

INDIVISIBLE LABOR, LOTTERIES AND EQUILIBRIUM

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This paper considers an economy where labor is indivisible and agents are identical. Although the discontinuity in labor supply at the individual level disappears as a result of aggregation, it is shown that indivisible labor has strong consequences for the aggregate behavior of the economy. It is also shown that optimal allocations involve lotteries over employment and consumption.

1. Introduction

During the last decade general equilibrium theory has become an increasingly common framework in which to study the aggregate properties of economies [see, e.g., Lucas (1981)]. In its original form [see Debreu (1959)] general equilibrium theory was developed in the context of convex economic environments, but has since been extended to non-convex environments [see Aumann (1966) and Mas-Colell (1977)]. The concern of these authors was the existence of equilibrium. They show that if there is a continuum of agents, then equilibrium exists even with non-convexities at the individual level. Their analysis left unanswered the question of whether or not these non-convexities had an important effect on the nature of equilibrium. The conclusion of this paper is that they may have major implications for the aggregate response of an economy to shocks. This claim is demonstrated in the context of an economy in which labor supply is indivisible. In particular it is shown that such an economy composed of identical agents behaves as if populated by a single agent whose preferences do not match the preferences of any individual in the economy. Furthermore the economy will display much larger fluctuations in hours of work in response to a given shock to technology.

The analysis also shows that attaining optimal allocations in such economies may involve holding lotteries over employment to determine which agents

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supply labor. It is demonstrated how these lotteries may be decentralized through markets and how the use of lotteries facilitates computation of equilibria. Prescott and Townsend (1984, 1985) found similar properties in economies with private information.

Although this paper only considers the case of indivisible labor, there are many other situations of practical importance where similar non-convexities may play an important role. Markets for durables such as houses or cars involve an element of indivisibility that may play an important role in understanding aggregate fluctuations in these markets. Decisions about mobility, marriage, fertility and occupational choice also involve a choice which is essentially a zero-one choice, and hence, the same types of considerations studied in this paper will be of interest in analysing those problems. Becker (1985) shows how human capital accumulation may lead to non-convexities.

2. The environment E

In this and the next three sections the problem to be studied is considered in its simplest form.

The economy consists of a continuum of identical agents with names in the interval $[0, 1]$. There are three commodities: labor, capital and output (although, as will become clear later, capital only serves the role of allowing for a constant returns to scale technology together with a diminishing marginal product of labor). All activity takes place in a single time period. Capital (K) and labor (N) are used to produce output according to the concave constant returns to scale technology $f(K, N)$, which is assumed to be strictly increasing and twice continuously differentiable in both arguments with $f_{11}(K, N)$ and $f_{22}(K, N)$ strictly negative.

Each agent (or worker) is endowed with one unit of time and one unit of capital. Time is indivisible: either the entire unit is supplied as labor or none of it is supplied as labor. All workers have an identical utility function specified by

$$u(c) - v(n),$$

where $c \geq 0$ is consumption and $n \in \{0, 1\}$ is supply of labor. It is assumed that u is twice continuously differentiable, increasing and strictly concave. If labor were perfectly divisible it would be natural to assume that $v(n)$ is convex, increasing and twice continuously differentiable. With labor assumed to be indivisible, the only values of $v(n)$ that matter are $v(0)$ and $v(1)$. It is assumed that $v(0) = 0$ and $v(1) = m$, where m is a strictly positive constant. For future reference it will be useful to define the consumption set X for each worker. According to the above specification,

$$X = \{(c, n, k) \in R^3: c \geq 0, n \in \{0, 1\}, 0 \leq k \leq 1\}.$$

It is of particular interest that the set X is non-convex. The above description will be referred to as the economy E .

3. Equilibrium and optimality in E

This section presents the standard notion of competitive equilibrium for E and displays some anomalies. First we define an allocation.

Definition. An allocation for E is a list $(c(t), n(t), k(t), K, N)$, where for each $t \in [0, 1]$, $(c(t), n(t), k(t)) \in X$ and $K, N \geq 0$.

