4 Inventive Activity, Industrial Organisation and Economic Growth*

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Empirical studies by Abramowitz, Denison, Kendrick, Solow and others have made it quite clear that the deepening of capital cannot in itself explain observed increases in productivity. While it is probably incorrect to attribute all the residual (unexplained increases in productivity) to 'technical progress', it is clear that inventive activity contributes importantly to increased productivity. (Although Griliches and Jorgenson in their recent production-function studies have been able to 'sop up' the unexplained residual with quality measures of inputs, hours worked and so forth, their results have not detracted from the importance of 'technical change' – as that expression is commonly understood.)

Spurred by these productivity studies, along with the realisation that *exogenous* theories of technical change are essentially confessions of ignorance, contemporary growth theorists have constructed a variety of models of *endogenous* technical change. Most prominent of these are the learning-by-doing models initiated by Arrow and the 'invention possibility set' models proposed by Hicks and Fellner and more fully elaborated by Kennedy, Samuelson, von Weizsäcker, Phelps and Drandakis, and others. (I shall skip over the planning models, such as Uzawa's study of 'optimal education' and Nordhaus's study of the optimal direction of invention, because my primary concern at this time is with the enterprise – or at least the mixed – economy.)

For the most part, in these contemporary growth models of the mixed or enterprise economy, either perfect competition is assumed or the specification of industrial organisation is vague. The Schumpeterian vision of capitalist development, that the level of inventive[1] activity and in turn growth in productivity are crucially dependent upon the prevailing form of industrial organisation, is

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[1] The distinction between invention and innovation is very important in the Schumpeterian theory. As a first approximation, this distinction is ignored in the present paper. In his paper for this Conference, C. C. von Weizsäcker examines anew the roles of invention and innovation in the growth process.

largely overlooked. In this paper I shall examine three substantially new models of invention and growth. At this writing, while I shall try to be very specific about the role of industrial organisation and growth, these models can only serve as a first step in the taxonomy of models of industrial organisation and inventive activity in the dynamic economy. In the first model, invention is financed solely from monopoly profits in the capital-goods industry. In the second model, inventive activity is financed solely by the government. These two models are in some sense polar cases. My hope is that by studying extreme cases light will be shed on the general problem. In the third model, I begin the analysis of a 'competitive' economy in which invention is primarily financed by the quasi-rents accruing to advanced technology. I shall also attempt to relate the new models to the existing literature on endogenous technical change.

I. MACRO-ECONOMIC MODELS OF GROWTH AND INVENTION: SOME GENERAL COMMENTS

It seems to me that if we are to develop a useful macro-economic theory of technical change, we shall be forced to employ the notion of an (aggregate) stock of technical knowledge. Output of the inventive process is accretion to the stock of technical knowledge. There are strong grounds for objection to this 'capital-theoretic' view of technical knowledge. While in life we can find two pieces of machinery that are essentially alike, if two inventions are very alike they are indeed the same invention. Possession of the first invention is enough; virtually nothing is gained by possession of a second scrap of paper describing an already known invention.[1]

Since there are important distinguishing differences among machines, our models of heterogeneous capital accumulation allow for several different types of machinery. Similarly, we can class technical knowledge by type, e.g. purely capital-augmenting inventions, purely labour-augmenting inventions (Hicks-neutral), output-augmenting inventions and so forth. Perhaps, if our models allowed for heterogeneity of *types* of inventions, then the basic point of Fellner and his followers – that the direction as well as the level of technical change is an endogenous economic variable – would be accounted for without resort to the invention-possibility-set construct.

Many important phenomena of economic development are missed when we study homogeneous (rather than heterogeneous) capital models. None the less, the one-sector growth theory served as an

[1] This point has important qualifications. Because of the costs of transmitting information and uncertainty, it is often socially desirable to pursue 'parallel projects'.

important first step in the study of capital accumulation. Similarly, much of the story of invention and growth will be left out of a model with *homogeneous technical knowledge*. It does seem to me, however, that this is the natural first step to be taken.

This is not to suggest that technical knowledge should be treated as merely another capital good. There are fundamental differences between the processes of invention and investment in physical capital which cannot be overlooked. In the study of the enterprise economy, there are four important facts with which we must contend.

(1) *Appropriability*

The cost of dissemination of technical knowledge is typically very low in comparison with its production cost. Furthermore, technical knowledge can be employed by an economic agent without altering either its quantity or its quality. Thus, we must think of technical knowledge as a public good – primarily a public good in production but also a public good in consumption. In order to promote the production of knowledge (invention), limited property rights (patents) are created, but patents reduce short-run allocational efficiency and enforcement costs are high in many cases.

(2) *Riskiness*

There is no doubt that the return on investment in machinery is substantially less risky than the return on inventive activity. While this is a fact that cannot be ignored, I do not think that it necessarily compels us, at this stage of research, to build models in which the stochastic element is explicitly accounted for. There are, however, important implications of this pervasive uncertainty, notably implications for the financing of R. & D., that must be considered.

(3) *Financial Aspects*

The financing of invention differs in an important way from the financing of more conventional investments, such as plant and equipment expenditure. This difference is only in part due to the greater riskiness of invention. The banker, say, who extends a loan for conventional investment holds a residual claim against tangible assets – buildings, machinery, inventory, accounts receivable and so forth. At each stage, the banker can assure himself that accounts are in order, that plants are being constructed and equipment is being installed. The financier of an inventive activity has far less assurance. Salaries are paid to technicians and scientists, inventories of test tubes and such are on hand, but after a while the main asset of the laboratory is the accumulation of 'experience' and 'intermediate knowledge' that is useful on the route to creating profitable inventions. It is difficult for the financier to judge the quality of the laboratory's 'experience' and 'intermediate knowledge'. If the pay-off is expected to be in the distant future, the financier is likely to worry about whether the laboratory is indeed pursuing its stated objectives. Thus, 'moral hazards' are inherent in the financing of inventive activity. For this reason, the financial markets are less efficient for R. & D. than for plant and equipment. To a greater extent than for conventional investment, we would expect that market R. & D. effort must be financed internally, either through internally generated profits or bankrolling by the inventor-entrepreneur.[1]

