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Source: *Sociological Perspectives*, Vol. 36, No. 2 (Summer, 1993), pp. 117-135
Published by: University of California Press
Stable URL: http://www.jstor.org/stable/1389425
Accessed: 10/12/2010 21:25

Sociological Perspectives Vol. 36, No. 2, pp. 117-135

ISSN 0731-1214

GAME-THEORY MODELS FOR EXCHANGE NETWORKS: Experimental Results

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ABSTRACT: *The goal of current exchange-network literature is to develop algorithms, loosely based on rational choice, that predict how resources are distributed through exchange networks and which positions have power to accumulate resources. These objectives closely resemble those of N-person cooperative games with transferable utility, which are based on formal explicit models of rational choice. Experimentally, power is exhibited when a position can amass a favorable proportion of available resources by negotiating a division with another network member. Game-theory solution concepts that address the question of power in networks are introduced and compared to network-exchange models to evaluate the effectiveness of the game-theory solutions and those of exchange theory in predicting results observed in experiments. Experimental data show that there is a utility in incorporating game theory into the discussion of exchange in negatively connected networks. Furthermore, the use of game theory leads to a more comprehensive understanding of many processes to which exchange theory is insensitive.*

There is much overlap in what is studied in the social sciences. Different disciplines have different theoretical orientations and different approaches. Not infrequently, in two fields the same work may be under investigation with two distinct theoretical bases and two separate vocabularies. When this occurs, it is often useful to take the strengths of both fields and try to develop a common vocabulary to understand the phenomenon. The contribution of this study is to incorporate the ideas, techniques, and solution concepts of *N*-person cooperative game theory to the study of exchange in negatively connected networks.

Cook, Emerson, Gillmore, and Yamagishi, in their article "The Distribution of Power in Exchange Networks" (1983), established that some nodes in networks of exchange could, because of structural position, command more resources in

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dyadic exchanges. Cook et al. proposed an algorithm to calculate which network positions have a structural advantage. Markovsky, Willer, and Patton, in their paper "Power Relations in Exchange Networks" (1988), also showed that some network positions have structural advantage. They proposed a different algorithm for determining which positions have advantage.

There are two reasons that game theory has been overlooked as a source of hypotheses for determining where power is located in networks of exchange. The first is that Cook et al. and Markovsky et al. share a misconception of game theory. Markovsky, Willer, and Patton (1988) specifically rule out using models of "coalition formation" because they are thought to be incompatible with Markovsky's basic assumptions. Markovsky et al. reject game theory because they wrongly assume that "coalition formation" implies actors would have the options of "temporarily accepting reduced resources while receiving increasingly favorable offers from others." (Markovsky et al. 1988:223, n. 5). Markovsky et al. overlook the fact that when any two subjects agree to an exchange, they are in effect forming a coalition.

In addition, the focuses of both Cook et al. and Markovsky et al. on power may have prevented them from seeing the relevance of rational-choice models. Rational-choice models seem to involve uncoerced decisions by independent, rational decision makers. "Power" seems to play no role. However, it is questionable to us whether research on exchange networks really needs the concept of power at all. The only consequence of unequal power in exchange networks is unequal outcomes. The game-theory models we will describe directly predict outcomes, without the unnecessary intervening variable of power.

Game theory is firmly based in rational choice. While it is true that the grand purpose of exchange theory may be to study social exchange in general, the experiments used by both Cook et al. (1983) and Markovsky et al. (1988) involve purely economic exchanges. Both studies assume their subjects will behave rationally (Cook et al. 1983:286; Markovsky et al. 1988:223). In addition, both studies attempted to control any experimental feature that could induce nonrational behavior.

In many ways, the assumptions of "rational" behavior that Cook et al. and Markovsky et al. propose are more constrained then those of game theory. Cook et al. and Markovsky et al. make specific prescriptions of exactly the type of bargaining strategy subjects should use if they are being rational (Cook et al. 1983:286; Markovsky et al. 1988:223). Any deviation from this predicted behavior is attributed to subjects being influenced by nonrational extraneous concerns. This limited perception of rational choice unnecessarily confines their theories.

Game theory is based on formal explicit assumptions of rational choice that are general enough to allow for adaptation to many different situations. Game theory is not one predictive theory; it is a collection of solutions that are parsimonious enough to have interpretive value for many situations. In addition, the different solutions, while all based on the assumption of rational choice, focus attention on different aspects of rational choice and have different emphases. The view of Cook et al. (1983:286) that discussion of equity concerns are outside the realm of rational choice is incorrect. The Shapley value, one game-theory-solution concept, is considered an equity solution (Rapoport 1970:48). In this article, game theory is incorporated into the literature of exchange in negatively connected networks, not only because it can predict patterns of exchange in negatively connected networks but also because game theory allows for insights into the implications of the structures of networks that current models are not sensitive to. This study is intended to show that game-theory-solution concepts do as good a job, if not better, than existing algorithms in predicting who will get more points in exchanges and who exchanges with whom. In addition, it shows that incorporating game theory into the discussion allows for a more insightful investigation of exchange networks.

