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# POWER RELATIONS IN EXCHANGE NETWORKS\*

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*Many theories address the problem of how a social structure affects the experiences and behaviors of its members. This paper offers a network-exchange theory to solve this problem. Previous research has shown that the nature and outcomes of negotiations among individual or corporate actors can be inferred from their network positions. The impact of this research has been limited because its theory does not enable the researcher to locate power positions in the networks. We offer a theory that is both consistent with all previously reported experimental research and is generalized to conditions not considered by other formulations. In addition to supporting derived hypotheses pertaining to network-based power, our experiments demonstrate, among other things, that certain unstable networks break down to form stable substructures and that some networks contain overlapping but autonomous domains of power and exchange.*

Although no single exchange theory dominates the social sciences, a fairly coherent social-exchange perspective exists. In this perspective, social structures and processes impinge on and emerge from resource and sanction transfers between individuals and/or collectivities.<sup>1</sup> Recently, some theories have moved beyond two-party exchange contexts to focus on networks of exchange relations. As structural theories, network-exchange theories attempt to explain how macro-properties bear upon micro-units within structures. Concretely, they try to show how network structures affect the power of actors to extract valued resources in their exchanges with others.

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<sup>1</sup> Theoretical statements have been provided by Thibaut and Kelley (1959), Blau (1964), Gergen (1969), Homans (1974), Ekeh (1974), Heath (1976), Blalock and Wilken (1979), Burgess and Huston (1979), and Cook (1987). Emerson (1976), Bredemeier (1978), and Tuner (1986) have written reviews. Applications involving ethnographic, institutional, and historical analyses are provided by Polanyi (1944), Elkin (1953), Sahlins (1972), Earle and Ericson (1977), and Emerson (1981). Recent applications of *network-exchange* theories to interorganizational relations, backward and forward integration of the firm, community structure, historical development of modern exchange relations, and exchange processes in antiquity are given by Hansen (1981), Loukinen (1981), Gilham (1981), Galaskiewicz (1985), Skinner and Guiltinan (1986), Lind (1987), and Willer (1987).

We propose and test a theory that predicts relative power for network positions. In so doing, we address several structural phenomena, including the breakdown of larger networks into smaller parts and the emergence of positions that simultaneously have one level of power in one part of the network and a different level in another. Our theory is further intended to provide higher levels of rigor, power, and specificity than are found in earlier approaches. We find that each such technical advancement produces a manifold increase in the array of potential applications.

Whenever a person or group negotiates with another person or group over the allocation of valued resources, a minimal social-exchange network exists. More elaborate (i.e., nondyadic) structures form when one member is involved in two or more such relations. For example, college students Al, Bea, and Cleo each want to date, and norms prohibit them from dating more than one person at a time. Suppose that Bea and Cleo both vie for Al's attention and have no other prospects, while Al would be happy to date either Bea or Cleo. This creates a B-A-C network, where A(1) may "negotiate" with B(ea) and C(leo), but only date one of them. Such circumstances actually do tip the balance of power (Peplau 1979) in dating relations: A is able to make greater demands than his chosen partner, and generally has greater influence in the relationship. But if B or C develop dating interests with a responsive D, A loses his structural advantage.

This type of analysis is applicable in other areas such as international, auctioneer-bidder, retailer-consumer, and manufacturer-retailer relations. A good example is the control that a manufacturer may impose upon retailer marketing strategies (Skinner and Guiltinan 1986). Suppose Ascii Ugetty (A) is the sole manufacturer of a line of computer games. Big Bytes (B), Chips-R-Down (C) and Data Dump (D) are independent retailers that want to carry the line. Even with fixed wholesale prices, A's position affords it power over B, C, and D. Skinner and Guiltinan found that retailer activities such as advertising expenditures, sales force training, and credit policies were under manufacturer control to a greater extent when retailers had no alternative suppliers. So if D can obtain the product from E-Z Access (E), A loses its ability to control D's policies. A may have to "outbid" E just to keep D's business.

Our purpose is to understand the structural logic manifested in all such exchange networks—a logic unbounded by empirical content. If the experiences of actors depend on their positions, this suggests a structural determination of behavior. At issue in this paper is the logic of that determination.

### AN EARLIER APPROACH

Recent work by Cook, Emerson, Gillmore, and Yamagishi (1983) clearly overlaps with our own in scope.<sup>2</sup> They showed that their approach could anticipate power distributions in some cases where alternative measures failed. Based on Emerson (1972b), Cook et al. (1983) defined *Exchange network* as

(1) a set of actors (either natural persons or corporate groups), (2) a distribution of valued resources among those actors, (3) for each actor a set of exchange opportunities with other actors in the network, (4) a set of historically developed and utilized exchange opportunities called exchange relations, and (5) a set of network connections linking exchange relations into a single network structure. (p. 277)

The set of exchange relations is a subset of exchange opportunities, and actors in the

system are assumed to be committed to exchanging within their relations, to the exclusion of alternative opportunities. The concept of *connection* permits networks to be considered from relations. Formally

Two exchange relations between actors A-B and actors A-C are connected to form the minimal network B-A-C to the degree that exchange in one relation is contingent on exchange (or nonexchange) in the other relation. (a) The connection is positive if exchange in one relation is contingent on exchange in the other. (b) The connection is negative if exchange in one relation is contingent on nonexchange in the other. (p. 277)

A negative connection exists if B and C can substitute as providers of A's resources. The authors cite as examples dating and friendship networks. In the case of a positive connection, A cannot benefit without exchanges from B and C. This is true if A is a brokerage agent, or if B and C are assembly-line workers who must exchange their labor for pay before the firm (A) can benefit.

Cook et al. (1983) define *power* as "In any dyadic exchange relation  $A_x;B_y$  (where A and B are actors, and x and y are resources introduced in exchange), the power of A over B is the potential of A to obtain favorable outcomes at B's expense" (p. 284). *Dependence* is given as: "The dependence of A on B in a dyadic exchange relation is a joint function (1) varying directly with the value of y to A, and (2) varying inversely with the availability of y to A from alternate sources" (pp. 284-85).

By *informally* applying power-dependence ideas, Cook et al. developed several hypotheses predicting relative power for positions in several types of negatively connected networks. Toward the end of the paper, a network *vulnerability* (V) method was suggested as a first step toward a formal procedure for predicting positions' relative power.

To determine V for the B-A-C network, assume that related actors negotiate over the division of 24 resource points, and a one-exchange rule creates the negative connection: A may exchange with B or C but not both in a given round. First, the maximum resource flow (MRF) for the network is calculated.  $MRF = 24$  since, by the one-exchange rule, only 24 points may be distributed per round. Next, the reduction in

<sup>2</sup> Comparisons among these theories are hindered by their lack of explicit scope conditions. Although some scope conditions can be inferred, at times it is not clear when theories are competitors (Wagner and Berger, 1985) with divergent predictions testable in the same empirical settings.

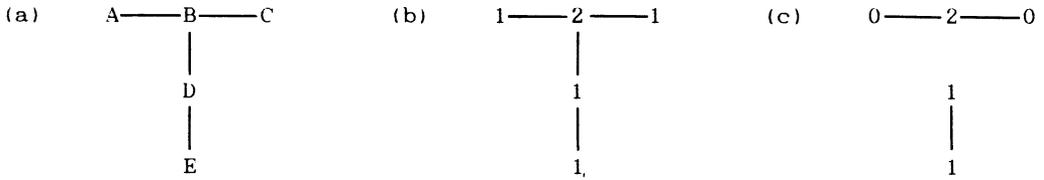


Fig. 1

maximum flow (RMF) is calculated for each position by noting the effect of its removal on MRF. If B or C is removed, RMF = 0, since A may still exchange with the other. However,  $RMF_A = 24$ . The network is then most vulnerable at A, and A is declared a power node.

