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AFFECTIVE DECISION MAKING AND THE ELLSBERG PARADOX

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Affective Decision Making and the Ellsberg Paradox*

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Abstract

Affective decision-making is a strategic model of choice under risk and uncertainty where we posit two cognitive processes — the "rational" and the "emoitonal" process. Observed choice is the result of equilibirum in this intrapersonal game.

As an example, we present applications of affective decision-making in insurance markets, where the risk perceptions of consumers are endogenous. We derive the axiomatic foundation of affective decision making, and show that affective decision making is a model of ambiguity-seeking behavior consistent with the Ellsberg paradox.

JEL Classification: D01, D81, G22

Keywords: Affective choice, Endogenous risk perception, Insurance, Ellsberg paradox, Variational preferences, Ambiguity-seeking

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1 Introduction

The theory of choice under risk and uncertainty is a consequence of the interplay between formal models and experimental evidence. The Ellsberg paradox (1961) introduced the notion of ambiguity aversion and inspired models such as maxmin expected utility (Gilboa and Schmeidler 1989), and variational preferences (Maccheroni Marinacci and Rustichini [MMR] 2006). In this paper we present a model of choice under risk and uncertainty that accommodates both optimism bias and ambiguity-seeking behavior.

Optimism bias is the tendency to overstate the likelihood of desired future outcomes and understate the likelihood of undesired future outcomes — even for events that are purely random (Irwin 1953; Weinstein 1980; Slovic et al. 1982; Slovic 2000). Hence, optimism bias is inconsistent with the independence of weights and payoffs found in most individual choice models, such as expected utility, subjective expected utility and prospect theory. To accommodate optimism bias, we endogenize decision weights — perceived risk.

In affective decision making, we envision two distinct psychological processes that mutually determine choice. This approach is inspired both by Kahneman (2003), who proposes two systems of reasoning that differ in several important aspects, such as emotion, and by the modular brain hypothesis in neuroscience (Damasio 1994; LeDoux 2000; Camerer, Loewenstein and Prelec 2004). Psychology distinguishes between two systems, such as analytical and intuitive processing (Chaiken and Trope 1999), and neuroscience suggests different brain modules that specialize in different activities. For instance, the amygdala is associated with emotions while the prefrontal cortex is associated with higher level, deliberate thinking (e.g., Reisberg 2001). Decision making is hypothesized, in both psychology and neuroscience, to be the result of an interaction between different modules of the brain (e.g., Sacks 1985; Damasio 1994; Epstein 1994; LeDoux 2000).

Decision making under risk and uncertainty derives naturally from the interplay between two cognitive processes, similar to the dual systems proposed by Kahneman and consistent with recent research in neuroscience. That is, decision making under risk can be modeled as a deliberate process choosing an optimal action, and an affective process forming risk perception. The deliberate process chooses a lottery or a sure amount, chooses insurance, or a spouse, while the affective process considers the probability of winning a lottery, being in a car accident, or getting married.

In our model, we call these systems of reasoning the *rational process* and the *emotional process*. The rational process coincides with the expected utility model. That is, for a given risk perception, i.e., perceived probability distribution, it maximizes expected utility. The emotional process is where risk perception is formed. In particular, the agent selects an optimal risk perception to balance two contradictory impulses: (1) affective motivation and (2) a taste for accuracy. This is a definition of motivated reasoning, a psychological mechanism where emotional goals motivate agent's beliefs, e.g., Kunda (1990), and is a source of psychological biases, such as optimism bias. Affective motivation is the desire to hold a favorable personal risk perception — optimism — and is captured by the expected utility term. The desire for accuracy is the mental cost incurred by the agent for holding beliefs other than her *base rate,* given her desire for favorable risk beliefs. The base rate is the belief that minimizes the mental cost function of the emotional process. This is the agent's *correct* risk belief, if her risks are objective such as mortality tables.

To reach a decision, the two processes interact to achieve consistency. This interaction is modeled as a simultaneous-move *intrapersonal* game, and consistency between the two processes, which represents the candidate for choice, is characterized by the pure strategy Nash equilibria of the game. The intrapersonal game is consistent with recent advances in neuroscience which suggest that simultaneous processes are complementary (Damasio 1994; LeDoux 2000) and show evidence for their integration (Gray et al. 2002; Pessoa 2008). Gray et al. conclude that "at some point of processing, functional specialization is lost, and emotion and cognition conjointly and equally contribute to the control of thought and behavior." Pessoa (2008) argues that ". . . emotions and cognition not only strongly interact in the brain, but that they are often integrated so that they jointly contribute to behavior," and he also makes this argument in the context of expectation formation. Hence, the ADM model may be viewed as a more descriptive model of choice, as it attempts to capture specialization and integration of brain activity.

As an application of affective decision-making, we present an example of the demand for insurance in a world with two states of nature: Bad and Good. The relevant probability distribution in insurance markets is personal risk, hence the demand for insurance may depend on optimism bias. Affective choice in insurance markets is defined as the insurance level and risk perception which constitute a pure strategy Nash Equilibrium of the ADM intrapersonal game.

The systematic departure of the ADM model from the expected utility model allows for both optimism and pessimism in choosing the level of insurance, and shows, consistent with consumer research (Keller and Block 1996), that campaigns intended to educate consumers on the loss size in the bad state may have the unintended consequence that consumers purchase less, rather than more, insurance. Hence, the ADM model suggests that the failure of the expected utility model to explain some data sets may be due to systematic affective biases.

An obvious question is what is the class of preferences over risky or ambiguous acts that are represented by the ADM model? The ADM intrapersonal game is a potential game and the potential function allows a representation of affective decision making as maximization of the affective agent's preferences .We provide an axiomatic foundation for affective decision making and show that an affective agent has ambiguity-seeking preferences, consistent with the Ellsberg paradox.

