A New Approach to Consumer Theory
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A NEW APPROACH TO CONSUMER THEORY*

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I. THE CURRENT STATUS OF CONSUMER THEORY

The theory of consumer behavior in deterministic situations as set out by, say, Debreu (1959, 1960) or Uzawa (1960) is a thing of great aesthetic beauty, a jewel set in a glass case. The product of a long process of refinement from the nineteenth-century utility theorists through Slutsky and Hicks-Allen to the economists of the last twenty-five years,[1] it has been shorn of all irrelevant postulates so that it now stands as an example of how to extract the minimum of results from the minimum of assumptions.

To the process of slicing away with Occam's razor, the author made a small contribution (1957). This brought forth a reply by Johnson (1958) which suggested, somewhat tongue-in-cheek, that the determinateness of the sign of the substitution effect (the only substantive result of the theory of consumer behavior) could be derived from the proposition that goods are goods.

Johnson's comment, on reflection, would seem to be almost the best summary that can be given of the current state of the theory of consumer behavior. All *intrinsic* properties of particular goods, those properties that make a diamond quite obviously something different from a loaf of bread, have been omitted from the theory, so that a consumer who consumes diamonds alone is as rational as a consumer who consumes bread alone, but one who sometimes consumes bread, sometimes diamonds (*ceteris paribus*, of course), is irrational. Thus, the only property which the theory can build on is the property shared by all goods, which is simply that they are goods.

Indeed, we can continue the argument further, since goods are simply what consumers would like more of; and we must be neutral with respect to differences in consumer tastes (some consumers might like more of something that other consumers do not want), that the ultimate proposition is that *goods are what are thought of as goods.*

In spite of the denial of the relevance of intrinsic properties to the pure theory, there has always been a subversive undercurrent suggesting that economists continue to take account of these properties. Elementary textbooks bristle with substitution examples about butter and margarine, rather than about shoes and ships, as though the authors believed that there was something intrinsic to butter and margarine that made them good substitutes and about automobiles and gasoline that made them somehow intrinsically complementary. Market re-

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[1] The American Economic Association *Index of Economic Journals* lists 151 entries under category 2.111 (utility, demand, theory of the household) over the period 1940–63.

searchers, advertisers, and manufacturers also act as though they believe that knowledge of (or belief in) the intrinsic properties of goods is relevant to the way consumers will react toward them.

The clearest case of conflict between a belief that goods do have intrinsic properties relevant to consumer theory but that they are not taken into account has been the long search for a definition of "intrinsic complementarity." The search was successful only where Morishima (1959) turned from traditional theory to an approach somewhat similar to that of the present paper.

Perhaps the most important aspects of consumer behavior relevant to an economy as complex as that of the United States are those of consumer reactions to new commodities and to quality variations. Traditional theory has nothing to say on these. In the case of new commodities, the theory is particularly helpless. We have to expand from a commodity space of dimension n to one of dimension $n + 1$, replacing the old utility function by a completely new one, and even a complete map of the consumer's preferences among the n goods provides absolutely no information about the new preference map. A theory which can make no use of so much information is a remarkably empty one. Even the technique of supposing the existence of a utility function for all possible goods, including those not yet invented, and regarding the prices of nonexistent goods as infinite—an incredible stretching of the consumers' powers of imagination—has no predictive value.

Finally we can note the unsuitability of traditional theory for dealing with many of the manifestly important aspects of actual relationships between goods and consumers in I. F. Pearce's (1964) recent heroic but rather unsuccessful attempts to deal with complementarity, substitution, independence, and neutral want associations within the conventional framework.

II. A NEW APPROACH

Like many new approaches, the one set out in this paper draws upon several elements that have been utilized elsewhere. The chief technical novelty lies in breaking away from the traditional approach that goods are the direct objects of utility and, instead, supposing that it is the properties or characteristics of the goods from which utility is derived.

We assume that consumption is an activity in which goods, singly or in combination, are inputs and in which the output is a collection of characteristics. Utility or preference orderings are assumed to rank collections of characteristics and only to rank collections of goods indirectly through the characteristics that they possess. A meal (treated as a single good) possesses nutritional characteristics but it also possesses aesthetic characteristics, and different meals will possess these characteristics in different relative proportions. Furthermore, a dinner party, a combination of two goods, a meal and a social setting, may possess nutritional, aesthetic, and perhaps intellectual characteristics different from the combination obtainable from a meal and a social gathering consumed separately.

In general—and the richness of the approach springs more from this than from anything else—even a single good will possess more than one characteristic, so that the simplest consumption activity will be characterized by joint outputs. Furthermore, the same characteristic (for example, aesthetic properties) may be included among the joint outputs of many consumption activities so that goods which are apparently unrelated in certain of their characteristics may be related in others.

We shall assume that the structure we have interposed between the goods themselves and the consumer's preferences is, in principle, at least, of an objective kind. That is, the characteristics possessed by a good or a combination of goods are the same for all consumers and, given units of measurement, are in the same quantities,[2] so that the personal element in consumer choice arises in the choice between collections of characteristics only, not in the allocation of characteristics to the goods. The objective nature of the goods-characteristics relationship plays a crucial role in the analysis and enables us to distinguish between objective and private reactions to such things as changes in relative prices.

The essence of the new approach can be summarized as follows, each assumption representing a break with tradition:

1. The good, per se, does not give utility to the consumer; it possesses characteristics, and these characteristics give rise to utility.

2. In general, a good will possess more than one characteristic, and many characteristics will be shared by more than one good.

3. Goods in combination may possess characteristics different from those pertaining to the goods separately.

A move in the direction of the first assumption has already been made by various workers including Strotz (1957, 1959) and Gorman (1959), with the "utility tree" and other ideas associating a particular good with a particular type of utility. The theory set out here goes much further than these ideas. Multiple characteristics, structurally similar to those of the present paper but confined to a particular problem and a point utility function, are implicit in the classical "diet problem" of Stigler (1945), and multidimensioned utilities have been used by workers in other fields, for example, Thrall (1954). The third assumption, of activities involving complementary collections of goods, has been made by Morishima (1959) but in the context of single-dimensioned utility.

A variety of other approaches with similarities to that of the present paper occur scattered through the literature, for example, in Quandt (1956), or in Becker (1965), or in various discussions of investment-portfolio problems. These are typically set out as *ad hoc* approaches to particular problems. Perhaps the most important aspect of this paper is that the model is set out as a general replacement of the traditional analysis (which remains as a special case), rather than as a special solution to a special problem.

It is clear that only by moving to multiple characteristics can we incorporate many of the intrinsic qualities of individual goods. Consider the choice between a gray Chevrolet and a red Chevrolet. On ordinary theory these are either the same commodity (ignoring what may be a relevant aspect of the choice situation) or different commodities (in which case there is no a priori presumption that they are close substitutes). Here we regard them as goods associated with satisfaction vectors which differ in only one component, and we can proceed to look at the situation in much the same way as the consumer—or even the economist, in private life—would look at it.

Traditional theory is forever being forced to interpret quite common real-life

[2] Since the units in which the characteristics are measured are arbitrary, the objectivity criterion relating goods and characteristics reduces to the requirement that the *relative* quantities of a particular characteristic between unit quantities of any pair of goods should be the same for all consumers.

happenings, such as the effects of advertising in terms of "change of taste," an entirely non-operational concept since there is no way of predicting the relationship between preference before and after the change. The theory outlined here, although extremely rich in useful ways of thinking about consumer behavior, may also be thought to run the danger of adding to the economist's extensive collection of non-operational concepts. If this were true, it need not, of course, inhibit the heuristic application of the theory. Even better, however, the theory implies predictions that differ from those of traditional theory, and the predictions of the new approach seem to fit better the realities of consumer behavior.

III. A MODEL OF CONSUMER BEHAVIOR

To obtain a working model from the ideas outlined above, we shall make some assumptions which are, on balance, neither more nor less heroic than those made elsewhere in our present economic theorizing and which are intended to be no more and no less permanent parts of the theory.

1. We shall regard an individual good or a collection of goods as a consumption activity and associate a scalar (the level of the activity) with it. We shall assume that the relationship between the level of activity k, y_k, and the goods consumed in that activity to be both linear and objective, so that, if x_j is the jth commodity we have

$$x_j = \sum_k a_{jk} y_k , \qquad (1)$$

and the vector of total goods required for a given activity vector is given by

$$x = Ay . \qquad (2)$$

Since the relationships are assumed objective, the equations are assumed to hold for all individuals, the coefficients a_{jk} being determined by the intrinsic properties of the goods themselves and possibly the context of technological knowledge in the society.