*Discussion*

This general approach has been corroborated in several experiments, including those published by Cook et al. (1983), Stolte and Emerson (1977), and Cook and Emerson (1978). However, V has not been systematically tested. Moreover, Willer (1986) determined that V produces untenable predictions for some relatively simple networks such as that in Figure 1a. V predicts high power for B, D, and E. But under Cook et al.'s experimental conditions, high profit for D would entail low profit for B, E, or both. Although Cook, Gillmore, and Yamagishi (1986) described V as only "a preliminary notion," it still provided the only explicit basis for deriving hypotheses. Without it, predictions were informal and not fully determined by the theory.

Cook et al. (1986, p. 447) later proposed a modified V-measure. Network-wide dependence ( $D_N$ ) weighs a position's RMF by the factor  $(1 - CRMF)$ , where CRMF is "no. of lines that need to be removed [for a position] to exercise power at its potential" divided by the number of lines connected to the position. By this measure, B in Figure 1 has higher power than D, and D higher power than A, C, and E. Although these predictions are tenable, they diverge from test results reported later in this paper and their derivation is indeterminate.<sup>3</sup>

<sup>3</sup> Using their model, we could not reproduce Cook et al.'s predictions. The authors stated "This measure is relevant only when RMF is not zero" (p. 447). But RMF = 0 for positions A and C, apparently making  $D_N$  inapplicable. Further, the expression "exercise power at its potential" is not defined, and it is not stated whether

Many of Cook et al.'s methodological choices were neither necessitated nor precluded by their theory. For instance, negotiations took place over a series of rounds; each relation had its own resource pool; each pool was replenished before every round; exchange consisted of mutually agreed on pool divisions; there was a one-exchange rule; resources did not move through positions; coalitions were prohibited; and actors had no information on negotiations in which they were not directly involved. At issue is whether the approach might have been falsified under alternative methodological conditions. Later we demonstrate that very different results are obtained under slightly different conditions.

A GRAPH-ANALYTIC THEORY

In his recent elaboration on the work of Cook and her associates, Marsden (1987) succinctly offered as unsolved problems several of the implications that may be drawn from our theory:

The difficulty in developing a more general measure is that an alternative [exchange partner] may be exploitable for two reasons: It may have few alternative relations, or all of its alternatives (irrespective of how many in all are available) may be in a position to exploit others. The second condition of exploitability can lead to consideration of quite distal features of network structure. (p. 147, note 5)

Building on an earlier exchange formulation (Willer and Anderson 1981; Willer 1987), our graph-analytic approach recognizes both types of "exploitability" and

the removed lines must stem from the position whose  $D_N$  is being assessed. Following the Cook et al. examples, it appears that in the 1a network, two lines must be removed from B to reduce the maximum flow of network resources, and one relation must be removed from D. The result is  $CRMF_B = 1/3$ ,  $CRMF_D = 1/2$ ,  $D_{NB} = 8$ , and  $D_{ND} = 12$ . D should be higher than B, contradicting Cook et al.'s prediction. In either case, the predictions diverge from those we will obtain from our model.

specifies conditions under which distal network properties will or will not influence proximal outcomes. We first present  $p(1)$ , an index for power in one-exchange networks. This allows us to test our predictions against those of Cook et al. (1983, 1986). Following this,  $p(e)$ , a generalized version, will be explicated and tested.

### *Conditions of Exchange*

Power and resource distributions are affected not only by network shapes, but also by the conditions under which exchanges transpire. The theory provides scope statements encompassing relatively broad conditions, some of which are later relaxed, others of which await future tests, theoretical extensions, and refinements. Scope conditions are not assumptions about human nature or frequencies of empirical circumstances. They are statements that, if satisfied (or approximated), commit the theory to critical examination and, if not satisfied, relieve it of any explanatory imperative (Walker and Cohen 1985).

Several important concepts must first be defined: *actors* are decision-making entities, e.g., organisms, collectivities, or even computer programs. *Positions* are network locations occupied by actors. A *relation* between two positions is an exchange opportunity for actors in those positions. In short, *actors* occupy *positions* linked by *relations*.<sup>4</sup> We will index both actors and positions using upper-case letters and at times refer to them interchangeably.

*Actor Conditions.* Four conditions delimit actors' behavior: (1) all actors use identical strategies in negotiating exchanges; (2) actors consistently excluded from exchanges raise their offers; (3) those consistently included in exchanges lower their offers; (4) actors accept the best offer they receive, and choose randomly in deciding among tied best offers.

Condition 1, requiring identical strategies, is nearly always implicit in exchange theories. In tests and applications, however, it is generally sufficient that actors adopt functionally *similar* strategies. Condition 1 also asserts that actors negotiate, i.e., they make offers and adjust their subsequent offers in

light of counter-offers they receive. Conditions 2 and 3 require that actors seek to enter exchange if previously denied, and to improve outcomes beyond those previously obtained. Finally, condition 4 rules out a range of strategies that may drive up the offers of excluded parties.<sup>5</sup>

*Position Conditions.* These apply to positions and their relations: (5) each position is related to, and seeks exchange with, one or more other positions; (6) at the start of an exchange round, equal pools of positively valued resource units are available in every relation; (7) two positions receive resources from their common pool if and only if they exchange; (8) each position exchanges with at most one other position per round.

Since isolates cannot exchange, Condition 5 omits them from consideration. Condition 6 reflects conditions in most prior research: a pool of *profit points* resides in every relation and is replenished with each new round. Condition 7 indicates that two actors will not exchange unless both benefit. Condition 8, relaxed later, asserts that actors may complete at most one exchange per round. This creates negative connections in a way consistent with all previously cited experimental research and Cook et al.'s (1983) simulations. It assumes that, for whatever reasons, actors only require a single exchange, or are only able or permitted to complete a single exchange in a given round.<sup>6</sup>

### *The Graph-theoretic Power Index*

Building upon simple arithmetic procedures, our graph-theoretic power index (GPI) deter-

<sup>5</sup> These conditions allow a variety of more determinate rational or quasi-rational strategies. For example, resistance theory (Heckathorn 1980; Willer 1981, 1987) provides an elegant model of joint-bargaining decision-making. Resistance is given as the ratio of an actor's interest in gaining a better exchange to interest in avoiding conflict. The conditions do, however, rule out strategies such as coalition formation (Kahan and Rapoport 1984; Shubik 1982; Willer 1987), in which some actors temporarily accept reduced resources while receiving increasingly favorable offers from others.

<sup>6</sup> We treat negative connection the same way as Cook et al. (1983), but diverge from Emerson's (1972a,b) original usage (Willer, Markovsky, and Patton forthcoming). In the earlier formulation, for an actor with multiple relations, exchange in one reduces the value of exchange in others because the actor's satiation level increases with each exchange. Exchange rates across the actor's relations are then negatively correlated, but as an *outcome* of the exchange process, not as an *initial condition*.

<sup>4</sup> The reason for distinguishing actors and positions is that actor properties (e.g., decision strategies) and position properties (e.g., number of relations) may affect power independently (Markovsky 1987a).