COOK AND EMERSON

By incorporating networks into exchange theory, Emerson (1972) was the first to extend exchange theory beyond the dyad. Cook, Emerson, Gillmore, and Yamagishi (1983) developed these ideas further. The purpose of their article was to discover how to "best integrate network-structural principles and power-dependence theory to explain the dynamics of power in exchange networks" (1983:289). They used Emerson's (1972) definition of exchange networks as a starting point:

An exchange network can be defined as consisting of:

1. A set of actors.
2. A distribution of valued resources among these actors.
3. For each actor, a set of exchange opportunities with other actors in the network.
4. A set of historically developed and utilized exchange opportunities called exchange relations.
5. A set of network connections linking exchange relations into a single network structure.

Thus, an "exchange network" is formed by two or more connected exchange relations between actors, with "connections" defined as follows:

> **Definition:** Two exchange relations between actors A—B and actors B—C are connected to form the minimal network A—B—C to the degree that exchange in one relation is contingent on exchange (or nonexchange) in the other relation. (a) The connection is positive if exchange in one relation is contingent on exchange in the other. (b) The connection is negative if exchange in one relation is contingent on nonexchange in the other. (Cook et al. 1983).

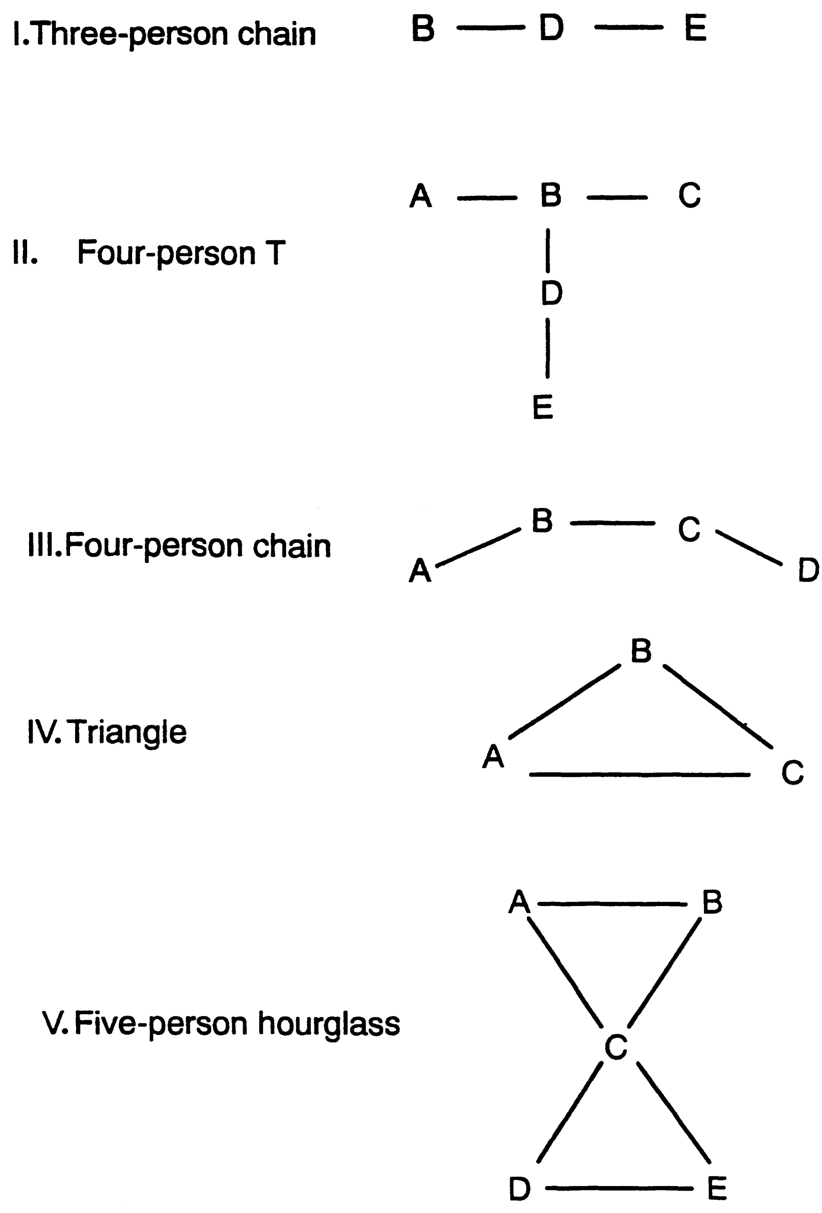

Figure 1
Illustrative and Experimental Networks

As an illustration of power differences in negatively connected networks, consider Network I in Figure 1. Imagine this is a monogamous world and the exchange relation is marriage or dating. 'D'ebbie has two options: 'B'ob and 'E'ddie will bid to make themselves more attractive to her. Since Debbie has alternatives, she is in a more powerful position then the men. All of the algorithms (network and game-theoretic) for determining power in negatively connected networks agree that D is the most powerful person in this simple three-person network.

