GAMES AND ECONOMIC BEHAVIOR 6, 299–311 (1994)

Rationalizable Conjectural Equilibrium: Between Nash and Rationalizability*

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Received March 6, 1991

Static equilibria can be viewed as steady states of recurring play of a game. Such steady states in which players do not perfectly observe the actions of others need not be Nash equilibria. This paper suggests a static solution concept, *rationalizable conjectural equilibrium*, that corresponds to such steady states. To present it, the basic model of a normal form game is enriched by specifying the signals that players get about others' actions. The solution is a profile of actions such that each player's action is optimal given that it is common knowledge that all players maximize utility given their signals. *Journal of Economic Literature* Classification Number C72.

1. INTRODUCTION

Games are used as models of two very different scenarios. Sometimes they model behavior in isolated situations in which players' expectations of what others might do are not based on any information beyond the

* We thank Paulo Battigali for his valuable comments. Research support from the BSF (Grant 90-00165/1) is gratefully acknowledged. Ariel Rubinstein thanks the Department of Economics at Northwestern University for its hospitality during which period some of this research was conducted. Asher Wolinsky gratefully acknowledges support from the NSF (Grant SES-9009509).

0899-8256/94 $6.00

basic description of the game. At other times, games are used to model behavior in recurring situations, where the game in question or similar ones repeat often enough and expectations about others' actions can be related to past experience.

The solution that we propose in this paper is motivated by scenarios of the second type. Although this solution is purely "static," what we have in mind is a game which is played repeatedly with no strategic links among the repetitions. That is, in each repetition the players are interested only in maximizing their instantaneous payoff—neither are they concerned about the effect their actions might have on future beliefs of others, nor are they interested in sacrificing current payoffs for learning more about their current opponents. To fix ideas, one may think that the same game is being played by a sequence of distinct players. Although each player plays the game only once, he has some information on how the game was played in the past and on this he bases his expectations about others' actions. A *steady state* of this repeated interaction is such that, in all repetitions, the players who occupy any position play the same strategy which is best response, given the available information about the past actions of others. We view such a steady state as an equilibrium of the stage game. If players perfectly observe the past actions of all players, such a steady state will be a Nash equilibrium of the stage game. If, however, players have only an imperfect knowledge of past actions, the steady state will not necessarily be a Nash equilibrium, since their information may be insufficient to figure out a profitable deviation. This may be the case, for example, if each player knows the actions and payoffs of the players who occupied his position in the past but not the actions or payoffs of others.

To formalize these ideas we enrich the model by specifying the signal that each player has about the actions taken by the others. The solution we present in this paper, which we call *rationalizable conjectural equilibrium (RCE)*, is a profile of actions such that each player's action is optimal, given that it is common knowledge that all players maximize their expected utility relative to their information. In other words each player chooses an action which maximizes his payoff given a conjecture regarding the actions of the others; each agent's conjectures are consistent with his signal and his own choice; the conjectures are also consistent with the understanding that everybody rationalizes his action in this manner. If players do not get any information, the RCE coincides with rationalizability, and if they get exact information about their opponents' actions, it coincides with Nash equilibrium. Thus, the RCE occupies an intermediary position between Nash equilibrium on the one hand and rationalizability style (Bernheim, 1984; Pearce, 1984) on the other hand.

There are two reasons why modelers may be interested in RCE. First, as a solution concept in its own right which refines the rationalizability concept in a nontrivial way and still is not equivalent to Nash equilibrium. Second, in games in which, given some natural signal, the RCE coincides with Nash equilibrium, the Nash equilibrium concept is more compelling because the equilibrium requires less information on the part of the players.

Some related concepts are suggested in the literature. First, Battigalli and Guaitoli (1988) present the notion of conjectural equilibrium. This concept requires a player's action to be a best response to some conjecture which is consistent with the signal he gets but does not impose any rationality conditions on the conjecture assignment about the other players' actions. Fudenberg and Kreps (1988) present a particular example of conjectural equilibrium. They point out that, if a player just observes the outcome of an extensive form game, then the RCE does not have to be a Nash equilibrium. Fudenberg and Levine (1990) present the notion of self-confirming equilibrium. This is an RCE in which each player's signal is the others' strategy in all information sets that are reached with positive probability.

