and $J(s|x) = \int_0^s j(\tilde{s}|x)d\tilde{s}$. Integrating by parts:

$$\begin{aligned} \mathcal{L}(x) &= \int_0^1 j(s|x) \left(l(1-s) - l(s) \right) ds \\ &= l(0) - l(1) + \int_0^1 J(s|x) \left(l'(1-s) + l'(s) \right) ds \end{aligned}$$

Therefore

$$\frac{d\mathcal{L}(x)}{dx} = \int_0^1 J_x(s|x) \left(l'(1-s) + l'(s) \right) ds > 0$$

since, by **(A1')**, we know that $F_s(x|s) < 0$ and therefore $J_x(s|x) < 0$. From (15), $\mathcal{L}(\hat{x}) = 0$, so $\mathcal{L}(x) \geq 0$ for all $x \geq \hat{x}$. This also proves that, for the purpose of the action to be taken, it is equivalent to learn c or to learn whether c is greater or smaller than \hat{x} .

A5. Proof of Proposition 5 and 6

They follow the exact same steps as the proofs of Propositions 2 and 3, and are therefore omitted for the sake of brevity.

A6. Proof of Proposition 8

Cognitive channel. Taking F.O.C. in (21), we obtain \underline{x} and \bar{x} . We also have $\left. \frac{\partial^2 V(x_1, x_2)}{\partial x_1^2} \right|_{\underline{x}} = -\frac{1}{2}pOf(\underline{x}) \left(\frac{h(\underline{x})}{f(\underline{x})} \right)' - \frac{1}{2}pAf(\underline{x}) \left(\frac{g(\underline{x})}{f(\underline{x})} \right)' < 0$, $\left. \frac{\partial^2 V(x_1, x_2)}{\partial x_2^2} \right|_{\bar{x}} = \frac{1}{2}pOg(\bar{x}) \left(\frac{h(\bar{x})}{g(\bar{x})} \right)' + \frac{1}{2}pBg(\bar{x}) \left(\frac{f(\bar{x})}{g(\bar{x})} \right)' < 0$, and $\frac{\partial^2 V(x_1, x_2)}{\partial x_1 \partial x_2} = 0$. Therefore \underline{x} and \bar{x} are maxima. Last, it can be easily checked that $\left(\Pr(A|c \in \mathcal{X}), \Pr(O|c \in \mathcal{X}), \Pr(B|c \in \mathcal{X}) \right)$ are such that it is indeed optimal to have $\tilde{\gamma} = 0$ if $\mathcal{X} = [0, \underline{x}]$, $\tilde{\gamma} = \frac{1}{2}$ if $\mathcal{X} = [\underline{x}, \bar{x}]$, and $\tilde{\gamma} = 1$ if $\mathcal{X} = (\bar{x}, 1]$.

Affective channel. The three candidates for optimal cutoffs are:

$$\begin{cases} x_a & \text{so that } \tilde{\gamma} = 0 & \text{if } c < x_a & \text{and } \tilde{\gamma} = 1 & \text{if } c > x_a \\ x_b & \text{so that } \tilde{\gamma} = 0 & \text{if } c < x_b & \text{and } \tilde{\gamma} = 1/2 & \text{if } c > x_b \\ x_c & \text{so that } \tilde{\gamma} = 1/2 & \text{if } c < x_c & \text{and } \tilde{\gamma} = 1 & \text{if } c > x_c \end{cases}$$

These cutoffs are formally defined by:

$$\begin{cases} x_a = \arg \max_x V^a(x) & \equiv \Pr(c < x) L(0; p(\cdot | c < x)) + \Pr(c > x) L(1; p(\cdot | c < x)) \\ x_b = \arg \max_x V^b(x) & \equiv \Pr(c < x) L(0; p(\cdot | c < x)) + \Pr(c > x) L(\frac{1}{2}; p(\cdot | c < x)) \\ x_c = \arg \max_x V^c(x) & \equiv \Pr(c < x) L(\frac{1}{2}; p(\cdot | c < x)) + \Pr(c > x) L(1; p(\cdot | c < x)) \end{cases}$$

It is straightforward to check that $x_a = \check{x}$, $x_b = \underline{x}$, $x_c = \bar{x}$. Now, fix p_O . Differentiating each first-order condition with respect to p_B , we get:

$$\frac{dx_a}{dp_B} > 0, \quad \frac{dx_b}{dp_B} > 0, \quad \frac{dx_c}{dp_B} > 0$$

Furthermore:

$$\begin{aligned} \frac{dV^a(x^a)}{dp_B} = F(x^a) + G(x^a) - 1 &\geq 0, & \frac{d^2V^a(x^a)}{dp_B^2} = [f(x^a) + g(x^a)] \frac{dx_a}{dp_B} &\geq 0, \\ \frac{dV^b(x^b)}{dp_B} = \frac{F(x^b) + G(x^b)}{2} &\geq 0, & \frac{dV^c(x^c)}{dp_B} = \frac{F(x^c) + G(x^c)}{2} - 1 &\leq 0 \end{aligned}$$

Also, $\lim_{p_B \rightarrow 0} V^a(x^a) = -\frac{p_O}{2} < \lim_{p_B \rightarrow 0} V^c(x^c)$ and $\lim_{p_B \rightarrow 1-p_O} V^a(x^a) = -\frac{p_O}{2} < \lim_{p_B \rightarrow 1-p_O} V^b(x^b)$.