If we add 'A'lice and 'C'arol to form a more complex network (Network II in Figure 1), different approaches yield different results. In this network, Bob has options and is no longer completely dependent on Debbie. It is now no longer obvious in which position power is concentrated.

To determine where power is located in a network, Cook et al. (1983) developed an experiment. The experiment placed people in network positions and allowed connected pairs to negotiate over the division of a preset allocation of points. If a pair could agree on a division of points, they would receive the points. Points were later converted to money. The assumption was that people in powerful positions would use that power to obtain more points.

Cook et al. (1983) developed a measure to determine network-wide dependence. In Network I in Figure 1, there is no reduction in the value of the network if either B or E (but not both) is removed. For example, if the value of an agreement between any two connected positions is worth 24 points, then the removal of either position B or E does nothing to reduce the value of the network. If either B or E is available, there is still the potential for a payoff of 24 points. In Cook 's terminology, there would be no reduction in maximum flow (RMF). However, when D leaves the network, no agreements can be made and the value of the network is 0. This means that B and E are dependent on D to complete the transaction and if B is not an active participant nobody profits. Looking at Network II in Figure 1, where it is possible to make two agreements, the maximum resource flow (MRF) is 48. If B or D or E is removed, for example, the reduction in maximum flow is 24; removing any one of these positions would make it impossible for two agreements to be completed. If either A or C is removed, the RMF is 0, and the network can still achieve a joint value of 48. In graph-theoretic terms, Cook et al. are using point vulnerability to model this process.

One problem with using reduction in maximum flow as the only indication of power is that to exercise power in a network, a person has to remove him/herself entirely from the transaction and accept no exchanges. In other words, by the RMF measure, power can only be expressed at the expense of gain. A better measure of power would give the powerful position the ability to exercise power and still profit in an exchange round. In other words, a powerful position could affect others' power without sacrificing its own profit by removing certain connections and depriving some members of the network of the benefit of its value while keeping other options open. Cook, Gillmore, and Yamagishi (1986) revised their measure by combining the concept of point vulnerability (RMF) with line vulnerability (CRMF), the Cost of RMF. Line vulnerability is the ratio of the minimum number of lines that need to be removed in order for a position to exercise power and its total number of connections.

The final measure is known as D_{Ni}.

$CRMF_i$ = (Number of lines that need to be removed to exercise power to its potential) / (Number of lines connected to point i).

$D_{Ni} = RMF_i \times (1 - CRMF_i)$.

As an illustration of how this is computed, observe the distribution of power in Network II of Figure 1:

$$
\begin{aligned}
C &= 0 \times (1 - 1/1) = 0\\
B &= 24 \times (1 - 2/3) = 8\\
A &= 0 \times (1 - 1/1) = 0\\
D &= 24 \times (1 - 1/2) = 12\\
E &= 24 \times (1 - 1/1) = 0
\end{aligned}
$$

In this network, position D seems to have the most power, followed by position B. Positions A, C, and E appear to have the same power. The hierarchy of power for this network according to the Cook, Gillmore, and Yamagishi (1986) algorithm is $D > B > A = C = E$. These result are counterintuitive, particularly because they find the power of E to be equal to that of A and B.

WILLER AND MARKOVSKY

Markovsky, Willer, and Patton (1988) have developed another method of determining power in negatively connected exchange networks, which they call Elementary Theory. Implicitly, their method is also based on power and dependence: the more options a position has, the more power the position has, if these options do not themselves have options. In Network I in Figure 1, Debbie has power because she is connected to two people without alternatives. Once Bob has an alternative, Debbie's position weakens. If Alice and Carol had alternatives, this would strengthen Debbie's position by weakening Bob's. The Markovsky-Willer-Patton approach is based on two steps: the computation of a power index and the predictions about who will trade with whom based on this index. In their power index, paths with an odd number of links have a weight of +1, paths with an even number of links have a weight of −1, and only paths of the same length with completely disjoint sets of members are counted. The power of a point i, $p(1)_i$, is indexed by the sum of the weights of the paths emanating from point i. Letting m_{ik} be the number of paths of length k that start at position i, Markovsky et al.(1988:224) state:

AXIOM I:

$$p(1)_i = \Sigma_k(-1)^{k-1} m_{ik} \tag{1}$$

The second step involves dividing the network into subgraphs of individuals who will want to trade with one another. This is done through the following axioms (Markovsky et al. 1988:225):

Axiom II: *i* seeks to trade with *j* if and only if *i*'s power is greater than *j*'s, or if *i*'s power relative to *j*'s equals or exceeds that in any of *i*'s other relations.