Before we proceed to the more formal parts of the discussion, it is useful to clarify the methodological position of this work. As emphasized above, we are proposing a purely "*static*" solution concept. While we do not analyze nor even pose any dynamic model, we allude informally to a dynamic scenario in order to motivate our interest in this concept. Thus, from a methodological point of view alone, this work resembles the work on the ESS concept, which is also the static concept motivated by reference to population dynamics. While it is easy to construct stylized dynamic scenarios that support these solutions, complementary research which looks at rich learning models might be very useful in assessing the robustness of these solutions.

2. Notation and Definitions

We consider games in normal form. The set of players is $N = \{1, 2, \ldots, n\}$; the set of player i's actions is A_i; the set of outcomes is $A = A_1 \times A_2 \times \cdots \times A_n$; and i's payoff function is u_i: $A \rightarrow R$. The new ingredients that we add to the description of the game are the signal functions g_i: $A \rightarrow S_i$. The element $s_i = g_i(a)$ is the signal that player i *privately* observes when all players choose the actions that make up the profile a. It is assumed that the functions g_i (as well as the other components of the model) are common knowledge (but the actual signals are not). In what follows we refer to $\langle N, (A_i)_{i\in N}, (u_i)_{i\in N}, (g_i)_{i\in N}\rangle$ as a game.

The solution concept that we are about to present is such that each player's action is optimal, given a player's conjecture about what the other players' actions will be. The player's conjecture has to be consistent with the signal that he has on the actions the others intend to take, with the knowledge of his own action and with the knowledge that the other players follow similar reasoning.

DEFINITION 1. The action–signal pair (a_i, s_i) is *g-rationalized* by a probability measure μ on $A_{-i} = \times_{j\neq i} A_j$ if

(i) for all $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ in the support of μ, $g_i(a_i, a_{-i}) = s_i$

(ii) a_i is best response, given μ.

DEFINITION 2. The sets of signal–action pairs $B_1, B_2, \ldots, B_n$ are *g-rationalizable* if, for all i, every (a_i, s_i) in B_i is g-rationalized by a probability measure μ on A_{-i} such that for all a_{-i} in the support of μ and for all j, $(a_j, g_j(a_i, a_{-i})) \in B_j$.

DEFINITION 3. A *rationalizable conjectural equilibrium (RCE)* is an n-tuple of actions $a^* = (a_1^*, \ldots, a_n^*)$ such that for some g-rationalizable sets $B_1, B_2, \ldots, B_n$ and for all i, $(a_i^*, g_i(a^*)) \in B_i$.

Note that, when $g_i(a) = a$ for all i, the concept of RCE coincides with a pure strategy Nash equilibrium. When $g_i(a)$ is independent of a_{-i}, this concept coincides with the correlated-conjectures version of Bernheim and Pearce's rationalizability.

Definition 2 requires that, for each pair $(a_i, s_i) \in B_i$, the action a_i is best response against a belief on vectors a_{-i} which are consistent with player i's action a_i and the signal s_i, and for each pair $(a_j, g_j(a_{-i}, a_i))$ the action a_j is j's best response against a belief on vectors b_{-j} which are consistent with player j's action a_j and the signal $g_j(a_{-i}, a_i)$ and so on.

A concept related to the RCE is Battigalli and Guaitoli's (1988) conjectural equilibrium (CE). The CE requires that a player behave rationally in the sense that his action is a best response given the knowledge of the player about his action, the signal, and the game, but it does not require common knowledge of rationality. Every RCE is a CE.

DEFINITION 4. A *conjectural equilibrium* (CE) is an n-tuple of actions $a^* = (a_1^*, \ldots, a_n^*)$ such that for all i the pair $(a_i^*, g_i(a^*))$ is g-rationalized by some probability measure A_{-i}.

Note that the existence of an RCE is not an issue of independent interest: since every Nash equilibrium is an RCE, existence is implied by existence of Nash equilibrium.

The rest of the paper is devoted mainly to examples of games which clarify the RCE concept and compare its predictions with those of other solution concepts.

3. Examples

The case of *public signals* is such that each player can infer from his own signal what signals are held by the other players (i.e., $g_i(a) = g_i(a')$ if and only if $g_j(a) = g_j(a')$). In the next three examples the signals are public.