(4) *Returns to Scale*

Contemporary growth theory relies heavily on the assumption of constant returns to scale. If technical knowledge is an argument of the production function, then constant returns in *all* factors is not an attractive hypothesis. If the firm doubles its conventional factors, capital and labour, output should be at least doubled since mere replication is always a possibility. Therefore, if the firm doubles its conventional factors and doubles its stock of knowledge (as measured, say, in patents held), then the firm's output must be more than doubled. If the firm does indeed face these increasing returns to scale, then it is glaringly obvious that specification of industrial organisation will not be straightforward. For example, the competitive model with free entry or costless adjustment of inputs will not work. By Euler's Theorem, if factors were rewarded their marginal products, then payments to conventional factors would exhaust output, leaving no room for inventive activity.[2]

II. THE PURE MONOPOLY MODEL

In what follows, I shall study an economy composed of three sectors: (i) consumption, (ii) investment and (iii) inventive sectors.[3] Output of the various sectors is given by

$$Y_j = \Phi_j(K_j, A, L_j) \quad j = I, C, R. \tag{4.1}$$

[1] The importance of non-market financing of inventive activity should not be forgotten.

[2] This paragraph on increasing returns to scale at the firm level is to be taken as argument by *reductio ad absurdum*. I wish to show the incompatibility of competition and frequently encountered technological assumptions. I do not mean to argue that decreasing returns to scale (especially at the economy level) are impossible.

[3] This section is based on the paper, 'A Schumpeterian Model of Induced Innovation and Capital Accumulation', that I presented to the Winter Meeting of the Econometric Society, San Francisco, December 1966.

The subscripts I, C and R denote respectively the investment, consumption and inventive sectors. At any instant of time the fixed total stock of physical capital, K, can be divided among the three sectors:

$$K \geqslant \sum_j K_j. \tag{4.2}$$

Similarly, the labour force, L, can be divided among the three sectors:

$$L \geqslant \sum_j L_j. \tag{4.3}$$

The parameter A is interpreted as the stock of (homogeneous) technical knowledge. No j subscript is attached to A because the use of knowledge in one sector does not preclude its use in another sector of the economy.

If capital depreciates at the constant rate $\mu > 0$, then

$$\dot{K} = Y_I - \mu K. \tag{4.4}$$

We can also assume that technical knowledge deteriorates at the constant rate $\rho > 0$, so that

$$\dot{A} = Y_R - \rho A. \tag{4.5}$$

Differential equation (4.5) can be interpreted as a crude long-run approximation to fundamental processes not treated in the model. For example, a positive value of ρ reflects the loss to the economy due to retirement of the technically trained members of the labour force.

In what follows, it will be assumed that workers consume all their wages and that the consumption-goods sector is competitive, so that workers' consumption, $Y_C{}^W$, is given by

$$Y_C{}^W = wL = L\frac{\partial \Phi_C}{\partial L_C} \tag{4.6}$$

where w is the market wage rate. It is assumed that the investment-goods sector and the inventive sector are controlled by a single monopolist who sets Y_I and Y_R subject to technological and market constraints in order to optimise his own infinite-lifetime consumption stream.

The monopolist's income is equal to the rentals on machines employed in the consumption-goods sector. Since it is assumed that there is no way to appropriate directly the fruits of inventive activity (no patent system, etc.), inventive activity is pursued by the monopolist in order to lower his own unit costs in machine-goods production and, if possible, to raise the rental rate on physical capital.

The monopolist's expenses are the wages paid (in units of the consumption good) to workers in the research and machine-goods departments. If $Y_C{}^M$ is monopolist consumption, then

$$Y_C = Y_C{}^M + Y_C{}^M \tag{4.7}$$

where

$$rK_C = wL_R + wL_I + Y_C{}^M \tag{4.8}$$

and

$$wL_R + wL_I + wL_C = Y_C{}^W \tag{4.9}$$

where r is the rental on physical capital in terms of consumption.

(1) *Monopoly Capitalism: A Digression*

It is assumed for the purposes of this section that the production functions defined in (4.1) are such that the production-possibility frontier in (Y_C, Y_I, Y_R) space is a plane surface along which all ratios of supply prices are equal to unity. This will simplify the analysis, since by a proper choice of units we can reduce all calculations to those involving a 'single' production function, so that

$$Y \equiv Y_C + Y_I + Y_R = \Phi(K, A, L). \tag{4.10}$$

In order to simplify the analysis further, it is assumed that there is no growth in the labour force, $\dot{L} = 0$. For the purpose of this digression, technical knowledge is assigned no role in production, $(\partial\Phi/\partial A) \equiv 0$ and thus $Y_R = 0$. Under the assumptions made, output per worker y is a function of capital per worker k, written as

$$y = f(k) \tag{4.11}$$

where

$$\left.\begin{array}{l} f(k) > 0, \quad f'(k) > 0 \\ f''(k) < 0, \quad f'''(k) < 0, \quad \text{for} \quad 0 < k < \infty. \end{array}\right\} \tag{4.12}$$

In addition to the usual curvature assumptions, (4.12) implies that the monopolist's profit, $Lkf'(k)$, is a concave function of k. (From here on, assign $L \equiv 1$ for simplicity.)

The capitalist (a *bon vivant*) desires to maximise

$$\int_0^\infty U[(1-s)kf']e^{-\delta t}dt \tag{4.13}$$

where $\delta > 0$ is his subjective rate of time discount. The functional (4.13) is constrained by

$$s(t) \in [0, 1] \tag{4.14}$$

and

$$\dot{k} = skf' - \mu k \tag{4.15}$$

for $0 \leqslant t < \infty$.

