A NONPARAMETRIC MEASURE OF CONVERGENCE TOWARD PURCHASING POWER PARITY

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Abstract

It has been claimed that the deviations from purchasing power parity are highly persistent and have quite long half-lives under the assumption of a linear adjustment of real exchange rates. However, inspired by trade cost models, nonlinear adjustment has been widely employed in recent empirical studies. This paper proposes a simple nonparametric procedure for evaluating the speed of adjustment in the presence of nonlinearity, using the largest Lyapunov exponent of the time series. The empirical result suggests that the speed of convergence to a long-run price level is indeed faster than what was found in previous studies with linear restrictions.

Keywords: Mean reversion; Nonlinear time series; Nonparametric regression; Purchasing power parity puzzle; Real exchange rates.

JEL classification: C14; C22; F31

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1 Introduction

Since Rogoff's (1996) observation on the volatile yet extremely persistent real exchange rate, the mean reversion to long-run purchasing power parity (PPP) has attracted considerable attention from researchers. To measure persistence, the half-life of deviations from PPP has been frequently employed as a quantity of interest. Summarizing the empirical evidence provided by Frankel (1986), Diebold et al. (1991), and Lothian and Taylor (1996), Rogoff claimed the consensus of three- to five-year half-lives of deviations. However, as recently pointed out by Taylor (2001), nonlinearity might possibly be a source of large half-life estimates, since it could cause an upward bias if a linear model were incorrectly employed in the estimation.

As emphasized by Obstfeld and Rogoff (2000), trade costs most likely play a central role in the persistence of international price differentials, as well as in many other empirical puzzles in international macroeconomics. With the inspiration of the trade cost models, estimating nonlinear time series models has become a very popular approach among the recent empirical studies on the real exchange rates dynamics (e.g., Michael et al., 1997; Obstfeld and Taylor, 1997; O'Connell, 1998; Sarantis, 1999; Taylor and Peel, 2000; Baum et al., 2001; and Taylor et al., 2001). One difficulty regarding the new approach is that, unlike the traditional linear approach, the interpretation of results in terms of the persistence of PPP deviations is not straightforward, since the trade cost models generally predict a slower rate of adjustment for smaller deviations from the steady state level. One may report the exact half-life based on the nonlinear impulse response functions (IRFs) to investigate the difference between linear and nonlinear results. However, since the nonlinear IRFs depend on the history of the time series and the size of the shocks, such a half-life cannot be uniquely determined. In practice, summarizing all the information of many different half-lives is not an easy task since evaluation of each nonlinear IRF usually requires computer-intensive simulation method.

The purpose of this paper is to propose a simple persistence measure of PPP deviations based on the largest Lyapunov exponent of the nonlinear time series. While this summary measure is certainly not a unique measure of persistence, there seems to be several advantages in PPP applications. First, the measure is simple in computation and does not rely on computer-intensive nonlinear IRFs. The evaluation of estimation uncertainty can also be easily incorporated into the analysis. Second, it is similar to a conventional linear half-life measure in the sense that it can be interpreted as the half-lives of the locally linearized nonlinear processes. By definition, it corresponds to the exact half-life concept if the true process is linear. This measure is therefore convenient for assessing the effect of nonlinearity in comparison with the previous results of linear half-lives of PPP deviations available in the literature. Third, the measure is well-defined even in the case of a simple trade cost model that predicts no price adjustment for some range of values. Fourth, the measure is estimated using the nonparametric regression technique without specifying the parametric functional form. In consequence, the method is robust to very general nonlinearity in the adjustment process.

The remainder of the paper is organized as follows: Section 2 reviews the half-life as a summary persistence measure of PPP deviations for both linear and nonlinear models. Section 3 proposes a nonparametric convergence measure based on the Lyapunov exponent. The finite sample properties of the proposed measure are also investigated by a Monte Carlo simulation. In Section 4, the proposed measure is applied to two different data sets, the annual historical exchange rate series originally constructed by Lee (1976), and the quarterly series during the current float. Comparison with the results from the conventional linear half-life measure and from the half-life based on nonlinear IRFs is also provided. Some concluding remarks are presented in Section 5.