2. More heroically, we shall assume that each consumption activity produces a fixed vector of characteristics[3] and that the relationship is again linear, so that, if z_i is the amount of the ith characteristic

$$z_i = \sum_k b_{ik} y_k , \qquad (3)$$

or

$$z = By . \qquad (4)$$

Again, we shall assume that the coefficients b_{ik} are objectively determined—in principle, at least—for some arbitrary choice of the units of z_i.

3. We shall assume that the individual possesses an ordinal utility function on characteristics $U(z)$ and that he will choose a situation which maximizes $U(z)$. $U(z)$ is provisionally assumed to possess the ordinary convexity properties of a standard utility function.

The chief purpose of making the assumption of linearity is to simplify the problem. A viable model could certainly be produced under the more general set of relationships

$$F_k(z, x) = 0 , \quad k = 1 \ldots m . \qquad (5)$$

The model could be analyzed in a similar way to that used by Samuelson (1953*b*) and others in analyzing production, although the existence of much jointness among outputs in the present model presents difficulties.

[3] The assumption that the consumption technology A, B is fixed is a convenience for discussing those aspects of the model (primarily static) that are the chief concern of this paper. The consequences of relaxing this particular assumption is only one of many possible extensions and expansions of the ideas presented and are discussed by the author elsewhere (Lancaster, 1966).

In this model, the relationship between the collections of characteristics available to the consumer—the vectors z—which are the direct ingredients of his preferences and his welfare, and the collections of goods available to him—the vectors x—which represent his relationship with the rest of the economy, is not direct and one-to-one, as in the traditional model, but indirect, through the activity vector y.

Consider the relationships which link z and x. These are the equation systems: $x = Ay$ (2) and $z = By$ (4). Suppose that there are r characteristics, m activities, and n goods. Only if $r = m = n$ will there be a one-to-one relationship between z and x. In this case both the B and A matrixes are square (the number of variables equals the number of equations in both sets of equations) and we can solve for y in terms of x, $y = A^{-1}x$, giving $z = BA^{-1}x$. $U(z)$ can be written directly and unambiguously as a function $u(x)$. Otherwise the relations are between vectors in spaces of different dimensions. Consider some x^* in the case in which $m > n$: equation (2) places only n restrictions on the m-vector y, so that y can still be chosen with $m - n$ degrees of freedom. If $r < m$, then there are $m - r$ degrees of freedom in choosing y, given some z, but whether the ultimate relationship gives several choices of z for a given x, or several x for a given z, and whether all vectors z are attainable, depends on the relationships between r, m, and n and the structures of the matrixes A, B. In general, we will expect that the consumer may face a choice among many paths linking goods collections with characteristics collections. The simple question asked (in principle) in the traditional analysis—does a particular consumer prefer collection x_1 or collection x_2—no longer has a direct answer, although the question, does he prefer characteristics collection z_1 or z_2, does have such an answer.

If we take the standard choice situation facing the consumer in a free market, with a linear budget constraint, this situation, in our model, becomes:

$$\begin{aligned} &\text{Maximize } U(z) \\ &\text{subject to } px \leqq k \\ &\text{with} \qquad z = By \\ &\qquad\qquad\; x = Ay \\ &\qquad\qquad\; x, y, z \geqq 0 \,. \end{aligned}$$

This is a non-linear program of an intractable kind. The problem of solution need not worry us here, since we are interested only in the properties of the solution.

IV. THE SIMPLIFIED MODEL

We shall simplify the model in the initial stages by supposing that there is a one-to-one correspondence between goods and activities so that we can write the consumer-choice program in the simpler form

$$\begin{aligned} &\text{Maximize } U(z) \\ &\text{subject to } px \leqq k \\ &\text{with} \qquad z = Bx \\ &\qquad\qquad\; z, x \geqq 0 \,. \end{aligned}$$

This is still, of course, a non-linear program, but we now have a single step between goods and characteristics.

The model consists of four parts. There is a maximand $U(z)$ operating on characteristics, that is, U is defined on characteristics-space (C-space). The budget constraint $px \leqq k$ is defined on goods-space (G-space). The equation system $z = Bx$ represents a transformation between G-space and C-space. Finally, there are non-negativity constraints z,

$x \geqq 0$ which we shall assume to hold initially, although in some applications and with some sign conventions they may not always form part of the model.

In traditional consumer analysis, both the budget constraint and the utility function are defined on G-space, and we can immediately relate the two as in the ordinary textbook indifference-curve diagram. Here we can only relate the utility function to the budget constraint after both have been defined on the same space. We have two choices: (1) We can transform the utility function into G-space and relate it directly to the budget constraint; (2) we can transform the budget constraint into C-space and relate it directly to the utility function $U(z)$.

Each of these techniques is useful in different circumstances. In the case of the first, we can immediately write $U(z) = U\ (Bx) = u(x)$, so we have a new utility function directly in terms of goods, but the properties of the function $u(x)$ depend crucially on the structure of the matrix B and this, together with the constraints $x \geqq 0$ and $z = Bx \geqq 0$ give a situation much more complex than that of conventional utility maximization. The second technique again depends crucially on the structure of B and again will generally lead to a constraint of a more complex kind than in conventional analysis.

The central role in the model is, of course, played by the transformation equation $z = Bx$ and the structure and qualitative[4] properties of the matrix B. Most of the remainder of the paper will be concerned with the relationship between the properties of B, which we can call the *consumption technology*[5] of the economy, and the behavior of consumers.

Certain properties of the transformations between G- and C-space follow immediately from the fact that B is a matrix of constants, and the transformation $z = Bx$ is linear. These can be stated as follows, proof being obvious.

a) A convex set in G-space will transform into a convex set in C-space, so that the budget constraint $px \leqq k$, $x \geqq 0$ will become a convex constraint on the z's.

b) An inverse transformation will not necessarily exist, so that an arbitrary vector z in C-space may have no vector x in G-space corresponding to it.

c) Where an inverse transformation does exist from C-space into G-space, it will transform convex sets into convex sets so that, for any set of z's which do have images in G-space, the convexity of the U function on the z's will be preserved in relation to the x's.

The properties are sufficient to imply that utility maximization subject to constraint will lead to determinate solutions for consumer behavior.

V. THE STRUCTURE OF CONSUMPTION TECHNOLOGY

The consumption technology, which is as important a determinant of consumer behavior as the particular shape of the utility function, is described fully only by the A and B matrixes together, but certain types of behavior can be related to more generalized descriptions of the technology. We shall distinguish broadly between structural properties of the technology, such as the relationship between the number of rows and columns of B and/or A and whether A, B are decomposable, and qualitative properties, such as the signs of the elements of A and B.

The leading structural property of the

[4] "Qualitative" is used here in a somewhat more general sense than in the author's work on the properties of qualitatively defined systems for which see Lancaster (1962, 1965).

[5] If the relationship between goods and activities is not one-to-one, the consumption technology consists of the two matrixes B, A, as in the technology of the Von Neumann growth model.

consumption technology is the relationship between the number of characteristics (r) and the number of activities (m), that is, between the number of rows and columns of B. It will be assumed that B contains no linear dependence, so that its rank is the number of rows or columns, whichever is less. We shall assume, unless otherwise stated, a one-to-one relationship between goods and activities.

1. The number of characteristics is equal to the number of goods. In this case, there is a one-to-one relationship between activities vectors and characteristics vectors. We have $z = Bx$, $x = B^{-1}z$. If B is a permutation of a diagonal matrix then there is a one-to-one relationship between each component of z and each component of y, and the model becomes, by suitable choice of units, exactly the same as the traditional model. If B is not a diagonal permutation, the objects of utility are composite goods rather than individual goods, and the model has some important differences from the conventional analysis. Note how specialized is the traditional case in relation to our general model.

If B is a diagonal permutation but there is not a one-to-one relationship between activities and goods so that A is not a diagonal permutation, we have a model similar to that of Morishima (1959).