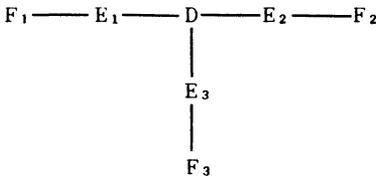


Fig. 2

mines relative power for all positions in any network that meets the scope conditions.<sup>7</sup> As also implied in the work of Kuhn (1974), Cook et al. (1983), Bonacich (1987a), Marsden (1983, 1987) and others, power is assumed to derive from the availability of alternative exchange relations, the unavailability of their relations' alternative relations, and so on. Power is then conceived as an unobservable, structurally determined potential for obtaining relatively favorable resource levels. Power use, as manifested in resource distributions, serves as an indicator of power. So while we theorize about potential power, we test our theory by observing its use.

The procedure for determining GPI involves counting path lengths. Thus, network B-A-C has two one-paths, A-B and A-C. B and C are linked by a two-path. As explained below, path counting is greatly simplified by only counting the number of nonintersecting paths of each length stemming from a given position. Nonintersecting paths stemming from position X have only X in common. In Figure 2, for example, three nonintersecting two-paths stem from D, but only one nonintersecting two-path stems from E<sub>1</sub> (connecting with either E<sub>2</sub> or E<sub>3</sub>).

An implication of this procedure is that it does not matter for X whether a position *m* steps away "branches" to one or a hundred positions *m* + 1 steps away. All that matters is whether or not there is a position *m* + 1 steps from X. This is a subtle, possibly nonintuitive, but incontrovertible assertion within our framework. The following example therefore bears careful study.

Imagine removing A and C from the Figure 1a network. D benefits greatly from the resulting three-actor chain: B and E must try to engage D, offering ever more favorable deals to D. Now restore A. B now has an alternative to bidding against E. But with B not bidding against E, D's advantage dis-

solves. Although still with two alternatives, D cannot play B and E against each other and so all positions are on an equal footing. Now restore C. B now benefits because A and C will try to outbid each other for B's exchange. This presents no further disadvantage for D, however, who may still exchange with E on an equal basis.

Note that A and C are on intersecting two-paths from D. The creation of one of those two-paths changed the minimum relative power in D's relations from high to equal. But the creation of the second two-path had no effect on this minimum. If we further attached F, G, and H to position B, these added two-paths from D will still not affect the minimum relative power that D would enjoy. This shows why only one nonintersecting path of a given length is counted.

It may now be apparent that X's odd-length nonintersecting paths are advantageous, and even-length nonintersecting paths are disadvantageous. Advantageous paths either provide direct exchange alternatives (in the case of one-paths), or counteract the advantage-robbing effects of disadvantageous paths.

The GPI simply tallies the number of advantageous paths and subtracts the number of disadvantageous paths to determine each position's potential power.

Position *i*'s GPI under the one-exchange condition is calculated as<sup>8</sup>

$$\begin{aligned}
 p(1)_i &= \sum_{k=1}^g (-1)^{(k-1)} m_{ik} \\
 &= m_{i1} - m_{i2} + m_{i3} \\
 &\quad - m_{i4} + \dots \pm m_{ig} \quad (1)
 \end{aligned}$$

and *i*'s power relative to *j* is

$$p(1)_{ij} = p(1)_i - p(1)_j.$$

The function  $(-1)^{(k-1)}$  produces + signs for advantageous paths and - signs for disadvantageous paths. These are attached to the  $m_{ik}$  values—the number of position *i*'s nonintersecting paths of length *k*. For now we may

<sup>7</sup> See Harary, Norman, and Cartwright (1965), Harary (1969), and Fararo (1973) for discussions of a variety of graph-theoretic tools.

<sup>8</sup> Readers familiar with our unpublished reports should note that we have referred to this measure as  $C_M(i)$ , position *i*'s centrality when allowed *N* exchanges. The present notation more accurately reflects our concern with power rather than centrality and adheres to the convention of displaying variable indices and parameters as, respectively, subscripts and parenthetical elements.

suppress the number-of-exchanges parameter for  $p(1)_i$  and refer to the index simply as  $p_i$ .

The values for  $g$  and the  $m$ s are obtained as follows:

$m_{i1}$  is the number of one-paths stemming from position  $i$ , which is the same as the number of  $i$ 's relations. In Figure 1a, for example,  $m_{D1} = 2$ .

$m_{i2}$  is the number of nonintersecting two-paths from  $i$ . As shown in the earlier example, D has only one nonintersecting two-path, so  $m_{D2} = 1$ .

$m_{i3}$  is the number of nonintersecting three-paths stemming from  $i$ ;  $m_{D3} = 0$ .

The largest path of length  $k$  for which  $m_{ik} > 0$  is the *geodesic* ( $g$ ) of the network. In Figure 1a, three-paths link A to E, and C to E, hence,  $g = 3$ .

The final step is to combine the  $m_{ik}$ s: take  $m_{i1}$ , subtract  $m_{i2}$ , add  $m_{i3}$ , and so on. We find that  $p_D = 2 - 1 + 0 = 1$ . Figure 1b shows this value and the  $p_i$  values for the other four positions.<sup>9</sup>

### Axioms and Theorems

The formal statement of our theory appears in the Appendix to this paper. In the statements below, "power" refers to  $p_{ij}$ , with  $i$  and  $j$  related.

**AXIOM 1:** given by equation (1) above.

**AXIOM 2:**  $i$  seeks exchange with  $j$  if and only if  $i$ 's power is greater than  $j$ 's, or if  $i$ 's power relative to  $j$  equals or exceeds that in any of  $i$ 's other relations.

**AXIOM 3:**  $i$  and  $j$  can exchange only if each seeks exchange with the other.

**AXIOM 4.** if  $i$  and  $j$  exchange, then  $i$  receives more resources than  $j$  if and only if  $i$  has more power than  $j$ .

In Axiom 2, "i seeks exchange with j" means that  $i$  makes competitive offers to  $j$ , i.e., offers that compete with others that  $j$  receives. A more psychological interpretation would be "i makes offers that  $j$  seriously considers." The axiom first claims that this occurs if  $i$ 's power is greater than  $j$ 's. Further, even if  $i$ 's power is less than  $j$ 's,  $i$  will seek exchange with  $j$  if  $i$ 's relative power is even lower in its other relations.<sup>10</sup> Note that Axiom 3 *does not* imply that two actors will exchange if they seek exchange with each other; actors may negotiate without exchanging. Finally, Axiom 4 asserts that potential power determines the use of power, i.e., GPI predicts final resource distributions.

Some of the theorems that can be derived from these axioms include

**Theorem 1:** If  $i$  has no alternative relations, then  $i$  seeks exchange with  $j$ .

**Theorem 2:** If  $i$  does not seek exchange with  $j$  or if  $j$  does not seek exchange with  $i$ , then  $i$  and  $j$  do not exchange.

**Theorem 3:** Actor  $i$  does not seek exchange with  $j$  if and only if  $i$ 's power is less than or equal to  $j$ 's and  $i$  has a better alternative to  $j$ .

**Theorem 4:** If  $i$ 's power is less than or equal to  $j$ 's and  $i$  has a better alternative to  $j$ , or if  $j$ 's power is less than or equal to  $i$ 's and  $j$  has a better alternative to  $i$ , then  $i$  and  $j$  will not exchange.

