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Appropriation and Efficiency: A Revision of the First Theorem of Welfare Economics

By LOUIS MAKOWSKI AND JOSEPH M. OSTROY*

The First Theorem of Welfare Economics rests on the assumption that individuals have neither price-making nor market-making capacities. We offer a revision in which individuals have such capacities. The revision emphasizes two keys for market efficiency: (i) the need to align private rewards with social contributions—called full appropriation, and (ii) the need for an assumption to counter the possibility of coordination failures in the choice of produced commodities—called noncomplementarity. We also emphasize that information about prices of unmarketed commodities involves decentralized knowledge available only to product innovators and that pecuniary externalities are important potential sources of market failure. (JEL D51, D60, D62)

The First Theorem of Welfare Economics provides a set of sufficient conditions for a price system to efficiently coordinate economic activity. It is a beautiful result, with a strikingly simple proof. But its reliance on price-taking and complete markets contributes to a lack of explicit emphasis on strategic/incentive issues. This paper offers an alternative, complementary set of sufficient conditions for efficient coordination, one that emphasizes the importance of full appropriation rather than price-taking behavior.

Once appropriation is given center stage, our understanding of the reasons for economic efficiency deepens. For example, the fit between the theory of market failure (which already emphasizes problems of appropriability, in the form of externalities) and the theory of market success becomes tighter. To give a second illustration, in the First Theorem the set of available markets

must be complete; hence, product innovation cannot occur. By contrast, in a model for achieving economic efficiency based on appropriation, innovative activity may be regarded as endogenous. The extent that the innovator can fully appropriate the consequences of his innovations becomes the central question, as far as efficiency is concerned.

As this suggests, complete markets will not be a maintained assumption in what follows. There is, however, an important limitation: the model below does not address issues associated with moral hazard and adverse selection. In later work we hope to show how these too can be usefully regarded as appropriation problems. To give a suggestive illustration, it is well known that “residual claimant contracts” can efficiently resolve moral-hazard problems when agents are risk-neutral. By making the agent the residual claimant, one forces him to appropriate fully the consequences of his actions.¹

The First Theorem is based on the Walrasian model of economic coordination,

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¹J. G. Head (1962) also argues for the central importance of appropriation for welfare economics. He pursues the argument broadly and vigorously. Here we take a complementary and more formal route, relating appropriation to the First Theorem.

where individuals have neither price-making nor market-making capabilities. By contrast, our revision is based on an extension that we call a model of *occupational choice*. It is related to mechanism design in that the market outcome can be described as a Walrasian mechanism in which prices as well as marketed commodities respond to individuals' occupational choices. Thus, in the model of occupational choice there is:

price-making: individuals may be able to influence market-clearing prices by their choice of occupations; and

market-making: individuals determine the set of available markets by their choice of occupations.²

To illustrate its workings, production at different scales can be modeled as the choice of different occupations, so the producer may be able to influence prices by choosing to operate at a smaller scale. Or to illustrate market-making, different occupations may involve the introduction of different *new* commodities (in the model, markets are open only for commodities that can be actually supplied given individuals' occupational choices; hence the choice of occupations has a market-making role). An equilibrium in the model is called an *occupational equilibrium*.

Our main result identifies conditions under which occupational equilibria will be efficient, in spite of the greater scope for individual choice and self-interested behavior. We show that, if the following two conditions are met, then the allocation of resources will be Pareto efficient:

full appropriation (FA): each individual's private benefit from any occupational choice coincides with his/her social contribution in that occupation; and

noncomplementarity (NC): a subadditivity condition is satisfied among occupational choices made by different individuals.

²The model of occupational choice collapses to a standard Walrasian model when each individual has only one occupational choice (i.e., no choice at all).

The central condition, full appropriation, represents an extension of Pigou's "appropriation logic" which underlies both market success and failure. Specifically, FA requires private and social benefits to be aligned. Its role is to give individuals the right incentives in their occupational choices, and hence in both their price-making and market-making.

Because market-making is endogenous, even with full appropriation, *coordination failures* can occur. (Early examples of this phenomenon are underinnovation traps pointed out by Tibor Scitovsky [1954].) The noncomplementarity condition rules these cases out. This result is based on conditions identified by Oliver Hart (1980) and Makowski (1980b) as sufficient for efficient product innovation under perfect competition. While both of these studies contain heuristics pointing to the importance of appropriation, their primary focus is on a particular application, rather than on incorporating their findings into standard welfare economics (i.e., the First Theorem).

Some recent developments in macroeconomics and industrial organization study the implications of *strategic complementarities* in imperfectly competitive models (see Russell Cooper and Andrew John, 1988; Xavier Vives, 1989; Walter P. Heller, 1986; Paul Milgrom and John Roberts, 1990).³ In terms of our revision of the First Theorem, inefficiencies due to strategic complementarities arise from an amalgam of failures of FA and failures of NC.

We call our main result a "revision" because its two assumptions are stated in a language unlike that used in the First Theorem. Instead of emphasizing price-taking and complete markets, FA and NC directly describe the structure of individual payoffs that give good incentives. Such payoffs may arise in either a thick-market or a thin-market setting. In either setting, perfect

³The term "strategic complementarities" is found in Jeremy I. Bulow et al. (1985).

competition is a key ingredient for efficiency: it leads to FA. By “perfect competition” we mean not just price-taking, but something stronger: that individuals actually face *perfectly elastic demands and supplies* (PEDS).

Consider a thick-markets setting first, where all commodities are *standardized* and there are many buyers and sellers of each commodity. In such a setting, intense competition among buyers and among sellers will typically lead to PEDS. This is probably the most natural setting for applying the standard First Theorem: both price-taking and complete markets make sense here. We show that in a thick-markets setting, FA and NC are satisfied. Thus, in this setting the revision complements the First Theorem by making explicit the reward scheme that induces efficiency. Of course, one could say that the standard presentation, where price-taking is a shortcut for PEDS and complete markets is a shortcut for standardization, yields a simpler statement and shorter proof. Our claim is that taking the shortcut means bypassing the central issue of appropriability underlying market efficiency. (See Remark 3 for an important instance where the link between appropriability and efficiency was bypassed.)

Consider next a more dynamic thin-markets setting, where product innovation is an issue. Suppose that, among all conceivable commodities (a huge set), most are never innovated. A difficulty with interpreting the standard First Theorem in such a setting is its “uneconomical” use of price information: why should there be market prices for all conceivable commodities? Alternatively put, while it might be an appealing fiction to have an auctioneer announce prices of standardized commodities, the auctioneer may have no idea of what needs to be priced in a world of personalized commodities.

The model of occupational choice permits a more appealing, decentralized description of pricing in such a setting. We assume that each seller only has access to the prices of currently marketed commodities and the prices of commodities that he can innovate. For example, a supplier of a

word-processing program who is capable of also producing a (not yet existing) spreadsheet program knows the price at which the latter could be sold because that is part of his decentralized knowledge: it comes from participating in his segment of the software market. But he may have no idea about the potential price of a new item of clothing apparel or even the potential price of new computer hardware because that is not part of his technological expertise. We call this “local price information.” Markets are “complete” in the sense that the potential price of any new commodity is known by someone, but price information is decentralized because—assuming that any one seller can innovate only a narrow range of commodities—no one knows most prices.

Local price information may be too decentralized to reflect accurate information about complementarities among the commodities not currently marketed. The difficulty can be traced to a kind of pecuniary externality in which innovations by one individual affect the market valuations of others’ potential innovations. Nevertheless, we show that FA and NC (hence efficiency) will result under perfect competition, provided that local price information satisfies a consistency condition. As this suggests, in contrast to a thick-markets setting, achieving FA and NC in a thin-markets setting is much more delicate. With complete markets, all prices are common knowledge, and hence, price consistency occurs automatically. One could argue that the complete-markets assumption in the standard First Theorem is a shortcut which yields a simpler statement and shorter proof; and again our claim is that taking the shortcut means bypassing another issue: the price decentralization problems associated with the allocation of nonstandardized commodities (see Remark 4).

In this paper we do not strive for the utmost generality, preferring to emphasize principles. One simplifying assumption deserves special mention. We shall assume that individuals have quasi-linear preferences. This allows for cardinal measures of individuals’ private rewards and their social marginal products; hence, it greatly facili-

tates emphasizing the appropriability theme. From our work on the *no-surplus* approach to perfect competition in both its ordinal and cardinal versions (Makowski, 1980a; Ostroy, 1980; Makowski and Ostroy, 1987), we strongly surmise that there are ordinal analogues of our current results, just as there is both an ordinal and cardinal version of the *no-surplus* condition. There is also a simplified treatment of firms in this paper, relative to the Arrow-Debreu version of the Walrasian model. While the “individuals” in the model of occupational choice may possess production possibilities and may be interpreted as single proprietary firms, the model does not include firms with multiple shareholders. This is just to avoid the notational complications involved in including shareholdings and the required redistributions of profits, complications that would distract from our main goal: an alternative presentation of the First Theorem.