Axiom III: *i* and *j* can exchange only if each seeks exchange with the other.

Axiom IV: If *i* and *j* exchange, then *i* receives more resources than *j* if and only if *i* has more power than *j*.

The results for the five-person network discussed above are:

$$\begin{aligned} A &= 1-1+1=1 \\ B &= 3-1 \quad\;\; =2 \\ C &= 1-1+1=1 \\ D &= 2-1 \quad\;\; =1 \\ E &= 1-1+1=1 \end{aligned}$$

In other words, only position B has power; all other positions are equal. Thus, the hierarchy of power by this measure for this network is B > A = C = D = E. This also seems counterintuitive. It seems that D and E should in some way have more power then A and B.

Like Cook et al. , Markovsky et al. designed an experiment to test their theory. While many details of the experiment differed from those of Cook et al. the basic structure of both experiments was the same. Subjects were asked to negotiate to decide how to divide a preset allocation of points, (usually 24 points,) that would later be converted to money. Only connected pairs could negotiate. Each subject received the amount agreed upon. Only one agreement per round was allowed.

ASSUMPTIONS OF RATIONALITY

Cook et al. and Markovsky et al. both had rational-choice models for how subjects would make decisions. Cook et al. predicted that subjects would act in a "rational way" and explained rational as follows:

> This assumption (rational choice) is necessary theoretically since it allows us to derive testable predictions concerning manifest power from principles dealing with potential power. In our experimental setting by "rational" we mean

> that each actor in the network explores alternative sources of benefit in the network (a) through extending offers to others and (b) by comparing offers and counter offers from others. Each actor maximizes benefit by (a) accepting the better of any two offers, (b) lowering offers when offers go unaccepted, and (c) holding out for better offers when it is possible to do so (Cook et al. 1983:286, n. 12).

These assumptions are very similar to the "axioms" of Markovsky et al. (1988:223, n. 5), who say of their "actor conditions," which presuppose their axioms: "These conditions allow a variety of more deterministic rational or quasi-rational strategies." Ironically, Markovsky et al. rule out the use of game theory (specifically, coalition theory) as an avenue.

GAME-THEORY SOLUTIONS

Networks I, III, and IV in Figure 1 illustrate how these solutions can be applied to negatively connected networks. All the solutions discussed are solutions to the game in characteristic function form.[1] Consider a set of players N. A characteristic function v assigns to every subset of players S a total payoff $v(S)$ that they can realize among themselves despite the actions of the other players (Kahan and Rapoport 1984:26-27). In the network games we are considering, the characteristic value for any subset of players will be their maximum total payoff if the network consisted only of the connections among these players. For example, for the three-person chain in Network I of Figure 1, $v(\{BE\}) = 0$ because B and E cannot trade with one another, but $v(\{BDE\}) = 24$ because this set could guarantee itself 24 points if D were to trade with B or E. Table 1 contains the characteristic function form of the negatively connected network game for Networks I and III of Figure 1.

Each game-theoretic solution is based on concepts of rationality. There are three kinds of rationality commonly distinguished in the coalition literature: individual rationality, coalition rationality, and group rationality (Rapoport 1970:88-90.) *Individual rationality* is the assumption that no individual in a coalition will accept less than what he can earn alone. If there is a payoff vector $x = (x_1, x_2, x_3, \ldots, x_n)$, then:

$$x_i \geq v(i);\ i = 1, 2, \ldots, n.$$

Coalition rationality is the same assumption with respect to coalitions; no set of actors S will accept less in total than what they can earn in a coalition together.

$$\textstyle\sum_{i \in S} x_i \geq v(S) \text{ for all } S \subseteq N$$

Finally, *group rationality* is the assumption that the set of all actors, the grand coalition, will maximize their total reward.

TABLE 1
The Characteristic Function for Two Networks

Network	*Characteristic Value Function*
3-person Chain	v(∅) = 0 v(B) = v(D) = v(E) = v(BE) = 0 v(BD) = v(DE) = v(BDE) = 24
4-person Chain	v(∅) = 0 v(A) = v(B) = v(C) = v(D) = v(AC) = v(BD) = 0 v(AB) = v(BC) = v(CD) = v(ABC) = v(ABD) = v(ACD) = v(BCD) = 24 v(ABCD) = 48

$$\sum_{i \in N} x_i = v(N)$$

All the theories discussed will assume individual rationality. How important or predictive the other types of rationality are will be a contribution of this experiment.

The Core

Intuitively, the Core is the "the set of all feasible outcomes that no coalition can improve upon" (Lloyd Shapley, personal communication, October, 1989). No group of players will accept an outcome if by forming a coalition they can do better. Formally, the core consists of outcomes that have individual, coalition, and group rationality.