More intuitively, Theorem 1 claims that an actor in a position with only one relation will seek exchange via that relation, whatever its relative power. Theorem 2 is a logical variant of Axiom 3. Theorem 3 specifies the conditions under which an actor will not seek exchange via one of its relations. Theorem 4 predicts when a network will break at the  $i$ - $j$  relation. It reveals that certain relations are expected to remain unused, leading some

<sup>9</sup> Exchange in one relation will often temporarily alter the relative power of nearby positions. This dynamic is captured through an iterative application of the GPI. In Figure 2, for example,  $p_i$  is first calculated for all positions in the network. In a given round of negotiation, if  $E_1$  and  $F_1$  exchange first,  $p_i$  is recalculated for the network with  $E_1$  and  $F_1$  removed. The new  $p_i$  values are then in force until the next exchange occurs or until the end of the round. In the relatively simple networks examined in this paper, initial  $p_i$  values provide accurate predictions for power use. In more complex networks, however, the iterative application of the GPI is required to obtain accurate predictions (Markovsky, Willer, and Patton 1987).

<sup>10</sup> After a sufficiently extended series of exchanges, an actor with  $p = 0$  should seek exchange in *all* of its relations, regardless of power differences. That is, to avoid complete exclusion, the actor will offer to keep just one resource unit and relinquish the balance of the pool to any other that is willing to exchange. This seems to violate Axiom 2; however, this actor is no longer engaged in negotiation. This violates the first actor condition and makes the theory inapplicable. This is hardly a limitation of the theory, however, for when exchanges reach this point of non-negotiability, the system (or subsystem) has run its course, exchange rates will remain fixed, and the theory is "finished" with its predictions for the application.

complex networks to break apart into smaller, stable subnetworks. When such a break occurs, power indices are recalculated within the resulting subnetworks. This is demonstrated in some of the applications below.

### Applications

We have applied the GPI, axioms, and theorems to a large number of networks of varying shape and size. This small sampling demonstrates the use of the theory.

For the A-B dyad,  $p_A = p_B = 1$ . No position has a structural advantage. The same is true for positions on any even-length chain, as verified in computer simulation research (Markovsky 1987b). In general, however, the longer the chain, the more rounds transpire before the predicted power relations stabilize.

For the B-A-C network,  $p_B = p_C = 1 - 1 = 0$  and  $p_A = 2 - 0 = 2$ . A's power advantage is 2 in both of its relations, while  $p_{BA} = p_{CA} = -2$ . In fact, for odd-length chains of any length,  $p = 2$  for even positions and  $p = 0$  for odd positions; low- and high-power positions alternate. This conforms with Cook et al.'s predictions and experimental results for the five-position chain and with our computer simulations for longer chains.<sup>11</sup> Similarly, in Figure 2,  $p_F = 1 - 1 + 1 - 1 = 0$ ,  $p_E = 2 - 1 + 1 = 2$ , and  $p_D = 3 - 3 = 0$ . Thus the center and periphery have low power, and the off-center positions have high power. This also conforms with Cook et al.'s predictions and simulation results.

Returning to Figure 1, we find that a decomposition is predicted. Figure 1b shows the  $p_i$  values as initially calculated. Applying Theorem 4, however, since D's index is less than B's, and since E is a "better" alternative for D (because  $p_E < p_B$ ), D and B are predicted not to exchange. Finally, Figure 1c shows the final  $p_i$  values recalculated for the resulting subnetworks.

## EXPERIMENT 1

Since the scope of our theory appears to overlap with that of Cook et al., we compare our predictions with those derived from their measure. We tested the Figure 1 network.

Based on our analysis, D-E will form an equal power dyad, the B-D relation will break, and B will have power over A and C. In contrast, Cook et al. (1986) order  $B > D > (A, C, E)$  with no breaks predicted.

### Method

Subjects were undergraduates at a large university. Before being taken to the laboratory, participants in a given session met as a group, received written instructions, and had any questions answered. In the research room, connections among network positions were clearly marked and, to limit collusion, temporary barriers separated positions among which exchange was prohibited. The setting minimally restricted the availability of information about the structure and the actions of others.<sup>12</sup>

Twenty-four counters were placed between related positions. These served as resources to be divided by mutual agreement, each valued at one profit point and worth 3 cents. Each position was limited to one agreement per round. Before starting, we emphasized that exchanges could only occur by mutual agreement between related positions, and long-term strategies were prohibited.

Experiments were organized by rounds, periods, and sessions. In all, five sessions were run, each with a different group of subjects. There were five periods per session, allowing each subject to occupy each position for one period before the session was over. Each period contained four negotiation rounds, each with a three-minute time limit. Each position's scores were announced after every round. At the close of a session, participants were paid according to points they obtained—around \$5.00 on the average. This design produced a total of 100 rounds of negotiation.

### Hypotheses

Below we present hypotheses derived from our theory, those obtained from Cook et al.'s (1986)  $D_N$  procedure, and the null hypotheses.

1. Our theory predicts that the network will break at the B-D relation, eliminating ex-

<sup>11</sup> Cook et al. had a "low profit" relation between the two end-points of the chain. While this places a lower limit on the profit that these positions can receive, it does not affect the relative power of positions in this network.

<sup>12</sup> Having information on negotiations other than one's own is expected to accelerate the use of power, but not affect relative power. For a more extended discussion of information effects, see Willer and Markovsky (1986).

Table 1. First Experiment: Profit by Position

| Session | Position |       |      |       |      | Test  |        |
|---------|----------|-------|------|-------|------|-------|--------|
|         | E        | D     | A    | B     | C    | $t^*$ | $p$    |
| 1       | 12.55    | 11.42 | 4.29 | 19.10 | 5.09 | 7.85  | <.0005 |
| 2       | 12.45    | 11.58 | 8.56 | 15.33 | 8.25 | 2.01  | <.025  |
| 3       | 12.00    | 12.00 | 3.29 | 20.95 | 3.29 | 3.50  | <.0005 |
| 4       | 12.05    | 11.95 | 3.75 | 21.55 | 3.75 | 11.15 | <.0005 |
| 5       | 11.80    | 12.20 | 4.17 | 19.16 | 4.17 | 5.77  | <.0005 |

\* The reported tests are for position B's actual profit points versus the null hypothesis of 12 profit points.

change between B and D.  $D_N$  provides no hypothesis in this regard. In contrast, if exchanges are distributed randomly in the network, B will turn to D one-third of the time, but half of those times D will turn to E. The null hypothesis, then, predicts  $.333 \times .500 \times 100 = 16.667$  exchanges between B and D.

2. B will exercise power over A and C, so B will receive more points per exchange than A and C. The  $D_N$  hypothesis also predicts  $B > (A,C)$ . The null hypothesis predicts no difference in the point accumulations of B, A, and C.

3. The GPI indicates that D and E have equal power, and so should have a 12-12 division of points.  $D_N$  predicts that D will obtain higher profits than E. Our prediction can be falsified either by  $D > E$ , as  $D_N$  predicts, or by  $E > D$ .

4. E's profits will exceed those of A and C since E is in an equipower dyad and the others are low-power positions. The  $D_N$  and null hypotheses predict no profit differences among E, A, and C.

## Results

In 100 negotiation rounds across five sessions, only three exchanges occurred between B and D.<sup>13</sup> The difference between this number and the null hypothesis of 16.667 was assessed with the  $z$ -test for proportions. The result,  $z = 3.666$ ,  $p < .0003$ , supports Hypothesis 1 and refutes the null hypothesis.