2. The number of characteristics is greater than the number of goods. In this case, the relationships $Bx = z$ contain more equations than variables x_i so that we cannot, in general, find a goods vector x which gives rise to an arbitrarily specified characteristics vector z. We can take a basis of any arbitrarily chosen n characteristics and consider the reduced $n \times n$ system $\bar{B} = \bar{z}$, which gives a one-to-one relationship between n characteristics and the n goods, with the remaining $r-n$ characteristics being determined from the remaining $r-n$ equations and the goods vector x corresponding to $\bar{z}$. In this case, it is generally most useful to analyze consumer behavior by transforming the utility function into G-space, rather than the budget constraint into C-space. What does the transformed utility function look like?

As shown in the Appendix, the utility function transformed into G-space retains its essential convexity. An intuitive way of looking at the situation is to note that all characteristics collections which are actually available are contained in an n-dimensional slice through the r-dimensional utility function, and that all slices through a convex function are themselves convex. The transformation of this n-dimensional slice into G-space preserves this convexity.

For investigation of most aspects of consumer behavior, the case in which the number of characteristics exceeds the number of goods—a case we may often wish to associate with simple societies—can be treated along with the very special case (of which conventional analysis is a special subcase) in which the number of characteristics and goods is equal. In other words, given the consumption technology, we concern ourselves only with the particular n-dimensional slice of the r-dimensional utility function implied by that technology[6] and, since the slice of the utility function has the same general properties as any n-dimensional utility function, we can proceed as if the utility function was defined on only n characteristics.

[6] Assuming no decomposability or singularities in the consumption technology matrix B, then, if z_n is the vector of any n components of z and B_n, the corresponding square submatrix of B, the subspace of C-space to which the consumer is confined, is that defined by $z_{r-n} = B_{r-n}B_n^{-1} z_n$, where z_{r-n}, B_{r-n} are the vector and corresponding submatrix of B consisting of the components not included in z_n, B_n.

3. In the third case, in which the number of goods exceeds the number of characteristics, a situation probably descriptive of a complex economy such as that of the United States, there are properties of the situation that are different from those of the two previous cases and from the conventional analysis.

Here, the consumption technology, $z = Bx$, has fewer equations than variables so that, for every characteristics vector there is more than one goods vector. For every point in his characteristics-space, the consumer has a choice between different goods vectors. Given a price vector, this choice is a pure efficiency choice, so that for every characteristics vector the consumer will choose the most efficient combination of goods to achieve that collection of characteristics, and the efficiency criterion will be minimum cost.

The efficiency choice for a characteristics vector z^* will be the solution of the canonical linear program

$$\begin{aligned} &\text{Minimize} \quad px \\ &\text{subject to } Bx = z^* \\ &\qquad\qquad\quad x \geqq 0 \,. \end{aligned}$$

Since this is a linear program, once we have the solution x^* for some z^*, with value k^*, we can apply a scalar multiple to fit the solution to any budget value k and characteristics vector $(k/k^*)z^*$. By varying z^*, the consumer, given a budget constraint $px = k$, can determine a characteristics frontier consisting of all z such that the value of the above program is just equal to k. There will be a determinate goods vector associated with each point of the characteristics frontier.

As in the previous case, it is easy to show that the set of characteristics vectors in C-space that are preferred or indifferent to z transforms into a convex set in G-space if it is a convex set in C-space; it is also easy to show that the set of z's that can be obtained from the set of x's satisfying the convex constraint $px \leqq k$ is also a convex set. The characteristics frontier is, therefore, concave to the origin, like a transformation curve. For a consumption technology with four goods and two characteristics, the frontier could have any of the three shapes shown in Figure 1. Note that, in general, if B is a positive matrix, the positive orthant in G-space transforms into a cone which lies in the interior of the positive orthant in C-space, a point illustrated in the diagrams.

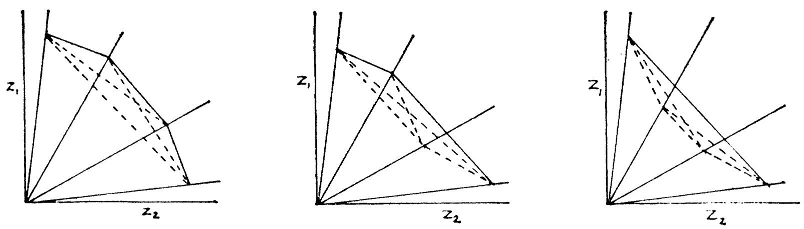

FIG. 1

A consumer's complete choice subject to a budget constraint $px \leqq k$ can be considered as consisting of two parts:

a) An efficiency choice, determining the characteristics frontier and the associated efficient goods collections.

b) A private choice, determining which point on the characteristics frontier is preferred by him.

The efficiency choice is an objective not a subjective choice. On the assumption that the consumption technology is objective, the characteristics frontier is also objective, and it is the same for all consumers facing the same budget constraint. Furthermore the characteristics frontier is expanded or contracted linearly and proportionally to an increase or decrease in income, so that the frontier has the same *shape* for all consumers facing the same prices, income differences simply being reflected in homogeneous expansion or contraction.

We should note that, if the consumption technology matrix has certain special structural properties, we may obtain a mixture of the above cases. For example, a matrix with the structure

$$B \equiv \begin{bmatrix} B_1 0 \\ 0 B_2 \end{bmatrix},$$

where B_1 is an $(s \times k)$ matrix and B_2 is an $(r-s) \times (n-k)$ matrix, partitions the technology into two disconnected parts, one relating s of the characteristics to k of the goods, the other separately relating $r-s$ of the characteristics to $n-k$ of the goods. We can have $s \geqq k$ and $r-s < n-k$ giving a mixed case.

Dropping the assumption of a one-to-one relationship between goods and activities does not add greatly to the difficulties of the analysis. We have, as part of the technology, $x = Ay$, so that the budget constraint $px \leqq k$ can be written immediately as $pAy \leqq k$. The goods prices transform directly into implicit activity prices $q = pA$. Interesting cases arise, of course. If the number of goods is less than the number of activities, then not all q's are attainable from the set of p's; and if the number of goods exceeds the number of activities, different p vectors will correspond to the same q vector. This implies that certain changes in relative goods prices may leave activity prices, and the consumer's choice situation, unchanged.

In most of the succeeding analysis, we will be concerned with the B matrix and the relationship between activities and characteristics, since this represents the most distinctive part of the theory.

VI. THE EFFICIENCY SUBSTITUTION EFFECT AND REVEALED PREFERENCE

At this stage, it is desirable to examine the nature of the efficiency choice so that we can appreciate the role it plays in the consumer behavior implied by our model. Consider a case in which there are two characteristics, a case that can be illustrated diagrammatically, and, say, four activities.

The activities-characteristics portion of the consumption technology is defined by the two equations

$$\begin{aligned} z_1 &= b_{11}y_1 + b_{12}y_2 + b_{13}y_3 + b_{14}y_4 \,; \\ z_2 &= b_{21}y_1 + b_{22}y_2 + b_{23}y_3 + b_{24}y_4 \,. \end{aligned} \qquad (6.1)$$

With activity 1 only, the characteristics will be obtained in proportion, b_{11}/b_{21} (the ray labeled 1 in Fig. 2). Similarly with activities 2, 3, 4, one at a time, characteristics will be obtained in proportions b_{12}/b_{22}, b_{13}/b_{23}, b_{14}/b_{24}, respectively, corresponding to the rays 2, 3, 4 in the diagram.

We are given a budget constraint in goods space of the form $\Sigma_i p_i x_i \leqq k$. If there is a one-to-one correspondence between goods and activities, the prices of the activities are given by p_i. If there is not a one-to-one relationship, but a goods-activities portion of the consumption technology

$$x_i = a_{i1}y_1 + a_{i2}y_2 + a_{i3}y_3 + a_{i4}y_4 \qquad i = 1 \ldots n\,, \qquad (6.2)$$

then the budget constraint can be transformed immediately into characteristics space

$$\left(\sum_i p_i a_{i1}\right) y_1 + \left(\sum_i p_i a_{i2}\right) y_2 + \left(\sum_i p_i a_{i3}\right) y_3 + \left(\sum_i p_i a_{i4}\right) y_4 \leqq k \tag{6.3}$$

where the composite prices $q_j = \Sigma_i p_i a_{ij}$, $j = 1..4$ represent the prices of each activity. The number of goods in relation to the number of activities is irrelevant at this stage, since each activity has a unique and completely determined price q_j, given the prices of the goods.