Let H be the discounted value of monopolist's profits so that

$$He^{\delta t} = U[(1-s)kf'] + q[skf' - \mu k] \tag{4.16}$$

where $q(t)$ is the capitalist's shadow demand price of investment at time t in terms of utility forgone at time t. We assume that $U' > 0$ and $U'' < 0$ with $U'[0] = \infty$. Therefore, constrained maximisation of (4.13) implies that

$$\dot{q} = (\delta+\mu)q - [f' + kf'']U' \tag{4.17}$$

where s is chosen such that

$$U' \geqslant q, \quad \text{with equality when} \quad s > 0. \tag{4.18}$$

Defining the set N by

$$N = \{(k, q) : U'(kf') \leqslant q\}$$

then in the set N (for non-specialisation)

$$\psi(k, q) \equiv U'[(1-s)kf'] - q = 0. \tag{4.19}$$

Thus, in N,

$$\left.\begin{aligned} &\frac{\partial\psi}{\partial q} = -1, \quad \frac{\partial\psi}{\partial s} = -kf'U'', \\ &\frac{\partial\psi}{\partial k} = (1-s)(f'+kf')U''. \end{aligned}\right\} \tag{4.20}$$

Hence along the capitalist's consumption-optimal trajectory,

$$\left(\frac{\partial s}{\partial q}\right) = \frac{-1}{kf'U''} > 0 \tag{4.21}$$

and

$$\left(\frac{\partial s}{\partial k}\right)_N = \frac{(1-s)(f'+kf'')}{kf'} < 0. \tag{4.22}$$

Stationaries, k^*, q^* and s^*, to the system (4.14), (4.15), (4.17) and (4.18) are given as solutions to

$$\Phi(k) \equiv f' + kf'' = \delta + \dot{\mu} \tag{4.23}$$

where, since $\partial\Phi/\partial k = kf''' + 2f'' < 0$, there exists at most one solution to (4.23). Assume that k^* solves (4.23), then stationarity of k implies that s takes on a value s^* given by

$$0 < s^* = \frac{\mu}{f'(k^*)} < \frac{\mu}{\delta+\mu} < 1. \tag{4.24}$$

And, of course, q is assigned a value q^* given by

$$q^* = U'[(1-s^*)k^*f'(k^*)]. \tag{4.25}$$

Consider, for purposes of exposition, an economy which begins with $k(0) = k^*$. The above shows that, since a programme satisfying (4.14), (4.15), (4.17) and (4.18) is optimal if the transversality condition

$$\lim_{t \to \infty} qe^{-\delta t} = 0 \tag{4.26}$$

holds, the capitalist will strive to maintain k at k^* for ever. Because of monopoly power, long-run accumulation under monopoly capitalism is less than it would have been had wealth been evenly distributed, had everyone's tastes been given as in (4.13), and had they acted upon them. In fact, as $\delta \to 0$, under monopoly capitalism, k^* approaches a value which is bounded below the golden rule capital–labour ratio.

The full-phase diagram in (k, q) space is quite exhausting to treat, especially since there are several qualitatively different cases to examine. Instead of detailing that analysis, I shall limit myself to examination of the 'small vibration' analysis about the point (k^*, q^*). The linear Taylor expansion about (k^*, q^*) is

$$\begin{bmatrix} \dot{k} \\ \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{\partial \dot{k}}{\partial k}\right)_* & \left(\dfrac{\partial \dot{k}}{\partial q}\right)_* \\ \\ \left(\dfrac{\partial \dot{q}}{\partial q}\right)_* & \left(\dfrac{\partial \dot{q}}{\partial q}\right)_* \end{bmatrix} \begin{bmatrix} (k-k^*) \\ \\ (q-q^*) \end{bmatrix}. \tag{4.27}$$

But

$$\frac{\partial \dot{k}}{\partial k} = skf'' + sf' + kf' \frac{\partial s}{\partial k} - \mu$$

or

$$\frac{\partial \dot{q}}{\partial k} = f' + kf'' - \mu$$

so that

$$\left(\frac{\partial \dot{k}}{\partial k}\right)_* = \delta > 0.$$

Also

$$\frac{\partial \dot{k}}{\partial q} = kf' \frac{\partial s}{\partial q} = \frac{-1}{U''} > 0$$

and

$$\frac{\partial \dot{q}}{\partial k} = \delta + \mu - (f' + kf'')$$

so that

$$\left(\frac{\partial \dot{q}}{\partial k}\right)_* = 0.$$

Finally,

$$\frac{\partial \dot{q}}{\partial k} = -q(kf''' + 2f'') > 0.$$

Defining

$$\beta \equiv \left(\frac{-1}{U''}\right)_* > 0 \quad \text{and} \quad \alpha \equiv -q^*[k^*f'''(k^*) + sf''(k^*)] > 0$$

gives the following characteristic equation to the associated linear system (4.18):

$$\begin{vmatrix} \delta - x & \beta \\ \alpha & -x \end{vmatrix} = 0 \tag{4.28}$$

where x is the characteristic root. (4.28) yields two roots and so, by completing the square,

$$x = \frac{-\delta \pm \sqrt{(\delta^2 + 4\alpha\beta)}}{2} \tag{4.29}$$

and thus the characteristic roots are real but of opposite signs. The unique equilibrium point (k^*, q^*) is a saddle-point, and thus we know that except for a finite initial time period the capital–labour ratio will be arbitrarily close to this 'k^* turnpike'.

In the Conference discussion, it was pointed out by Mirrlees and Stiglitz that the capitalist with sufficiently large initial endowments will withhold capital for some initial period of time. Since profits, $kf'(k)$, are concave in k, capital will be withheld if and only if $k > k^{**}$ where k^{**} is defined by $f'(k^{**}) + k^{**}f''(k^{**}) = 0$. Since k^* is defined $f'(k^*) + k^*f''(k^*) = \delta + \mu > 0$, k^{**} is larger than k^*. The capitalist withholds capital, holding investment at zero, until k falls to k^{**}. Thereafter, capital is fully employed and growth is as described above. (At any instant, capital employment will be $\min(k, k^{**})$.) The capital–labour ratio k^* retains the turnpike property.

The Mirrlees–Stiglitz objection causes somewhat more difficulty in the analysis of the model with induced innovation. Because of the interaction of k and A, there may be several isolated episodes in which capital is not fully employed. To do full justice to the calculus of variations problem, one must explicitly allow for unemployment of capital. The added constraint will have an associated shadow price yielding jump conditions for transferring from regimes of unemployment to full employment.

If we allow the capitalist to withhold technological knowledge, then some very interesting cases can occur. Just as critics of the monopolistic invention system have alleged, new technological knowledge adversely affecting current profits would be secreted by the monopolist.

(2) *Monopoly Profits and Induced Innovation*

It is assumed as before that the amount of homogeneous output is dependent upon the size of the labour force and the level of the stock of physical capital. Now we turn to the more interesting case where, in addition, it is assumed that output is an increasing function of the stock of technical knowledge A. For compatibility with the assumptions of non-appropriability of technical knowledge and of competition in the consumption-goods sector, it is assumed that there are constant returns to scale in physical capital and labour and thus increasing returns to scale in all three factors. Output y can be written as

$$y = g(k, A) \qquad (4.30)$$

where g is an increasing concave function and profits, $\pi = kg_1$, are also concave in k and A.