As is well known, the subjective expected utility (SEU) models of Savage (1954) and Anscombe and Aumann (1963) are refuted by the Ellsberg paradox (1961). In the Ellsberg experiment, individuals are asked to bet on a draw from an urn with 100 balls, some red and the rest black, where the distribution is unknown or bet on a draw from an urn with 50 black balls and 50 red balls. This experiment partitions the subjects into three disjoint groups: A, B, and C. Individuals in Group A preferred to bet on a black draw from the urn with the known distribution, rather than bet on a black draw from the urn with the unknown distribution and similarly for bets on drawing a red ball. Individuals in Group B, were indifferent between betting on draws from either urn. Individuals in Group C preferred to bet on the ambiguous urn.

In his thought experiment, Ellsberg (1961, p. 651) suggests that the majority of people are in group A, but a small minority are in group C and he ignores the people in group B. As he points out, both Group A and C violate Savage's axioms for the SEU model. Subjects in Group A are said to be ambiguity-averse and subjects in Group C are said to be ambiguity-seeking.

A number of alternative models of choice under risk and uncertainty have been proposed as models that rationalize ambiguity-averse choices, such as the maxmin expected utility model of Gilboa and Schmeidler (1989) or more recently the multiplier preferences of Hansen and Sargent (2000). Recently, Maccheroni, Marinacci and Rustichini[MMR] (2006) proposed variational preferences as a general class of preferences that rationalize ambiguity-averse choices. MMR (2006) show that variational preferences subsume both maxmin preferences and multiplier preferences and are characterized by six axioms, where axiom 5, due to Schmeidler (1989), is the axiom for ambiguity aversion. This axiom has the simple geometric interpretation that the preference relation over acts is quasi-concave. Moreover, if axiom 5 is replaced by axiom $\hat{5}$ where the preference relation over acts is quasi-linear, then axioms 1–4, $\hat{5}$ and 6 characterize the SEU model. Both of these results are proven in MMR (2006).

Another possibility is that the preference relation over acts is quasi-convex. If so, then is the behavioral interpretation of this axiom ambiguity-seeking choice — a possibility anticipated by Ellsberg's thought experiment (1961), where the decision-makers in Group C are ambiguity-seeking? Do these preferences share with variational preferences a penalized SEU representation? We show that the answer to both questions is yes.

In the variational preferences models the decision maker is playing a sequential game against a malevolent nature, where nature moves last. Hence the solution concept is maxmin. In the affective decision making (ADM) model proposed in this paper the rational and the emotional process of the decision-maker are engaged in a simultaneous move, potential game, where the solution concept is Nash equilibrium. Both classes of models are penalized SEU models. In the variational preferences models the penalty reflects the decision maker's uncertainty that her "subjective" beliefs about the states of the world are the correct state probabilities. In the ADM model, the penalty reflects the mental cost of her "optimistic" beliefs about preferred outcomes.

We suggest that the outcomes of Ellsberg's thought experiment are not paradoxical, but allow for three mutually exclusive formulations of Schmeidler's axiom. That is, preferences over acts can be quasi-concave, quasi-linear or quasi-convex. If in addition preferences satisfy axioms 1–4 and axiom 6 in MMR (2006), then the corresponding classes of preferences over acts are: variational preferences, SEU preferences and ADM preferences. We show that if axiom 5: $f \sim g \Rightarrow \alpha f + (1-\alpha)g \succcurlyeq f$, the axiom that the preference relation over acts is quasi-concave, is replaced with axiom $\hat{5}$: $f \sim g \Rightarrow \alpha f + (1-\alpha) g \preccurlyeq f$, the axiom that the preference relation over acts is quasi-convex, then the preference relation has an ADM representation if and only if it satisfies axiom $\hat{5}$ and axioms 1–4 and 6 for variational preferences.

Both preference over beliefs and dual processes have previously been considered in the economic literature, but have been modeled separately. Models with preference over beliefs, such as Akerlof and Dickens (1982), Yariv (2002), Koszëgi (2006), Bénabou and Tirole (2002), Bodner and Prelec (2001), Caplin and Leahy (2004), and others, all assume that an agent chooses beliefs in a strategic manner to resolve a trade-off between a standard instrumental payoff and some notion of psychologically based belief utility.[1] In contrast, this model formulates the trade-off by introducing an intrapersonal game between two processes. In this game, the emotional process chooses optimal beliefs (for a given action) to maximize mental profit from those beliefs. The rational process chooses optimal action (for a given belief) to maximize expected utility. Choice is an equilibrium outcome determined by the two processes.

Dual processes models such as Thaler and Shefrin's (1981), Bernheim and Rangel's (2004), and Benhabib and Bisin's (2005), Fudenberg and Levine's (2006), and Brocas and Carrillo's (2008), conceive the two systems, or decision modes, as mutually exclusive. However, the two processes in ADM are simultaneously active, and mutually determine choice. Thus, ADM offers a model of a single decision-making *mode* composed of two inner processes. This difference results from the different questions that previous and the current research address. In the models of addiction and self-control, both processes determine *action*, whereas herein, one process chooses action while the other forms perceptions, and both are necessary for decision making.

In Brunnermeier and Parker (2005) the agent chooses both beliefs and actions. They propose a dynamic model where the agent chooses beliefs for all future periods at period one, and in each period thereafter the agent chooses an action. In contrast, our model is a static model, where beliefs and actions mutually determine choice, and is a potential game. Finally, we show that affective decision making is characterized by six axioms. This characterization allows an interpretation of the ADM model as a model of ambiguity-seeking choice behavior. An interpretation not shared by Brunnermeier and Parker (2005) or the other dual process models.