Table 1 shows the average number of points per session for each position. B clearly obtained favorable exchange rates, above 19.5 in all but one session. The  $t$ -tests show

<sup>13</sup> The three B-D exchanges occurred in three different experimental groups, on second, third, and fourth rounds. In two cases, B received 12 points, in the third, 11. This indicates that the Bs were checking their alternatives, but quickly found no reason to continue such explorations.

that in every session, B's mean profits were significantly above 12 (and, by necessity, A's and C's significantly below). The null hypothesis is rejected and Hypothesis 2 and the  $D_N$  prediction are supported.

Table 1 shows that the mean D-E exchange rates for each session differed only slightly from the 12-12 split;  $t$ -tests indicate that none of these differences was statistically significant. Therefore, Hypothesis 3 is also confirmed and the  $D_N$  hypotheses rejected.

As for Hypothesis 4, the mean point total for position E was 12.12, A's was 4.81, and C's was 4.91. Combining session means for the latter two positions and testing against E's scores,  $t = 7.522$ ,  $p < .0005$ . Hypothesis 4 is supported and the null and  $D_N$  hypotheses refuted.

In sum, this study provided strong support for the  $p(1)$  measure as tested against its null hypotheses and the revised vulnerability measure. In the next section we present  $p(e)$ , a generalization for *multi-exchange* networks, that is, networks in which actors exchange more than once per round.

## DOMAINS OF POWER AND MULTI-EXCHANGE NETWORKS

### Identifying Domains

The concept of *domain* simplifies GPI calculations in multi-exchange networks. Domains are independent subnetworks—independent in the sense that structural changes in one cannot affect power in another.

First, let  $e$  be the maximum number of *unique exchanges* that positions can make in a given round. Two exchanges are unique for  $i$  only if they involve different relations. To identify domains we will need to distinguish  $e^+$  and  $e^-$  positions:  $e^+$  positions have more than  $e$  relations, and  $e^-$  positions have  $e$  or fewer. In Figures 3–5,  $e^+$  positions are boxes,  $e^-$  positions are circles.

There are two types of domains. A *dyadic domain* is two related  $e^-$  positions. A *power domain* is a set of one or more related  $e^+$  positions, along with all  $e^-$  positions related to any member of this set. Formally,

DOMAINS: Given the set  $V$  of all positions on a path between  $i$  and  $j$ ,  $i$  and  $j$  are in the same domain if and only if there exists a path such that either (1)  $V = \{ \}$ , or (2) all members of  $V$  are  $e^+$  positions.

For example, both positions in the one-exchange network of Figure 3a are in the same domain since the set of positions ( $V$ ) on the path connecting them is empty. They form a dyadic domain. Network 3b, in which  $e = 1$ , forms a single-power domain: all pairs of positions are either related or can be reached through a path containing only  $e^+$  positions (boxes). Network 3c is also a single-power domain and, as noted earlier, no position has a structural advantage. This shows that being an  $e^+$  position is necessary but not sufficient to produce high power (Willer and Patton 1987). Network 3d also forms a single-power domain.

By comparing 3c to 3d, we can see how a change in one part of the power domain can have distal effects. Note that 3d is the 3c network with E added to the D position. In 3c, A was in an equipower relation with B. But A becomes a low-power position when E is attached. In fact, the relative power of positions in all relations in the network change when E is added.

We can draw two implications at this point. (1) If there is differential power in a domain, then there is an  $e^+$  position. This yields the useful contrapositive assertion: the absence of  $e^+$  positions implies no power differentiation. So for power to exist in a domain (or in a network, for that matter), at least one position must have an excess of available partners. (2) All one-exchange networks form single domains. The reason will be clear as we next show that when  $e > 1$ , a network can have multiple domains.

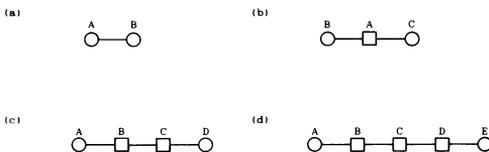


Fig. 3.  $e = 1$

When  $e > 1$ , by the unique-exchange restriction, a position can exchange  $e$  times only if it has  $e$  or more relations. Some positions—those with fewer than  $e$  relations—can have effective maxima less than  $e$ . Since  $e = 2$  in Figure 4a, for example, A can exchange twice, but B and C have effective maxima of one.

In Figure 4a, B-A-C now has two dyadic domains, (AB) and (AC); there is no core of one or more  $e^+$  positions. By the assertion given above, since there are no  $e^+$  positions, there is no power differentiation. This is reasonable since neither B nor C is excluded from exchanging with A in a given round. No position has excess exchange opportunities, and no position may garner favorable profit divisions. The same logic holds, in fact, for chains of any length, including the 4b network. This network contains four dyadic domains.

The manifestation of distal effects depends on the extent of domains. For instance, since B and C in 4a are in different domains, neither removing C nor adding new relations to C can affect B's power, and vice versa. The same is true for any two positions, e.g., B and D, lying in different domains in the 4b chain. In contrast, 4c shows that attaching F to the center of the 4b chain changes C from an  $e^-$  to an  $e^+$  position—from a circle to a box. This creates a (BCDF) domain. C now has power over B, D, and F since it can exclude one of them in each round. Attaching a new position to D would remove C's power and benefit B, further demonstrating that B and D are in the same domain.

Calculating  $p(e)$

Every position in a multi-exchange network will have a  $p$  index for each of its domains:  $p_{id}(e_d)$  is position  $i$ 's power in domain  $d$ , under the condition that  $i$  can make  $e_d$  exchanges per round within this domain.

Let  $m_{idk}$  be the number of nonintersecting paths of length  $k$  from position  $i$  in domain  $d$ ,

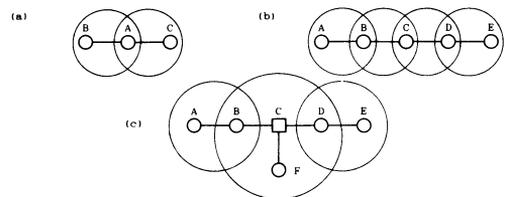


Fig. 4.  $e = 2$

and  $h$  the longest such path from  $i$  in that domain. Only paths within a domain's boundaries are counted. As illustrated in the graphs, each path begins and ends with circles, between which there are either no positions or only boxes. Position  $i$ 's GPI within the domain is<sup>14</sup>

$$p_{id}(e_d) = [1/e_d] \sum_{k=1}^h (-1)^{(k-1)} m_{idk} \quad (2)$$

$p(e)$  is closely related to  $p(1)$  and similarly calculated. Multiplying the summation by  $1/e_d$  simply places  $p(e)$  and  $p(1)$  values on the same scale.

Let us apply equation 2 (which now substitutes for Axiom 1) to network 4a, with  $e = 2$ . The two dyadic domains are indicated by (AB) and (AC) subscripts. We see that  $p_B = p_C = p_{A(AB)} = p_{A(AC)} = (1/1)(1) = 1$ . Each position has, in each of its domains, exactly one one-path and one exchange. Therefore, A has no power advantage in either of its domains. Similar results obtain in Figure 4b.

The 4c network has (AB) and (DE) dyadic domains and power domain (BCDF). Again,  $p = 1$  for members of dyadic domains. However, for the power domain we calculate  $p_{C(BCDF)} = (1/2)(3) = 3/2$ , and for B, D, and F,  $p = (1/1)(1-1) = 0$ . Thus, C has a power advantage in both of its exchanges, B and D have low power in one of their exchanges and equal power in the other, A and E have equal power in their one exchange, and F has low power in its one exchange.

