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'First-order' risk aversion and the equity premium puzzle*

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This paper integrates Yaari's dual theory of choice under uncertainty into a multiperiod context and examines its implications for the equity premium puzzle. An important property of these preferences is that of 'first-order risk aversion' which implies, in our model, that the risk premium for a small gamble is proportional to the standard deviation rather than the variance. Since the standard deviation of the growth rate in aggregate consumption is considerably larger than its variance, the model can generate both a small risk-free rate and a moderate equity premium.

1. Introduction

It is common practice in macroeconomics and finance to employ a representative agent model in order to organize aggregate data on consumption and asset returns. Because of the smoothness of aggregate consumption data for the U.S., the way in which the agent's risk preferences evaluate small gambles about certainty is critical for providing a good fit to the data. On the other hand, the plausibility of utility specifications is often evaluated informally on the basis of their evaluation of moderate or large gambles derived from thought experiments or 'real world' risks. Thus it is desirable to have a functional form for risk preferences which can model plausible risk attitudes

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over a broad range of gamble sizes. The common constant-relative-risk-averse, expected utility function fails in this respect, as pointed out by Kandel and Stambaugh (1989), for example. In this paper we describe an alternative nonexpected utility functional form for risk preferences that is better able to satisfy the above desideratum, while retaining the constancy of the degree of relative risk aversion and the tractability which that constancy affords. Then we examine the extent to which the new functional form helps to resolve the equity premium puzzle posed by Mehra and Prescott (1985).

Mehra and Prescott argue that the representative agent, expected additive utility model, sensibly restricted, cannot account for both the 0.8 percent average real return on debt and the nearly 7.0 percent average real return on equity that the U.S. data show for the 1889–1978 period. The qualification 'sensibly restricted' is essential for the existence of a puzzle. For example, as the authors describe (p. 154), with a sufficiently large degree of relative risk aversion 'virtually any pair of average equity and risk-free returns can be obtained by making small changes in the process on consumption'. In related modelling exercises, a number of authors have shown that with a degree of relative risk aversion in the 20–30 range, the representative agent model performs fairly well [see, for example, Grossman, Melino, and Shiller (1987), Kandel and Stambaugh (1989), and Cecchetti and Mark (1990)]. On the other hand, as shown by Kocherlakota (1988), allowing negative time preference also helps the model to match the above historical averages. To some extent what is 'sensible' or 'plausible' is subject to personal judgement, though as advocated by Mehra and Prescott, evidence from other areas can and should be brought to bear upon the choice of parameter values. In this paper we maintain the common assumption of positive discounting of the future, an assumption which needs no defense here. Our specification of risk preferences, which is where we deviate from the standard model, is appealing on two grounds. First, as previously described, for the same parameter values, our specification implies a degree of risk aversion for a broad range of gambles that is plausible on introspective grounds. Second, our model of risk preferences can explain evidence, such as Allais-type behavior, which contradicts expected utility theory, thereby providing a unifying framework for organizing observations of individual behavior in the laboratory and data on aggregate market behavior.

Following Epstein and Zin (1989) and Weil (1990), we assume that intertemporal utility is recursive thus permitting the notions of risk aversion and intertemporal substitutability to be partially disentangled [see also Epstein (1988)]. Roughly speaking, recursive utility specifications have two components, corresponding to certainty preferences (substitution) and risk preferences, respectively. An integral part of our model is the assumption that risk preferences exhibit 'first-order' risk aversion in the sense recently defined by Segal and Spivak (1990). In an expected utility model of choice amongst monetary gambles, the risk premium for small risks about certainty is proportional to the variance of the gamble, at least if the von Neumann–Morgenstern index is twice differentiable. That is because, as Pratt (1964, p. 126) notes, to the first-order, utility is linear and thus the risk premium is determined by second-order terms. Correspondingly, we think of this case as one of 'second-order' risk aversion. On the other hand, if the risk premium is proportional to the standard deviation of the gamble, then we speak of 'first-order' risk aversion. Given the smoothness of consumption data, the standard deviation of the consumption growth rate is considerably larger than its variance. Therefore, the role played by first-order risk aversion in generating a sizeable equity premium is intuitive. A nonexpected utility theory of risk preference which exhibits first-order risk aversion and which we adopt here is rank-dependent expected utility, where the rank ordering of outcomes plays a critical role. A special case that is useful below is Yaari's (1987) dual theory of choice. The rank-dependent theory has respectable theoretical credentials [for axiomatizations see Quiggin (1982), Yaari (1987), and Segal (1989)] and some empirical support (see section 4).

We conclude this introduction by adding to the papers already cited a number of other relevant studies of the equity premium puzzle and related asset pricing issues. Reitz (1988) posits the possibility of disasters with small but positive probabilities. [See Mehra and Prescott (1988) for a discussion.] Constantinides (1990) 'resolves' the puzzle by means of a habit-formation specification but only by assuming negative time preference. Weil (1989) adopts a recursive intertemporal utility specification in which risk preferences exhibit second-order risk aversion. He argues that the separation of substitution from risk aversion does not improve upon the performance of the Mehra–Prescott model [see also Kocherlakota (1990)]. Weil shows that the risk premium increases as the elasticity of substitution falls (independently of risk aversion), but with the adverse consequence of inflating the risk-free rate, e.g., 1.31% premium with a 21.68% risk-free rate. On the other hand, with risk preferences that exhibit first-order risk aversion, we show that a modest risk premium, e.g., 1.6%, is compatible with a risk-free rate on the order of 2.6%. Finally, Epstein and Zin (1990) estimate the Euler equations implied by the same parametric specification employed by Weil. Using U.S. aggregate monthly data on consumption and asset returns, they find some support for the general utility specifications while the data generally reject the standard intertemporally additive expected utility function. However, the general equilibrium interrelations between consumption and asset returns, which are implied by the theory and which are central to the present study, are not tested there.

The paper proceeds as follows: Section 2 describes the asset-pricing model with recursive utility and general risk preferences. Next rank-dependent preferences and first-order risk aversion are defined and discussed. In section

4 we discuss the plausibility of our utility specification and its calibration. Finally, a number of numerical simulations are described in section 5. Concluding remarks are offered in section 6.