The remainder of the paper is organized as follows. The application of the ADM intrapersonal game to insurance markets is presented in Section 2. In Section 3, we present the axiomatic foundation of ADM. In the final section we review rationalizations of the Ellsberg paradox and present the ADM model of ambiguity-seeking behavior. All proofs are in the Appendix.

2 The ADM Intrapersonal Game

Affective decision-making (ADM) is a theory of choice, which generalizes expected utility theory by positing the existence of two cognitive processes — the *rational* and the *emotional process*. Observed choice is the result of their simultaneous interaction. This theory accommodates endogenity of beliefs, probability perceptions and tastes.

[1] The axiomatic foundation for this is provided by Caplin and Leahy (2001) and Yariv (2001).

In this paper, we present a model of affective choice in insurance markets, where probability perceptions are endogenous.

Consider an agent facing two possible future states of the world, Bad and Good with associated wealth levels ω_B and ω_G, where $\omega_B < \omega_G$. The agent has a strictly increasing, strictly concave, smooth utility function of wealth, $u(W)$, with $\lim_{w\to-\infty} Du(W) = \infty$, $\lim_{w\to\infty} Du(W) = 0$.[2] Risk perception is defined as the perceived probability $\beta \in [0,1]$ of the Bad state occurring. To avoid (perceived) risk, the agent can purchase or sell insurance $I \in (-\infty, \infty)$ to smooth her wealth across the two states of the world. The insurance premium rate, $\gamma \in (0,1)$ is fixed for all levels of insurance.

The *rational process* chooses an optimal insurance (I^*) to maximize expected utility given a perceived risk β. Specifically, the rational process maximizes the following objective function:

$$\max_I \{\beta u(\omega_B + (1-\gamma)I) + (1-\beta)u(\omega_G - \gamma I)\}.$$

The *emotional process* chooses an optimal risk perception (β^*) given an insurance level I, to balance affective motivation and taste for accuracy. Specifically, the emotional process maximizes the following objective function:

$$\max_\beta \{\beta u(\omega_B + (1-\gamma)I) + (1-\beta)u(\omega_G - \gamma I) - c(\beta;\beta_0)\}.$$

Affective motivation is captured with the expected utility term — the agent would like to assign the highest possible weight to her preferred state of the world. Taste for accuracy is modeled by introducing a mental cost function $c(\beta;\beta_0)$ that is a nonnegative, and smooth function of β. It is strictly convex in β, and reaches a minimum at $\beta = \beta_0$, where β_0 is the objective probability. See Figure 1.

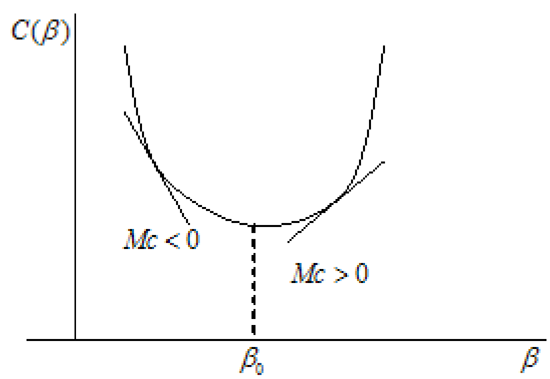

Figure 1

The farther away β is from β_0, the greater are the psychological cost. This is because to justify favorable beliefs agents need to use strategies such as the availability heuristic, which can be unconsciously manipulated to arrive at the desired beliefs.

[2] All qualitative results remain the same for the case of $\lim_{W\to 0} Du(W) = \infty$, $\lim_{W\to\infty} Du(W) = 0$.

Such mental strategies, or justification processes, are likely to be costly and are captured by the cost function. We assume that biased recall becomes increasingly more costly as the distance between desired beliefs β and the objective odds β_0 increases. We will assume that $c(\beta;\beta_0)$ is a smooth function of β_0. It is well known that agents attribute a special quality to situations corresponding to the extreme beliefs $\beta \in \{0,1\}$ (Kahneman and Tversky 1979). Hence we assume that there exist limits $\underline{\beta}, \overline{\beta} \in (0,1)$ such that for $\beta \in (\underline{\beta}, \overline{\beta})$, $c(\beta;\beta_0)$ is finite, and $\lim_{\beta \to \underline{\beta}} c(\beta;\beta_0) = \lim_{\beta \to \overline{\beta}} c(\beta;\beta_0) = +\infty$.

The interaction of the two processes in decision-making is modeled using an intrapersonal simultaneous-move game. Modeling the interaction of the two processes as a simultaneous move game reflects a recent view in cognitive neuroscience; namely, both processes mutually determine the performance of the task at hand (Damasio 1994).

Definition 1 *An intrapersonal game is a simultaneous move game of two players, namely, the rational and the emotional processes. The strategy of the rational process is an insurance level,* $I \in (-\infty, \infty)$*, and the strategy of the emotional process is a risk perception,* $\beta \in (\underline{\beta}, \overline{\beta})$*. The payoff function for the rational process* $g : (\underline{\beta}, \overline{\beta}) \times (-\infty, \infty) \to R$ *is* $g(\beta, I) \equiv \beta u(\omega_B + (1-\gamma)I) + (1-\beta)u(\omega_G - \gamma I)$*. The payoff function for the emotional process* $\psi : (\underline{\beta}, \overline{\beta}) \times (-\infty, \infty) \to R$ *is* $\psi(\beta, I) \equiv g(\beta, I) - c(\beta;\beta_0)$*, where* $c(\cdot)$ *is the mental cost function of holding belief* β*, which reaches a minimum at* β_0*.*

The pure strategy Nash equilibria of this game, if they exist, are the natural candidates for the agent's choice, as they represent mutually determined choice and reflect consistency between the rational and emotional processes. The intrapersonal game defined above is a potential game, where the potential function allows a representation of the affective agent's preferences and suggests sufficient conditions for a unique pure strategy Nash equilibrium.