The single capitalist maximises the functional

$$\int_0^\infty U[(1-s)kg_1(k, A)]e^{-\delta t}dt \qquad (4.31)$$

subject to

$$\dot{k} = \sigma skg_1 - \mu k \qquad (4.32)$$

$$\dot{A} = (1-\sigma)skg_1 - \rho A \qquad (4.33)$$

$$s \in [0, 1] \quad \text{and} \quad \sigma \in [0, 1] \qquad (4.34)$$

where s is the saving fraction and σ is the proportion of saving devoted to capital investment.

Let H be the present value to the capitalist of profits, then

$$He^{\delta t} = U[(1-s)kg_1] + \xi(\sigma skg_1 - \mu k) + \eta[(1-\sigma)skg_1 - \rho A] \qquad (4.35)$$

where ξ and η are respectively his demand valuation of a unit of investment and a unit of invention. It is necessary for maximisation of (4.31) that s and σ be chosen such that:

$$U'[(1-s)kg_1] \geqslant \max(\xi, \eta) \equiv \gamma, \quad \text{with equality if} \quad s > 0 \qquad (4.36)$$

$$\left.\begin{array}{ll} \sigma = 1, & \text{when} \quad \xi > \eta \\ \sigma \in [1, 0], & \text{when} \quad \xi = \eta \\ \sigma = 0, & \text{when} \quad \xi < \eta \end{array}\right\} \qquad (4.37)$$

$$\dot{\xi} = (\delta+\mu)\xi-[kg_{11}+g_1]U' \tag{4.38}$$

$$\dot{\eta} = (\delta+\rho)\eta-kg_{12}U'. \tag{4.39}$$

Conditions (4.38) and (4.39) state that the demand valuation of an asset must change so as to compensate the capitalist for loss due to depreciation plus a reward for 'waiting' less the value (in terms of utility) of the marginal product of that asset.

Defining the set N by

$$N \equiv \{(k, A, \xi, \eta):U'[kg_1(k, A)] \leqslant \gamma\} \tag{4.40}$$

yields from (4.36) that

$$\left(\frac{\partial s}{\partial \gamma}\right)_N = \frac{-1}{kg_1U''} > 0 \tag{4.41}$$

$$\left(\frac{\partial s}{\partial k}\right)_N = \frac{(1-s)[g_1+kg_{11}]}{kg_1} \tag{4.42}$$

$$\left(\frac{\partial s}{\partial A}\right)_N = \frac{(1-s)g_{12}}{g_1} > 0. \tag{4.43}$$

Stationaries to (4.32), (4.33), (4.38) and (4.39) are given by solving the system:

$$kg_{11}+g_1 = \delta+\mu \tag{4.44}$$

$$kg_{12} = \delta+\rho \tag{4.45}$$

$$\sigma skg_1 = \mu k \tag{4.46}$$

$$(1-\sigma)skg_1 = A. \tag{4.47}$$

Defining $D = kg_{11}+g_1-\delta-\mu$, and implicitly differentiating (4.44), yields

$$\left(\frac{dA}{dk}\right)_{D=0} = -\left(\frac{kg_{111}+2g_{11}}{kg_{112}+g_{12}}\right) > 0 \tag{4.48}$$

by the concavity of $g(k, A)$ and $\pi(k, A)$. Defining

$$E = kg_{12}-\delta-\rho$$

and implicitly differentiating (4.45) yields

$$\left(\frac{dA}{dk}\right)_{E=0} = -\frac{kg_{112}+g_{12}}{kg_{122}} > 0 \tag{4.49}$$

by the concavity assumptions. But notice that

$$\frac{\left(\frac{dA}{dk}\right)_{E=0}}{\left(\frac{dA}{dk}\right)_{D=0}} = \frac{(kg_{112}+g_{12})^2}{kg_{122}(kg_{112}+2g_{11})} < 1 \tag{4.50}$$

by the concavity of $\pi(k, A) = kg_1(k, A)$.

By (4.50) we know that there is at most one solution to the system (4.44) and (4.45) in (k, A) space. Assume that such a solution exists and denote it by (k^*, A^*). Now if $(A^*/k^*) < (\delta/\rho)$, then (4.46) and (4.47) yield

$$s^* = \frac{\mu k^* + \rho A}{k^* g_1(k^*, A^*)} < 1$$

and thus

$$\frac{\sigma^*}{1-\sigma^*} = \frac{\mu k^*}{\rho A^*}$$

ensuring that $\sigma^* \in [0, 1]$.

Also, notice that if development tends to (k^*, A^*), then the transversality conditions

$$\lim_{t\to\infty} \xi e^{-\delta t} = \lim_{t\to\infty} \eta e^{-\delta t} = 0 \tag{4.51}$$

are satisfied. Except for a finite initial time period, growth of the economy is arbitrarily close to the (k^*, A^*) turnpike.

This mathematical argument has been terse and may have led to some confusion. It should be worth while to take some time to elaborate.

I do not mean to say that transversality conditions such as (4.26) and (4.51) are necessary conditions for utility maximisation. The Ramsey optimal-growth problem with zero impatience and zero population growth is a well-known counter-example. We do know that because of the concavity of $U(\cdot)$ and $g(\cdot)$, the utility-maximising programme is unique. Because of the concavity of $U(\cdot)$ and $g(\cdot)$ and because δ is positive, a feasible path satisfying Euler equations (4.36)–(4.39) and transversality conditions (4.51) will be preferred by the monopolist to any other feasible path. In the neighbourhood of (k^*, A^*) a path satisfying the Euler equations and the transversality conditions does indeed exist. I have not shown existence of such a trajectory for all initial endowments vectors, (k, A). Existence could be established by a constructive argument. One would need to show that in (k, A, ξ, η) space the manifold of Euler solutions tending to (k^*, A^*) covers the entire positive orthant of (k, A) plane.

III. TECHNICAL KNOWLEDGE AS A PURE PUBLIC GOOD OF PRODUCTION

Because of space limitations, I was unable in the preceding section to develop many specific conclusions. (Even the concavity assumptions are made more for mathematical convenience than because they are realistic, or the reverse.) While more study is needed before the analysis will lead to definite results (such as the pattern of optimal social control), it is my hope that we have gained some insight into the basic dynamics of a model in which monopoly profits fuel inventive activity.