(a) *The Distance Game*

Two players choose locations in the unit interval. Their payoffs are decreasing in the distance between their locations, so there is no conflict of interest and both want to coordinate their location at the same point. Thus, $N = \{1, 2\}$, $A_i = [0, 1]$, and $u_i(a_1, a_2) = L(d(a_1, a_2))$, where L is any decreasing function and $d(a_1, a_2) = |a_1 - a_2|$. It is immediately verifiable that the set of Nash equilibria includes all pairs (a_1, a_2) such that $a_1 = a_2$ and that all possible pairs of locations are consistent with rationalizability.

If the signal is the distance, i.e., $g_i(a_1, a_2) = d(a_1, a_2)$, then the set of conjectural equilibria includes all (a_1, a_2) such that neither of the locations is a distance of less than $d(a_1, a_2)$ from the edges of the interval.

Claim. *The* RCE *are all pairs* (a_1, a_2) *such that* $a_1 = a_2$.

Proof. Suppose there is an RCE (a_1, a_2) such that $d(a_1, a_2) = d > 0$. This means that there are g-rationalizable signal–action sets B_1 and B_2, as described in the definition, such that (a_i, d) belongs to B_i. The pair (a_i, d) can be g-rationalized only by a conjecture which puts positive probabilities on both $a_i - d$ and $a_i + d$. This is because the action a_i is not a best response against a conjecture which puts probability zero on one of the actions $a_i - d$ or $a_i + d$. Thus, since $(a_i, d) \in B_i$, both $(a_i - d, d)$ and $(a_i + d, d)$ must be in B_j. Define $m_i = \max\{a_i: (a_i, d) \in B_i\}$. (If there is no maximum, follow the argument with sup.) From the above, $(m_i + d, d)$ must belong to B_j and therefore $(m_i + 2d, d)$ must belong to B_i, in contradiction to the maximality of m_i. ∎

Thus, the RCE coincide here with the Nash equilibria, although players have less information than is required by the Nash equilibrium.

(b) *Two-Characteristic Quality Competition Game*

Two firms compete on the business of a fixed population of buyers by selecting for their products two characteristics (x_1, x_2) in $\{1, 2, 3\} \times \{1, 2, 3\}$. The action of firm i is denoted by (x_1^i, x_2^i). The total measure of the buyer population is 1. The population is evenly split so that half of it attributes lexicographic priority to the first characteristic and the other to the second characteristic. Thus, if for example $x_1^i < x_1^j$ and $x_2^i \leq x_2^j$ then firm j gets the whole market, while if $x_1^i < x_1^j$ and $x_2^i > x_2^j$ then both firms split the market equally. When choosing (x_1, x_2), a firm incurs a fixed cost of $x_1 + x_2$. A firm's gross profit (before subtracting the fixed cost) as a function of its market share is as follows:

Market share	Gross profit	Marginal profit
1	5.5	1.5
0.5	4	4
0	0	

This pattern of profit will obtain, for example, if the prices of the products are fixed and the marginal cost is increasing in quantity.

The only Nash equilibrium of this game is $(x_1^1, x_2^1) = (x_1^2, x_2^2) = (3, 3)$. To see this, note first that in Nash equilibrium there may not be a characteristic k such that $x_k^i - x_k^j = 2$ or such that $x_k^i - x_k^j = 1$ and $x_l^i - x_l^j = 1$ where $k \neq l$. This is because in either case player i can profit by reducing k by 1. Next, observe that firm i's equilibrium market share cannot be 1, since then, by the previous observation, it has to be that for some k, $x_k^i - x_k^j = 1$ and $x_l^i - x_l^j = 0$. But then j can gain 3 (= 4 − 1) by deviating to i's position. Thus, in equilibrium the firms split the market so that either $(x_1^j, x_2^j) = (x_1^i, x_2^i)$ or $x_k^i - x_k^j = 1$ and $x_l^j - x_l^i = 1$. Therefore, unless both players choose (3, 3), player j can gain 1.5 by increasing k by one.

