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Author(s): Gary E Bolton

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# A Comparative Model of Bargaining: Theory and Evidence

By GARY E BOLTON\*

*Recent laboratory studies of alternating-offer bargaining find many empirical regularities that are inconsistent with the standard theory. In this paper, I postulate that bargainers behave as if they are negotiating over both "absolute" and "relative" money. Absolute money is measured by cash, relative money by the disparity between absolute measures. The resulting model is consistent with previously observed regularities. New experiments provide further support as well as evidence against several alternative explanations. Also finding some support is an extension which predicts that the equilibrium of the standard theory will be observed when bargaining is done in a "tournament" setting. (JEL C78, C92)*

A controversy has developed over what role, if any, "fairness" plays in laboratory alternating-offer bargaining. Experimental investigators have come to markedly different conclusions about the ability of perfect equilibrium, in conjunction with the assumption that utility is measured by monetary payoffs, to predict behavior. A possible role for fairness arises because settlements regularly differ from those predicted in the direction of the equal money division.

Jack Ochs and Alvin E. Roth (1989) suggest that a model in which utility functions contain an argument reflecting tolerance for deviations from equal divisions may be useful in explaining these phenomena. Not everyone agrees:

We strictly reject the idea to include results of analyzing a social decision problem into the utility functions of the interacting agents. ... Furthermore, all our experiences from ultimatum bargaining experiments indicate that subjects do not "maximize" but are guided by sometimes conflicting behavioral norms. The utility approach necessarily neglects the dy-

namic nature of the intellectual process which subjects apply to derive their decision behavior...

(Werner Guth and Reinhard Tietz, 1990 p. 440).

Instead, Guth and Tietz favor a model in which bargainers shift between strategic and equity considerations in a hierarchical manner. Still other experimental investigators see little or no role for distributional concerns. Kenneth Binmore et al. (1985) suggest that experience is sufficient to turn "fairmen" into perfect-equilibrium "game-men." Janet Neelin et al. (1988) conjecture that when perfect equilibrium predicts inaccurately it is because bargainers fail to do backwards induction. The data sets from these investigations are broadly consistent with one another; it is the data interpretations that differ. Ochs and Roth (1989) survey these studies and reconcile some of the conclusions. They trace some of the discrepancies to differences in the scope of experimental design and trace others to differences in the focus of the data analysis.

This paper reports on experimental work that addresses some of the unreconciled disparities. The paper also describes a *comparative model* in which distributional concerns are incorporated into utility functions. While Ochs and Roth (1989) suggested this sort of model, they neither fully described it nor did a full analysis.

\*Department of Management Science, The Smeal College of Business Administration, Pennsylvania State University, University Park, PA 16802. The author thanks Maria Herrero, Alvin Roth, and Sanjay Srivastava for their helpful comments.

The comparative model explains previously observed behavior in some detail. Of course, if this were all it could do, it would be of marginal interest: the litmus test is the veracity of the model's added implications. The comparative model predicts with considerable accuracy in two previously unstudied environments. Even so, one might question the prudence of inserting distributional concerns into utility functions.<sup>1</sup> For instance, why not maintain the conventional behavioral postulate and modify one of the usual suspects (say, the complete-information assumption)? The answer, detailed below, is that observed behavior is in direct conflict with the supposition that bargainer utility is measured exclusively by monetary payoffs (irrespective of informational considerations), thereby necessitating the type of modification embodied in the comparative model.

The comparative model is a modification of the conventional theory, deriving predictions, as does the conventional theory, from utility maximization and perfect equilibrium. Thus, the comparative model demonstrates that game theory can provide useful explanations for the behavior observed in this type of laboratory bargaining. More specifically, the comparative model demonstrates that there is a strategic, as well as a "fairness," aspect to the way in which subjects bargain. (Vesna Prasnikar and Roth [1991] draw a similar conclusion from a rather different sort of experiment.)

In the basic bargaining environment to be considered, there are two bargainers,  $\alpha$  and  $\beta$ . They seek mutual agreement on sharing a pie. Attention will be restricted to a two-period version of the model. In the first period,  $\alpha$  proposes a division of the pie which  $\beta$  either accepts or rejects. If  $\beta$  accepts, the pie is divided in accordance with  $\alpha$ 's proposal; otherwise, the game proceeds to the second period, and due to delay, the pie shrinks. Now roles are reversed:  $\beta$  makes a proposal. If  $\alpha$  accepts, the pie is divided

accordingly; otherwise, the game ends in "disagreement," with both bargainers receiving nothing.

Alternating-offer bargaining of this sort has been analyzed by Ingolf Stahl (1972) and Ariel Rubinstein (1982), among others. The standard analysis, which I will refer to as the *pecuniary model*, assumes that bargainer utility is equivalent to the amount of pie that the bargainer receives. The predicted outcome, (subgame) perfect equilibrium, is derived from backwards induction. For example, suppose that the pie is one dollar, and suppose that  $\alpha$  and  $\beta$  have respective discount factors  $\delta_\alpha$  and  $\delta_\beta$ , both contained on the interval  $[0, 1]$ . Consider the subgame beginning in the second period. Since  $\alpha$ 's utility depends exclusively on the amount of money he receives,  $\beta$  need offer  $\alpha$  at most one cent in order to induce  $\alpha$  to accept. Consequently,  $\beta$  may ask for and receive virtually the entire dollar. Now back up to the first period. Since  $\beta$ 's utility depends only on the amount of money he receives,  $\alpha$  need offer  $\beta$  at most one cent more than  $\beta$  can expect from rejecting and moving the game into the second period. Bargainer  $\beta$ 's first-period valuation of a second-period dollar is  $\delta_\beta$  dollars. Therefore,  $\alpha$ 's equilibrium strategy is to offer  $\delta_\beta$  dollars to  $\beta$  and, if the game goes to the second period, to accept any nonnegative offer. Bargainer  $\beta$ 's equilibrium strategy is to accept any offer of  $\delta_\beta$  dollars or more, to reject otherwise, and if the game goes to the second period, to offer  $\alpha$  no more than one cent. So the perfect-equilibrium allocation has  $\alpha$  receiving  $\delta_\beta$  dollars and  $\beta$  receiving  $1 - \delta_\beta$  dollars, both in the first period.

### I. Experimental Study: Test of the Pecuniary Model

I conducted a laboratory test of the pecuniary model. One purpose was to check whether my experimental design replicated, in qualitative terms, the data of previous studies (i.e., Binmore et al., 1985; Guth and Tietz, 1988; Neelin et al., 1988; Ochs and Roth, 1989), which it does. Another purpose was to generate benchmark data against which new hypotheses could be

<sup>1</sup>Although this is by no means a novel approach. Harold M. Hochman and James D. Rodgers (1969) is a significant example.

TABLE 1—EXPERIMENTAL DESIGN

Structure	$(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$	$(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$
Direct money split	cell 1: inexperienced, 16 subjects cell 4: experienced, 14 subjects (10 from cell 1, 4 from cell 2)	cell 2: inexperienced, 14 subjects
Tournament	cell 5: inexperienced, 14 subjects cell 7: experienced, 12 subjects (7 from cell 5, 5 from cell 6)	cell 6: inexperienced, 14 subjects cell 8: experienced, 14 subjects (5 from cell 5, 9 from cell 6)
Structural variation	cell 3 (rotating positions): inexperienced, 12 subjects cell 9 (truncation): inexperienced, 16 subjects	cell 10 (truncation): inexperienced, 16 subjects

tested (reported on below). A summary of these experiments is placed at this point in the exposition in order to demonstrate how the new hypotheses are suggested by the data.

#### A. Design and Methodology

The original experiment comprised ten cells, each distinguished by three treatment variables: *structure*, *subject experience*, and *discount factors* (see Table 1). The games played in cells 1 and 2 conform precisely to the alternating-offers structure described in the Introduction. This structure will be referred to as the *direct money split*. Subjects who participated in cells 1 and 2 were *inexperienced* (i.e., the subjects had no previous experience with bargaining experiments). Nor did any subject who participated in cell 1 participate in cell 2. The discount factors used in cell 1 were  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$ ; in cell 2,  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$ .

The basic design of cells 1 and 2 is due to Ochs and Roth (1989): the "pie" is a predetermined amount of money which, in accordance with the relevant discount factors, diminishes from one period to the next. To implement unequal discount factors, subjects negotiate over how to split an intermediate commodity: 100 "chips." In both cells 1 and 2, the period-1 value of each chip to each bargainer was \$0.12. In cell 1, the

period-2 value for  $\alpha$  was \$0.08, and the value for  $\beta$  was \$0.04; these were reversed in cell 2.

For the sake of brevity, the pecuniary-model perfect equilibrium will be referred to as the *pecuniary equilibrium*. Table 2 identifies the pecuniary equilibria for both cells 1 and 2. Multiple equilibria are due to the discrete nature of the pie.

With the noted exceptions, the methodology was the same for all 10 cells: all observations for a given cell were collected in a single session.<sup>2</sup> Subjects were recruited from the undergraduate population of Carnegie Mellon University.<sup>3</sup> Cash was the only incentive offered.<sup>4</sup> In order to participate, subjects had to appear at a special time and

<sup>2</sup>The longest session was 1 hour and 15 minutes, the shortest was 45 minutes, and the average was approximately 1 hour. Cells 1–4, 9, and 10 began at 3:30 P.M. Cells 5–8 began at 3:00 P.M. All cells were run on weekdays, between 20 April and 15 September 1989.

<sup>3</sup>Participants for cells 1–4 were recruited from various undergraduate economics classes. Most of the participants in cells 5–10 were recruited via university bulletin boards, and the rest were recruited from undergraduate economics classes. Of the 102 participants, all but two were undergraduates. Of the two that were not, one was a graduate student in public policy and engineering (participated in cells 1 and 4), and the other was a graduate student in mechanical engineering (participated in cells 6–8).

<sup>4</sup>Subjects were told explicitly that this was the only reason they should consider participating.

TABLE 2—MODEL PREDICTIONS AND TESTS, DIRECT MONEY SPLIT

A. Predictions:		
Pecuniary model	Both models	Comparative model
$\mu_1 = \mu_4 = \$3.96\text{--}\$4.08$ (33–34 chips)	$\mu_1, \mu_4 < \$6.00$ (50 chips)	$\mu_2 < \$7.92$ (66 chips)
$\mu_2 = \$7.92\text{--}\$8.04$ (66–67 chips)	$\mu_1, \mu_4 < \mu_2$	
B. Tests:		
Hypothesis ( $H_0$ )	$t$ statistic	$d.f.$
$\mu_1 = \$4.08$	1.658	7.00
$\mu_4 = \$4.08$	3.382	6.00
$\mu_1 \geq \$6.00$	–8.447	7.00
$\mu_4 \geq \$6.00$	–8.847	6.00
$\mu_2 \geq \$7.92$	–12.580	6.00
$\mu_1 = \mu_4$	0.878	12.84
$\mu_1 \geq \mu_2$	–5.510	12.98
$\mu_4 \geq \mu_2$	–5.131	11.93

Notes: The observed opening offer in cell  $i$  is denoted  $\mu_i$ . All  $t$  statistics were calculated using means and standard errors from the last round of the cell(s). Welch's two-mean test was used for two-mean comparisons (see Peter J. Bickel and Kjell A. Doksum, 1977 pp. 218–9).

place. After random seating, they read directions (see Appendix B).<sup>5</sup> This was followed by 10–15 minutes of practice games with the computer as bargaining partner. As subjects were aware, the computer generated random (hence meaningless) offers and responses.<sup>6</sup> After practice, important portions of the directions (italicized in Appendix B) were read aloud, and all chip values were publicly announced. Finally, subjects were assigned  $\alpha$  and  $\beta$  roles (equal numbers of each). These did not change for

the duration of the cell (except in cell 3, where roles were alternated).

As subjects were apprised, each  $\alpha$  anonymously played each  $\beta$  exactly once. The computer communicated all offers and responses. It computed and displayed the monetary value of offers, whether actually made or just under consideration. Also displayed was the history of the game in progress (for a facsimile of a typical computer screen see Appendix B).<sup>7</sup> First-period proposals and tentative second-period counteroffers appeared on the screen together. Bargainers playing  $\beta$  sent first-period rejections and second-period counteroffers simultaneously. At the end of each game, subjects recorded the complete game history (blank in Appendix B), making it avail-

<sup>5</sup>Because of differences in treatment variables, directions necessarily differed across cells. The directions presented in Appendix B are a composite.

<sup>6</sup>During practice, the computer and the subject would take turns at the roles of  $\alpha$  and  $\beta$ . When the computer was called upon to respond to a proposal, "accept" and "reject" would be randomly chosen, regardless of what the actual proposal was. When the computer was called upon to propose, a random-number generator would produce a proposal of the form  $(x, 100 - x)$ . One might worry that this procedure would bias participants toward making Pareto-optimal proposals. However, the previous studies cited in this paper provide overwhelming evidence that subjects would do so anyway.

<sup>7</sup>The facsimile in Appendix B pertains to the direct money split. The only alterations made for the tournaments were that chip values were measured in points and at the conclusion of each game the total points a participant had made during the experiment was displayed. The only alteration made for the structural-variation cells was that in cells 9 and 10,  $\alpha$ 's second-period option to accept or to reject was eliminated.

able for later reference. Subjects had no access to information about games in which they were not participants.<sup>8</sup> At the conclusion of the session, two games were randomly chosen for immediate cash payment of earnings (tournament payoffs were different: see Section III-C).<sup>9</sup>

Investigators must always be concerned that superfluous aspects of the experimental design might affect the results (this is true in all experimental sciences). The remedy is to repeat the experiment, varying the superfluous aspects. Therefore, while the basic design follows Ochs and Roth (1989), details of my experiment differed. For one, no fee was paid to subjects for arriving on time. This was done because some commentators had expressed concern that Ochs and Roth's data might reflect subjects' interest in the fee rather than the bargaining payoffs. Second, discount factors were altered in such a way that the pecuniary equilibria and the fifty-fifty money division are spread relatively farther apart. This was done to make it easier to interpret opening offers. Third, chip values and the number of rounds for which subjects were actually paid were chosen so that the expected value of a given proposal was about the same across experiments (actually a bit higher in mine),<sup>10</sup> but the probability that the proposal would actually be paid on was higher in mine (25–29 percent compared to 10 percent). This addresses, at least partially, the concern that subjects in the Ochs and Roth study might not have taken certain situations too seriously because of a low probability that they would matter, cashwise. (See Sections II-E and III-C for additional, strong evidence that this is not a problem.)

### B. Results

The pecuniary model suggests a data analysis along several dimensions. Point

<sup>8</sup>In the tournaments, however, final point counts were publicly announced (without attribution) at the completion of the session.

<sup>9</sup>The average payout was slightly less than \$10.

<sup>10</sup>If expected values are prorated for the running time of the experiment, they are much higher for mine.

predictions are made about first-period offers. In addition, there are predictions about what should happen if play deviates from the equilibrium path: about conditions under which  $\beta$  should reject and about what the subsequent counteroffer should be.

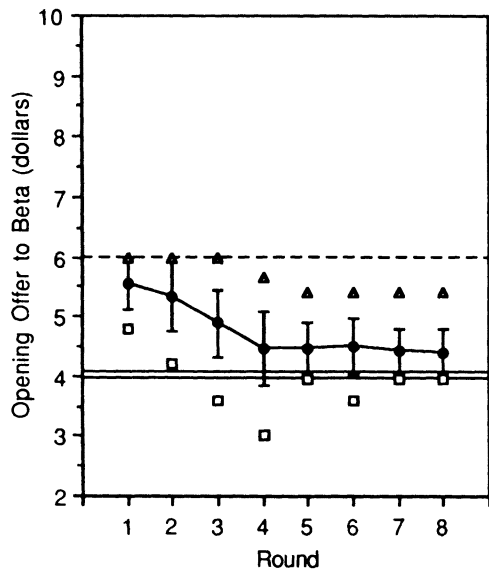
Begin with first-period offers: Figure 1 provides a graphical comparison of cell-1 mean observed opening offers with the pecuniary equilibrium. Figure 2 does the same for cell 2. For cell 1, the hypothesis that last-round means are the same as the equilibrium cannot be rejected at the 0.05 level of significance, but it is rejected at the 0.025 level. For cell 2, the hypothesis can be rejected at all conventional levels (see Table 2 for the  $t$  statistics). In fact, cell-2 mean offers are consistently less than half the pie, even though pecuniary equilibrium calls for an offer of about two-thirds. Note that, for both cells, the deviations are in the direction of the equal money split.

It might be argued that the proposed offers, in cell 1 at least, are close to the pecuniary equilibrium, statistics aside. After all, the difference between the equilibrium and the last-round mean is only about \$0.50, not a big difference, particularly when one considers that offers must be made in \$0.12 intervals. The following explanation is seemingly consistent with the pecuniary model: if the game should go to the second period, and assuming that  $\alpha$  prefers ending in disagreement to accepting nothing,  $\beta$  must offer  $\alpha$  a fraction of the pie. It would be no surprise if this fraction were higher than \$0.04 (one chip). Using a slightly higher value than \$0.04 will alter the backwards-induction calculation by a small amount, hence the observed slight deviation. However, the explanation implies that the deviations should be consistently negative, while the observed deviations are consistently positive. This inconsistency, combined with the very substantial deviations observed in cell 2, suggests that an alternative explanation is necessary. Others will be considered below.