In this section, we focus on a model in which production of consumption and investment is competitive, with technical knowledge entering each firm's production function as a pure public good. Inventive activity must therefore be supported by non-market institutions. In the present model, the government imposes an excise tax, and the revenue is used to finance government-controlled research.[1]

As before, we simplify by assuming a technology with equal capital intensities, so that we can write

$$Y \equiv Y_C + Y_I + Y_R = \Phi(K, A, L). \tag{4.52}$$

Assume further that for firm i, output, Y^i, is given by

$$Y^i = AF(K^i, L^i) \tag{4.53}$$

where $F(\cdot)$ is positively homogeneous of degree one. In the aggregate,

$$Y = \sum_i Y^i = A \sum_i F(K^i, L^i) \tag{4.54}$$

so that at the economy level the production function under our particular specification is positively homogeneous of degree two in the three factors:

$$A, K = \sum_i K^i, \quad \text{and} \quad L = \sum_i L^i.$$

Since each firm is small, it cannot substantially affect either aggregate A or aggregate Y_R. The competitive price of knowledge is zero although its marginal (and average) social product is equal to $F(\cdot)$.

To repair this market failure, the government imposes an excise tax on the output of consumption and investment. If the tax rate is $0 < \alpha < 1$, then the competitive wage rate, w, and rental rate, r, are given by

$$r = (1-\alpha)AF_K$$
$$w = (1-\alpha)AF_L.$$

[1] The treatment here is condensed since it is based on some earlier work. See Shell (1966, 1967).

Tax revenue, $\alpha(Y_C + Y_I + Y_R) = \alpha Y$, is equal to government production (or purchases) of inventive output, $\alpha Y = Y_R$. If the research department hires factors at competitive prices, then by Euler's Theorem

$$rK + wL = (1-\alpha)AF_K K + (1-\alpha)AF_L L = (1-\alpha)Y.$$

Rewards to capital and labour fully exhaust the output of private goods, $rK + wL = Y_C + Y_I$, while the output of public goods, Y_R, is community property.

If individuals save a constant fraction, $0 < s < 1$, of disposable income, then equation (4.4) can be rewritten as

$$\dot{k} = s(1-\alpha)Af(k) - \mu k \tag{4.55}$$

ignoring labour-force growth. Differential equation (4.5) can be rewritten as

$$\dot{A} = \alpha Af(k) - \rho A. \tag{4.56}$$

Motion of the mixed economy is given by differential equations (4.55) and (4.56). If s and α are constants, and $f(\cdot)$ satisfies the usual regularity conditions, then there exists a unique stationary state (k^*, A^*), which is a saddle-point.

This model – although very primitive – presents two important departures from that of the standard growth paradigm. (i) The rest point (k^*, A^*) is not stable. *The model economy is morphogenetic rather than morphostatic*, i.e. long-run development is very sensitive to initial conditions. (ii) In particular, for the regime of perpetual growth, *the rate of growth in productivity is increasing through time.*[1]

These two basic properties are not independent of the particular forms of the production function, consumption function and so forth. It seems to me, however, that morphogeneticism and the related possibility of an increasing rate of productivity growth are 'likely' for economies exhibiting increasing returns to scale in A, K and L.

[1] See Weizsäcker (1969). The wildly increasing productivity gains that my model predicts may be offset in life by exhaustion of fixed natural resources. This is especially likely if income and consumption are correctly measured to reflect the decreasing quality of the environment that seems to go along with industrial development. (The growth models presented at this Conference assume without exception that $\dot{L}/L = n$, an exogenous constant. It surprises me that, while we study technology so carefully, we have been little interested in demography.)

Notice the important change in the specification of the production function. In the preceding sections the production function is assumed to be concave in k and A, while in this section the function is quasi-concave but not jointly concave in k and A. Even if $f(\cdot)$ is bounded, the analysis of optimal growth based on the technology of this section does not appear to be easy. Without concavity, questions of uniqueness, sensitivity to initial conditions, and so forth, are all open.

IV. A COMPETITIVE MODEL IN WHICH INVENTIVE ACTIVITY IS FINANCED FROM QUASI-RENTS ON ADVANCED TECHNOLOGY

In what we have done so far, invention is either a pure public good financed by government expenditure or is financed by monopoly profits in the production of capital goods. Now we turn our attention to a model which can be thought of as lying between these two extreme models. The present model allows for government intervention in the R. & D. process, but its most salient feature is the financing of R. & D. by competitive firms.[1]

I begin the story with a partial-equilibrium analysis of an industry in which the level of technology may differ over firms. There are several reasons for technological possibilities to be different for two firms in the same industry. While in the long run transmission costs are typically low relative to production costs, it is very costly to transmit information at a rapid *rate*. Firms with advanced technologies have incentives for not revealing their technologies, and employ secrecy to achieve this end. Patents can give some limited legal protection to the 'advanced' firm.

In life there is usually a spectrum of technologies that are employed by the different firms. It will make the story simpler without seriously affecting the basic argument if we assume that there are two types of firms: those capable of operating at the 'advanced' technology (denoted by A_1) and those capable of operating at the 'backward' technology (denoted by A_2). The number of firms (actual and potential) capable of operation at the backward technology is infinite. The number of firms capable of operating at the advanced technology is some finite number, say n. Although finite, n is large enough so that all firms consider themselves to be price-takers.

In order to make things simple, assume that there is only one factor of production, say labour, that the firm can vary in the short run. In Fig. 4.1B, short-run U-shaped average cost curves are drawn for firms of each type. (AC_1 for an advanced firm; AC_2 for a backward firm. Q denotes output of the firm in question.) Also shown in Fig. 4.1B is an advanced firm's marginal cost schedule (MC_1). Ignoring second-order indivisibilities, we can construct from Fig. 4.1B the industry's supply schedule (shown in Fig. 4.1A). If the price of a unit of output is less than the minimum average cost for the advanced firm, $AC_1(Q^*)$, then, of course, supply of output is zero. If the output price is equal to $AC_1(Q^*)$, then supply will be elastic

[1] Some of the fundamental ideas in this section were worked out some time ago in a conversation with Joseph Stiglitz. He bears no responsibility, however, for what I have done with these ideas.

up to the level nQ^*, at which point all firms capable of operating at the advanced technological level will have entered the industry.