The importance of a core for the experiments described above is that Cook et al. and Markovsky et al. assume that their networks will reach an equilibrium. Cook et al. assume that over time, the more powerful members of networks will exercise their power over the others and eventually an equilibrium will be reached. Once this is obtained, Cook et al. predict that the values will remain the same. Markovsky et al. predict that, based on their axioms, a group of rational actors will quickly decide how to "ration" the points. They also predict an equilibrium.

The game theorist would have to look at the characteristic function of the network game to decide if there is a core before making predictions about reaching an equilibrium. If there is a core, the game theorist would predict that, once a core solution is stumbled upon, it is unlikely that any individual or coalition would be willing and able to produce a disruptive change. If there is no core, the game theorist would have no reason to expect that the game would ever be stable.

Network IV in Figure 1 is an example of a network without a core. If all the pairings have the same value, the network is unstable for the following reason: If A agrees to exchange with B, then C is excluded and receives no points. C would want to improve his/her situation and receive more than zero points so C would make an offer to either A or B that promised more points than the other exchange. If either A or B would agree to exchange with C, the other would receive no points. If B was excluded, B would then make an offer to either C or A that would improve their take. This could go on forever. The excluded person, by offering a player engaged in an exchange more than s/he would receive in that exchange, could cause the coalition to unravel. When there is no solution that some player or set of players cannot improve on, there is no core. When there is no core, there should be no expectation of stability. If there is no core, it would be difficult to decide if any position could exercise power and command resources.

To compute the core, the characteristic function is plugged into the equations for rationality. The objective is to find a solution that is simultaneously individually, coalition- and group-rational. For Network I in Figure 1, the core is D = 24, B = E = 0. No other set of values satisfies all the inequalities. For Network III, there is a range of possibilties in the core. A and B exchange, C and D exchange, and $B + C \geq 24$: the middle positions B and C have an advantage.

The Shapley Value

The Shapley value is based upon four assumptions that are desirable criteria for a solution:

1. The value should be feasible and efficient.
2. The value should be symmetric if the game is symmetric.
3. The value should award nothing to "dummy" players who contribute nothing to the coalition.
4. If two characteristic functions are added, then the sum of their values should be the value of their sum.

Intuitively, it can be understood as the payoff configuration that awards to each player that player's marginal worth to the coalition. A computational method for finding the Shapley value that uses this intuitive definition is the method of random orders. Consider all $n!$ random orders of the n individuals in the set of players N. For each ordering, tally the additional value made by each player as s/he enters a coalition. For each player, sum his/her total contribution for all $n!$ orderings. Dividing this sum by $n!$ gives the Shapley value for each player.

For example, consider the three-person chain network in Figure 1. There are six permutations. Each node is entered sequentially. When a node entered increases the value of the coalition, that node receives the value added. Table 2 is an example for the three-person chain network. In the first permutation, B enters first and contributes no value since the characteristic function value is B = 0. Next D enters. Because $v(BD) = 24$, the added value of D is 24 points. Last D enters. D contributes nothing so D receives credit for no points. The order of entry into the agreement is on the left side of the table; the values received for each node are tallied on the right. The Shapley value is computed by taking the total for each node and dividing it by the total possible points contributed.[2]

TABLE 2
Shapley Value for the 3-person Chain

Sequence entered			*Points added*		
First	*Second*	*Third*	*B*	*D*	*E*
B	D	E	0	24	0
B	E	D	0	24	0
D	B	E	24	0	0
D	E	B	0	0	24
E	B	D	0	24	0
E	D	E	0	24	0
Totals:			24	96	24
Shapley value:			1/6	2/3	1/6

The Kernel

Unlike the core and the Shapley value, which assume that any coalition structure that forms will be group-rational, the kernel makes no predictions about which coalitions will form and does not assume group rationality. The kernel predicts only the distribution of rewards, given some assumption about the memberships of all coalitions (Kahan and Rapoport 1984:128-134).

To calculate the kernel, we assume a complete coalition structure and a hypothetical distribution of rewards within each coalition. We then ask whether this distribution is in the kernel. Consider two players k and l in the same coalition. In this context, it means that the two players have agreed to trade with one another. Both k and l consider alternative trading partners. The maximum surplus of k over l, s_{kl}, is the maximum increase in reward to k and to any alternative trading partner j with respect to the present distribution if k and j agree to trade. Similarly, s_{lk} is the maximum increase in reward to l and some alternative trading partner j with respect to the present distribution of rewards if l were to agree to trade with j. A reward distribution is in the kernel if $s_{kl} = s_{lk}$ for every pair of players who are trading.