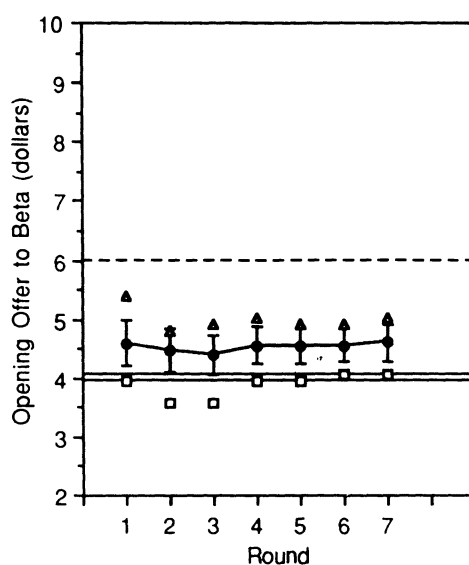
Next, consider data on rejections and counteroffers (see Figs. 3, 4). In both cells, about 20 percent of all opening offers were rejected. Rejections are not on the pecuniary-equilibrium path, nor are *disadvanta-*

**Direct Money Split**

Cell 1 (Inexperienced)

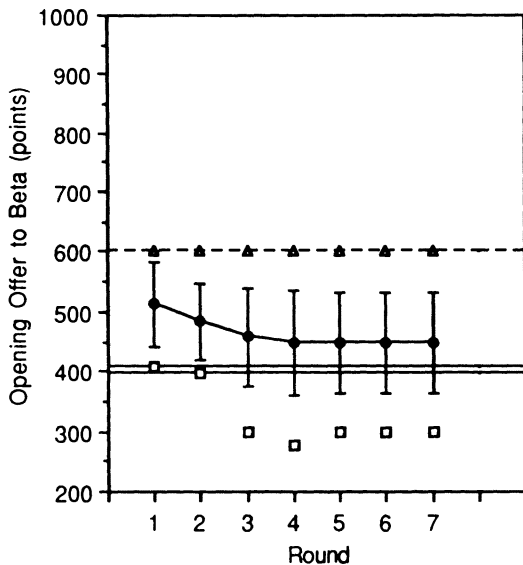


Cell 4 (Experienced)

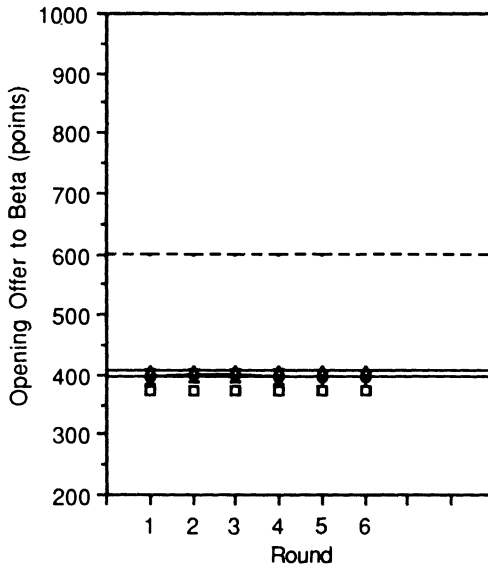


**Tournament**

Cell 5 (Inexperienced)



Cell 7 (Experienced)



- △ maximum observed offer
- mean observed offer
- mean plus 2 standard errors
- mean minus 2 standard errors

- minimum observed offer
- equal division
- ==== pecuniary equilibrium interval

FIGURE 1. MEAN OBSERVED OPENING OFFERS WITH THE PECUNIARY EQUILIBRIUM:  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

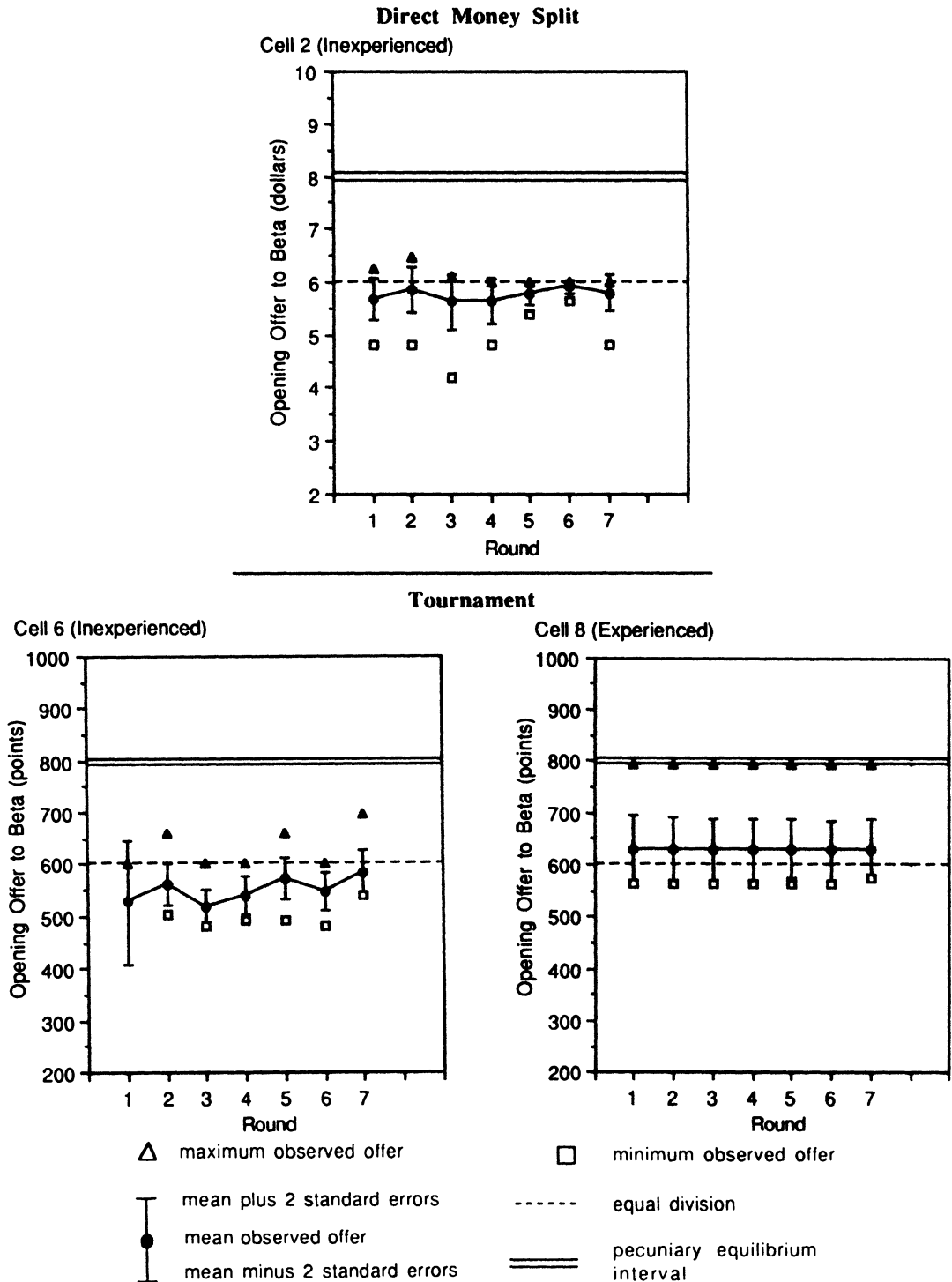


FIGURE 2. MEAN OBSERVED OPENING OFFERS WITH THE PECUNIARY EQUILIBRIUM:  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$



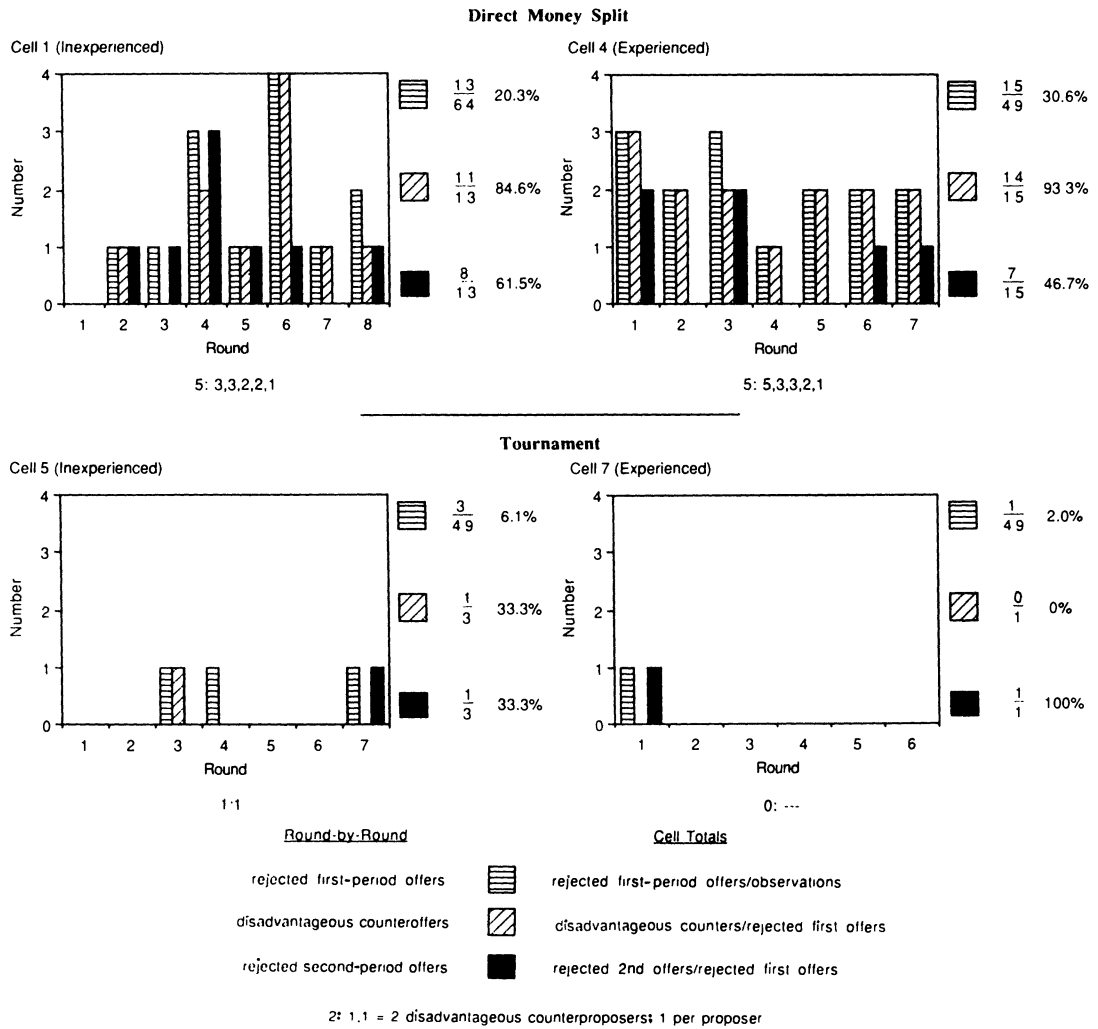


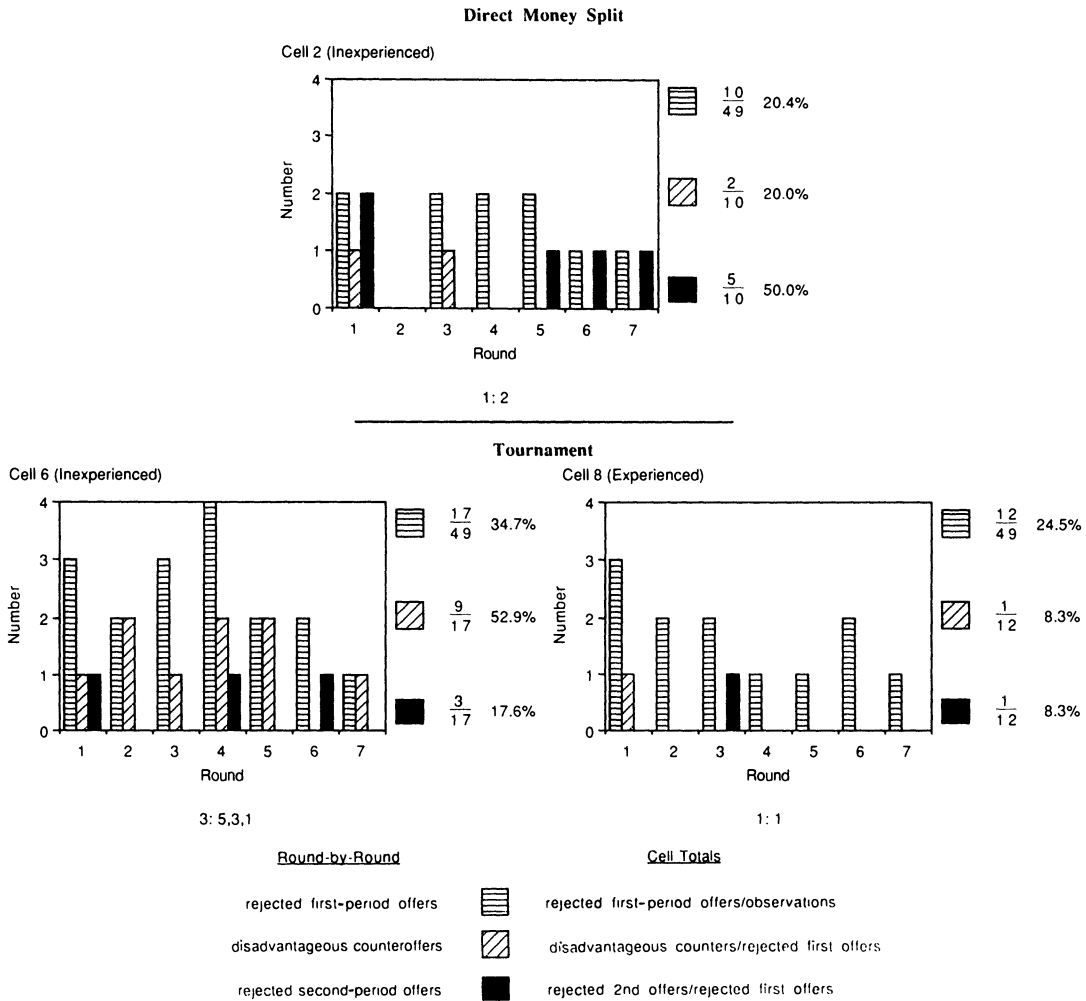
FIGURE 3. REJECTED OFFERS:  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

geous counteroffers (second-period offers that give the  $\beta$  proposer less money than the first-period offer he rejects). In cell 1, 85 percent of rejected first-period offers were followed by disadvantageous counteroffers. For cell 2, the figure is 20 percent.<sup>11</sup> This behavior was not restricted

to a very few: disadvantageous counteroffers were made by a majority of cell-1  $\beta$  bargainers, no one being responsible for a very large proportion (Fig. 3). Finally, note the high percentage of second-period rejections. This is so in spite of the fact that most second-period offers gave  $\alpha$  a positive, often

<sup>11</sup>It is not surprising that a smaller proportion of first-period rejections in cell 2 are followed by disadvantageous counteroffers: In cell 2,  $\alpha$ 's almost always offered  $\beta$ 's less than the pecuniary equilibrium, and

vice versa in cell 1. This almost always leaves room for advantageous counteroffers in cell 2 but almost never does in cell 1.



Ochs and Roth study had sufficient scope to capture. Recall that, by the pecuniary equilibrium, the proportional allocation should be dependent exclusively on the value of  $\delta_\beta$ .

R5: The value of  $\delta_\alpha$  influences the outcome.

Disadvantageous counteroffers appear to be the key to understanding why the empirical data differ from the pecuniary model's predictions: a key auxiliary assumption of the model is that each bargainer's utility corresponds to his monetary payoff. If this is true, then disadvantageous counteroffers should never be observed, and the model's predictions should follow easily (at least in the two-period model, where the backwards induction is trivial). However, disadvantageous counteroffers are observed in relatively large numbers in all of the cited studies. Note that no amount of incomplete information will explain this: when subjects make disadvantageous counteroffers, they have sufficient information to know that they are turning down money. Other explanations come to mind, but as discussed in the next section, many are either in conflict with the data or theoretically problematic.

### *C. Some Plausible, but Flawed, Hypotheses About Disadvantageous Counteroffers*

It might be thought that disadvantageous counteroffers are evidence that subjects were confused by the experimental design. In particular,  $\beta$  bargainers may have committed themselves to rejecting opening offers before considering feasible counteroffers, or maybe subjects made calculation errors when translating chips into money. However, in my experiments, as well as those of Ochs and Roth (1989), Guth and Tietz (1988), and Neelin et al. (1988), counteroffers appeared on the computer screen (or message paper) along with the original offers, before any commitment was made on the part of  $\beta$ . In my experiment, the computer calculated the value of every proposal for both players, and it appeared on the screen whenever the proposal did. (Further evidence that subjects understood the game

on a cognitive level is detailed in Appendix C.)

Another possibility is that bargainers prefer disagreement to accepting offers that are "insultingly low." However, as Ochs and Roth point out, this does not explain why a bargainer would reject an offer and come back with one that gives him less money. If anything, the argument, unembellished, would seem to rule out disadvantageous counteroffers.

A more subtle version of the last argument conjectures that bargainers prefer disagreement to receiving less than  $\$x$  and, because of the value of the discount factors, the second-period pie is worth less than  $\$x$  to at least one bargainer. As a consequence, this bargainer does not care about the second-period pie, and disadvantageous counteroffers are simply evidence of the resulting capricious behavior. This argument, however, implies that the bargaining reduces to a one-period demand game. It follows that the opening equilibrium offer is  $\$x$  regardless of the values of bargainer discount factors. However, definite shifts in opening offers are observed as discount factors are varied in all of the relevant studies (compare cells 1 and 2 in Figs. 1 and 2; see Guth and Tietz, 1988; Ochs and Roth, 1989).

The above argument might be taken one step further: perhaps the money involved in the experiment *as a whole* is not enough to induce subjects to take it seriously.<sup>12</sup> In the lab, I manipulated the structure of the basic bargaining game in several ways. In some of these variations, disadvantageous counteroffers virtually disappear (see Sections II-E and III-C). The payoffs were comparable across all variations, implying that something else is responsible for disadvantageous counteroffers. Also, if subjects did not take the game seriously, one would expect to see erratic behavior in the data. This is not the

<sup>12</sup>The question of whether the amount of money is sufficient to induce subjects to take the game seriously is distinct from the question of what would happen if the stakes were raised. The latter question is discussed in Section II-D.

case. Indeed, the consistency of the data, not only within the various experiments, but across them, is quite remarkable (for a discussion see Ochs and Roth [1989]).