If the price of output is slightly greater than the minimum of AC_1, then the n advanced firms will be of equal size and the quantity produced by a representative firm, Q, can be found by solving $MC_1(Q) = P$, where P is the price of output. This regime persists until marginal cost for the advanced firm is equal to minimum average cost for the backward firm, $AC_2(Q^{**})$.

Therefore, if output is less than nQ^{**} (indicated in Fig. 4.1A), then only advanced firms are operating. If industry output is greater than nQ^{**}, then both backward and advanced firms are operating

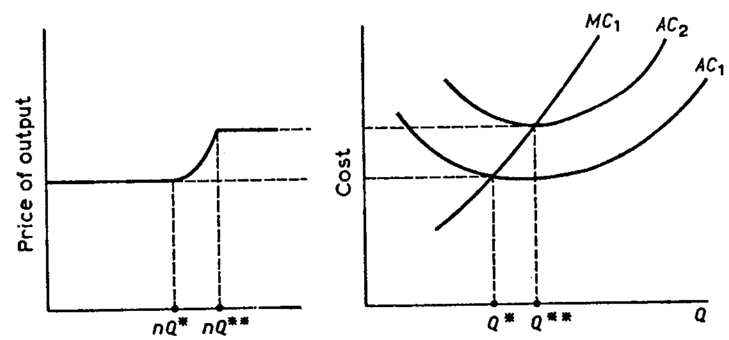

FIG. 4.1A FIG. 4.1B

and the industry supply price of output is equal to the minimum average cost for the backward firms. It is important to observe that *when industry output is greater than nQ^* the advanced firms are reaping positive quasi-rents on advanced technology*. This possibility of positive quasi-rent for an industry in which all producers are price-takers will play a central role in the further analysis of this problem.[1]

We can assume that there are three basic sources of improvement in the ith firm's technology: (i) The firm can devote some of its own resources to the invention of improved technique. (ii) Spillovers from more advanced firms in the same industry. (iii) Spillovers from

[1] It is worth noting that this model is anti-Chamberlinian. One might think of the regime to the right of nQ^* in Fig. 4.1A as imperfectly competitive. But in this imperfectly competitive regime, advanced firms operate to the right of the minimum average cost point – rather than to the left, as in the celebrated case of monopolistic competition.

other industries in the economy, including the socialised sectors. This can be formalised by

$$\dot{A}_i/A_i = \phi'[R_i, \quad (A_1 - A_i)/A_i, \quad \dot{A}/A] \tag{4.57}$$

where A_i is the index of technology for firm i, A_1 is the index of technology for the most advanced firm in the industry, A is an economy-wide index of accumulated technical knowledge and R_i is the number of man-hours devoted to invention by the ith firm. $\phi'[\cdot]$ is then an increasing function of its three arguments.

In the long run, because of technological progress, the wage rate, w, and income per head, y, grow at the proportionate rate $\alpha > 0$:

$$\dot{w}/w = \dot{y}/y = \alpha. \tag{4.58}$$

While in the aggregate the economy may tend to some quasi-stationary state, the composition of output is likely to be changing substantially through time. To understand the implications of this point, consider first the 'standard' industry, which in the long run is experiencing factor-augmenting technical progress at the same proportionate rate as the economy-wide rate, α; i.e.

$$\dot{A}_1/A_1 = \alpha = \dot{A}_2/A_2$$

where $A_1 > A_2$. Supply (*SS*) and demand (*DD*) schedules for the standard industry are shown in Fig. 4.2. *SS* does not shift through time since increases in productivity exactly offset increasing factor

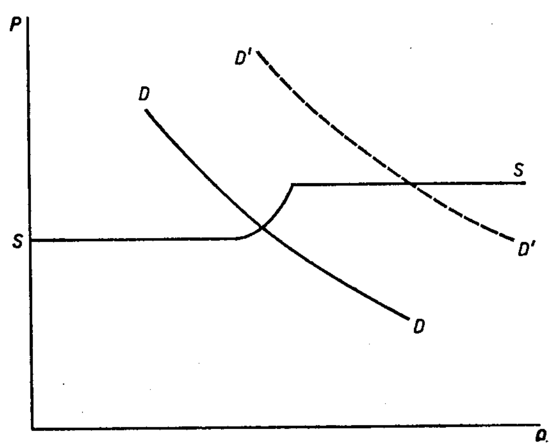

FIG. 4.2 The 'Standard Industry'

costs. But if the industry does not produce an inferior good, then the demand schedule must be shifting rightward through time (to $D'D'$) because of the positive income elasticity of demand. Consequently, quasi-rents are non-decreasing through time, allowing for continuing financing of research, $R_1 > 0$.

If for some industry the previous assumptions hold except that research is even more productive than in the standard industry, so that

$$\dot{A}_1/A_1 = \dot{A}_2/A_2 = \beta > \alpha$$

then the same qualitative conclusion holds, namely, in the long run financing will be available to permit R_1 to be positive.

Consider, on the other hand, the industry in which long-run technical progress for the advanced firm proceeds at a rate slower than the economy average:

$$\dot{A}_1/A_1 = \beta < \alpha$$

even when output is great enough to generate the maximum amount of quasi-rent. This is described in Fig. 4.3. On the vertical axis, we measure $P(t)e^{(\alpha-\beta)t}$, where $P(t)$ is the price of a unit of the industry's output.

Since wages are growing at the rate α while productivity is only increasing at the rate β, the SS schedule will not be shifting in Fig. 4.3. However, the demand schedule (initially DD) will in general shift through time. The direction and manner of shifting will depend on

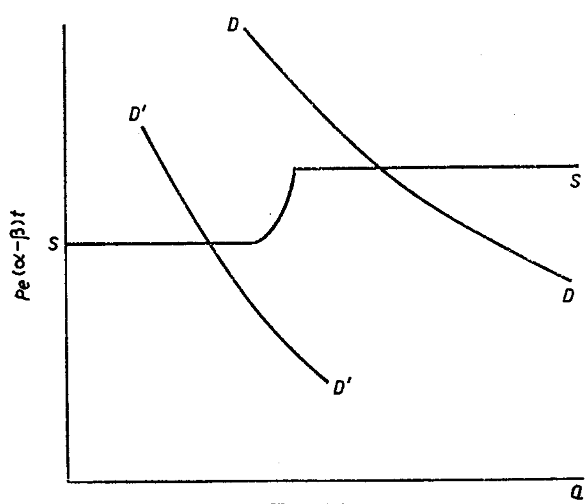

Fig. 4.3

the income elasticity of demand and the price elasticity of demand. If, through time, equilibrium Q is increasing, then positive quasi-rents will be generated and R_1 will be positive. If, however, equilibrium Q is falling, then quasi-rents and research expenditures will fall to zero. With no research expenditures, $R_1 = 0$, the gap between A_1 and A_2 declines so that in the long run the SS schedule becomes everywhere horizontal.