The contents of the rest of the paper are as follows. The model of occupational choice and a basic example are described in Section I. Section II gives the main result. First, it is proved that rewarding individuals with their social marginal products (full appropriation) is good for incentives: it leads to efficient occupational choices, excepting perhaps for some coordination problems. Second, when the changes in the gains from trade are subadditive (the noncomplementarity condition), no coordination problems will arise. Section III gives the thick- and thin-markets applications of our revision. The ultimate goal of any formalization of an invisible-hand theorem is to guide our understanding of market success/failure. With this in mind, Section IV concludes with a brief discussion of some implications of our method of proof.

I. The Model and an Example

Although the Walrasian model permits a broad range of possible interpretations, the Walrasian conception of the coordination of economic activity fosters a certain point of view that might be termed a “thick-markets mentality.” According to this vision, the world is described by a fixed set of commod-

ity markets as the paved highways of economic travel. In contrast to this, we will take a “thin-markets” approach. What we mean by this is that we shall try to avoid the fixed set of roads upon which individuals travel. The aim is to portray a world in which economic actors are connected not by several main highways, but by a myriad of individual byways of their own construction. It is this alternative vision that underlies the following.⁴

We pose the problem of the coordination of economic activity by supposing that each individual can be one of several different types. Call these types the possible “occupations” for the individual. More formally, there are n individuals, indexed by i . For each individual i there is a given set of possible occupations V_i from which he must choose exactly one. An *assignment* of individuals to occupations is a $\mathbf{v} = (v_1, \dots, v_i, \dots, v_n) \in \times_i V_i$. $\mathcal{V} \equiv \times_i V_i$ represents the set of all possible assignments.

In order to include both pure exchange and production-and-exchange economies, it will be simpler to work in *trade space*. Thus we leave implicit i 's consumption and production decisions, which are his private information, to focus on what is essential for the model, his trade relationships. A trade for individual i is a point $\mathbf{z}_i \in \mathbb{R}^\ell$ with the sign convention that positive (negative) components of \mathbf{z}_i represent his purchases (sales). Observe that i 's preferences over trades will generally change when his occupation changes, even if his consumption tastes remain constant (e.g., if he becomes a baker then he will value the purchase of 1,000 bushels of wheat more than if he becomes a candlestick-maker). Thus, when i chooses an occupation $v_i \in V_i$, he chooses both a trading possibility set $Z(v_i)$ and preferences over the possible trades in $Z(v_i)$.

To capture both aspects of occupational choice, we view an occupational choice v_i as an extended real-valued function (i.e., $v_i: \mathbb{R}^\ell \rightarrow \mathbb{R} \cup \{-\infty\}$). Our convention is that

⁴Formally, thick markets will be a special case; see Section III-A.

those trades $\mathbf{z}_i \in \mathbb{R}^\ell$ which are infeasible for i are assigned a utility level of $-\infty$. Thus, i 's trading possibility set in occupation v_i is given by the effective domain of v_i , that is,

$$\mathbf{Z}(v_i) = \{\mathbf{z}_i: v_i(\mathbf{z}_i) > -\infty\}.$$

Thus the function v_i does double duty: it identifies both i 's trading possibilities in occupation v_i and also his preferences over possible trades. Examples illustrating the flexibility of the setup will be given. Notice that since we are in trade space, the zero vector in \mathbb{R}^ℓ corresponds to no trade, which we shall assume is always an option.⁵

In addition to trade in the ℓ commodities, there is also a money commodity that the individual can use to establish quid pro quo in exchange. Utility from these $\ell + 1$ commodities depends only on i 's characteristics v_i because all individuals have *quasi-linear* utility functions with respect to the money commodity. That is, i 's utility from $(z_i, m_i) \in \mathbb{R}^\ell \times \mathbb{R}$ when he is in occupation v_i is given by

$$v_i(\mathbf{z}_i) + m_i.$$

To preserve the quasi-linearity of the model we put no limitation on the amount of money i can supply (the spirit is that i never hits the boundary of his money endowment).

⁵To give an illustration, suppose that individual i 's consumption set is \mathbb{R}_+^ℓ and that he has an endowment $\omega_i \in \mathbb{R}_+^\ell$ and preferences over consumption bundles given by $u_i: \mathbb{R}_+^\ell \rightarrow \mathbb{R}$. Now suppose that, if he becomes a baker, his production possibilities set will be $Y_i \subset \mathbb{R}^\ell$. Then for any given trade $z_i \in \mathbb{R}^\ell$, z_i is not feasible for i as a baker if it calls on him to deliver more of some good than he could possibly supply as a baker—for example, some candlesticks (assuming he has no endowment of candlesticks). That is $v_i(z_i) = -\infty$ if and only if $\omega_i + z_i + y_i \notin \mathbb{R}_+^\ell$ for any production decision $y_i \in Y_i$. On the other hand, if a trade z_i is feasible for i , his utility from z_i if he is a baker, $v_i(z_i)$, is simply the maximum utility in consumption he can achieve given the trade, that is,

$$v_i(z_i) = \max_{y_i \in Y_i} u_i(\omega_i + z_i + y_i).$$

The set of commodities which can be potentially supplied in the economy is restricted by individuals' occupational choices. To express this formally, let h index commodities, $h = 1, \dots, \ell$. So for any given trade $\mathbf{x} = (x_1, \dots, x_h, \dots, x_\ell) \in \mathbb{R}^\ell$, x_h represents the amount of commodity h purchased (if $x_h > 0$) or sold (if $x_h < 0$). Let

$$\mathbf{H}(\mathbf{v}) = \{h: z_{ih} < 0\}$$

for some i and some trade $\mathbf{z}_i \in \mathbf{Z}(v_i)$

represent the set of commodities that can be potentially supplied in \mathbf{v} . We make the harmless assumption that all commodities can be potentially supplied: $\bigcup_{\mathbf{v} \in \mathcal{V}} \mathbf{H}(\mathbf{v}) = \{1, \dots, \ell\}$. Define the subspace

$$\mathbb{R}^{\ell(\mathbf{v})} = \{\mathbf{x} \in \mathbb{R}^\ell: x_h = 0 \text{ for all } h \notin \mathbf{H}(\mathbf{v})\}.$$

Once the assignment \mathbf{v} to occupations is made, trading is restricted to $\mathbb{R}^{\ell(\mathbf{v})}$.

Trades $\mathbf{z} = (\mathbf{z}_i)$ are *feasible* for \mathbf{v} if each $\mathbf{z}_i \in \mathbf{Z}(v_i) \cap \mathbb{R}^{\ell(\mathbf{v})}$ and $\sum_i \mathbf{z}_i = \mathbf{0}$. Let $\mathbf{Z}(\mathbf{v})$ be the set of all such trades.

Definition: Given an assignment \mathbf{v} , a *Walrasian equilibrium* for \mathbf{v} is a pair (\mathbf{z}, \mathbf{p}) such that \mathbf{z} is feasible for \mathbf{v} , $\mathbf{p} \in \mathbb{R}^\ell$, and for all i ,

$$v_i(\mathbf{z}_i) - \mathbf{p}\mathbf{z}_i \geq v_i(\mathbf{z}'_i) - \mathbf{p}\mathbf{z}'_i \text{ for all } \mathbf{z}'_i \in \mathbb{R}^{\ell(\mathbf{v})}.$$

That is, i maximizes $v_i(\mathbf{z}'_i) + m'_i$ subject to the (trading) budget constraint $\mathbf{p}\mathbf{z}'_i + m'_i = 0$. Note that \mathbf{z}'_i belongs to $\mathbb{R}^{\ell(\mathbf{v})}$; therefore, the values of p_h for $h \notin \mathbf{H}(\mathbf{v})$ are irrelevant since \mathbf{z}'_i is zero there.

Exploiting the quasi-linearity of the model, define the maximum potential *gains from trade* in \mathbf{v} as follows:⁶

$$g(\mathbf{v}) = \max \left\{ \sum_i v_i(\mathbf{z}_i) : \mathbf{z} \in \mathbf{Z}(\mathbf{v}) \right\}.$$

⁶We assume throughout that for all i and all $v_i \in \mathcal{V}_i$, v_i is continuous on $\mathbf{Z}(v_i)$, $\mathbf{Z}(v_i)$ is closed, and $0 \in \mathbf{Z}(v_i)$. We also assume for all $\mathbf{v} \in \mathcal{V}$, $\mathbf{Z}(\mathbf{v})$ is compact. Thus the maximum (in the definition) exists since $\sum v_i$ is a continuous function on a compact and nonempty set.

As is well known, in quasi-linear economies maximizing the gains from trade is both necessary and sufficient for achieving efficiency.