In some studies, including mine, subjects negotiated in round-robin style. While they never played the same person twice, each  $\beta$  bargainer played the same set of  $\alpha$  bargainers. It might be argued that either the  $\alpha$ 's or  $\beta$ 's are colluding via "feedback effects." For instance, suppose that perfect equilibrium calls for  $\beta$  to receive one-third of the first-period pie. A  $\beta$  bargainer might reject offers of less than, say, 40 percent of the pie, thinking that this will influence the rejected  $\alpha$  bargainers to play softer in the future to the benefit of all  $\beta$  bargainers. A similar argument can be fashioned for  $\alpha$  collusion when  $\alpha$ 's equilibrium share of the pie is less than 50 percent. This argument has the unraveling problem that most collusion arguments have when applied to finite games. Moreover, it is quite awkward to argue that  $\beta$  bargainers are colluding when it is observed that they receive more than their perfect-equilibrium share while  $\alpha$  bargainers are colluding when the situation is reversed.

#### D. Experience Hypothesis

From examining Figure 1 it might be conjectured that, at the conclusion of play in cell 1, subjects are still learning (i.e., behavior has not yet stabilized). Specifically, there is a downward trend to opening offers, which, if it continued, might lead to pecuniary equilibrium play. How a learning argument would apply to cell 2 is less apparent (compare Fig. 2). Nevertheless, one cannot dismiss out-of-hand the hypothesis that more experience might lead to pecuniary-model results, or at least to results that differ substantially from the less-experienced case.

Cell 4 was designed with the experience hypothesis in mind. Experienced bargainers were recruited by first inviting all cell-1 participants. They were given first priority because the cell-4 bargaining game is identical to that in cell 1 (same discount factors). It was necessary to recruit a few subjects from

cell 2.<sup>13</sup> These were telephoned in random order, until the desired number of subjects was obtained (see Table 1 for numbers).<sup>14,15</sup> Except for subject experience, cells 1 and 4 are identical in terms of both methodology and design. In particular, bargainer roles for cell 4 were randomly assigned.

Comparing cells 1 and 4 provides a test of the experience hypothesis. As indicated in Table 2 and displayed in Figure 1, the average observed opening offers for each round of cell 4 are virtually identical to those of the final rounds of cell 1. The standard errors are smaller for cell 4 than for cell 1, making it possible to reject the hypothesis of pecuniary-equilibrium play in cell 4 at all conventional levels of significance.

Contrary to the experience hypothesis, this suggests that play has "stabilized" by the end of cell 1, away from the pecuniary equilibrium. However, note that the aggregate data on rejections and disadvantageous counteroffers are very similar for both cells (Fig. 3). While this is clear evidence against the hypothesis that experience will produce pecuniary play, it might also be taken as evidence against the argument that play has stabilized. However, if one rejects the idea that subjects are playing equilibrium, it is no longer clear what is meant by "stabilized." A discussion of what this might mean, as well as whether players are doing it, is postponed until Section II-C. The conclusion to be drawn here is that experience does not seem to produce pecuniary play, nor does it lead to any substantial changes, with the exception of the shrinking of standard errors. (In a personal communication, Alvin E. Roth has informed me that he and Claudia Garcia have obtained very similar results using experienced subjects.)

<sup>13</sup>With one exception, all cell-1 participants that could be contacted agreed to return. The one who could not said she had a scheduling conflict but offered to return at another time.

<sup>14</sup>After the running of cell 1, one of the computer terminals broke and was not repaired for many months. As a result, for cells 2-8, the maximum feasible number of subjects fell from 16 to 14.

<sup>15</sup>Subjects were not previously told that they would be invited back to play.

E. Simple Fairness Hypotheses

It might be argued that the deviations from pecuniary equilibria observed in cells 1, 2, and 4 are due to equity considerations implicit in the experimental design. At least two distinct testable hypotheses fall under this heading. One hypothesis states that play deviates in the direction of the fifty-fifty money split because of an asymmetry of opportunity; for example,  $\beta$  bargainers in cell 1 never get to be  $\alpha$  bargainers, thus putting them at a strategic disadvantage. The  $\beta$  bargainers react to this by demanding more than the pecuniary equilibrium prescribes to them. According to this hypothesis, if each player had an equal number of opportunities to be  $\alpha$ , as well as  $\beta$ , pecuniary equilibrium should result. A second hypothesis states that, by randomly assigning subjects to be  $\alpha$  or  $\beta$ , the experimenter is inadvertently suggesting fair outcomes to the subjects (i.e., the experimenter is treating subjects in an egalitarian manner, thereby influencing subjects to act similarly; for an example of this sort of phenomenon, see Elizabeth Hoffman and Matthew L. Spritzer [1982, 1985]).

Both hypotheses are tested in cell 3, which was identical to cell 1 with the exception that subjects rotated between  $\alpha$  and  $\beta$  roles. Twelve subjects played 11 rounds. No subject played any other subject more than once. In the first ten rounds, each subject was an  $\alpha$  half the time and a  $\beta$  the other half. Subjects strictly alternated roles for the first six rounds (because of the nature of the permutations, strict alternation is not possible beyond six rounds). In the 11th and last round, roles were randomly picked by the computer.<sup>16</sup>

The test is this: if either of the fairness hypotheses is correct, then cell-3 data should be closer to pecuniary equilibrium than cell-1 data. If neither the asymmetry of design nor the randomization has any impact, then cell-3 and cell-1 data should be very similar. The influence of broader experience on

<sup>16</sup>The 11th round was necessary to insure that each subject played every other subject exactly once.

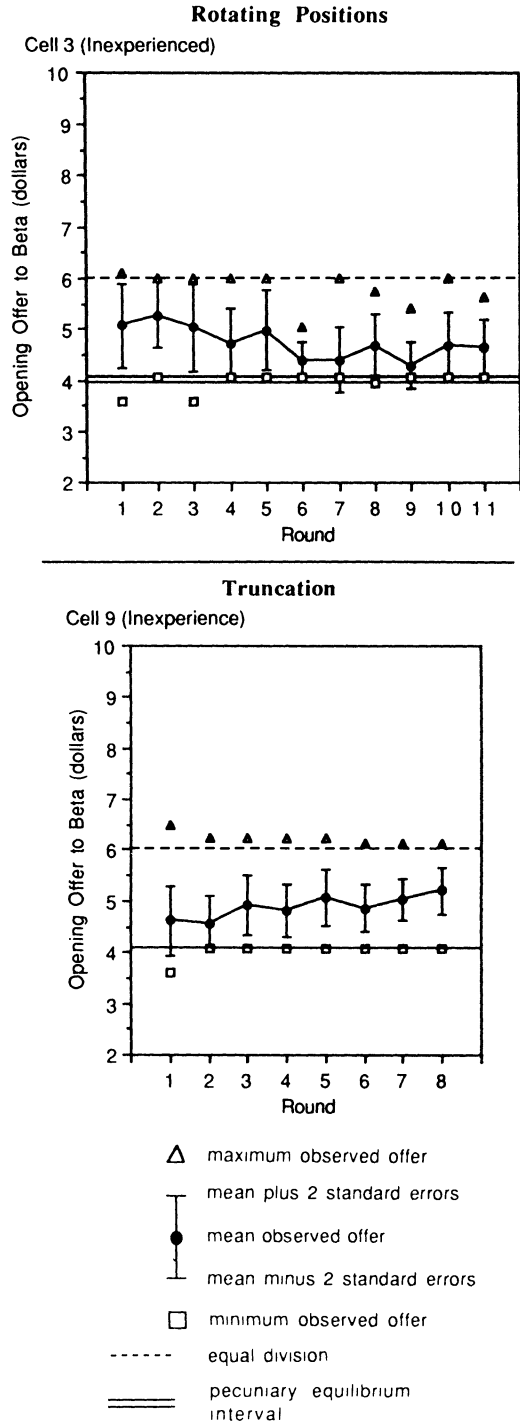
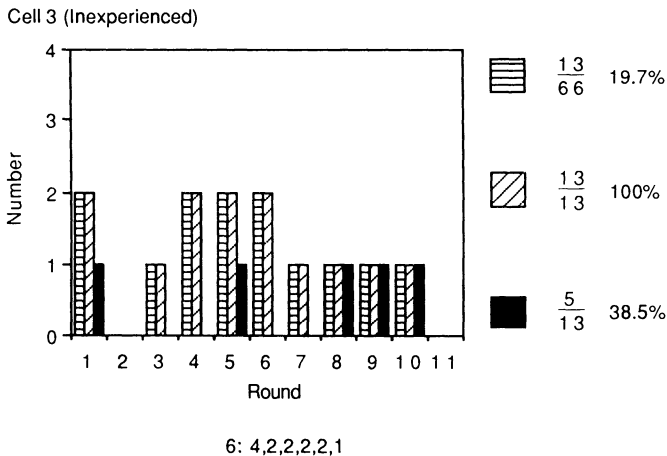
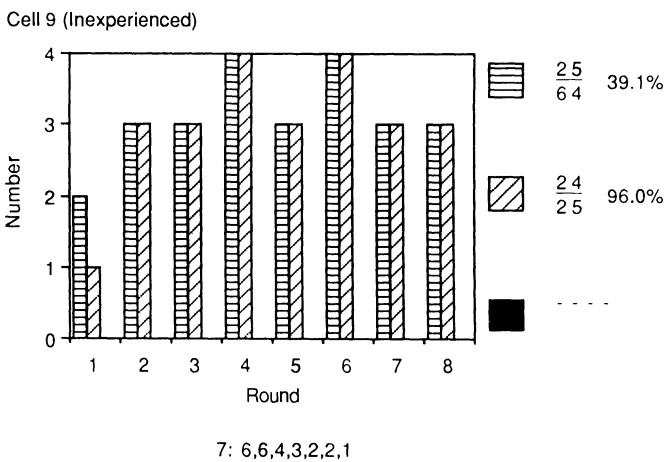


FIGURE 5. MEAN OBSERVED OPENING OFFERS, ROTATING POSITIONS AND TRUNCATION:  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

**Rotating Positions**



**Truncation**



Round-by-Round

Cell Totals

- |                               |  |  |
|-------------------------------|--|--|
| rejected first-period offers  |  | rejected first-period offers/observations      |
| disadvantageous counteroffers |  | disadvantageous counters/rejected first offers |
| rejected second-period offers |  | rejected 2nd offers/rejected first offers      |

2 1,1 = 2 disadvantageous counterproposers: 1 per proposer

FIGURE 6. REJECTED OFFERS, ROTATING POSITIONS AND TRUNCATION:  
 $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

cell-3 subjects is controlled for by the cell-4 comparison.<sup>17</sup>

Figure 5 graphically summarizes the data on cell-3 opening offers. Note that, except for the spiked nature of the averages (which is explained by the alternating roles), the graph is very similar to that for cell 1 (Fig. 1). In fact, Welch's two-mean test of the hypothesis that  $\mu_1 = \mu_3$  results in a  $t$  statistic of  $-0.8208$  with 8.39 degrees of freedom, so the hypothesis cannot be rejected at any conventional level of significance.<sup>18</sup> As shown in Figure 6, the statistics for rejections and disadvantageous counteroffers for the two cells are also very similar. Note that fully 100 percent of the counteroffers for cell 3 are disadvantageous. From this evidence, it would not appear that rotating the first-mover role has much impact on bargainer behavior, leading to the rejection of both hypotheses.

## II. The Comparative Model

### A. Intuition

There is an explanation for disadvantageous counteroffers that is consistent with a wide range of empirical observations, including R1–R5: contrary to an auxiliary assumption of the pecuniary model, bargainers care about the relative split of money as well as their own cash payoff. Put another way, bargainers measure what they receive by both an absolute and a relative yardstick. The absolute yardstick measures the cash payoff. The relative yardstick measures the disparity between the two bargainers' abso-

<sup>17</sup>Close readers of Binmore et al. (1985) may take up those authors' assertion that it is the type of experience acquired, not experience per se, which makes for a perfect-equilibrium "gamesman." They may feel that cell 3 is a stronger test of the experience hypothesis and that the test in cell 4 is inadequate or inappropriate. These readers are welcome to apply this interpretation.

<sup>18</sup>The test was done using the round-8 average from cell 1 and the round-10 average of cell 3. If the round-11 average from cell 3 is substituted instead, a  $t$  statistic of  $-0.7370$  with 9.41 degrees of freedom results, and again the hypothesis is not rejected at any conventional level of significance.

lute measures (no altruism: utility is nondecreasing in the relative measure's self-favorability). Although cash is the only commodity involved in negotiations, bargainers act as if there are two: absolute and relative money. It is assumed that bargainers find the monies substitutable for one another. The explanation for disadvantageous counteroffers (R4) is immediate: bargainers are trading away absolute money in order to gain relative money.

As shown in the next section, the comparative model offers partial explanations for why settlements consistently deviate in the direction of the equal money division (R2) as well as for the first-mover advantage (R1). The complete-information comparative model also explains virtually all of the shifts in mean offers observed by Guth and Tietz (1988), Ochs and Roth, and myself when bargainer discount factors were varied (including R5). An extension to incomplete information, presented in the next section, explains rejected first-period offers (R3). I first present the model with complete information so as not to distract from the main thrust driving the results: an explanation for disadvantageous counteroffers.

### B. Formal Model

Consider a two-period alternating-offer bargaining game in which the pie is worth  $k$  dollars. Let  $\alpha$  and  $\beta$  be the respective first- and second-period proposers. Offers take the form  $(x_\alpha, x_\beta)$  where  $x_\alpha + x_\beta \leq 1$ . Let  $\delta_\alpha, \delta_\beta \in (0, 1]$  be the respective per-period discount factors, so, for example, an offer of  $(x_\alpha, x_\beta)$  is worth  $kx_\alpha$  dollars to  $\alpha$  in the first period, but only  $\delta_\alpha kx_\alpha$  dollars in the second period.

Bargainers receive utility from two sources. One source is the amount of money obtained from the settlement. The other source is a relative comparison of money earnings, incorporated into the utility function by way of a proportional index:

$$i_{n,t}(x_\alpha, x_\beta) = \begin{cases} 1 & \text{if } x_n = x_{\sim n} = 0 \\ \frac{\delta_n^{t-1} x_n}{\delta_{\sim n}^{t-1} x_{\sim n}} & \text{otherwise} \end{cases}$$

where  $t \in \{1, 2\}$  is the period of settlement and  $n \in \{\alpha, \beta\}$ . Note that  $i_{n,t} \in [0, +\infty]$ .

Bargainers  $\alpha$  and  $\beta$ 's utility from a settlement of  $(x_\alpha, x_\beta)$  in period  $t$  are respectively given by

$$A(\delta_\alpha^{t-1}x_\alpha k, i_{\alpha,t})$$

and

$$B(\delta_\beta^{t-1}x_\beta k, i_{\beta,t})$$

which are assumed to have the following characteristics.

**ASSUMPTION 1:** *A and B are continuous and right-differentiable in both arguments.*

**ASSUMPTION 2:**  $A_1 > 0$  and  $B_1 > 0$ .

**ASSUMPTION 3:** *For all  $i_{n,t} < 1$ ,  $A_2 > 0$  and  $B_2 > 0$ , where  $n = \alpha, \beta$  and  $t = 1, 2$ .*

**ASSUMPTION 4:** *If  $\delta_\alpha^{t-1}x_\alpha^* > \delta_\alpha^{s-1}x_\alpha^{**}$  and  $i_{\alpha,t}(\delta_\alpha^{t-1}x_\alpha^*) \geq 1$ , then  $A(\delta_\alpha^{t-1}x_\alpha^* k, i_{\alpha,t}) > A(\delta_\alpha^{s-1}x_\alpha^{**} k, i_{\alpha,s})$ , where  $s, t \in \{1, 2\}$ ; if  $\delta_\beta^{t-1}x_\beta^* > \delta_\beta^{s-1}x_\beta^{**}$  and  $i_{\beta,t}(\delta_\beta^{t-1}x_\beta^*) \geq 1$ , then  $B(\delta_\beta^{t-1}x_\beta^* k, i_{\beta,t}) > B(\delta_\beta^{s-1}x_\beta^{**} k, i_{\beta,s})$ .*

Assumption 2 says that, all other things equal, bargainers prefer more money to less. Assumption 3 says that the closer the split is to fifty-fifty, the better off is the bargainer who receives the smaller share. For example, suppose  $\alpha$  receives \$2. Then, he is better off when  $\beta$  receives \$4 than when  $\beta$  receives \$5. On the other hand, Assumption 4 states that, if a bargainer receives a share that is larger than or equal to the share received by his partner, the only way to make him better off is to increase his money holdings. Put another way, once he obtains parity, a bargainer's only concern is with absolute money. The latter interpretation is formalized in the following lemma (proof in Appendix A).

**LEMMA 1:**  $A_2 = 0$  and  $B_2 = 0$  whenever  $i_{n,t} \geq 1$ , where  $n = \alpha, \beta$  and  $t = 1, 2$ .

Figure 7 illustrates the type of preferences

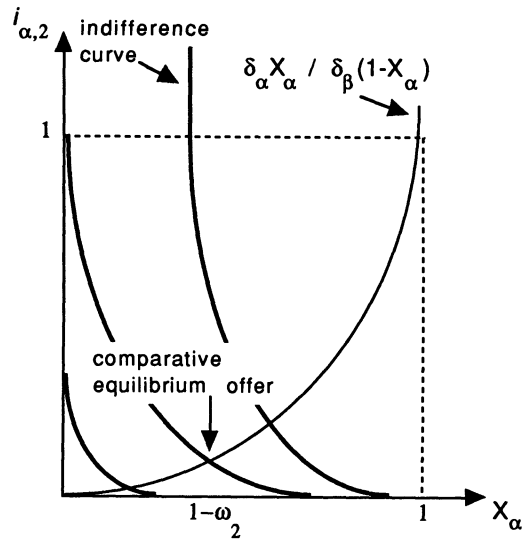


FIGURE 7. SECOND-PERIOD  $\alpha$  INDIFFERENCE CURVES

described. Evidence supporting the assumptions made about  $i$  is discussed in Section II-E.