Combining these results, we have that there exist p^* such that x^c dominates x^b if $p_B < p^*$ and x^b dominates x^c if $p_B > p^*$. Also, there exist p^{**} and p^{***} such that x^c dominates x^a if $p_B < p^{**}$ and x^b dominates x^a if $p_B > p^{***}$. The ranking between p^* , p^{**} and p^{***} will depend on the relative values of p_O and p_A .

A7. Proof of Proposition 9

$$\left(\frac{g_\alpha(c)}{g_\beta(c)}\right)' > 0, \quad \left(\frac{f_\beta(c)}{g_\beta(c)}\right)' < 0, \quad \left(\frac{f_\alpha(c)}{f_\beta(c)}\right)' < 0 \Rightarrow \frac{g'_\alpha(c)}{g_\alpha(c)} > \frac{g'_\beta(c)}{g_\beta(c)} > \frac{f'_\beta(c)}{f_\beta(c)} > \frac{f'_\alpha(c)}{f_\alpha(c)} \Rightarrow \left(\frac{f_\alpha(c)}{g_\alpha(c)}\right)' < 0.$$

Now, suppose there exists $\hat{c} \in (0, 1)$ such that $\frac{f_\alpha(\hat{c})}{g_\alpha(\hat{c})} = \frac{f_\beta(\hat{c})}{g_\beta(\hat{c})}$. Then,

$$\frac{d}{dc} \left[\frac{f_\alpha(c)}{g_\alpha(c)} - \frac{f_\beta(c)}{g_\beta(c)} \right]_{c=\hat{c}} = \frac{f_\alpha(\hat{c})}{g_\alpha(\hat{c})} \left(\frac{f'_\alpha(\hat{c})}{f_\alpha(\hat{c})} - \frac{g'_\alpha(\hat{c})}{g_\alpha(\hat{c})} \right) - \frac{f_\beta(\hat{c})}{g_\beta(\hat{c})} \left(\frac{f'_\beta(\hat{c})}{f_\beta(\hat{c})} - \frac{g'_\beta(\hat{c})}{g_\beta(\hat{c})} \right) < 0$$

so $\frac{f_\alpha(c)}{g_\alpha(c)}$ and $\frac{f_\beta(c)}{g_\beta(c)}$ cross at most once. Furthermore, $\left(\frac{g_\alpha(c)}{g_\beta(c)}\right)' > 0$ and $\left(\frac{f_\alpha(c)}{f_\beta(c)}\right)' < 0 \Rightarrow \frac{f_\alpha(0)}{g_\alpha(0)} > \frac{f_\beta(0)}{g_\beta(0)}$ and $\frac{f_\alpha(1)}{g_\alpha(1)} < \frac{f_\beta(1)}{g_\beta(1)}$. Together with the previous result, it means that there exists $\hat{x} \in (0, 1)$ such that $\frac{f_\alpha(x)}{g_\alpha(x)} \geq \frac{f_\beta(x)}{g_\beta(x)}$ for all $x \leq \hat{x}$. Finally, given (5), there exists \hat{p} such that $\frac{f_\alpha(x_\alpha^*(\hat{p}))}{g_\alpha(x_\alpha^*(\hat{p}))} = \frac{f_\beta(x_\beta^*(\hat{p}))}{g_\beta(x_\beta^*(\hat{p}))} = \frac{\hat{p}}{1-\hat{p}}$, that is, $x_\alpha^*(\hat{p}) = x_\beta^*(\hat{p}) = x^*(\hat{p}) \equiv \hat{x}$. For all $p \geq \hat{p}$, $\frac{f_\alpha(x_\alpha^*(p))}{g_\alpha(x_\alpha^*(p))} = \frac{f_\beta(x_\beta^*(p))}{g_\beta(x_\beta^*(p))} = \frac{p}{1-p} \Rightarrow x_\beta^*(p) \leq x_\alpha^*(p) \leq x^*(\hat{p}) \leq \hat{x}$.