This is the story of a 'sick' industry – an industry with low income elasticity of demand and high price elasticity of demand in relation to the average profitability of research, ϕ'/R. Such 'sick' industries present a case for social support of industry-related inventive activity. In recent years, the 'sick' industry phenomenon has also provided an opening wedge for expansion of 'conglomerates'. Contrary to the usual view of the conglomerate, 'sick industry' expansion is one important source of growth. The present analysis helps us in understanding this. Financing flows from industries generating high quasi-rents to profitable opportunities, including industries where invention is profitable. An example is the case where quasi-rents from an industry with high income elasticity of demand are invested in technique improvement in a 'sick' (low income elasticity of demand) industry; e.g. from petroleum extraction to coal mining, from the chemical industry to the textile industry.[1]

If A_1 is considered to be an index of the level of economy-wide advanced technology, A_2 an index of economy-wide backward technology, we have from aggregation of equation (4.57) that

$$\frac{\dot{A}_1}{A_1} = G\left[R(\beta), \frac{\dot{A}}{A}\right] \tag{4.59}$$

where $G[\cdot]$ is increasing in both arguments ($G_1 > 0$, $G_2 > 0$) and R is increasing in β where

$$\beta \equiv \frac{A_1 - A_2}{A}. \tag{4.60}$$

Also, from an aggregation based on (4.57),

$$\frac{\dot{A}_2}{A_2} = H\left[\beta, \frac{\dot{A}}{A}\right] \tag{4.61}$$

where $H_1 > 0$ and $H_2 > 0$.

[1] The present framework can be easily employed in the study of a variety of important policy questions concerning invention, industrial organisation and growth, e.g. the question of infant industry protection and so forth. Tempting as such diversions may be, our main task at present is not partial-equilibrium micro-economics but rather general-equilibrium macro-economics.

In the quasi-stationary state

$$\dot{A}_1/A_1 = \dot{A}_2/A_2 = \dot{A}/A \equiv \alpha.$$

Thus stationaries to (4.59) and (4.61) solve

$$\left.\begin{aligned} &G[R(\beta), \alpha] - \alpha = 0 \\ \text{and} \quad &H(\beta, \alpha) - \alpha = 0 \end{aligned}\right\} \tag{4.62}$$

two equations in two unknowns (β and α). From (4.57)–(4.62) we know that

$$\text{sign}\left(\frac{d\beta}{d\alpha}\right)_{\dot{A}_1/A_1 = \alpha} = \text{sign}\,(1 - G_2)$$

and

$$\text{sign}\left(\frac{d\beta}{d\alpha}\right)_{\dot{A}_2/A_2 = \alpha} = \text{sign}\,(1 - H_2).$$

Without a deeper study of the problem, we can say no more about these two slopes. Consequently, detailed analyses of existence and uniqueness of long-run equilibrium as well as comparative dynamics and stability must be postponed until we know more about the $G(\cdot)$, $H(\cdot)$ and $(R\cdot)$ functions.

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C. C. von Weizsäcker, 'Forschungsinvestitionen und Macroökonomische Modelle: Ein Wirtschaftstheoretisches Dilemma?', *Kyklos*, vol. XXII, pt. 3 (1969) pp. 454–65.

Discussion of the Paper by Karl Shell

Professor Shell said that in order to explain economic development satisfactorily, an endogenous theory of technological progress is required. There seems to be no satisfactory explanation linking inventive activity at the enterprise level with growth in aggregative productivity. Previous studies, while concentrating on constant-returns-to-scale technologies and on competitive markets, have missed the crucial role of industrial organisation in the inventive process.

He offered here three models in which invention is undertaken by 'non-competitive' economic agents. The three cases of industrial organisation may not be realistic, but it is hoped that they are at least internally consistent and that (as polar cases) they may shed light on the more general problem of invention and growth in the enterprise economy.

He emphasised that investment in inventive activity was crucially different from that in machinery for three reasons. Firstly, assumptions concerning returns to scale need to be different. Secondly, the returns to machinery are less risky. Finally, investment in inventions is more difficult to finance since bankers have very little tangible to claim if the investment fails.

Dr Berglas began his discussion of the paper by welcoming the emphasis on industrial organisation and said he would welcome an examination of further cases. He thought that more discussion of the stock of technical progress was needed to clarify its meaning. He did not like the idea of all technical knowledge being produced with a production function with payment to the factors involved exhausting product. Much technical knowledge is not produced in firms but in universities and government establishments. If technical progress was included as a function of time, most of the results (apart from the last model) would disappear.

He pointed to the extreme nature of the assumptions necessary for the most thoroughly analysed case, the first. The consumption-goods sector was competitive – this seemed incongruous with complete monopoly in the capitalist sector. There was a single production function and no growth in the labour force. All wages were saved and the monopolist maximised the present value of a consumption stream.

He thought the assumption in the second model that the government has a constant tax rate to finance inventions inappropriate. It ought to be optimising in some way.

He said that the assumption in the third model of a fixed finite number of advanced firms was inappropriate in the long run. This assumption could be replaced by a pair of assumptions that determine the relative number of advanced and backward firms: (i) research and development units are not perfectly divisible and have an optimum size for an advanced firm; (ii) the cost function of backward firms depends on R. & D. in the industry as a whole, i.e. when research in the advanced sector increases, costs of the backward firms decline. These assumptions ensure that pure profits to the advanced sector imply an increase in the number n of advanced firms, a decrease in the costs of backward firms and therefore a lowering of product prices. n ceases to increase when pure profits to advanced firms are eliminated. In this model inventive activity can continue in a declining industry. As in the paper, n is constant in a growing industry. This feature is eliminated if returns to advanced firms depend on the size of the industry, for then advanced firms may wish to sell patents to backward firms. Without complicating the analysis these assumptions make the model more compatible with long-run competitive equilibrium.