As with the pecuniary model, (subgame) perfection will be used as the solution concept. It will be clear from the analysis that, starting in any subgame, all equilibrium splits,  $(x_\alpha^e, x_\beta^e)$ , satisfy  $x_\alpha^e + x_\beta^e = 1$ . It is convenient to assume this up front because it allows for a simplification of the notation: a period- $t$  equilibrium offer is completely characterized by  $\omega_t$ , the proportion of the pie that  $\beta$  will receive if the offer is accepted (then, the proportion that  $\alpha$  receives is  $1 - \omega_t$ ).

The (subgame) perfect equilibrium is characterized by two equations, both derived from backwards induction: suppose that the first-period offer is turned down by  $\beta$ , who then must make a second-period offer. Since  $\alpha$  can achieve utility level  $A(0, 1)$  by turning down  $\beta$ 's offer (both bargainers leave the game with nothing),  $\beta$ 's second-period equilibrium offer,  $\omega_2$ , must satisfy

$$(1) \quad A\left(\delta_\alpha(1 - \omega_2)k, \frac{\delta_\alpha(1 - \omega_2)}{\delta_\beta\omega_2}\right) = A(0, 1).$$



Equation (1) has an “=” sign, not a “≥” sign, because (i) in equilibrium, β never offers α more than necessary and (ii) by Assumption 1, A is continuous in ω<sub>2</sub> (see Fig. 7).

Furthermore, β will accept α’s first-period offer, ω<sub>1</sub>, only if β receives at least as much utility from α’s offer as she can expect by rejecting and receiving ω<sub>2</sub>:

$$(2) \quad B\left(\omega_1 k, \frac{\omega_1}{1-\omega_1}\right) = B(\delta_\beta \omega_2 k, i_{\beta,2}(\omega_2)).$$

Equation (2) involves an “=” sign, not a “≥” sign, for reasons analogous to those given for (1).

Equation (2) can be simplified: define  $x_2^*$  by  $\delta_\alpha(1-x_2^*) = \delta_\beta x_2^*$ ; that is,  $x_2^*$  is the second-period split that gives each bargainer the same amount of money. Thus,  $i_{\alpha,2}(x_2^*) = 1$ . Monotonicity implies that  $A(\delta_\alpha(1-x_2^*), 1) > A(0, 1)$ , so in the second period, β need never offer α more than  $1-x_2^*$ ; that is,  $\omega_2 > x_2^*$ . Consequently, it is always the case that  $i_{\beta,2}(\omega_2) \geq 1$ , and by Lemma 1,  $B(\delta_\beta \omega_2 k, i_{\beta,2}(\omega_2)) = B(\delta_\beta \omega_2 k, 1)$ . Thus, (2) becomes

$$(2') \quad B\left(\omega_1 k, \frac{\omega_1}{1-\omega_1}\right) = B(\delta_\beta \omega_2 k, 1).$$

**PROPOSITION 1:** *There exists  $\omega_1, \omega_2 \in [0, 1]$  satisfying (1) and (2'). Further, if  $0 < \delta_\alpha$  and  $\delta_\beta < 1$ , then  $(\omega_1, \omega_2)$  is the unique subgame-perfect-equilibrium strategy combination and  $\omega_1$  is the unique subgame-perfect-equilibrium allocation.<sup>19</sup>*

(See Appendix A for the proof.) Regularities R1 and R2 are partially explained by the next proposition.

<sup>19</sup>When  $\delta_\alpha = \delta_\beta = 1$ , a slight modification of the proof shows that the stated equilibrium outcome is unique, although players will be indifferent about the period of settlement.

**PROPOSITION 2:** *If  $\delta_\beta \geq \frac{1}{2}$ , then  $\omega_1 < \delta_\beta$ ; if  $\delta_\beta < \frac{1}{2}$ , then  $\omega_1 < \frac{1}{2}$ .*

**PROOF:**

*Statement 1:* Since  $\omega_2 < 1$ , then  $\delta_\beta \omega_2 < \delta_\beta$ . Then, by monotonicity,  $B(\delta_\beta k, 1) > B(\delta_\beta \omega_2 k, 1)$ . Using the continuity of B, this implies that (2') is satisfied by  $\omega_1 < \delta_\beta$ . The proof of statement 2 is analogous.

Proposition 2 predicts that the settlement will deviate from the pecuniary equilibrium in the direction of the equal money split when  $\delta_\beta \geq \frac{1}{2}$ . The direction of deviation for  $\delta_\beta < \frac{1}{2}$  is ambiguous. For example, suppose that  $k = 1$  and that bargainer  $n$  has the utility function

$$\delta_n^{t-1} x_n + 0.49 i_{n,t}$$

where  $n = \alpha, \beta$  and  $t = 1, 2$ , and suppose that  $\delta_\alpha = \delta_\beta = 0.49$ . Solving (1) yields  $\omega_2 \cong 0.62$ , and using this to solve (2') yields  $\omega_1 \cong 0.43 < \delta_\beta$ . On the other hand, replacing 0.49 in the utility functions with 0.35 and setting  $\delta_\alpha = \delta_\beta = 0.35$ , one obtains  $\omega_1 \cong 0.37$ . Therefore, whether  $\omega_1$  is less than or greater than  $\delta_\beta$  when  $\delta_\beta < \frac{1}{2}$  depends on the values of the discount factors in conjunction with utility-function characteristics undetermined by Assumptions 1–4.

Proposition 2 predicts a first-mover advantage when  $\delta_\beta \leq \frac{1}{2}$ . In the case of  $\delta_\beta > \frac{1}{2}$ , a first-mover advantage is guaranteed if  $\delta_\alpha = \delta_\beta = 1$  [examine (1) and (2')]. Under any other circumstances, a first-mover advantage is uncertain: from (2'),  $\omega_1 \geq \frac{1}{2}$  only if it is the case that  $\delta_\beta \omega_2 \geq \frac{1}{2}$ . On the other hand, satisfaction of (1) requires that  $i_{\beta,2}(\omega_2) \geq 1$ . Combining these conditions yields

$$i_{\beta,2}\left(\frac{1}{2\delta_\beta}\right) = \delta_\alpha\left(2 - \frac{1}{\delta_\beta}\right) \geq 1$$

which is true only if  $\delta_\beta = \delta_\alpha = 1$ .

Both the Ochs-Roth and Guth-Tietz data sets exhibit clear shifts in opening offers as discount factors are varied. The comparative model accurately predicts the

direction of virtually all of them<sup>20</sup> and provides simple intuitive explanations. In addition, regularity R5 is explained.

**PROPOSITION 3:** *For all  $\delta_\alpha$ , if  $d\delta_\alpha > 0$  then  $d\omega_1 > 0$ .*

(See Appendix A for the proof.) The intuition behind Proposition 3 is as follows. Suppose the game goes to the second period. An increase in  $\alpha$ 's discount factor means that a fixed proportion of the pie is worth more to  $\alpha$  in absolute as well as relative terms. It follows that  $\beta$  can reduce the share of the second-period pie that she offers and still get the offer accepted (i.e.,  $\omega_2$  increases). Therefore, to  $\beta$ , the value of the period-2 subgame increases, meaning that  $\alpha$  must increase his first-period offer in order to get it accepted.

A generalization of the intuition for Proposition 3 leads to the following.

**PROPOSITION 4:** *Suppose that  $\delta_\alpha \leq \delta_\beta$ . If  $d\delta_\alpha \geq d\delta_\beta \geq 0$ , then  $d\omega_1 > 0$ .*

(See Appendix A for the proof.) Both the pecuniary and the comparative model predict that  $\beta$ 's equilibrium share will increase as  $\delta_\beta$  increases.

**PROPOSITION 5:** *For all  $\delta_\beta$ , if  $d\delta_\beta > 0$ , then  $d\omega_1 > 0$ .*

(See Appendix A for the proof.) The intuition for Proposition 5 is as follows. An increase in  $\delta_\beta$  decreases the comparative value to  $\alpha$  of any period-2 split. Consequently,  $\omega_2$  must decrease. However, as the math shows,  $\omega_2$  decreases at a slower rate

than  $\delta_\beta$  increases. Consequently,  $\delta_\beta \omega_2$  increases, implying [look at (2')] that  $\omega_1$  must rise.

Write  $\omega_1(\delta_\alpha, \delta_\beta)$  to denote that the value of  $\omega_1$  is dependent on both discount factors. The comparative model yields a prediction of what will happen when bargainers' discount factors are switched, as they are from cell 1 to cell 2.

**PROPOSITION 6:** *Suppose that  $0 < q < Q < 1$ . Then  $\omega_1(q, Q) > \omega_1(Q, q)$ .*

(See Appendix A for the proof.) Proposition 6 is true because, given  $\omega_2(Q, q)$ , there is always a second-period split under  $(\delta_\alpha, \delta_\beta) = (q, Q)$  that makes both  $\alpha$  and  $\beta$  better off. Consequently  $\alpha$  must offer  $\beta$  more in the first period when  $(\delta_\alpha, \delta_\beta) = (q, Q)$  than when  $(\delta_\alpha, \delta_\beta) = (Q, q)$ . Comparing cells 1 and 2, the observed mean offers are as Proposition 6 predicts, and the difference across cells is statistically significant at all conventional levels (Table 2).

### C. Incomplete Information

As described above, the comparative model assumes that subjects have complete information about one another's utility functions. In reality, however, they do not. More specifically, although somewhat roughly, the marginal rate of substitution between absolute and relative money most likely varies by individual, making utility functions private information. The observed dynamics of the experimental cells suggest an explanation for how subjects handle this problem.

Suppose that each  $\alpha$  bargainer is either risk-averse or risk-neutral. If there exists a first-period proposal of  $\frac{1}{2}k$  or more, which both maximizes the expected monetary value of the game and is the minimum proposal acceptable to all  $\beta$  bargainers (with probability 1), then such a proposal should be preferred to any other by all  $\alpha$  bargainers.<sup>21</sup>

<sup>20</sup>In the case of the Ochs and Roth study, I made my comparisons on the basis of the final round of play. The only observation that is inconsistent with the theory is that for Ochs and Roth's two-period cells in which the discount factors are flipped. However, judging from the figures in their paper, the observed difference in the average opening offers for round 10 is not statistically significant. Cells 1 and 2, however, provide an example that is consistent with the theory, and the shift in offers is statistically significant at all conventional levels.

<sup>21</sup>For the moment, suppose bargainers are risk-neutral. Then,  $\alpha$ 's expected utility function over the set of outcomes  $\{x_1: i_{\alpha,1}(x_1) \geq 1\}$  can be written as

Intuitively, the reason for expecting the two conditions to overlap is that offers that are rejected lead to very small payoffs relative to those that are accepted, so even a small probability of rejection greatly diminishes expected value.

The data corroborate this intuition. In Figures 8–10, average earnings for  $\alpha$  are plotted against opening offers to  $\beta$  (Tables 3–5 show the acceptances and rejections for these opening offers). The plots for cells 1, 3, and 4 (Figs. 8, 10) are all very similar, having peak values right around an offer of \$4.80 (40 chips). For cell 2 (Fig. 9), the peak value is around \$5.75 (48 chips). As shown in Tables 3 and 5 (cell-3 portion), for cells 1, 3, and 4, offers of greater than or equal to

\$4.80 were almost always accepted (only one exception in 90 observations). In every other column there is a substantial percentage of rejections. For cell 2 (Fig. 9, Table 4), the offer with the highest expected payoff is about \$5.50, and offers at or above this value were accepted 93.5 percent of the time, with a much more substantial percentage of rejections for lower offers. Thus, while the offer with maximum expected value is not literally acceptable every time, such an assumption seems reasonably accurate.

Proposers do not know the offer with the peak expected value when they begin play, so they must search. Searching can be conceptualized as a fairly straightforward exercise in hill-climbing, made even easier if proposers assume (correctly) that the offer with maximum expected value is also the minimum offer that is accepted with probability 1. Then, searching proceeds roughly as follows. Based on a subject's priors, he makes an offer. If it is rejected, he makes a more generous offer in the next round; if it is accepted, his offer is less generous in the next round. If searching is over a single-peaked hill, one would expect the process to converge on the peak.

A smoothing of the curves presented in Figures 8–10 shows that they may all be thought of as single-peaked. The observable implication is that experience should lead to a greater concentration of offers around the peak expected value. Actually, some evidence of this has already been mentioned: it happens that mean offers, at least for the later rounds of play, correspond closely to the offer that maximizes expected value (compare Figs. 1, 2, and 5 to their counterparts in Figs. 8–10). The standard errors around the mean offer shrink when moving from cell 1 (inexperience) to cell 4 (experience). Another way of seeing this is to note how, in Figure 8, the dispersion of offers shrinks when moving from cell 1 to cell 4.

Table 3 displays the *distribution of opening offers* for each cell, by  $\alpha$  bargainer. Across cells 1 and 4, the distributions of opening offers, by category, are quite similar (a slightly higher proportion in cell 4 is concentrated in the upper two categories

---

$A((1-x_1)k, 1) = (1-x_1)k$ . Let  $F_\beta(x_1) = \Pr\{\text{first-period offer of } x \geq x_1 \text{ will be accepted by a randomly chosen } \beta\}$ . Then, the expected utility to  $\alpha$  from proposing  $x_1$  is given by

$$F_\beta(x_1)A((1-x_1)k, i_{\alpha,1}(x_1)) + (1-F_\beta(x_1)) \times \max\{A(\delta_\alpha(1-\omega_2)k, i_{\alpha,2}(\omega_2)), A(0,1)\}$$

where  $\omega_2$  is the (expected) second-period equilibrium proposal. Restricting attention to the discount factors used in the experiments, it can be shown that  $\delta_\beta \omega_2 k \leq \frac{2}{5}k$ , meaning that  $\alpha$  may restrict consideration to  $x_1 < \frac{1}{2}$ . Then, the expected utility to  $\alpha$  from proposing  $x_1$  can be written as

$$F_\beta(x_1)(1-x_1)k + (1-F_\beta(x_1)) \times \max\{A(\delta_\alpha(1-\omega_2)k, i_{\alpha,2}(\omega_2)), A(0,1)\}$$

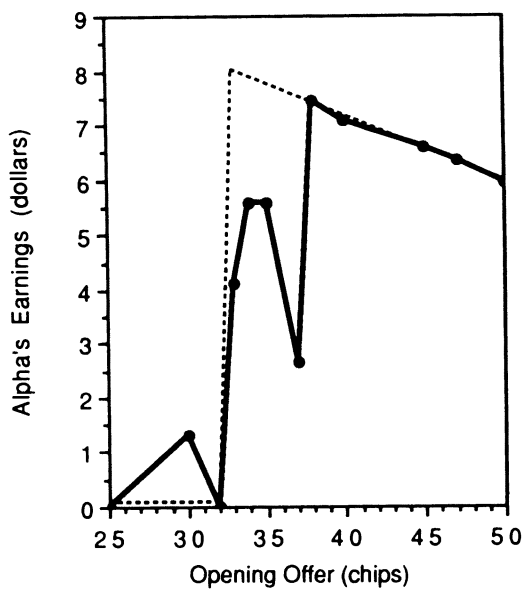
where  $\max\{A(\delta_\alpha(1-\omega_2)k, i_{\alpha,2}(\omega_2)), A(0,1)\} \in [0, \frac{2}{5}k]$ . Therefore, a rough approximation of  $\alpha$ 's expected utility is given by

$$F_\beta(x_1)(1-x_1)k + (1-F_\beta(x_1))\frac{1}{5}k$$

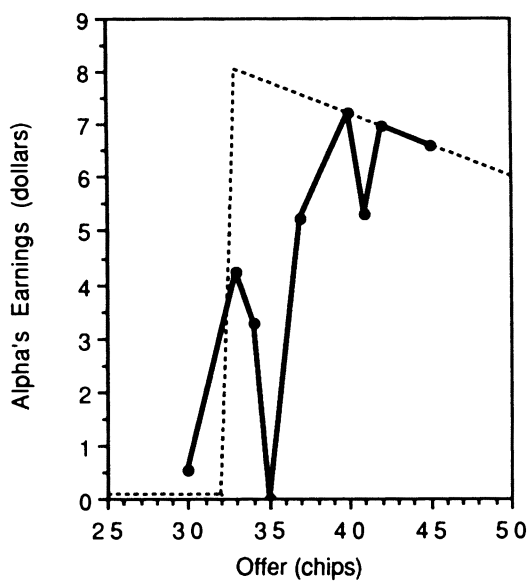
(i.e.,  $\alpha$ 's expected utility is *approximately* equal to the expected value of his first-period proposal. Let  $x_1^*$  be the offer that maximizes expected value. If  $x_1^*$  maximizes this expected value and  $F_\beta(x_1^*) = 1$ , then risk-averse proposers should prefer  $x_1^*$  as well.

