# Ideology and Competence in Alternative Electoral Systems.*

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**Abstract**

We develop a model of elections in proportional (PR) and majoritarian (FPTP) electoral systems. The model allows an endogenous number of candidates, differentiation in a private value dimension, or ideology, and in a common value dimension, which we interpret broadly as quality or competence. Voters are fully rational and strategic. We show that the quality of candidates running for office in PR elections is lower than that of any candidate running for office in FPTP, and that in all equilibria in which candidates are ideologically differentiated, the number of candidates running for office is larger than in majoritarian electoral systems (where exactly two candidates run). Moreover, we provide conditions under which the rankings are strict. We consider several extensions of the model, which include introducing majority premiums in PR, thresholds for representation, and multiple electoral districts.

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# 1 Introduction

Electoral systems translate votes cast in elections to the number of seats won by each party in the national legislature. By affecting how voters' preferences are ultimately mapped into policy outcomes, they are one of the fundamental institutions in representative democracies. Electoral systems also influence indirectly - through the choices they induce in voters and politicians - most key features of modern political systems: from the number of alternatives faced by voters and the diversity of ideological positions they represent, to the quality of the candidates, of their staff and of their platforms.

These key political outcomes are themselves naturally intertwined. On the one hand, the more diverse are the policy positions represented by candidates running for office, the larger is the incentive for a new candidate to run representing an intermediate ideological alternative. On the other hand, the less diverse the ideological positions represented by candidates running for office, the larger is the number of voters that will be swayed by a quality differential among them. In this paper, we tackle jointly the effect of alternative electoral systems on the number of candidates running for office, and the quality and ideological diversity of their platforms. To do so, we develop a model of electoral competition in Proportional (PR) and Majoritarian (FPTP) electoral systems that integrates three different approaches in formal models of elections, allowing free entry of candidates, differentiation in a private value dimension, or *ideology*, and in a common value dimension, or *quality*.

In our model, each potential candidate is endowed with an ideological position that he can credibly implement if he chooses to run and gets elected. With the field of competitors given, candidates running for office can then invest money, time, or effort to develop an attribute that is valued by all voters alike (e.g., the probability that their staff is competent or non-corrupt), which we interpret as quality. We assume that in deciding whether to run for office or not, each potential candidate cares only about the spoils he can appropriate from being in office, and that voters are risk averse and fully rational, and therefore vote strategically.

The incentives of voters and politicians are shaped by the electoral system under consideration. Focusing on PR and FPTP is a natural starting point for both practical and theoretical purposes. First, PR and FPTP are two of the most commonly used electoral systems in modern democracies around the world.[1] Second, proportional and majoritarian systems represent *ideal* entities at the opposite side of the spectrum of what is possibly the main attribute of electoral systems: how they map maps votes into seats. While in its purest form PR translates the share of votes obtained by each party in the election to an equal share of seats in the legislature, FPTP gives a disproportionate representation to the candidate obtaining a plurality of votes.[2] Extending the comparison to the broader link between votes and *policy outcomes*, these alternative electoral systems differ also in a second dimension: given a voting outcome in the electorate, the process of post-election bargaining in PR introduces more uncertainty for voters in the selection of the policy outcome.[3] To capture these two fundamental differences in a stylized manner within our model, we assume that in FPTP the candidate who wins a plurality of votes appropriates all rents from office and implements the policy he represents, while in PR systems the policy outcome is the result of a lottery between the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly).

The central result of the paper shows that the maximum quality among all candidates running for office in proportional elections is always lower than the quality of any candidate running for office in majoritarian elections, and that in all equilibria in which candidates are differentiated, the number of candidates running for office is larger than in majoritarian electoral systems. We show, moreover, that under mild conditions these rankings are in fact strict; i.e., for a "large" set of parameters, PR leads to strictly more candidates, each with strictly less quality, than FPTP. The

[1]About one fourth of all countries use FPTP electoral systems, and about one third use PR systems. These proportions change from 23.6 percent to 32.4 per cent for FPTP, and from 35 percent to 30.9 percent for PR when the universe is the class of established democracies, see IDEA (2005).

[2]This is, of course, a very stylized representation of a complex and diverse array of electoral institutions. As Cox (1997) argues, however, "much of the variance in two of the major variables that electoral systems are thought to influence - namely, the level of disproportionality between each party's vote and seat shares, and the frequency with which a single party is able to win a majority of seats in the national legislature - is explained by this distinction."

[3]As Lizzeri and Persico (2001) note: "Proportional systems are usually associated with many parties having an influence on policymaking, through the process of post-election bargaining [. . . ] Majoritarian systems are thought to favor the party with the highest share of the vote, in the sense that more power of policy setting is conferred to that party [. . . ]."

diversity and polarization of the ideological positions represented in the election can in general be larger or smaller in PR than in FPTP. In the most efficient equilibrium, however, FPTP restricts the electoral competition to two centrist top quality candidates. This equilibrium maximizes voters' welfare, and dominates in fact in this regard all admissible equilibria in PR.

To prove our main result, we begin by characterizing equilibria of the model for FPTP elections. Voters' risk aversion over policies implies that in equilibrium only two candidates compete for office, and the winner-takes-all nature of FPTP induces candidates to invest as much as possible in quality, thus ruling out mediocre candidates/platforms. Since voters vote sincerely between the two candidates on the equilibrium path, and candidates running for office must anticipate winning with positive probability, equilibrium candidates must be symmetrically located around the median ideological position in the electorate (they do *not* need however to be centrist - although this is possible - and in fact can be completely polarized). On the contrary, PR elections admit multi-candidate equilibria in which no candidate offers a top quality alternative. The number of candidates running for office and the degree of ideological differentiation between candidates are determined in equilibrium by two opposing forces. First, in any electoral equilibrium in PR, candidates must be sufficiently differentiated in the ideological spectrum, because of the basic tension that emerges in our model between quality and differentiation in policies: the closer candidates are in terms of their ideological position, the larger is the number of voters that can be attracted by a given increase in quality by one of the candidates. This implies in turn that candidates will invest more aggressively in quality the closer they are to one another, eventually competing away their rents. Second, the maximum degree of horizontal differentiation among candidates is bounded by entry: candidates cannot be too differentiated in PR elections without triggering the entry of an additional candidate, who would be able to attain the support of a sufficiently large niche of voters.

In the second part of the paper, we consider several extensions of our main model. In Section 5.1, we introduce a modified version of PR elections (PR-Plus), in which the candidate with the largest number of votes obtains a majority premium in both the probability with which his policy is implemented, and in the proportion of office rents he attains after the election. We show that for a given majority premium, but sufficiently large electorates, equilibrium behavior in PR-Plus resembles that in FPTP. For a fixed size of the electorate, instead, if the majority premium is sufficiently small (approximating PR), PR-Plus elections admit equilibria with more than two candidates choosing non-maximal quality, as in the case of pure PR.

In Section 5.2, we show that our main comparison holds under alternative specifications of the policy function mapping elected representatives to policy outcomes. In particular, we show that PR elections also admit an electoral equilibrium in which more than two candidates run for office choosing non-maximal quality if the policy outcome is selected as the median policy of all elected representatives in the ideological space. In Section 5.3, we consider a variant of the main model that allows us to compare PR and FPPT when the relevant aspects of candidates' quality can not be easily modified or acquired exerting effort - as in our benchmark *choice* model - but instead are innate characteristics of the candidates (we call this the *selection* model). We show that the selection model leads to higher quality candidates than the choice model in PR, and allows mediocre candidates to run for office in FPTP. The results suggest that we should expect alternative electoral systems to have different effects on the quality of policies and on the quality or competence of politicians: a FPTP electoral system is more effective in inducing candidates to offer better choices to voters, while a PR system can be more effective in selecting more attractive politicians.

In Section 5.4, we introduce thresholds for representation in PR elections. We show that a threshold for representation can have a large impact on electoral outcomes, both directly, by restricting the number and characteristics of candidates competing for office, and indirectly, through strategic voting. We argue that it is this latter feature that can most significantly affect behavior in PR elections, as it expands the set of electoral equilibria to include both the efficient outcome and the worst possible admissible equilibrium in PR, in which two extreme and mediocre candidates run for office. In Section 5.5, we introduce multiple districts (and district magnitudes) in PR and FPTP, and note that if we extend the lottery mechanism for the determination of policy outcomes to both PR and FPTP in this institutional setting, our main result is unaffected. Finally, in Section 5.6, we consider a possible source of strategic uncertainty for voters, introducing behavioral voters who do not vote strategically.

The rest of the paper is organized as follows. We review related literature in Section 2. Section 3 introduces the model. We present the basic comparison of electoral systems in Section 4, and the extensions in Section 5. Section 6 concludes. All proofs are in the appendix.

## 2 Related Literature

Our paper is related to three strands of literature. A first group of papers focuses on the effect of different electoral systems on the number of candidates running for office. This literature provides several formalizations of the well-known *Duvergerian* predictions, namely that majoritarian elections leads to a two-party system (Duverger's law), and that PR tends to favor a larger number of parties than FPTP (Duverger's hypothesis). A relatively large literature focuses on Duverger's law, studying the equilibrium number of candidates in FPTP elections.[4] Among these papers, the closest to our work are Feddersen (1992) and Feddersen, Sened, and Wright (1990) (FSW). Our model of FPTP elections differs from these papers on two accounts. First, while in our set up candidates are endowed with an ideological position that they can credibly implement if elected, in FSW candidates can adjust their ideological positions after entry without costly consequences.[5] Second, while in FSW candidates can only differ in an ideological dimension, in our model candidates can also differentiate themselves through the quality of the alternative they offer to voters. Finally, two papers compare the effect of alternative electoral systems on the number of candidates competing for office. Osborne and Slivinski (1996) compare plurality and plurality with runoff under sincere voting, and Morelli (2004) compares majoritarian and proportional electoral systems under strategic voting. Differently than in our paper, Morelli focuses on how different electoral systems influence the incentives of politicians to coordinate their

[4]For papers that study entry in FPTP under the assumption of sincere voting see, e.g., Palfrey (1984), and Greenberg and Shepsle (1987). For papers that study entry in FPTP under strategic voting see, e.g., Palfrey (1989), Besley and Coate (1997), and Patty (2006). For models of differentiation and entry in industrial organization, see d'Aspremont, Gabszewicz, and Thisse (1979), Shaked and Sutton (1982), and Perloff and Salop (1985).