Proposition 2 *The intrapersonal game is a potential game, in which the emotional process's objective function is the potential function for the game. Because the potential function is strictly concave in each variable (risk perception and insurance), its critical points are the pure strategy Nash equilibria of the game.*

Excluding the case of tangency between the best responses of the two processes, we have the following existence theorem. See Figure 2.

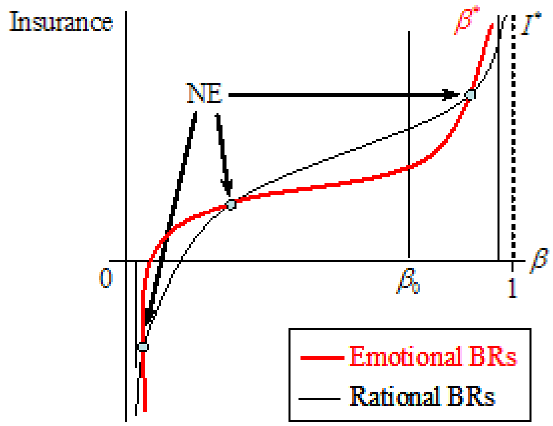

Figure 2

Proposition 3 *The ADM intrapersonal game has an odd number of pure strategy Nash equilibria. The set of Nash equilibria is a chain in R^2, under the standard partial order on points in the plane.*[3]

To derive the predictive properties of the ADM model, we require the intrapersonal game to have a unique pure strategy Nash equilibrium. A sufficient condition for uniqueness follows:.

Proposition 4 *A sufficient condition for a unique pure strategy Nash equilibrium of the intrapersonal game is:*

$$\frac{\partial^2 c(\beta;\beta_0)}{\partial \beta^2} > -\frac{[Du(\omega_B+(1-\gamma)I)(1-\gamma)+Du(\omega_G-\gamma I)\gamma]^2}{[\beta D^2 u(\omega_B+(1-\gamma)I)(1-\gamma)^2+(1-\beta)D^2u(\omega_G-\gamma I)\gamma^2]},$$
$$\forall\,(I,\beta)\in[I^*(\underline{\beta}'),I^*(\overline{\beta}')]\times[\underline{\beta}',\overline{\beta}'],$$

where $\underline{\beta}' \equiv \beta^*(I^*(\underline{\beta}))$ *and, similarly,* $\overline{\beta}' \equiv \beta^*(I^*(\bar{\beta}))$.

Hence, for large mental costs, the equilibrium is unique (think of $\lambda > 0$, $\hat{c}(\cdot) = \lambda c(\cdot)$). Moreover, for very large mental costs, the ADM model reduces to the expected utility model.[4]

However, considering the general case, where the mental costs are not very large, risk perceptions are endogenous and the ADM model systematically departs from the expected utility model. How exactly does affective choice in insurance markets differ from the demand for insurance in the expected utility model? Proposition 5 below shows that the expected utility outcome in the case of an actuarially fair insurance market (full insurance) falls within the choice set of the ADM agent. However, if the insurance market is not actuarially fair, then this is no longer the case.

[3] The existence of a pure strategy Nash equilibrium also can be derived for the case of a logarithmic utility function, in which the agent's income in each state is not negative.

[4] As $c \to \infty$, $\beta^* \to \beta_0$ for all values of I. As a result, the ADM model converges to the expected utility model.

Proposition 5 *If* $\gamma = \beta_0$, *there exists at least one Nash equilibrium* (β^*, I^*) *with* $\beta^* = \beta_0 = \gamma$, *and* $I^* =$ *full insurance.*

If $\gamma > \beta_0$, *there exists at least one Nash equilibrium* (β^*, I^*) *with* $\beta^* < \beta_0$ *and* $I^* < I^*(\beta_0)$

If $\gamma < \beta_0$, *there exists at least one Nash equilibrium* (β^*, I^*) *with* $\beta_0 < \beta^*$ *and* $I^* > I^*(\beta_0)$.

To understand the intuition behind these results, consider a standard myopic adjustment process where the processes alternate moves. If $\gamma > \beta_0$, at β_0 the rational process, similar to the expected utility model, prescribes buying less than full insurance. The emotional process, in turn, leads the decision maker to believe "this is not going to happen to me" and determines that she is at a lower risk. This effect causes a further reduction in the insurance purchase, with a result of less than full insurance, even less than what the expected utility model would predict. Note that Proposition 5 also implies that, from the viewpoint of an outside observer, both optimism and pessimism (relative to β_0) are possible. This is due to the characteristics of insurance: if an agent purchases more than full insurance, then the "bad" state becomes the "good" state, and vice versa. Consequently, if there is no effective action, i.e., one cannot change the bad state to a good state, we would observe optimism and less-than-optimal insurance.

Here is another example of the difference between affective choice and the demand for insurance in the expected utility model. In the expected utility model, if people realize that they face a higher potential loss, due to educational campaigns that make them aware of the possible catastrophe, then they purchase more insurance. In the ADM model, if an agent realizes she faces higher possible loss, then she might purchase less insurance. Because the increased loss size affects both the emotional and the rational processes in different directions; the rational process prescribes more insurance, the emotional process prescribes lower risk belief to every insurance level (due to greater incentives to live in denial). If the emotional effect is stronger the agent will buy less insurance than previously. That is, if the loss is great, agents might prefer to remain in denial and ignore the possible catastrophes altogether, which will lead them to take fewer precautions such as buying insurance. This is consistent with consumer research showing that high fear arousal in educating people on the health hazards of smoking leads to a discounting of the threat (Keller and Block 1996). Proposition 6 and Figure 3 below summarizes the conditions for educational campaigns to produce the counter-intuitive affective result.