Direct Money Split

Cell 1 (Inexperienced)

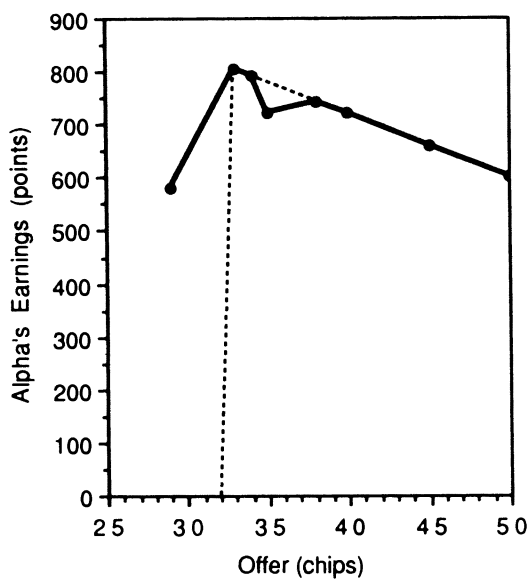


Cell 4 (Experienced)

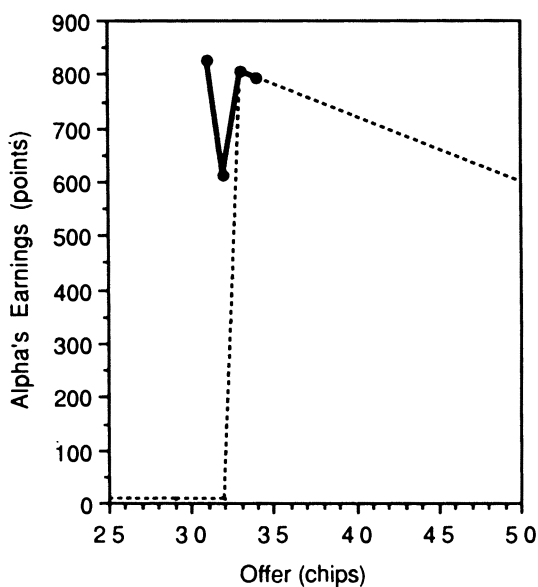


Tournament

Cell 5 (Inexperienced)



Cell 7 (Experienced)



—●— observed average  
 - - - - - pecuniary equilibrium prediction

FIGURE 8. AVERAGE EARNINGS FOR  $\alpha: (\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

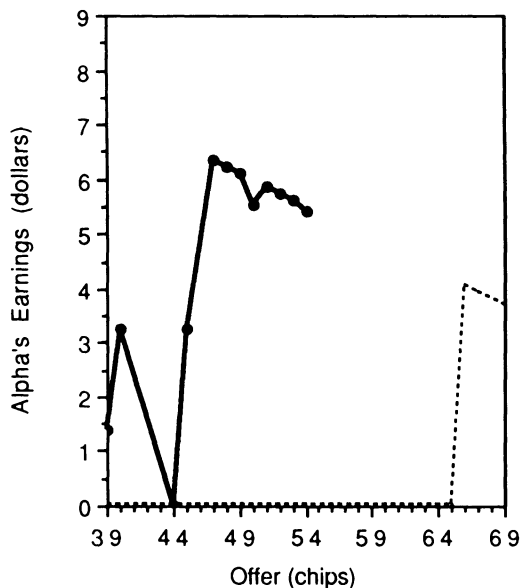
TABLE 3—OPENING OFFERS BY  $\alpha$  PLAYER:  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

A. Direct money split:						
Cell	Value of opening offer (dollars)				Total earnings	
	$\leq \$3.84$	$\$3.96-\$4.08$	$\$4.20-\$4.68$	$\leq \$4.80$		
1 (inexperienced, 8 rounds)		aaaa	a	aaa	\$56.40	
		aa		aaaaaa	\$53.40	
				aaaaaaaa	\$51.36	
			xxa	aaaaa	\$46.08	
	x	xaa		aaaa	\$44.72	
		xxxxaaa	a	a	\$43.48	
	x	x		aaaaaa	\$42.00	
	xx	xxa	a	aa	\$28.44	
	<u>a total</u>	0	12	4	35	
	<u>a + x total</u>	4	19	6	35	
4 (experienced, 7 rounds)			a	aaaaaa	\$50.52	
				aaaaaaa	\$49.20	
			x	aaaaaa	\$44.24	
				xaaaaaa	\$42.96	
			xxxxaaa		\$33.28	
		xxxxaaa			\$27.76	
	xx	xxxxa			\$15.32	
	<u>a total</u>	0	4	5	25	
	<u>a + x total</u>	2	12	9	26	
	B. Tournament:					
Cell	Value of opening offer (points)				Total points	
	$\leq 384$	396-408	420-468	$\geq 480$		
5 (inexperienced, 7 rounds)		aaaaaaa			5,544	
		aaaa	a	aa	5,304	
		aaa	aa	aa	5,280	
			xaaaaaa		4,896	
		xxaaa	a	a	4,484	
				aaaaaaa	4,200	
				aaaaaaa	4,200	
	<u>a total</u>	3	15	10	18	
	<u>a + x total</u>	5	15	11	18	
	7 (experienced, 6 rounds)		aaaaaa			4,968
			aaaaaa		4,824	
			aaaaaa		4,776	
			aaaaaa		4,752	
			aaaaaa		4,752	
		xaaa	aa		4,056	
<u>a total</u>		9	26	—	—	
<u>a + x total</u>		10	26	—	—	

Notes: a = accepted offer; x = rejected offer.

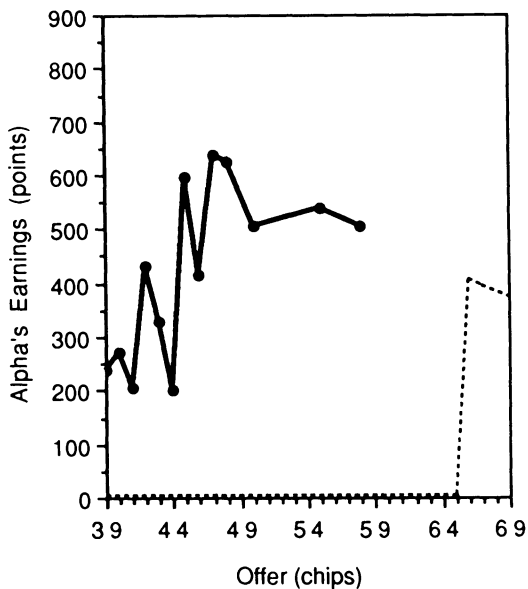
**Direct Money Split**

Cell 2 (Inexperienced)

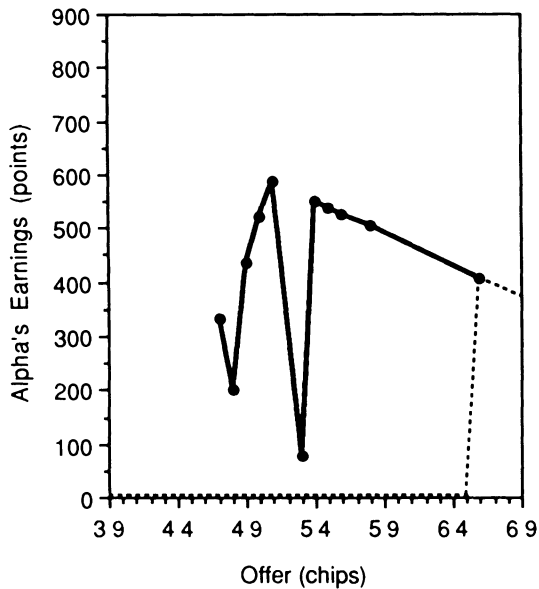


**Tournament**

Cell 6 (Inexperienced)



Cell 8 (Experienced)



—●— observed average  
 - - - - - pecuniary equilibrium prediction

FIGURE 9. AVERAGE EARNINGS FOR  $\alpha: (\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

TABLE 4—OPENING OFFERS BY  $\alpha$  PLAYER:  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

A. Direct money split:					
Cell	Value of opening offer				Total earnings
	≤ \$4.68	\$4.80–\$5.40	\$5.52–\$6.00	≥ \$6.12	
2 (inexperienced, 7 rounds)			aaaaaaa		\$42.00
			aaaaaaa		\$42.00
		x	aaaa	aa	\$37.64
		x	aaaa	aa	\$36.00
			xxaaaaa		\$30.01
	x	xxaa	aa		\$29.20
		xxxxaaa			\$27.32
<u>a total</u>	0	6	29	4	
<u>a + x total</u>	1	13	31	4	
B. Tournament:					
Cell	Value of opening offer (points)				Total points
	≤ 468	480–540	552–600	≥ 612	
6 (inexperienced, 7 rounds)			xxxxaaa		3,760
		xxaaaa	a		3,540
		xxa	aaaa		3,512
		a	xxaaaa		3,388
		xxa	aaaa		3,384
		xxa	x	aaa	2,504
	x	xxxxaa			2,204
<u>a total</u>	0	10	19	3	
<u>a + x total</u>	1	22	23	3	
8 (experienced, 7 rounds)			xxxxaaa		3,696
				xxxxaaa	3,248
			xxxxaaa		3,176
			xxxxaaa		3,176
				xxxxaaa	2,856
			xxxxaaa		2,856
			xxxxaaa	a	2,604
<u>a total</u>	—	—	23	14	
<u>a + x total</u>	—	—	34	15	

Notes: a = accepted offer; x = rejected offer.

[71 percent] than in cell 1 [64 percent]).<sup>22</sup> However, on a bargainer-by-bargainer basis, there is much less deviation across cate-

gories in cell 4 than in cell 1.<sup>23</sup> Thus, the similarity of the total distributions is deceiving: cell-4  $\alpha$  bargainers experimented much less with opening offers than did those in cell 1. In addition, the table sorts bargainers by their total earnings, and in cell 4, nearly

<sup>22</sup>Performing Pearson's chi-square test on the hypothesis that the distribution of offers is the same across categories yields a test statistic of 5.0749 (*d.f.* = 3). The hypothesis cannot be rejected at any conventional level of significance. Since there is no theoretical justification for the category definitions, the test has little statistical power, but nonetheless it provides some idea of how much variation there is between the two distributions.

<sup>23</sup>In fact, taking the absolute value of the difference between first- and last-round offers for each player, the average for cell 4 is approximately \$0.16 (less than 1.3 chips), while the average for cell 1 is approximately \$1.16 (about 10 chips).

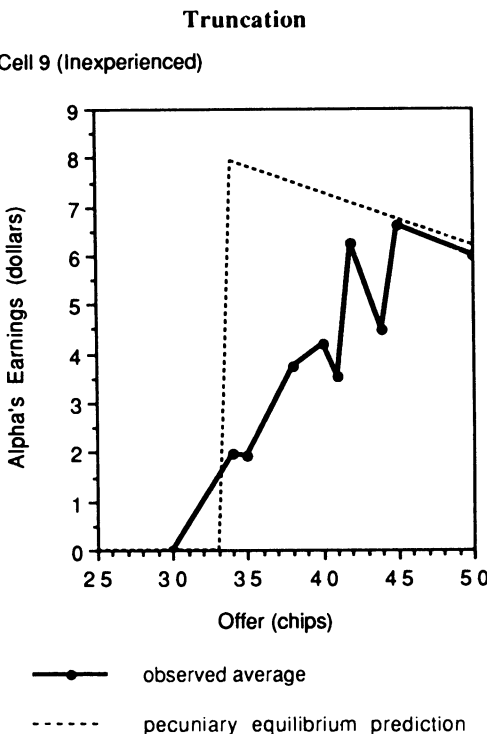
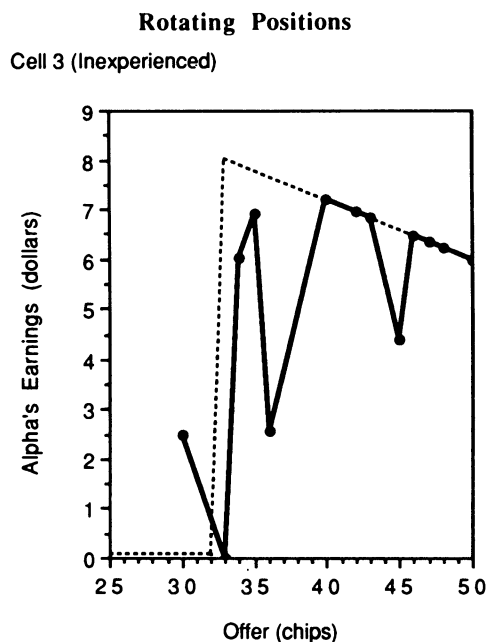


FIGURE 10. AVERAGE EARNINGS FOR  $\alpha$ , ROTATING POSITIONS AND TRUNCATION:  
 $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

all of the offers of the four bargainers making the most money had values of \$4.80 or more. Almost all of the offers of the three bargainers making the least money had lesser values. In fact, the three bargainers making the least, including the one who made nothing but pecuniary-equilibrium offers, would have made substantially more money (\$42.00) by always offering the equal money split (assuming this would always be accepted—a reasonable assumption, judging from the data).

The four top bargainers appear to have been aware of the information contained in Table 3: offers below \$4.80 get turned down a substantial proportion of the time,<sup>24</sup> and since rejections are very costly, it is best to keep offers at about \$4.80. This information, however, appears to be unevenly distributed among experienced bargainers, possibly due to the fact that some may have been  $\beta$  bargainers in cell 1 or may have participated in cell 2, where the discount parameters were different, and hence their experience was not quite as helpful. For a few cases, other explanations may be required.<sup>25</sup>

The description of the dynamic process as one of hill-climbing roughly characterizes the behavior of most subjects. The vast majority only move their offers up when they experience rejection and move them down only after acceptance, albeit, some are very

<sup>24</sup>It might be objected that the  $\alpha$  bargainer who made the most money in cell 1 also made a substantial number of pecuniary-model equilibrium offers. Note, however, that the other players who made a substantial number of equilibrium offers all finished in the bottom 50 percent in terms of money earnings. In short, the bargainer who made the most money got lucky.

<sup>25</sup>Unfortunately, it is very difficult to trace individual bargainers from inexperienced to experienced cells. However, due to particular circumstances, it was possible to track the perfect-equilibrium bargainer of cell 4 (who made \$27.76). He made mostly perfect-equilibrium offers in cell 1 (and made \$43.48). Apparently his experience in cell 1, in which he was rejected 43 percent of the time, did not have much of an impact on his thinking. One possible explanation is that, due to the luck of the draw, he was actually paid for two of the rounds in cell 1 in which his perfect-equilibrium offers were accepted (almost \$16 total). He was not so fortunate in cell 4 (making less than \$5).



TABLE 5—OPENING OFFERS BY  $\alpha$  PLAYER,  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$

*A. Rotating positions:*

Cell	Value of opening offer				Total earnings
	$\leq \$3.84$	$\$3.96-\$4.08$	$\$4.20-\$4.68$	$\geq \$4.80$	
3 (inexperienced, 11 rounds)		aaaa	a	a	\$37.56
		a	a	aaa	\$35.28
		xaaaa			\$34.32
		xaaa	a	a	\$33.24
			x	aaaaa	\$31.36
				aaaaaa	\$31.08
		aa	x	aa	\$30.40
				aaaaa	\$30.00
	x	xxaa	a		\$26.20
				xaaaaaa	\$25.80
	x	xxaa			\$22.48
		xx	a	aa	\$21.84
<i>a total</i>	0	18	5	30	
<i>a + x total</i>	$\frac{0}{2}$	$\frac{18}{26}$	$\frac{5}{7}$	$\frac{30}{31}$	

*B. Truncation:*

Cell	Value of opening offer				Total earnings
	$\leq \$3.84$	$\$3.96-\$4.08$	$\$4.20-\$4.68$	$\geq \$4.80$	
9 (inexperienced, 8 rounds)				aaaaaaaa	\$46.20
			xa	xaaaaa	\$42.84
			xa	xaaaaa	\$42.60
				xxxxxxxx	\$40.92
		xa	x	xaaaa	\$35.52
	x	x		xxxxxxx	\$25.92
		xxxxaa	x	xa	\$22.56
		xxxxxxa			\$7.92
<i>a total</i>	0	3	2	33	
<i>a + x total</i>	$\frac{0}{1}$	$\frac{3}{16}$	$\frac{2}{6}$	$\frac{33}{41}$	

Notes: a = accepted offer; x = rejected offer. For cell 3 (rotating positions), because there were 11 rounds, half the subjects made one more proposal than the other half. For the purpose of comparability, earnings for the last round, in which proposers were randomly chosen, are omitted.

slow to change (similar observations were made by Ochs and Roth [1989]).

*D. Changing the Value of the Pie*

It is easy to show that, if the value of the pie ( $k$ ) is increased, then the value of the first-period equilibrium offer ( $\omega_1 k$ ) must also increase [from equations (1) and (2)]. The interesting question, however, is how the  $\omega_1$  term shifts. The comparative model does not offer a clear-cut prediction. As an example of the type of additional assumption necessary to get determinance, let

$B_1(\omega_1)$  be shorthand for  $B_1(\omega_1 k, \omega_1 [1 - \omega_1])$ , and similarly let  $B_1(\omega_2)$  be shorthand for  $B_1(\delta_\beta \omega_2 k, 1)$ .

**PROPOSITION 7:** *Suppose that  $B_1(\omega_1) = B_1(\omega_2)$  and suppose that  $\omega_1 > \delta_\beta$ . If  $dk > 0$ , then  $d\omega_1 < 0$ .*

(See Appendix A for the proof.) For example, when  $\beta$ 's utility function is quasi-linear in the absolute money variable,  $B_1(\omega_1) = B_1(\omega_2)$ . In such a case, the marginal rate of substitution between absolute and relative money is independent of the value of  $k$

(i.e., the amount of absolute money that  $\beta$  is willing to exchange for more relative money is independent of the initial value of the pie). Consequently, when  $\omega_1 > \delta_\beta$ , an increase in the value of  $k$  allows for a decrease in the proportion of the pie offered to  $\beta$ .

For the more general case, the movement of  $\omega_1$  as  $k$  increases is indeterminant. However, the comparative model does shed light on the determining factors: from the perspective of a  $\beta$  bargainer, an increase in  $k$  has the impact of increasing the utility he expects if the game progresses to the second period [i.e., the value of the right-hand side of (2') increases]. Therefore, whether  $\omega_1$  rises or falls depends on a kind of income effect: it depends on how  $\beta$ 's relative preference for absolute and relative money is affected by a rise in the level of reservation welfare. Thus,  $\omega_1$  rises if fairness (relative money) is a "normal" good and falls if it is an "inferior" good.

So far the discussion has assumed complete information. Under incomplete information, there may be additional considerations: if fairness is an inferior good for most but not all bargainers,  $\omega_1$  might not fall and might even rise. To see why, recall that in cells 1 and 4, 40 chips appears to be the optimal offer. If chip values increase, inferiority of fairness for most  $\beta$ 's implies that fewer bargainers will reject offers just under 40. On the other hand, as the value of the chips increases so do the losses to an  $\alpha$  bargainer from a rejected offer (losses relative to a "noncontroversial" offer like 40). Even if the probability of a rejection decreases, it may be optimal for  $\alpha$  to continue to offer either the same or maybe even a slightly larger proportion of chips. It all depends on the new expected-value-of-offers curve in conjunction with  $\alpha$ 's risk posture.