Definition. A competitive equilibrium for E is a list $(c(t), n(t), k(t), K, N, w, r)$ such that:

(i) For each $t \in [0, 1]$, $(c(t), n(t), k(t))$ is a solution to

$$\max_{c, n, k} u(c) - mn,$$

subject to

$$c \leq nw + rk, \quad c \geq 0, \quad n \in \{0, 1\}, \quad 0 \leq k \leq 1.$$

(ii) N and K are a solution to

$$\max_{N, K} f(K, N) - rK - wN,$$

subject to

$$K \geq 0, \quad N \geq 0.$$

(iii)

$$K = \int_0^1 k(t) dt,$$

$$N = \int_0^1 n(t) dt,$$

$$f(K, N) = \int_0^1 c(t) dt.$$

The above definition is standard. Conditions (i)–(iii) are, respectively, utility maximization, profit maximization and market clearing.

It is possible to show that a competitive equilibrium exists for E , however this is not central to the discussion here. The interesting feature of E that

distinguishes it from purely neoclassical economies is the indivisibility in labor supply. One of the implications of this feature is that it is possible for identical agents to receive different allocations in equilibrium. If consumption and technology sets are convex and preferences are strictly convex, then in equilibrium identical agents always receive identical allocations. This result follows from the fact that budget sets are convex and hence if two distinct points each in the budget set give equal utilities then there necessarily exists a third point also in the budget set providing a greater level of utility. In the economy E being considered here the consumption set X is not convex and hence this argument no longer holds. As the following example demonstrates, neither does the result that identical agents receive identical bundles.

Example 1. Consider the following specification for E :

$$f(K, N) = K^\alpha N^{1-\alpha}, \quad \alpha = 0.5.$$

$$u(c) = \ln c,$$

$$m = \ln 2.5.$$

The only variable to be solved for is N : all capital will necessarily be supplied, the consumption of those supplying labor will be $(w+r)$ and of those not supplying labor it will be r ; also, w is the *MPL* and r is the *MPK*. All workers must receive the same utility even if they receive different allocations. So solving the following equation will determine the equilibrium:

$$u(MPL + MPK) - m = u(MPK),$$

which, if $K = 1$, is simply an equation in N . For the given functional forms this becomes

$$\ln [\alpha N^{1-\alpha} + (1-\alpha)N^{-\alpha}] - m = \ln [\alpha N^{1-\alpha}].$$

Solving this equation gives

$$N = \frac{1-\alpha}{\alpha} \frac{1}{e^m - 1}.$$

The equilibrium for the above specification is

$$(c(t), n(t), k(t)) = (1.1067972, 1, 1), \quad t \in [0, 0.4],$$

$$= (0.3162278, 0, 1), \quad t \in [0.4, 1],$$

$$(w, r) = (0.7905694, 0.3162278).$$

One of the surprising features of this equilibrium is that identical agents are receiving different allocations. Another surprising feature is that there are allocations which Pareto-dominate the above allocation. In particular, consider the following allocation rule. Give every individual a consumption of

$$c = 0.6324555,$$

(which is the average consumption in the previous example) but hold a lottery to randomly choose a fraction 0.4 of workers to supply labor. The utility obtained by an individual in the equilibrium allocation is given by

$$V = 0.5623413,$$

whereas the expected utility of the alternative allocation described above is

$$\bar{V} = 0.5993895.$$

At first this result seems to be troubling. As commonly stated [see, e.g., Debreu (1959, ch. 6)] the first welfare theorem applies to the economy E . The solution to this apparent logical inconsistency is that the alternative allocation described above does not belong to the set X and hence is outside the scope of the set of allocations considered by the first welfare theorem. Recall that the standard method of proof for the first welfare theorem involves an argument that if individuals prefer an allocation to their individual allocation then it must cost too much relative to their budget or else they would have purchased it. This argument does not hold here because the allocation involving lotteries is not viewed as a feasible one by consumers with consumption set X . However, the result is still troubling because it suggests that not all gains to trade are being realized even though apparently all markets are in operation. In the remainder of this paper it is shown how the consumption set X can be modified so that allocations like the one described above can be obtained as a competitive allocation.