We may also calculate an *average power index*,  $\bar{p}_i$ , as the mean of  $i$ 's indices across domains. In 4c,  $\bar{p}_C = 3/2$ ;  $\bar{p}_A = \bar{p}_E = 1$ ;  $\bar{p}_B = \bar{p}_D = (1 + 0)/2 = .5$ ;  $\bar{p}_F = 0$ .

The Figure 5a network is the same as Figure 2, but redrawn using the circle and box notation. When  $e = 1$ , the network is a single domain and only the Es are high-power positions. In 5b, where  $e = 2$ , the situation is drastically altered. Only D has power advantages, with the Es all having low power relative to D. Furthermore, the E-F relations form three equipower dyadic domains.

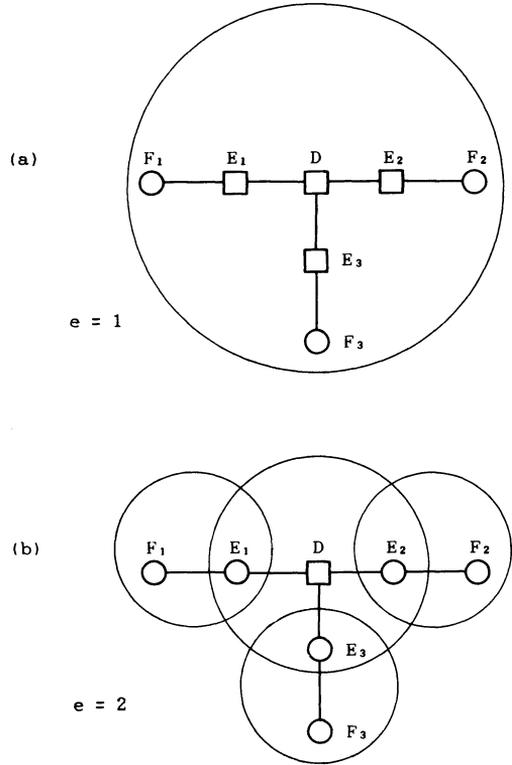


Fig. 5

The 5a and 5b networks tested the GPI generalization. The two networks have identical shapes. Only the number of exchanges per round differs. Cook et al.'s (1983) simulations found the Es to be high-power positions in this network;  $p(e)$  concurs, but only for the special case of  $e = 1$ .

EXPERIMENT 2

Experiment 2 tests the Figure 5 networks under  $e = 1$  and  $e = 2$  conditions. In spite of their identical shape, our analysis indicates that these networks should exhibit radically different profit distributions.

Method

Procedures for this experiment were similar to those used in Experiment 1. In this case, however, each subject negotiated from the different network positions under both one-exchange and two-exchange conditions, controlling for any personal characteristics of subjects that might confound the test.

Instructions for the one- and two-exchange conditions were identical, save for the

<sup>14</sup> For clarity,  $i$  subscripts have been suppressed for the  $e$  and  $h$  variables,  $d$  is suppressed for  $h$ , and  $p_{id}(e_d)$  will be written as  $p(e)$  or  $p$ . Note that Axiom 1 is now comprised of the more general equation (2).

number of exchanges allowed per round. In the two-exchange condition only, D and E could exchange with up to two different partners in the same round.

Four groups were run. Each group had seven subjects, one for each of the seven network positions. Two of the groups had the one-exchange condition first, followed by the two-exchange condition. The other two groups had the order of conditions reversed. As in the previous experiment, each subject occupied each network position over a series of four negotiation rounds. The design produced a total of 224 negotiations, 112 under each exchange condition.

After completing both parts of the experiment, subjects were paid according to the number of points they had accumulated, around \$7.00 on average.

*Hypotheses*

The following hypotheses apply to the Figure 5 networks. All are tested against the null hypotheses that every relation would average 12-12 divisions.

1. In the one-exchange condition, the Es will exercise power over the Fs and D, and so the Es will all receive higher point totals than the others.

2. In the two-exchange condition, only D will exercise power. D will obtain higher point accumulations than the Es.

3. In the two-exchange condition, Es exchange in two domains. In the power domain, they will receive unfavorable profit divisions with D. In their respective dyadic domains, they will receive 12-12 divisions with the Fs.

*Results*

Table 2a and Figure 6a show the mean number of profit points obtained by each position under the one-exchange condition. The position labels for Figure 5a are also the

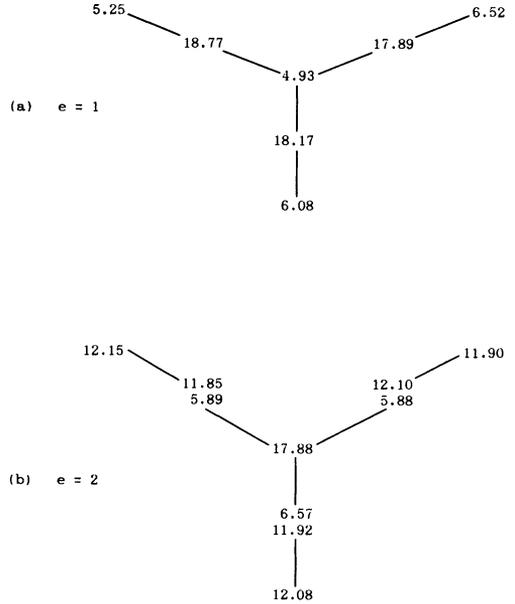


Fig. 6. Results of a Second Experiment

column headings of Table 2a. The Es clearly obtained favorable profits, around an 18-6 split on the average. The *t*-tests show that the Es' profits were significantly greater than a 12-12 split. Moreover, the Es exercised power over both D and the Fs. Hypothesis 1 is supported.

Table 2b and Figure 6b show results for the two-exchange condition. Now the power relationships have been reversed from the one-exchange condition, with the Es losing power and D gaining. As was the case for the Es under the one-exchange condition, D was able to gain approximately 18-6 profit divisions—significantly greater than the 12-12 split. Hypothesis 2 is confirmed.

Hypothesis 3 predicted equipower relations between the Es and their adjacent Fs under two-exchange conditions. The Es and the Fs should then have 12-12 profit-point divisions. As Table 2b shows, the 12-12 split was approximated. None of the differences were significant. Hypothesis 3 is also supported.

Table 2a. Second Experiment: Profit by Position, One-Exchange Condition

|             | Position |                |                |                |                |                |                |
|-------------|----------|----------------|----------------|----------------|----------------|----------------|----------------|
|             | D        | E <sub>1</sub> | E <sub>2</sub> | E <sub>3</sub> | F <sub>1</sub> | F <sub>2</sub> | F <sub>3</sub> |
| mean profit | 4.93     | 18.77          | 17.89          | 18.17          | 5.25           | 6.52           | 6.08           |
| s.d.        | 4.55     | 4.58           | 4.82           | 4.64           | 4.56           | 4.86           | 4.65           |
| <i>t</i>    | 13.65    | 15.53          | 12.91          | 13.87          | 13.65          | 10.64          | 11.66          |

Note: All tests significant at *p* < .001, 1-tailed.