Every action (x_1, x_2) is rationalizable. To verify this, note that (1, 1) is best response to (3, 3) and any other (x_1, x_2) is best response to either $(x_1 - 1, x_2)$ or $(x_1, x_2 - 1)$.

Assume that each firm's signal, g_i is its own market share. Then, the RCE outcome must be an equal split of the market.

CLAIM. *All* RCE *are the action–signal pairs* $([x_1^1, x_2^1), \frac{1}{2}], [(x_1^2, x_2^2), \frac{1}{2}])$ *such that* (x_1^1, x_2^1) *and* (x_1^2, x_2^2)

(i) *split the market equally*

(ii) *belong to* {(2, 1), (1, 2), (3, 1), (2, 2), (1, 3), (3, 3)}.

Proof. There is no RCE in which one of the firms has market share 0. If there is an RCE in which firm i's share is 0, then the only action–share pair in B_i in which the share is 0 must be [(1, 1), 0]. Otherwise, firm i can profit by deviating to (1, 1). Therefore, the only action–share pairs in B_j in which firm j's share is 1 are [(1, 2), 1] and [(2, 1), 1]. But then [(1, 1), 0] is not g-rationalizable by any belief on B_j since a deviation to (2, 2) is profitable.

Consider next an RCE with equal split of the market. There is no such

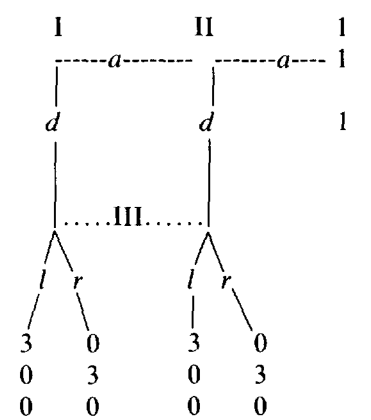
RCE in which firm i chooses $(1, 1)$, since equal shares imply that player i believes that j's action is $(1, 1)$ and then i can profitably deviate by increasing any coordinate by 1. Also, there is no such RCE in which firm i chooses $(3, 2)$ or $(2, 3)$, since then the equal share implies that i believes with probability 1 that j's action is not $(3, 3)$; by deviating to $(3, 3)$ firm i increases its share from $\frac{1}{2}$ to 1 and profit. Finally, $B_1 = B_2 = \{[(2, 1), \frac{1}{2}], [(1, 2), \frac{1}{2}], [(3, 1), \frac{1}{2}], [(2, 2), \frac{1}{2}], [(1, 3), \frac{1}{2}], [(3, 3), \frac{1}{2}]\}$ are g-rationalized: $[(3, 3), \frac{1}{2}]$ is g-rationalized by the belief concentrated on $(3, 3)$. $[(2, 1), \frac{1}{2}]$ is g-rationalized by $(1, 3)$, and $[(1, 3), \frac{1}{2}]$ is g-rationalized by an equal mixture of $(1, 3)$ and $(3, 1)$ which also g-rationalizes $[(2, 2), \frac{1}{2}]$. Analogously, $[(1, 2), \frac{1}{2}]$ and $[(3, 1), \frac{1}{2}]$ are also g-rationalized. ∎

Note that this is a Bertrand-like game in which the incentives for "undercutting" lead to a unique Nash equilibrium in which the joint profit is minimized. The RCE relaxes the "Bertrand Paradox" by not insisting that each player know perfectly his opponent's action, yet it rules out some of the rationalizable outcomes by allowing a player to have the information that can be derived from observing its own payoff.

Note that the set of CE is quite large and includes the configuration where one firm chooses $(1, 1)$ and the other chooses any $(x_1, x_2) \neq (1, 1)$. The additional requirement of the RCE concept, that the signal–action pairs which g-rationalize an action should be g-rationalizable themselves, is used in the first part of the proof to rule out configurations in which one of the firms dominates the market.

(c) *Fudenberg and Kreps' Example*

Fudenberg and Kreps (1988) bring the following example of an extensive form game:



The pure strategy Nash equilibria are (d, a, l), (d, d, l), (d, d, r), and (a, d, r). That is, in all Nash equilibria at least one of the players I or II chooses d, and so there is no Nash equilibrium in which both I and II play a. Also, every configuration is rationalizable.