Given q_1, q_2, q_3, q_4, and k, the maximum attainable level of each activity in isolation can be written down (corresponding to the points E_1, E_2, E_3, E_4 in Fig. 2,) and the lines joining these points represent combinations attainable subject to the budget constraint. In the diagram it has been assumed that prices are such that combinations of 1 and 2, 2 and 3, 3 and 4 are efficient, giving the characteristics frontier, while combinations 1 and 3, 2 and 4, or 1 and 4 are inefficient.

Suppose that the consumer chooses characteristics in the combination represented by the ray z^*, giving a point E^* on the frontier. Now suppose that relative prices change: in particular, that the price of activity 2 rises so that, with income still at k, the point E_2 moves inward on ray 2. If the movement is small enough, the characteristics frontier continues to have a corner at E_2, and the consumer will continue to obtain characteristics in proportion z^* by a combination of activities 1 and 2. If income is adjusted so that the new frontier goes through E^*, the consumer will use the same activities in the same proportions as before.

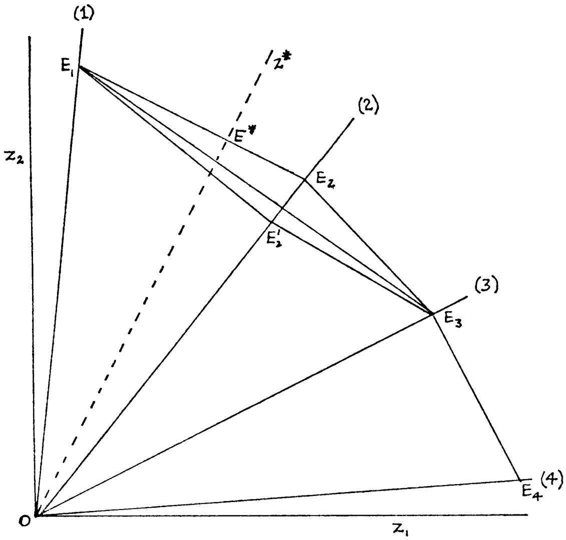

FIG. 2

If the price of activity 2 rises sufficiently, however, the point E_2 will move inward past the line joining E_1 and E_3 to E_2'. Combinations of 1 and 2 and of 2 and 3 are now inefficient combinations of activities, their place on the efficiency frontier being taken by a combination of 1 and 3. The consumer will switch from a combination of activities 1 and 2 to a combination of 1 and 3.

Thus there is an efficiency substitution effect which is essentially a switching effect. If price changes are too small to cause a switch, there is no efficiency substitution effect: If they are large enough, the effect comes from a complete switch from one activity to another.

The manifestation of the efficiency substitution effect in goods space depends on the structure of the A (goods-activities) matrix. There are two polar cases:

a) If there is a one-to-one relationship between goods and activities, the efficiency substitution effect will result in a complete switch from consumption of one good to consumption of another. This might be regarded as typical of situations involving similar but differentiated products, where a sufficiently large price change in one of the products will result in widespread switching to, or away from, the product.

b) If there is not a one-to-one relationship between goods and activities and, in particular, if all goods are used in all activities, the efficiency substitution effect will simply result in less consumption of a good whose price rises, not a complete disappearance of that good from consumption. If all cakes require eggs but in different proportions, a rise in the price of eggs will cause a switch from egg-intensive cakes to others, with a decline in the consumption of eggs, but not to zero.

The existence of an efficiency substitution effect depends, of course, on the number of activities exceeding the number of characteristics (otherwise switching of activities will not, in general, occur[7]) but does not require that the number of goods exceed the number of characteristics. In fact, with two goods, two characteristics, and three activities, the effect may occur. With two goods, two characteristics and one hundred activities (well spread over the spectrum), an almost smooth efficiency substitution effect would occur.

Since the efficiency substitution effect implies that consumers may change goods collections as a result of compensated relative price changes, simply in order to obtain the same characteristics collection in the most efficient manner, it is obvious that the existence of substitution does not of itself either require or imply convexity of the preference function on characteristics. In other words, the axiom of revealed preference may be satisfied even if the consumer always consumes characteristics in fixed proportions (and possibly even if the consumers had *concave* preferences), so that the "revelation" may be simply of efficient choice rather than convexity. A formal proof is given in the Appendix.

VII. OBJECTIVE AND SUBJECTIVE CHOICE AND DEMAND THEORY

In an economy or subeconomy with a complex consumption technology (many goods relative to characteristics), we have seen that there are two types of substitution effect:

1. Changes in relative prices may result in goods bundle I becoming an *in-*

[7] This is a somewhat imprecise statement in that, if the B matrix is partitionable into disconnected subtechnologies, for some of which the number of activities exceeds the number of characteristics and for others the reverse, an efficiency-substitution effect may exist over certain groups of activities, although the number of activities is less than the number of characteristics over-all.

efficient method of attaining a given bundle of characteristics and being replaced by goods bundle II even when the characteristics bundle is unchanged.

2. Changes in relative prices, with or without causing efficiency substitutions as in type 1, may alter the slope of the characteristics frontier in a segment relevant to a consumer's characteristics choice. The change in the slope of the frontier is analogous to the change in the budget line slope in the traditional case and, with a convex preference function, will result in a substitution of one characteristics bundle for another and, hence, of one goods bundle for another. Note that, even with smoothly convex preferences, this effect may not occur, since the consumer may be on a corner of the polyhedral characteristics frontier, and thus his characteristics choice could be insensitive to a certain range of slope changes on the facets.

The first effect, the efficiency substitution effect, is universal and objective. Subject to consumer ignorance or inefficiency,[8] this substitution effect is independent of the shapes of individual consumers' preference functions and hence of the effects of income distribution.

The second effect, the private substitution effect, has the same properties, in general, as the substitution effect in traditional theory. In particular, an aggregately compensated relative price change combined with a redistribution of income may result in no substitution effect in the aggregate, or a perverse one.

These two substitution effects are independent—either may occur without the other in certain circumstances—but in general we will expect them both to take place and hence that their effects will be reinforcing, if we are concerned with a complex economy. Thus, the consumer model presented here, in the context of an advanced economy, has, in a sense, more substitution than the traditional model. Furthermore, since part of the total substitution effect arises from objective, predictable, and income-distribution-free efficiency considerations, our confidence in the downward slope of demand curves is increased even when income redistribution takes place.

Since it is well known that satisfaction of the revealed preference axioms *in the aggregate* (never guaranteed by traditional theory) leads to global stability in multimarket models (see, for example, Karlin, 1959), the efficiency substitution effect increases confidence in this stability.

In a simple economy, with few goods or activities relative to characteristics, the efficiency substitution effect will be generally absent. Without this reinforcement of the private substitution effect, we would have some presumption that perverse consumer effects ("Giffen goods," backward-bending supply curves) and lower elasticities of demand would characterize simple economies as compared with complex economies. This seems to be in accord with at least the mythology of the subject, but it is certainly empirically verifiable. On this model, consumption technology as well as income levels differentiate consumers in different societies, and we would not necessarily expect a poor urban American to behave in his consumption like a person at the same real-income level in a simple economy.

[8] One of the properties of this model is that it gives scope for the consumer to be more or less efficient in achieving his desired characteristics bundle, although we will usually assume he is completely efficient. This adds a realistic dimension to consumer behavior (traditional theory never permits him to be out of equilibrium) and gives a rationale for the Consumers' Union and similar institutions.

VIII. COMMODITY GROUPS, SUBSTITUTES, COMPLEMENTS

In a complex economy, with a large number of activities and goods as well as characteristics, and with a two-matrix (A, B) consumption technology, it is obvious that taxonomy could be carried out almost without limit, an expression of the richness of the present approach. Although an elaborate taxonomy is not very useful, discussion of a few selected types of relationships between goods can be of use. One of the important features of this model is that we can discuss relationships between goods, as revealed in the structure of the technology. In the conventional approach, there are, of course, no relationships between goods as such, only properties of individual's preferences.