A8. Proof of Proposition 10

Cooperation. Taking the FOC in (22) and then differentiating it with respect to the other agent's threshold, we get:

$$\frac{\partial x_i^+}{\partial x_j} = -\frac{\partial^2 V_i^+ / \partial x_i^+ \partial x_j}{\partial^2 V_i^+ / \partial (x_i^+)^2} = -\mu \frac{f(x_i^+) / g(x_i^+)}{(f(x_i^+) / g(x_i^+))'} \left[\frac{g(x_j)}{\lambda + \mu(1 - G(x_j))} + \frac{f(x_j)}{\lambda + \mu F(x_j)} \right] > 0$$

Because reaction functions are positive and identical for both individuals, equilibria can only be symmetric. Let $R_i^+(x_j)$ be i 's reaction function. Sufficient conditions for a unique, stable interior solution are: $R_i^+(0) > 0$, $R_i^+(1) < 1$ and $\left. \frac{\partial x_i^+}{\partial x_j} \right|_{x_j=x_i^+} < 1$. From (24), the first and second inequalities can be written as $\frac{f(0)}{g(0)} > \frac{p}{1-p} (1 + \kappa)$ and $\frac{f(1)}{g(1)} < \frac{p}{1-p} \frac{1}{1+\kappa}$, which are strengthened versions of the conditions determined in footnote ???. The third inequality boils down to **(A3.1)**. From (5) and (24), we finally get:

$$\frac{f(x^+)}{g(x^+)} \frac{\lambda + \mu F(x^+)}{\lambda + \mu(1 - G(x^+))} = \frac{f(x^*)}{g(x^*)} = \frac{p}{1-p} \quad (27)$$

Competition. Taking the FOC in (23) and then differentiating it with respect to the other agent's threshold, we get:

$$\frac{\partial x_i^-}{\partial x_j} = -\frac{\partial^2 V_i^- / \partial x_i^- \partial x_j}{\partial^2 V_i^- / \partial (x_i^-)^2} = \mu \frac{f(x_i^-) / g(x_i^-)}{(f(x_i^-) / g(x_i^-))'} \left[\frac{g(x_j)}{\lambda + \mu G(x_j)} + \frac{f(x_j)}{\lambda + \mu(1 - F(x_j))} \right] < 0$$

Because reaction functions are negative there can only exist one symmetric equilibrium. Let $R_i^-(x_j)$ be i 's reaction function. Sufficient conditions for the symmetric equilibrium to be stable and interior are: $R_i^-(0) > 0$, $R_i^-(1) < 1$ and $\left. \frac{\partial x_i^-}{\partial x_j} \right|_{x_j=x_i^-} > -1$. From (25), the first and second inequalities can be written as $\frac{f(0)}{g(0)} > \frac{p}{1-p} \frac{1}{1+\kappa}$ and $\frac{f(1)}{g(1)} < \frac{p}{1-p} (1 + \kappa)$, which are weaker than the conditions in footnote ???. The third inequality boils down to **(A3.2)**. From (5) and (25), we finally get:

$$\frac{f(x^-)}{g(x^-)} \frac{\lambda + \mu(1 - F(x^-))}{\lambda + \mu G(x^-)} = \frac{f(x^*)}{g(x^*)} = \frac{p}{1-p} \quad (28)$$

Let \tilde{x} be the value that solves $F(\tilde{x}) + G(\tilde{x}) = 1$. Combining (27) and (28), we have:

$$\frac{f(z)}{g(z)} \frac{\lambda + \mu F(z)}{\lambda + \mu(1 - G(z))} \geq \frac{f(z)}{g(z)} \geq \frac{f(z)}{g(z)} \frac{\lambda + \mu(1 - F(z))}{\lambda + \mu G(z)} \Leftrightarrow F(z) + G(z) \geq 1 \Leftrightarrow z \geq \tilde{x}$$

There exists \tilde{p} such that $\frac{f(x^+(\tilde{p}))}{g(x^+(\tilde{p}))} \frac{\lambda + \mu F(x^+(\tilde{p}))}{\lambda + \mu(1 - G(x^+(\tilde{p})))} = \frac{f(x^-(\tilde{p}))}{g(x^-(\tilde{p}))} \frac{\lambda + \mu(1 - F(x^-(\tilde{p})))}{\lambda + \mu G(x^-(\tilde{p}))} = \frac{f(x^*(\tilde{p}))}{g(x^*(\tilde{p}))} = \frac{\tilde{p}}{1-\tilde{p}}$. For all $p \geq \tilde{p}$, then $x^+(p) \leq x^*(p) \leq x^-(p) \leq \tilde{x}$.