Definition: The trade \mathbf{z} is *efficient for* \mathbf{v} (synonymously, “efficient relative to \mathbf{v} ”) if \mathbf{z} is feasible for \mathbf{v} and $\sum v_i(\mathbf{z}_i) = g(\mathbf{v})$. An allocation (\mathbf{v}, \mathbf{z}) is (globally) *Pareto efficient* if \mathbf{z} is efficient for \mathbf{v} and $g(\mathbf{v}) \geq g(\mathbf{v}')$ for all $\mathbf{v}' \in \mathcal{V}$.

As an application of the standard First Theorem of Welfare Economics, we have the following.

PROPOSITION 1: *If (\mathbf{z}, \mathbf{p}) is a Walrasian equilibrium for \mathbf{v} then \mathbf{z} is efficient for \mathbf{v} .*

PROOF:

Let \mathbf{z}' be any other feasible allocation for \mathbf{v} . Then from the condition for Walrasian equilibrium, summing over the i and recalling $\sum \mathbf{z}'_i = \mathbf{0}$ since \mathbf{z}' is feasible:

$$\sum v_i(\mathbf{z}_i) \geq \sum v_i(\mathbf{z}'_i) \quad \text{for all feasible } \mathbf{z}'.$$

That is, $\sum v_i(\mathbf{z}_i) = g(\mathbf{v})$.

Nevertheless, a Walrasian equilibrium for \mathbf{v} can evidently be very inefficient—not globally Pareto efficient—since the set of feasible trades may be restricted to a very inefficient subset of commodities: people may be in the wrong occupations. We will be interested in how the “invisible hand” may be able to lead the economy to a Pareto-efficient outcome.

Suppose occupational choice is the Nash equilibrium outcome of a game in which people hold rational conjectures about how Walrasian prices will change when they change occupations. Let $\vartheta: \mathcal{V} \rightarrow \mathbb{R}^\ell$ be a Walrasian price selection in the sense that for each $\mathbf{v} \in \mathcal{V}$, there are trades \mathbf{z} such that $(\mathbf{z}, \vartheta(\mathbf{v}))$ is a Walrasian equilibrium for \mathbf{v} ; and let

$$\pi_i(\mathbf{v}) = \max\{v_i(\mathbf{z}_i) - \vartheta(\mathbf{v})\mathbf{z}_i: \mathbf{z}_i \in \mathbb{R}^{\ell(\mathbf{v})}\}$$

represent i 's payoff (synonymously, “profit” or “utility”) in the assignment \mathbf{v} under prices $\vartheta(\mathbf{v})$.

Definition: An *occupational equilibrium* (OE) is a triple $(\vartheta, \mathbf{v}, \mathbf{z})$ such that $(\mathbf{z}, \vartheta(\mathbf{v}))$ is a Walrasian equilibrium for \mathbf{v} , and for all i and all $v'_i \in \mathcal{V}_i$,

$$\pi_i(\mathbf{v}) \geq \pi_i(v'_i, \mathbf{v}^i)$$

where $\mathbf{v}^i \equiv (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$ is the assignment \mathbf{v} with individual i omitted; and consequently, (v'_i, \mathbf{v}^i) represents the assignment \mathbf{v} with only i 's occupation changed from v_i to v'_i .

The displayed condition expresses the idea that \mathbf{v} is a Nash equilibrium in occupational choice. In terms of traditional economics, it picks up the idea of resources flowing into their (privately) most profitable uses. We will be interested in identifying conditions under which OE's are Pareto efficient. Note that if $(\vartheta, \mathbf{v}, \mathbf{z})$ is an occupational equilibrium, then $\pi_i(\mathbf{v}) = v_i(\mathbf{z}_i) - \vartheta(\mathbf{v})\mathbf{z}_i$.

In an occupational equilibrium, the market outcome for \mathbf{v} is obtained from a predetermined selection among the Walrasian, and therefore price-taking, equilibria for \mathbf{v} . This should be regarded as a convenient simplification in which we ignore the monopoly problems in a given \mathbf{v} to focus on the monopoly issues across \mathcal{V} . Note, however, that the more variation there is in the choice of “occupations,” the closer this fiction will come to mimicking conventional monopoly. For example, consider a seller with occupations/activities that distinguish between different quantities of the same good supplied. Then, the seller can observe the Walrasian outcome from selling one unit, from selling two units, and so on (i.e., the seller can observe the aggregate demand schedule just as a simple monopolist would). If buyers are permitted to have similar quantity-varying “occupations,” they will attempt to exercise their monopsony power. An illustration along these lines follows.

Example 1 (Simple monopoly as an occupational equilibrium): Let $\ell = 1$ and partition individuals into one seller, s , and $B \equiv n - 1$ buyers indexed by b . The seller only likes

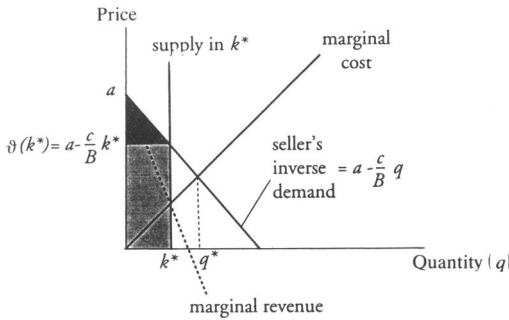


FIGURE 1. THE OCCUPATIONAL EQUILIBRIUM IN EXAMPLE 1

money. His possible occupations are parameterized by $k \in [0, K]$; when in occupation k , he can supply up to k units of the commodity at a cost of $\frac{1}{2}q^2$ for any $q \in [0, k]$. Thus, $\mathcal{V}_s = \{v_s^k: k \in [0, K]\}$ where

$$v_s^k(z_s) = \begin{cases} -\frac{1}{2}z_s^2 & \text{if } z_s \in [-k, 0] \\ -\infty & \text{otherwise.} \end{cases}$$

Buyers are identical. Each buyer has no initial endowment of the commodity and values its consumption according to a quadratic utility function; further, we view buyers as passive here, and so we model them with only one occupation. Thus, for each buyer b , $\mathcal{V}_b = \{v_b\}$, where

$$v_b(z_b) = \begin{cases} az_b - \frac{1}{2}cz_b^2 & \text{if } z_b \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

and where a and c are positive constants.

Let q^* be the output where the seller's inverse demand curve intersects his marginal cost curve (see Fig. 1), and let us assume $K > a$. Writing $\vartheta(k)$ for $\vartheta(v_s^k, \mathbf{v}^s)$, it is easy to check that $\vartheta(k)$ is unique and given by

$$\vartheta(k) = \begin{cases} a - \frac{c}{B}k & \text{if } k < q^* \\ a - \frac{c}{B}q^* & \text{otherwise.} \end{cases}$$

For efficiency, we want the seller to produce q^* and thus to choose an occupation

$k \geq q^*$. But the seller's profit, $\pi(\cdot)$, is maximized in occupation

$$k^* = \frac{aB}{2c + B}$$

where his marginal revenue equals his marginal cost (again see Fig. 1). His equilibrium occupational choice exhibits the usual inefficiency associated with simple monopoly: he can influence market-clearing prices $\vartheta(k)$ by his choice of occupation (quantity). Hence, he enters the wrong occupation (undersupplies).

II. The Main Result

A. A Divergence between Private Profit and Social Benefit

The market failure that Example 1 illustrates may be explained in terms of a failure of appropriation at the individual margin (i.e., as arising from a divergence between the seller's private reward and his social marginal product). To see this, we shall need some new terminology.

As a preliminary observe that, for any individual i and any assignment to occupations \mathbf{v} , the maximum potential gains from trade in \mathbf{v} without i is given by

$$g^i(\mathbf{v}^i) = \max \left\{ \sum_{j \neq i} v_j(z_j) : \sum_{j \neq i} z_j = \mathbf{0} \right\}.$$

[Recall that $\mathbf{v}^i \equiv (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$, represents the occupations of all individuals except i .] Thus, individual i 's contribution to society is naturally defined as the difference between the gains from trade with him and without him.