The simplest taxonomy is that based on the zero entries in the technology matrixes. It may be that both matrixes A, B are almost "solid," in which case there is little to be gained from a taxonomic approach. If, however, the B matrix contains sufficient zeros to be decomposable as follows,

$$B \equiv \begin{bmatrix} B_1 0 \\ 0 B_2 \end{bmatrix}, \qquad (7.1)$$

so that there is some set of characteristics and some set of activities such that these characteristics are derived only from these activities and these activities give rise to no other characteristics, then we can separate that set of characteristics and activities from the remainder of the technology. If, further, the activities in question require a particular set of goods which are used in no other activities (implying a decomposition of the A matrix), then we can regard the goods as forming an *intrinsic commodity group.* Goods within the group have the property that efficiency substitution effects will occur only for relative price changes within the group and will be unaffected by changes in the prices of other goods. If the utility function on characteristics has the conventional properties, there may, of course, be *private* substitution effects for goods within the group when the prices of other goods changes. For an intrinsic commodity group, the whole of the objective analysis can be carried out without reference to goods outside the group.

Goods from different intrinsic commodity groups can be regarded as *intrinsically unrelated,* goods from the same group as *intrinsically related.*

If, within a group, there are two activities, each in a one-to-one relationship with a different good, and if the bundles of characteristics derived from the two goods differ only in a scalar (that is, have identical proportions), we can regard the two goods in question as *intrinsic perfect substitutes.* If the associated characteristics bundles are similar, the goods are *close substitutes.* We can give formal respectability to that traditional butter-margarine example of our texts by considering them as two goods giving very similar combinations of characteristics.

On the other hand, if a certain activity requires more than one good and if these goods are used in no other activity we can consider them as *intrinsic total complements* and they will always be consumed in fixed proportions, if at all.

Many goods within a commodity group will have relationships to each other which are partly complementary and partly substitution. This will be true if two goods, for example, are used in different combinations in each of several activities, each activity giving rise to a similar combination of characteristics. The goods are complements within each activity, but the activities are substitutes.

IX. LABOR, LEISURE, AND OCCUPATIONAL CHOICE

Within the structure of the present theory, we can regard labor as a reversed activity, using characteristics as inputs and producing commodities or a commodity as output. This is similar to the standard approach of generalized conventional theory, as in Debreu (1959).

We can add to this approach in an important way within the context of the present model by noting that a work activity may produce characteristics, as well as the commodity labor, as outputs. This is structurally equivalent to permitting some of the columns of the B matrix to have both negative and positive elements, corresponding to activities that "use up" some characteristics (or produce them in negative quantities) and produce others. In a work activity, the corresponding column of the A matrix will contain a single negative coefficient for the commodity labor, or, more differentiated, for one or more types of labor. If a work activity corresponds to a column of mixed signs in the B matrix, it is a recognition of the obvious truth that some work activities give rise to valued characteristics directly from the work itself.

Consider a very simple model of two characteristics with two commodities, labor and consumption goods. Both labor and consumption goods correspond to separate activities giving rise to the two characteristics in different proportions—perhaps negative in the case of labor. With no income other than labor, and only one good available to exchange for labor, we can collapse work and consumption into a single work-consumption activity. Given the wage rate in terms of the consumption good, the characteristics resulting from the work-consumption activity are given by a linear combination of the characteristics from work and consumption separately, the weights in the combination being given by the wage rate.

Add another activity, leisure, which gives rise to the two characteristics, and the constraint that the weighted sum of the levels of activity labor and activity leisure is a constant.

The model is illustrated in Figure 3. W represents a work-consumption activity giving positive levels of both characteristics, l represents a leisure activity, also giving positive levels of both characteristics. The constraint on total time (so that a linear combination of w and l is a constant) is represented by some line joining w, l.

If the constraint line has, like AB in the diagram, a negative slope, then individual consumers' utility functions will be tangent to the constraint at different points (like m, m') and we will have a neoclassical type of labor-leisure choice in which the proportions depend on individual preferences. Some consumers' preferences may be such that they will choose A (maximum work) or B (maximum leisure), but it is a private choice.

In this model, however, for a certain level of the wage, given the coefficients of the technology, the constraint may have a positive slope as in $A'B$, or AB'. If the constraint is $A'B$ (corresponding, *ceteris paribus*, to a sufficiently low real wage), *all* individuals will choose B, the only efficient point on the constraint set $OA'B$. At a sufficiently high wage, giving constraint set OAB', A, the maximum labor choice, is the only efficient choice and will be chosen by *all* individuals.

The above effect, in which for some wage range there is a private labor-leisure choice between efficient points while outside the range all individuals will take maximum work or maximum leisure, can only occur if both the work-consumption and leisure activities give both characteristics in positive amounts. If the using up of characteristic 2 in labor exceeded the amount of that characteristic gained by consumption, then the work-consumption activity might lie outside the positive quadrant, like w'. In this case, a constraint like $A'B$ can exist, but not one like AB'. Furthermore, if the consumer will choose only positive characteristics vectors, no consumer will choose maximum work.

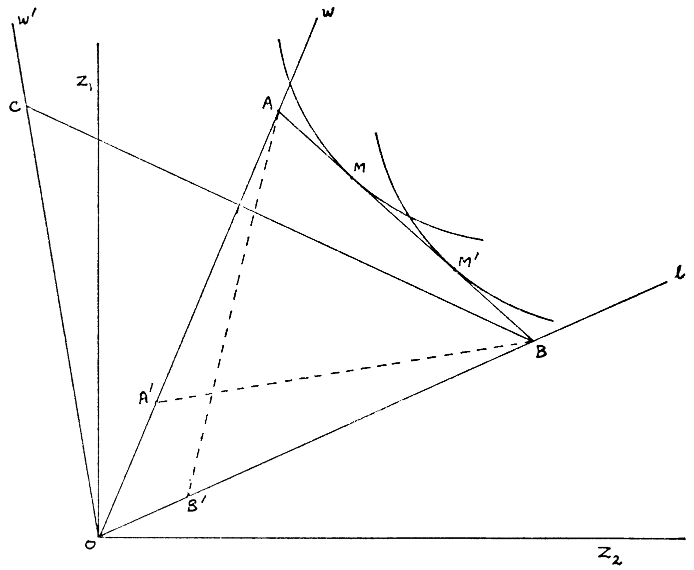

FIG. 3

This model of the labor-leisure choice, which provides for objective and universal efficiency choices as well as private choices, may be the basis for a useful working model for an underdeveloped area. If the "leisure" be defined as "working one's own field," the work-consumption activity as entering the market economy, we see that there will be wages below which no peasant will offer himself as paid labor and that this is an *efficiency* choice and not a private choice.

We can use the same type of model also to analyze occupational choice. Suppose that we have two types of work (occupations) but otherwise the conditions are as above. If and only if the characteristics arising from the work itself are different in the two occupations, the two work-consumption activities will give rise to activities in different combinations. If the work characteristics are in the same proportion, the characteristics of the work-consumption activity will be in the same proportions and one or the other occupation will be the only efficient way to achieve this characteristics bundle.

Figure 4 illustrates one possible set of relationships for such a model. In the diagram, w_1, w_2 represent the characteristics combinations from work-consumption activities in occupations 1 and 2, l the characteristics combinations from leisure. The frontier consists of the lines AC (combinations of w_1 and leisure) and AB (combinations of w_2 and leisure). We shall impose the realistic restriction that an individual can have only a single occupation so that AB is not a possible combination of activities.

The choice of occupation, given the relationships in the figure, depends on personal preferences, being M_1 (combination of w_2 and leisure) for an individual with preferences skewed towards z_2 and M_2 for an individual with preferences skewed towards z_1. But note a special effect. For some individuals whose indifference curves cannot touch BC but can touch AC, the efficient choice will be the corner solution M_3 (= B). There is, in fact, a segment of AC to the left of w_2 (the part of AC to the right of w_2 is dominated by BC), lying below the horizontal through B which is inefficient relative to B and will never be chosen.

In a configuration like the above we have the very interesting effect, where those who choose occupation 1 will work very hard at it; leisure-lovers will choose private combinations of occupation 2 and leisure—surely a good description of effects actually observed.

The loss to certain individuals from confinement to a single occupation is obvious. Could he choose a combination of occupations 1 and 2, the individual at M_2 would do so and be better off than with a combination of occupation 1 and leisure. In a two-characteristic, three-activity model, of course, two activities will be chosen at most, so that leisure plus both occupations will not appear.