[5]Reality, of course, in somewhere in between these two polar assumptions. For a model exploring this tradeoff see Banks (1990) and Callander (2008)

candidacies, addressing more directly the issue of party formation.[6]

A second group of papers analyzes how variations in the electoral system affect policy outcomes. Myerson (1993b) focuses on how the nature of electoral competition affects promises of redistribution made by candidates in the election. Building on this work, Lizzeri and Persico (2001) consider redistribution and provision of public goods in PR and FPTP electoral systems. In both papers, the emphasis is not on differentiation (in ideology or quality) but rather on the vote-buying strategies of the candidates. Austen-Smith and Banks (1988) and Baron and Diermeier (2001) consider models of elections and legislative outcomes in PR, were rational voters anticipate the effect of their vote on the bargaining game between parties in the elected legislature. In these papers, however, the number of parties is exogenously given. Finally, several recent papers consider the effects of alternative electoral systems and strategic voting when the relevant policy outcome is not bargaining over a fixed prize, but instead taxation and redistribution (e.g., Austen-Smith (2000) and Persson, Roland, and Tabellini (2003)), or corruption (e.g., Myerson (1993a) and Persson, Tabellini, and Trebbi (2006)). In particular, Myerson (1993a) considers a model where potential candidates are known to differ in their level of corruption (which all voters dislike) but also in a second policy dimension, over which there is disagreement among voters. Myerson (1993a) concludes that a PR electoral system is more effective in reducing the probability of selecting a corrupt candidate than a FPTP system. It is interesting to note that - interpreting the investement in quality in our model as an endogenous choice of the level of corruption - our model yields the opposite result. The reason is that in Myerson (1993a) the level of corruption is an exogenous characteristic of electoral candidates. Together with strategic voting, this assumption is enough to guarantee the existence of an equilibrium in FPTP where exactly two corrupt candidates tie, even if non corrupt alternatives are available to voters. This cannot occur in a PR system, where voting sincerely for non corrupt candidates is a dominant strategy. In our model, candidates' quality (or corruption level) is endogenous and candidates choose quality for a given set of electoral competitors. As a result the winner-takes-all nature of FPTP elections provides the strongest incentive to invest

[6] See also Cox (1997) for an empirical discussion of the Duvergerian predictions.

in quality (reduce the level of corruption) as compared to PR electoral systems.[7]

Our paper is also related to the large literature that, following Stokes (1963)'s original critique to the Downsian model, incorporates competition in *valence* issues, typically within FPTP, and with a given number of candidates (two). For recent papers see Ashworth and Bueno de Mesquita (2007), Carrillo and Castanheira (2006), Eyster and Kittsteiner (2007), Herrera, Levine, and Martinelli (2008), and Meirowitz (2007).[8] Of these, the closest paper to ours is Ashworth and Bueno de Mesquita (2007). They show that in FPTP elections with two candidates, candidates have an incentive to "diverge" in order to soften valence competition. Although this effect is also present in our model for PR elections, this does not occur in our set up in FPTP elections, since here the set of candidates is endogenous, candidates are endowed with fixed policy positions, and voters are strategic.

# 3 The Model

Let $X = [0, 1]$ be the ideology space. In any $x \in X$ there are at least two potential candidates, each of whom will perfectly represent policy $x$ if elected. There are three stages in the game. In the first stage, all potential candidates simultaneously decide whether or not to run for office. Potential candidates only care about the spoils they can appropriate from being in office, and must pay a fixed cost $F$ to participate in the election. We denote the set of candidates running for office at the end of the first stage by $\mathcal{K} = \{1, \ldots, K\}$. In a second stage, all candidates running for office simultaneously invest in quality $\theta_k \in [0, 1]$. Candidates can acquire quality $\theta_k$ at a cost $C(\theta_k)$, $C(\cdot)$ increasing and convex. We let $C(1) \equiv \overline{c}$ and - to allow competitive elections in all electoral systems - we assume that $F + \overline{c} \leq \frac{1}{2}$. In the third stage, $n$ fully strategic voters vote in an election, where we think as $n$ being a large finite number. A voter $i$ with ideal point $z^i \in X$ ranks candidates according to the utility function $u(\cdot; z^i)$, which assigns to candidate $k$ with characteristics $(\theta_k, x_k)$ the payoff $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$, with $v$ increasing and concave. The parameter $\alpha$ captures

[7]We further explore this point in Section 5.3, where we consider the case in which quality cannot be acquired but instead is an innate characteristics of the candidates.

[8]See also Groseclose (2001), Aragones and Palfrey (2002), Schofield (2004), and Kartik and McAfee (2007) for models where one candidate has an exogenous valence advantage.

voters' responsiveness to quality. Voters' ideal points are uniformly distributed in $[0, 1]$.[9]

The electoral system determines the mapping from voting profiles to policy outcomes and the allocation of rents. In FPTP the candidate with a plurality of votes appropriates all rents from office and implements the policy he represents. In PR systems, instead, the policy outcome is a lottery between the ideologies championed by the candidates participating in the election, with weights equal to their vote shares in the election (or seat share in the assembly). The (expected) share of rents captured by each candidate is also proportional to his vote share in the election. Let $s_k$ denote the proportion of votes for party $k$, $m_k$ the proportion of rents captured by party $k$. Also, let $\theta_{\mathcal{K}} \equiv \{\theta_k\}_{k\in\mathcal{K}}$, and $x_{\mathcal{K}} \equiv \{x_k\}_{k\in\mathcal{K}}$ denote the quality and policy positions of the candidates running for office. Normalizing total political rents in both systems to one, the expected payoff of a candidate $k$ running for office in electoral system $j$ can then be written as

$$\Pi_k^j(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = m_k^j(\theta_{\mathcal{K}}, x_{\mathcal{K}}) - C(\theta_k) - F. \tag{1}$$

We assume that $m_k^{PR}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = s_k(\theta_{\mathcal{K}}, x_{\mathcal{K}})$, and that in FPTP ties are broken by the toss of a fair coin, so that letting $H \equiv \{h \in \mathcal{K} : s_k = s_h\}$,

$$m_k^{FP}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = \begin{cases} \frac{1}{|H|} & \text{if } s_k \geq \max_{j\neq k}\{s_j\} \\ 0 & \text{o.w.} \end{cases}$$

A strategy for candidate $k$ is a running decision $e_k \in \{0, 1\}$, and a quality decision $\theta_k(\mathcal{K}, x_{\mathcal{K}}) \in [0, 1]$. A strategy for voter $i$ is a function $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) \in \mathcal{K}$, where $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = k$ indicates the choice of voting for candidate $k$, and $\sigma = \{\sigma_1(\cdot), \ldots, \sigma_N(\cdot)\}$ denotes a voting strategy profile. An *electoral equilibrium* is a Subgame Perfect Nash Equilibrium in pure strategies of the game of electoral competition, i.e., a strategy profile such that *(i)* voters cannot obtain a better policy outcome by voting for a different candidate in any voting game (on and off the equilibrium path), *(ii)* given the location and quality decisions of other candidates, and given voters' voting strategy, candidates cannot increase their expected rents by modifying their

[9]The choice of a uniform distribution for voters' ideal points is not crucial for our results, but simplifies considerably the analysis of PR elections.

investment in quality, *(iii)* candidates running for office obtain non-negative rents, and *(iv)* candidates not running for office prefer not to enter: would obtain negative rents in an equilibrium of the continuation game.

An *outcome* of the game is a set of candidates running for office $\mathcal{K}$, policy positions $x_{\mathcal{K}}$, and quality choices $\theta_{\mathcal{K}}$. A *polity* is a triplet $(\alpha, \overline{c}, F) \in \Re^3_+$. We say that the model *admits* an electoral equilibrium with outcome $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$ if there exists a set of polities $P \subseteq \Re^3_+$ with positive measure such that if a polity $p \in P$, then there exists an electoral equilibrium with outcome $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$.

# 4 The Basic Comparison of Electoral Systems

In this section we state our main result regarding the comparison between alternative electoral systems (Theorem 1). We show that the maximum quality among all candidates running for office in PR elections is always at most as high as the quality of any candidate running for office in majoritarian elections, and that in all equilibria in which candidates are ideologically differentiated, the number of candidates running for office is at least as large as in majoritarian electoral systems. We show, moreover, that under mild conditions these rankings are in fact strict; i.e, for a relatively *large* set of parameters, PR leads to more candidates, each with strictly less quality, than FPTP.

We begin our analysis by considering majoritarian/ FPTP electoral systems. We show that in any electoral equilibrium in FPTP elections, the number of candidates running for office and their choice of quality is uniquely determined. Moreover, although there are multiple equilibria (in fact a continuum), these equilibria can be ranked according to a utilitarian social welfare function. The following result characterizes equilibria in FPTP elections.

**Proposition 1** *Consider elections in FPTP electoral systems. An electoral equilibrium always exists. In any equilibrium in which candidates represent different ideological positions: (i) exactly two candidates compete for office, (ii) candidates are symmetrically located around the median in the policy space (i.e.,* $x_1 = 1 - x_2$*), and (iii) they choose maximal quality (i.e.,* $\theta_1^* = \theta_2^* = 1$*).*

Proposition 1 shows that in our setting, Duverger's law holds in almost all electoral equilibria. Although many candidates can run for office, majoritarian elections trim down competition between differentiated candidates to two top quality candidates. The degree of ideological differentiation between candidates, however, is not pinned down by equilibrium: FPTP elections admit both the Pareto optimal equilibrium with two centrist candidates, and an equilibrium in which candidates are maximally polarized (as well as any symmetric configuration). The centrist two-candidates equilibrium is efficient because of the concavity of voters' preferences over policy. This also implies that these continuum of symmetric equilibria can be ranked in terms of aggregate voters' welfare, with equilibria in which candidates are less polarized dominating those in which candidates are more polarized.[10] For some parameter values, there also exists an equilibrium in which more than two *perfectly centrist* (and in all respects identical) candidates run for office.

To see the intuition for the result, note first that given the winner-takes-all nature of FPTP elections, all candidates running for office must tie in equilibrium. From this it follows that *(a)* voters must vote sincerely, and that *(b)* candidates must be choosing maximal quality. Given that voters are uniformly distributed in $[0, 1]$, these facts also imply that *(c)* in any equilibrium, the set of candidates running for office must be symmetrically located with respect to the median ideological position. To see that there cannot be an electoral equilibrium with $K > 2$ differentiated candidates running for office, note that if this were the case, *(a)* and *(b)* imply that by deviating and voting for any candidate $j$ other than her preferred candidate, a voter could get candidate $j$ elected with probability one. Revealed preference from equilibrium therefore implies that this voter must prefer the lottery among all $\mathcal{K}^*$ candidates running for office to having $j$ elected for sure. But voters' risk-aversion and *(c)* imply that any voter must prefer a centrist candidate (i.e., located at the median) to the equilibrium lottery. As a result, any voter must also prefer a centrist candidate to any other candidate that

[10]To see this notice that when $x_1 = 1 - x_2 = \frac{1}{2} - y$ for $y \in [0, \frac{1}{2}]$ and $\theta_1 = \theta_2 = 1$, aggregate welfare equals

$$\int_0^1 \left( \frac{1}{2} u\left(1, \frac{1}{2} + y; z_i\right) + \frac{1}{2} u\left(1, \frac{1}{2} - y; z_i\right) \right) dz_i = \alpha v(1) - \left( y^2 + \frac{1}{12} \right),$$

which is decreasing in $y$ and maximized at $y = 0$ or $x_1 = x_2 = \frac{1}{2}$.

is not her most preferred choice, and in particular a candidate with an ideological position that is between the median and her most preferred ideological position. But this leads to a violation of single-peakedness, which is not consistent with the assumption of a strictly concave utility function.[11] As a result, in equilibrium we must have exactly two symmetrically located candidates chosing maximal quality.

In the proof we show that such an equilibrium exists, and in fact that there is a continuum of two-candidate symmetric equilibria, with candidates chosing maximal quality. The fact that the set of FPTP electoral equilibria cannot be further restricted follows from strategic voting, and from the fact that in our setting, potential candidates care only about the spoils they can appropriate from being in office. To see how strategic voting operates in this context, suppose that following entry of a third, out of the equilibrium path candidate, all voters vote for their preferred candidate among the two equilibrium candidates. Given this voting behavior in the population, any voter $i$ would be better off voting for her preferred candidate among the two equilibrium candidates rather than supporting the entrant. Even if the entrant were voter $i$'s most preferred candidate, voting for the entrant would only cause her least preferred equilibrium candidate to win the race for sure. The assumption that candidates do not care about the policies per se, on the other hand, rules out situations where potential candidates may choose not to run for office simply because another candidate championing a "close" ideological position is already running.[12]

We consider next elections within PR electoral systems. The characterization of equilibria in PR leads to dramatically different results with respect to the FTPT case. We begin by establishing a useful lemma.

**Lemma 1** *In (any voting subgame of) any electoral equilibrium in PR elections, voters vote sincerely.*

Recall that in PR each candidate running for office is elected and implements

[11] Feddersen, Sened, and Wright (1990) use a similar argument in a pure private values model, in which candidates decide both whether to enter or not and which policy position they will represent.