A9. Proof of Proposition 11

Let $\alpha \equiv P(A)P(1|A) = p(1 - G(x))$, $\beta \equiv P(B)P(1|B) = (1 - p)(1 - F(x))$, $\gamma \equiv P(A)P(0|A) = pG(x)$, and $\delta \equiv P(B)P(0|B) = (1 - p)F(x)$. Also, cell firing being informative, we need $\bar{p}(x) = \Pr(A|c > x) > 1/2 \Rightarrow \frac{\alpha}{\alpha + \beta} > 1/2 \Rightarrow \alpha > \beta$. Similarly, $\underline{p}(x) = \Pr(A|c < x) < 1/2 \Rightarrow \frac{\gamma}{\gamma + \delta} < 1/2 \Rightarrow \delta > \gamma$. We have:

$$\bar{p}(x, l_H) - \bar{p}(x) \equiv \bar{C}^+ = \frac{\alpha\beta(2\theta - 1)}{[\alpha\theta + \beta(1 - \theta)](\alpha + \beta)},$$

$$\bar{p}(x) - \bar{p}(x, l_L) \equiv \bar{C}^- = \frac{\alpha\beta(2\theta - 1)}{[\alpha(1 - \theta) + \beta\theta](\alpha + \beta)}$$

It is immediate to see that $\bar{C}^- - \bar{C}^+ \propto \alpha - \beta \geq 0$. Furthermore, $\frac{\partial \bar{C}^+}{\partial \theta} \propto \alpha\beta \geq 0$ and $\frac{\partial \bar{C}^-}{\partial \theta} \propto \alpha\beta \geq 0$. Also,

$$\frac{\partial \bar{C}^+}{\partial x} \propto \left((1 - \theta)\beta^2 - \alpha^2\theta \right) \left(\beta \frac{\partial \alpha}{\partial x} - \alpha \frac{\partial \beta}{\partial x} \right).$$

Given $\alpha > \beta$ and $\theta > 1 - \theta$, we have $(1 - \theta)\beta^2 - \alpha^2\theta < 0$. Also, $\beta \frac{\partial \alpha}{\partial x} - \alpha \frac{\partial \beta}{\partial x} = (1 - p)p[(1 - G(x))f(x) - (1 - F(x))g(x)] \geq 0$ for all x given MLRP. Therefore $\partial \bar{C}^+ / \partial x \leq 0$. Last,

$$\frac{\partial \bar{C}^-}{\partial x} \propto \left((1 - \theta)\alpha^2 - \beta^2\theta \right) \left(\alpha \frac{\partial \beta}{\partial x} - \beta \frac{\partial \alpha}{\partial x} \right)$$

so for each x there exists $\hat{\theta}$ such that $\frac{\partial \bar{C}^-}{\partial x} < 0$ for all $\theta < \hat{\theta}$ and $\frac{\partial \bar{C}^-}{\partial x} > 0$ for all $\theta > \hat{\theta}$. Similarly,

$$\underline{p}(x) - \underline{p}(x, l_H) \equiv \underline{C}^+ = \frac{\delta\gamma(2\theta - 1)}{[\delta\theta + \gamma(1 - \theta)](\delta + \gamma)},$$

$$\underline{p}(x, l_L) - \underline{p}(x) \equiv \underline{C}^- = \frac{\delta\gamma(2\theta - 1)}{[\delta(1 - \theta) + \gamma\theta](\delta + \gamma)}$$

It is again immediate to see that $\underline{C}^- - \underline{C}^+ \propto \delta - \gamma \geq 0$. Furthermore, $\frac{\partial \underline{C}^+}{\partial \theta} \propto \delta\gamma \geq 0$ and $\frac{\partial \underline{C}^-}{\partial \theta} \propto \delta\gamma \geq 0$. Also,

$$\frac{\partial \underline{C}^+}{\partial x} \propto \left((1 - \theta)\gamma^2 - \delta^2\theta \right) \left(\gamma \frac{\partial \delta}{\partial x} - \delta \frac{\partial \gamma}{\partial x} \right).$$

Given $\delta > \gamma$ and $\theta > 1 - \theta$, we have $(1 - \theta)\gamma^2 - \delta^2\theta < 0$. Also, $\gamma \frac{\partial \delta}{\partial x} - \delta \frac{\partial \gamma}{\partial x} = (1 - p)p[G(x)f(x) - F(x)g(x)] \leq 0$ for all x given MLRP. Therefore $\partial \underline{C}^+ / \partial x \geq 0$. Last,

$$\frac{\partial \underline{C}^-}{\partial x} \propto \left((1 - \theta)\delta^2 - \gamma^2\theta \right) \left(\delta \frac{\partial \gamma}{\partial x} - \gamma \frac{\partial \delta}{\partial x} \right)$$

so for each x there exists $\tilde{\theta}$ such that $\frac{\partial \underline{C}^-}{\partial x} > 0$ for all $\theta < \tilde{\theta}$ and $\frac{\partial \underline{C}^-}{\partial x} < 0$ for all $\theta > \tilde{\theta}$.

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