When the signal g_i is the actual terminal node reached, there is an RCE in which both I and II play a. Let aa denote that terminal node. Observe that the sets $B_1 = \{(a, aa)\}$, $B_2 = \{(a, aa)\}$, and $B_3 = \{(r, aa), (l, aa)\}$ are g-rationalizable: player I g-rationalizes a by the belief that player III plays r, and player II g-rationalizes a by the belief that player III plays l.

The reason that aa is not reached in a Nash equilibrium is that I and II must have the same belief on III's strategy, but any such belief that will induce I to prefer aa to playing d will induce II to prefer d. Now, since the signal aa does not reveal III's strategy, the RCE does not impose any such restriction on I and II's beliefs regarding III.

As this example establishes, an RCE outcome might not be Nash even when the signal reveals the actual path being played. Note, however, that there is no such distinction between CE and RCE: when the signal is the actual path being played, in any game any CE outcome is also RCE outcome (To see this, for any CE, define B_i as the set of strategy–signal pairs whose signal coordinate is this CE's path and whose strategy coordinate goes over all of i's strategies which agree with this CE's strategy on the path, and observe that this particular signal implies that these B_i's are g-rationalizable). The reason again is that RCE does not impose any restrictions off the equilibrium path and so there is nothing to distinguish it from a CE when the path is known.

In the next examples the signals are not public and each player forms conjectures regarding the other's signal as well as the other's action.

(d) *A Discrete Nash's Demand Game*

The players simultaneously announce numbers between 1 and K. If the sum is less than or equal to $K + 1$, they get their respective demands. Otherwise, they get zero. All (a_1, a_2) such that $a_1 + a_2 = K + 1$ are Nash equilibria and all entries are consistent with rationalizability. Taking the signal function g_i to be i's payoff, any pair (a_1, a_2) with $a_1 + a_2 \leq K + 1$ is an RCE since a_i is best response to $a_j = K + 1 - a_i$. Thus, unlike the Nash equilibrium outcomes, the RCE outcomes include inefficient ones such as (1, 1). But the RCE (and even the CE) rules out the no-trade outcome of both players submitting incompatible demands, since then player i can profitably deviate to demand 1.

(e) *Location Choice Game*

Each of N players chooses a location $k = 1, \ldots, K$ from among K locations (suppose that K divides N). Let n_k be the number of players in location k. Each player's payoff is a decreasing and convex function of the number of players in his location. For example, suppose that the players are firms and in each location there is a unit of profit potential to be split equally among those locating there. The payoff to a firm locating at k is in this case $1/n_k$.

The Nash equilibria are such that there are exactly N/K players in each location. All possible configurations of players in locations are rationalizable.

Let player i's signal, g_i, be the number of players in his location. Observe that all RCE as well as all CE are such that there are exactly N/K players in each location. This is because a player who is at a location with more than N/K knows that the expected number of players at any other location is less than N/K. Since the payoff is convex in the number of neighbors, such a player profits by deviating. Thus, the RCE and the CE coincide with Nash equilibrium, but they require less demanding assumptions on what the players know.

(f) *The Aggression Game*

Player 0 assigns a distinct natural number k_i to each of the players 1, . . . , n and each of these players chooses one of two possible actions from A_i = {attack, defend}. Their payoffs are

$$u_i(\text{defend}; k_1, \ldots, k_n) = 0$$

and

$$u_i(\text{attack}; k_1, \ldots, k_n) = \begin{cases} 1 & \text{if } k_i = \min\{k_j : j = 1, \ldots, n\} \\ -1 & \text{otherwise.} \end{cases}$$

Player 0 is indifferent among all outcomes. He is just a device to model deterministic but unknown information, without giving the players common beliefs about it.

An interpretation of this model is that the player with the lowest number is the strongest party. He can take advantage of this strength only if he attacks. However, if he attacks when he is not the strongest party, he is beaten. In any (pure strategy) Nash equilibrium only the player with the lowest number chooses to attack. The set of rationalizable outcomes includes all possible configurations since a player can believe that player 0 assigns to him the lowest number or he can believe that player 0 assigns it to somebody else.

$$\text{Define } F(a) = \begin{cases} 1 & \text{if there is an } i \text{ s.t. } a_i = \text{attack} \\ 0 & \text{otherwise.} \end{cases}$$

Let $g_i(a, k_1, \ldots, k_m) = (k_i, u_i(a, k_1, \ldots, k_n), F(a))$. That is, each player knows his number, his payoff, and if there is any player who chooses to attack.