[12] Note that without this assumption perfect convergence in FPTP elections cannot be supported in equilibrium (see the "citizen-candidate" models of Osborne and Slivinski (1996) and Besley and Coate (1997)). Stated differently, assuming policy-motivated candidates immediately implies some policy divergence in equilibrium. We show, however, that even ruling out these externalities, convergence cannot be supported in equilibrium in PR elections.

his ideology with a probability proportional to the share of votes received in the election. As a consequence, when a voter votes for a certain candidate, she is affecting the lottery among all candidates running for office by increasing the weight on that particular candidate's position. But this implies that voting for a candidate other than the most preferred one is always a strictly dominated strategy. In fact, by switching her vote to his most preferred candidate, a voter only affects the lottery's weights of exactly two candidates. But we know that with two alternatives strategic voting and sincere voting coincide. The fact that strategic or sophisticated voting boils down in PR to sincere voting greatly simplifies the characterization of electoral equilibria. In a nutshell, sincere voting assures uniquely determined, smooth and well behaved vote share functions for all candidates on and off the equilibrium path.

We can now state our main results for PR elections. First, we show that for a *large* set of parameters there exists an electoral equilibrium in PR elections in which more than two candidates run for office choosing non-maximal quality. Moreover, we show that PR elections do not admit electoral equilibria in which different candidates represent the same policy; i.e., these equilibria can exist only for a set of parameters of measure zero.

**Proposition 2** *PR elections (i) admit electoral equilibria in which more than two candidates run for office choosing non-maximal quality, and (ii) do not admit electoral equilibria in which two or more centrist candidates run for office.*

To prove this result we construct equilibria of a simple class - that we call Location Symmetric (LS) equilibria - in which candidates are equally spaced in the ideological dimension.[13] In particular, we show that for a LS equilibrium with $K \geq 3$ candidates choosing non-maximal quality to exist it is sufficient that *(i)* the responsiveness of voters to quality is not too high (i.e., $\alpha < \overline{\alpha}(K) \equiv \frac{1}{2K}\frac{C'(1)}{v'(1)}$), that *(ii)* the fixed cost of running for office is always larger than the cost of acquiring quality (i.e., $F > \overline{c}$), and that *(iii)* the fixed cost of running for office is not too low (to deter entry) or too high (for nonnegative rents); i.e., $\frac{1}{2K} < F < \frac{1}{K} - \overline{c}$. Note in particular that we can

[13]In Iaryczower and Mattozzi (2008) we show that the class of LS equilibria is relatively simple to analyze, since whenever rents cover variable costs, first order conditions in the quality subgame completely characterize best response correspondences.

support equilibria with an increasingly larger number of candidates given sufficiently lower costs of running for office and of investing in quality, and a sufficiently smaller responsiveness of voters to quality - equivalently, a sufficiently larger *ideological focus* of voters (Stokes (1963)).

The number of candidates running for office and the degree of ideological differentiation between candidates are determined in equilibrium by two opposing forces. First, in any electoral equilibrium in PR, candidates must be sufficiently differentiated in the ideological spectrum, because of the basic tension that emerges in our model between quality and differentiation in policies: the closer candidates are in terms of their ideological position, the larger is the number of voters that can be attracted by a given increase in quality by one of the candidates. This implies in turn that candidates will be more aggressive in the game of quality competition the closer they are to one another, eventually competing away their rents. Candidates that are sufficiently differentiated in the ideological dimension, instead, are not close substitutes for voters. In this case, PR leads to low powered incentives, quality competition is relaxed, and candidates running for office can choose non-maximal quality while still getting a positive share of office rents in equilibrium. Second, the limit to the degree of horizontal differentiation among candidates is entry: candidates cannot be too differentiated in PR elections without triggering entry of an additional candidate, who would be able - given sincere voting in the electorate - to attain the support of a sufficiently large niche of voters. The same logic implies in fact that PR elections do not admit an electoral equilibrium in which two or more perfectly centrist candidates run for office. If all candidates running for office are centrist, it is always possible for a candidate representing a policy position close to the median to run for office, capturing almost half of the votes. Since the centrist candidates were making non-negative rents in the proposed equilibrium, the entrant's expected payoff from running must be positive as well, and there is no way to deter his entry. As a result, the Pareto efficient equilibrium in FPTP cannot be supported in PR.[14]

Combining the results of Proposition 2 together with our earlier results in Proposition 1, we can conclude that quality is always at least as large, and the number of differentiated candidates at most as large, in FPTP elections as in PR elections, and

[14]More precisely, it can only be supported for a set of parameters of measure zero.

that there exists a large set of parameters for which these inequalities are strict. The next theorem, which follows as a corollary of Propositions 2 and 1, summarizes the comparison.

**Theorem 1**

(1) *In any admissible electoral equilibrium under PR, the quality of any candidate running for office is at most as high as in any admissible equilibrium in FPTP, and the number of candidates running for office is at least as large as in any admissible equilibrium in FPTP in which candidates are differentiated.*

(2) *PR elections admit electoral equilibria in which the number of candidates running for office is (strictly) larger, and the quality of any candidate running for office is (strictly) lower, than in any admissible equilibrium in FPTP elections in which candidates are differentiated.*

We conclude this section by suggesting a possible welfare comparison between electoral systems. Given the multiplicity of equilibria under both systems, we confine our comparison to be between the most efficient equilibrium in terms of aggregate voters' welfare in FPTP, which we label $\tilde{\Gamma}^{FP}$, and the most efficient equilibrium in PR, which we label $\tilde{\Gamma}^{PR}$. Then we have:

**Proposition 3** $\tilde{\Gamma}^{FP}$ *dominates* $\tilde{\Gamma}^{PR}$ *in terms of aggregate voters' welfare.*

Note that if we consider the class of LS equilibria under PR, the welfare comparison comes as an immediate corollary of our previous results. In fact, we already know that it is not possible to have convergence in PR elections and, given the same level of quality, concavity of voters' preferences implies that any voter strictly prefers the *expected candidate* with ideological position corresponding to the expected value of the equilibrium lottery to the lottery itself. The result follows from noticing that in any LS equilibrium the expected candidate is in fact centrist, and in the most efficient equilibrium in FPTP all candidates running for office are centrists. However, the result of Proposition 3 holds more generally for any electoral equilibrium in PR. To see this, note that first that for *any* equilibrium in PR, any voter prefers the expected candidate of the equilibrium lottery to the lottery itself. If this expected candidate is centrist, we are done. Otherwise, by concavity of voters' preferences, a centrist candidate will always be preferred by a majority of voters to the expected candidate.

# 5 Discussion and Extensions

In this section, we consider several extensions of our main model. We begin in Section 5.1 by considering a modified version of PR elections, in which the candidate with the largest number of votes obtains a majority premium in both the probability with which his policy is implemented and in the proportion of office rents he attains after the election. In Section 5.2, we consider alternative specifications of the policy function mapping elected representatives to policy outcomes. In Section 5.3, we consider a variant of the main model that allows us to compare PR and FPPT when the relevant aspects of candidates' quality can not be easily modified or acquired exerting effort, but instead are innate characteristics of the candidates. In Section 5.4 we introduce thresholds for representation in PR elections. In Section 5.5 we introduce multiple districts (and district magnitudes) in PR and FPTP. Finally, in Section 5.6, we consider a possible source of strategic uncertainty for voters, introducing behavioral voters who do not vote strategically.

## 5.1 A Majority Premium in PR

We have assumed up to now that in PR elections each candidate running for office captures a proportion of office rents equal to his share of votes in the election. In various political systems, however, it might be reasonable to expect that the majority party can obtain an additional reward over and above its share of votes in the election. In several parliamentary democracies adopting some form of PR, for instance, the *formateur* is typically the head of the majority party.[15] To capture this feature, we consider next a modified version of the model, in which the majority candidate is elected and captures all the rents from office with a probability more than proportional to his vote share. In particular, we assume that the candidate with the largest number

[15]In Greece, for example, the fact that the *formateur* has to be the head of the majority is mandated by law.

of votes obtains a majority premium $\gamma \in (0,1)$ in both the probability with which his policy is implemented and in the proportion of office rents he attains after the election. We call this new environment PR-plus (PRP). In PRP, letting as before $H \equiv \{h \in \mathcal{K} : s_k = s_h\}$, $k$'s proportion of office's rents after the election, $m_k$, is given by

$$m_k = \begin{cases} s_k(1-\gamma) + \frac{\gamma}{|H|} & \text{if } s_k \geq \max_{j \neq k}\{s_j\}, \\ s_k(1-\gamma) & \text{o.w.} \end{cases} \tag{2}$$

PRP can then be thought of as an intermediate electoral system between PR ($\gamma = 0$), and FPTP ($\gamma = 1$). The next proposition characterizes PRP elections in large electorates. We show that in large electorates there exists an electoral equilibrium with two top quality candidates, symmetrically located around the median voter, provided that the candidates are not too polarized. We also show that for any majority premium $\gamma$, in large elections electoral equilibria are either of this kind, or such that a single candidate appropriates the majority premium with certainty.

**Proposition 4**

*(1) There exists $\overline{n}$ such that for all $n \geq \overline{n}$, there is an electoral equilibrium in which two top quality candidates, symmetrically located around the median voter, run for office.*

*(2) Fix any sequence of equilibria $\{\tilde{\Gamma}_n\}_{n_0}^{\infty}$. There exists $\overline{n}$ such that if $n \geq \overline{n}$, then in $\tilde{\Gamma}_n$, either two top quality, symmetrically located candidates run for office, or a single candidate appropriates the majority premium with certainty.*

The main intuition for the existence of equilibria with two top quality candidates is that for any majority premium $\gamma$, the strategic problem of individual voters in PRP resembles - for sufficiently large electorates - the analogous problem in FPTP. As a result, we can support an equilibrium with two candidates, 1 and 2, by having voters coordinate on voting for their preferred choice among these candidates, even after entry of a third candidate $\ell$. To see this, consider without loss of generality a voter $i$ with preferences $\ell \succ_i 1 \succ_i 2$ (note that we only need strategic voting among voters whose preferred candidate in $\{1, 2, \ell\}$ is the entrant, $\ell$). Voter $i$ faces the following tradeoff. On the one hand, by switching to vote sincerely in favor of the entrant, the voter is transfering $1/n$ probability mass from her second best candidate ($k = 1$) to her most preferred candidate ($\ell$). On the other hand, she is also inducing a jump of $\gamma/2$ in the probability that the policy of her least favorite candidate in $\{1, 2, \ell\}$ emerges as the policy outcome, to be "financed" by a parallel decrease in the probability of her second best candidate's policy being chosen. For large $n$, the second effect dominates, and $i$ has incentives to vote strategically.[16]

The previous result should not be interpreted as implying a complete discontinuity with the PR environment. Note that for fixed $n$, and given a strategy profile for all other voters, the incentive to vote strategically in the way described above increases monotonically in the majority premium $\gamma$, and in the polarization of candidates 1 and 2: for any strategy profile of the remaining voters, if $i$ has an incentive to vote strategically given some $\gamma$, then $i$ also has an incentive to vote strategically given $\gamma' > \gamma$. Similarly, if $i$ has an incentive to vote strategically for some given degree of ideological differentiation between candidates 1 and 2, then $i$ also has an incentive to vote strategically for a larger payoff differential among equilibrium candidates. In fact, it is easy to see that if candidates running for office are not differentiated at all, then there cannot be strategic voting of this type, as in this case supporting the preferred candidate $\ell$ comes at not cost. But this implies that there cannot be electoral equilibria with perfect convergence in PRP. On the other hand, in general candidates cannot be too polarized either, for otherwise a deviation by one of the candidates to lower quality, forgoing the majority premium, can be profitable for sufficiently small $\gamma$. All in all, while equilibrium behavior of voters and candidates in PRP can resemble behavior in FPTP, the set of equilibria of this class has to be pruned to rule out complete convergence and under some conditions also extreme polarization.