2. Asset pricing with recursive utility[1]

A representative agent consumes a single perishable consumption good in each period. In period t, current consumption c_t is known with certainty but future consumption levels are generally uncertain. The intertemporal utility functional is recursive in the sense that the utility U_t, derived from consumption in period t and beyond, satisfies the recursive relation

$$U_t = W(c_t, \mu_t), \qquad t \geq 0, \tag{1}$$

where $\mu_t = \mu(\tilde{U}_{t+1})$ is the certainty equivalent of random future utility $\tilde{U}_{t+1}$ and W is called an aggregator. The latter aggregates current consumption c_t with a certainty equivalent index of future consumption in order to determine current utility U_t.

We restrict W to have the CES form

$$W(c, z) = (c^\rho + \beta z^\rho)^{1/\rho}, \qquad 0 \neq \rho < 1, \quad 0 < \beta < 1. \tag{2}$$

Then the utility of deterministic consumption paths is given by the CES intertemporal utility function

$$U_0 = \left(\sum_{t=0}^{\infty} \beta^t c_t^\rho \right)^{1/\rho},$$

having elasticity of substitution $(1-\rho)^{-1}$.

The functional μ assigns a certainty equivalent to any real-valued random variable and is the risk preference function referred to in the introduction. For example, in EZ (1990) and in Weil (1989, 1990) the following specification is considered:

$$\mu(\tilde{U}_{t+1}) = \begin{cases} \left(\mathrm{E}_t \tilde{U}_{t+1}^\alpha\right)^{1/\alpha}, & 0 \neq \alpha < 1, \\ \exp\left(\mathrm{E}_t \log(\tilde{U}_{t+1})\right), & \alpha = 0, \end{cases} \tag{3}$$

where E_t is the expectation conditional on period-t information. Mehra and Prescott (1985) further impose $\alpha = \rho$, which leads to the standard intertem-

[1]For the details which support the discussion in this section, the reader is referred to Epstein and Zin (1989). Those authors are henceforth EZ.

poral expected utility function

$$U_0 = \left(\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^\rho \right)^{1/\rho},$$

if $\rho \neq 0$. Alternative specifications will be adopted below. In all cases, it is assumed that μ reflects *constant relative risk aversion*, i.e.,

$$\mu(\lambda \tilde{U}) = \lambda \mu(\tilde{U}), \qquad \forall \lambda > 0. \tag{4}$$

The function μ embodies the agent's risk aversion in the following sense: the intertemporal preference ordering represented by U becomes more risk-averse if ρ is held fixed and μ becomes more risk-averse as a functional of real-valued random variables or associated distribution functions.

The agent's economic environment is identical to that in Mehra and Prescott (1985) and similar to that in Lucas (1978). In particular, the endowment process $\{\tilde{y}_t\}_0^\infty$ is such that growth rates $x_{t+1} \equiv y_{t+1}/y_t$ follow a first-order Markov process. The ex-dividend price of the single equity asset may be described by the time-invariant and positive function $p(x_t, y_t)$. In light of the homogeneity of preferences it follows that price is linearly homogeneous in current output, i.e.,

$$p(x, y) = p(x, 1)y \equiv P(x)y. \tag{5}$$

Denote by M_{t+1} the return to equity, and therefore to the market, over the interval from t to $t+1$, i.e.,

$$M_{t+1} \equiv \frac{p(x_{t+1}, y_{t+1}) + y_{t+1}}{p(x_t, y_t)} = x_{t+1} \frac{(P(x_{t+1}) + 1)}{P(x_t)}. \tag{6}$$

In equilibrium, the agent maximizes utility, markets clear, and price expectations are fulfilled. Thus [from EZ (1989, eq. (5.9))],

$$\beta \mu^\rho \left(x_{t+1}^{(\rho-1)/\rho} \tilde{M}_{t+1}^{1/\rho} \mid I_t \right) = 1,$$

where we have made explicit the conditioning information $I_t = (x_t, y_t)$. If we substitute (6) and apply (4), we obtain immediately the following recursive relation which must be satisfied by any equilibrium price function:

$$P(x_t) = \beta \mu^\rho \left(\tilde{x}_{t+1} (P(\tilde{x}_{t+1}) + 1)^{1/\rho} \mid I_t \right), \qquad t \geq 0. \tag{7}$$

Note that when μ is defined by (3) with $\alpha = \rho$, (7) reduces to the familiar recursive relation in Mehra and Prescott (1985). In general, (6) and (7) determine the return to equity in this economy.

To determine the risk-free return r_f (whose time dependence is suppressed in this notation) implied by our model, add a risk-free asset to the choice set of the individual. From EZ (1989, eq. (5.10)), the individual's portfolio choice is determined by solving

$$\max_{0 \leq a \leq 1} \mu\Big((\tilde{x}_{t+1}/\tilde{M}_{t+1})^{(\rho-1)/\rho} \big(a\tilde{M}_{t+1} + (1-a) r_f \big) \mid I_t \Big). \tag{8}$$

Of course, a denotes the proportion of savings invested in equity. The risk-free return r_f is fixed by the requirements that at an optimum of (8) $a^* = 1$ and the constraint $a \leq 1$ should not be binding.

A useful special case of the model is one where growth rates are i.i.d. In that case, P is constant and, therefore by (6), $M_{t+1}/x_{t+1} = K$ is also constant. Given the homogeneity of μ, this constant can be factored out of (8) and we conclude that (8) can be replaced by the following myopic portfolio choice problem with risk preferences corresponding to μ:

$$\max_{0 \leq a \leq 1} \mu\big(a\tilde{M} + (1-a) r_f \big). \tag{9}$$

Conditioning information and the time subscript have been deleted since $\tilde{M}_t$ is i.i.d. In addition, (6) and (7) imply that

$$\mathrm{E}\tilde{M} = \beta^{-1} \mathrm{E}\tilde{x} / \mu^{\rho}(\tilde{x}). \tag{10}$$

In particular, the constant of proportionality K between the market return $\tilde{M}$ and the consumption growth rate $\tilde{x}$ is

$$K = \beta^{-1} \mu^{-\rho}(\tilde{x}). \tag{11}$$

The conditions for existence of an equilibrium are readily derived in the i.i.d. case. A positive price, P, solving (7) exists if and only if

$$\beta \mu^{\rho}(\tilde{x}) < 1. \tag{12}$$

In the non-i.i.d. experiments of section 5 we prove existence of a positive solution to (7) by direct computation.[2]

[2]A sufficient condition for the existence of such a $P(\cdot)$ is that $\beta\mu^{\rho}(\tilde{x}_{t+1} \mid I_t) < 1$ for all $t \geq 0$. If μ is given by (3) and $\alpha = \rho$ and if a finite-state Markov chain is assumed for $\tilde{x}_t$, then this implies the condition for equilibrium given by Mehra and Prescott (p. 151).