Definition: The (social) marginal product of individual i in occupation v_i when others are in occupations \mathbf{v}^i is given by

$$MP_i(\mathbf{v}) = g(\mathbf{v}) - g^i(\mathbf{v}^i).$$

By contrast, the private marginal product of individual i in occupation v_i when others

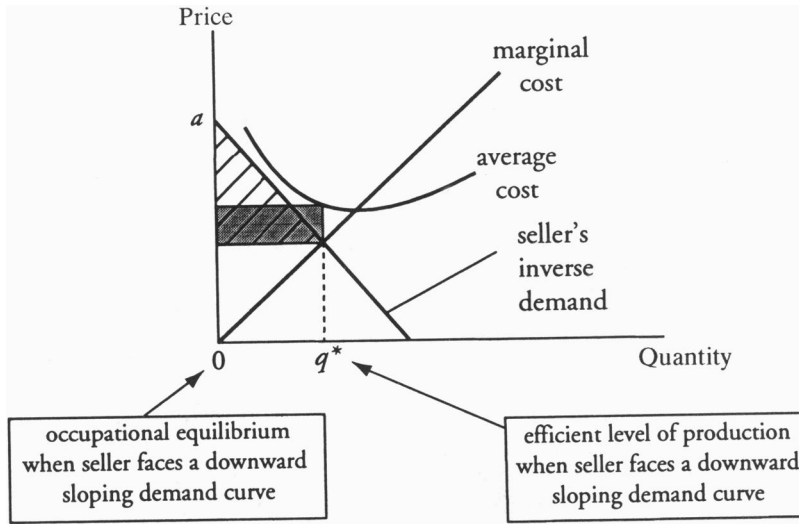


FIGURE 2. VARIANT OF EXAMPLE 1 WITH A FIXED COST

are in occupations \mathbf{v}^i is given by

$$PMP_i(\mathbf{v}) = \pi_i(\mathbf{v}).$$

In an occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$, since $(\mathbf{z}, \vartheta(\mathbf{v}))$ is Walrasian for \mathbf{v} , \mathbf{z} is efficient relative to \mathbf{v} ; that is, $\sum v_i(\mathbf{z}_i) = g(\mathbf{v})$. Thus, $\sum PMP_i(\mathbf{v}) = g(\mathbf{v})$. Further, we have the following.

THEOREM 1 (Inappropriability Theorem): *If $(\mathbf{z}, \vartheta(\mathbf{v}))$ is a Walrasian equilibrium for \mathbf{v} then, for each individual i ,*

$$PMP_i(\mathbf{v}) \leq MP_i(\mathbf{v}).$$

Thus, $\sum MP_i(\mathbf{v}) \geq g(\mathbf{v})$.

PROOF:

Let \mathbf{z}' be any set of trades that are feasible without i (i.e., that satisfy $\sum_{j \neq i} z'_j = 0$). Then as in the proof of Proposition 1, from the definition of a Walrasian equilibrium for \mathbf{v} ,

$$\sum_{j \neq i} v_j(\mathbf{z}_j) - \sum_{j \neq i} \vartheta(\mathbf{v})\mathbf{z}_j \geq \sum_{j \neq i} v_j(\mathbf{z}'_j).$$

Thus, $\sum_{j \neq i} v_j(\mathbf{z}_j) - \sum_{j \neq i} \vartheta(\mathbf{v})\mathbf{z}_j \geq g^i(\mathbf{v}^i)$. Multiplying both sides of this inequality by -1

and adding $\sum_{j=1}^n v_j(\mathbf{z}_j)$ to both sides shows

$$\begin{aligned} & \sum_{j=1}^n v_j(\mathbf{z}_j) - \sum_{j \neq i} v_j(\mathbf{z}_j) \\ & + \sum_{j \neq i} \vartheta(\mathbf{v})\mathbf{z}_j \leq g(\mathbf{v}) - g^i(\mathbf{v}^i). \end{aligned}$$

However, recalling the feasibility of \mathbf{z} , the left-hand side just equals $v_i(\mathbf{z}_i) - \vartheta(\mathbf{v})\mathbf{z}_i$, that is, $PMP_i(\mathbf{v})$; while the right-hand side equals $MP_i(\mathbf{v})$. Hence, $PMP_i(\mathbf{v}) \leq MP_i(\mathbf{v})$, as claimed. The second assertion of the theorem now follows immediately from the fact that $\sum PMP_i(\mathbf{v}) = g(\mathbf{v})$.

So “at best” in an OE, everyone will be rewarded with his full social marginal product. We call the result the “inappropriability theorem” to emphasize that usually some individuals will be rewarded with strictly less than their MP’s. This was illustrated in Example 1. In this example, for any assignment \mathbf{v} , the seller’s social marginal product in \mathbf{v} is the *whole* gain from trade in \mathbf{v} since no one else has any of the commodity to trade; for example, when $k = k^*$, then the seller’s MP is the entire shaded area in

Figure 1. But his PMP, his profit, is just a fraction of $g(v)$ since he faces a downward-sloping demand curve and so must give up some of $g(v)$ to the buyers as consumer surplus; for example, when $k = k^*$, then he must give the darker-shaded consumer surplus triangle in Figure 1. Thus, in the example, the seller appropriates less than his MP. This explains why he undersupplies in the OE: beyond k^* , the change in his PMP is negative, even though the change in his social marginal product is still positive.

While the undersupply equilibrium in Example 1 is bad, things could get worse: the unique seller may not want to produce at all. Specifically, consider the variant of Example 1 in which the seller, in addition to his marginal cost, has a fixed cost C that he must suffer if he enters any occupation $k > 0$. Suppose this fixed cost *exceeds* his equilibrium profit in Example 1; that is, his (now) U-shaped average cost curve lies strictly above his downward-sloping inverse demand curve (see Fig. 2). Thus, while $\vartheta(k)$ remains unchanged from Example 1, the unique occupational equilibrium now involves autarky: the seller does not produce any of the commodity.⁷ But also suppose that the sum of producer and consumer surplus would be strictly positive for some output levels (i.e., the area of the dark

shaded rectangle in Fig. 2 [his losses in occupation q^*] is smaller than the shaded consumer surplus triangle in the figure), so the no-production equilibrium is Pareto inefficient. In accord with traditional teaching, the source of the inefficiency is that the seller cannot *appropriate* the consumer surplus his commodity would produce. Or, in our language, he would not get the full social marginal product of his commodity.

Remark 1 (Imperfect competition and appropriation logic): As is well known, the market failures illustrated in Figures 1 and 2 would disappear if we allowed the seller to act as a perfectly discriminating monopolist, not just as a simple monopolist. But this extension of appropriation logic to imperfect competition is somewhat misleading. Apart from the well-known informational demands confronting the perfectly discriminating monopolist, Theorem 1 can be used to show a fundamental difficulty. To illustrate, consider the case of bilateral monopoly. While each of the two parties could appropriate all the surplus from the other, certainly both could not simultaneously appropriate. That is, the sum of their MP's is strictly greater than the total gains from trade between them—there just is not enough surplus to go around. This is always the case when there is imperfect competition (see Makowski and Ostroy, 1987).

B. Giving Individuals Their Marginal Products Is Good for Incentives

Traditional appropriation logic, as amended here to emphasize individuals rather than commodities, says that any discrepancy between private and social marginal products will typically be accompanied by market inefficiency, as illustrated by Example 1 and its variant. But it also says that, if there is no such discrepancy, private initiative leads to socially efficient allocations. Let us now formally examine this second assertion, that giving individuals their marginal products is good for incentives. Accordingly, let us suppose that, in an OE (ϑ, v, z) , private and social marginal products coincide at v in the sense that the following condition is

⁷The reader may have expected a nonexistence problem. Indeed, such a problem does occur in the Walrasian version of this variant because of the discontinuity in the firm's supply curve caused by the fixed cost. In the Walrasian version the firm's occupational choices are trivial, say $\mathcal{V}_s = \{v_s^i\}$, where

$$v_s^i(z_s) = \begin{cases} -\frac{1}{2}z_s^2 - C & \text{if } z_s \in [-K, 0) \\ 0 & \text{if } z_s = 0 \\ -\infty & \text{otherwise.} \end{cases}$$

There is no nonexistence problem in the occupational-choice version because the firm takes into account that the equilibrium price will change when it changes occupations (quantity); and when it enters any occupation $k > 0$, the fixed cost C is a bygone cost for the firm, so its supply curve in any *given* occupation k is continuous. Nevertheless, the occupational-choice model does not guarantee existence of equilibria even if each v_i is concave (see Roberts and Hugo Sonnenschein, 1979).

met:

Full appropriation (FA): For every individual i such that $\mathcal{V}_i \neq \{v_i\}$ and every $v'_i \in \mathcal{V}_i$,

$$\text{PMP}_i(v'_i, \mathbf{v}^i) = \text{MP}_i(v'_i, \mathbf{v}^i).$$

(Notice that any individual for whom $\mathcal{V}_i = \{v_i\}$, a singleton, cannot influence prices by his occupational choice since his choice set is trivial; hence, we need not worry about his incentives.)