At this point, a natural question to ask is whether equilibria with more than two candidates running for office choosing non-maximal quality - which we have shown can be supported in equilibrium in PR - can survive in the case of PRP elections. The answer is yes, provided that the size of the majority premium is not too big. To see this, note first that whenever a candidate is ahead by at least two votes in a PRP

[16]The intuition for the second part of the proposition follows along the same lines, and is only slightly more involved.

election, strategic voting must be sincere, since in this case any individual deviation in the voting strategy cannot affect the identity of the majority candidate. With this result in mind, consider a location symmetric equilibrium in PR ($\gamma = 0$) such that three candidates run for office choosing non-maximal quality, and the centrist candidate obtains the sincere vote of slightly more than a third of the electorate. Consider now the case of a positive but small majority premium $\gamma$, fixing all other parameters of the model. From our previous remark, sincere voting remains a best response when other voters vote sincerely. Moreover, with small enough $\gamma$, winning a plurality of the vote is not worth a deviation from the optimal quality choice in the pure PR environment. Finally, note that if the entry of a forth candidate was not profitable in the case of $\gamma = 0$, this has to be true also in the case of a small majority premium. In fact, it is enough for this that when $\gamma = 0$, the equilibrium candidates' rents in the continuation game following entry are strictly positive, but we know that this will be the case generically.

## 5.2 From Representation to Policy Outcomes

The central element of any model of elections is the mapping from votes in the electorate to a set of elected representatives. With fully rational and strategic voters, however, a second element of the model becomes equally important. In order for rational voters to be able to link their vote choices to payoffs, we need to provide them with a mapping from the characteristics of the set of elected representatives to final policy outcomes. In this paper, we have maintained the simplifying assumption that the policy outcome in PR comes about as the realization of a lottery between the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly). This assumption attempts to capture, in a stylized manner, the added uncertainty for voters introduced by the process of post-election bargaining in PR. Introducing the lottery assumption, however, has important consequences for the analysis of electoral equilibria: with this assumption, all voters find voting for their most preferred candidate to be a dominant strategy, and thus sincere voting is rational on and off the equilibrium path (an equilibrium in every voting subgame). This, in turn, produces demand functions for candidates that are uniquely determined, continuous, and well behaved, on and off the equilibrium path, greatly simplifying the analysis of electoral equilibria.[17] In this section we show that our main results hold under alternative specifications of the policy function mapping elected representatives to policy outcomes, and therefore do not depend on this assumption in any crucial way.

First, within the lottery framework, it is apparent that there is nothing special about the weights being exactly equal to election shares, or even linear functions of the shares. In fact, any lottery in which the weights are a nondecreasing, anonymous/symmetric function of the election shares would leave all results unchanged. More generally, any alternative mechanism inducing sincere voting will lead to the same results. We show next, moreover, that Proposition 2 and Theorem 1 hold unchanged under a simple, non-stochastic protocol for the determination of policy in the elected assembly, which does encourage voters to vote strategically under some conditions.

**Remark 1** *Suppose that the policy outcome is* $(x_{\tilde{k}}, \theta_{\tilde{k}})$*, where* $\tilde{k}$ *is the smallest* $k$ *such that* $\sum_{j=1}^{k} s_j \geq \frac{1}{2}$*; i.e.,* $x_{\tilde{k}}$ *is the median policy of all elected representatives (where a party with a share* $s_k$ *of seats and policy* $x_k$ *is assumed to be equivalent to a mass* $s_k$ *of individuals representing policy* $x_k$*). Call this the median protocol for policy determination. Then PR elections admit an electoral equilibrium in which more than two candidates run for office choosing non-maximal quality. Moreover, any candidate strategy profile that can be supported in a LS equilibrium in the benchmark PR elections can still be supported as an equilibrium with the median protocol without using weakly dominated strategies.*

To see the intuition behind this remark, first note that on the equilibrium path of a LS equilibrium, sincere voting is a rational voting strategy profile. In a LS equilibrium with $K \geq 3$ candidates, extreme candidates can never become the median legislator, and all non-extreme candidates choose equal quality $\theta^*$. Since voters have single-peaked preferences in the ideological dimension, this implies that voters have single-peaked preferences among all relevant options. As a result, any voter $i$ can never gain by not voting for her preferred candidate: either her deviation produces

[17] At the same time, this constraints the set of equilibria, and as a consequence stacks the deck, if anything, against Theorem 1

no change in the median (if $i$ votes for any candidate on the same side of the median in the ideological space) or produces a detrimental change in the outcome (if $i$ votes for a candidate on the opposite side of the median in the ideological space).

If the profile of candidates' quality is not symmetric, however, as would occur off the equilibrium path following deviations by an equilibrium candidate in the quality competition stage (or in the quality continuation game after entry of a non-equilibrium candidate) it is possible for voters to vote strategically. To see this, consider three candidates, 1, 2 and 3, such that $x_1 < x_2 < x_3$, and suppose that $\theta_1 > \theta_2 = \theta_3$. Then some voter $i$ who would rank candidates $3 \succsim_i 2 \succsim_i 1$ on a purely ideological dimension could possibly rank candidates $1 \succsim_i 3 \succsim_i 2$ when taking into consideration both their ideology and quality, leading to a non-single-peaked preference profile (this requires of course the quality differential to be sufficiently high given the responsiveness of voters to quality, $\alpha$). In this circumstance, our previous analysis of the rationality of the sincere voting profile would not necessarily apply: if $i$ is decisive for the median between 1 and 2 she would prefer to select 1, so sincere voting is rational for $i$. But if $i$ were decisive for the median among candidates 2 and 3, then $i$ would find it optimal to deviate from sincere voting and to vote for 3.[18]

While the previous argument shows the potential for strategic voting in this setting, it does not imply that sincere voting will not be rational (not a best response for all voters). In the appendix we show, in fact (proof of Remark 1), that *(i)* sincere voting is rational in any voting subgame following a deviation in quality by an equilibrium candidate in a LS equilibrium, and moreover that *(ii)* for every deviation at the entry stage, there is an equilibrium in the continuation voting subgame in which either all or all but a small number of voters vote sincerely, and for which out of equilibrium entry is not sequentially rational.

To sum up, we have shown that our main results hold under alternative specifications of the policy function mapping elected representatives to policy outcomes, and therefore are not driven by our assumption that policy outcomes are determined as the result of a lottery among elected representatives. In particular, any alternative mech-

[18] Similarly, $i$ could also rank candidates as $3 \succsim_i 1 \succsim_i 2$ when taking into consideration both their ideology and quality, and in this case $i$ would prefer to deviate from sincere voting if she is decisive for the median between candidates 1 and 2.

anism inducing sincere voting will leave Proposition 2 and Theorem 1 unchanged. As the previous result shows, even alternative protocols for the determination of policy that do not lead to sincere voting being rational in all continuation games can still preserve the main results. The logic of entry deterrence in proportional representation works easily with sincere voting but does not require it.

## 5.3 Quality as an Innate Characteristic: A Model of Candidate Selection.

In our benchmark model, we assume that candidates are endowed with an ideological position and choose first whether to run or not for office, and if so then how much to invest in developing an attribute valued by all voters alike (quality). Under some circumstances, however, other closely related games - in which either the set of choices available to the candidates or the sequence in which candidates choose actions are different - can better serve us to understand the effects of alternative electoral systems. This could be the case, for example, if what it is relevant to voters is not the quality of the candidate's platform or the competence of his staff but instead innate characteristics of the candidates which can not be easily modified or acquired by investing effort or money (i.e., valence).

In this section we consider a variation of the benchmark model that allows us to explore this possibility. We assume that candidates do not choose quality after entering electoral competition, but instead are endowed with both an ideological position they represent and a level of quality. In particular, we assume that the set of candidates coincides fully with the set of policies; i.e., that there is one candidate representing each point in the quality-ideology space, and that higher quality candidates have a higher opportunity cost of running for office. To make the results comparable to the benchmark *choice* model, we represent the opportunity cost of types in the alternative *selection* model with the same same cost function $C(\cdot)$ of the benchmark model. The action space of candidates is therefore restricted to a decision of whether to run or not for office.

How does this change the comparison between electoral systems? Consider first FPTP elections. It is easy to see that any configuration of candidates' characteristics that can be supported as an equilibrium of the choice model in FPTP elections, can also be supported as an equilibrium of the selection model (by suitably coordinating voters' behavior upon entry). However, the converse is not true. Clearly, some properties must hold irrespective of timing. First, the winner-takes-all nature of FPTP elections implies that potential candidates will run for office only if they have a strictly positive expected probability of winning. Second, in any equilibrium in which candidates are differentiated, only two candidates can run for office (the argument used in the proof of Proposition 1 builds on deviations by voters for a given set of candidates, and can therefore be applied in this case as well). These properties imply that voters must vote sincerely between the two candidates running for office on the equilibrium path, and therefore that these candidates must be symmetrically located around the median voter. Contrary to the choice model, however, every symmetric configuration of candidates (in both location and quality) can also be supported as an equilibrium of the selection model. In this alternative timing specification - a simultaneous game of entry - strategic voting is effective in deterring entry of any third candidate irrespectively of his quality level (the analogy with the intuition behind Myerson (1993a)'s result is immediately apparent).

Consider now location symmetric equilibria in PR. First, note that Lemma 1 still applies, so that voting is sincere on and off the equilibrium path in all equilibria. Second, note that if there is a candidate running for office with policy position $x_k$ and $\theta_k < 1$, who obtains strictly positive rents, i.e., $\Pi_k^{PR}(\theta_k, \theta_{-k}, x_{\mathcal{K}}) > 0$, then also $\Pi_k^{PR}(\theta_k', \theta_{-k}, x_{\mathcal{K}}) > 0$ for an alternative candidate with identical ideological position $x_k$ and higher quality $\theta_k' > \theta_k$. Therefore in any LS equilibrium of the selection model, either $\theta_k = 1$ or $\Pi_k^{PR}(\theta_k, \theta_{-k}, x_{\mathcal{K}}) = 0$ for all $k \in \mathcal{K}$. Now, if in a LS equilibrium of the choice model with $|\mathcal{K}| \geq 3$, candidates are sufficiently differentiated so that (interior) candidates would obtain positive rents even choosing maximal quality (i.e., $x_k - x_{k-1} \equiv \Delta > \bar{c} + F$), then in the selection model all interior candidates must be top quality candidates.[19] If, on the other hand, in a LS equilibrium of the choice model

[19]We have two possible scenarios: either *(i)* in the equilibrium of the choice model candidates choose maximal quality, in which case the same thing must be true in the selection model, or *(ii)* in the choice model candidates choose a non-maximal level of quality $\theta^* < 1$, in which case for any ideological position $x_k$, rents must be positive for $\theta_k \in [\theta^*, 1)$, which implies that in the selection model $\theta_k = 1$ for all interior candidates. If also $\Delta < 1/K$, extreme candidates must be top quality candidates too, for $\Delta < 1/K$ implies that extreme candidates obtain higher rents than interior candidates.

$K \geq 3$ and $\Delta < \bar{c} + F$, so that candidates choosing maximal quality would obtain negative rents, then non-maximal quality would be the equilibrium outcome of both the choice and the selection model. If candidates obtain no rents at the equilibrium quality $\theta^*$ in the choice model, then $\theta^*$ must also be the equilibrium quality of the selection model; if instead candidates obtain positive rents in equilibrium in the choice model, then the equilibrium of the selection model will be characterized by a higher level of quality with respect to the choice model.[20]

Two main conclusions follow. First, regarding the number of equilibrium candidates, our results still hold in the selection model studied in this section. Second, regarding the quality of candidates/policies, the selection model allows mediocre candidates to run for office in FPTP, and leads to higher quality candidates than the choice model in PR, reversing to some extent the stark comparison of the choice model. The driving force behind this result is that the selection model introduces more competition among candidates in PR elections. In fact, given sincere voting on and off the equilibrium, if a "mediocre" candidate is not completely dissipating his rents in an equilibrium of the selection model in PR, a better candidate would find it profitable to run for office as well. This is not the case in the quality choice model, where the quality competition takes place among a given set of candidates running for office. On the other hand, in the selection model under FPTP, strategic voting can prevent the entry of any third candidate irrespectively of his quality level, as in Myerson (1993a).