How  $\omega_1$  will shift is ultimately an empirical question. There is little relevant data.<sup>26</sup>

<sup>26</sup>In Guth and Tietz (1988), the size of the pie is varied, but no clear trend emerges. Also, I do not think the study yields an appropriate test of the hypothesis, because ending the game in disagreement was the only disadvantageous counteroffer allowed.

### E. Truncation Games

A test of whether bargainer preferences are correctly specified by the comparative model is provided by cells 9 and 10. In these cells,  $\alpha$ 's second-period option of rejecting  $\beta$ 's offer was removed, in effect giving  $\beta$  dictatorial power over second-period settlements. The comparative model predicts that any second-period split will have  $\beta$  taking all 100 chips.

In the case of  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$ , the comparative-equilibrium first-period offer for the truncated game is greater than that for the nontruncated game: for the truncated game, equation (2') becomes

$$B(\omega_1 k, i_{\beta,1}(\omega_1)) = B(\frac{1}{3}k, 1).$$

Let  $(\omega'_1, \omega'_2)$  be the equilibrium of the corresponding nontruncated game. The offers must satisfy

$$B(\omega'_1 k, i_{\beta,1}(\omega_1)) = B(\delta_\beta \omega'_2 k, 1) < B(\frac{1}{3}k, 1).$$

The inequality follows because, in the nontruncated game,  $\beta$ 's second-period equilibrium offer must give  $\alpha$  a positive amount of the pie. It follows that  $\omega_1 > \omega'_1$ .

In the case of  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$ , the comparative equilibrium is identical to the pecuniary equilibrium: for the truncated game, equation (2') becomes

$$B(\omega_1 k, i_{\beta,1}(\omega_1)) = B(\frac{2}{3}k, 1)$$

and  $\omega_1 = \frac{2}{3}$  is the unique proposal satisfying the equation.

Intuitively, the truncation lowers the cost of relative money faced by  $\beta$ . Consequently,  $\beta$  will demand more relative money (i.e.,  $\alpha$ 's first-period equilibrium offer must be greater in the truncated case).

Note that the only effect truncation has on pecuniary-model predictions is to narrow the set of equilibria from two to one. Therefore, an alternative hypothesis is that no change will result from the truncation. The truncation of second-period play was the only design feature that differentiated cells

TABLE 6—MODEL PREDICTIONS AND TESTS: TRUNCATION

Predictions (comparative model only):

$$\mu_9 > \mu_1 \quad \text{and} \quad \mu_9 > \mu_4$$

$$\mu_{10} = \$8.04 \text{ (67 points)}$$

Tests:

Hypothesis ( $H_0$ )	<i>t</i> statistic	<i>d.f.</i>
$\mu_9 \leq \mu_1$	2.736	13.5
$\mu_9 \leq \mu_4$	2.153	12.0
$\mu_{10} = \$8.04$	0.000	7.0

Notes:  $\mu_i$  = observed opening offer in cell *i*. All *t* statistics were calculated using means and standard errors from the last round of the cell(s). Welch's two-mean test was used for two-mean comparisons (see Bickel and Doksum, 1977 pp. 218–9).

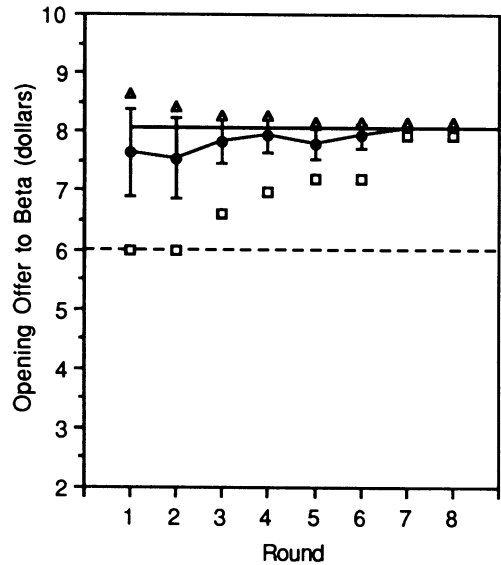
9 and 10 from their respective counterparts, cells 1 and 2.

First, consider cell 9. The statistical tests of the comparative-model predictions of opening offers are presented in Table 6, and the information is displayed graphically in Figure 5. The observed mean offer in the last round of play is greater in cell 9 than in either cells 1 or 4, and the difference is significant at all conventional levels. Figure 10 shows why  $\alpha$  bargainers increased their offers: whereas  $\beta$  bargainers in cells 1 and 4 almost always accepted offers of 40 chips or more, in cell 9 they began turning down offers of less than 45 chips on a fairly regular basis (24 percent of all offers between 40 and 44 chips were turned down). More than 39 percent of all initial offers were turned down (see Fig. 6), and all but one of these was for more than the pecuniary equilibrium, meaning that, for all but one case, no advantageous counteroffer was even possible. Seven of eight  $\beta$  bargainers made disadvantageous counteroffers. In all 25 relevant observations,  $\beta$ 's counteroffer gave him all 100 chips, so the comparative model performs well in cell 9.

In cell 10, the observed mean offers for the last two rounds are identical to the point prediction of the comparative model (Fig. 11). In fact, in these rounds there were only two offers that differed from the pre-

Truncation

Cell 10 (Inexperienced)



- $\Delta$  maximum observed offer
- mean plus 2 standard errors
- mean observed offer
- mean minus 2 standard errors
- minimum observed offer
- equal division
- ==== pecuniary equilibrium interval

FIGURE 11. MEAN OBSERVED OPENING OFFERS, TRUNCATION:  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

diction: one for one chip more and one for one chip less.

Cell 10 affords an opportunity to test whether bargainers have the types of preferences that are postulated by the comparative theory. Bargainers are assumed, all other things equal, to prefer more money to less, meaning (a)  $\beta$  bargainers should turn down all offers of less than 67 chips and (b) all second-period offers should have  $\beta$  demanding and receiving all 100 chips. Of the

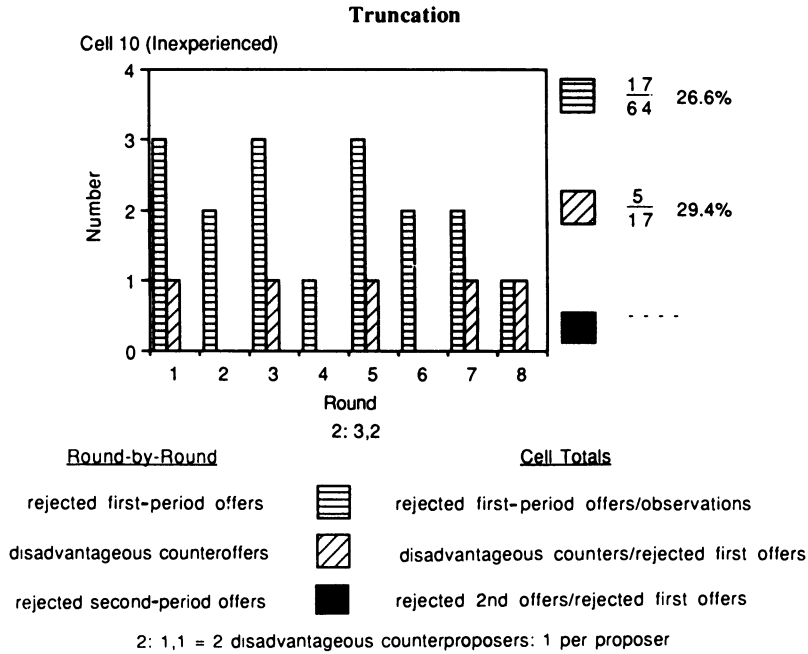


FIGURE 12. REJECTED OFFERS, TRUNCATION:  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

20 first-period offers of less than 67 chips made to  $\beta$ , 12 were turned down. Of the eight accepted, just one deviated more than two chips from the equilibrium, and only two others deviated by more than a single chip (the average deviation was two chips or \$0.24). Thus, while there seems to have been some token altruism, (a) appears to be reasonably consistent with the data. Of the 17 initial offers that were rejected, all but three (made by two bargainers) were followed by  $\beta$  taking all of the chips (average second-period offer to  $\alpha$  was slightly less than two chips; see Fig. 12), so (b) appears to be reasonably consistent as well.

Assumption 4 of the comparative model asserts that, once bargainers have achieved parity with their bargaining partners, they are no longer concerned with relative comparisons, only with earning more money. An alternative hypothesis is that bargainer utility is actually monotonically increasing over the entire domain of the relative argument. The difference is significant because, while weaker versions of most of the propositions

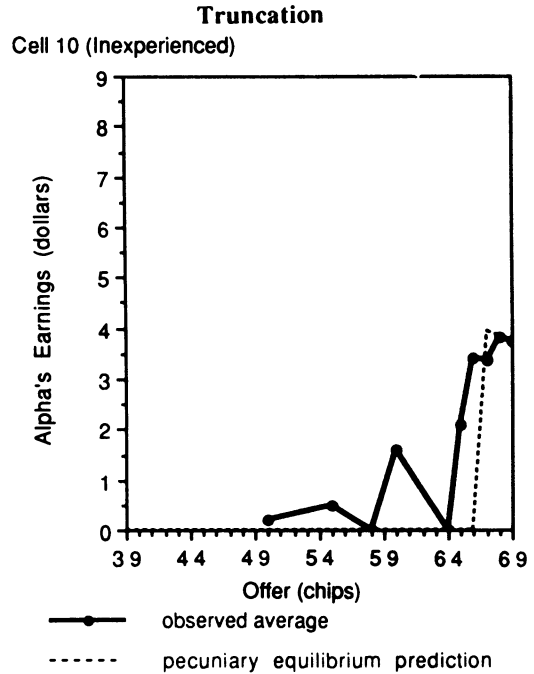


FIGURE 13. AVERAGE EARNINGS FOR  $\alpha$ , TRUNCATION:  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

TABLE 7—OPENING OFFERS BY  $\alpha$  PLAYER, TRUNCATION:  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

Cell	Value of opening offer				Total earnings
	$\leq \$7.08$	$\$7.20-\$7.92$	$\$8.04$	$\$8.16$	
10 (inexperienced, 8 rounds)			aaaaaaaa		\$31.68
				aaaaaaaa	\$29.76
		xaa	xaaa	a	\$24.36
		xaa	xaaaa		\$24.12
			xxaaaaaa		\$23.76
	x	xxa	aaaa		\$19.96
	x	xa	xaaa	a	\$19.80
	xxxx	xaa	a		\$13.52
a total	0	8	29	10	
a + x total	6	14	34	10	

Notes: a = accepted offer; x = rejected offer.

in Section II would still hold, Proposition 2 would be lost. Cell 10 provides a test. If Assumption 4 is valid, then offers of 67 or more chips should always be accepted. On the other hand, if utility is always monotonically increasing, then one would expect offers of 67 chips to be turned down, since doing so (and taking all 100 chips in the second period) would cost  $\beta$  only \$0.04 (i.e., for the cost of just \$0.04,  $\beta$  can obtain the highest possible relative value; see Fig. 13). Of the 44 offers in cell 10 that were at or above 67 chips, only five were rejected (see Table 7). All of those rejected were for exactly 67 chips, and these came from just two  $\beta$  bargainers. Neither of these bargainers was consistent: each accepted 67 chips on one occasion. Further, each accepted the one offer of more than 67 chips that she received: one for 69 chips and the other for 70 chips. Even if these two bargainers did receive utility for relative comparison values greater than parity, the additional utility from achieving the highest relative comparison possible would appear to be worth little to them—less than \$0.36. Assumption 4 appears to be reasonably accurate.

### III. Extended Version of the Comparative Model

#### A. Intuition

The assumption that a bargainer's welfare is determined, at least in part, by com-

paring the slice of pie he receives to that received by those with whom he splits the pie is key to the comparative model. An interesting implication of this assumption is that the behavior of the bargainer might be manipulated by changing the identity of his fellow pie-slicers. Consider, for instance, placing the standard two-period bargaining game in a *tournament* in which each  $\alpha$  bargainer's payoff depends on how successful he is at bargaining relative to other  $\alpha$  bargainers (same for  $\beta$ 's).<sup>27</sup> A bargainer now shares a payoff pie with his bargaining counterparts, rather than his bargaining partner, in the sense that a larger slice for one  $\alpha$  means the other  $\alpha$ 's will have less to share (the same is true for  $\beta$ 's). The way for a bargainer to obtain as large a *relative* slice as possible is to maximize his bargaining earnings (i.e., the bargainer should never make disadvantageous counteroffers). Since everyone will be doing the same, the pecuniary equilibrium should result.

#### B. Formal Model

Formalizing these ideas requires a little notation. A two-period alternating-offer bargaining game,  $g$ , consists of two players,

<sup>27</sup>A similar device was employed by Roger C. Kormendi and Charles R. Plott (1982) in a different sort of experiment.

$\alpha$  and  $\beta$ , playing either the direct-money-split or truncation game for 100 chips (note that the definition of  $g$  is distinct from the payoff function which assigns monetary values to the chips obtained). Define a *bargaining round-robin* as the quintuple,

$$G = \{g, N, M(t), S, H\};$$

where

- $g$  is a two-period alternating-offer bargaining game;
- $N = \{1, \dots, n\}$  is the set of bargainers;
- $M(t)$  is a mechanism, matching bargaining partners for each round ( $t = 1, 2, \dots, 0$ ), no two bargainers matched together more than once;
- $S = \{s | (s_1, \dots, s_n)\}$  is the strategy set induced by  $\{g, N, M\}$ ; and
- $H$  is the *monetary* payoff function,  $H(s) \rightarrow \mathbb{R}^n, s \in S$ .

Let  $U = \{U_1, \dots, U_n\}$  be the bargainers' utility functions defined over the payoff space  $H(S)$ .  $\{G, U\}$  constitutes a noncooperative game. Since no two bargainers are ever matched together more than once,  $\{G, U\}$  is finite. Without loss of generality,  $M(t)$  may be taken to determine all matches prior to the start of the round-robin.

A bargaining round-robin may be composed of several smaller bargaining round-robins. A *component* is a bargaining round-robin that cannot be decomposed into smaller ones. For example, for cells 1-4 and 9-10, each two-person bargaining game is a component. Only bargaining round-robins with a unique decomposition will be considered.

Suppose that  $G$  is composed of components  $G^1, \dots, G^c, \dots, G^r$ ; where  $G^c = \{g^c, N^c, M^c(t), S^c, H^c\}$ . Let  $P^c$  be the Pareto frontier of  $H^c(S^c)$ . A *pie*,  $D$ , is a subset of  $N^c$  such that (a)  $\sum_{j \in D} p_j$  is constant for all  $p \in P^c$ , and (b) there is no subset of  $D$  that satisfies (a). Intuitively, bargainers share a pie if a bigger slice for one means that the others necessarily receive a smaller slice. Attention will be restricted to bargaining round-robins in which, for any component,

$G^c$ , the set of all pies,  $K^c$ , is a nonintersecting cover of  $N^c$ .<sup>28</sup>

A description of  $U$  requires a characterization of the appropriate comparative indexes. This, in turn, requires a theory about what relative yardstick bargainers use to measure their earnings. My conjecture is that each bargainer compares his earnings to those of the bargainers with whom he shares a pie. Suppose that  $j \in D \in K^c$ . Denote a comparison index by  $i_j^D(q^D)$ , where  $q^D$  is the payoff vector associated with  $D$ . Let  $\{D_1, \dots, D_r\}$  be the set of pies that bargainer  $j$  participates in dividing during the course of  $G$ . Then bargainer  $j$ 's utility from receiving  $(q_j^{D_1}, \dots, q_j^{D_r})$  is given by

$$U_j(q_j^{D_1} + \dots + q_j^{D_r}, i_j^{D_1}, \dots, i_j^{D_r}).$$

For cells 1-4 and 9-10, each two-person bargaining game is a component. All of the results derived in Section II can be supported in the extension.<sup>29</sup>

In the tournament cells, chips were valued in points (detailed below). The number obtained by each bargainer was totaled at the end of the round-robin. The  $\alpha$  who obtained the most points relative to all  $\alpha$ 's received a fixed first prize, the  $\alpha$  with the second-most points received a fixed second prize, and so forth (the same was true for  $\beta$  bargainers). Each tournament therefore has only one component with two pies: one contains all  $\alpha$  bargainers; the other contains all

<sup>28</sup> $K^c$  is a nonintersecting cover of  $N^c$  if (i) for all  $D, E \in K^c, D \cap E = \emptyset$  and (ii) for  $j \in N^c, j \in D$  for some  $D \in K^c$ .

<sup>29</sup>A simple way to do this is to suppose that each  $\alpha$  has a utility function of the form

$$U_\alpha(q_j^{D_1} + \dots + q_j^{D_r}, i_j^{D_1}, \dots, i_j^{D_r}) = A(q_j^{D_1}, i_j^{D_1}) + \dots + A(q_j^{D_r}, i_j^{D_r}).$$

Suppose  $\beta$ 's utility function is additively separable in the same manner. Both  $A(\cdot)$  and  $B(\cdot)$  satisfy Assumptions 1-4. As in Section II-B, bargaining partners take each other's utility function as common knowledge. Then, the equilibrium of the round-robin game can be constructed directly from the comparative equilibria of the individual games. All derivations of Section II-B are valid in the extension.

$\beta$  bargainers. Since each pie contains more than two bargainers, it is difficult to say exactly what characteristics the comparison indexes should have. Even if the indexes are completely specified, there is a further problem: there are several ways to extend Assumptions 3 and 4 beyond the two-bargainer case. Fortunately, in order to specify the equilibrium of the tournament, these questions can be finessed: very weak assumptions are sufficient. Invoke Assumptions 1 and 2 plus the following two assumptions.

ASSUMPTION 5:  $i_j^D(p^D)$  is nondecreasing in  $p_j^D$ ;  $j \in D, D \in \{\alpha, \beta\}$ .

ASSUMPTION 6:  $U_j$  is nondecreasing in  $i_j^D$ ;  $j \in D, D \in \{\alpha, \beta\}$ .