3. Risk preferences

In this section we consider functional forms for the risk preference or certainty-equivalent function μ and then relate the specifications to the notion of first-order risk aversion. The functional forms are all consistent with monotonicity (in the sense of first-degree stochastic dominance), risk aversion (in the sense of aversion to mean-preserving spreads), and constant relative risk aversion [in the sense of (4)]. They also satisfy the normalization

$$\mu(x) = x, \qquad x > 0, \tag{13}$$

where x in $\mu(x)$ refers to the random variable that equals x with certainty.

It suffices for our purposes to consider only random variables $\tilde{x}$ having finitely many positive distinct outcomes, $x_1, \ldots, x_n$, with associated probabilities $p_1, \ldots, p_n$. The following specification corresponds to Yaari's (1987) dual theory of choice:

$$\mu_Y(\tilde{x}) \equiv \sum_{i=1}^{n} \left[g\left(\sum_1^i p_j \right) - g\left(\sum_1^{i-1} p_j \right) \right] x_i, \tag{14}$$

if $x_1 < x_2 < \cdots < x_n$. The function g: $[0,1] \to [0,1]$ is onto, strictly increasing, and concave. If g is linear, then $\mu_Y(\tilde{x})$ is simply the expected value of $\tilde{x}$. But if g is strictly concave, then μ_Y exhibits strict risk aversion [Yaari (1986)].

The nature of μ_Y is further clarified by looking at binary gambles. Then

$$\mu_Y(\tilde{x}) = g(p_1)x_1 + (1 - g(p_1))x_2, \qquad x_1 < x_2. \tag{15}$$

Under the above assumptions for g, $g(p_1) > p_1$ if the degenerate case of linear g is excluded. Thus the inferior outcome x_1 is given more weight in (15) than in the computation of $\mathrm{E}\tilde{x}$. Consequently, $\mu_Y(\tilde{x}) < \mathrm{E}\tilde{x}$, which is a form of risk aversion. If p_1 is fixed, μ_Y defines indifference curves in outcome space. A typical indifference curve is given by the piecewise linear curve in fig. 1.

A generalization of μ_Y in which the piecewise linearity is eliminated is provided by rank-dependent expected utility theory, according to which

$$\mu_{\mathrm{rd}}(\tilde{x}) = v^{-1}\left\{ \sum_{i=1}^{n} \left[g\left(\sum_1^i p_j \right) - g\left(\sum_1^{i-1} p_j \right) \right] v(x_i) \right\}, \tag{16}$$

where v is a continuous, strictly increasing, and concave function on the

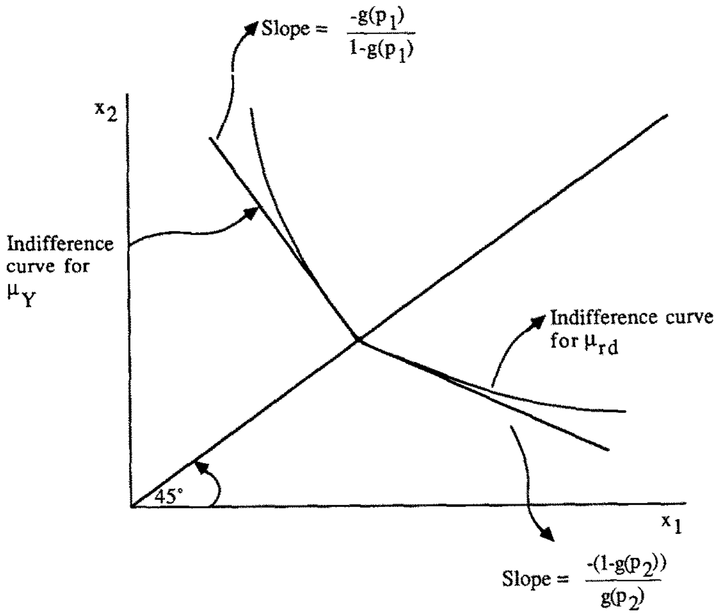

Fig. 1

positive real line.[3] To ensure the homogeneity (4), we also assume that

$$v(x)=\begin{cases}\dfrac{x^{\alpha}-1}{\alpha}, & 0\neq\alpha\leq 1,\\ \log(x), & \alpha=0.\end{cases}\tag{17}$$

If $\alpha=1$, μ_{rd} specializes to μ_{Y}. On the other hand, if $g(p)\equiv p$, the expected utility certainty-equivalent (3) is obtained.

A typical indifference curve of μ_{rd} is shown in fig. 1. Piecewise linearity is eliminated (if $\alpha\neq 1$), but there is still, significantly for our purposes, a kink at the certainty line. The presence of the kink corresponds to the fact that neither one-sided marginal rate of substitution at certainty equals the ratio of

[3]Unless otherwise specified the rank ordering $x_1<x_2<\cdots<x_n$ is assumed. For the axiomatic underpinnings of μ_{rd}, see Quiggin (1986) and Segal (1989). For properties, such as risk aversion, see Chew, Karni, and Safra (1987).

probabilities. This is in contrast to an expected utility model with a differentiable von Neumann–Morgenstern utility index, where indifference curves and the lines of constant expected value are mutually tangent at certainty. As a consequence, the first-order term in the Taylor-series expansion of the risk premium vanishes and for small risks the risk premium is proportional to the variance of the gamble [Pratt (1964)]. In such a case we say that preferences exhibit second-order risk aversion.