He said the author's policy conclusion that more help for sick industries might be necessary, depended on the fixed number of firms assumption. He did not see why research activities in expanding sectors might not be more helpful to society.

In general he found it difficult to compare the three models since the discussion was not carried on in the same style for each.

Professor Mirrlees noted that different production possibilities for the first two models had been assumed. In the first model, $g(A, k)$ had been assumed concave (p. 86) so that maximisation was made easy. However, the corresponding function in the second model was $Af(k)$. In this model no optimisation was performed, however. If f were Cobb–Douglas and we optimised, we would find that we would get infinite A and k in finite time. This can arise when $g(A, k)$ is not concave. If we assumed f were bounded, however, we would get an interesting steady-state solution.

Professor Shell agreed that the concavity assumptions were made for convenience, but felt that we let in the explosive solutions by abstracting from other constraints such as land.

Dr Bliss said it was unclear to him that investment in innovation *was* more risky than investment in physical capital. Physical capital, if it was very specific, might be worthless on failure of an enterprise. There may well be something left to sell after an innovation project had not produced its intended results – e.g. an alternative product or the knowledge that it is not worth looking further in a particular direction.

Mr Atkinson asked what was meant by a deterioration in technical knowledge (equation (4.5)). *Professor Shell* replied that he was thinking of skilled people dying off. If $\rho = 0$, the model is more likely to be morphogenetic. *Professor Mirrlees* suggested that $\rho > 0$ when we prove more general theorems and discard special cases. *Professor Weizsäcker* pointed out that this would be a net accretion to knowledge, however. *Professor Hahn* said we may lose the processes by which we arrived at theorems, e.g. now that we have replaced the labour theory of value by the non-substitution theorem, we may not be able to see how Marx arrived at the theory. *Professor Weizsäcker* thought that we forgot that which was not useful.

Professor Stiglitz suggested that the two polar cases (monopoly and public sector) had been used to avoid difficulties in specifying how returns from research are captured. We should like to examine a situation where technical knowledge is neither a pure public good nor a pure monopolistic good, e.g. by a patent system where the flow of knowledge is reduced to promote research.

Professor Weizsäcker said Nordhaus had done something like this.

Professor Shell said a difficulty with a competitive model that included a patent system was that we had increasing returns to all factors taken together if we had constant returns to conventional factors. He thought that the importance of patents was low – probably only 5–10 per cent of research and development output passed through the Patent Office; they refuse to handle many types of application, and other people can see filed patents.

Professor Weizsäcker said we should try to develop a theory in which monopolistic rents were returns to investment in invention or special knowledge of some kind. We could then develop an efficiency and equilibrium theory about monopolists.

Mrs Bharadwaj thought there was a danger of implicit theorising if we tried to explain monopolistic rents as returns to an unquantifiable factor like 'knowledge'.

Professor Shell said that the problem was no different from that of the identification of $\dot{A}$.

Professor Mirrlees suggested that firms would use patents more if courts did not uphold contracts enforcing secrecy on employees.

Dr Boussard noted that the consumption behaviour assumed in the first two models was different – in the first case we had discounting of utility and in the second a constant propensity to save. He thought this might be the main reason for the different results.

Professor Shell said some sort of optimisation of α might aggravate the morphogenetic problems.

Professor Stiglitz asked whether the monopolist would necessarily maximise his profits by renting out all the capital at his disposal. He also remarked that the $f''' < 0$ assumption was peculiar – if this did not hold, $kf'(k)$ (the monopolist profit) might have several local maxima.

Professor Mirrlees suggested that if k were below the k^* at which long-run consumption was maximised, then the monopolist would want to accumulate until k was equal to k^*. In that case, capital would never be withheld.

Professor Shell said he would look into the question raised by Professor Stiglitz.

Professor Uzawa wondered why the capitalist was a monopolist and why he had the objective function of (4.13).

Professor Shell replied that the monopolistic assumption was made to study a special case. We could think of the monopolist as a good family man or as the committee of the bourgeoisie.

Professor Hahn thought some embodiment ideas were necessary to study the relations between innovations and monopolistic situations. Schumpeter thought innovation would be less in old firms than in new ones. For instance, General Electric tried to suppress the neon light to protect the returns on capital embodied elsewhere, but were eventually forced to invest in it by a small firm carrying out the innovation. More rigorous empirical studies of, for instance, when it paid a monopolist to introduce an innovation were needed in this field.

Professor Shell agreed, but noted that even without enbodiment there are relations $g(\cdot)$ such that with $y = g(k, A)$ inventions do not increase profit.

Professor Stiglitz said that older theories had competitive pressure forcing innovations. Some interaction between firms is needed to capture the flavour of the problem.

Professor Spaventa thought that the author was too quick to jump to conclusions about, for example, conglomerates (p. 95).

Professor Shell said he was not advocating special policies. He was pointing to the possibility that it may in some circumstances be of both social and private benefit that resources be moved from high-growth to low-growth industries.

Professor Weizsäcker said the distinction between sick and other industries depended on there being no switch between the two types of firms.

Professor Shell said he recognised that there was a spectrum of firms in real life. He was pointing out that technical change may not be factor-augmenting. We can allow firms to become advanced by the growth processes of the model: with high quasi-rents a poor firm could improve; with low quasi-rents firms might drift together.

Dr Berglas said that this answer was different from that of the paper – there quasi-rents arose only because of the difference between firms. With the changes he had recommended earlier the number of advanced firms could change.

He said the problem posed by Professor Stiglitz (the withholding of capital) was similar to the contradiction between a monopolistic capital sector and a competitive consumption sector. With the wage equal to the marginal product in the consumption sector, homogeneity gives us $r = f'(k)$. However, a monopolist would be able to make $r > f'(k)$ and thus the wage less than the marginal product of workers.

Professor Shell concluded the discussion and said that a backward firm could become an advanced firm in his model. He agreed that the stark contrast between a monopolised capital sector and a competitive consumption sector was a problem.

He had looked at the optimal control of α. It was not clear whether the model was morphogenetic or morphostatic – it depended on the concavity of the production function. With wildly increasing returns we could have morphogeneticity and the usual criteria for optimality did not apply.