Proposition 6 *An educational campaign result in less insurance if*

$$\frac{r(\omega_B - \gamma I)}{Du(\omega_B - \gamma I)} > \frac{r(\omega_G + (1-\gamma)I)}{Du(\omega_G + (1-\gamma)I)},$$

where $r(\cdot)$ *is the absolute risk aversion property of the utility function* $u(\cdot)$

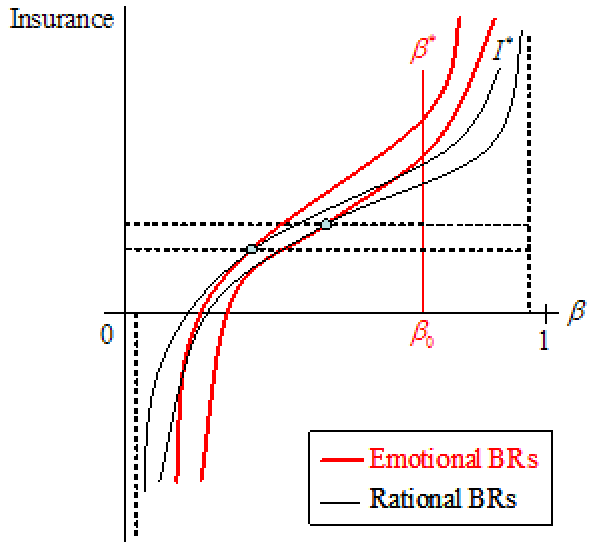

Figure 3

In Proposition 6, if the utility function $u(\cdot)$ exhibits constant or increasing absolute risk aversion, educational campaigns will lead to higher insurance purchase if and only if initially the agent buys more than full insurance. Insurees who initially buy less than full insurance will buy even less after the educational campaign. Hence, for such utility functions, educational campaigns divide the insurance market into a set of agents who purchase more insurance — the intended consequence — and a set of agents who purchase less insurance — the unintended consequence.

3 Axiomatic Foundation of ADM

This section addresses the question: What preferences over risky or uncertain acts are represented by the ADM model? The axioms over acts that give rise to an ADM representation are suggested by a duality property of the ADM potential function. This duality is analogous to the dual relationship between the cost and profit functions of a price taking, profit maximizing firm producing a single good. That is, the profit function, $\phi(p) = \sup_{y \geq 0}\{py - c(y)\}$ where p is the price of output, y is the output, and $c(y)$ is the continuous, convex cost function. As is well known $c(y) = \sup_{p \geq 0}\{py - \phi(p)\}$. In convex analysis $\phi(p)$ is called the Legendre–Fenchel conjugate of $c(y)$ and $c(y)$ is the biconjugate of $\phi(p)$, where we have invoked the theorem of the biconjugate. Returning to the ADM model, we note that the potential function $\Pi(f,p) = \int u(f)dp - c(p)$ where $c(p)$ is the smooth, convex cost function of the emotional process. The Legendre–Fenchel conjugate of the ADM potential function and the theorem of the biconjugate suggest axioms on preferences over acts that admit an ADM representation. Moreover, these axioms allow an interpretation of affective decision-making as ambiguity-seeking choice behavior.

The axiomatic foundation of the ADM model follows the setup in MMR, where: S is the set of states of the world; Σ is an algebra of subsets of S, the set of events;

and X, the set of consequences, is a convex subset of some vector space. F is the set of (simple) acts, i.e., finite-valued Σ-measureable functions $f : S \to X$. $B(\Sigma)$ is the set of all bounded Σ-measureable functions, and endowed with the sup-norm it is an AM-space with unit, the constant function 1. $B_0(\Sigma)$ the set of Σ-measureable simple functions is norm dense in $B(\Sigma)$. The norm dual of $B(\Sigma)$ is $ba(\Sigma)$, finitely additive signed measures of bounded variation on Σ (see Aliprantis and Border 1999 for further discussion).

The potential function for the ADM intrapersonal game is $\Pi(f,p) = \int u(f)dp - c(p)$. To make our notation consistent with the notation in convex analysis, we define $J^*(p) = c(p)$, and write the potential function as $\Pi(f,p) = \{\langle u(f), p\rangle - J^*(p)\}$. If $W(f) = \max_{p\in\Delta}\{\langle u(f), p\rangle - J^*(p)\}$, and the decision-maker maximizes $W(f)$ over her choice set K, then $\max_{f\in K} W(f) = \max_{f\in K}\max_{p\in\Delta}\{\langle u(f), p\rangle - J^*(p)\} = \max_{f\in K, p\in\Delta}\Pi(f,p)$. It follows from the Envelope theorem that $\arg\max_{f\in K,p\in\Delta}\Pi(f,p)$ is a subset of the pure strategy Nash equilibria of the ADM intrapersonal game, defined by $\Pi(f,p)$.

If $W(f) = \max_{p\in\Delta}\Pi(f,p) = \max_{p\in\Delta}\{\langle u(f), p\rangle - J^*(p)\}$, then $J(u(f)) = W(f)$ for some convex function J, by definition of the Legendre–Fenchel conjugate. Below,we present axioms on preferences over risky and uncertain acts that characterize $J(u(f))$, where it follows from the theorem of the biconjugate that $J(u(f)) = \max_{p\in\Delta}\{\langle u(f), p\rangle - J^*(p)\}$.