In contrast, in a model such as (16)–(17), the risk premium for a binary gamble is proportional to the standard deviation, rather than the variance of the gamble. To see this, fix p_1 and let the outcomes $x_1 < x_2$ of the binary gamble $\tilde{x}$ vary near certainty. Then

$$\mu_{\mathrm{rd}}(\tilde{x}) = \mathrm{E}\tilde{x} - k\sigma + \mathrm{o}(\sigma), \tag{18}$$

where σ denotes the standard deviation of $\tilde{x}$ and

$$k = (g(p_1) - p_1)/p_1^{1/2}(1 - p_1)^{1/2}. \tag{19}$$

If the risk premium π satisfies

$$\mu(\tilde{x}) = \mu(\mathrm{E}\tilde{x} - \pi),$$

it follows that

$$\pi = k\sigma + \mathrm{o}(\sigma). \tag{20}$$

When $k > 0$, which is true in any rank-dependent model except the 'degenerate' expected utility special case, we refer to *first-order risk aversion*. [See Segal and Spivak (1990) for a definition and analysis in the context of general probability distributions.]

For small risks, σ is much larger than σ^2. Thus the potential usefulness of first-order risk aversion for rationalizing a sizeable equity premium is evident. We conjecture that any theory of risk preferences which implies indifference curves of the sort in fig. 1 would produce comparable numerical results regarding the equity premium. This is readily verified for the Chew (1989) and Gul (1988) risk preferences, which are axiomatically distinct from the specifications described above but which also exhibit first-order risk aversion.

Of course, kinked indifference curves can appear also in expected utility theory. Consider the general (i.e., nonhomogeneous) expected utility model:

$$\mu(\tilde{x}) = u^{-1}(\mathrm{E}u(\tilde{x})).$$

If u is not differentiable at a point $\bar{x}$, then the indifference curve through $\bar{x}$ will resemble that in fig. 1 and (20) will apply for gambles having expected value $\bar{x}$. But this can happen only rarely since any increasing and concave function u can fail to be differentiable at most at a countable number of points. In contrast, for μ_{rd} the indifference curve in fig. 1 is representative of all indifference curves and (20) holds for all binary gambles.

4. Calibration of preferences

Intertemporal utility of the representative agent has the recursive structure (1), where the aggregator W has the CES form (2) and the risk preference function is μ_{rd}. For the parameters of W, β, and ρ, we consider a standard range of values.

We need to select a functional form for g and a parameter α for v defined in (17). For g we adopt the functional form

$$g(p) = p^{\gamma}, \tag{21}$$

where $0 < \gamma \leq 1$ is a parameter. When $\gamma = 1$ in (21), $g(p) = p$ and μ_{rd} is the expected utility certainty equivalent defined in (3). The corresponding intertemporal utility function is that explored by EZ (1989, 1990) and Weil (1989, 1990). If further $\alpha = \rho$, then the standard intertemporal expected utility model (e.g., Mehra and Prescott) is implied. Weil (1989) has shown that allowing $\alpha \neq \rho$, while maintaining $\gamma = 1$, does not substantially improve the explanation of the equity premium. We attribute this finding to the fact that as long as $\gamma = 1$ and regardless of whether or not $\alpha = \rho$, risk preferences exhibit only second-order risk aversion.

On the other hand, if $\gamma < 1$, there is first-order risk aversion with the coefficient k in (18)–(20) given by

$$k = (p^{\gamma} - p)/p^{1/2}(1-p)^{1/2}. \tag{22}$$

Since k is decreasing in γ, we see that a smaller γ implies greater aversion to small risks. [In fact, it implies increased aversion to all risks; see Chew, Karni, and Safra (1987).] The numerical value of γ is obviously of critical importance.

It is natural to enquire whether there exists empirical support for first-order risk aversion in general, or for $\gamma < 1$ in our particular specification. It is shown in EZ (1989) that the agent's preference functional μ defines attitudes towards timeless gambles, i.e., those which are resolved before further consumption takes place. Since the bulk of the behavioral/experimental evidence regarding individual choice under uncertainty is based on choices amongst timeless gambles, that evidence can be brought to bear upon the nature of μ. For a survey of the evidence we have in mind see Machina (1982), who argues that widespread and systematic violations of expected utility are indicated. Segal (1987a, b) has shown that the rank-dependent model (16), with the functional form (21) for g, can explain the Allais and Ellsberg paradoxes if (and only if) $\gamma < 1$. In particular, the Mehra and Prescott specification ($\gamma = 1$ and $\alpha = \rho$) is *incompatible* with such evidence.

Segal and Spivak (1990) describe some behavioral distinctions between first- and second-order risk aversion. Only in the latter case is it true that any actuarially favorable bet would be accepted at some sufficiently small scale. Two related behavioral observations that would support first-order risk aversion are: (i) specialization in the safe asset in a two-asset portfolio choice in spite of a positive expected excess return to the risky asset and (ii) demand for full insurance (zero deductible) given positive but small marginal loading. Borch (1974, pp. 27–28) claims that there is empirical support for (ii). It is widely thought that the degree of portfolio diversification falls short of that predicted by the standard expected utility portfolio choice model; but admittedly, transactions costs, rather than misspecified preferences, could be the explanation.

Even if it is admitted that $\gamma < 1$, a magnitude for γ must still be selected. Before addressing that problem we consider briefly v and its parameter α [eq. (17)]. As fig. 1 reveals, regardless of the choice of α, the corresponding μ_{rd} agrees with μ_{Y} (linear v, $\alpha = 1$) to the first-order near-certainty, i.e., the respective indifference curves are tangent to one another on either side of the certainty line. Consequently, since our numerical simulations involve only small gambles about certainty, the choice of v is of little consequence for those simulations and we adopt Yaari's specification $v(x) \equiv x$.