Introduce the following suggestive notation. Denote a change from v_i to some other occupation v'_i by Δv_i . Let

$$\frac{\Delta \text{PMP}_i}{\Delta v_i} = \text{PMP}_i(v'_i, \mathbf{v}^i) - \text{PMP}_i(\mathbf{v})$$

and let

$$\frac{\Delta \text{MP}_i}{\Delta v_i} = \text{MP}_i(v'_i, \mathbf{v}^i) - \text{MP}_i(\mathbf{v}).$$

In this notation, FA implies that, in any OE,

$$\frac{\Delta \text{PMP}_i}{\Delta v_i} = \frac{\Delta \text{MP}_i}{\Delta v_i} \quad \text{for all } \Delta v_i.$$

But notice that

$$\begin{aligned} \frac{\Delta \text{PMP}_i}{\Delta v_i} &\equiv \text{MP}_i(v'_i, \mathbf{v}^i) - \text{MP}_i(\mathbf{v}) \\ &= [g(v'_i, \mathbf{v}^i) - g^i(\mathbf{v}^i)] \\ &\quad - [g(\mathbf{v}) - g^i(\mathbf{v}^i)] \\ &= g(v'_i, \mathbf{v}^i) - g(\mathbf{v}) \equiv \frac{\Delta g(\mathbf{v})}{\Delta v_i}. \end{aligned}$$

Hence, FA implies that, in any OE,

$$\frac{\Delta \text{PMP}_i}{\Delta v_i} = \frac{\Delta g(\mathbf{v})}{\Delta v_i} \quad \text{for all } \Delta v_i.$$

But in any OE, individuals choose their occupations to maximize their private payoffs; hence, in any OE satisfying FA,

$$\frac{\Delta \text{PMP}_i}{\Delta v_i} = \frac{\Delta g(\mathbf{v})}{\Delta v_i} \leq 0 \quad \text{for all } \Delta v_i.$$

That is, the assignment \mathbf{v} is not Pareto-dominated by any other assignment \mathbf{v}' involving an occupational switch by only one individual, Δv_i . Stated as a theorem, we have proved the following.

THEOREM 2 (Partial Optimality): *If it is the case that $(\partial, \mathbf{v}, \mathbf{z})$ is an occupational equilibrium satisfying FA, then for all i and all $v'_i \in \mathcal{V}_i$,*

$$g(\mathbf{v}) \geq g(v'_i, \mathbf{v}^i).$$

The word “partial” in the name of the theorem is to suggest two ideas. First and most obvious, the theorem is only a “partial” optimality result in that it does not claim that \mathbf{v} is globally Pareto efficient. Second, the word “partial” suggests in what sense \mathbf{v} is efficient; here the word is intended to be suggestive of partial derivatives. The assignment \mathbf{v} cannot be Pareto-dominated by any changes of occupation in the individual “coordinate directions” (i.e., by any Δv_i); but it may be Pareto-dominated by coordinated changes in the “diagonal directions” (i.e., by some $\Delta \mathbf{v} = (\Delta v_1, \dots, \Delta v_n)$ that involves several individuals changing occupations simultaneously). Thus the theorem does not preclude the possibility of *coordination failures* in an OE, even when everyone is rewarded with his or her social marginal product. We will examine this possibility in Section II-C below.

Remark 2 (The mechanism-design connection): Readers familiar with Vickrey-Clarke-Groves mechanisms (William Vickrey, 1961; Edward H. Clarke, 1971; Theodore Groves, 1973) from the theory of mechanism design will see an intimate connection between the proof of Theorem 2 and the proof that such mechanisms efficiently solve the revelation problem. This is no accident; see Makowski and Ostroy (1987, 1992) for an interpretation of these mechanisms as mimicking the logic of the perfectly competitive market. The difference is that, while in the mechanism literature there is a central allocator who can costlessly find an efficient allocation once it

knows the true types of individuals, here individuals must find such an allocation on their own. Thus there is the possibility of coordination failures, to be discussed in the next subsection.

Combining Proposition 1 and Theorem 2, we are immediately able to state the following corollary.

COROLLARY 1 (A Partial Extension of the First Theorem): *Suppose that only one individual has a nontrivial occupational choice, that is, $\mathcal{V}_i = \{v_i\}$, a singleton, for all individuals except one. Then, any occupational equilibrium satisfying FA is Pareto efficient.*

The corollary implies that we can construct an example of a Pareto-efficient OE by modifying Example 1 so that the seller always earns his social marginal product. Since the discrepancy between his PMP and his MP resulted from facing a downward-sloping demand curve, hence having to give up a part of $g(v)$ to the buyers as consumer surplus, it should suffice if we modify the example so that the seller faces a perfectly elastic demand for his product.

Example 2 (An efficient occupational equilibrium): This example is the same as Example 1 except that each buyer's preferences now exhibit a constant marginal utility from consuming the commodity equal to a for the first d/B units, where $d > K \equiv$ the seller's maximum potential supply.⁸ Since $d > K$, the seller's inverse demand curve is now perfectly elastic in his operating range; that is,

$$\vartheta(k) = a \quad \text{for all } k \in [0, K].$$

⁸That is, now $\mathcal{V}_b = \{v_b\}$, where

$$v_b(z_b) = \begin{cases} az_b & \text{if } z_b \in \left[0, \frac{d}{B}\right] \\ a\left(z_b - \frac{d}{B}\right) - \frac{1}{2}c\left(z_b - \frac{d}{B}\right)^2 & \text{if } z_b > \frac{d}{B} \\ -\infty & \text{otherwise.} \end{cases}$$

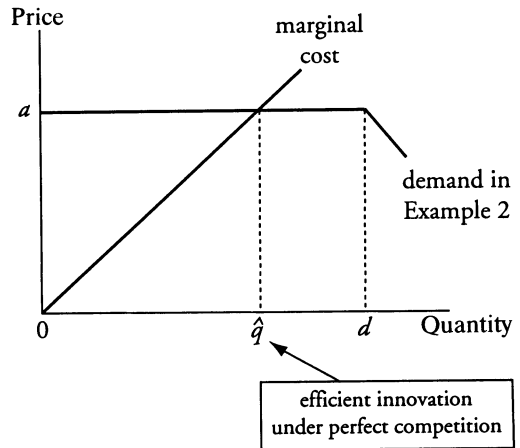


FIGURE 3. THE OCCUPATIONAL EQUILIBRIUM IN EXAMPLE 2

Hence, the seller's profit $\pi_s(v_s^k, v^s) \equiv \pi_s(k)$ is maximized by choosing any occupation $k \in [\hat{q}, K]$ and producing where his marginal cost curve intersects the perfectly elastic portion of his demand curve (see Fig. 3). So, in accord with Corollary 1, the equilibrium is efficient. Notice that FA is satisfied since for any occupation k he may choose

$$\pi_s(k) = g(v_s^k, v^s) = MP_s(v_s^k, v^s).$$

It is interesting to observe that this efficient outcome is the limiting outcome of the occupational equilibria in Example 1 as one replicates the number of buyers. As B increases, the inverse demand curve in Figure 1 rotates around point a on the vertical axis, becoming more and more elastic. Hence asymptotically the seller's profits would equal his full social marginal product (the entire shaded area in Fig. 1). Given this context, one can regard Example 2 as a finite "magnification" of the limiting economy (notice that the length of the flat segment in any buyer's utility function, d/B , goes to zero as B approaches infinity; hence, buyers' preferences approach v_b , the preferences of the buyers in Example 1, as B approaches infinity). A similar, but asymptotic, example appears in Hart (1979).

Either the finite “magnification” or the asymptotic version of the example tells an interesting moral: a unique seller of a product may still be a perfect competitor (in the sense of facing a perfectly elastic demand for his product in the relevant region), provided his desired supply is less than the demands of the highest-valuing buyers. Viewing the seller as innovating a new commodity, the example illustrates that the phrase “a perfectly competitive innovator” is not an oxymoron. This is the lesson of the literature on product innovation under perfect competition (e.g., Hart, 1979, 1980; Makowski, 1980b, 1983).

C. The Coordination Problem

Continuing with our suggestive notation, let $\Delta g(\mathbf{v})/\Delta \mathbf{v} = g(\mathbf{v}') - g(\mathbf{v})$, where $\Delta \mathbf{v} = (\Delta v_1, \dots, \Delta v_n)$ denotes a change from \mathbf{v} to some other assignment $\mathbf{v}' = (v'_1, \dots, v'_n)$. If \mathbf{v} is an equilibrium assignment, FA ensures

$$\frac{\Delta g(\mathbf{v})}{\Delta v_i} \leq 0 \quad \text{for all } \Delta v_i$$

but it does not ensure

$$\frac{\Delta g(\mathbf{v})}{\Delta \mathbf{v}} \leq 0 \quad \text{for all } \Delta \mathbf{v}.$$

For some $\Delta \mathbf{v}$ the changes in the gains from trade may be strictly *superadditive*:

$$\sum_i \frac{\Delta g(\mathbf{v})}{\Delta v_i} < \frac{\Delta g(\mathbf{v})}{\Delta \mathbf{v}}.$$

Example 3 (Computer hardware and software): Suppose a computer hardware manufacturer could supply a powerful machine, the value of which would be enhanced by a sophisticated graphics program, and a software manufacturer could supply a sophisticated graphics program whose value would increase if operated on a powerful machine. But either one without the other is sufficiently costly that it does not cover the price buyers are willing to pay. Therefore, neither commodity is produced, although the sum

of their costs is less than the value of joint innovation. (A numerical illustration will be given in Section III below.) The upshot is that if \mathbf{v} is the assignment in which neither hardware nor software is innovated and i is a potential hardware innovator, then

$$\frac{\Delta g(\mathbf{v})}{\Delta v_i} \leq 0 \quad \text{for all } \Delta v_i$$

where Δv_i is any occupational switch that involves i innovating hardware. Thus, i will stay out of the hardware business, even if he can fully appropriate his contribution. A similar statement holds if i is a potential software innovator. Nevertheless

$$\frac{\Delta g(\mathbf{v})}{\Delta \mathbf{v}} > 0$$

for some $\Delta \mathbf{v}$ that involves both the hardware and software innovators producing; so the OE is inefficient.