The appeal of the kernel is that it might model the way players in these networks actually determine how much they are willing to ask. In Network I in Figure 1, for example, D will trade with B or E and will try to take the entire 24 points. This is calculated as follows. Assume D is considering an exchange with B in which B receives x points and D receives 24−x. Being left out of any trade, E would receive nothing. If D trades with E instead of B, his/her potential additional profit from the change is $24 - (24 - x) - 0 = x = s_{DB}$. B has no one else to trade with, so s/he would suffer a loss of $-x = s_{DB}$. Equating these two surpluses ($x = -x$), it follows that $x = 0$. The kernel for this network is B = 0, D = 24, E = 0. The kernel seems an elegant formalization of the power/dependence model of power. The quantity $-s_{ab}$ is a's dependence on b; it shows how much a might lose in the best alternative relation. Equating s_{ab} and s_{ba} shows the outcome after power has been equalized (Emerson 1972).

THE EXPERIMENT

As subjects arrived, they were ushered into private cubicles with terminals. They were asked to sign a human subject's release briefly describing the experiment and to wait until the experimenter returned. When all the subjects had arrived, the experimenter went to each cubicle with index cards that were to be selected by the subjects to determine their network position. In this way, the subject's position in the network was randomly determined. The experimenter then started the terminal and gave brief verbal instructions on how to progress through the instructions and to notify the experimenter in the event of a computer malfunction or some other problem. After all the subjects had been oriented in this way, the experimenter observed the transactions from the control room. The game commenced when all subjects completed the instructions. There were three networks: the four-person chain (Network III in Figure 1), the five-person T (Network II), and the five-person hourglass (Network V).

There were 10 sessions for each of the networks. Each session was divided into a series of games. The number of games per session varied. Each group played as many games as it could conclude in one hour. Each game consisted of from one to five rounds where bargaining occurred. Each round consisted of three stages: the offer stage, the acceptance stage, and the confirmation stage.

To avoid confusion, a rigid structure was developed within which subjects could bargain that would ensure no subject could make more then one agreement per game. In the first stage, subjects could made offers to all eligible (connected) positions. At the next stage, all subjects were informed regarding who made them offers and reminded what offers they had made. They would then either allow the offers they had made to stand, keeping them available to other subjects, or they would accept an offer. If they accepted an offer, their standing offers were nullified. The last phase of the round was the confirmation stage. At this phase, the person who made the offer could either chose among competing offers or decide not to confirm any offers. This guaranteed that each person made at most one exchange, and it gave the one making an offer a chance, in a game, to rethink his/her original offer. If a dyad agreed, they sat out subsequent rounds until the end of the game. All subjects were informed of which positions were no longer eligible. Subsequent rounds continued so that players could negotiate an agreement until either all players were ineligible or five rounds had been completed. At the end of a game, if there was time remaining, another game commenced.

RESULTS

For each of the three networks in the experiment, we examine who traded with whom and which positions had the most power (earned the most points) in exchanges.

The Four-person Chain Network

Who Trades with Whom?

Three of the competing theories make predictions regarding who exchanges with whom: Elementary Theory, the Shapley value, and the core. The core and the Shapley value theories assume group rationality; therefore, no exchanges between positions B and C should occur. The prediction of Elementary Theory is that there should be no trading preferences at all: B and C should not desire to trade with each other any less than with the outer positions.

To test whether positions B and C exchanged more frequently with the outer positions than with each other, it was necessary to show that positions B and C exchanged with each other less often than they would based on a chance model. Position B has a 50 percent chance of choosing position C. Similarly, position C has a 50 percent chance of choosing position B. Since to conclude an agreement, both must agree to choose each other simultaneously, the probability of B and C engaging in an exchange with one another is 25 percent. If the Elementary Theory solution is to be confirmed, the two inner positions should exchange with each other 25 percent of the time. If the core and Shapley value predictions are to be confirmed, the data must show that the two center positions exchange with each other less often than 25 percent.

If the only theory under consideration were the Elementary Theory prediction, a two-tailed test would be used. However, the purpose of this research is to compare hypotheses. The core predicts that trades between positions B and C should occur less than 25 percent of the time. Therefore, the hypothesis is stated as a one-tailed test. A rejection of the null hypothesis would confirm the core and Shapley value predictions. Not rejecting the null hypothesis would support the Elementary Theory prediction.

The actual average percent of B—C trades across the ten groups in this condition was .16. The null and alternative hypotheses were:

Hypothesis ∅: $p \geq .25.$

Hypothesis 1: $p < .25.$

The results were $t_9 = -2.05$, $\alpha < .05$, one-tailed. The t-test shows that the null hypothesis can be rejected, lending support to the core prediction. The Elementary Theory prediction is rejected at a .05 level. Trades between one outer position and one inner position are preferred to exchanges between two inner positions.

Who Gets More?