The expected utility model has been thoroughly studied and applied and there exists a tradition concerning what constitutes a plausible value for the (constant) degree of relative risk aversion, namely that it be no greater than 10. In contrast, the preference specification adopted here is much less familiar and it is not immediately clear what constitutes a plausible magnitude for γ. To shed some light on the magnitude of γ we compare μ_{Y}, now denoted $\mu^{\gamma}_{\mathrm{Y}}$, and the expected utility function (3), denoted $\mu^{\alpha}_{\mathrm{eu}}$, in the following way: For any binary gamble with $m = \mathrm{E}\tilde{x}$, $x_1 < x_2$, and $p_1 = p$, write

$$x_1 = m - \left(\frac{1-p}{p}\right)^{1/2} \sigma, \qquad x_2 = m + \left(\frac{p}{1-p}\right)^{1/2} \sigma, \tag{23}$$

where σ is the standard deviation of $\tilde{x}$. By homogeneity (4), we can

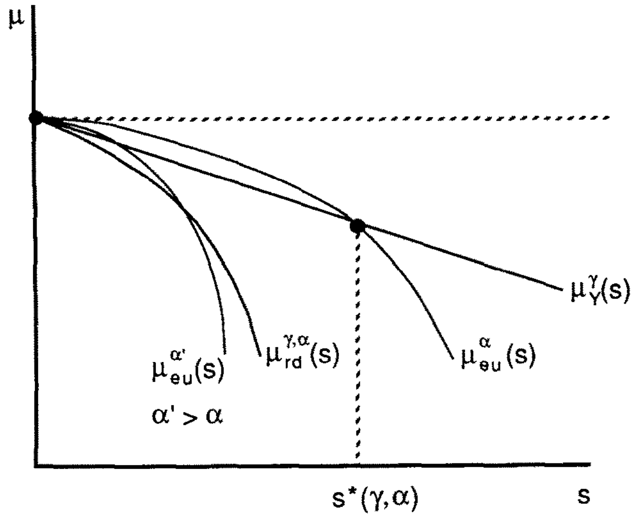

Fig. 2

normalize by the mean and thus restrict attention to gambles with unit mean. For fixed p, the gamble is completely specified by $s = \sigma/m$, the coefficient of variation, which we use as a measure of the riskiness of $\tilde{x}$. For simplicity, therefore, we may write $\mu(s)$ instead of $\mu(\tilde{x})$, where $\tilde{x}$ is defined in (23) with $(m, \sigma) = (1, s)$ and where the dependence upon p is suppressed in the notation.

By the nature of first-order risk aversion, if $\gamma < 1$, then μ_Y^γ is more risk-averse than μ_{eu}^α for gambles concentrated near certainty, for any α, i.e.,

$$\mu_Y^\gamma(s) < \mu_{eu}^\alpha(s) \quad \text{for } s \text{ near } 0.$$

On the other hand, the inequality is reversed for sufficiently large s. Moreover, there is a unique critical coefficient of variation $s^*(\gamma, \alpha)$ at which comparative degrees of risk aversion reverse, where

$$\mu_Y^\gamma(s^*(\gamma, \alpha)) = \mu_{eu}^\alpha(s^*(\gamma, \alpha)). \tag{24}$$

If $s^*(\gamma, \alpha) = \bar{s}$, then μ_Y^γ and μ_{eu}^α exhibit comparable degrees of risk aversion for binary gambles having coefficient of variation near $\bar{s}$.

Table 1

Risk aversion comparison; coefficient of variation, $s^*(\gamma,\alpha)$.[a]

		α						
p	γ	0	−1	−2	−3	−4	−9	−20
0.50	0.50	0.71	0.41	0.29	0.22	0.18	0.09	0.04
	0.75	0.37	0.19	0.13	0.10	0.08	0.04	0.02
	0.90	0.14	0.07	0.05	0.04	0.03	0.01	0.01
0.25	0.50	0.51	0.35	0.26	0.20	0.17	0.09	0.04
	0.75	0.33	0.19	0.13	0.10	0.08	0.04	0.02
	0.90	0.15	0.08	0.05	0.04	0.03	0.02	0.01

[a] $s^*(\gamma,\alpha)$ is the coefficient of variation that equates the certainty equivalent associated with Yaari preferences with risk aversion parameter γ and expected utility preferences with risk aversion parameter α. The probability of state one is given by p.

The above procedure is illustrated in fig. 2. Note that the curve for μ_Y^γ falls as γ falls toward zero and similarly for μ_{eu}^α as α falls. Also included in this figure is the curve for $\mu_{rd}^{\gamma;\alpha}$ defined by (16) and (17). It lies everywhere below the curve for μ_Y^γ and is tangent to it at $s = 0$, which has the implication noted earlier that in our model with small risks, the Yaari certainty equivalent and the rank-dependent certainty equivalent yield similar numerical results.

Table 1 presents values of s^* for a number of (γ,α) pairs and, respectively, for symmetric bets ($p=\frac{1}{2}$) and for bets having $p=\frac{1}{4}$. For symmetric gambles, the specification of μ_Y corresponding to $\gamma=\frac{1}{2}$ is comparable to an expected utility ordering with relative risk aversion coefficient of 10 for gambles having σ/m near 0.09. Thus a nontrivial degree of aversion towards a small amount of risk near certainty is compatible with a degree of aversion to moderate risks that lies within the range considered by Mehra and Prescott. We will shortly have reason to consider also the rank-dependent specification having $\gamma=\frac{1}{2}$ and $\alpha=\frac{3}{4}$ (rather than $\alpha=1$ as in μ_Y). Thus for completeness we note here that this μ_{rd} function produces a certainty-equivalent value equal to that implied by the above borderline specification of Mehra and Prescott for symmetric gambles having coefficient of variation equal to 0.095.