4. Equilibrium with lotteries

In this section the consumption set is expanded by introducing a specific class of lotteries. All objects will remain the same except for the consumption sets and preferences. Define:

$$X_1 = \{(c, n, k) \in X: n = 1\},$$

$$X_2 = \{(c, n, k) \in X: n = 0\},$$

$$\bar{X} = X_1 \times X_2 \times [0, 1].$$

\bar{X} will be the new consumption set for the workers. The set X_1 represents allocations where the individual is supplying labor and the set X_2 represents allocations where the individual is not supplying labor. Motivated by example one and the discussion that followed, it is desirable to allow workers to randomize the labor supply decision. Hence the third element, a number in the interval $[0, 1]$ represents the probability that the X_1 allocation is realized, whereas one minus this number is the probability that the X_2 allocation is realized. Note that the set \bar{X} is convex. An element of \bar{X} will be written $((c_1, 1, k_1), (c_2, 0, k_2), \phi)$. Preferences must be defined over this set and the natural extension is to compute the expected utility of the lottery, i.e., the utility obtained from receiving the above allocation is

$$\phi[u(c_1) - m] + (1 - \phi)[u(c_2)].$$

The economy produced by making these changes will be denoted by \bar{E} and a competitive equilibrium for \bar{E} is defined by:

Definition. A competitive equilibrium for \bar{E} is a list $(c_1(t), k_1(t), c_2(t), k_2(t), \phi(t), K, N, w, r)$ such that:

(i) For each $t \in [0, 1]$, $(c_1(t), k_1(t), c_2(t), k_2(t), \phi(t))$ is a solution to

$$\max_{c_1, c_2, k_1, k_2, \phi} \phi[u(c_1) - m] + (1 - \phi)[u(c_2)],$$

subject to

$$\phi c_1 + (1 - \phi)c_2 \leq w\phi + r[\phi k_1 + (1 - \phi)k_2],$$

$$c_i \geq 0, \quad 0 \leq k_i \leq 1, \quad i = 1, 2,$$

$$0 \leq \phi \leq 1.$$

(ii) K and N are a solution to

$$\max_{K, N} f(K, N) - rK - wN.$$

subject to

$$K \geq 0, \quad N \geq 0.$$

(iii)

$$K = \int_0^1 (\phi(t)k_1(t) + (1 - \phi(t))k_2(t)) dt,$$

$$N = \int_0^1 (\phi(t)) dt,$$

$$f(K, N) = \int_0^1 (\phi(t)c_1(t) + (1 - \phi(t))c_2(t)) dt.$$

Although the nature of the above three conditions is standard there is a non-standard element imbedded in them. This arises because individuals are buying and selling commodities contingent upon the outcome of an individual specific lottery. It is worthwhile to discuss possible institutional descriptions corresponding to the formal notion of equilibrium described above. A description of a fully decentralized equilibrium is as follows. The prices of output, labor, and capital are given by 1, w , and r , respectively. An individual chooses a lottery where with probability ϕ they work and supply k_1 units of capital and with probability $(1 - \phi)$ they don't work and supply k_2 units of capital. Hence, with probability ϕ they will receive income $w + k_1r$, and with probability $(1 - \phi)$ they will receive income k_2r . It is assumed that the individual can purchase insurance in the face of this income uncertainty. In particular, the individual can purchase consumption contingent upon the outcome of the lottery. Assuming a zero profit condition for the firm offering this insurance implies that relative prices between the work and don't work outcomes will be given by $(\phi/(1 - \phi))$. Hence the budget constraint is simply given by that which appears in condition (i) of the definition.

It is also worth noting that it is implicitly assumed that the wage rate that an individual faces is independent of the probability ϕ of working that is chosen. This occurs because there is a continuum of agents and each individual is choosing an individual specific random variable. Under these conditions a given agent's decision has no impact on the distribution of total labor supplied. In particular there is no difference between the case where all workers supply labor with probability one-half and the case where half the workers supply labor with probability one and the other half supply labor with probability zero. This is not the case with a finite number of workers: the two situations provide the same expected value of labor supply but the variance is different. In this case we might expect that the wage rate will depend on the value of ϕ chosen. With a continuum of agents the above mentioned variance is always zero.¹

Call the maximization problem in condition (i) of equilibrium problem (P-1).