Table 2b. Second Experiment: Profit by Domain and Position, Two-Exchange Condition

| <i>Power Domain</i> | D-E <sub>1</sub> | D-E <sub>2</sub> | D-E <sub>3</sub> |
|---------------------|------------------|------------------|------------------|
| mean profit         | 18.05            | 18.12            | 17.43            |
| s.d.                | 4.61             | 4.50             | 4.65             |
| <i>t</i>            | 11.66            | 11.63            | 9.77             |

Note: D's profit shown, E's profit = 24 - D's. All results significant at  $p < .001$ , 1-tailed.

| <i>Dyadic Domains</i> | E <sub>1</sub> -F <sub>1</sub> | E <sub>2</sub> -F <sub>2</sub> | E <sub>3</sub> -F <sub>3</sub> |
|-----------------------|--------------------------------|--------------------------------|--------------------------------|
| mean profit           | 12.15                          | 11.90                          | 12.08                          |
| s.d.                  | 1.46                           | 1.46                           | .73                            |
| <i>t</i>              | 1.05                           | .72                            | 1.17                           |

Note: E's profit shown, F's profit = 24 - E's. No significant test results.

In sum, this experiment provided strong support for the hypotheses testing the GPI generalization to multi-exchange networks. The presence of domains within the larger network under the two-exchange conditions strongly influenced the exchanges transpiring within those domains. As far as we know, such phenomena are not anticipated by alternative network-exchange theories.

## NEW THEORETICAL DIRECTIONS

In addition to making its predictions more precise, the formality of our theory has made it easier to develop extensions. We briefly note five that are in varying stages of development and corroboration.

### *M-Exchange Networks*

After developing the  $p(e)$  model, we discovered that, with no loss of precision, different positions may seek different maximum numbers of exchanges per round—what we call the “*M-Exchange*” condition. This admits networks in which actors may seek exchange in one, some, or all of their relations. No reformulation of the GPI is needed to deal with this extension. The analysis predicts a new class of previously unanticipated power shifts.

### *No-Round Exchange*

Allowing nonunique exchanges lets positions exchange more than once per round *within relations*. This effectively eliminates the need for exchange rounds. This is the *M-Exchange No-Round* condition. Now  $i$  may exchange up to  $e_i$  times or until those to which it is related

have exchanged up to their limits. Though this is a more complex situation, it is still true that only  $e^-$  positions can have low power and only  $e^+$  positions can have high power.

### *Resource-Pool Values*

If resource pools are different sizes in different relations, then there is another source for network breaks (Bonacich 1987b; Willer and Patton 1987). For example, in the one-exchange B-A-C network, let B and A negotiate over the division of 30 points, while A and C negotiate over 10. At first A will benefit from the bids of B and C. Eventually C will offer 9 points to A, keeping 1. Then B will offer 10 to A, keeping 20. C cannot meet this bid and still receive profit. Therefore, exchange should continue exclusively between B and A, with C excluded from the network. With the loss of C, only an A-B dyad remains and profits should reach a 15-15 split. Thus, power relations can be affected by variations in resource-pool values.

### *Flow-Networks*

So far, we have focused on exchange conditions under which resources cannot transfer across relations. We have done so primarily because this is the condition under which most of the relevant research was conducted. However, as others have indicated (Marsden 1983; Bonacich 1987a), it is worthwhile to relax this restriction and consider networks with transferable resources—those in which resources may flow through positions.

A consequence of extending into the realm of flow-networks is that positions may have power over others to which they are not related, depending on the initial distributions of resources and on which actors seek which resources. This is similar to Marsden's view. The foremost difference between that view and our approach is that we incorporate explicit assumptions about individual negotiation strategies and the conditions of exchange—factors that affect exchange outcomes, breaks, and domains.

### *Positive Connection*

While negative connections place an upper limit on the number of exchanges in which a

“hub” position may engage per round, positive connections place a lower limit on the number of exchanges in which the position must engage to realize a profit (Patton 1986). An example is the manufacturer who must obtain *all* components for a synthetic product before that product becomes a viable source of revenue. New research shows that the exchange dynamics that occur in positive connections differ markedly from those in negatively connected networks, and power advantages belong to peripheral positions in branches such as B-A-C (Patton and Willer 1987). This work on positive connections only begins to uncover a range of phenomena at least as broad and interesting as those associated with negative connections.

## CONCLUSION

Our findings indicate that by only focusing on the effects of networks *per se*, alternative network theories do not recognize that power and resource distributions depend as much on prevailing exchange conditions as they do on configurations of positions and relations. We introduced a model that considers both structural form and exchange conditions, anticipating and explaining such phenomena as relative power, network breakage, power reversals and domain-specific effects. The studies that we described are only the first of many that could investigate stability and instability in exchange networks.

Future developments aside, we have found the present incarnation of the theory quite useful for understanding many real-world power struggles in exchange networks—from international disputes over geographical control to toddlers’ negotiations over the sharing of playthings. Whatever the application, the theory directs us to specify the relevant actors and resources, identify other pertinent relations in which the actors are engaged, observe who seeks exchange with whom, identify which actors risk exclusion from valued resources, consider temporal constraints such as ultimatums or deadlines that create exchange rounds and, in general, determine the

extent to which the exemplar departs from the idealized scope conditions of the theory.

Our work also has implications for two very general questions that are relevant to structural approaches: (1) what is the appropriate unit of analysis for structural theories; and (2) how are characteristics of structures and the social units within them mutually determined?

Regarding the first question, we eschew the designation of one unit of analysis as, in general, more or less appropriate than another. Our theory explains certain actor and network behaviors. In any given instance, the network may be an organization, as may the actor. It follows that actors may or may not be individual persons. All that matters is that the units considered have the necessary properties. Therefore, no unit of analysis is generally most appropriate for structural approaches.

We can offer no universal solution to the question of how social structures and constituent units each determine properties of the other. Our approach does, however, point to *excludability* as a linchpin securing individual and network realms. That is, structures and exchange conditions at times bar some actors from procuring the resources they value and desire. Thus, power *happens* to those whose positions allow them to dodge the struggle to avoid exclusion.

As the foregoing review of extensions-in-progress implies, we do not claim that our theory is finished or unimprovable. Nor do we claim that it explains all phenomena within the purview of alternative formulations. It is, however, consistent with the findings of *all* previous experimental research on exchange networks. Moreover, it addresses a range of conditions and generates predictions that are either beyond the range of alternative formulations or simply contradict them, depending upon how one interprets their scope. Our long-term goal is to continue incremental extensions and systematic tests of increasingly refined network-exchange models.

## APPENDIX

*The Axiomatic Theory*Symbols:

$i, j$  : actors in relation  $i \rightarrow j$   
 $V$  : all positions related to  $i$ , other than  $j$   
 $Z$  : all positions related to  $j$ , other than  $i$   
 $v$  : a member of  $V$   
 $z$  : a member of  $Z$   
 $r_{ij}$  : resources received by  $i$  from  $j$  from an exchange  
 $p_i$  :  $i$ 's power index  
 $p_{ij}$  :  $p_i - p_j$   
 $E_{ij}$  :  $i$  and  $j$  exchange  
 $S_{ij}$  :  $i$  seeks exchange with  $j$   
 $k$  : empirical constant

Logical Operators:

$x \& y$  : conjunction ("x and y")  
 $x \text{ or } y$  : inclusive disjunction ("x and/or y")  
 $\neg x$  : negation ("not x")  
 $x \rightarrow y$  : implication ("If x, then y")  
 $x \leftrightarrow y$  : biconditional ("x if and only if y")  
 $(x)$  : universal quantifier ("For all x such that ...")  
 $(\exists x)$  : existential quantifier ("There is an x such that ...")

