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COMMENT: GENERAL COMPETITIVE ANALYSIS IN AN ECONOMY WITH PRIVATE INFORMATION*

BY HAROLD LINH COLE

This note corrects a misunderstanding in Prescott and Townsend (1984) by pointing out that random allocation rules can lead to Pareto dominating allocations even when the set of incentive compatible and resource feasible allocations are convex under deterministic allocation rules.

1. INTRODUCTION

Prescott and Townsend (1984) extend the literature on general equilibrium theory to economies in which agents have private information about their preferences. They examine the extent to which the classical welfare theorems carry over in this environment. The existence of nonverifiable information imposes limits on the economy's contract structure. Contracts cannot be written to require an agent to reveal information unless truthful revelation is optimal. Incentive compatiblity constraints, typically nonlinear, are thereby introduced into the corresponding planner's problem. As a result, the space of feasible deterministic allocations is typically not convex. The authors circumvent this problem by allowing for random allocations, where the planner is choosing over the space of probability distributions of consumption allocations. The incentive compatibility constraints in this space are linear in the planner's choice variables, rendering the set of feasible consumption allocations convex. Some of the attainable random allocations Pareto dominate the set of incentive-feasible allocations attainable via deterministic allocation rules.

Randomized allocations have two functions. They convexify the planner's choice set, and they may allow for Pareto superior allocations to be achieved. Prescott and Townsend's presentation may leave readers with the misleading conclusion that the nonconvexity of the choice set under deterministic allocations is necessary for random allocations to be Pareto superior. But, one can attain Pareto superior allocations with lotteries even when the choice set under deterministic allocation rules is convex. This note demonstrates that Prescott and Townsend's own example has this property.

2. MODEL

Consider an economy with two periods, one consumption good, and a continuum of agents. In the second period agents may be of two types. In the first period agents are endowed with e of the perfectly storable consumption good.

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Agents care only about their second period consumption. Agents of type 1 have preferences given by the strictly concave function $V(c(1))$, where $c(1)$ denotes the consumption of the type 1 agent. Agents of type 2 have preferences given by $\theta c(2)$. Assume that $V'(e) < \theta$, $V'(\infty) = 0$, and $V'(0) = \infty$. Assume that a proportion α of the agents will be of type 1, and that $(1 - \alpha)$ will be of type 2.

The resource constraint of the economy can be expressed as:

$$\alpha c(1) + (1 - \alpha)c(2) \leq e. \tag{1}$$

The ex-ante Pareto optimal allocation, $[c^*(1), c^*(2)]$ (when all of the agents are given equal weight in the planner's objective function), would maximize an agent's expected utility conditional on his knowing the probabilities of each type. When agents' types are public knowledge in period two, the optimal allocation would be such that equation (1) and the following condition were simultaneously satisfied:

$$V'(c^*(2)) = \theta. \tag{2}$$

Given our assumptions about preferences, this would imply that $c^*(2) > c^*(1)$.

In the optimal allocation when agents' types are public information we may differentiate between the two types of agents in a manner which is not feasible if agents' types are private information. To see this note that an agent of type 1 would strictly prefer to receive the type 2 bundle since $c^*(2) > c^*(1)$. In order for an allocation to be incentive compatible when agents' types are private information it must be the case that agents weakly prefer their own bundle to that of any other agent. Thus we must have:

$$V(c(1)) \geq V(c(2)) \tag{3}$$

$$\theta c(2) \geq \theta c(1). \tag{4}$$

With one consumption good the incentive feasible set is always convex since these constraints will simply imply that:

$$c(1) \geq c(2) \tag{3'}$$

$$c(2) \geq c(1). \tag{4'}$$

As long as agents' preferences are strictly increasing in the consumption good, these two constraints imply that $c(1) = c(2)$. In other words, the set of incentive compatible allocations is the 45 degree line in Diagram 1. The feasible set of consumption allocations is the intersection of the incentive and resource feasible sets, and consists of the line segment from $(0, 0)$ to (e, e). Note that this is a convex set. Since agents' utilities are increasing in the consumption good, the constrained Pareto optimal allocation will consist of the point (e, e).