Lemma 1. If $(c_1, c_2, k_1, k_2, \phi)$ is a solution to (P-1) and $\phi \in (0, 1)$, then $c_1 = c_2$.

Proof. The first-order conditions for this problem are

$$\phi u'(c_1) = \phi \theta \quad \text{and} \quad (1 - \phi) u'(c_2) = (1 - \phi) \theta,$$

¹ The case of a continuum of i.i.d. random variables can cause some problems. See Judd (1985) for a treatment of this problem.

where θ is the multiplier on the budget constraint. If ϕ is not zero or one then the result follows immediately. ■

Note that if $\phi \notin (0, 1)$, there is no harm in requiring that $c_1 = c_2$. We can also assume that $k_1 = k_2 = 1$. Then problem (P-1) becomes

$$(P-2) \max_{c, \phi} u(c) - \phi m,$$

subject to

$$c = w\phi + r, \quad c \geq 0, \quad 0 \leq \phi \leq 1.$$

Note that by the strict concavity of $u(\cdot)$ this problem has a unique solution. Since all agents are identical it follows that c and ϕ are independent of t . Hence, finding an equilibrium now reduces to finding a list (c, ϕ, K, N, r, w) such that

- (i) (c, ϕ) solves problem (P-2).
- (ii) (K, N) solves profit maximization problem.
- (iii) $\phi = N, K = 1, c = F(K, N)$.

This is identical to the equilibrium one would obtain for an economy with technology $f(K, N)$, with one agent whose utility is specified by $u(c) - mn$ with consumption set

$$X = \{(c, n, k) \in R^3: c \geq 0, 0 \leq n \leq 1, 0 \leq k \leq 1\}.$$

This economy is entirely neoclassical; in particular it has no non-convexity. Define the problem

$$(P-3) \max_{c, \phi} u(c) - m\phi,$$

subject to

$$c \leq f(1, \phi), \quad c \geq 0, \quad 0 \leq \phi \leq 1.$$

This is the social planning problem for \bar{E} which maximizes utility. Because this economy now appears identical to one without any non-convexities, the standard results on equivalence of competitive and optimal allocations can be applied [see, e.g., Negishi (1960)].

Proposition 1. If $(c^*, \phi^*, K^*, N^*, w^*, r^*)$ is a competitive equilibrium for \bar{E} , then (c^*, ϕ^*) is the solution to problem (P-3).

Proposition 2. If (c^*, ϕ^*) is the solution to problem (P-3), then there exists K^*, N^*, w^* , and r^* such that $(c^*, \phi^*, K^*, N^*, w^*, r^*)$ is a competitive equilibrium for \bar{E} .

Since (P-2) is a strictly concave programming problem, the above two results imply the existence of a unique equilibrium.

One of the reasons for adding lotteries to the consumption set was the potential gain in welfare. In essence, making labor indivisible creates a barrier to trade and the introduction of lotteries is one way to overcome part of this barrier. It should be noted that adding lotteries of the type considered here to an environment similar to E but without the indivisibility in labor would have no effect on equilibrium. Finally, in Example 1 it was demonstrated that an allocation involving lotteries Pareto-dominated the equilibrium allocation for E . That this result is general should be clear, but a formal statement is:

Proposition 3. If $(c^*(t), n^*(t), k^*(t), K^*, N^*)$ is an equilibrium allocation for E and $(\bar{c}_1(t), \bar{k}_1(t), \bar{c}_2(t), \bar{k}_2(t), \bar{\phi}(t), \bar{K}, \bar{N})$ is an equilibrium allocation for \bar{E} , then

$$u(c^*(t)) - n^*(t)m \leq \bar{\phi}(t)[u(\bar{c}_1(t)) - m] + (1 - \bar{\phi}(t))u(\bar{c}_2(t)),$$

with strict inequality if $n^*(t)$ is not constant for all t .