It is important to put in perspective the differences in results between the choice and the selection model in light of the different implications of the two underlying environments. The benchmark choice approach is more appropriate to model attributes that can be affected by candidates, either deploying resources or exerting effort. This, we believe, is the natural setting to think of factors leading to corruption or inefficiencies in the implementation of policies. The selection model seems better suited to represent situations in which voters care fundamentally about innate attributes of candidates that cannot be affected by money or effort. The results of this section

[20] Note that in this case the level of quality in the equilibrium of the selection model will be still not maximal since rents are decreasing in $\theta_k$ for $\theta_k \in [\theta^*, 1)$, and negative at $\theta_k = 1$, so must be zero at some $\overline{\theta}_k < 1$. In Iaryczower and Mattozzi (2008) we show that PR elections admit equilibria such that $K \geq 3$ and $\Delta < \bar{c} + F$.

suggest that we should expect alternative electoral systems to have different (and possibly opposite) effects on the quality of policies rather than the quality or competence of politicians. In particular, in an environment where voters are strategic, a FPTP electoral system is more effective in selecting better policies, while a PR system can be more effective in selecting better politicians (and in fact is more effective in the worst equilibrium for voters).

## 5.4 Vote Thresholds

We consider next imposing a *vote threshold for representation* $\mu$ in PR elections, such that candidate lists must receive a proportion of at least $\mu > 0$ of the total vote to get any seats in the legislature.[21] It is immediate to see that introducing a threshold for representation in FPTP would not affect the equilibrium behavior of voters or candidates. In PR elections, instead, introducing a threshold of representation can have a large impact on electoral outcomes. First, even with all voters voting sincerely as in our benchmark PR elections, the threshold for representation has a direct impact on the size of the smallest party allowed to be represented in parliament, and thus (possibly) on the number of candidates competing for office. Most notably, however, introducing a threshold for representation allows strategic voting along the lines of FPTP elections, and it is this feature which can most significantly affect behavior in PR elections.

Consider, for example, a political environment in which voters are highly ideologically focused (small $\alpha$), and the fixed and variable costs of running for office are small (small $F$ and $\bar{c}$). In this political environment, the electoral equilibrium in the benchmark PR model would have a relatively large number of candidates running for office, say $K$. Introducing a sufficiently large threshold for representation (say larger than $1/K$) would destroy this equilibrium. With a smaller threshold, however, this equilibrium remains, but now the set of equilibria expands dramatically. First, it is now possible to support in PR elections the Pareto optimal FPTP equilibrium, with

[21] This is a common feature of many PR electoral systems. This threshold is five percent in Poland's Sejm, Germany's Bundestag, and New Zealand's House of Representatives, two percent in Israel's Knesset, and as high as ten percent in the Turkish parliament. Portugal, South Africa, Finland, and the Netherlands, on the other hand, are examples of PR systems without a threshold.

two top quality candidates representing the ideology most preferred by the median voter. Since the two candidates are perfect substitutes for voters, the low powered incentives typical of PR play no role here, and candidates have an incentive to choose maximal quality. The threat of entry that breaks this equilibrium in the benchmark PR model with no thresholds is ruled out here by coordinating voters' behavior so that an entrant would receive no support if such a deviation were to come about. This is entirely due to the threshold for representation, which allows a *spoiler* effect much as in FPTP to be in play. Second, it is also possible to support the worst possible PR equilibrium (the worst equilibrium for voters under any political environment in PR): two extreme parties running for office, representing the most outward positions in the ideological space, investing as low as possible in quality as it is consistent with electoral competition in PR. As before, entry is ruled out by coordinating voters' behavior away from a possible out of equilibrium entrant. In this case, however, competition on the equilibrium path is restricted to a contest among highly differentiated candidates, and as a result the low powered incentives of PR competition produce low quality candidates.

To sum up, we have shown that introducing a threshold of representation in PR can have a large impact on electoral outcomes, both directly, restricting the number and characteristics of candidates competing for office, and indirectly, through strategic voting. We argued that it is this latter feature which can most significantly affect behavior in PR elections, as it expands the set of electoral equilibria to include both the efficient outcome and the worst possible PR equilibrium, in which two extreme low quality candidates run for office. Since in all equilibria in FPTP elections competition is restricted to top quality candidates, it follows that the worst equilibrium in FPTP in terms of voters' welfare dominates the worst equilibrium in PR with a threshold of representation.

## 5.5 Multiple Electoral Districts

Up to now we have considered electoral systems in which all votes are aggregated in a single *electoral district*. Most electoral systems currently used to select representatives to a national legislature, however, admit several electoral districts, each selecting some members to the national legislature. In Argentina, for example, the election of the 257 members of the Camara de Diputados is ruled by a PR electoral system with twenty four electoral districts, of which ten select five members each, twenty select ten or less members each, twenty three select twenty five or less, and one (Buenos Aires) selects seventy members. Similarly, the US elects the 435 members of the House of Representatives by FPTP in 435 different electoral districts. We now extend the model to consider this feature, and show that this does not affect our main result.

Consider a PR electoral system with $D$ districts, $PR(D, \{\eta_d\}_{d=1}^D)$, where voters in district $d = 1, \ldots, D$ elect a share $\eta_d$ of representatives to a national legislature. We call $FPTP(D)$ an electoral system in which the voters in each one of $D$ districts elect one representative to a national legislature in a FPTP election. As before, we assume that the policy outcome in the legislature in both systems is the result of a seat-weighted lottery among selected representatives (a fair lottery in FPTP). Let $k(d)$ denote the $k_{th}$ candidate in district $d$. The expected payoff of a candidate $k(d)$ running for office in electoral system $j$ is still given by (1), with expected rents given by $m_{k(d)}^{PR} = s_{k(d)}\eta_d$ in PR with $D$ districts, and $m_{k(d)}^{FP(D)} = \frac{1}{D} m_{k(d)}^{FP(1)}$ in FPTP with $D$ districts.

Note first that the expected payoffs of candidates in $FP(D)$ are exactly the expected payoffs of candidates in FPTP (or here $FP(1)$) times a constant $(1/D)$ to reflect the fact that the candidate elected in district $d$ only obtains the office rents with probability $1/D$ (or, equivalently, $1/D$ of the rents). Second, note that voters have the same incentives as in FPTP. Given that in the second stage the policy outcome in the legislature is the result of a seat-weighted lottery among selected representatives, voters always prefer to select the same candidate that they would have selected in FPTP. But then the proof of Proposition 1 still applies, and all equilibria in each district are exactly as in Proposition 1. Exactly the same is true in PR elections with $D$ districts, and as a result, we have an immediate extension of Theorem 1 to the multiple district setting. First, in any admissible electoral equilibrium in $PR(D, \{\eta_d\}_{d=1}^D)$, the number of candidates running for office in any PR-district $d$ is (weakly) larger, and the quality of any candidate running for office is (weakly) lower than in any FP-district $d'$, in any admissible equilibrium in $FP(D')$ with differentiated candidates. Moreover, $PR(D, \{\eta_d\}_{d=1}^D)$ elections admit electoral equilibria such that the above comparison is strict.

### 5.6 Behavioral Sincere Voters

We have assumed throughout that voters face no strategic uncertainty. In this section, we consider a possible source of strategic uncertainty for voters, introducing behavioral voters who do not vote strategically. Suppose that with a small but positive probability $\xi$, each voter can be of a behavioral type, who votes sincerely for her preferred candidate, independently of both the electoral institutions and of the strategy profile of the remaining voters. How does this change affect our results? Note first that the existence of these sincere behavioral voters changes nothing in our benchmark PR elections, in which strategic voting is sincere. Consider then FPTP. Note that on the equilibrium path in an environment with $\xi = 0$ all voters vote sincerely, and therefore nothing changes here either. After entry, however, it is now possible for some voters to end up voting for the entrant. With a large electorate and a small $\xi$, however, the probability that a voter is pivotal between the entrant and one of the incumbents vanishes vis a vis the probability of her being pivotal between the two incumbents, and the strategy profile remains an equilibrium. A similar logic sustains the argument ruling out equilibria with more than two differentiated candidates in FPTP elections. Applying the same perturbation to PR elections with a threshold of representation establishes a lower bound (given $\xi$) for the threshold to support strategic voting in PR.

## 6 Conclusion

The contribution of this paper is to tackle jointly the effect of alternative electoral systems on the number, the quality, and the ideological diversity of candidates running for office. In doing so, we integrate a large and influential literature about the effect of alternative electoral systems on the number of candidates, on policy outcomes (for a fixed field of candidates), and on competition in *valence* issues, typically within FPTP. The joint approach that we pursue in this paper uncovers that these key political outcomes are themselves naturally intertwined.

The central result of the paper is a comparison between PR and FPTP electoral systems. We show that in any admissible electoral equilibrium under PR *(i)* the number of candidates running for office is larger, and *(ii)* the quality of any candidate running for office is lower, than in any admissible equilibrium in FPTP in which candidates are differentiated. Moreover, PR elections admit electoral equilibria in which the number of candidates running for office is (strictly) larger, and the quality of any candidate running for office is (strictly) lower, than in any admissible equilibrium in FPTP elections in which candidates are differentiated. We show that our main comparison also holds under alternative specifications of the policy function mapping elected representatives to policy outcomes, and in electoral systems with multiple electoral districts. We also consider a variant of the main model that allows us to compare PR and FPTP when the relevant aspects of candidates' quality can not be easily modified or acquired exerting effort but instead are innate characteristics of the candidates. The results suggest that we should expect alternative electoral systems to have different effects on the quality of policies and on the quality or competence of politicians: a FPTP electoral system is more effective in inducing candidates to offer better choices to voters, while a PR system can be more effective in selecting more attractive politicians. Finally, we show that introducing a threshold of representation in PR can have a large impact on electoral outcomes, both directly, restricting the number and characteristics of candidates competing for office, and indirectly, through strategic voting. It is this latter feature that can most significantly affect behavior in PR elections, expanding the set of electoral equilibria to include both the efficient outcome and the worst possible admissible equilibrium in PR, in which two extreme low quality candidates run for office.

Many interesting aspects remain to be addressed, and are left for future research. Possibly the most important among these is to better understand how strategic voters resolve coordination on alternative voting strategy profiles (on and off the equilibrium path). As we have seen, the behavior of voters is a key element in establishing the nature of the competitiveness of elections, and therefore on the quality of candidates and the set of policy positions represented in elections. At the same time, it is also the element about which we know the least.

# 7 Appendix

**Proof of Proposition 1.** Note first that in any equilibrium all candidates that are running for office must tie, since otherwise there would be at least one candidate who would lose for sure and - given the fixed cost of running for office $F > 0$ - would prefer not to run. Since candidates are tying, in equilibrium voters must vote sincerely. If this were not the case, there would exist some voter who is not voting for her most preferred candidate in equilibrium but who could have this candidate winning with probability one by deviating to voting sincerely. Third, note that in any equilibrium it must be that $\theta_k^* = 1$ for all $k \in \mathcal{K}^*$. In fact, since all candidates that are running for office must tie in equilibrium, if $\theta_h^* < 1$ for some $h \in \mathcal{K}^*$, candidate $h$ can profitably deviate by choosing $\tilde{\theta}_h = \theta_h^* + \nu$, for some sufficiently small $\nu > 0$ (winning the election with probability one). The previous results and the assumption that voters' preferences are uniformly distributed in $X$ imply that in any equilibrium the set of candidates running for office must be located symmetrically with respect to $\frac{1}{2}$. We have then established that in any equilibrium *(i)* candidates running for office must tie, *(ii)* voting is sincere, and *(iii)* $\theta_k^* = 1$ for all $k \in \mathcal{K}^*$, and that *(iv)* candidates must be symmetrically located.