2 Half-Life of PPP Deviations

2.1 Linear Model

Let q_t be the (log of the) real exchange rate series defined by

$$q_t = s_t + p_t^* - p_t \tag{1}$$

where s_t, p_t^*, and p_t are the (log of the) nominal exchange rate, the (log of the) foreign price level, and the (log of the) domestic price level, respectively. Researchers are interested in investigating the adjustment process of q_t toward its long-run level q provided that the PPP holds in the long run. The conventional approach is to employ a simple linear time series model, such as an autoregressive (AR) model of order one,

$$q_t = \mu + \rho q_{t-1} + \varepsilon_t \tag{2}$$

where $0 < |\rho| < 1$, $\mu = (1 - \rho)q$ and ε_t is a white noise. While the constant term is included in the model, we can let $\mu = 0$ by assuming the long-run level $q = 0$ without loss of generality.[1] There are

[1] For absolute prices, long-run PPP (or the law of one price) implies that the mean of the process is zero. However, for price indexes, a non-zero constant term is usually included in practice to allow for the heterogeneous base years.

several different measures that characterize the mean reverting structure of the model. The most informative strategy is to show the entire shape of the IRF of q_t to a shock of size δ. For the AR(1) example provided above, n steps ahead IRF is simply $\rho^n\delta$. Alternatively, one can report a summary measure of persistence, such as the cumulated IRFs, the sum of AR coefficients, and the half-life of deviations. Since the half-lives are the most frequently used summary measure of persistence in the literature of PPP, we will mainly focus on this type of measure in this paper.

The half-life of deviations is the number of years (for annual data) required for the deviation at an initial level q_0 to dissipate by half. Using the IRF of the AR(1) model above, $\rho^h\delta = \delta/2$ implies the half-life of $h = \ln(1/2)/\ln\rho$ for $\rho > 0$. For the AR model of higher order, or other linear models, with monotonic decreasing IRFs, the half-life is the value of h that satisfies $IRF_h(\delta) = \delta/2$ where $IRF_h(\delta)$ is the h steps ahead IRF of q_t to the shock of size δ. It should be noted that the IRF of a linear model does not depend on the initial level q_0, and is a homogeneous function of order one, $IRF_h(\delta) = \delta IRF_h(1)$. Therefore, the condition can be also rewritten as $IRF_h(1) = 1/2$. This independence of half-lives to q_0 and δ is a very convenient feature of the linear time series model.

When the linear models have nonmonotonically decreasing IRFs, such as the hump-shaped curve or oscillation, the notion of half-lives becomes somewhat ambiguous. A practically relevant definition would be the time required for $IRF_h(1)$ to be permanently below 0.5, or the smallest h that satisfies $IRF_n(1) < 1/2$ for all $n > h$. There is a convenient approximation formula for the AR(1) model above that allows for an oscillation with negative ρ. By using the absolute value of the condition, $|\rho|^h = 1/2$, yields the half-life of $h = \ln(1/2)/\ln|\rho|$.[2] Since the denominator $\ln|\rho|$ can be

[2]In PPP applications, the estimated AR(1) coefficients are almost always positive, suggesting no need for this absolute value transformation.

interpreted as the speed of adjustment (in absolute value), h becomes greater than unity only if the speed of adjustment is slower than that of the AR(1) model with $|\rho| = 0.5$. As $|\rho|$ approaches unity, the speed of adjustment $\ln|\rho|$ approaches zero from the left, and half-life h approaches infinity, implying the absence of convergence toward PPP. In practice, this half-life can be estimated by

$$\widehat{h} = \frac{\ln(1/2)}{\ln|\widehat{\rho}|} \tag{3}$$

where $\widehat{\rho}$ is an OLS estimator of ρ in (2).

2.2 Nonlinear Model

The idea of nonlinear adjustment of deviations from PPP is mainly justified by the presence of trading costs, including transportation costs, insurance costs, information costs, tariffs, and nontariff barriers. Theoretical models of exchange rates with trade costs have been developed by many researchers, including Dumas (1992); Sercu et al. (1995); Betts and Kehoe (1999); and Sercu and Uppal (2003), among others. These models generally predict the slower speed of adjustment when the deviation from PPP is smaller. Recall that the speed of adjustment for a linear model (as well as its half-life) is constant and does not depend on the initial level q_0, or the size of shock δ. The nonlinear model of PPP adjustment, in contrast, implies that the time needed for the initial deviation δ to become $\delta/2$ is shorter than the time for $\delta/2$ to become $\delta/4$, and both lengths now depend on q_0 and δ. This is the main reason why it causes some difficulties in using half-lives as a measure of persistence in the nonlinear model.[3]