The configuration in the diagram (Fig. 4) represents the situation for some set of technical coefficients and specific wages in the two occupations. A large number of other configurations is possible. In particular, if the wage rate in occupation 2 fell sufficiently, BC would lie inside AC and occupation 2 would cease to be chosen

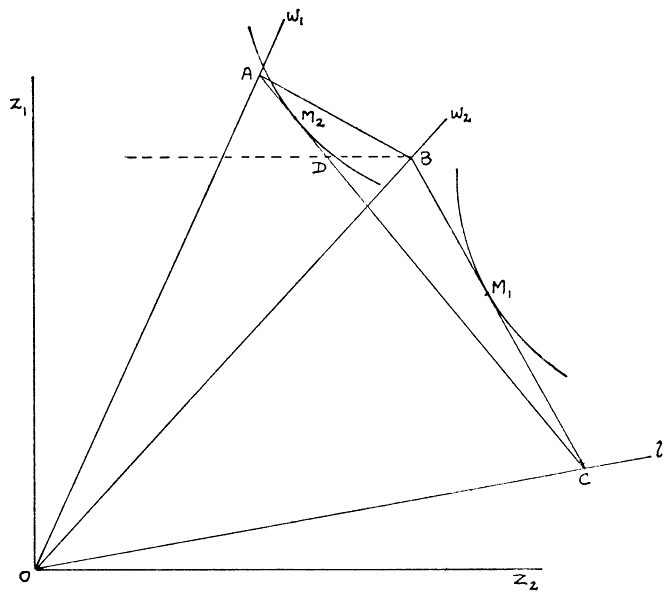

FIG. 4

by any individual. All individuals, in this case, would choose their various personal combinations of occupation 1 and leisure.

Confinement to a single occupation need not result in a welfare loss, even when neither occupation dominates the other in an efficiency sense. If the technical coefficients were different, so that the characteristics vectors representing occupation 2 and leisure changed places, then the work-leisure combinations would be given by AB and BC, both efficient relative to any combination of occupations 1 and 2. In this case, all individuals would optimize by some combination of leisure and any one of the occupations.

Approaches similar to those outlined above seem to provide a better basis for analysis of occupational choice than the traditional, non-operational, catch-all "non-monetary advantages."

X. CONSUMER DURABLES, ASSETS, AND MONEY

Within the framework of the model, we have a scheme for dealing with durable goods and assets. A durable good can be regarded simply as giving rise to an activity in which the output consists of dated characteristics, the characteristics of different dates being regarded as different characteristics.

Given characteristics as joint outputs and two types of dimension in characteristics space—cross-section and time—any asset or durable good can be regarded as producing a combination of several characteristics at any one time, and that combination need not be regarded as continuing unchanged through time. In the decision to buy a new automobile, for example, the characteristic related to "fashion" or "style" may be present in relative strength in the first season, relatively less in later seasons, although the characteristics related to "transportation" may remain with constant coefficients over several seasons.

Elementary textbooks stress the multidimensional characteristics of money and other assets. The present model enables this multidimensionality to be appropriately incorporated. "Safety," "liquidity," and so forth become workable concepts that can be related to characteristics. We can use analysis similar to that of the preceding sections to show why efficiency effects will cause the universal disappearance of some assets (as in Gresham's Law) while other assets will be held in combinations determined by personal preferences. It would seem that development along these lines, coupled with development of some of the recent approaches to consumer preferences over time as in Koopmans (1960), Lancaster (1963), or Koopmans, Diamond, and Williamson (1964) might eventually lead to a full-blooded theory of consumer behavior with respect to assets—saving and money—which we do not have at present.

In situations involving risk, we can use multiple characteristics better to analyze individual behavior. For example, we might consider a gamble to be an activity giving rise to three characteristics—a mathematical expectation, a maximum gain, and a maximum loss. One consumer's utility function may be such that he gives more weight to the maximum gain than to the maximum loss or the expected value, another's utility function may be biased in the opposite direction. All kinds of models can be developed along these lines, and they are surely more realistic than the models (Von Neumann and Morgenstern, 1944; Friedman and Savage, 1952) in which the expected value, alone, appears in the utility-maximizing decisions.

XI. NEW COMMODITIES, DIFFERENTIATED GOODS, AND ADVERTISING

Perhaps the most difficult thing to do with traditional consumer theory is to introduce a new commodity—an event that occurs thousands of times in the U.S. economy, even over a generation, without any real consumers being unduly disturbed. In the theory of production, where activity-analysis methods have become widely used, a new process or product can be fitted in well enough; but in consumer theory we have traditionally had to throw away our n-dimensional preference functions and replace them by totally new $(n + 1)$ dimensional functions, with no predictable consequences.

In this model, the whole process is extraordinarily simple. A new product simply means addition of one or more activities to the consumption technology. Given the technology (or the relevant portion of it) and given the intrinsic characteristic of the activity associated with the new good, we simply insert it in the appropriate place in the technology, *and we can predict the consequences.*

If a new good possesses characteristics in the same proportions as some existing good, it will simply fail to sell to anyone if its price is too high, or will completely replace the old good if its price is sufficiently low.

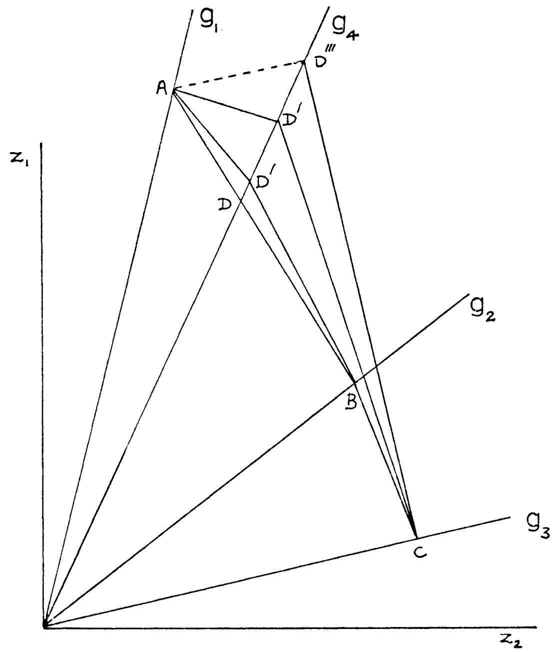

FIG. 5

More usually, we can expect a new good to possess characteristics in somewhat different proportions to an existing good. If its price is too high, it may be dominated by some *combination* of existing goods and will fail to sell. If its price is sufficiently low, it will result in adding a new point to the efficiency frontier. In Figure 5, *ABC* represents the old efficiency frontier, on which some individuals will consume combinations of goods g_1 and g_2 in various proportions, some combinations of g_2 and g_3. If the price of the new good, g_4, is such that it represents a point, D, on the old efficiency frontier, some persons (those using combinations of g_1 and g_2) will be indifferent between their old combinations and combinations of either g_1 and g_4 or g_2 and g_4. If the price of g_4 is a little lower, it will push the efficiency frontier out to D'. Individuals will now replace combinations of g_1 and g_2 with combinations of g_1 and g_4 or g_2 and g_4, depending on their preferences. The new good will have taken away some of the sales from both g_1 and g_2, but completely replaced neither.

If the price of g_4 were lower, giving point D'', then combinations of g_4 and g_3 would dominate g_2, and g_2 would be replaced. At an even lower price, like D''', combinations of g_4 and g_3 would dominate g_2, and the corner solution g_4 only would dominate all combinations of g_1 and g_4 (since AD''' has a positive slope), so that g_4 would now replace both g_1 and g_2.

Differentiation of goods has presented almost as much of a problem to traditional theory as new commodities. In the present analysis, the difference is really one of degree only. We can regard a differentiated good typically as a new good within an existing intrinsic commodity group, and within that group analyze it as a new commodity. Sometimes there appear new commodities of a more fundamental kind whose characteristics cut across those of existing groups.

We may note that differentiation of goods, if successful (that is, if the differentiated goods are actually sold) represents a welfare improvement since it pushes the efficiency frontier outward an enables the consumer more efficiently to reach his preferred combination of characteristics.

Many economists take a puritanical view of commodity differentiation since their theory has induced them to believe that it is some single characteristic of a commodity that is relevant to consumer decisions (that is, automobiles are only for transportation), so that commodity variants are regarded as wicked tricks to trap the uninitiated into buying unwanted trimmings. This is not, of course, a correct deduction even from the conventional analysis, properly used, but is manifestly incorrect when account is taken of multiple characteristics.

A rather similar puritanism has also been apparent in the economist's approach to advertising. In the neoclassical analysis, advertising, if it does not represent simple information (and little information is called for in an analysis in which a good is simply a good), is an attempt to "change tastes" in the consumer. Since "tastes" are the ultimate datum in welfare judgments, the idea of changing them makes economists uncomfortable.