To see how one can achieve Pareto superior allocations when one allows for random allocation rules, consider the following allocation rule which offers an agent a choice between a certain level of the consumption good, denoted by n, and a lottery in which he receives consumption $m(1)$ with probability π or consumption $m(2)$ with probability $(1 - \pi)$. If we assume that type 1 agents choose the certain level of consumption and type 2 agent choose the lottery, the resource

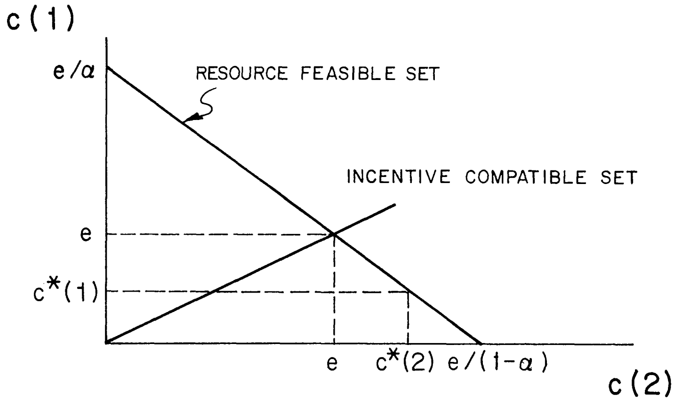

DIAGRAM 1

constraint will be given by:

$$\alpha n + (1 - \alpha)[\pi m(1) + (1 - \pi)m(2)] \leq e. \tag{5}$$

The incentive compatibility constraint take the following form:

$$V(n) \geq \pi V(m(1)) + (1 - \pi)V(m(2)), \tag{6}$$

$$\theta[\pi m(1) + (1 - \pi)m(2)] \geq \theta n. \tag{7}$$

The second incentive compatibility constraint implies that the expected level of consumption associated with the lottery must be at least as great as the sure level of consumption if the type 2 agents are to choose the lottery. Because the type 1 agents are risk averse while the type 2 agents are risk neutral, the type 2 agents will always pick the lottery if the type 1 agents do. There exist however certain lotteries and levels of n such that the type 1 agent picks the certain level of consumption while the type 2 agent picks the lottery. Following Prescott and Townsend, set $n = c^*(1)$, $m(1) = 0$, and $(1 - \pi) = c^*(2)/m(2)$. The expected utility of the lottery to a type 2 agent is, by construction, equal to $\theta c^*(2)$. As $m(2)$ goes to infinity, the variance of the lottery also goes to infinity. There exists a $m(2)$ large enough so that the type 1 agents prefers $c^*(1)$ with certainty to the lottery, even though the expected return of the lottery is higher. Because they are risk neutral, the type 2 agents always prefers the lottery.

By allowing for randomization, allocation rules are attained with associated expected utilities that match those when agents' types are public information. If we were to graph the expected level of consumption of the type 1 agent against that of the type 2 agent, the diagram would look exactly like that of Diagram 1 except that the incentive feasible set would be everything on or below the 45 degree line, excluding the horizontal axis. The feasible set under deterministic allocation rules would be unchanged. Thus, within the space of expected levels of consumption the use of random allocation rules has expanded the set of feasible allocations. Note that this allowed for Pareto superior allocations because it was optimal to have the expected consumption of the more risk averse agent be less than that of the less risk averse agent. There is probably some general sense in which this must be true for randomized allocations to dominate deterministic allocations.

3. CONCLUSION

Differences in preferences among agents allow one to elicit information about them from their choices. Agents can be separated by type even when their types are private information. By allowing for random allocation rules, one not only shifts from a potentially nonconvex to a convex space, as noted by Prescott and Townsend, but also enables the planner to screen agents based on their degree of risk aversion. To the extent that agents have differences in their attitude towards risk, lotteries can be used to separate agents and attain Pareto superior allocations. The incentive feasible set need not be nonconvex for there to be gains from introducing lotteries.

University of Pennsylvania, U.S.A.

REFERENCES

PRESCOTT, EDWARD C. AND ROBERT M. TOWNSEND, "General Competitive Analysis in an Economy with Private Information," *International Economic Review* 25 (February 1984), 1–20.