Note that Assumptions 5 and 6 are weaker versions of Assumptions 3 and 4.

It will be assumed, in the following proof, that any time a bargainer is indifferent between accepting or rejecting an offer of zero chips, the offer is rejected with probability 1. This rule allows for the "uniqueness" result. Note that the solution concept is *trembling-hand* perfect equilibrium. Because of the nature of the information sets, there are no proper subgames in a tournament round-robin.

PROPOSITION 8: *Let*

$$G = \{g, N, M(t), S, H\}$$

*be a tournament bargaining round-robin. The unique perfect equilibrium for  $G$  has each bargainer playing the pecuniary equilibrium of the bargaining game  $g$ .*

PROOF:

The proof makes use of the facts that weakly dominated strategies are never played in perfect equilibrium and that all perfect equilibria are sequential equilibria (see David M. Kreps and Robert Wilson, 1982). Since players compare themselves to their counterparts and not to their bargaining partners, tournament utility functions are nondecreasing in the number of

points accumulated during the course of the round-robin.

Consider the final round of play and the information set at which  $\alpha$  responds to a given second-period offer. Because it is a weakly dominant strategy, in any perfect equilibrium,  $\alpha$  accepts an offer of a positive number of chips with probability 1 and, as assumed, rejects an offer of zero chips with probability 1. Now consider the information set at which  $\beta$  makes the second-period offer. Sequential equilibrium requires that, in conjunction with Bayes's rule and  $\alpha$ 's strategy choice,  $\beta$ 's strategy choice must maximize her expected utility. Since  $\alpha$  will accept any positive offer but reject zero chips, offering one chip and keeping the rest for herself is  $\beta$ 's unique perfect-equilibrium strategy. Now consider the information set at which  $\beta$  decides whether to accept or reject  $\alpha$ 's first-period offer. By once more applying the requirements of sequential equilibrium, it can be seen that  $\beta$ 's unique perfect-equilibrium strategy is to accept any offer greater than or equal to the pecuniary-equilibrium offer and to reject otherwise.

Now move to the next-to-last round of the tournament. Iterated application of the above reasoning completes the proof.

### C. Tournament Cells

Cells 5–8 were "tournaments": subjects played the basic bargaining game, except now they negotiated over points. In each game, the first-period value of each chip was 12 points. For cells 5 (inexperience) and 7 (experience),  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$ , so for  $\alpha$  and  $\beta$  second-period chip values were, respectively, 8 and 4 points. These values were reversed in cells 6 (inexperience) and 8 (experience). As with the direct money split, bargainer roles were not changed during the session, and each  $\alpha$  played each  $\beta$  exactly once. Individual points were summed across games, and totals were private information<sup>30</sup>

<sup>30</sup>A subject's accumulated total was displayed on the screen after every game.

until the conclusion of the session, when they were announced without attribution. Payoffs were made according to the number of total points a bargainer accumulated relative to other bargainers having the same type ( $\alpha$  or  $\beta$ ). The  $\alpha$  who obtained the most points relative to all  $\alpha$ 's received a fixed first prize, the  $\alpha$  with the second-most points received a fixed second prize, and so forth (the same was true for  $\beta$  bargainers). Payoff schedules were provided to subjects prior to play and were identical for both types (see Appendix B). Payoffs were designed to be comparable to the actual payoffs made in the direct-money-split cells with respect to maximum, minimum, and mean. Tied bargainers received the average of the awards assigned to the positions in which they finished. Experienced subjects were recruited in much the same way as for cell 4 (see Table 1 for breakdown).

Table 8 displays the predicted comparative-equilibrium bargaining splits for the tournament games. They are the same (in terms of chips) as those predicted by the pecuniary model for the analogous direct-money-split game.

Figure 1 presents (graphically) information about the average observed opening offers for cells 5 and 7. Offer behavior in the inexperienced tournament cell and the inexperienced direct-money-split cell is very similar. However, with experience, tournament subjects play in accordance with the predicted equilibrium. In fact, in cell 7, the observed means are virtually identical to the prediction and have very low standard errors. These observations are all confirmed statistically (see Table 8).

The equilibrium prediction for cells 6 and 8 are rejected at all conventional levels of significance (Table 8). Nevertheless, Figure 2 displays movement towards the predicted equilibrium as bargainers gain experience. The movement is particularly notable because in cell 8 the observed mean offer crosses the fifty-fifty monetary line (i.e., on average, the first-mover advantage disappears).

Thus,  $\alpha$  behavior is clearly different in the tournaments, and so is  $\beta$  behavior: there is only one disadvantageous counteroffer in

TABLE 8—MODEL PREDICTIONS AND TESTS: TOURNAMENT

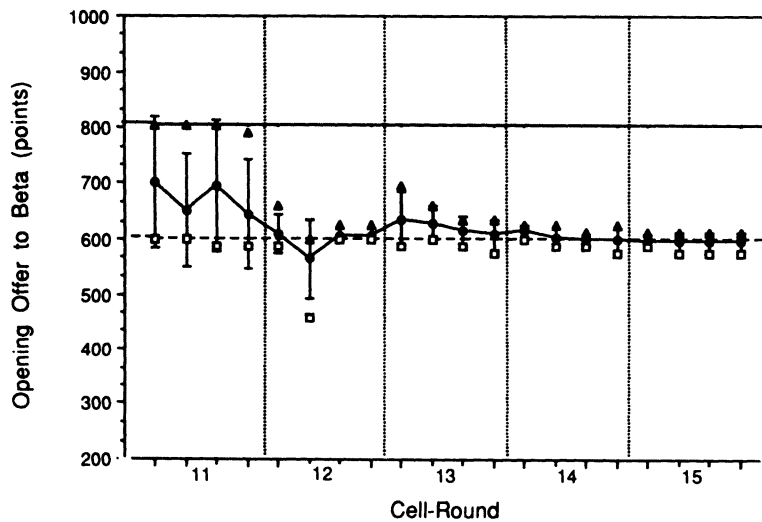
Predictions:		
$\mu_5 = \mu_7 = 396-408$ points (33-34 chips)		
$\mu_6 = \mu_8 = 792-804$ points (66-67 chips)		
Tests:		
Hypothesis ( $H_0$ )	<i>t</i> statistic	<i>d.f.</i>
$\mu_5 = 408$	0.934	6.00
$\mu_5 = 396$	1.219	6.00
$\mu_7 = 408$	-2.443	5.00
$\mu_7 = 396$	-0.349	5.00
$\mu_6 = 792$	-9.768	6.00
$\mu_8 = 792$	-5.621	6.00
$\mu_1 = \mu_5$	-0.171	8.37
$\mu_4 = \mu_7$	4.018	7.55
$\mu_2 = \mu_6$	-0.127	11.38
$\mu_2 = \mu_8$	-1.546	9.72

Notes:  $\mu_i$  = observed opening offer in cell *i*. All *t* statistics were calculated using means and standard errors from the last round of the cell(s). Welch's two-mean test was used for two-mean comparisons (see Bickel and Doksum, 1977 pp. 218-9).

cell 5 and none in cell 7. While there are several disadvantageous counteroffers in inexperienced cell 6, there is only one in experienced cell 8. Final rejection rates are also generally much lower than in the direct-money-split cells.

The dynamics that were developed earlier can be applied here as well: as in the direct money split,  $\alpha$  bargainers search for the offer that returns them the highest expected value. Now, however,  $\beta$  bargainers are not willing to make disadvantageous counteroffers. Consequently,  $\alpha$  bargainers are led to perfect-equilibrium offers. The movement is visible in the data: in cell 5, the peak of the average-observed-earnings curve has shifted to the equilibrium prediction, and it is actually just below this in cell 7 (Fig. 8). The story is somewhat more complex for cells 6 and 8. Note that, with experience, the earnings peak moves forward across the fifty-fifty split line and that the distribution of offers moves forward with the peak. (Tables 3 and 4 tell the same story on a disaggregated level.)





First Rejections	0	7	2	1	1
Disadvantageous Counteroffers	0	0	0	0	0
Second Rejections	0	6	2	0	1

—●— observed average  
 - - - - - pecuniary equilibrium prediction

FIGURE 14. RESULTS OF THE SUPEREXPERIENCE TOURNAMENTS:  
 $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$

The dynamics suggest that, if play in cell 8 were iterated a few more times, maybe offers would move to the predicted equilibrium. In order to test this, eight subjects were invited to play a series of five new tournaments, all identical in design to cell 8. Subjects were randomly chosen from a list of those who had participated in at least two prior tournaments (superexperience). The new tournaments were run in a single session, but the procedures were otherwise identical to the previous ones. In particular, subject types were randomly reassigned prior to each tournament. New tournament payoff schedules were comparable to the previous ones in terms of maximum, minimum, and mean. At the conclusion of the session, two tournaments were randomly chosen for payoff.

The results of the superexperience tournaments are summarized in Figure 14. Opening offers in the first tournament are very similar to those in cell 8, but over the next four tournaments they converge to the fifty-fifty split. Therefore, the equilibrium prediction fails. Nevertheless, behavior in these tournaments is very different from that in the direct money split (cell 2). In particular, over the course of the five tournaments there is not a single disadvantageous counteroffer, nor is there a first-mover advantage.

Of course, these data may imply that there is a problem with the comparative-model extension. On the other hand, they may be indicative of a flaw in the experimental tournament design: in the new tournaments, an  $\alpha$  bargainer whose offer was rejected

usually rejected the counteroffer. A majority of these rejected counteroffers came in tournament 2 and were spread fairly uniformly among three of the four  $\alpha$  bargainers. Suppose that tournament-2  $\alpha$  bargainers, upon experiencing a rejection, concluded that the number of points they were offered (usually around 80) was not sufficient to change how they finished relative to other  $\alpha$  bargainers and, therefore, not sufficient to change their payoff (remember that they were unaware that other  $\alpha$  bargainers were experiencing similar difficulties). Being otherwise indifferent, it would not be surprising if these  $\alpha$  bargainers rejected in order to hurt the  $\beta$  who put them in this situation (lexicographic preferences of this sort would not be inconsistent with the comparative model). As a result of this experience,  $\beta$  bargainers might conclude that, in the future, they should reject only if there exists an advantageous counteroffer giving  $\alpha$  a very substantial number of points. This would allow  $\alpha$  bargainers to be more aggressive. Indeed, after tournament 2, first rejections quickly tail off, and opening offers settle down to fifty-fifty.

The reason such scenarios are possible has to do with the ordinal nature of the design of the tournament payoff schedules. Since only the order of finish counts, a few points may not make a difference in an individual's payoff. The extended comparative model does not include such a feature. This incongruity between experiment and theory could be avoided by paying each subject according to the proportion of the total points accumulated by bargainers of the same type for which she is responsible. For example, if  $\alpha$  bargainers make a total of 100 points and a given  $\alpha$  bargainer is responsible for 30 of those points, then he would receive 30 percent of the payoff money allotted for  $\alpha$  types.

It should be stressed that the tournament data, discreteness and all, are very different than the direct-money-split data. In particular, experienced tournament players do not make first-period disadvantageous counteroffers, while experienced money-split players do. Moreover, the direction of data shifts is always consistent with the predicted direction, and in one case, the point predic-

tions are accurate. Thus, by several measures, the extended comparative model is a good, if somewhat rough, description of tournament play.

In addition, there is some evidence against one possible alternative explanation: in the tournaments, the payoff schedules for  $\alpha$  and  $\beta$  bargainers are identical and, hence, more equitable, at least in the sense of opportunity, than they are in the direct money split, where randomly assigned  $\alpha$  bargainers have a first-mover advantage. Thus, in the direct money split,  $\beta$  bargainers compensate for the inequity of opportunity by demanding equitable splits, while in the tournaments, they are content to take as many points as they can get, since doing so does not mean that they must settle for a smaller payment than the  $\alpha$  bargainers. This possibility is tested, albeit indirectly, by cell 3, where subjects took turns being  $\alpha$  bargainers, thus having equal opportunities to exploit the first-mover advantage. If the hypothesis were true, one would expect to see pecuniary-equilibrium results in cell 3, but this is not the case (see Section I-E).

#### IV. Conclusions

The key idea driving the comparative model is that bargainers appear to desire fairness *for themselves*, treating fairness for their partners as their partners' problem. Obtaining fairness does not appear to be a moral imperative: subjects consider the pecuniary price and have varying reservation values. Bargainers making proposals must take this into consideration or suffer the consequences. In fact, coping with this situation is the dominant strategic aspect of the game. The resulting behavior can be captured in a subgame-perfect-equilibrium model in which money and fairness (relative money) are incorporated into bargainer utility functions as substitutable goals.

The comparative model fits well with the qualitative regularities observed in previous experimental studies. In the experiments reported here, it predicts accurately when discount factors are switched between proposer and responder. It also predicts well when the second-period accept/reject node is truncated from the game. Some cells were

designed to test alternative explanations. In the resulting data, there is no evidence that greater subject experience leads to pecuniary-equilibrium play. In fact, with experience, mean offers remain unchanged, while standard errors shrink. Nor is there evidence that the equity of the experimental design has explanatory power.

Standard-model equilibrium results were obtained in both a truncation game and a tournament. The amount of money available in these games was comparable to that available in the direct money split. This implies that nonstandard-model play cannot be attributed to capriciousness resulting from insubstantial payoffs. This is not to say that there would be no change if the amount of money bargained over were increased. From the perspective of the comparative-model analysis, any such change pivots on whether fairness is a "normal" or "inferior" good, and if inferior, the risk posture of  $\alpha$  bargainers may also be a factor. What, if any, change would occur is presently a matter of speculation which can only be settled by further testing.

The tournaments provide some evidence that behavior can be manipulated by altering a bargainer's comparison group. The data have a competitive look: there are few disadvantageous counteroffers, and in one case, play was almost identical to the standard theory's prediction. Among other things, this suggests that the comparative model may be quite consistent with the standard economic theory of competitive markets, at least in some environments.

One might take the view that the experiments reported on in this paper simply demonstrate that there is an uncontrolled nonpecuniary variable present in utility functions. While it is true that the nonpecuniary variable is not suppressed (in spite of the pecuniary incentives offered), to come to such a conclusion is to miss the intended point: the nonpecuniary variable can be isolated and characterized. The proof is that outcomes can be manipulated on the basis of the characterization. Understanding the systematic influence of this variable on one class of experiments is a step toward understanding its influence in a larger domain. Although understanding does not imply the

ability to suppress,<sup>31</sup> it is not clear that suppression is (always) desirable: to assume that the lab results are interesting only if the nonpecuniary variable is suppressed is to assume that the nonpecuniary variable is not significant in the field. In fact, absent evidence to the contrary, the broader significance of the fairness motive is an open question.

I do not think it prudent to conclude that subjects who make disadvantageous counteroffers are acting irrationally simply because this behavior finds no ready explanation in standard theory. The fact that people vote in national elections, in spite of the virtually zero chance that one vote will influence the outcome, does not find ready explanation either. Nevertheless, voting is not considered irrational. Why are people, at least in some situations, willing to pay for fair treatment? It is a key question, as of yet without an answer.

#### APPENDIX A: PROOFS AND DERIVATIONS

##### PROOF OF LEMMA 1:

Suppose the statement is not true. Then, by Assumption 1 there exists open interval  $(i', i'')$ ,  $i' \geq 1$ , such that  $A_2 > 0$  for all  $i \in (i', i'')$  (the proof for the case of  $A_2 < 0$  is analogous). Then, it must be the case that, for arbitrary  $z$ ,

$$A(z, i'') - A(z, i') = \varepsilon > 0.$$

On the other hand, by Assumption 4, for all  $\delta > 0$  it is the case that

$$A(z + \delta, i') - A(z, i') = \varepsilon' > 0$$

and by Assumption 1,  $\delta$  can be chosen such that  $\varepsilon' < \varepsilon$ . Substituting the second expression into the first yields

$$A(z, i'') - A(z + \delta, i') = \varepsilon - \varepsilon' > 0$$

<sup>31</sup>Understanding may imply methods for suppression. For instance, the tournament design may be thought of as a restructuring of bargaining payoffs in an effort to suppress fairness concerns (although I find it more useful to think of the design in terms of fairness-manipulation).

which contradicts Assumption 4. The proof for utility function  $B$  is analogous.

**PROOF OF PROPOSITION 1:**

Define  $x_2^*$  by  $\delta_\alpha(1 - x_2^*) = \delta_\beta x_2^*$ . Monotonicity implies that  $A(\delta_\alpha(1 - x_2^*)k, 1) > A(0, 1)$ . It follows from continuity that there exists  $\omega_2$  satisfying (1). Let

$$x_1^* = \max\left[\delta_\beta \omega_2, \frac{1}{2}\right].$$

Consequently,  $x_1^*/(1 - x_1^*) \geq 1$ . By monotonicity,

$$B\left(kx_1^*, \frac{x_1^*}{1 - x_1^*}\right) > B(\delta_\beta \omega_2 k, 1).$$

By continuity there exists  $\omega_1 < x_1^*$  satisfying (2').

That  $\omega_2$  is the unique subgame-perfect-equilibrium strategy starting in the second period is clear from the monotonicity of the utility functions. The only thing to worry about with  $\omega_1$  is that it might be the case that

$$\begin{aligned} A(\delta_\alpha(1 - \omega_2)k, i_{\alpha,2}(\omega_2)) \\ > A(1 - \omega_1, i_{\alpha,1}(\omega_1)). \end{aligned}$$

That is,  $\alpha$  prefers the second-period offer to offering  $\omega_1$  in the first period, implying an incentive to deviate. To see that this can never happen, consider two cases:

*Case 1:*  $\delta_\beta \omega_2 \geq \frac{1}{2}$ .—From (2') it follows that  $\omega_1 = \delta_\beta \omega_2$ . Since  $0 < \delta_\beta < 1$ ,

$$\omega_2 > \omega_1 \Rightarrow 1 - \omega_1 > 1 - \omega_2 > \delta_\alpha(1 - \omega_2).$$

Thus, in absolute terms,  $\alpha$  gets more in the first period. On the other hand, from  $\omega_1 = \delta_\beta \omega_2$  and  $1 - \omega_1 > \delta_\alpha(1 - \omega_2)$ , it follows that

$$i_{\alpha,1}(\omega_1) = \frac{1 - \omega_1}{\omega_1} > \frac{\delta_\alpha(1 - \omega_2)}{\delta_\beta \omega_2} = i_{\alpha,2}(\omega_2)$$

which means that  $\alpha$  also gets more in comparative terms. Therefore,  $\alpha$  prefers receiving  $\omega_1$  in the first period to receiving  $\omega_2$  in the second.