On the analysis presented here, there is much wider scope for informational advertising, especially as new goods appear constantly. Since the consumption technology of a modern economy is clearly very complex, consumers require a great deal of information concerning that technology. When a new version of a dishwashing detergent is produced which contains hand lotion, we have a product with characteristics different from those of the old. The consumption technology is changed, and consumers are willing to pay to be told of the change. Whether the new product pushes out the efficiency frontier (compared, say, with a combination of dishwasher and hand lotion consumed separately) is, of course, another matter.

In any case, advertising, product design, and marketing specialists, who have a heavy commitment to understanding how consumers actually do behave, themselves act as though consumers regard a commodity as having multiple characteristics and as though consumers weigh the various combinations of characteristics contained in different commodities in reaching their decisions. At this preliminary stage of presenting the model set out here, this is strong evidence in its favor.

XII. GENERAL EQUILIBRIUM, WELFARE, AND OTHER MATTERS

Since the demand for goods depends on objective and universal efficiency effects as well as on private choices, we can draw some inferences relative to equilibrium in the economy.

A commodity, especially a commodity within an intrinsic commodity group, must have a price low enough relative to the prices of other commodities to be represented on the efficiency frontier, otherwise it will be purchased by no one and will not appear in the economy. This implies that if there are n viable commodities in a group, each in a one-to-one relation to an activity, the equilibrium prices will be such that the efficiency frontier has $n-1$ facets in the two-characteristic case. In Figure 6, for example, where the price of commodity 3 brings

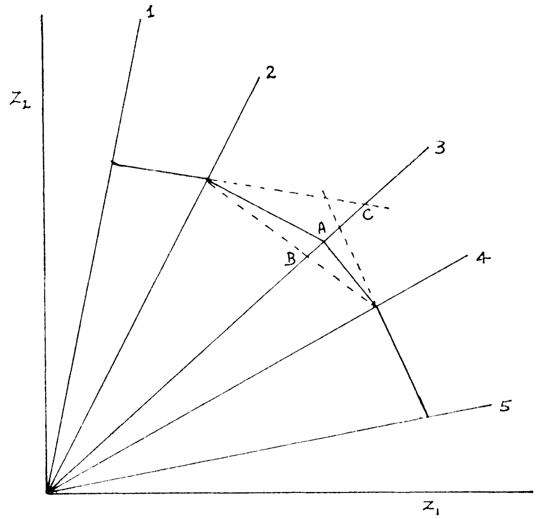

FIG. 6

it to point A on the efficiency frontier, that price could not be allowed to rise to a level bringing it inside point B, or it would disappear from the market; and if its price fell below a level corresponding to C, commodities 2 and 4 would disappear from the market. Thus the limits on prices necessary for the existence of all commodities within a group can be established (in principle) from objective data. Only the demand within that price range depends on consumer preferences.

With a large number of activities relative to characteristics, equilibrium prices would give a many-faceted efficiency frontier that would be approximated by a smooth curve having the general shape of a production possibility curve. For many purposes it may be mathematically simple to analyze the situation in terms of a smooth efficiency frontier. We can then draw on some of the analysis that exists, relating factor inputs to outputs of goods, as in Samuelson (1953*b*). Goods in our model correspond to factors in the production model, and characteristics in our model to commodities in the production model.

The welfare implications of the model set out here are quite complex and deserve a separate treatment. We might note several important aspects of the welfare problem, however, which arise directly from a many-faceted, many-cornered efficiency frontier:

1. Consumers whose choices represent a corner on the efficiency frontier are not, in general, *equating* marginal rates of substitution between characteristics to the ratio of any parameters of the situation or to marginal rates of substitution of other consumers.

2. Consumers whose choices represent points on different facets of the efficiency frontier are equating their marginal rates of substitution between characteristics to different implicit price ratios between characteristics. If there is a one-to-one relationship between goods and activities, the consumers are reacting to relative prices between different sets of goods. The traditional marginal conditions for Paretian exchange optimum do not hold because the price ratio relevant to one consumer's decisions differs from the price ratio relevant to another's. In common-sense terms, the price ratio between a Cadillac and a Continental is irrelevant to my decisions, but the price ratio between two compact cars is relevant, while there are other individuals for whom the Cadillac/Continental ratio is the relevant datum. If the A matrix is strongly connected, however, the implicit price ratios between different activities can correspond to price ratios between the same sets of goods, and the Paretian conditions may be relevant.

Finally, we may note that the shape of the equilibrium efficiency frontier and the existence of the efficiency substitution effect can result in demand conditions with the traditionally assumed properties, even if the traditional, smooth, convex utility function does not exist. In particular, a simple utility function in which characteristics are consumed in constant proportions—the proportions perhaps changing with income—can be substituted for the conventional utility function.

XIII. OPERATIONAL AND PREDICTIVE CHARACTERISTICS OF THE MODEL

In principle, the model set out here can be made operational (that is, empirical coefficients can be assigned to the technology). In practice, the task will be more difficult than the equivalent task of determining the actual production technology of an economy.

To emphasize that the model is not simply heuristic, we can examine a simple scheme for sketching out the efficiency frontier for some commodity group. We shall assume that there is a one-to-one relationship between activities and goods, that at least one characteristic shared by the commodities is capable of independent determination, and that a great quantity of suitable market data is available.

In practice, we will attempt to operate with the minimum number of characteristics that give sufficient explanatory power. These may be combinations of fundamental characteristics (a factor-analysis situation) or fundamental characteristics themselves.

Consider some commodity group such as household detergents. We have a primary objective characteristic, cleaning power, measured in some chosen way. We wish to test whether one or more other characteristics are necessary to describe the consumer-choice situation.

We take a two-dimensional diagram with characteristic "cleaning power" along one axis. Along the axis we mark the cleaning power per dollar outlay of all detergents observed to be sold at the same time. If this is the same for all detergents, this single characteristic describes the situation, and we do not seek further. However, we shall assume this is not so. From our observed market data, we obtain cross-price elasticities between all detergents, taken two at a time. From the model, we know that cross-price elasticities will be highest between detergents with adjacent characteristics vectors, so that the order of the characteristics vectors as we rotate from one axis to the other in the positive quadrant can be established.

The ordering of "cleaning power per dollar" along one axis can be compared with the ordering of the characteristics vectors. If the orderings are the same, an equilibrium efficiency frontier can be built up with two characteristics as in Figure 7a. The slopes of the facets can be determined within limits by the limiting prices at which the various detergents go off the market. If the ordering in terms of cleaning power does not agree with the ordering in terms of cross-elasticity, as in Figure 7b, two characteristics do not describe the market appropriately, since detergent with cleaning power 3 in the figure cannot be on the efficiency frontier. But with a third characteristic, detergent 3 could be adjacent to detergents 2 and 1 in an extra dimension, and we could build up an efficiency frontier in three characteristics.

Other evidence could, of course, be used to determine the efficiency frontier for a given market situation. Among this evidence is that arising from ordinary activity-analysis theory, that, with r characteristics we would expect to find some consumers who used r commodities at the same time, unless all consumers were on corners or edges of the efficiency frontier.

Last, but possibly not least, simply asking consumers about the characteristics associated with various commodities may be much more productive than attempts to extract information concerning preferences within the context of conventional theory.

In general, if consumer preferences are well dispersed (so that all facets of the efficiency frontier are represented in some consumer's choice pattern), a combination of information concerning interpersonal variances in the collections of goods chosen and of the effects of price changes on both aggregate and individual choices can, in principle, be used to ferret out the nature of the consumption technology. Some of the problems that arise are similar to those met by psychologists in measuring intelligence, personality, and other multidimensional traits, so that techniques similar to those used in psychology, such as factor analysis, might prove useful.

Even without specification of the consumption technology, the present theory makes many predictions of a structural kind which may be contrasted with the predictions of conventional theory. Some of these are set out in Chart 1.

XIV. CONCLUSION

In this model we have extended into consumption theory activity analysis, which has proved so penetrating in its application to production theory. The crucial assumption in making this application has been the assumption that goods possess, or give rise to, multiple characteristics in fixed proportions and that it is these characteristics, not goods themselves, on which the consumer's preferences are exercised.