To see this point more in detail, let us consider a following variation of smooth transition autoregressive (STAR) models,

$$q_t = \begin{cases} \mu + \rho q_{t-1} + \varepsilon_t & q_{t-1} > c \\ q_{t-1} - q_{t-1} F(q_{t-1}) + \varepsilon_t & -c \leq q_{t-1} \leq c \\ -\mu + \rho q_{t-1} + \varepsilon_t & q_{t-1} < -c. \end{cases} \tag{4}$$

where $F(q_{t-1}) = 1 - \exp(-q_{t-1}^2)$, $0 < \rho < 1$ and $\varepsilon_t \sim$iid $N(0, \sigma^2)$. $c(> 0)$ is a threshold value that satisfies $G'(c) = \rho$ where $G(q_{t-1}) = q_{t-1} - q_{t-1} F(q_{t-1}) = q_{t-1} - q_{t-1}\{1 - \exp(-q_{t-1}^2)\}$. The linear AR structure outside the $(-c, c)$ band is introduced here to ensure that the speed of adjustment is always positive. The intercept for the outside regime is selected as $\mu = G(c) - \rho c$. The inner regime has a simple STAR structure with the speed of adjustment becoming slower as q_{t-1} approaches the steady state level $q = 0$. The class of STAR models has been popularly employed in recent studies on PPP, including Michael et al. (1997); Sarantis (1999); Taylor and Peel (2000); Baum et al. (2001); and Taylor et al. (2001). To consider the half-life of (4), we first need to define the IRF of a nonlinear model.

The notion of nonlinear IRFs is developed by Gallant et al. (1993); Potter (1995, 2000); and Koop et al. (1996). In this paper, we focus on the IRFs of the class of nonlinear AR(1) model that

[3]Assumption of a constant speed of adjustment is still appropriate in many other applications. For example, in nuclear physics, half-life is often used to characterize radioactive materials. Since the probability of decay of an atom is constant, the proportion of survived nuclei in a fixed period of time is constant. Therefore the half-life does not depend on the total number of initial nuclei.

can be written as

$$q_t = m(q_{t-1}) + \varepsilon_t \tag{5}$$

where $m(q_{t-1})$ is a nonlinear conditional mean function $E(q_t|q_{t-1})$. The n steps ahead conditional mean function will be further denoted by $m_n(q_{t-1}) = E(q_{t+n-1}|q_{t-1})$. Then, the most frequently used definition of the nonlinear IRF is given by

$$IRF_n(q_0, \delta) = m_n(q_0 + \delta) - m_n(q_0). \tag{6}$$

By analogy to the linear model, one can compute the exact half-life by obtaining h that satisfies $IRF_h(q_0, \delta) = \delta/2$ for a monotonic IRF, and by obtaining the smallest h that satisfies $IRF_n(q_0, \delta) = \delta/2$ for all $n > h$ for a non-monotonic IRF. However, since the nonlinear IRF depends on the initial value q_0 (or past history) and the size of shock δ, the system does not have a unique value of half-life.

Table 1 shows the exact half-lives of the STAR model (4) with $\rho = 0.5$, $\sigma = 0.1$ and various combination of q_0 and δ. The threshold value under this specification is computed as $c = 0.4426$. Conditional expectation required for the nonlinear IRF is obtained based on simulation using 100,000 iteration. Note that only the case with positive δ is reported in the table because of the symmetric structure of our STAR model. In general, however, nonlinear IRF and the half-life depend on the sign of the shock as well as its size. The table clearly shows the tendency of the shorter half-lives when the shocks are the smaller and when the initial value is closer to the long-run level $q = 0$. For a fixed value of $\delta = \sigma$, the half-life varies from 1 year with $q_0 = 0.5$ to 9.2 years with $q_0 = 0$

reflecting the difference in the speed of adjustment of inner and outer regimes.

The variation of half-lives becomes even larger if we consider discrete transition rather than smooth transition between the regimes. By setting $F(q_{t-1}) = 0$ (and thus $\mu = G(c) - \rho c = (1-\rho)c$), (4) becomes a threshold autoregressive (TAR) model which may be appropriate to describe the price adjustment of a single traded good. The threshold parameter c in such a case can be interpreted as the transaction cost in a simple "iceberg" model (e.g., see Sercu et al., 1995), and the model implies the random walk (no price adjustment) inside the band.[4] Table 2 shows the half-lives of the TAR model using the same parameter values of ρ, σ, c, q_0 and δ as in Table 1. For the shock of $\delta = \sigma$, the half-life now varies from 1 year to 23.4 years when initial value approaches from 0.5, a value in the outer regime, to 0, a value in the inner regime.