D. A Revision of the First Theorem

Our main result says that, provided everyone is rewarded with his or her social marginal product, such superadditivity is the *only* possible source of inefficiency. Say that the changes in the gains from trade are *subadditive* at \mathbf{v} or, synonymously, satisfy the *noncomplementarity* condition if the following holds:

Noncomplementarity (NC): For any assignment switch $\Delta \mathbf{v} = (\Delta v_1, \dots, \Delta v_n)$,

$$\sum_i \frac{\Delta g(\mathbf{v})}{\Delta v_i} \geq \frac{\Delta g(\mathbf{v})}{\Delta \mathbf{v}}.$$

THEOREM 3 (A Revision of the First Theorem of Welfare Economics): *Any occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$ satisfying FA and NC is globally Pareto efficient.*

PROOF:

By definition of an OE, for all i and all Δv_i ,

$$\frac{\Delta \text{PMP}_i}{\Delta v_i} \leq 0.$$

Thus,

$$\sum \frac{\Delta \text{PMP}_i}{\Delta v_i} \leq 0.$$

But by FA, the left-hand side equals $\sum \Delta \text{MP}_i / \Delta v_i = \sum \Delta g(\mathbf{v}) / \Delta v_i$. Thus, using NC,

$$\frac{\Delta g(\mathbf{v})}{\Delta \mathbf{v}} \leq 0.$$

That is, no assignment \mathbf{v}' Pareto-dominates \mathbf{v} . Hence, since \mathbf{z} is efficient for \mathbf{v} , (\mathbf{v}, \mathbf{z}) is globally Pareto efficient.

III. Two Settings for the Revision

FA and NC characterize a reward scheme that gives individuals good incentives. What market structures induce such payoffs? We highlight two: a thick-markets setting and a thin-markets setting. The former builds a bridge to the standard First Theorem; the latter takes us into a more dynamic environment.

Common to both settings, perfect competition is a key ingredient for efficiency. It leads to FA. The following definition captures the notion that there is perfect competition in an occupational equilibrium.

Definition: All individuals face *perfectly elastic demands and supplies* (PEDS) in the occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$ if there exists a price vector \mathbf{p} such that for all i and all $v'_i \in \mathcal{V}_i$,

$$\vartheta_h(v'_i, \mathbf{v}^i) = p_h$$

for all commodities $h \in \mathbf{H}(v'_i, \mathbf{v}^i)$.

PEDS says that no one individual can influence market-clearing prices by switching occupations. It is stronger than the hypothesis of price-taking: Since $\vartheta(v'_i, \mathbf{v}^i)$ is a Walrasian selection, PEDS implies that if i switched occupations from v_i to v'_i , then prices actually would not change; so his price-taking is rational.

To establish the link from PEDS to FA, we will need a technical assumption, that individuals' occupational choices \mathcal{V}_i are sufficiently rich in variety. A precise statement of the assumption appears in the Appendix, preceding the proof of Theorem 4. We call an occupational equilibrium *regular* if it satisfies the richness assumption. (The Appendix contains the proofs of all remaining results.)

THEOREM 4: *For any regular occupational equilibrium,*

$$\text{PEDS} \Rightarrow \text{FA}.$$

Thus, under perfect competition, each economic agent is rewarded with his full social contribution in whatever occupation he may enter:

$$\pi_i(\cdot) = \text{MP}_i(\cdot).$$

The injunction to "profit-maximize" (i.e., to seek to maximize one's selfish interests) agrees with the injunction to "maximize one's contribution to society." As we have already emphasized, such a reward scheme gives good incentives, absent coordination problems (failures of NC). As we are about to show, such problems cannot arise in thick markets.

A. Thick Markets

Suppose there is a fixed number of homogeneous commodities traded, with many buyers and sellers of each. In such a thick-markets setting, competition among buyers and sellers implies that no one individual will be able to influence market-clearing prices. Price-taking behavior and complete markets—the twin assumptions that drive the standard First Theorem—make sense. We will show that

thick markets \Rightarrow FA and NC \Rightarrow efficiency.

If we interpret the standard First Theorem as saying that thick markets lead to efficiency (the first and last items in the schema), then in this setting the revision

may be viewed as supplementing the First Theorem by specifying the reward scheme that induces efficiency.

Definition: Given an occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$, we will say that all commodities are *standardized* if

$$\mathbf{H}(\mathbf{v}) = \mathbf{H}(v_i^i, \mathbf{v}^i) = \{1, \dots, \ell\}$$

for all i and all $v_i^i \in \mathcal{V}_i$.

Markets are *thick* if both (i) all commodities are standardized and (ii) all individuals face PEDS.

The formal definition highlights two features of thick markets: (i) all commodities can be supplied irrespective of any one individual's occupational choice, and (ii) there is perfect competition. Condition (i) is weaker than the hypothesis of many buyers and sellers of each commodity. As emphasized above, many buyers and sellers would lead to PEDS. Thus, the two conditions are at least informally linked.

We have already seen that PEDS implies FA. The next theorem states that when PEDS is combined with standardized commodities then NC will be satisfied automatically. That is, in the absence of product innovation, coordination failures cannot arise. This helps explain the notable neglect of such failures in discussions of welfare economics starting from a thick-markets perspective.

THEOREM 5: *In any regular occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$,*

thick markets \Rightarrow FA and NC.

Remark 3 (Market socialism): The thick-markets interpretation of the standard First Theorem is perhaps the most natural one. But it is not the only interpretation. In the market-socialism tradition, price-taking is assumed to be independent of any market structure (e.g., even in one-firm, socialized sectors). The First Theorem is interpreted as showing that market socialism will lead to efficiency. The outcome results from the

fact that Walrasian prices measure the social values of all resources at the margin—even in the absence of full appropriation. Therefore, the price system will coordinate an efficient outcome, provided agents act as price-takers. To illustrate, consider the Walrasian version of Example 1 in which the firm does not consider shading its production; hence its occupational choices are trivial, $\mathcal{V}_s = \{v_s^K\}$. The Walrasian equilibrium for this economy will be efficient—the firm will produce q^* units—since it is *required* to act as a price-taker even though in reality it faces a downward-sloping demand curve. Thus, the standard First Theorem “goes through” in spite of the fact that the firm only appropriates a small fraction of its social marginal product when it produces q^* in the Walrasian equilibrium. This illustrates that “coordination via prices” is possible even in the absence of “full appropriation via prices”—provided agents act as price-takers even if it is not in their self-interest. It is through the logic of coordination via prices, *not* appropriation via prices, that the First Theorem is traditionally proved. For a critique of market socialism as relying too heavily on coordination rather than appropriation logic, see Makowski and Ostroy (1993).

B. Thin Markets

Suppose now that commodities are not all standardized; rather, commodities are heterogeneous because sellers have the ability to personalize their product lines. Call this a thin-markets setting. Further, suppose there is only a limited number of commodities produced in any assignment to occupations (e.g., specializing in the production of software A means not specializing in the production of software B), and as a result a large number of commodities, even most, are not traded in equilibrium.

Because the set of all conceivable innovations is huge, the hypothesis of complete markets becomes problematic. Further, with decentralized knowledge, although each agent knows best what products he could innovate, the set of possible innovations is not common knowledge. In such a setting,

to assume that a Walrasian auctioneer could know of all possible commodities and publicly announce a price for each is not only heroic, but also inconsistent with the hypothesis of decentralized knowledge.

In a thin-markets setting, it is more appropriate to view each individual as only possessing "local price information." Recall that at any assignment \mathbf{v} , individuals can only register demands for commodities in $\mathbf{H}(\mathbf{v})$; it is only as sellers that they can change the commodity space from $\mathbb{R}^{\ell(\mathbf{v})}$ to $\mathbb{R}^{\ell(v_i, \mathbf{v}^i)}$. Therefore, as buyers, the relevant prices are $\vartheta_h(\mathbf{v})$, $h \in \mathbf{H}(\mathbf{v})$. We shall assume the following:

Local Price Information: At \mathbf{v} , seller i only has access to prices $\vartheta_h(v_i, \mathbf{v}^i)$, $h \in \cup_{v_i} \{\mathbf{H}(v_i, \mathbf{v}^i)\}$.