The result, as this brief survey of the possibilities has shown, is a model very many times richer in heuristic explanatory and predictive power than the con-

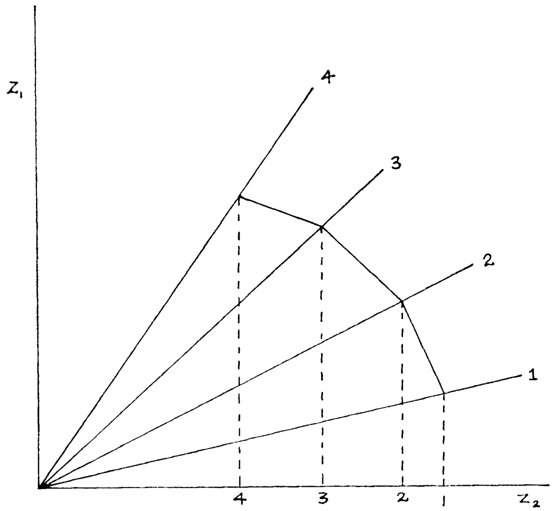

FIG. 7*a*

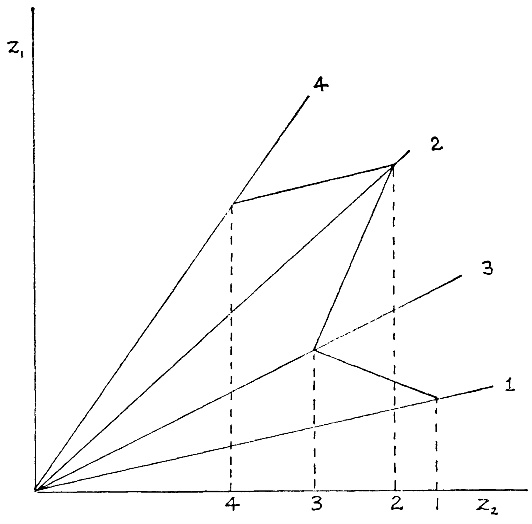

FIG. 7*b*

ventional model of consumer behavior and one that deals easily with those many common-sense characteristics of actual behavior that have found no place in traditional exposition.

This paper is nothing more than a condensed presentation of some of the great number of possible ways in which the model can be used. It is hoped that a door has been opened to a new, rich treasure house of ideas for the future development of the most refined and least powerful branch of economic theory, the theory of the consumer himself.

CHART 1

This Theory	Conventional Theory
Wood will not be a close substitute for bread, since characteristics are dissimilar	No reason except "tastes" why they should not be close substitutes
A red Buick will be a close substitute for a gray Buick	No reason why they should be any closer substitutes than wood and bread
Substitution (for example, butter and margarine) is frequently intrinsic and objective, will be observed in many societies under many market conditions	No reason why close substitutes in one context should be close substitutes in another
A good may be displaced from the market by new goods or by price changes	No presumption that goods will be completely displaced
The labor-leisure choice may have a marked occupational pattern	Labor-leisure choice determined solely by individual preferences; no pattern, other than between individuals, would be predicted
(Gresham's Law) A monetary asset may cease to be on the efficiency frontier, and will disappear from the economy	No ex ante presumption that any good or asset will disappear from the economy
An individual is completely unaffected by price changes that leave unchanged the portion of the efficiency frontier on which his choice rests	An individual is affected by changes in all prices
Some commodity groups may be intrinsic, and universally so	No presumption that commodities forming a group (defined by a break in spectrum of cross-elasticities) in one context will form a group in another context

APPENDIX

I. TRANSFORMATION OF THE UTILITY FUNCTION INTO G-SPACE

Consider some characteristics vector z^* which does have an image x^* in G-space, and consider the set P of all vectors z preferred or indifferent to z^*. If U has the traditional properties, the set P is convex with an inner boundary which is the indifference surface through z^*. Now $z \geqq z^*$ implies z is in P so that every x such that $Bx \geqq z^*$, a set S, is preferred or indifferent to x^*. If we take some other z' in P, every x in S' such that $Bx \geqq z'$ is also preferred or indifferent to x'^*. Similarly for z'' in P and S'' such that that $Bx \geqq z''$, and so on. From the theory of inequalities, the sets S, S', S'' . . . are all convex, and since P is convex, a linear combination of z', z'' is in P, so that a linear combination of x's in S', S'' is also preferred or indifferent to x^*. Hence the set $\bar{P}$ of all x preferred or indifferent to x^* is the linear combination of all the sets $S, S', S'', \ldots$ and so is convex.

Thus the utility function transformed into G-space retains its essential convexity. A more intuitive way of looking at the situation is to note that all characteristics collections which are actually available are contained in an n-dimensional slice through the r-dimensional utility function and that all slices through a convex function are themselves convex. The transformation of this n-dimensional slice into G-space preserves this convexity.

II. "REVEALED PREFERENCE" IN A COMPLEX ECONOMY

We shall use the structural properties of the consumption technology A, B (dropping the assumption of a one-to-one relationship between goods and activities) to show that in a complex economy with more activities than characteristics the efficiency choice always satisfies the weak axiom of revealed preference and will satisfy the strong axiom for sufficiently large price changes, so that satisfaction of even the strong axiom does not "reveal" convexity of the preference function itself.

Consider an economy with a consumption technology defined by

$$z = By ,$$

$$x = Ay ,$$

and a consumer subject to a budget constraint of the form $p^*x \leqq k$ who has chosen goods x^* for activities y^*, giving characteristics z^*.

We know that if the consumer has made an efficient choice, y^* is the solution of the program (the value of which is k).

$$\text{Minimize } p^*Ay \ (= p^*x): \quad By = z^* , y \geqq 0 , \qquad (8.1a)$$

which has a dual (solution v^*).

$$\text{Maximize } vz^*: vB \leqq p^*A . \qquad (8.1b)$$

The dual variables v can be interpreted as the implicit prices of the characteristics themselves. From the Kuh-Tucker Theorem, we can associate the vector v with the slope of the separating hyperplane between the set of attainable z's and the set of z's preferred or indifferent to z^*.

For the same satisfactions vector Z^* and a new price vector p^{**} the efficiency choice will be the solution y^{**} (giving x^{**}), v^{**}, of

$$\begin{aligned} &\text{Min } p^{**}Ay: By = z^* , y \geqq 0 , \\ &\text{Max } vz^*: vb \leqq p^{**}A . \end{aligned} \qquad (8.2)$$

Since z^* is the same in (8.1) and (8.2), y^{**} is a feasible solution of (8.1) and y^* of (8.2). From the fundamental theorem of linear programing we have

$$p^{**}Ay^* \geqq v^{**}z^* = p^{**}Ay^{**} , \qquad (8.3)$$

$$p^*Ay^{**} \geqq v^*z^* = p^*Ay^* . \qquad (8.4)$$

A program identical with (8.2) except that z^* is replaced by hz^* will have a solution hy^{**}, v^{**}. Choose h so that $hp^{**}Ay^{**} = p^{**}Ay^*$. From (8.3) $h \geqq 1$. From (8.4),

$$hp^*Ay^{**} \geqq p^*Ay^{**} \geqq p^*Ay^* . \qquad (8.5)$$

If we now write p for p^*, p' for p^{**}; $x = Ay^*$, $x' = hAy^{**}$, we have

$$p'x' = p'x \text{ implies } px' \geqq px , \qquad (8.6)$$

satisfying the *weak axiom of revealed preference.*

The equality will occur on the right in (8.6) only if equalities hold in *both* (8.3) and (8.4), and these will hold only if y^{**} is optimal as well as feasible in (8.1), and y^* is optimal as well as feasible in (8.2). In general, if the number of activities exceeds the number of characteristics, we can always find two prices p^*, p^{**} so related that neither of the solutions y^{**}, y^* is optimal in the other's program.

Hence, if the number of activities exceeds the number of characteristics (representing the number of primary constraints in the program), we can find prices so related that the strong axiom of revealed preference is satisfied, even though the consumer has obtained characteristics in unchanged proportions (z^*, hz^*) and has revealed nothing of his preference map.

The above effect represents an *efficiency substitution effect* which would occur even if characteristics were consumed in absolutely fixed proportions. If the consumer substitutes between different satisfactions bundles when his budget constraint changes, this private substitution effect is additional to the efficiency substitution effect.

Just as the conceptual experiment implicit in revealed preference implies "over-compensation" in the conventional analysis (see Samuelson 1948, 1953*a*), so the efficiency effect leads to "external overcompensation" additional to private overcompensation.

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