For a concrete example, table 2 provides, for five different specifications of risk preferences and for a range of alternative values of ε, the willingness to pay to avoid a symmetric gamble with outcomes $\pm\varepsilon$ given wealth equal to 75,000. (Alternatively, think of 75,000 as annual income and and $\pm\varepsilon$ as perturbations to that income.) The first three columns extend the calculations reported in Kandel and Stambaugh (1989) and substantiate their argument that for the commonly used expected utility functional form it seems possible to construct a gamble that makes any degree of relative risk aversion appear

Table 2

Some 'willingness-to-pay' calculations.[a]

	Risk preferences				
ε	μ^{α}_{cu} $(\alpha = -1)$	μ^{α}_{cu} $(\alpha = -9)$	μ^{α}_{cu} $(\alpha = -29)$	μ^{γ}_{Y} $(\gamma = \frac{1}{2})$	$\mu^{\gamma,\alpha}_{rd}$ $(\gamma = \frac{1}{2}, \alpha = \frac{3}{4})$
250	0.83	4.17	12.48	103.55	103.64
2,500	83.33	410.34	1,091.17	1,035.53	1,044.19
25,000	8,333.33	21,008.72	23,790.52	10,355.34	11,262.27
40,000	21,333.33	37,198.00	39,153.37	16,568.54	19,037.86
50,000	33,333.33	47,998.51	49,395.26	20,710.68	24,815.13
60,000	48,000 00	58,799.10	59,637.16	24,852.81	31,323.84
74,000	73,013.33	73,919.94	73,975.81	30,651.80	43,809.83

[a]Entries give the willingness to pay to avoid a gamble with equally likely outcomes $\pm\varepsilon$, given initial wealth equal to 75,000. Thus, for each μ and ε, the appropriate entry is $75{,}000 - \mu(\tilde{x})$, where $\tilde{x}$ equals $75{,}000 \pm \varepsilon$ with probability $\frac{1}{2}$.

unreasonable on introspective grounds. For example, if relative risk aversion is 2, the individual would pay 'only' 0.83 to avoid the smallest gamble and 'only' 83.33 to avoid the gamble with $\varepsilon = 2{,}500$. Note that the latter gamble has a coefficient of variation roughly equal to that of the U.S. per capita consumption growth rate series used by Mehra and Prescott (1985) and in section 5 below. If relative risk aversion equals 30, the willingness to pay rises to 12.48 for the small gamble. But then the individual would pay 23,790.52 to avoid the gamble having $\varepsilon = 25{,}000$.

The functional form μ_Y implies reasonable risk attitudes for a broader range of gamble sizes, though the willingness to pay seems too large for the smallest gamble and too small for the largest gamble. The former problem can be ameliorated by taking γ close to 1. The latter problem is due to the piecewise linearity of the indifference curves of the Yaari functional and is ameliorated by considering the rank-dependent form with $\alpha \neq 1$ in (17). For example, if $\alpha = 0.75$ (and $\gamma = 0.5$), then μ_{rd} implies much more reasonable levels for willingness to pay to avoid large gambles. Moreover, the change from $\alpha = 1$ to $\alpha = 0.75$ is of little consequence for the evaluation of the gamble having $\varepsilon = 2{,}500$ which, as noted, is the gamble size that corresponds most closely to the equity premium puzzle. This supports our contention, which we noted earlier but which bears repetition, that virtually the identical numerical results for the model economy of section 5 would be obtained if μ^{γ}_{Y} were replaced by $\mu^{\gamma,\alpha}_{rd}$ and $\alpha = 0.75$. The significance of this observation is that the levels of the risk-free rate and the equity premium obtained below are compatible with plausible risk attitudes over a very broad range of gamble sizes as demonstrated in the final column of table 2.[4]

[4]We chose to formulate the analysis of section 5 in terms of μ_Y rather than μ_{rd} because the former delivers some elegant and intuitive closed-form expressions. See particularly (26) below.

5. The equity premium

5.1. I.i.d. consumption growth[5]

Consider first the following simple endowment process: growth rates $\tilde{x}$ are i.i.d. and can assume the symmetric values $\mathrm{E}\tilde{x}-\varepsilon$ and $\mathrm{E}\tilde{x}+\varepsilon$. Typically, these states occur with equal probability, although we will consider some asymmetric distributions. As in Mehra and Prescott, we choose these states to approximately match the first two sample moments of aggregate U.S. data, i.e., $\mathrm{E}\tilde{x}=1.018$ and $\varepsilon=(\operatorname{var}\tilde{x})^{1/2}=\sigma=0.036$. We also consider experiments with roughly double the variance, i.e., $\sigma=0.051$.

Adopt the Yaari risk preference function μ_{Y} in which case (9) simplifies to the linear problem

$$\max_{0\leq a\leq 1} a\mu(\tilde{M})+(1-a)r_{\mathrm{f}}. \tag{9'}$$

We conclude that

$$r_{\mathrm{f}}=\mu(\tilde{M}). \tag{25}$$

Since $\tilde{M}=K\tilde{x}$, where K is given by (11), it follows that

$$r_{\mathrm{f}}=\beta^{-1}\mu^{1-\rho}(\tilde{x}). \tag{26}$$

Relation (26) is very intuitive and provides considerable insight into the workings of our model economy. For example, it resolves the 'risk-free rate puzzle' emphasized by Weil (1989). If consumption grows on average, say, 2% per year and consumption is not very substitutable over time (as much of the

[5]Kocherlakota (1990) has observed that in an i.i.d world it is impossible to distinguish empirically between the standard expected intertemporal utility model and recursive intertemporal utility. Similarly, risk preferences are not recoverable from observations of equilibrium prices in an i.i.d. economy. On first reflection, therefore, one might question how we can hope to improve upon the Mehra and Prescott analysis, at least in the context of an i.i.d. economy However, as was discussed in the introduction, the reasonableness of the utility specification is an integral part of the puzzle posed by Mehra and Prescott. While an expected utility specification with negative time preference and/or very large degree of relative risk aversion may be able to match the data as well as or even more closely than our model, such a specification is widely perceived with misgivings for reasons that were in part described above. The plausibility of our utility specification was argued in section 4.

Of course, the question of whether or not alternative utility functions are empirically distinguishable, given observations from a single dynamic equilibrium, is important in deciding on the broader usefulness of new models of preference. Lest the reader be misled by this reference to Kocherlakota's observation, we refer the reader to EZ (1990) where the expected utility and recursive intertemporal utility models are distinguished econometrically, and to Wang (1990) who shows that generically in the space of economies the Kocherlakota observation fails; that is, the i.i.d. case is very special rather than representative These issues are discussed more fully in Epstein (1990).

empirical literature suggests), then how can the equilibrium real interest rate be small? Short of assuming negative time preference, expected utility models have a great deal of difficulty dealing with this question. On the other hand, (26) shows that since, in our model, it is the certainty equivalent of consumption growth that determines the equilibrium level of the risk-free rate, use of simple averages for gaining intuition about the relationship between consumption, the real interest rate, and the elasticity of intertemporal substitution, may be misleading. However, the basic intuition about consumption and interest rates that is derived from deterministic models holds in our model, provided appropriate certainty equivalents are computed. Preferences that exhibit first-order risk aversion yield a certainty equivalent of consumption growth that is significantly smaller than its mean. If this certainty-equivalent growth rate is near one, then a moderate level of the risk-free rate is implied for a range of values for ρ. This consequence of (26) is central to the ability of this model to generate both a low risk-free rate and a nontrivial equity premium without resorting to negative time preference.