An individual therefore knows the prices of commodities in $\mathbf{H}(\mathbf{v})$ as well as the prices of the other commodities he can supply. Assuming PEDS, the idea of local price information can also be expressed in terms of the price vector \mathbf{p} : any seller i only knows the prices, p_h , of the commodities he can innovate. This information might come from test marketing or simply from an accurate estimate picked up as a result of being "in the business." Note that when $\cup_{v_i} \{\mathbf{H}(v_i, \mathbf{v}^i)\}$ is only a small subset of $\{1, \dots, \ell\}$, we encounter a problem not present in the thick-markets setting: no individual has access to most prices. Nevertheless, as we shall see, efficient outcomes may occur, provided individuals' local price information is consistent.

The possibility of inconsistent local price information is intimately connected with the possibility of coordination failures (failures of NC). We can illustrate using the hardware–software example introduced in Section II-C. Suppose that individual 1 can produce only hardware, while individual 2 can produce only software. Assume that it takes 1.25 units of money to produce each unit of hardware or software. All potential buyers of hardware and software have identical tastes:

$$v_i(r, s) = \min(r + 2s, s + 2r).$$

That is, each buyer is willing to pay \$1 per unit of hardware (r) or software (s) if the other commodity is unavailable; but each buyer is willing to pay \$3 per hardware–software package.

It is easy to check that no innovation of either hardware or software is an occupational equilibrium. In the absence of software, individual 1 perceives that he can only get \$1 per unit of hardware, less than his marginal cost. Similarly, in the absence of hardware, individual 2 perceives that he can only get \$1 per unit of software. Both remain out of business, in spite of the fact that the value of a hardware–software package exceeds the cost of such a package. Observe that each individual's local price information is correct: \$1 is the market-clearing price per unit of hardware (in the absence of software), and similarly \$1 is the market-clearing price of software (in the absence of hardware). But the individuals' local price information is inconsistent in the sense that if they pieced their information together, the price vector $\mathbf{p} = (1, 1)$ would not clear the market for *both* hardware and software. Indeed, since buyers are willing to pay \$3 for each hardware–software package, at $\mathbf{p} = (1, 1)$ there would be an excess demand for both commodities.

The example motivates the following definition.

Definition: Suppose $(\vartheta, \mathbf{v}, \mathbf{z})$ is an occupational equilibrium satisfying PEDS. Individuals' local price information is *consistent* if the price vector \mathbf{p} (defined in PEDS) satisfies the following condition for each individual i and each possible occupation v_i' :

$$\pi_i(v_i', \mathbf{v}^i) \geq v_i'(\mathbf{z}_i) - \mathbf{p}\mathbf{z}_i \quad \text{for all } \mathbf{z}_i \in \mathbb{R}^{\ell}.$$

Consistency says that local price information can be pieced together to form one vector of prices which, if it were known to all, would not change trade decisions even if each individual could trade *any* combination of commodities he wishes (not just those in $\mathbf{H}(v_i, \mathbf{v}^i)$). In the hardware–software example, PEDS is satisfied: individual 1 can sell as much hardware as he likes at a price

of \$1 (if software is unavailable).⁹ A similar statement holds for individual 2. But the consistency condition is violated by buyers: they could increase their payoffs if they could trade anywhere in \mathbb{R}^2 at prices $\mathbf{p} = (1, 1)$. See Hart (1980) and Makowski (1980b) for sufficient conditions for consistency; not surprisingly, differentiability of preferences (absence of strict complementarities) plays a key role.

THEOREM 6: *In any regular occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$,*

PEDS and price consistency \Rightarrow FA and NC.

The hypotheses of Theorem 6 are strictly weaker than those of Theorem 5: in thick-market environments, price consistency is trivially satisfied since $\mathbf{H}(v_i^j, \mathbf{v}^i) = \mathbb{R}^l$. Basically, with thick markets, local and global price information coincide.

The (more general) thin-market conditions for efficiency may be fruitfully interpreted using the language of externalities. If an individual imposes externalities on others, then he does not fully appropriate the consequences of his actions. The model of occupational choice does not include the possibility of real externalities, but it does allow individuals to impose pecuniary externalities: any individual, by switching occupations, may affect the terms of trade that others face and hence, indirectly, affect others' welfare.

It is useful to distinguish two sorts of pecuniary externalities, "market-price" and "reservation-price" externalities. The significance of PEDS (i.e., perfect competition) is that it rules out the possibility of market-price externalities: under perfect competi-

tion no one individual can affect the market prices others face. With thick markets, this is the only type of pecuniary externality possible. But with innovation, another type of pecuniary externality may occur: an individual, by innovating one commodity, may be able to influence the reservation prices other innovators see for their potential innovations. To illustrate, in the case of the hardware–software example, if individual 1 marketed his hardware and sold it at \$1 each—even though he would lose money doing so—then individual 2 would see that buyers' willingness to pay (i.e., reservation price) for his software has increased from \$1 to \$2, and so he would find it profitable to innovate his software. The hypotheses of Theorem 6, namely, PEDS and price consistency, rule out, respectively, market-price and reservation-price externalities.

Let us say that there are *no pecuniary externalities* in an occupational equilibrium if it satisfies both PEDS and price consistency. Then, our analysis may be summarized by:

absence of pecuniary externalities
 \Rightarrow FA and NC
 \Rightarrow efficiency.

That is, market settings that preclude pecuniary externalities (e.g., all thick-market settings and some thin-market settings) will induce payoffs consistent with efficient behavior.

Remark 4 (Decentralized knowledge of prices): There is an interesting contrast between our concept of local price information and Friedrich A. Hayek's view of the price system in "The Use of Knowledge in Society" (1945). Hayek, while stressing the local character of economic knowledge of time and place, views the price system as common knowledge guiding individuals in the socially efficient use of their local information (e.g., his famous illustration of how different individuals would cope with an economy-wide scarcity of tin reflected in its higher price). In a world of nonstandardized commodities, however, the sharp distinction

⁹PEDS implies that, as in Example 2, any innovator of any commodity h will always receive the buyers' reservation price for his commodity, no matter how many units he sells. The intuition is that since the price p_h must continue to clear the market for h even if the innovator switched to occupations that allow him to produce less and less of it (hence, occupations in which h is getting scarcer and scarcer) p_h must equal the buyers' reservation price. The argument is formalized in the proof of Theorem 4.

between local knowledge of individual circumstances but “global” knowledge of prices needs to be blurred to recognize local price information. Local price information, although it economizes on price information, introduces the possibility of inconsistent perceptions of the value of innovations. Hence the possibility of coordination failures.

IV. Concluding Remarks

In what sense is this a revision? By substituting FA (or PEDS) for price-taking and by substituting NC (or consistent local price information) for complete markets, we took a much longer route to get to more or less the same conclusions as the First Theorem. This was our goal. In our view, *how* that theorem is proved is at least as important as *what* is proved. The current statement and proof of the First Theorem is too concise; the argument does not exhibit sufficient potential complications to allow one to grasp the essentials of why competition leads to efficiency. We call attention to two features of our proof: one is how individual behavior is modeled, and the other is the role of pecuniary externalities. Below we briefly indicate why these features are important.

Individual Behavior.—In the last decade, a gap has developed between general-equilibrium theory and other branches of economics due largely to differences in sophistication about the meaning of “pursuit of self-interest.” With the recent spread of game/information theoretic techniques, the price-taking behavior of general equilibrium appears to be naively simplistic (cf. Samuel Bowles and Herbert Gintis, 1993; Joseph Stiglitz, 1993). From the more sophisticated perspective, general equilibrium would seem to be fine for the more traditional issues of determining the relative prices of standardized commodities (e.g., in the Heckscher-Ohlin approach to international trade), but when it comes to the many economic phenomena based on contracting, asymmetric information, and strategic behavior, one must look elsewhere. In our view, this perspective is incorrect because general equi-

librium need not be identified with naive price-taking behavior. More importantly, it is also ill-advised: the full appropriation underpinning of perfect competition provides both a canonical model of the kind of incentive system that efficiently channels potentially opportunistic behavior and also a canonical reason why—in the absence of full appropriation—such behavior can become socially inefficient.

Pecuniary and Real Externalities.—In general equilibrium, inappropriability is associated with real externalities and is modeled as the incompleteness of markets associated with incompletely defined property rights. In the revision, property rights to all conceivable commodities are well-defined. The only kind of inappropriability permitted is of the pecuniary-externalities kind, associated either with the absence of PEDS or the absence of consistent local price information.

Scitovsky (1954) to the contrary notwithstanding, pecuniary externalities have not been taken very seriously since A. C. Pigou (1912) mistakenly identified as efficiency-reducing appropriability problems what turned out to be welfare-benign price changes (Allyn Young, 1913; Frank Knight, 1924). The moral drawn from Pigou’s error was that pecuniary externalities should be distinguished from welfare-relevant ownership externalities (Howard S. Ellis and William Fellner, 1943).

