TOWARD A THEORY OF INVENTIVE ACTIVITY AND CAPITAL ACCUMULATION*

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I. *Introduction*

In at least two recent models of economic growth, the rate of technical change depends upon other economic variables. The first, a model introduced by Kaldor [3] [4] [5], assumes a positive relationship (the technical progress function) between relative changes in productivity per worker and relative changes in gross investment. The technical progress function is an eclectic amalgam summarizing basic technical and institutional forces in a free enterprise economy. Kaldor takes the Schumpeterian view that the creation of new ideas largely occurs at an autonomous rate, but that the implementation of these new techniques by entrepreneurs can be explained by economic phenomena. Obviously, if the implementation of a new technique requires new capital equipment as opposed to mere organizational change, increased productivity can only be transmitted through new (gross) investment. In addition, Kaldor argues that for a capitalist economy the higher the relative rate of gross investment the higher the degree of "technical dynamism." Technical dynamism is a mass measure of entrepreneurial psychology including the readiness to adopt new methods of production.

In the second model with endogenous technical change, Arrow [1] concentrates upon the relation between learning and experience. Economic learning results in higher productivity, while cumulative gross investment is the measure of economic experience. Therefore, in refining the technical progress function, Arrow explicitly postulates that productivity per worker is determined by accumulated gross investment. The production of new technical knowledge (invention) and the transmission and application of that knowledge (innovation) are treated as by-products in the production and adoption of new capital goods.

While it is doubtlessly true that technical change is related to gross investment both as a by-product of capital goods production and as a vehicle for embodying new techniques in new capital equipment, it is also true that the rate of production of technical knowledge can be increased by increasing the allocation of economic resources explicitly devoted to inventive activity.

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At least two peculiar properties of technical knowledge require special study. First, technical knowledge can be used by many economic units without altering its character. Thus, for the economy in which technical knowledge is a commodity, the basic premises of the classical welfare economics are violated, and the optimality of the competitive mechanism is not assured. Typically, technical knowledge is very durable and the cost of transmission is small in comparison to the cost of production. Second, at least on the microeconomic level, the inventive process is characterized by extreme riskiness.[1]

II. *The Model*

I have argued that increases in technical knowledge are fundamentally related to the amount of resources explicitly devoted to inventive activity. In order to study the respective roles of invention and investment in economic growth, I assume that current aggregate output $Y(t)$ is determined by the relation

$$Y(t) = F[A(t), K(t), L(t)], \tag{1}$$

where $A(t)$ and $K(t)$ denote the current levels of the stocks of technical knowledge and physical capital, respectively; $L(t)$ denotes the current size of the labor force inelastically offered for employment.[2]

The growth in the stock of technical knowledge satisfies the differential equation

$$A(t) = \sigma\alpha(t)Y(t) - \rho A, \tag{2}$$

where $0 \leq \alpha(t) \leq 1$ is the fraction of output currently devoted to invention and $0 < \sigma \leq 1$ is the fraction of inventions that are "successful." For the case where ρ is positive, equation (2) should be understood as a long-run approximation to processes not explicitly treated in the model. For example, decay in technical knowledge is observed because of the imperfect transmission of technical information from one generation of the labor force to the next.

If capital is subject to evaporative decay at the given technical rate $\mu > 0$, then

$$\dot{K}(t) = s(t)(1 - \alpha(t))Y(t) - \mu K(t), \tag{3}$$

where $0 \leq s(t)(1 - \alpha(t)) \leq 1$ is the fraction of output currently devoted to investment.

[1] Cf., e.g., [8], especially Arrow's contribution on pp. 609-25.

[2] I treat the one-sector model for ease of exposition. This implies that the production possibility frontier is a hyperplane in the consumption-investment-invention space. If the model is disaggregated to two or three sectors, then the frontier can possess greater curvature. Also notice that increases in efficiency are shared by all vintages of capital and labor.

III. *A Stylized Economy*

In the United States, intervention in behalf of inventive activity has taken two basic forms: (1) the establishment of a legal device, the patent, designed to bestow property rights on certain of the outputs of the inventive process; (2) direct nonmarket support of research and development. The universities and the Department of Agriculture, for example, have contributed to our economy in the second role. Recently the Department of Commerce has initiated industrial research programs modeled after the programs of the agricultural research stations, while the Department of Defense favors the device of contracting research tasks to private enterprises on a cost-plus-fixed-fee basis.

Consider a model economy in which production is undertaken by many individual firms. The level of technical knowledge enters each firm's production function as a pure public good of production. Hence, the competitive price of invention is zero—suggesting the desirability of social intervention in the market process.[3] Assume that the only form of intervention is the imposition of a tax upon output at a given constant rate $0<\alpha<1$, the proceeds of which are used to support invention. The private sector saves (and invests in capital accumulation) the constant fraction $0<s<1$ of disposable income.