Proof. Define

$$\bar{\phi} = \int_0^1 n^*(t) dt \quad \text{and} \quad \bar{c} = \int_0^1 c^*(t) dt.$$

Define an allocation for \bar{E} as follows:

$$c_1(t) = c_2(t) = \bar{c}, \quad \text{all } t,$$

$$\phi(t) = \bar{\phi}, \quad \text{all } t,$$

$$k_1(t) = k_2(t) = 1, \quad \text{all } t,$$

$$K = K^*, \quad N = N^*.$$

This allocation is clearly feasible, and by definition of an equilibrium for E

$$\begin{aligned} u(c^*(t)) - n^*(t)m &= \int_0^1 (u(c^*(t)) - n^*(t)m) dt \\ &\leq u\left(\int_0^1 c^*(t) dt\right) - m \int_0^1 n^*(t) dt = u(\bar{c}) - \bar{\phi}m, \end{aligned}$$

where the inequality follows from Jensen's inequality and is strict if $c(t)$ is not constant. Note that the right-hand side is simply the utility resulting from one particular feasible allocation in \bar{E} . By Proposition 1, the equilibrium for \bar{E} must result in utility at least this large. ■

5. Stochastic environments and computation

The previous analysis continues to hold for stochastic environments. Suppose that there is a random variable s taking values in a finite subset S of R^N . Let s_i index the realizations of s and p_i be the probability that $s = s_i$. In state i assume that preferences are given by

$$u(c, s_i) - m(s_i)n,$$

and technology is given by

$$f(K, N, s_i),$$

where these functions are assumed to have the same properties as before for each value of s_i . If lotteries are introduced then optimal allocations and equilibrium allocations are given by:

$$\max_{c_i, \phi_i} \sum_i p_i (u(c_i, s_i) - m(s_i)\phi_i),$$

subject to

$$0 \leq c_i \leq f(1, \phi_i, s_i), \quad 0 \leq \phi_i \leq 1.$$

Note that this is equivalent to solving the following problem separately for each value of i :

$$\max u(c_i, s_i) - m(s_i)\phi_i,$$

subject to

$$0 \leq c_i \leq f(1, \phi_i, s_i), \quad 0 \leq \phi_i \leq 1.$$

This property generally applies to all static models with homogeneous agents in convex environments but will not hold for the case of indivisible labor in the absence of lotteries.

To see this assume that S contains two elements. Computing separate equilibria for the two realizations produces vectors (c_w^i, c_n^i, ϕ^i) , where c_w^i is consumption for individuals who work in state i , c_n^i is consumption for individuals who don't work in state i , and ϕ^i is the fraction of individuals who

work in state i . In equilibrium it must be that

$$u(c_w^i, s_i) - m(s_i)n = u(c_n^i, s_i).$$

Take two individuals, one who works in state 1 but not state 2 and one who works in state 2 but not in state 1. (There is always an equilibrium of this form.) These two individuals will have uneven consumption streams across states of nature. In state 1 the first individual consumes relatively more and in state 2 the second individual consumes relatively more. Because utility is concave in consumption these individuals can become better off by trading claims for state i consumption. Hence the above cannot be an equilibrium. If there are M states of nature, this implies that there are $2M$ markets which need to be operated, one each for labor and output in each state. And because of the discrete choice in labor supply the aggregate demands will generally be correspondence, not functions. It appears that even simple stochastic versions of the indivisible labor economy will be excessively demanding computationally in the absence of lotteries.

This is not to imply that models which are easier to compute are inherently better. Rather, the point being made is that the preceding analysis suggests that our understanding of non-convexities will be facilitated by assumptions like this that facilitate computation even if ultimately a more refined or sophisticated notion of equilibrium is to be adopted for non-convex environments.

6. Implications for aggregate fluctuations

A recurring problem in attempts to produce equilibrium models of aggregate fluctuations has been the inability of these models to account for observed relative magnitudes of fluctuations in total labor supply and real wages. [For example, see Altonji and Ashenfelter (1980), Kydland and Prescott (1982).] In particular, the estimates of the elasticity of labor supply found using micro data are much smaller than that required to reconcile aggregate fluctuations with equilibrium theory. This paper demonstrates that non-convexities may be of substantial interest for this problem.

Consider the alternative specification of preferences for E :

$$u(c) - v(n), \quad c \geq 0, \quad n \in \{1, 0\}, \quad v(0) = 0.$$

From problem (P-3) we have the result that if labor is indivisible this economy behaves as though there is a single agent with preferences given by

$$u(c) - m \cdot n.$$

Hence there is a discrepancy between the true preferences of agents and the preferences of the hypothetical representative consumer generating aggregate fluctuations. In particular, the second of these has preferences linear in n , indicating a higher elasticity of labor supply.