The second question addressed is: who receives more points? The levels of prediction of the different competing theories differ. Below is a summary of the predictions of each theory for this network, the four-person chain:

TABLE 3
Average Points in Exchanges in Four-person Chain

	Middle-End (B-A and C-D)	*Middle-Middle (B-C)*
Difference	4.64	.72
df.	9	6
T-test	2.48	.41
α	.05	ns

Core: $B + A \geq 24$. The sum of the earnings of B and C will be greater than or equal to 24.
Shapley Value: A = 10, B = 14, C = 14, D = 10.
Kernel: A = 8, B = 16, C = 16, D = 8, when A exchanges with B and C with D.
Power/Dependence: The interior positions have power.
Elementary Theory: All positions have equal power.

This can be summarized by the following two hypotheses:

Hypothesis 2: The core, kernel, Shapley value, and power/dependence theories all predict that the center positions have an advantage.

Hypothesis 3: Elementary Theory predicts that there are no advantages to any position.

The average points for trades between dyads are given in Table 3. In all ten groups, there were middle-end trades. The average of all groups' average differences was 4.64 points in favor of the central position. This difference is statistically significant ($t_9 = 2.475$, $\alpha < .05$). In seven groups, there were trades between the middle positions B and C. The average $B - C$ difference was, not surprisingly, insignificant. These results support Hypothesis 2.

The Five-Person T Network

Who Trades With Whom?

Three theories make predictions regarding who exchanges with whom: the core, the Shapley value, and Elementary Theory. All make the same prediction about trading preferences: they predict that there is no reason for positions B and D to exchange; therefore, no exchanges between positions B and D should occur. This is tested against the null hypothesis that there are no trading preferences.

Position B has a 1/3 chance of choosing position D; position D has a 1/2 chance of choosing position B. Since to conclude an agreement, both must agree to choose each other simultaneously, the probability of B and C engaging in an exchange with one another is 1/6. If the null hypothesis is to be confirmed, positions B and

D should exchange 1/6 of the time. If the prediction of the two theories (the core and Elementary Theory) are to be confirmed, the data must show that these two positions exchange with each other less often the 1/6.

For the Five-person T, there is only one hypotheses under investigation:

Hypothesis 4: The core and Elementary Theory predict that positions B and D will exchange less often than chance: less than 1/6 of the time.

The average percent of games in which B and D traded was 13 percent. This is less than 1/6, but not significantly so. In eight of the ten groups, the proportion of games with B—D trades was less than 1/6. The failure to achieve significance is largely due to one outlying group. In this group, 54 percent of the games involved B—D exchanges. Using a one-tailed sign test, the null hypotheses that the probability of B—D trades is greater than or equal to 1/6 can be rejected at the almost significant .056 level.

Who Gets More?

The predictions of the theories are as follows:

Core: A = C = 0: B = 24: D + E = 24. Position B receives all points in trades with A and C. D and E trade on equal terms.

Shapley Value: A = C = 3.2, E = 11.2, D = 13.2, B = 17.2. Position B is more powerful than A or C, and D is more powerful than E.

Kernel: When B and D do not exchange, A = C = 0, B = 24, D = E = 12. When B and D do trade, B = D = 12.

Power/Dependence: A, C, E < B < D. In trades with A and C, B receives more points. In a D—E trade, D should do better than E. In a B—D trade, D should do better than B.

Elementary Theory: A, C < B, D = E. B should receive more than half the points in trades with A or C. D and E should trade on equal terms.

The predictions are summarized in Table 4. A broken line means that the theory does not make a prediction about that dyad. An asterisk means that the

TABLE 4
Dyadic Predictions of Theories

	B versus A and C	*B versus D*	*D versus E*
Core	B > A or C *	——	D = E *
Shapley Value	B > A or C *	——	——
Kernel	B > A or C *	B = D *	D = E *
Power/Dependence	B > A or C *	D > B	D > E
Elementary Theory	B > A or C *	——	D = E *

TABLE 5
Five Person T: Dyadic Comparisons for each Session

	B—A	*B—C*	*B—D*	*D—E*
Difference	3.66	4.40	.66	1.03
df.	9	9	6	9
T-test	2.22	2.19	.26	.67
α	.05, one-tailed	.05, one-tailed	ns	ns

corresponding null or alternative hypothesis was confirmed, according to the results presented in Table 5. These results show that all the theories performed about equally. All of them correctly predicted B's power over A and C.

The Five-person Hourglass: A Different Type of Network

The four-person chain and the five-person T networks had cores, but the five-person hourglass network does not. Consequently, the core makes no predictions concerning resource distribution in this network. The absence of the core suggests no stable solution. An assumption of the exchange theory experiments was that there was a "stable phase of power use" (Cook et al. 1983:285) or an "equilibrium point" (Cook et al. 1983:287). It was assumed that over time some pattern would stabilize that would ensure powerful positions would receive more of the good. The absence of a game-theoretic core suggests that no stable pattern should emerge.

Who Trades With Whom?