Scope Conditions for Relations

$SC_5.$   $(i)(j)(\exists v)(S_{iv} \text{ or } S_{ij})$   
 $SC_6.$   $(i)(j)[E_{ij} \rightarrow (r_{ij} + r_{ji} = k)], k > 0;$   
 $(i)(j)[\neg E_{ij} \rightarrow (r_{ij} + r_{ji} = 0)]$   
 $SC_7.$   $(i)(j)[E_{ij} \leftrightarrow [(r_{ij} > 0) \& (r_{ji} > 0)]]$

Axioms

$A_1.$   $(i)(p_i = \dots)$  (see equations 1 and 2 in text)  
 $A_2.$   $(i)(j)[S_{ij} \leftrightarrow [(p_{ij} > p_{ji}) \text{ or } (v)(p_{ij} \geq p_{iv})]]$   
 $A_3.$   $(i)(j)[E_{ij} \rightarrow (S_{ij} \& S_{ji})]$   
 $A_4.$   $(i)(j)[E_{ij} \rightarrow [(p_i > p_j) \leftrightarrow (r_{ij} > r_{ji})]]$

Theorems

$T_1.$   $(i)(j)[(V = \{ \}) \rightarrow S_{ij}]$   
 $T_2.$   $(i)(j)[(\neg S_{ij} \text{ or } \neg S_{ji}) \rightarrow \neg E_{ij}]$   
 $T_3.$   $(i)(j)[\neg S_{ij} \leftrightarrow [(p_{ij} \leq p_{ji}) \& (\exists v)(p_{ij} < p_{iv})]]$   
 $T_4.$   $(i)(j)[[(p_{ij} \leq p_{ji}) \& (\exists v)(p_{ij} < p_{iv})] \text{ or } [(p_{ji} \leq p_{ij}) \& (\exists z)(p_{ji} < p_{iz})] \rightarrow \neg E_{ij}]$

Proofs\*Theorem 1

$(1)$   $V = \{ \}$  premise (P)  
 $(2)$   $\neg S_{iv}$  (1), definition of  $V$   
 $(3)$   $\neg(\exists v)S_{iv}$  (2), Interchange of Quantifiers (IQ)  
 $(4)$   $(\exists v)(S_{iv} \text{ or } S_{ij})$   $SC_5$

-----  
 $S_{ij}$  (3), (4), disjunctive syllogism

\* For clarity, most universal quantifiers have been suppressed.

Theorem 2

|  |                                     |
|--|-------------------------------------|
| (1) $\neg S_{ij}$ or $\neg S_{ji}$                           | P                                   |
| (2) $\neg(S_{ij} \ \& \ S_{ji})$                             | (1), DeMorgan's Law (DL)            |
| (3) $E_{ij} \ \text{---} \rightarrow (S_{ij} \ \& \ S_{ji})$ | A <sub>3</sub>                      |
| -----  |                                     |
| $\neg E_{ij}$  | (2), (3), <u>modus tollens</u> (MT) |

Theorem 3

|   |   |
|---|---|
| (1) $\neg S_{ij}$   | P                                       |
| (2) $[(p_{ij} > p_{ji}) \ \text{or} \ (\forall)(p_{ij} \geq p_{iv})] \ \text{---} \rightarrow S_{ij}$ | A <sub>2</sub> , Biconditional Law (BL) |
| (3) $\neg[(p_{ij} > p_{ji}) \ \text{or} \ (\forall)(p_{ij} \geq p_{iv})]$                             | (1), (2), MT                            |
| (4) $\neg(p_{ij} > p_{ji}) \ \& \ \neg(\forall)(p_{ij} \geq p_{iv})$                                  | (3), DL                                 |
| (5) $\neg(p_{ij} > p_{ji})$   | (4), Law of Simplification              |
| (6) $\neg(\forall)(p_{ij} \geq p_{iv})$   | (4), Law of Simplification              |
| (7) $p_{ij} \leq p_{ji}$  | (5), Law for Inequalities               |
| (8) $(\exists v)(p_{ij} < p_{iv})$  | (6), IQ                                 |
| -----   |   |
| $(p_{ij} \leq p_{ji}) \ \& \ (\exists v)(p_{ij} < p_{iv})$  | (7), (8), Law of Adjunction             |

Theorem 4

|   |  |
|---|--|
| (1) $[(p_{ij} \leq p_{ji}) \ \& \ (\exists v)(p_{ij} < p_{iv})] \ \text{or} \ [(p_{ji} \leq p_{ij}) \ \& \ (\exists z)(p_{ji} < p_{jz})]$   | P  |
| (2) $(p_{ij} \leq p_{ji}) \ \text{---} \leftrightarrow \neg(i)\neg(j)(p_{ij} > p_{ji})$   | Law for Inequalities (LI),<br>Double Negation (DN) |
| (3) $(p_{ji} \leq p_{ij}) \ \text{---} \leftrightarrow \neg(j)\neg(i)(p_{ji} > p_{ij})$   | LI, DN   |
| (4) $(\exists v)(p_{ij} < p_{iv}) \ \text{---} \leftrightarrow \neg(\forall)(p_{ij} \geq p_{iv})$   | LI, DN, IQ   |
| (5) $(\exists z)(p_{ji} < p_{jz}) \ \text{---} \leftrightarrow \neg(\forall)(p_{ji} \geq p_{jz})$   | LI, DN, IQ   |
| (6) $[\neg(p_{ij} > p_{ji}) \ \& \ \neg(p_{ij} \geq p_{iv})] \ \text{or} \ [\neg(p_{ji} > p_{ij}) \ \& \ \neg(p_{ji} \geq p_{jz})]$   | (2), (3), (4), (5) substituted<br>into (1)         |
| (7) $[\neg(p_{ij} > p_{ji}) \ \& \ \neg(p_{ij} \geq p_{iv})] \ \text{---} \rightarrow \neg[(p_{ij} > p_{ji}) \ \text{or} \ (p_{ij} \geq p_{iv})]$   | DL   |
| (8) $[\neg(p_{ji} > p_{ij}) \ \& \ \neg(p_{ji} \geq p_{jz})] \ \text{---} \rightarrow \neg[(p_{ji} > p_{ij}) \ \text{or} \ (p_{ji} \geq p_{jz})]$   | DL   |
| (9) $\neg[(p_{ij} > p_{ji}) \ \text{or} \ (p_{ij} \geq p_{iv})] \ \text{or} \ \neg[(p_{ji} > p_{ij}) \ \text{or} \ (p_{ji} \geq p_{jz})]$   | (6), (7), (8)                                      |
| (10) $S_{ij} \ \text{---} \leftrightarrow [(p_{ij} > p_{ji}) \ \text{or} \ (p_{ij} \geq p_{iv})]$   | A <sub>2</sub>                                     |
| (11) $S_{ji} \ \text{---} \leftrightarrow [(p_{ji} > p_{ij}) \ \text{or} \ (p_{ji} \geq p_{jz})]$   | A <sub>2</sub>                                     |
| (12) $(S_{ij} \ \& \ S_{ji}) \ \text{---} \leftrightarrow \{[(p_{ij} > p_{ji}) \ \text{or} \ (\forall)(p_{ij} \geq p_{iv})] \ \& \ [(p_{ji} > p_{ij}) \ \text{or} \ (\forall)(p_{ji} \geq p_{jz})]\}$ | (10), (11)   |
| (13) $\neg(S_{ij} \ \& \ S_{ji})$   | (9), (12)  |
| (14) $\neg S_{ij}$ or $\neg S_{ji}$   | (13), DL   |
| -----   |  |
| $\neg E_{ij}$   | (14), T <sub>2</sub>                               |

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