Hansen (1985) has shown that this feature has implications which are empirically relevant. Whereas many other individuals have found that movements in aggregate hours are too small relative to movements in real wages of productivity, Hansen's indivisible labor economy delivers too much movement in aggregate hours relative to real wages and productivity. It is important to know that the results of this paper do not depend critically upon the assumption of identical agents. It may be thought that having all agents simultaneously being indifferent between working and not working is what causes the large response in employment relative to productivity. A parametric example is offered to illustrate that heterogeneity need not affect the results of this paper. The important feature of the example is that each agent has a different reservation wage. Consider the following specification: there is a continuum of agents with total mass equal to one. Each agent has a utility function of the form

$$C^\alpha - m_i l^\beta,$$

where C is consumption, l is labor supply, $0 < \alpha < 1$, $\beta > 1$, and m_i is an individual specific parameter. The values of m are uniformly distributed on an interval $[\underline{m}, \bar{m}]$. If the wage rate is W and labor is divisible, then individual labor supply is given by

$$l_i = \left(\frac{W^\alpha \cdot \alpha}{m_i \beta} \right)^{1/\beta - \alpha}.$$

Integrating over $[\underline{m}, \bar{m}]$ to obtain aggregate labor supply gives

$$l = \left(\frac{W^\alpha \cdot \alpha}{\beta} \right)^{1/\beta - \alpha} \cdot \frac{1}{\gamma} \left(\frac{\bar{m}^\gamma - \underline{m}^\gamma}{\bar{m} - \underline{m}} \right),$$

where

$$\gamma = 1 - (1/\beta - \alpha).$$

In logs this gives

$$\ln l = \frac{\alpha}{\beta - \alpha} \ln W + \text{constant}.$$

If it is assumed that labor is indivisible and agents trade lotteries as outlined in section 4, then the labor supply of individual i is given by

$$\phi_i = \left(\frac{W^\alpha \alpha}{m_i} \right)^{1/1-\alpha}.$$

Integrating gives

$$\phi = (W^\alpha \alpha)^{1/1-\alpha} \cdot \frac{1}{\gamma} \left(\frac{\bar{m}^\gamma - \underline{m}^\gamma}{\bar{m} - \underline{m}} \right),$$

where

$$\gamma = \frac{1}{1-\alpha} + 1.$$

In logs this gives

$$\ln \phi = \frac{\alpha}{1-\alpha} \ln W + \text{constant}.$$

As can be seen by comparison of these two expressions, the case where labor supply is indivisible produces a slope which is larger. In fact, in this example the heterogeneity has no impact on the elasticity of labor supply. Note that the elasticity in the indivisible case is found by setting β equal to one in the corresponding expression for the divisible labor case. Hence, the same result concerning linearity holds in this case.

Care should be taken not to misinterpret this result. It is derived in the context of a static deterministic environment. Computing equilibrium allocations for dynamic stochastic environments (like that of Hansen) is ultimately the object of interest, but is too difficult a task to be undertaken here for the case of heterogeneous consumers.

If there were no lotteries in the economy with indivisible labor, then each agent simply decides whether or not to work. Because the m_i 's differ across agents, this decision will differ across agents. In particular, an agent supplies labor if $W^\alpha - m_i > 0$ and doesn't supply labor if $W^\alpha - m_i < 0$. When equality holds the individual is indifferent. In this economy each individual has a different reservation wage which causes them to enter the market, given by $\bar{w}_i = m_i^{1/\alpha}$. Note that in the economy with lotteries individuals always have the choice of $\phi_i = 1$ or $\phi_i = 0$, so implicitly it follows that lotteries are improving welfare.

7. Conclusion

This paper has analyzed the problem of indivisible labor in an economy with identical individuals.

The main conclusion of this paper is that non-convexities at the individual level may have important aggregate effects even if there are a large number of individuals. In the case studied here, the aggregate economy behaves as if there were no non-convexities but all individuals have preferences which are linear in leisure even though no individual in the economy has such preferences.

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