The set of CE includes the outcome that all players get numbers of at least 2 and choose to defend.

CLAIM. *In all* RCE *the player with the smallest number is the only player who chooses to attack.*

Proof. There is no CE in which a player other than the lowest numbered one chooses to attack. This is because each player's signal includes his payoff, which would be negative if he attacked when his number was not the lowest. To see that there is no RCE in which no player attacks, note first that there is no such RCE in which one of the players is assigned the number 1, since that player always benefits from attacking. Therefore, there is no RCE in which the lowest number is 2 and nobody attacks, since a player numbered 2 cannot g-rationalize his decision to defend by a belief that puts positive probability on the existence of a player numbered 1 who does not attack. By induction, for any n, there is no such RCE in which the lowest number is n and hence there is no RCE in which nobody attacks. ∎

4. COMMENTS

(a) *Many Players*

Recall definitions 1 and 2. The fact that the measure μ which g-rationalizes a pair (a_i, s_i) is defined over $\times_{j \neq i} A_j$ means that we allow for correlated conjectures. That is, for example, in a three person game, player 3 may hold a belief which assigns probability $\frac{1}{2}$ to each of two pairs of actions (a_1, a_2) and (b_1, b_2). The following two reasons may give rise to correlated beliefs, even where there is no external correlating device:

(i) Player 3 learns from his signal that players 1 and 2 definitely do not play (a_1, b_2) or (b_1, a_2).

(ii) Player 3 excludes (a_1, b_2) from the support of his belief because one of the action–signal pairs $(a_1, g_1(a_1, b_2, a_3))$ or $(b_2, g_1(a_1, b_2, a_3))$ does not belong to B_1 or B_2 accordingly.

If we are interested in a situation without external correlating devices so that the only possible correlation is due to these reasons, then the proper modification of definitions 1 and 2 is that player i g-rationalizes a pair $(a_i, s_i) \in B_i$ by a conjecture μ satisfying the following condition: If there are sets $(C_j)_{j \neq i}$ such that for all $c_{-i} \in \times_{j \neq i} C_j$, $g_i(a_i, c_{-i}) = s_i$ and for all j, $(c_j, g_j(c_{-i}, a_i)) \in B_j$, then the belief μ conditional on $\times_{j \neq i} C_j$ is a product measure.

(b) *Random Signal and Mixed Strategies*

So far we have considered only pure strategies and deterministic signals. The extension to random signals, g_i, is straightforward: after identifying S_i with the set of probability measures on the original space of signals, the basic definitions are valid without modification.

Note however that, in this case, thinking of the RCE as a steady state in which the information is accumulated through repetition is less straightforward than it was with deterministic signals. This is because it requires one to envision a more complicated learning process—players have to learn a whole distribution by observing its realizations. An explicit modeling of such a process, which we do not peruse, would be a more delicate task than the modeling of the deterministic case. For example, players can obtain long strings of atypical observations, and so such a learning process should not place too much weight on recent observations; however, this would slow down the pace at which deviations from the right beliefs can be corrected. As we are not modeling in this paper the underlying learning processes, there is no point in discussing such issues at length beyond noting that, in terms of the interpretation, the extension to random signals involves an extra leap.

Next, consider the possibility of mixed strategies. Let $g_i(m_1, \ldots, m_n)$ be the distribution of signals arising when the players use the mixed strategies $(m_1, \ldots, m_n)$. Distinguish two scenarios: (i) a player does not observe the realization of his own mixed strategy (i.e., he chooses the roulette but somebody else spins it and plays the realized action); (ii) a player observes the action realized by his own mixed strategy. Note that in scenario (i) what the player can presumably learn through repetition is $g_i(m_1, \ldots, m_n)$. Therefore, the appropriate model in this case is the random-signal model discussed above with random signal $g_i(m_1, \ldots, m_n)$. In scenario (ii) a player can presumably learn $g_i(a_i, m_{-i})$, for each a_i in the support of m_i. For this case the appropriate definitions are as follows:

Let M_i denote the set of mixed strategies of player i and let K_i be the set of probability measures on S_i. The random signal $g_i(a_i, m_{-i})$ is an element of K_i interpreted as the distribution of signals when player i chooses a_i while the rest choose m_{-i}. Denote by L_i the set of all pairs $(m_i, (k_i(a_i)_{a_i \in \mathrm{Supp}(m_i)}))$ where $m_i \in M_i$ and $k_i(a_i) \in K_i$.