AXIOMS:

A.1 (Weak Order): If $f, g, h \in F$, (a) either $g \succsim f$ or $f \succsim g$, and (b) $f \succsim g$ and $g \succsim h \Rightarrow f \succsim h$.

A.2 (Weak Certainty Independence): If $f, g \in F$, $x, y \in X$ and $\alpha \in (0,1)$, then $\alpha f + (1-\alpha)x \succsim \alpha g + (1-\alpha)x \Longrightarrow \alpha f + (1-\alpha)y \succsim \alpha g + (1-\alpha)y$.

A.3 (Continuity): If $f, g, h \in F$, the sets$\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succsim h\}$ and $\{\alpha \in [0,1] : h \succsim \alpha f + (1-\alpha)g\}$ are closed.

A.4 (Monoticity): If $f, g \in F$ and $f(s) \succsim g(s)$ for all $s \in S$, the set of states, then $f \succsim g$.

A.$\hat{5}$ (Quasi-Convexity): If $f, g \in F$ and $\alpha \in (0,1)$, then $f \sim g \Rightarrow \alpha f + (1-\alpha)g \precsim f$.

A.6 (Nondegeneracy): $f \succ g$ for some $f, g \in F$.

These axioms where A.$\hat{5}$ is replaced by A.5 (quasi-concavity) are due to MMR (2006).

Proposition 7 *Let $\succsim$ be a binary order on F. The following conditions are equivalent:*

1. *The relation $\succsim$ satisfies axioms A.1–A.6.*

2. *There exists a nonconstant affine function* $u : X \to R$ *and a continuous, convex function* $J^* : \Delta \to [0, \infty]$ *where for all* $f, g \in F$, $f \succsim g \Leftrightarrow W(f) \geq W(g)$ *and* $W(h) = \max_{p\in\Delta} \{\langle u(h), p\rangle - J^*(p)\}$ *for all* $h \in F$.

If $W(f) = \max_{p\in\Delta} \Pi(f, p) = \max_{p\in\Delta}\{\langle u(f), p\rangle - J^*(p)\}$, then $J(u(f)) = W(f)$ for some convex function J, by definition of the Legendre–Fenchel conjugate. Below,we present axioms on preferences over risky and uncertain acts that characterize $J(u(f))$. It follows from the theorem of the biconjugate that $J(u(f)) = \sup_{p\in(R^k)^*_{++}}\{\langle u(f), p\rangle - J^*(p)\}$.

4 Rationalizations of the Ellsberg Paradox

The Ellsberg paradox poses a challenge to expected and subjective expected utility theories. Both theories have additive probabilities, that is $V(x_1, ..., x_k) = \sum_{i=1}^k u(x_i)p_i$, where $\sum_{i=1}^k p_i = 1$ is either objective or subjective probability measure. Models that are consistent with the Ellsberg paradox can be divided into two broad categories: models that introduce non-additive probabilities measures, known as capacities, $c(\cdot)$, and models with multiple priors.

Examples of the first approach are Choquet expected utility (CEU) and the special case of rank-dependent expected utility (RDEU). Choquet expected utility of an act x over n states $w_1, ..., w_n$ such that $u(x(w_1)) \geq u(x(w_2)) \geq \cdots \geq u(x(w_n))$ is $CEU(x; c) = \sum_{i=1}^n [u(x(w_i)) - u(x(w_{i+1}))]c(w_1, ..., w_n) + u(x(w_n)))$, where $c(\cdot)$ is a capacity. A capacity $c(\cdot)$ is convex, or super-additive, if for all events A, B, $c(A) + c(B) \geq c(A \cap B) + c(A \cup B)$. A capacity $c(\cdot)$ is concave, or sub-additive, if for all events A,B $c(A) + c(B) \leq c(A \cap B) + c(A \cup B)$. A CEU defined with a convex capacity is ambiguity-averse, an affine capacity is ambiguity-neutral, and a concave capacity is ambiguity-seeking. Similarly, RDEU is defined as $CEU(x; c) = \sum_{i=1}^n [u(x(w_i)) - u(x(w_{i+1}))]\phi(p(w_1, ..., w_n)) + u(x(w_n)))$, where $\phi(\cdot)$ is a capacity with $\phi(0) = 0$, $\phi(1) = 1$, and $p(\cdot)$ is an additive probability measure. If $\phi(\cdot)$ is convex, affine, or concave capacity, RDEU represent ambiguity-averse, ambiguity-neutral, or ambiguity-seeking, respectively. Hence, if capacities are convex or concave,the choice behavior of a CEU or a $RDEU$ agent exhibits ambiguity-aversion, or ambiguity-seeking respectively, and is consistent with the Ellsberg paradox.

An example of the second approach is maxmin expected utility of Gilboa and Schmeidler (1989), a model of ambiguity-aversion. MMR (2006) show that maxmin expected utility can be represented by variational preferences where the decision maker maximizes $V(f) = \inf_{p\in\Delta} \{\langle u(f), p\rangle - J^*(p)\} = J(u(f))$, and $J(\cdot)$ is concave. Föllmer and Shied (2004) show that this representation of preferences over acts exhibits ambiguity-aversion — see example 2.75 on page 88.

In contrast to maxmin, we show that the ADM model is an example of a multiple prior model consistent with ambiguity-seeking choice behavior. It follows from the axiomatic foundation of the ADM model that preferences over acts in the ADM model can be represented by $W(h) = \max_{p\in\Delta} \{\langle u(h), p\rangle - J^*(p)\}$, and $J^*(\cdot)$ is convex. Hence the general representation of preference over acts for the ADM model is $J(u(h)) = W(h) = \max_{p \in \Delta} \{\langle u(h), p \rangle - J^*(p)\}$, where $J(\cdot)$ is convex. Hence $-J(u(h))$ is concave and represents an ambiguity-averse decision maker and therefore for convex $J(\cdot)$, $J(u(h))$ represents an ambiguity-seeking decision maker.