We show next that there cannot be an electoral equilibrium with $K > 2$ candidates running for office representing different ideological positions. If this were the case, *(i)* and *(iii)* imply that by deviating and voting for any candidate $j$ other than her preferred candidate, a voter could get candidate $j$ elected with probability one. But then equilibrium implies that this voter must prefer the lottery among all $\mathcal{K}^*$ running candidates to having $j$ elected for sure. This implies, in particular, that

$$\frac{1}{|\mathcal{K}^*|} \sum_{k \in \mathcal{K}^*} u(1, x_k^*; z^i) \geq u(1, x_{K-1}^*; z^i) \tag{3}$$

for all voters such that $z^i > \frac{x_{K-1}^* + x_K^*}{2}$, i.e., all voters whose most preferred winning candidate is $k = K$ and next most preferred winning candidate is $k = K - 1$. On the other hand, strict concavity of $u(\cdot; z^i)$ with respect to policy and *(i)*, *(iii)*, and *(iv)* imply that for all $z^i$

$$u(1, \frac{1}{2}; z^i) > \frac{1}{|\mathcal{K}^*|} \sum_{k \in \mathcal{K}^*} u(1, x_k^*; z^i). \tag{4}$$

Combining (3) and (4), we obtain

$$u(1,\frac{1}{2};z^i) > u(1,x^*_{K-1};z^i)$$

for all voters such that $z^i > \frac{x^*_{K-1}+x^*_K}{2}$. But $K>2$ and *(iv)* imply that $\frac{1}{2} \le x^*_{K-1}$. Hence $u(\cdot;z^i)$ cannot be single-peaked for all voters such that $z^i > \frac{x^*_{K-1}+x^*_K}{2}$, contradicting the strict concavity of $u(\cdot;z^i)$.[22]

Finally, note that $\overline{c}+F \le \frac{1}{2}$ implies that a unique candidate equilibrium cannot be supported, since otherwise a second candidate, symmetrically located with respect to the median, will always find it profitable to run. As a result, the only possible equilibrium must have exactly two symmetrically located candidates chosing maximal quality. We are only left to show that such an equilibrium exists. So consider a strategy profile in which two top quality candidates, 1 and 2, symmetrically located around the median voter (i.e., $x_1 = 1 - x_2 < 1/2$), compete for office. Voters vote sincerely among these two candidates on the equilibrium path. If, off the equilibrium path, a third candidate $\ell$ enters the electoral competition, then we require that voters vote sincerely among candidates in $\{1,2\}$ for all $(\theta_1,\theta_2,\theta_3)$ for which $\max\{\theta_1,\theta_2\} = 1$.[23] We show that this strategy profile is an electoral equilibrium. First note that on the equilibrium path, voters are best responding, since with two candidates strategic voting is sincere. Next note that given that $\overline{c}+F \le \frac{1}{2}$, equilibrium rents of the two candidates running for office are always non-negative. Since candidates are choosing maximal quality in equilibrium, $\theta^*_1 = \theta^*_2 = 1$, the only possible deviation in the quality game is downwards. But any such deviation would entail sure loss, and is thus not profitable. Suppose now that a third candidate $\ell$ such that $x_\ell \in [0,1]$ decides to enter. Recall that voters vote sincerely among candidates in $\{1,2\}$ for all $(\theta_1,\theta_2,\theta_3)$ for which $\max\{\theta_1,\theta_2\} = 1$. But given these strategies, there is no voter which can benefit from a deviation. In fact, since candidates 1 and 2 are tying, any deviation from sincere voting between candidate 1 and candidate 2 in order to support the entrant will determine a victory of the least preferred candidate instead of having

[22]It should be noted that property *(iv)*, which follows from the assumption that voters' preferences are uniformly distributed, is in fact not needed to show that an equilibrium with more than two candidates cannot exist. Indeed, the argument can be slightly modified in order to account for a general continuous distribution of voters' preferences.

[23]It is not necessary to specify the strategy profile any further.

a lottery between $k = 1$ and $k = 2$. But then the strategy profile $(x_1^*, \theta_1 = 1)$, $(x_2^* = 1 - x_1^*, \theta_2 = 1)$, $(x_3, \theta_3 = 0)$, together with the same strategy profile for voters is an equilibrium in the continuation, and entry is not profitable. ■

**Proof of Lemma 1.** Suppose voter $i$'s preferred candidate is $k^*(i) \in \mathcal{K}$, and that $\tilde{k} \in \mathcal{K}$ and $\tilde{k} \neq k^*(i)$. Let $t_k(\sigma_{-i}^v)$ denote the number of votes for candidate $k$ given a voting strategy profile $\sigma_{-i}^v$ for all voters other than $i$. The payoff for $i$ of voting for $\tilde{k}$ given $\sigma_{-i}^v$, $U(\tilde{k}; \sigma_{-i}^v)$, is

$$\sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{[t_{\tilde{k}}(\sigma_{-i}^v) + 1]}{N} u(x_{\tilde{k}}; z^i) + \frac{t_{k^*(i)}(\sigma_{-i}^v)}{N} u(x_{k^*(i)}; z^i).$$

Similarly, the payoff for $i$ of voting for $k^*(i)$ given $\sigma_{-i}^v$, $U(k^*(i); \sigma_{-i}^v)$, is

$$\sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{t_{\tilde{k}}(\sigma_{-i}^v)}{N} u(x_{\tilde{k}}; z^i) + \frac{[t_{k^*(i)}(\sigma_{-i}^v) + 1]}{N} u(x_{k^*(i)}; z^i).$$

Thus

$$U(k^*(i); \sigma_{-i}^v) - U(\tilde{k}; \sigma_{-i}^v) = \frac{1}{N}[u(x_{k^*(i)}; z^i) - u(x_{\tilde{k}}; z^i)],$$

which is positive by definition of $k^*(i)$. Since $\sigma_{-i}^v$ was arbitrary, this shows that voting sincerely strictly dominates voting for any other available candidate and is thus a dominant strategy for voter $i$. It follows that in all Nash equilibria in the voting stage voters vote sincerely among candidates running for office. ■

**Proof of Proposition 2.**

Proof of Part 1. The first step towards proving the result is to provide conditions for the existence of electoral equilibria of a simple class, which we call location symmetric (LS) equilibria. In equilibria of this class, all candidates running for office are located at the same distance to their closest neighbors; i.e., $x_{k+1} - x_k = \Delta$ for all $k = 1, \ldots, K-1$, $x_1 = 1 - x_K = \Delta_0$, and all interior candidates $k = 2, \ldots, K-1$ choose the same level of quality. Within this class, the relevant competitors for any candidate $k$'s decision problem are $k$'s neighbors, $k+1$ and $k-1$. This is enough to show that payoff functions are twice differentiable in the relevant set (non-differentiabiliies can only arise for quality choices that are not optimal), and that whenever rents cover variable costs, first order conditions in the quality subgame completely characterize best response correspondences (see Iaryczower and Mattozzi (2008) for more details).

With this in mind, consider then two candidates $k$ and $j > k$ with policy positions $x_k$ and $x_j > x_k$, and choosing quality levels $\theta_k$ and $\theta_j$, and let $\tilde{x}_{k,j} \in \mathcal{R}$ denote the (unique) value of $x$ for which $u(\theta_k, x_k; x) = u(\theta_j, x_j; x)$, so that $u(\theta_k, x_k; z^i) > u(\theta_j, x_j; z^i)$ if and only if $z^i > \tilde{x}_{k,j}$,

$$\tilde{x}_{k,j} = \frac{x_k + x_j}{2} + \alpha \frac{[v(\theta_k) - v(\theta_j)]}{|x_j - x_k|}. \tag{5}$$

Provided that all cutpoints $\tilde{x}_{k,k+1}$ are in $(0, 1)$ for $k = 1, \ldots, K-1$, $k$'s vote share given $\{(\theta_\ell, x_\ell)\}_{\ell \in \mathcal{K}}$ is $s_k(\theta_k; \theta_{-k}, x) = \tilde{x}_{k,k+1} - \tilde{x}_{k-1,k}$, and therefore from (1) for PR, the payoff for an interior candidate $k = 2, \ldots, K-1$ is

$$\Pi_k(\theta_\mathcal{K}, x_\mathcal{K}, \mathcal{K}) = \Delta + \alpha \left[ \frac{v(\theta_k) - v(\theta_{k+1})}{\Delta} + \frac{v(\theta_k) - v(\theta_{k-1})}{\Delta} \right] - C(\theta_k) - F.$$

Defining $\Psi(\theta) \equiv \frac{v'(\theta_k)}{C'(\theta_k)}$, and noting that $\Psi(\cdot)$ is a decreasing function, $k$'s best response is then

$$\theta_k^* = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) & \text{if } \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \leq 1, \\ 1 & \text{if } \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) > 1. \end{cases} \tag{6}$$

Similarly, for an extreme candidate (say $k = 1$),

$$\Pi_1(\theta_\mathcal{K}, x_\mathcal{K}, \mathcal{K}) = \Delta_0 + \frac{\Delta}{2} + \alpha \left[ \frac{v(\theta_1) - v(\theta_2)}{\Delta} \right] - C(\theta_1) - F,$$

and $k = 1$'s best response is then

$$\theta_1^* = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) & \text{if } \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) \leq 1, \\ 1 & \text{if } \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) > 1. \end{cases} \tag{7}$$

Now, define $L \equiv \max\{2\overline{c}, \overline{c} + F, \frac{1-2F}{K-1}\}$ and $U \equiv \min\{2(\overline{c} + F), \frac{1}{K}\}$. We can now show that:

**Lemma 2** *If* $\max\{2\alpha\Psi(1), L\} < U$, *then there exists a LS equilibrium with* $K$ *candidates running for office and choosing a non-maximal quality.*

**Proof of Lemma 2.** Consider first the interior candidates $k = 2, \ldots, K-1$. If $\theta_j^* = \theta_r^* < 1$ for all $j, r \neq k$, then $k$'s marginal vote share is differentiable, and $k$'s FOC is given by $\frac{2\alpha}{\Delta} v'(\theta_k^*) = C'(\theta_k^*)$. Therefore,

$$\theta_k^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \quad \text{for all } k = 2, \ldots, K-1.$$

Moreover, since $\theta^* < 1$, it must be that $\Delta > 2\alpha\Psi(1)$. Non-negative rents for interior candidates requires that $\Pi_k^* = \Delta - C(\theta^*) - F \geq 0$, or equivalently $\theta^* \leq C^{-1}(\Delta - F)$. Substituting $\theta^*$ we get $\Delta \geq 2\alpha\Psi(C^{-1}(\Delta - F))$. Note that $2\alpha\Psi(1) \geq 2\alpha\Psi(C^{-1}(\Delta - F))$ if and only if $\Delta \geq \overline{c} + F$. Then, as long as in equilibrium $\Delta \geq \overline{c} + F$ (i.e., $\Pi_k(1) \geq 0$ for $k = 2, \ldots, K-1$), $\Delta \geq 2\alpha\Psi(1)$ implies $\Delta \geq 2\alpha\Psi(C^{-1}(\Delta - F))$; i.e., if interior candidates are choosing (the same) non-maximal quality, they obtain non-negative rents. It will be sufficient for our result to look for equilibria in which $\Delta \geq \overline{c} + F$, and therefore we require that

$$\max\{\overline{c} + F, 2\alpha\Psi(1)\} < \Delta. \tag{8}$$

Next, we consider the possibility of entry. First, we require that all equilibrium candidates have an incentive not to drop from the competition in any continuation game. For this it is sufficient that $\max\{\Delta_0, \frac{\Delta}{2}\} \geq \overline{c}$. Since $2\Delta_0 + (K-2)\Delta = 1$, then $\Delta_0 = \frac{1-(K-2)\Delta}{2}$, and the previous condition can be written as

$$2\overline{c} \leq \Delta \leq \frac{1 - 2\overline{c}}{K - 2}. \tag{9}$$

Suppose now that $j$ enters at $x_j \in (x_k, x_{k+1})$ for $k = 1, \ldots, K-1$, and define $\delta_j^r \equiv \frac{x_{k+1} - x_j}{\Delta}$. Suppose first that in the continuation $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = 1$. Then it must be that

$$\alpha v'(1)\left[\frac{1}{\delta_j^r \Delta} + \frac{1}{\Delta}\right] \geq C'(1),$$

$$\alpha v'(1)\left[\frac{1}{(1-\delta_j^r)\Delta} + \frac{1}{\Delta}\right] \geq C'(1).$$