DEFINITION 1. The pair $(m_i, (k_i(a_i)_{a_i \in \text{Supp}(m_i)}))$ in L_i is *g-rationalized* by a conjecture (probability measure) μ on M_{-i} if

(i) for all a_i in the support of m_i and m_{-i} in the support of μ, $k_i(a_i) = g(a_i, m_{-i})$

(ii) m_i is i's best response, given μ.

Condition (i) requires that, for any action a_i that i takes, i's conjecture be consistent with the distribution of signals, $k_i(a_i)$, that he receives. The fact that this requirement is made for *any* m_{-i} in the support of μ means that the signal distribution that i receives does not allow him to distinguish between the alternative patterns of behavior that he conjectures as possible for the others.

DEFINITION 2. The sets $B_1, B_2, \ldots, B_n$, where B_i is a subset of L_i are *g-rationalizable* if, for all i, every $(m_i, (k_i(a_i)_{a_i \in \text{Supp}(m_i)}))$ in B_i is g-rationalized by a probability measure μ on M_{-i} such that for all m_j in the support of μ the pair $(m_j, (g_j(a_j, m_{-j})_{a_j \in \text{Supp}(m_j)})) \in B_j$.

DEFINITION 3. A *RCE* is a profile of mixed strategies $m^* = (m_1^*, m_2^*, \ldots, m_n^*)$ satisfying that there are g-rationalizable sets $B_1, \ldots, B_n$, such that $(m_i^*, (k_i(a_i)_{a_i \in \text{Supp}(m_i^*)})) \in B_i$ for all i.

Note that, under the last definitions, it follows from the existence of Nash equilibrium that in finite games, for any signal functions g_i, there exists an RCE.

(c) *Experimentation*

Since we motivate the RCE by referring to a steady-state scenario in which the information is accumulated through repetition, this concept can be criticized on the grounds that steady states which are not Nash equilibria are "unstable." This is in the sense that, if players unconsciously tremble or consciously experiment, they may learn information that will induce them to change their behavior. Let us point out, however, that this criticism is somewhat sweeping. First, in order to fully expose the strategy profile being played, the unconscious trembles have to be of a rather special form (see Fudenberg and Kreps, 1988). Second, since conscious experimentation is costly, players will engage in it only if they are sufficiently long lived and patient to realize the benefits (see Fudenberg and Levine, 1990). Thus, while the possibility of trembles and experimentation may suggest some restrictions on the form of the signal function g_i, it does not empty, by any means, the RCE concept.

(d) *Equivalence to Nash Equilibrium*

In three of the examples considered above (the distance, location, and aggression games) the RCE coincided with Nash equilibrium. In games with this property the Nash equilibrium concept is more compelling, because in a sense the equilibrium requires less information on the part of the players. It may therefore be of interest to identify conditions under which, for some natural signal function such as one's own payoff, RCE and Nash equilibria are equivalent. Bernheim (1984) analyzes the analogous question for rationalizability—he presents necessary and sufficient conditions for such equivalence between rationalizable and Nash equilibrium. As a corollary of his analysis he shows that, in a two-player Cournot game, the unique pair of rationalizable strategies is the unique Nash equilibrium. This equivalence fails for $n \geq 3$, where any outputs between zero and the monopoly output are rationalizable. In contrast, the RCE lends some support to the Cournot equilibrium for $n \geq 3$ as well: when each firm's signal is its own profit, the unique RCE coincides with the Nash equilibrium for any n. Note that in this example the CE already coincides with the Nash equilibrium so that the power of RCE is not used. In the distance and aggression games the RCE coincides with the Nash equilibrium while the CE does not. This research direction provides well-defined analytic questions.

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