I assume that the production function given in (1) exhibits constant returns in capital and labor and consequently increasing returns in all three factors. Then if the special assumption is made that there are constant returns to (Hicks-neutral) technical knowledge,[4] then (1) can be rewritten as

$$y = Af(k), \tag{4}$$

where lower-case letters denote quantities per worker and $f(k)$ is shorthand for $F(k, 1)$. $f(\cdot)$ is twice-continuously differentiable with $f(k)>0, f'(k)>0, f''(k)<0$ for $0<k<\infty$; $f(0)=0, f(\infty)=\infty, f'(0)=\infty$, $f'(\infty)=0$. For simplicity, I assume that there is no change in the labor force, thus setting $L=1$ and writing

$$A = \alpha\sigma y - \rho A, \tag{5}$$

$$k = s(1-\alpha)y - \mu k. \tag{6}$$

From (4) and (5), $\dot{A}=0$ if and only if

[3] In practical applications, the distinction between private and public goods is fuzzy. What is considered to be a public good under one set of legal and social arrangements may be considered to be a private good under a different set of arrangements. In choosing among differing arrangements, society should include in its calculation the buying and selling costs that they imply.

[4] Qualitative long-run behavior does not depend upon this special assumption. It does allow a simple aggregation congenial to the competitive hypothesis: $Y=\Sigma_j Y_j=A\Sigma_j F(K_j, L_j)$ where, for example, K_j is the quantity of capital employed by the jth firm.

$$(7) \qquad f(k) = \frac{\rho}{\alpha\sigma} \cdot$$

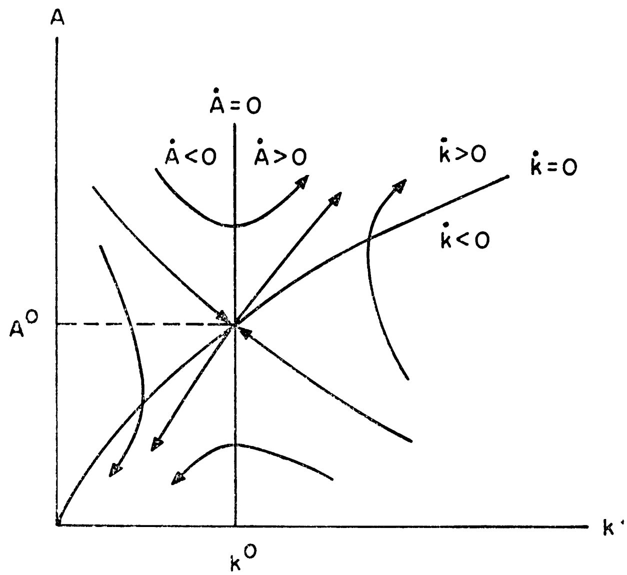
Call the unique solution to (7) k^0. But from (4) and (6), $\dot{k}=0$ if and only if

$$(8) \qquad A = \frac{\mu k}{s(1-\alpha)f(k)},$$

which yields

$$\left(\frac{dA}{dk}\right)_{\dot{k}=0} = \frac{\mu[f(k) - kf'(k)]}{s(1-\alpha)[f(k)]^2} > 0.$$

The laws of motion for the stylized economy are shown in the phase diagram below. To verify that (A^0, k^0), the unique solution to (7) and



(8), is a saddlepoint, solve the characteristic equation for the linear Taylor approximation to (5) and (6) about (A^0, k^0). The characteristic roots are real and opposite in sign, guaranteeing that (A^0, k^0) is a local saddlepoint. Twice-continuous differentiability of $f(\cdot)$ then guarantees that (A^0, k^0) is a global saddlepoint. Notice that for initial endowments sufficiently large (small), the stylized economy explodes (decays).[5]

[5] In the multisector model, it is possible that there are many equilibrium points. Some would be saddlepoints; the others would be stable or surrounded by limit cycles. See [10]. Global stability is an interesting property for a descriptive growth model but should not be thought to be essential. In fact, Maruyama [7] argues that social systems are basically morphogenetic rather than morphostatic.

IV. *The Controlled Economy*

In what follows, it will be convenient to assume that equation (1) can be rewritten as

$$y = g(A, k), \tag{9}$$

where $g(\cdot)$ is an increasing strictly concave function of A and k. This assumption is made in order to avoid complications by insuring that the usual necessary conditions for optimality are also sufficient conditions. Suppose that the economic planning board desires to maximize the sum of future discounted utility (of per capita consumption)

$$\int_0^\infty U[c(t)]e^{-\delta t}dt. \tag{10}$$

subject to given initial endowments $A(0) = A_0$, $k(0) = k_0$, and technology given by (5), (6), (9). $U[c(t)]$ is the utility of consumption at time t, with $U'[c] > 0$, $U''[c] < 0$ for $0 < c < \infty$, $U'[0] = \infty$, $U'[\infty] = 0$. $\delta > 0$ is the (constant) pure rate of social discount. For simplicity, set $L(t) \equiv 1$.