One can see the influence of this tradition in the property-rights approach to externalities (Ronald Coase, 1960). The message of the Coase Theorem is similar to the First Theorem. Once property rights are fully articulated, efficiency will be achieved. Besides complete property rights (the replacement for complete markets), the other key assumption of the Coase Theorem is *zero transactions costs* (the replacement for price-taking behavior). In addition to eliminating the typical frictions ignored in much of economic theory (e.g., the need for a title search in property transactions), this assumption is used to eliminate the “transactions costs” which are due to imperfect competition—as if zero transactions costs make individuals with monopoly power be-

have as efficiently as price-takers.¹⁰ The main conclusion of the Coase Theorem is that all appropriability problems stem from ownership problems. This is more or less supported by conventional interpretations of the First Theorem which trace departures from efficiency to incompleteness of markets.

It could be argued that any failure of price consistency is an ownership problem; for example, the hardware–software example above would not cause any difficulties if one firm could supply both commodities. Recalling an earlier contribution by Coase (1937), enlarging the boundaries of the firm is one possible response to the limitations of local price information. But carried to its logical conclusion, this remedy would lead to one firm; the coordinating role of the price system would be dramatically attenuated.

To conclude, there is a basic contrast between our revision and the exclusive emphasis on ownership externalities in the First Theorem and the Coase Theorem. In the revision, well-defined property rights are a necessary but not a sufficient condition for FA. Instead of drawing a line around incompletely defined property rights as the sole source of appropriation problems, the revision emphasizes the essential similarities between real and pecuniary externalities. Both are instances of the malincentive consequences of inappropriability. Alternatively put, the economic rationale for property rights is that it helps to achieve, but does not automatically establish, FA and NC. That requires real—not just price-taking—perfect competition, as well as the consistency of individuals' local price information.

APPENDIX

This section contains proofs of the results in Section III. As mentioned there, to estab-

¹⁰George Stigler (1966) coined the term "Coase theorem" but added the qualification that efficiency required perfect competition.

lish the link from PEDS to FA, we will need to assume that individuals' occupational choices \mathcal{V}_i are sufficiently rich in variety. Specifically, we will assume that sellers have the ability to limit their capacities, as in Example 1, by appropriate occupational choices.

Definition: Individual i can choose his capacity if for any $v'_i \in \mathcal{V}_i$ and any capacity $k > 0$ there exists an occupation $v_i^k \in \mathcal{V}_i$ such that (i) $\mathbf{H}(v_i^k, \mathbf{v}^i) = \mathbf{H}(v'_i, \mathbf{v}^i)$, (ii) $\mathbf{Z}(v_i^k) \subseteq \mathbf{Z}(v'_i)$, and (iii) $z_{ih} \geq -k$ for all commodities h and all trades $\mathbf{z}_i \in \mathbf{Z}(v_i^k)$.

The proviso says that i can (i) supply the same commodities in v_i^k as in v'_i , but (ii) his trading possibilities are more restricted in occupation v_i^k , and in particular (iii) he cannot supply more than k units of any commodity. Let v_i^0 denote the *dummy occupation* for individual i , that is, the occupation in which $\mathbf{Z}(v_i^0) = \{0\}$. In the dummy occupation, i cannot trade with others, so he effectively withdraws from the economy. The occupational equilibrium $(\vartheta, \mathbf{v}, \mathbf{z})$ will be called *regular* if each individual i can choose his capacity and also can choose the dummy occupation (i.e., $v_i^0 \in \mathcal{V}_i$).

PROOF OF THEOREM 4:

Suppose $\mathcal{V}_i \neq \{v_i\}$ and choose an arbitrary $v'_i \in \mathcal{V}_i$. Let $(\mathbf{z}', \mathbf{p})$ be Walrasian for $\mathbf{v}' \equiv (v'_i, \mathbf{v}^i)$; and let $G = \sum_{j \neq i} [v_j(\mathbf{z}'_j) - \mathbf{p}\mathbf{z}'_j]$. Since $\sum_j [v_j(\mathbf{z}'_j) - \mathbf{p}\mathbf{z}'_j] = g(\mathbf{v}')$ and since $v_i(\mathbf{z}'_i) - \mathbf{p}\mathbf{z}'_i \leq \text{MP}_i(\mathbf{v}') \equiv g(\mathbf{v}') - g^i(\mathbf{v}')$ (recall Theorem 1), subtracting shows

$$G \geq g^i(\mathbf{v}').$$

It will suffice to show that the weak inequality is really an equality, for then $\sum_j [v_j(\mathbf{z}'_j) - \mathbf{p}\mathbf{z}'_j] = g(\mathbf{v}')$ and $\sum_{j \neq i} [v_j(\mathbf{z}'_j) - \mathbf{p}\mathbf{z}'_j] = g^i(\mathbf{v}')$. Subtracting shows $\pi_i(\mathbf{v}) \equiv v_i(\mathbf{z}'_i) - \mathbf{p}\mathbf{z}'_i = g(\mathbf{v}') - g^i(\mathbf{v}')$, as required.

To verify the equality, consider a sequence of occupations for i in which his capacity gets smaller and smaller: $\{v_i^k\}$, with $k \rightarrow 0$. Let $(\mathbf{z}^k, \mathbf{p})$ be Walrasian for $\mathbf{v}^k \equiv (v_i^k, \mathbf{v}^i)$ (such a Walrasian equilibrium exists since ϑ is defined as a Walrasian price

selection and PEDS implies that prices remain at \mathbf{p}). Notice that since both \mathbf{z}_j^k and \mathbf{z}_j^k are optimal for individual j under prices \mathbf{p} , for any k

$$v_j(\mathbf{z}_j^k) - \mathbf{p}\mathbf{z}_j^k = v_j(\mathbf{z}_j') - \mathbf{p}\mathbf{z}_j' \quad \text{for each } j \neq i.$$

Further, since all allocations \mathbf{z}^k are in a compact set (recall footnote 6), \mathbf{z}^k approaches some limiting allocation \mathbf{z}^* as $k \rightarrow 0$ (at least on a subsequence). Hence, since $\mathbf{Z}(v_j)$ is closed and v_j is continuous on $\mathbf{Z}(v_j)$, $v_j(\mathbf{z}_j^*) - \mathbf{p}\mathbf{z}_j^* = v_j(\mathbf{z}_j') - \mathbf{p}\mathbf{z}_j'$ for each $j \neq i$. Summing shows

$$\sum_{j \neq i} [v_j(\mathbf{z}_j^*) - \mathbf{p}\mathbf{z}_j^*] = G.$$

Let v_i^0 be i 's dummy occupation, let $\mathbf{v}^0 = (v_i^0, \mathbf{v}^i)$, and let $(\mathbf{z}^0, \mathbf{p})$ be Walrasian for \mathbf{v}^0 . By construction, each $\mathbf{z}_j^* \in \mathbb{R}^{\ell(v^0)}$. Hence, $v_j(\mathbf{z}_j^0) - \mathbf{p}\mathbf{z}_j^0 \geq v_j(\mathbf{z}_j^*) - \mathbf{p}\mathbf{z}_j^*$ for each $j \neq i$. Since $\sum_j \mathbf{z}_j^0 = \sum_{j \neq i} \mathbf{z}_j^0 = \mathbf{0}$, summing shows

$$\sum_{j \neq i} v_j(\mathbf{z}_j^0) \geq G.$$

But $\sum_{j \neq i} \mathbf{z}_j^0 = \mathbf{0}$ implies $g^i(\mathbf{v}^i) \geq \sum_{j \neq i} v_j(\mathbf{z}_j^0)$. Thus,

$$g^i(\mathbf{v}^i) \geq G.$$

This establishes that $g^i(\mathbf{v}^i) = G$, as was to be proved.

PROOF OF THEOREM 6:

We already know from Theorem 4 that PEDS implies FA. To verify NC, let $\mathbf{v}' \in \mathcal{V}$ and let $\mathbf{z}' \in \mathbf{Z}(\mathbf{v}')$ satisfy $\sum v'_i(\mathbf{z}'_i) = g(\mathbf{v}')$. Price consistency implies

$$\pi_i(v'_i, \mathbf{v}^i) \geq v'_i(\mathbf{z}'_i) - \mathbf{p}\mathbf{z}'_i.$$

Note that, by FA, the left-hand side equals $\text{MP}_i(v'_i, \mathbf{v}^i)$. Summing shows

$$\sum \text{MP}_i(v'_i, \mathbf{v}^i) \geq g(\mathbf{v}').$$

Thus, since $\sum \text{MP}_i(\mathbf{v}) = g(\mathbf{v})$, subtracting we see that for any assignment switch $\Delta \mathbf{v}$ from

\mathbf{v} to \mathbf{v}' :

$$\sum \frac{\Delta \text{MP}_i}{\Delta v_i} \geq \frac{\Delta g(\mathbf{v})}{\Delta \mathbf{v}}.$$

Since the left-hand side equals $\sum \Delta g(\mathbf{v}) / \Delta v_i$, we have arrived at NC.

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