5 Appendix

Proof of Proposition 2. Denote the rational process's payoff function as (R) and the emotional process's payoff function as (E). A necessary and sufficient condition for the intrapersonal game to have a potential function (Monderer and Shapley 1996) is $\frac{\partial^2 R}{\partial\beta\partial I} = \frac{\partial^2 E}{\partial\beta\partial I}$. This condition clearly is satisfied in the ADM model. The potential function $P(\beta, I)$ is a function such that (Monderer and Shapley 1996): $\frac{\partial P}{\partial\beta} = \frac{\partial E}{\partial\beta}$, $\frac{\partial P}{\partial I} = \frac{\partial R}{\partial I}$. Because $\frac{\partial E}{\partial I} = \frac{\partial R}{\partial I}$, (E) can serve as a potential function. The critical points of the potential function are $\frac{\partial P}{\partial\beta} = \frac{\partial E}{\partial\beta} = 0$, $\frac{\partial P}{\partial I} = \frac{\partial R}{\partial I} = 0$. The potential function is strictly concave in each variable, so at each critical point, each process is maximizing its objective function, given the strategy of the other process. Therefore, the critical points of the potential function are the pure strategy Nash equilibria of the intrapersonal game, and all pure strategy Nash equilibria are critical points of the potential function. ■

Proof of Proposition 3. By the boundaries on risk perception, $0 < \underline{\beta} < \overline{\beta} < 1$, $\beta^* \in (\underline{\beta}, \overline{\beta})$, and insurance $I^* \in [I^*(\underline{\beta}), I^*(\overline{\beta})]$. Hence, all Nash equilibria will have perceived probabilities in the interval $[\beta^*(I^*((\underline{\beta})), \beta^*\left(I^*(\overline{\beta})\right)]$ where $0 < \underline{\beta} < \beta^*(I^*((\underline{\beta})) < \beta^*\left(I^*(\overline{\beta})\right) < \overline{\beta} < 1$. Define $\beta^*(I^*((\underline{\beta})) \equiv \underline{\beta}'$, $\beta^*\left(I^*(\overline{\beta})\right) \equiv \overline{\beta}'$; because all the Nash equilibria of the intrapersonal game for $\beta \in (\underline{\beta}, \overline{\beta})$ are $\in [\underline{\beta}', \overline{\beta}']$ the focus can remain on the latter probability space.

The existence and chain results can be shown by defining a restricted intrapersonal game in which the insurance pure strategy space is restricted to $[I^*(\underline{\beta}), I^*(\overline{\beta})]$ and the perceived probabilities are restricted to $\beta \in [\underline{\beta}', \overline{\beta}']$, such that the equilibria points of the intrapersonal game are not altered. The restricted game is a supermodular game, and thus, these results follow from the properties of this class of games (see Topkis 1998). To Show that the game admits odd number of equilibria, think of the geometry of the game. As $\beta \to \overline{\beta}$, the best response of the emotional process is above the best response of the rational process, while this relationship is reversed for $\beta \to \underline{\beta}$. Since the best responses are monotonically increasing, it follows that there exists odd number of Nash equilibria. ■

Proof of Proposition 4. The emotional process's objective function $\beta u(\omega_B + (1-\gamma)I) + (1-\beta)u(\omega_G - \gamma I) - c(\beta; \beta_0)$ is the potential function of the game. The maximization of (P) with respect to a pair (I, β) gives rise to a pure strategy Nash equilibria of the game. $\beta \in [\underline{\beta}', \overline{\beta}']$ and $I \in [I^*(\underline{\beta}'), I^*(\overline{\beta}')]$ (see proof of Proposition 3), hence only the restricted intrapersonal game in which both players' strategy spaces are compact need be considered. Neyman (1997), proved that a potential game with a strictly concave, smooth potential function, in which all players have compact, convex strategy sets, has a unique pure strategy Nash equilibrium. That is, the Hessian of the potential function is negative definite, as follows from the condition given above. ■

Proof of Proposition 5. Consider the case in which $\gamma = \beta_0$. At full insurance, there is no mental gain for holding beliefs $\beta \neq \beta_0$ but there exists mental cost.

Therefore, at full insurance, the mental process's best response is $\beta = \beta_0$. Given that $\gamma = \beta_0 = \beta$, the rational process's best response is full insurance. Consequently, full insurance and $\beta = \beta_0$ is a Nash equilibrium of this case. Next, consider the case $\gamma > \beta_0$; because the insurance premium is higher than β_0, $I^*(\beta = \beta_0) < z$. Also, $\beta^* = \beta_0$ only at full insurance, where $I = z$. Therefore, at $\beta = \beta_0$ the mental process's best response falls above the rational process's best response. This relationship is reversed at the limit $\beta \to \underline{\beta}$, and both the mental and the rational best responses increase; therefore, there exists a Nash equilibrium with $\beta < \beta_0$ and less insurance than predicted by the expected utility model. A similar argument can be used to prove the result when $\gamma < \beta_0$. ■

Proof of Proposition 6. Define $\tilde{I}(\beta;\beta_0)$ as the inverse function β^{*-1}. Define $\Pi(\beta;\beta_0) = I^*(\beta) - \tilde{I}(\beta;\beta_0)$, $\Pi : [\underline{\beta}', \overline{\beta}'] \to R$

Educational campaigns on impending catastrophes increase the loss size, z. Because $\Pi(\beta;\beta_0) = 0$ is a NE, $\frac{\partial \Pi}{\partial z} < 0$ represent the unintended consequence of such campaigns.