Because the marginal utility of consumption is infinitely large when consumption is zero, consumption must be everywhere positive on the optimal program. Further assume that the initial endowments (A_0, k_0) are sufficiently small that the optimal program will never be specialized to consumption. A feasible consumption program $\{c(t) : 0 \leq t \leq \infty\}$ satisfying (5), (6), and (9) is optimal[6] if, and only if, there exist continuous functions $q(t)$ and $v(t)$ such that

$$\dot{q} = (\delta + \mu)q - \max(q, v\sigma)g_k(A, k), \tag{11}$$

$$\dot{v} = (\delta + \rho)v - \max(q, v\sigma)g_A(A, k), \tag{12}$$

$$\lim_{t \to \infty} q(t)e^{-\delta t} = \lim_{t \to \infty} v(t)e^{-\delta t} = 0, \tag{13}$$

where $0 \leq \alpha(t) \leq 1$ and $0 \leq s(t) \leq 1$ are chosen at each instant to maximize

$$U[(1 - s)(1 - \alpha)g(A, k)] + \{qs(1 - \alpha) + v\alpha\sigma\}g(A, k). \tag{14}$$

If utility is the numeraire, then q and v are the social demand prices for investment and invention, respectively. Conditions (11) and (12) imply that the planning board has perfect foresight with respect to marginal products. Transversality condition (13) requires the present value of a unit of future investment or invention to become small as the future date becomes distant. Expression (14) is simply the certainty

[6] Cf. [9], especially theorem 7, p. 69, and pp. 188-91, 298-300.

equivalent of the imputed value of gross national product. Maximization of (14) implies that if $U'=q>v\sigma$ then $\alpha=0$, and if $U'=v\sigma>q$ then $s=0$. Further if the certainty equivalent net marginal products are equal, $\sigma g_A-\rho=g_k-\mu$, then $q=v\sigma$.

Notice that (11) and (12) have a special interpretation in a decentralized economy in which factors are rewarded by their marginal products. From (11), for example, the change in the demand price of investment should be such as to compensate the representative rentier for "abstinence" and depreciation loss net of rewards from the employment of his capital. Of course, for a simple decentralization of the economy treated above, condition (11) will not necessarily hold. If output is taxed to support invention, then private factors will not be paid their full marginal products. At any rate, if an optimal program exists in the fully controlled economy, the stocks of technical knowledge and physical capital will approach limiting values (A^*, k^*) such that $\sigma g_A(A^*, k^*)-\rho=g_k(A^*, k^*)-\mu=\delta$. The limiting value of consumption c^* is given by $c^*=g(A^*, k^*)-\mu k^*-\rho A^*/\sigma$.

It may be that the planning board treats the process of private saving and investment in physical capital as institutionally given leaving the choice of $\{\alpha(t):0\leq t\leq\infty\}$ as the remaining policy instrument. Assume, for example, that private capital accumulation follows (6) with savings $0<s<1$ a given fixed fraction of disposable income. The planning board desires to maximize (10) subject to given initial endowments and subject to technological and behavioral relations (5), (6), (9). It is implicit in this formulation that the free play of the private capital market does not necessarily yield a socially preferred result. There could be several reasons for this divergence, including the existence of conventional externalities and certain intrinsic impediments to borrowing and lending in a risky world.[7]

In the partially controlled economy, maximization of (14) implies that $(1-s)U'+qs\geq v\sigma$, or with equality if $\alpha>0$. The optimal consumption program $\{c(t):0\leq t\leq\infty\}$ in the partially controlled economy is such that the stocks of technical knowledge and physical capital approach limiting values (A^{**}, k^{**}) where $\sigma g_A(A^{**}, k^{**})-\rho=\delta$. Notice that asymptotically the marginal products of technical knowledge are identical for the partially and fully controlled economies. This is a (long-run) dynamic generalization of the Rule of the Second Best. If, for example, long-run private savings are too low, then long-run inventive activity will be greater in the partially controlled economy than it would be in the fully controlled economy.

[7] Arrow in [8] refers to such impediments as "moral hazards." I have found an exposition very similar to what follows in an unpublished paper by Arrow [2].

V. *Concluding Comments*

I have argued that in the study of aggregative models it is useful to think of technical knowledge as a public good of production, while the level of inventive activity (the process of production of knowledge) is dependent upon the amount of economic resources devoted to that activity. Of course, invention is a particularly risky form of social investment. In my model, a given fraction of inventions is "successful," thus removing the difficult decision problems associated with uncertainty. Perhaps this is a legitimate approximation in a macroeconomic model. On the other hand, differential equation (2) represents the most unsatisfactory simplification in the model. A complete theory should explicitly treat the problems of transmission of knowledge, e.g., education, book publishing, etc., and its effect upon the efficiency of different generations of the labor force and different vintages of capital. Finally, the recent contribution of Kennedy [6] warns us that additions to technical knowledge should not be thought of as increasing efficiency in any specified way. That is, the "bias of technical progress," whether in a stylized economy or in a planned economy, should be a subject for economic decision.

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