The equity premium $\mathrm{E}\tilde{M} - r_f$ can be computed from (26) and (10) once parameter values are selected. For example, if $\gamma = \frac{1}{2}$, then $\mu(\tilde{x}) = 1.003$ for the symmetric case with $\sigma = 0.036$. If further, $\rho = -1.0$ and $\beta = (1.02)^{-1}$, then $r_f = 1.026$ and $\mathrm{E}\tilde{M} - r_f = 0.016$. Note that, as anticipated, the certainty equivalent of consumption growth is substantially smaller than the mean in this case. Table 3 summarizes the results for a number of i.i.d. experiments. The preference parameters, γ and ρ, vary across experiments as do the parameters of the consumption growth process. It is interesting to note the effect increasing the negative skewness (lowering the probability of the unfavorable state, $p = \frac{1}{4}$), while holding the standard deviation fixed at 0.036, has on the certainty equivalent associated with $\gamma = \frac{1}{2}$: it falls below one. Therefore, by (26), for this certainty equivalent, the level of the risk-free rate is *increasing* in the elasticity of intertemporal substitution. In contrast to the results in Weil (1989), a smaller elasticity, in this case, generates a smaller risk-free rate. This reduction in the level of the risk-free rate, however, does not imply a reduction in the equity premium. For example, when $\gamma = \frac{1}{2}$ and $\rho = -1$, the risk-free rate is 1.4% and the equity premium is 2.1%. Lowering ρ to -4 results in a risk-free rate of 0.6% with an equity premium that is still 2.1%.

In the sense of the last section, our risk preference specification with $\gamma = 0.5$ is comparable to a degree of relative risk aversion equal to 10 given the usual expected utility specification for risk preferences. Thus compare our results with for $\gamma = 0.5$ with those implied by the Kreps–Porteus model used in Weil (1989), fixing the coefficient of relative risk aversion at 10. If the elasticity of intertemporal substitution is equal to 0.5 ($\rho = -1$), then the risk-free rate equals 3.8% and the equity premium is 1.3% (in the symmetric case with $\beta = (1.02)^{-1}$ and $\sigma = 0.036$). Lowering the substitution elasticity to

Table 3

Equilibrium returns and equity premia (i.i.d. consumption growth).[a]

ρ	p	σ	γ	E(M)	Sd(M)	r_f	E(M) − r_f
−1	0.50	0.036	0.50	1.042	0.037	1.026	0.016
			0.75	1.050	0.037	1.043	0.007
			0.90	1.054	0.037	1.052	0.002
		0.051	0.50	1.035	0.051	1.014	0.021
			0.75	1.047	0.052	1.037	0.010
			0.90	1.053	0.053	1.049	0.004
	0.25	0.036	0.50	1.035	0.021	1.014	0.021
			0.75	1.048	0.037	1.039	0.009
			0.90	1.054	0.037	1.051	0.003
		0.051	0.50	1.027	0.051	0.997	0.030
			0.75	1.044	0.052	1.032	0.012
			0.90	1.053	0.053	1.048	0.005
−4	0.50	0.036	0.50	1.051	0.037	1.036	0.015
			0.75	1.086	0.038	1.078	0.008
			0.90	1.104	0.039	1.101	0.003
		0.051	0.50	1.026	0.051	1.004	0.022
			0.75	1.074	0.054	1.063	0.011
			0.90	1.099	0.055	1.095	0.004
	0.25	0.036	0.50	1.027	0.036	1.006	0.021
			0.75	1.078	0.038	1.069	0.009
			0.90	1.102	0.039	1.098	0.004
		0.051	0.50	—[b]	—[b]	—[b]	—[b]
			0.75	1.063	0.053	1.050	0.013
			0.90	1.096	0.055	1.091	0.005
−9	0.50	0.036	0.50	1.068	0.038	1.052	0.016
			0.75	1.148	0.041	1.140	0.006
			0.90	1.192	0.042	1.189	0.003
		0.051	0.50	—[b]	—[b]	—[b]	—[b]
			0.75	1.119	0.056	1.109	0.010
			0.90	1.180	0.059	1.176	0.004
	0.25	0.036	0.50	—[b]	—[b]	—[b]	—[b]
			0.75	1.129	0.039	1.120	0.009
			0.90	1.186	0.042	1.183	0.003
		0.051	0.50	—[b]	—[b]	—[b]	—[b]
			0.75	1.094	0.055	1.081	0.013
			0.90	1.173	0.059	1.168	0.005

[a]The random endowment growth follows a two-state Markov process with states $m-(p/(1-p))^{1/2}\sigma$ and $m+((1-p)/p)^{1/2}\sigma$, where p is the unconditional probability of state 1, m is the unconditional mean, and σ is the unconditional standard deviation. The discount factor for preferences, β, is set to $(1.02)^{-1}$ and the autocorrelation coefficient coefficient, θ, is set equal to zero, i.e., these are all i.i.d. experiments.

[b]No equilibrium exists for these parameter values.

0.2 ($\rho = -4$) results in a risk-free rate of 7.5% and an equity premium that is still approximately 1.3%. Finally, lowering the substitution elasticity to 0.1 ($\rho = -9$) reduces this model to the Mehra–Prescott expected utility model and results in a risk-free rate of 14.0% and an unchanged equity premium of approximately 1.3%.[6] For the asymetric distribution, the level of the risk-free rate is slightly smaller and the equity premium is slightly larger. Therefore, comparable models with only second-order risk aversion consistently generate high levels of returns and do not typically predict a large equity premium for these experiments.