Only Elementary Theory makes a prediction about who will trade with whom. The core is empty and there is more than one pattern satisfying group rationality. Therefore, the core and Shapley value do not predict a pattern. Position C initially has more power than the other positions and thus two outside positions are the first to form an agreement. Once this agreement has formed, the remaining triad consists of three equal-power positions. The implication is that positions A, B, D, and E should have a 5/6 probability of being included in an exchange, while the central C position should have only a 2/3 probability. The baseline random model is that the central position should have a slightly greater probability, .821 versus .795, of being included in an exchange.[3]

The actual results showed that the central position was .186 less likely to be included in an exchange. As a test of Elementary Theory, this difference is not significant at the .05 level (t_9 = 1.72). However, the actual difference (−.186) is significantly different from the baseline random difference (.026) at the .05 level (t_9 = 2.00). These results support Elementary Theory.

Who Gets More?

Core: Makes no predictions since there is no core.
Shapley value: A = B = D = E = 7.4, C = 11.6.
Kernel: No power differences.
Power/Dependence: No power differences.
Elementary Theory: All trades should be between equally powerful positions.

The Shapley value predicts differences between the positions, but the differences could be either due to C being included in more exchanges or because it gets more in each exchange; the Shapley value does not tell us which is the case. However, position C cannot average 12.8 without earning more than half the points in exchanges with A, B, D, or E. The core makes no prediction. The kernel, Power/Dependence, and Elementary Theory all predict equal exchanges.

The three theories predicting equal exchange are wrong in this instance. On average, C earned 3.79 more points than his/her trading partners, and this difference is significant ($\alpha < .05$, one-tailed *t*-test with $df = 9$). This is consistent with the predictions of the Shapley value.

It was also suggested that coreless groups should be less stable in their coalition pattern. A reasonable measure of instability would be the proportion of games in which the group changed its coalition pattern. Unfortunately, these experimental groups did not play enough games to estimate the stability of coalition patterns. An examination of this issue will require experiments in which groups play more games.

CONCLUSIONS

The objective of this article was to show that there is a utility in incorporating game theory into the discussion of exchange in negatively connected networks. This was achieved. The game-theory models did at least as well as the exchange-theory models. In addition, there is much that game theory can contribute to an understanding of the processes that the exchange-theory solutions are insensitive to. Not only was it as successful at correctly predicting where power was located in negatively connected networks than either of the exchange-theory predictions, but it also provides a basis for deciding when there are any stable power relations to predict; coreless games should not have stable power relations.

Table 6 is a summary of the performance of the different solutions. A plus "+" symbol shows that the theory was confirmed. A minus "−" means that the theory's predictions were incorrect, a plus/minus shows that the theory made both correct and incorrect predictions. The symbol "---" shows that the theory made no prediction.

The first network, the four-person chain, confirmed the predictions of both the game-theory-solution and power/dependence theories. Elementary theory was inaccurate in its prediction regarding which positions would exchange and which

TABLE 6
Summary of Results for all Theories and All Networks

	4-Person Chain	*5-Person T*	*Hourglass*	*4-Person Chain*	*5-Person T*	*Hourglass*
Core	+	+	---	+	+	---
Kernel	---	−	---	+	+	−
Shapley value	+	+	---	+	±	+
Power/Dependence	---	−	---	+	±	+
Elementary theory	−	+	+	−	+	−

Note: + means that the theory was confirmed.
− means that the theory's predictions were incorrect.
± means correct and incorrect predictions.
--- means that the theory made no prediction.

positions would be powerful. All the theories did about equally well on the five-person T network. There were the fewest successful predictions about the figure without a core, the hourglass. In many instances, no predictions were made, and in other cases, the predictions were incorrect. As measured by the proportion of sucessful predictions (where ± counts as half a success), the only completely successful theory is the core, closely followed by the Shapley value. Elementary Theory was the least successful.

The main point we wish to make is that both the major sociological approaches to exchange networks, Cook's and Markovsky's, do assume rational behavior. Game theory offers a variety of sophisticated rational-choice models for exchange networks that perform at least as well and are worthy of further exploration.

ACKNOWLEDGMENTS

Work on this article was supported in part by a dissertation support grant received from the National Science Foundation.

NOTES

1. Further extensions of the models presented in this article are described in Bonacich and Bienenstock (in press) and Bienenstock and Bonacich (in press).

2. The Shapley value assumes complete "interpersonal transfer of utility." This means that any coalition can split its value between its members in any way. This is not true in these exchange networks. For example, in network III, $v(ABCD) = 48$, but $A = B = C = 0$, $D = 48$ is impossible. This means that sometimes (for example, Network II), the Shapley value cannot occur. Our experience is that this is not a major problem in its application as an ordinary measure of power.

3. In the baseline model, each position chooses randomly from its alternative trading partners. Reciprocated choices produce exchanges. Positions that exchange are removed from the network and choices are made by the remaining positions from the reduced graph. A game ends when no additional exchanges can be made.

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