Then if $\delta_j^r \geq \frac{1}{2}$ ($j$ enters in $(x_k, x_{k+1})$ closer to $x_k$ than to $x_{k+1}$) the first two inequalities above hold if and only if $\Delta \leq \alpha\Psi(1)\left[1 + \frac{1}{\delta_j^r}\right]$, or $\delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}$. Thus, the continuation strategy profile is a Nash equilibrium for $\frac{1}{2} \leq \delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}$, which is feasible if and only if $\Delta \leq 3\alpha\Psi(1)$. When instead $\delta_j^r \leq \frac{1}{2}$ ($j$ enters closer to $x_k$) then we need $\Delta \leq \alpha\Psi(1)\left[1+\frac{1}{(1-\delta_j^r)}\right]$, or $\delta_j^r \geq \frac{\Delta-2\alpha\Psi(1)}{\Delta-\alpha\Psi(1)}$. Thus, the continuation strategy profile is a Nash equilibrium for $\frac{\Delta-2\alpha\Psi(1)}{\Delta-\alpha\Psi(1)} \leq \delta_j^r \leq \frac{1}{2}$, which is feasible if and only if $\Delta \leq 3\alpha\Psi(1)$. Therefore, the strategy profile $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = 1$ is a Nash equilibrium in the continuation for entrants such that

$$\frac{\Delta - 2\alpha\Psi(1)}{\Delta - \alpha\Psi(1)} \leq \delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}, \tag{10}$$

where $2\alpha\Psi(1) < \Delta \leq 3\alpha\Psi(1)$. Since the entrant in this case obtains $\hat{\Pi}_j = \frac{\Delta}{2} - [\bar{c}+F]$, then as long as in equilibrium

$$\Delta < 2[\bar{c} + F], \tag{11}$$

entry in an "interior" region as in (10) is not profitable. It should be clear that this rules out "interior" entrants only, since $2\alpha\Psi(1) < \Delta$ from (8) implies with (10) that $\delta_j^r \in (0,1)$.

Consider then $\delta_j^r > \frac{\alpha\Psi(1)}{\Delta-\alpha\Psi(1)}$ ($j$ enters close to $x_k$; the other case is symmetric). Consider the continuation $\hat{\theta}_k = \hat{\theta}_j = 1$, $\hat{\theta}_{k+1} = \Psi^{-1}(\frac{\delta_j^r}{1+\delta_j^r}\frac{\Delta}{\alpha}) < 1$. This is clearly an equilibrium in the continuation ($j$ and $k$ have even a greater incentive to choose 1 than in the previous case since they are now closer substitutes). For entry not to be profitable, we need

$$\hat{\Pi}_j = \frac{\Delta}{2} + \frac{\alpha}{\delta_j^r\Delta}[v(1) - v(\hat{\theta}_{k+1})] - [\bar{c} + F] < 0,$$

and a sufficient condition for the above inequality to be true is

$$\Delta \leq 2F. \tag{12}$$

To see this, suppose that the division of the electorate between $k$ and $j$ were fixed, with cutpoint $\tilde{x}_{kj} = \frac{x_k+x_j}{2}$. Then $j$ would optimally choose $\tilde{\theta}_j = \Psi^{-1}(\frac{\delta_j^r\Delta}{\alpha}) < \hat{\theta}_{k+1}$, and we have that

$$\hat{\Pi}_j \leq \frac{\Delta}{2} - \frac{\alpha}{\delta_j^r\Delta}[v(\hat{\theta}_{k+1}) - v(\tilde{\theta}_j)] - [C(\tilde{\theta}_j) + F] < \frac{\Delta}{2} - [C(\tilde{\theta}_j) + F].$$

Consider next optimality and non-negative rents for extreme candidates, and no-entry conditions at the extremes. Note first that given that interior candidates are choosing non-maximal quality, then optimal quality by extreme candidates must be non-maximal as well. In particular, it must be that $\theta_1^* = \theta_K^* = \Psi^{-1}(\frac{\Delta}{\alpha})$. For no entry at the extremes it is sufficient as before that $\Delta_0 < F$, and since $\Delta_0 = \frac{1-(K-1)\Delta}{2}$ this can be written as

$$\frac{1-2F}{K-1} < \Delta. \tag{13}$$

For non-negative rents we need $\Pi_1^* = \Delta_0 + \frac{\Delta}{2} - \frac{\alpha}{\Delta}[v(\theta^*) - v(\theta_1^*)] - C(\theta_1^*) - F \geq 0$. Since $\Pi_1^*$ is maximized at $\theta_1^*$, then $\Pi_1(\theta_1^*) \geq \Pi_1(\theta_1)$ for all $\theta_1 \neq \theta_1^*$ and, as a result, it suffices to show that $\Pi_1(\theta^*) > 0$, or equivalently, $\frac{(K-2)}{2}\Delta + [C(\theta^*) + F] \leq \frac{1}{2}$. But since in equilibrium $\Delta \geq C(\theta^*) + F$, then it is sufficient that

$$\Delta \leq \frac{1}{K}. \tag{14}$$

We have then shown that the strategy profile specified above is an electoral equilibrium (in which all candidates choose non-maximal quality) if $\Delta$ satisfies conditions (8) - (14). Now, (8) and (12) imply that for this to be feasible it is necessary that $\bar{c} < F$ $(*)$. From $(*)$, $\bar{c} + F < \Delta$ in (8) and (14) imply (9), and (11) implies (12). The relevant conditions on the degree of policy differentiation, $\Delta$, can then be written as $\max\{2\alpha\Psi(1), L\} \leq \Delta < U$, as we wanted to show. ∎

Lemma 2 shows that if $\max\{2\alpha\Psi(1), L(K)\} < U(K)$, then there exists a LS equilibrium in which $K \geq 3$ run for office choosing non-maximal quality. After some algebra,[24] we note that for $K \geq 3$ given, this condition is implied by the following four inequalities

$$\bar{c} < F, \tag{15}$$

$$\frac{1}{2K} \leq F \leq \frac{1}{K} - \bar{c}, \tag{16}$$

and

$$\alpha < \frac{1}{\Psi(1)K}. \tag{17}$$

[24]The condition $\max\{2\alpha\Psi(1), L(K)\} < U(K)$ embodies six relevant inequalities: *(a)* $\alpha\Psi(1) < \bar{c}+F$, *(b)* $2\alpha\Psi(1) < 1/K$, *(c)* $2\bar{c} < 1/K$, *(d)* $(\bar{c}+F) < 1/K$, *(e)* $\frac{1-2F}{K-1} < 1/K$ and *(f)* $\frac{1-2F}{K-1} < 2[\bar{c}+F]$. Note that *(e)* can be written as $F > \frac{1}{2K}$, and *(f)* as $F > \frac{1}{2K} - \frac{K-1}{K}\bar{c}$. Thus *(e)* implies *(f)*. Moreover, from this it follows that $\frac{1}{K} < 2[\bar{c} + F]$, and that therefore *(b)* implies *(a)*. Finally, given (15), *(d)* implies *(c)*. Inequalities *(d)* and *(f)* give (16).

Take $K \geq 3$ given. If $\bar{c} < \frac{1}{2K}$, there exists an interval $[\underline{F}(K), \overline{F}(K)]$ such that $F \in [\underline{F}(K), \overline{F}(K)]$ satisfies (15), and (16). Finally, any $\alpha < \frac{1}{\Psi(1)K}$ satisfies (17). This concludes the proof of part 1 of the Theorem.

Proof of Part 2. Consider first the case of $K = 2$. Note that since identically located candidates are perfect substitutes, in equilibrium quality must be maximal. Otherwise candidate $k$ can increase rents discretely (in fact capturing all votes) by increasing $\theta_k$ (and costs) only marginally. The rents of candidates are non-negative if and only if $\frac{1}{2} - \bar{c} \geq F$. To show that an equilibrium cannot exist it is enough to show that there exists a small positive $\nu$ such that entry of a third candidate at $x' = \frac{1}{2} - \nu$ is always profitable. Note that if a third candidate $j$ enters at $x'$ with $\theta_j = 1$ either $\hat{\theta}_k = 1$ for $k = 1, 2$, or $\hat{\theta}_k = 1$ and $\hat{\theta}_{-k} = 0$, $k = 1, 2$ ($\frac{1}{2} - \bar{c} \geq F$ implies that the case $\hat{\theta}_k = 0$, $k = 1, 2$ can never happen). If $\frac{1}{2}(1 - \frac{x'+1/2}{2}) - \bar{c} = \frac{3-2x'}{8} - \bar{c} \geq 0$, we have that in the continuation game $\hat{\theta}_k = 1$, $k = 1, 2$, and to deter entry at $\tilde{x}$ we need $\frac{x'+\frac{1}{2}}{2} - \bar{c} < F$. When $\nu \to 0$ the two last inequalities become $\frac{1}{2} - \bar{c} \in \left[\frac{1}{4}, F\right]$. Together with the above condition for non-negative rents for candidates, the last expression implies that a two candidate equilibrium with perfectly centrist candidates exists if and only if $F \geq \frac{1}{4}$ and $\frac{1}{2} - \bar{c} = F$. If instead $\frac{3-2\tilde{x}}{8} - \bar{c} < 0$, we have that in the continuation game one of the two running candidates will drop, i.e., $\hat{\theta}_k = 1$, and $\hat{\theta}_{-k} = 0$, $k = 1, 2$. Since to deter entry at $\tilde{x}$ it must be that $\frac{x'+\frac{1}{2}}{2} - \bar{c} < F$, in this case when $\nu \to 0$ we need $\frac{1}{2} - \bar{c} \leq \min\left\{\frac{1}{4}, F\right\}$. Once again combining the last expression with the above condition for non-negative rents for candidates we get that a two candidate equilibrium with perfectly centrist candidates exists if and only if $F \leq \frac{1}{4}$ and $\frac{1}{2} - \bar{c} = F$. If $K > 2$ we need $\frac{x'+\frac{1}{2}}{2} - \bar{c} < F$ and $\frac{1}{K} - \bar{c} \geq F$, which leads to a contradiction when $\nu \to 0$. ■