5.2. Autocorrelated consumption growth

More generally, we consider serially dependent, two-state Markov processes for endowment growth with transition probabilities given by

$$p_{ij} = \mathrm{Prob}\left[x_t = x_j \mid x_{t-1} = x_i\right].$$

Further, without loss of generality in the two-state economy, we parameterize the serial dependence with the autocorrelation parameter θ such that

$$p_{ij} = p_j(1-\theta) + \delta_{ij}\theta, \qquad -1 < \theta < 1,$$

where p_j is the unconditional (or equilibrium) probability of being in state j and $\delta_{ij} = 1$ for $i = j$ and is zero otherwise. The two states are determined as in eq. (23) for given values for the mean, m, the standard deviation, σ, and the probability of the first state, p, for the unconditional distribution of the endowment growth process.

Table 4 presents means and standard deviations for equilibrium equity and bond returns computed for a variety of endowment processes and values of the preference parameters. The persistence parameter is varied across simulations to allow for both negative ($\theta = -0.2$) and positive ($\theta = 0.2$) serial dependence. The risk-preference parameter, γ, is chosen to be 0.5, 0.75, or 0.9 with 0.5 being the most risk-averse and 0.9 the least risk-averse. The simulation results in table 4 relate to a symmetric unconditional distribution ($p_1 = p_2 = p = \frac{1}{2}$) with unconditional mean and variance of 1.018 and 0.036, respectively. The substitution parameter, ρ, is chosen to be -1, -4, or -9 corresponding to intertemporal substitution elasticities in consumption of 0.5, 0.2, and 0.1, respectively. The rate of time preference is fixed at 0.02 so that

[6] Weil (1988) shows that if (one plus) the equity premium is defined as the ratio of the gross equity return to the gross bond return, the elasticity of intertemporal substitution has no effect on the premium in the i.i.d. economy. Since we have defined the premium as the difference in the returns, this elasticity can have an effect on the premium, though this effect is negligible for the typical i.i.d. experiment.

Table 4

Equilibrium returns and equity premia (autocorrelated consumption growth).[a]

ρ	γ	θ	E(M)	Sd(M)	E(r_f)	Sd(r_f)	E($M-r_f$)	Sd($M-r_f$)
−1	0.50	−0.2	1.044	0.043	1.027	0.010	0.017	0.041
		0.2	1.039	0.033	1.027	0.011	0.012	0.030
	0.75	−0.2	1.051	0.044	1.043	0.013	0.008	0.042
		0.2	1.049	0.032	1.044	0.013	0.005	0.029
	0.90	−0.2	1.055	0.045	1.051	0.014	0.004	0.042
		0.2	1.054	0.032	1.052	0.014	0.002	0.028
−4	0.50	−0.2	1.060	0.062	1.037	0.026	0.023	0.055
		0.2	1.043	0.030	1.038	0.027	0.005	0.012
	0.75	−0.2	1.091	0.070	1.078	0.034	0.013	0.060
		0.2	1.082	0.035	1.080	0.034	0.002	0.007
	0.90	−0.2	1.107	0.072	1.101	0.038	0.006	0.062
		0.2	1.103	0.038	1.103	0.038	0.000	0.005
−9	0.50	−0.2	1.089	0.098	1.054	0.053	0.036	0.078
		0.2	1.050	0.055	1.050	0.054	0.000	0.017
	0.75	−0.2	1.162	0.117	1.141	0.072	0.021	0.090
		0.2	1.141	0.078	1.141	0.074	0.000	0.029
	0.90	−0.2	1.200	0.125	1.189	0.081	0.011	0.094
		0.2	1.192	0.088	1.192	0.081	0.000	0.033

[a] Random consumption growth follows a two-state Markov process with states $m-(p/(1-p))^{1/2}\sigma$ and $m+((1-p)/p)^{1/2}\sigma$, where p is the unconditional probability of state 1, m is the unconditional mean, σ is the unconditional standard deviation, and θ is the autocorrelation coefficient. The parameters p, m, and σ are set at 0.5, 1.018, and 0.036, respectively, and the discount factor for preferences, β, is set at $(1.02)^{-1}$.

the discount factor, β, is approximately equal to 0.98. The largest average equity premium generated by the simulations in table 4 is 3.5% (when $\gamma=0.5$, $\rho=-9$, $\theta=-0.2$) and the smallest is 0.0% (when $\rho=-9$, $\theta=0.2$). The typical premium, however, is between 1% and 2% so that the i.i.d. examples above are fairly representative.

The following patterns emerge upon examination of these tables: The average premium on equity gets larger (i) as the agent becomes more risk-averse, (ii) as the endowment growth process gets more negatively autocorrelated, (iii) as the distribution for endowment growth becomes more skewed, (iv) as the variance of the endowment growth becomes larger, (v) as substitutability decreases when endowment growth is negatively autocorrelated, and (vi) as substitutability increases when the endowment growth is positively autocorrelated.

The model also tends to underpredict the second moments of equity return data. The largest standard deviation for the equity return in table 4 is 12.5%, which is smaller than the 16.5% reported in Mehra and Prescott for historical data. The largest standard error for the risk-free bond is 8.1%, which is larger than the historical estimate of 5.7%. The largest equity premium is not necessarily associated with the largest standard deviations for returns. The experiment that generates the largest premium generates a standard deviation for equity returns of 9.8% and a standard deviation for bond returns of 5.3%. The model, therefore, is unable to account for all of the variance in stock returns but does account for most of the variance in bond returns.

6. Conclusion

We have explored whether the specification of first-order risk aversion for risk preferences can help to resolve the equity premium puzzle posed by Mehra and Prescott (1985). Our findings indicate that we can account for a low risk-free rate and an average equity premium of roughly 2%. This is in contrast to the historical average risk premium of 6.2% and the largest premium obtainable by Mehra and Prescott of 0.35%. Thus our utility specification can only partially resolve the puzzle. That it cannot resolve the puzzle completely, we find neither surprising nor discouraging. We expect that features of real economies that have frequently been mentioned, such as money and incomplete markets, will be required for a complete resolution. But this paper does provide some reason to believe that first-order risk aversion may be part of such a complete explanation.

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