*Case 2:*  $\delta_\beta \omega_2 < \frac{1}{2}$ .—In the second period,  $\beta$  never receives less than  $\alpha$ , so

$$\delta_\alpha(1 - \omega_2) < \frac{1}{2}.$$

On the other hand, from (2'), it must be that  $\omega_1 \leq \frac{1}{2}$  or equivalently  $1 - \omega_1 \geq \frac{1}{2}$ . Combining inequalities,

$$1 - \omega_1 > \delta_\alpha(1 - \omega_2).$$

Thus, in absolute terms,  $\alpha$  gets more in the first period. Since  $\omega_1 \leq \frac{1}{2}$ , then  $i_{\alpha,1}(\omega_1) \geq 1$ , and therefore, in comparative terms,  $\alpha$  gets at least as much in the first period. It follows that  $\alpha$  prefers receiving  $\omega_1$  in the first period to receiving  $\omega_2$  in the second.

Formal derivations of Propositions 3, 4, 5, and 7 require the total differentials of (1) and (2'):

$$\begin{aligned} \text{(A1)} \quad & \left[ -\delta_\alpha k A_1 - \frac{\delta_\alpha}{\delta_\beta \omega_2^2} A_2 \right] d\omega_2 \\ & + \delta_\alpha(1 - \omega_2) A_1 dk \\ & + \left[ (1 - \omega_2)k A_1 + \frac{(1 - \omega_2)}{\delta_\beta \omega_2} A_2 \right] d\delta_\alpha \\ & = \frac{\delta_\alpha(1 - \omega_2)}{\delta_\beta^2 \omega_2} A_2 d\delta_\beta \end{aligned}$$

$$\begin{aligned} \text{(A2)} \quad & \left[ kB_1(\omega_1) + \frac{1}{(1 - \omega_1)^2} \right] d\omega_1 \\ & + [\omega_1 B_1(\omega_1) - \delta_\beta \omega_2 B_1(\omega_2)] dk \\ & = \omega_2 kB_1(\omega_2) d\delta_\beta + \delta_\beta kB_1(\omega_2) d\omega_2 \end{aligned}$$

where  $B_1(\omega_1)$  is shorthand notation for  $B_1(\omega_1 k, \omega_1/[1 - \omega_1])$  and  $B_1(\omega_2)$  is shorthand for  $B_1(\delta_\beta \omega_2 k, 1)$ .

**PROOF OF PROPOSITION 3:**

Reduce (A1) and (A2) by setting  $dk = d\delta_\beta = 0$ . Signing the terms of (A1) yields  $d\omega_2 > 0$ . The proposition is then established by signing the terms of (A2).

**PROOF OF PROPOSITION 4:**

Reduce (A1) and (A2) by setting  $dk = 0$ . Signing the terms of (A1) yields  $d\omega_2 > 0$ .

The proposition is then established by signing the terms of (A2).

**PROOF OF PROPOSITION 5:**

Reduce (A1) and (A2) by setting  $dk = d\delta_\alpha = 0$ . Use (A1) to substitute  $d\omega_2$  out of (A2). Signing the result proves the proposition.

**PROOF OF PROPOSITION 6:**

Let  $\omega_2^* = \omega_2(Q, q)$ . Choose  $\lambda$  such that

$$Q\lambda = q\omega_2^* \Rightarrow \lambda = \frac{q}{Q}\omega_2^*$$

and since  $q < Q$ , then  $\lambda \in (0, 1)$  and is well defined as a potential offer. In fact, think of  $\lambda$  as a potential second-period offer when  $(\delta_\alpha, \delta_\beta) = (q, Q)$ . Note that  $\lambda$  has been chosen to give  $\beta$  the same money value under  $(q, Q)$  as  $\omega_2^*$  does under  $(Q, q)$ . Recall that in the second period,  $\beta$  never offers  $\alpha$  more money than  $\beta$  receives. Consequently,

$$\begin{aligned} q\omega_2^*k > Q(1 - \omega_2^*)k &\Rightarrow \omega_2^* > \frac{Q}{Q + q} \\ \Rightarrow (Q^2 - q^2)\omega_2^* &> Q^2 - Qq \\ \Rightarrow q(Q - q\omega_2^*) &> Q^2 - Q^2\omega_2^* \\ \Rightarrow q\left(1 - \frac{q}{Q}\omega_2^*\right) &> Q(1 - \omega_2^*) \\ \Rightarrow q(1 - \lambda) &> Q(1 - \omega_2^*). \end{aligned}$$

In absolute terms,  $\alpha$  gets more money from  $q(1 - \lambda)k$  than from  $Q(1 - \omega_2^*)k$ . Also, since  $\lambda$  was chosen so that  $\beta$  gets the same amount under either set of discount factors, it follows that  $\alpha$  prefers  $\lambda$  from a relative point of view as well. This means that

$$\begin{aligned} A\left(q(1 - \lambda)k, \frac{q(1 - \lambda)}{Q\lambda}\right) \\ > A\left(q(1 - \omega_2^*)k, \frac{Q(1 - \omega_2^*)}{q\omega_2^*}\right) \\ = A(0, 1). \end{aligned}$$

The continuity properties of  $A$  imply that there exists an equilibrium offer  $\lambda^*(q, Q) > \lambda$  such that  $Q\lambda^* > q\omega_2^*$ . Therefore, relative to the game with  $(\delta_\alpha, \delta_\beta) = (Q, q)$ ,  $\alpha$ 's first-period equilibrium offer in the game with  $(\delta_\alpha, \delta_\beta) = (q, Q)$  must be greater.

**PROOF OF PROPOSITION 7:**

Reduce (A1) and (A2) by setting  $d\delta_\alpha = d\delta_\beta = 0$ . Use (A1) to substitute  $d\omega_2$  out of (A2). By suitable rearrangement of the resulting equation, it is clear that the coefficient of  $dk$  will be positive if

$$\omega_1 - \delta_\beta\omega_2 - \delta_\beta(1 - \omega_2) > 0$$

which simplifies to  $\omega_1 > \delta_\beta$ , and this is assumed true. The proposition follows from signing the terms of (A2).

**Appendix B: Experiment Materials**

*Instructions*

Below are the complete instructions (exact transcript) for the tournament cells (5–8). Alterations for nontournament cells appear in brackets. Italicized sections were read aloud just prior to the beginning of the experiment.

Welcome to Simulab! Please read the instructions carefully. If at any time you have questions or problems, raise your hand and the monitor will assist you. From now until the end of the experiment, communication of any nature, with other participants, is prohibited.

This experiment is part of a study having to do with bargaining behavior. During the experiment you will participate in a series of bargaining games. For each game, you will be matched with one of the other participants present in the room. You will never be matched

with the same person more than once. All matches will be anonymous: you will not know the identity of the person you are matched with, nor will they know yours, nor will these identities be revealed after the experiment is completed.

*Each game is played by one "Alpha" player and one "Beta" player. If you are an Alpha player for one game, you will be an Alpha player for all games. The same applies to Beta players. The actual type you are assigned to be will be determined by a coin flip just prior to the beginning of the experiment. [For cell 3, the following was substituted for the last two sentences: All participants will alternate between types in such a way that for half the games they play they will be an Alpha type and for the other half they will be a Beta type. (If we play an odd number of games, then types for the last game played will be established by the flip of a coin.)]*

Each game consists of two playing periods. During these periods, players take turns proposing ways to divide between them 100 (abstract) chips. In Period 1, Alpha proposes a division. If Beta accepts this division the game ends and each player receives the number of chips designated by Alpha's proposal. However, if Beta rejects Alpha's proposal, the game proceeds to Period 2 and now Beta proposes a division of the 100 chips. If Alpha accepts the new division, the game ends and each player receives the number of chips designated by Beta's proposal. However, if Alpha rejects the Period 2 proposal, the game ends and both players receive zero chips.

*Note that the number of chips to be split is always constant at 100. So you may always use 100 chips in a proposal. You may use less than 100 chips, if you like, but you may never use more than 100 chips.*

Each chip has a point value. [For all non-tournament cells: Each chip has money value.] Alpha's chip values may differ from those of Beta's and chip values are always higher in Period 1 than in Period 2. At the beginning of the game, the computer will inform you what the chip values will be (the chip values will be the same from game to game). Bargaining partners will know each other's chip values as well as their own.

You can calculate the value of a proposal to you by multiplying the relevant chip value to the number of chips that you will receive if the proposal is accepted. For example, suppose a Period 1 proposal calls for you to receive  $Z$  chips and your Period 1 chip value is  $P$  Points per chip [for all non-tournament cells:  $P$  dollars per chip]. Then, provided it is accepted, the proposal's worth to you is  $(P \times Z)$  Points [dollars]. A totally analogous calculation determines the value of the proposal to your bargaining partner. For your convenience, the computer will automatically calculate and display the value of any proposal for both you and your bargaining partner. Scratch paper and a pen have been provided to you for any other calculations that you might wish to make.

*At the completion of each game, fill out a Bargaining Record form (several blanks should be laying [sic] next to the computer). Completed Bargaining Records provide you with a history of the past games that you participated in and you may reference them at any time during the experiment.*

*You will play enough games so that each Alpha player will be matched exactly once with each Beta player. At the end of the experiment, the number of points that you receive for each game will be summed up. The Alpha player and the Beta player with the highest number of total points will each receive a cash award of equal value. A smaller cash award will go to the Alpha player and the Beta player with the second highest number of total points, etc. A complete list of the cash awards has been provided to you. Note that the cash awards for Alpha players are identical to those for Beta players.*

*In case of a tie, the players involved will each receive the average of the relevant awards. For example, if two Alpha players tie for first place, each would receive a cash award equal to the average of the first and second place awards. The Alpha player with the third highest total points would then receive the third place award, etc.*

[For all non-tournament cells, the following was substituted for the last two paragraphs: *You will be paid the money that you make for two of the games that you play. We will play more*

than two games. The two that you are paid for will be determined by a lottery to be held at the conclusion of the experiment. Since you will not know in advance which games will count, it is in your interest to make as much money as you can in each and every game played. The amount you make will be completely confidential. The money is yours to do with as you please.]

[For tournament cells 11–15: We will repeat this experiment several times. You will be paid for two runs which will be determined by a lottery at the conclusion of the session.] Since the amount of money you make is determined by the total number of points that you accumulate, it is in your interest to make as many points as you can in each and every game that you play [in each and every experiment]. You will be paid your cash award immediately upon completion of the experiment. The amount that you make will be completely confidential. The money is yours to do with as you please.

A note about operating the computer: whenever it is your turn to make a proposal or to respond to one, you are not committed to any particular course of action until you have pressed the “y” key when the “Verification” message is on the screen. Until then, you may freely experiment with alternative courses of action without any commitment or loss of options. However, once you have pressed the “y” key while the Verification message is on the screen, your proposal or response is sent to your bargaining partner and it cannot be recalled. So before pressing the “y” key, be sure to check the screen to see that the computer is sending the message that you think it is sending.

In the time remaining before the session begins, play some practice games. Practice until you feel comfortable with how the game is played. Be aware that the practice games differ from the real games in three ways. First, no money will be paid for the practice games. Second, in the practice games you will be able to experience being both an Alpha type and a Beta type. In the actual experiment you will be either one or the other every game. Third, in the practice games your bargaining partner will be the computer. [For cell 3, the following was substituted for the last five sentences: Be aware that the practice games differ from the real games in two ways. First, no money will be paid for the practice games. Second, in the practice games your bargaining partner will be the computer.] The computer’s responses and proposals are generated randomly, so they won’t make any sense. This, however, should not deter you from becoming accustomed to how the game is played and to how data is entered into the computer.

You may return to these directions between practice games if you wish to do so.

#### *Tournament Payoff Schedules*

*Cells 5, 6, and 8:*

Alpha		Beta	
Total points	Award	Total points	Award
Highest	\$16	Highest	\$16
Second	\$14	Second	\$14
Third	\$12	Third	\$12
Fourth	\$10	Fourth	\$10
Fifth	\$ 8	Fifth	\$ 8
Sixth	\$ 5	Sixth	\$ 5
Lowest	\$ 5	Lowest	\$ 5

*Cell 7:*

Alpha		Beta	
Total points	Award	Total points	Award
Highest	\$16	Highest	\$16
Second	\$13	Second	\$13
Third	\$11	Third	\$11

Fourth	\$ 9	Fourth	\$ 9
Fifth	\$ 6	Fifth	\$ 6
Lowest	\$ 5	Lowest	\$ 5

Cells 11–15:

Alpha		Beta	
Total points	Award	Total points	Award
Highest	\$16	Highest	\$16
Second	\$12	Second	\$12
Third	\$ 8	Third	\$ 8
Lowest	\$ 5	Lowest	\$ 5

*Screen Facsimile and Postsession Questionnaire (Exact Transcript)*

Suppose you are going to play one more round of the game. You are the Alpha player and the computer is the Beta player. You know that the computer has been programmed to adhere to the following strategy: reject Alpha's Period 1 proposal only if there is some Period 2 proposal which, *if* accepted, would give Beta more points.

Below is a representation of what the screen would look like for this game. At the bottom, fill in the Period 1 proposal that you, as an Alpha player, would make.

Bargaining Record: you are an *Alpha* player

*Alpha Chip Values*

Period 1: 12 Points per Chip

Period 2: 8 Points per Chip

*Beta Chip Values*

Period 1: 12 Points per Chip

Period 2: 4 Points per Chip

All proposals must involve 100 chips or less

\* \* \*

Period 1 Proposal: Enter the number of chips Alpha receives: \_\_\_\_\_

Enter the number of chips Beta receives: \_\_\_\_\_

*Bargaining Record (Exact Transcript)*

Game \_\_\_\_\_ Bargaining Record

You are a (*circle one*) Alpha Beta player.

*Alpha Chip Values*

Period 1: \$0.12 per chip

Period 2: \$0.08 per chip

*Beta Chip Values*

Period 1: \$0.12 per chip

Period 2: \$0.04 per chip

Period 1      Alpha's proposal is:  
                  Alpha receives \_\_\_\_\_ chips      Beta receives \_\_\_\_\_ chips  
                  Value: Alpha receives \$ \_\_\_\_\_      Beta receives \$ \_\_\_\_\_  
                  Beta (*circle one*) accepts    rejects    Alpha's proposal.

Period 2      Beta's proposal is:  
                  Alpha receives \_\_\_\_\_ chips      Beta receives \_\_\_\_\_ chips  
                  Value: Alpha receives \$ \_\_\_\_\_      Beta receives \$ \_\_\_\_\_  
                  Alpha (*circle one*) accepts    rejects    Beta's proposal.

Summary: You receive \_\_\_\_\_ chips with a value of \$ \_\_\_\_\_.



## APPENDIX C:

## A NOTE ON COGNITIVE UNDERSTANDING

It has been suggested that disadvantageous counteroffers are signs that participants did not fully understand the game. The argument seems to be that disadvantageous counteroffers are the result of subject confusion about the values of alternative available actions. This assertion would seem to be refuted by the results of the truncation and tournament cells, in which a slight modification of the game dramatically reduces the frequency of disadvantageous counteroffers. Also, the computer calculated the value of all proposals and displayed them to subjects, so it is not clear to me what the source of confusion is supposed to be. Nevertheless, there is the following further evidence.

At the end of each session, subjects were asked, in writing, to consider playing the game once more, as an  $\alpha$  bargainer, with a computer as  $\beta$  partner (see screen facsimile and postsession questionnaire in Appendix B). Subjects were told that the computer was programmed to reject  $\alpha$ 's offer only if there existed some second-period counteroffer that would, if accepted, yield  $\beta$  more money (more points in the case of tournament cells); this is just a description of  $\beta$ 's pecuniary-equilibrium strategy.

In inexperienced cells with discount factors  $(\delta_\alpha, \delta_\beta) = (\frac{2}{3}, \frac{1}{3})$  (cells 1, 3, 5, and 9), 71 percent of the participants gave the pecuniary-equilibrium answer (79 percent if one allows for a one-chip error). Also, there was no correlation between making disadvantageous counteroffers and giving the equilibrium answer: 74 percent of the 23 subjects who made disadvantageous counteroffers answered with the pecuniary-equilibrium response.

Due to a clerical error, participants in two inexperienced cells with discount factors  $(\delta_\alpha, \delta_\beta) = (\frac{1}{3}, \frac{2}{3})$  (cells 2 and 6) received the same question as participants in cells 1, 3, and 5 (i.e., the chip values were not adjusted to reflect the discount factors for these sessions). Nevertheless, 46 percent gave the pecuniary-equilibrium answer (61 percent if one allows for a one-chip error

and answers that gave the reverse of the equilibrium, which is the correct equilibrium for cells 2 and 6). None of the disadvantageous counteroffers answered with the pecuniary-equilibrium response, but there were only four such bargainers. Participants in cell 10 received the question with the same discount factors that they played with. Fully 100 percent answered the question correctly.

Of course, the question was not a test; there is no right answer. Nevertheless, the answers given indicate that a high percentage of participants were capable of calculating the pecuniary-equilibrium offer. In addition, there is some evidence that the play of the preceding cell influenced many of the nonequilibrium answers: 76 percent of these were in the direction of the observed average settlement. A similar trend is observed when moving from inexperienced to experienced cells: in cell 4, where average observed opening offers were greater than the pecuniary equilibrium, the percentage of equilibrium answers actually decreased relative to cell 1. In cells 7 and 8, where offers were at or closer to the equilibrium, the percentage rose relative to cells 5 and 6. These results suggest that some subjects may have interpreted the question as a request for advice on how to play the game with other people.

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