**Proof of Proposition 4.**

(1) For given $n$, consider a strategy profile in which two top quality candidates, 1 and 2, symmetrically located around the median voter (i.e., $x_1 = 1 - x_2 < 1/2$), compete for office. Voters vote sincerely among these two candidates on the equilibrium path. If, off the equilibrium path, a third candidate $\ell$ enters the electoral competition, then we require that voters vote sincerely among candidates in $\{1, 2\}$

for all $(\theta_1, \theta_2, \theta_3)$ for which $\max\{\theta_1, \theta_2\} = 1$.[25] We show that a strategy profile of this class, with $\Delta \equiv x_2 - x_1$ sufficiently small, is an electoral equilibrium for large $n$. First note that on the equilibrium path, voters are best responding, since with two candidates strategic voting is sincere. Next note that given $\bar{c} + F \leq \frac{1}{2}$, equilibrium rents of the two candidates running for office are always non-negative. Since candidates are choosing maximal quality in equilibrium, $\theta_1^* = \theta_2^* = 1$, the only possible deviation in the quality game is downwards. So suppose that candidate 1 deviates to some $\theta_1 < 1$. Note that since candidates were tying in equilibrium, and that voters must vote sincerely, this deviation entails the loss of the majority premium $\gamma$ for sure. Given $\theta_2^* = 1$, and $\theta_1 < 1$, the payoff of candidate 1, $\Pi_1 = (1-\gamma)\tilde{x}_{12}(\theta_1, 1) - C(\theta_1)$ is continuous and differentiable (as before, $\tilde{x}_{12}(\theta_1, \theta_2)$ represents the voter who is indifferent between candidates 1 and 2 given $\theta_1, \theta_2$). Extending the choice set to include $\theta_1 = 1$, but assuming away the possibility of obtaining the majority premium $\gamma$, the most profitable "deviation" is then to play

$$\hat{\theta}_1 = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{\alpha(1-\gamma)}\right) & \text{if } \Delta > \alpha(1-\gamma)\Psi(1), \\ 1 & \text{if } \Delta \leq \alpha(1-\gamma)\Psi(1). \end{cases} \tag{18}$$

It follows that if $\Delta \leq \alpha(1-\gamma)\Psi(1)$, 1 prefers not to deviate. To deter this deviation, therefore, it suffices to consider strategy profiles such that $\Delta \leq \alpha(1-\gamma)\Psi(1)$. Suppose now that a third candidate $\ell$ such that $x_\ell \in [0,1]$ decides to enter. Recall that voters vote sincerely among candidates in $\{1,2\}$ for all $(\theta_1, \theta_2, \theta_3)$ for which $\max\{\theta_1, \theta_2\} = 1$. But given these strategies, no voter can benefit from a deviation, provided that $n$ is large enough. To see this, suppose without loss of generality that voter $i$ prefers candidate 1 to candidate 2, and note that $i$'s equilibrium payoff, voting for $k = 1$, is

$$U(1; \sigma_{-i}^v) = \left(\frac{1}{2}(1-\gamma) + \frac{\gamma}{2}\right)[u(x_1; z_i) + u(x_2; z_i)].$$

Deviating and voting for an entrant $\ell$, $i$ obtains

$$U(\ell; \sigma_{-i}^v) = \frac{n-2}{2n}(1-\gamma)u(x_1; z_i) + \left(\frac{1}{2}(1-\gamma) + \gamma\right)u(x_2; z_i) + \frac{1}{n}(1-\gamma)u(x_\ell; z_i).$$

[25] It is not necessary to specify the strategy profile any further.

For equilibrium, it is necessary that $U(\ell;\sigma^v_{-i}) - U(1;\sigma^v_{-i}) < 0$, which is always true if $u(x_\ell;z_i) < u(x_1;z_i)$. If instead $u(x_\ell;z_i) > u(x_1;z_i)$, this occurs if and only if

$$\frac{1-\gamma}{\gamma} < \frac{n}{2}\frac{[u(x_1;z_i)-u(x_2;z_i)]}{[u(x_\ell;z_i)-u(x_1;z_i)]},$$

but this is satisfied for large enough $n$, since $x_1 \neq x_2$. This concludes the proof of part *(i)*.

(2) Suppose, contrary to the statement of the proposition, that there does not exist such $\overline{n}$. Then for any $n$ there exists $n' > n$ such that $K \geq 3$ candidates tie for the win in $\tilde{\Gamma}_n$. We show that this is not possible. First, note that if a set of candidates $W \subseteq \mathcal{K}$ tie for the win, then all voters voting for candidates in $W \subseteq \mathcal{K}$ vote for their preferred candidate within $W$ (for otherwise a voter could induce a strictly preferred lottery over outcomes by voting for her preferred candidate in $W$). But then $\theta_k = 1$ for all $k \in W$, for otherwise there exists a candidate $\ell \in W$ with $\theta_\ell < 1$, who would gain from deviating to $\theta'_\ell = \theta_\ell + \eta$ for sufficiently small $\eta > 0$. So suppose first that in equilibrium all $K > 2$ candidates in $\mathcal{K}$ tie, with $\theta_k = 1$ for all $k$, and let $k^*(i)$ denote $i$'s preferred candidate in $\mathcal{K}$. It is immediate here that all voting is sincere, for otherwise any voter not voting sincerely would induce a strictly preferred lottery over outcomes by voting for their preferred candidate $k^*(i)$. Since all candidates are tying choosing maximal quality and voting is sincere, candidates must be equally spaced. Next, note that equilibrium implies that all voters $i \in N$ must prefer the equal probability lottery among all $k \in \mathcal{K}$ induced in equilibrium to the lottery that is implied after a deviation to any candidate $\ell \neq k^*(i)$. Now, if for any $n$ there exists $n' > n$ such that this strategy profile is an equilibrium, it must be that all voters $i \in N$ must prefer the equal probability lottery among all $k \in W$ induced in equilibrium to the degenerate lottery in which they get any candidate $\ell \neq k^*(i)$ for sure. To see this, note that $i$'s equilibrium payoff, voting for $k^*(i)$, is

$$U(k^*(i);\sigma^v_{-i}) = \sum_{k\in\mathcal{K}}\left[\frac{1}{K}\frac{n-1}{n}(1-\gamma)+\frac{\gamma}{K}\right]u(x_k;z_i) + \frac{1}{n}(1-\gamma)u(x_{k^*(i)};z_i).$$

Deviating and voting for $\ell \neq k^*(i)$, $i$ obtains

$$U(\ell;\sigma^v_{-i}) = \sum_{k\in\mathcal{K}}\left[\frac{1}{K}\frac{n-1}{n}(1-\gamma)\right]u(x_k;z_i) + \left[\frac{1}{N}(1-\gamma)+\gamma\right]u(x_\ell;z_i).$$

The deviation gain $U(\ell;\sigma^v_{-i}) - U(k^*(i);\sigma^v_{-i}) < 0$ implies then that

$$u(x_\ell;z_i) - \frac{1}{K}\sum_{k\in\mathcal{K}}u(x_k;z_i) < \frac{1}{n}\frac{(1-\gamma)}{\gamma}\left[u(x_{k^*(i)};z_i) - u(x_\ell;z_i)\right],$$

but since for any $n$ there exists $n' > n$ such that this strategy profile is an equilibrium, it must be that $u(x_\ell;z_i) < \frac{1}{K}\sum_{k\in\mathcal{K}}u(x_k;z_i)$, for otherwise, we can always find an $n'$ that would reverse this inequality. Thus, if there does not exist a largest finite $n$ for which all $K > 2$ candidates in $\mathcal{K}$ can tie in equilibrium, it must be that all voters $i \in N$ must prefer the equal probability lottery among all $k \in W$ induced in equilibrium to the degenerate lottery in which they get any candidate $\ell \neq k^*(i)$ for sure. But then the same argument as in Theorem 1 shows that this can not be an equilibrium.

Next suppose that $2 \leq |W| < K$ candidates tie for the win in equilibrium, where again $W$ denotes the set of winning candidates and $L$ the set of losing candidates. This cannot be an equilibrium either for sufficiently large $n$, since otherwise a voter $i$ voting for one of the losing candidates $\ell_0 \in L$ could gain by breaking the tie among the candidates in $W$ in favor of her favorite candidate among $W$, $w_0$. To see this, denote the fraction of votes obtained by candidate in $W$ by $\omega$, and note that $i$'s equilibrium payoff, voting for $\ell_0 \in L$, is

$$U(\ell_0;\sigma^v_{-i}) = \sum_{w\in W}\left[\omega(1-\gamma) + \frac{\gamma}{|W|}\right]u(x_w;z_i) + \sum_{\ell\in L}\frac{t_\ell}{n}(1-\gamma)u(x_\ell;z^i).$$

The expected payoff of deviating and voting for $w_0 \in W$ is instead

$$U(w_0;\sigma^v_{-i}) = \sum_{w\in W}\omega(1-\gamma)u(x_w;z^i) + \left[\frac{1}{n}(1-\gamma)+\gamma\right]u(x_{w_0};z^i) +$$
$$\sum_{\ell\neq\ell_0\in L}\frac{t_\ell}{n}(1-\gamma)u(x_\ell;z^i) + \frac{(t_{\ell_0}-1)}{n}(1-\gamma)u(x_{\ell_0};z^i).$$

But then $U(w_0; \sigma^v_{-i}) - U(\ell_0; \sigma^v_{-i}) > 0$ if and only if

$$\frac{\gamma}{1-\gamma} > \frac{1}{n} \frac{[u(x_{\ell_0}; z^i) - u(x_{w_0}; z^i)]}{\left[u(x_{w_0}; z^i) - \frac{1}{|W|}\sum_{w \in W} u(x_w; z_i)\right]},$$

which holds for sufficiently large $n$. ■

**Proof of Remark 1.** It remains to show that sincere voting is rational in any voting subgame following a deviation in quality by an equilibrium candidate in a LS equilibrium, and that for every deviation at the entry stage, there is an equilibrium in the continuation voting subgame in which either all or all but a small number of voters vote sincerely, and such that out of equilibrium entry is not sequentially rational.

Consider first voting subgames following a deviation in quality by an equilibrium candidate in a LS equilibrium. Suppose that candidate $k$ deviates to $\theta_k \neq \theta^*$. We know from the proof of Proposition 2 that this cannot be a profitable deviation for $k$ if voters vote sincerely. Moreover, given that candidates care exclusively about vote shares this cannot be a profitable deviation if all but a small number of voters vote sincerely either. As a result, a sufficiently large number of voters must be voting strategically for this to be a profitable deviation. On the other hand, if any voter is to vote strategically, it must be that $k$ is either tying or contending for the median position by at most one vote. But this implies that if all voters vote sincerely, $k$ can't be close to contending for the median, and therefore no voter can have an incentive to vote strategically for candidate $k$. Since all other relevant candidates have equal quality, then there cannot be strategic voting for any other candidate either, and sincere voting is rational.[26] Thus choosing $\theta^*$ is a best response for $k$ in the quality competition stage.

Similarly, we can show that for every deviation at the entry stage, there is an equilibrium in the continuation voting subgame in which either all or all but a small number of voters vote sincerely, and for which out of equilibrium entry is not sequentially rational. Consider then a deviation at the entry stage. Note that if voters

[26] Moreover, voting sincerely is not a weakly dominated strategy for any voter $i$, as it is always possible to find a voting profile for the remaining voters for which $i$'s vote can be decisive between $i$'s favorite candidate and some other candidate running for office.

vote sincerely after every continuation, or if all but a small number of voters vote sincerely after every continuation, then entry is not profitable, in the sense that for every possible entry there exists an equilibrium in the continuation game such that the entrant obtains a negative payoff. Now suppose that after a deviation at the entry stage, candidates play the continuation strategy profile that deters entry in the proof of Proposition 2, and suppose that all voters vote sincerely. Then the event in which two candidates contend for the median position by a one vote difference given sincere voting and given this particular strategy profile by candidates has probability zero. But if no two candidates are contending for the median position by a one vote difference, sincere voting is rational. Now consider a deviation from this profile by one of the candidates. By our previous argument, this can only be a profitable deviation if a sufficiently large number of voters is voting strategically in the voting subgame following this deviation. But then we can always choose a voting strategy profile in which all but a small number of voters vote sincerely. Then no voter can be decisive for the median, and no voter will have an incentive to deviate. All voters, moreover, are using undominated strategies (we know that voters voting sincerely are not using weakly dominated strategies, but neither are the voters who continue to vote as in the strategic voting profile, since in fact this was a best response against this strategy profile by the other voters). Since candidates only care about voting shares, and since with a large electorate the impact of a small number of votes on payoffs is negligible, this cannot be a profitable deviation. This concludes the argument. ■

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