$$\frac{\partial \Pi}{\partial z} < 0 \Leftrightarrow \frac{\frac{\partial \tilde{I}}{\partial z}}{\frac{\partial I^*}{\partial z}} > 1$$

.

$$\frac{\partial I^*}{\partial z} = \frac{[u''(w_2 - z + (1-\gamma)I^*)][u'(w_2 - \gamma I^*)]^2}{[u'(w_2-\gamma I^*)][u''(w_2 - z+(1-\gamma)I^*)u'(w_2-\gamma I^*)(1-\gamma) + u'(w_2-z+(1-\gamma)I^*)u''(w_2-\gamma I^*)\gamma]};$$

$$\begin{aligned} \frac{\partial \tilde{I}}{\partial z} &= \frac{\left[u'(w_2 - z + (1-\gamma)\tilde{I})\right]}{\left[u'(w_2 - z + (1-\gamma)\tilde{I})(1-\gamma) + u'(w_2 - \gamma\tilde{I})\gamma\right]} \Rightarrow \frac{\partial \Pi}{\partial z} < 0 \\ &\Leftrightarrow \frac{r(w_2 - \gamma I)}{u'(w_2 - \gamma I)} > \frac{r(w_1 + (1-\gamma)I)}{u'(w_1 + (1-\gamma)I)}, \text{where } r(x) = -\frac{u''(x)}{u'(x)}. \end{aligned}$$

■

Proof of Proposition 7. Axioms 1–4 are used in MMR to derive a nonconstant affine utility function, u, over the space of consequences, X. u is extended to the space of simple acts, F, using certainty equivalents. That is, $U(f) = u(x_f) \in B_0(\Sigma)$ for each $f \in F$, where x_f is the certainty equivalent of f. This is Lemma 28 in MMR, where $I(f) = U(f)$ is a niveloid on $\Phi = \{\varphi : \varphi = u(f) \text{ for some } f \in F\}$. Niveloids are functionals on function spaces that are monotone: $\varphi \leq \eta \Rightarrow I(\varphi) \leq I(\eta)$ and vertically invariant: $I(\varphi + r) = I(\varphi) + r$ for all φ and $r \in R$ — see Dolecki and Greco (1995) for additional discussion. Φ is a convex subset of $B(M)$ and by Schmeidler's axiom 5, I is quasi-concave on Φ. We also assume axioms1–4, so Lemma 28 in MMR holds for the niveloid J in the ADM representation theorem. By axiom $\hat{5}$, J is quasi-convex on Φ.

MMR show in Lemma 25 that I is concave if and only if I is quasi-concave. Hence J is convex if and only if J is quasi-convex, since J is convex (quasi-convex) if and only if $-J$ is concave(quasi-concave). MMR extend I to a concave niveloid $\widehat{I}$ on all of $B(\Sigma)$ — see Lemma 25 in MMR. Epstein, Marinacci and Seo [EMS] (2007) show in Lemma

A.5 that niveloids are Lipschitz continuous on any convex cone of an AM-space with unit and concave (convex) if and only if they are quasi-concave (convex). Hence, since $B(\Sigma)$ is a convex cone in an AM-space with unit, $\widehat{I}$ is Lipschitz continuous. It follows from the theorem of the biconjugate for continuous, concave functionals that $I(\varphi) = \inf_{p\in ba(\Sigma)}\{\int \varphi dp - \widehat{I}^*(p)\}$, where $\widehat{I}^*(p) = \inf_{\varphi\in B_o(\Sigma)}\{\int \varphi dp - \widehat{I}(\varphi)\}$ is the concave, conjugate of $\widehat{I}(\varphi)$ — see Rockafellar (1970, p. 308) for finite state spaces. MMR (2006, p. 1476) that we can restrict attention to Δ, the family of positive, finitely additive measures of bounded variation in $ba(\Sigma)$. Hence $I(\varphi) = \min_{p\in\Delta}\{\int \varphi dp - \widehat{I}^*(p)\} = \min_{p\in\Delta}\{\int u(f)dp + c(p)\}$, where $\varphi = u(f)$ and $c(p) = -\widehat{I}^*(p)$. $c(p)$ is convex since $\widehat{I}^*(p)$ is concave.

Extending $-J$ to $-\widehat{J}$ on $B(\Sigma)$, using lemma 25 in MMR, it follows from the theorem of the biconjugate for continuous, convex functionals that $J(\varphi) = \max_{p\in ba(\Sigma)}\{\int \varphi dp - \widehat{J}^*(p)\}$ where $\widehat{J}^*(p) = \max_{\varphi\in B_o(\Sigma)}\{\int \varphi dp - \widehat{J}(\varphi)\}$ is the convex, conjugate of $\widehat{J}(\varphi)$ — see Rockafellar (1970, p. 104) for finite state spaces and Zălinescu (2002, p. 77) for infinite state spaces.

Again it follows from MMR that $J(\varphi) = \max_{p\in\Delta}\{\int \varphi dp - \widehat{J}^*(p)\} = \max_{p\in\Delta}\{\int u(f)dp - c(p)\} = W(f)$, where $\varphi = u(f)$ and $c(p) = \widehat{J}^*(p)$. $c(p)$ is convex since $\widehat{J}^*(p)$ is convex.

$f \succsim g \Leftrightarrow J(u(f)) \geq J(u(g)) \Leftrightarrow W(f) \geq W(g)$. Hence $\arg\max_{f\in F} J(u(f)) \subseteq$ set of pure strategy Nash equilibria of the ADM intrapersonal game, where $u(\cdot)$ is the Bernoulli utility function of the rational process and $\widehat{J}^*(\cdot)$ is the cost function of the emotional process. ■

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