The examples in Tables 1 and 2 show the inconvenient feature of the half-lives of the nonlinear time series model, namely, the sensitivity of half-lives to the initial conditions and shocks. This issue is closely related to the difficulty in summarizing the information contained in the nonlinear IRFs produced by all the possible different histories and shocks, pointed out by Gallant et al. (1993) and Potter (1995). One possibility is to report a table of half-lives similar to Tables 1 and 2. For example, Taylor et al. (2001) reported tables of half-lives of their estimated STAR model for several different δ's and q_0's. However, reporting the full table may not be suitable for the purpose of comparison of persistence in PPP deviations among different countries or different time periods. Furthermore, each entry in table requires simulation and thus incorporating the effect of sampling variability or estimation error becomes even more difficult. Another possibility is to report some

[4]This TAR model has been estimated by Obstfeld and Taylor (1997) and O'Connell (1998) and has been used in Taylor (2001) to illustrate the problem of misspecification with the linear half-life measure.

summary measures that can be use for direct comparisons. For example, Potter (2000) proposed using a stochastic dominance of cumulated nonlinear IRFs to measure the persistence. In what follows, we also use the latter approach and construct a summary measure of persistence suitable for nonlinear PPP applications. In particular, we consider an alternative summary measure of persistence based on the largest Lyapunov exponent of the time series.[5] The notable feature of our measure is that it is closely related to the notion of half-life reviewed in this section. This feature seems to be advantageous for the comparison with the half-lives of linear model which were often reported in the previous studies on PPP.

3 An Alternative Persistence Measure of PPP Deviations

3.1 Lyapunov Exponent of Nonlinear Time Series

One possibility of constructing a nonlinear summary measure analogous to the linear half-lives is to evaluate the exact half-lives of the nonlinear model using the distribution of all possible shocks and initial conditions (or history). While such a measure is certainly feasible, it requires the evaluation of many nonlinear IRFs and thus is not appealing from the computational point of view. Our goal is to construct a summary measure of a nonlinear model while maintaining the simplicity in computation as in the case of the half-life of a linear model. To achieve this goal, let us first note that ρ in the definition of linear half-life $h = \ln(1/2)/\ln|\rho|$ can be considered as the first derivative of the conditional mean function in (2). Furthermore, note that the first derivative

[5] Potter (2000, footnote 10) also mentioned the possiblity of using the largest Lyapunov exponent as an alternative to his summary measure based on nonlinear IRFs.

of the conditional mean function $m(q_{t-1})$ in (5) is proportional to the one step ahead nonlinear IRF for small δ since

$$Dm(q_0) = \lim_{\delta \to 0} \frac{m(q_0 + \delta) - m(q_0)}{\delta} = \lim_{\delta \to 0} \frac{IRF_1(q_0, \delta)}{\delta}.$$

By combining the two facts, we can introduce the notion of a local half-life at q_0 defined by

$$h(q_0) = \frac{\ln(1/2)}{\ln |Dm(q_0)|}. \tag{7}$$

This is nothing but the half-life of a linear model from the linearization of (5) around the initial level q_0, and thus it corresponds to the linear half-life h under the linearity assumption $Dm(q_0) = \rho$. A summary measure of persistence may then be constructed by averaging the local half-life using the distribution of the initial condition, or $E[h(q_{t-1})]$. Unfortunately, this average local half-life turns out to be inappropriate for PPP applications. The final columns of Tables 1 and 2 show the local half-life $h(q_0)$ with various q_0 using the same specification of STAR and TAR models considered in the previous section. The local half-life is infinity with $q_0 = 0$ for the STAR model and with $q_0 \in (-c, c)$ for the TAR model. This outcome follows from the fact that half-life becomes infinity under the absence of convergence with $\rho = 1$ in linear model, a situation causing some difficulties in averaging the local half-lives. In the TAR example with $\rho = 0.5$, $h(q_{t-1}) = 1$ if $|q_{t-1}| > c$ and $h(q_{t-1}) = \infty$ if $|q_{t-1}| \leq c$. Let $\mathbf{1}_{\{|q_{t-1}| \leq c\}}$ be an indicator function which takes one, if $|q_{t-1}| \leq c$, and zero, otherwise. When $E[\mathbf{1}_{\{|q_{t-1}| \leq c\}}] > 0$ the average half-life is always infinity regardless of the size of c and $E[\mathbf{1}_{\{|q_{t-1}| \leq c\}}]$. For a fixed size of c, a preferable summary measure seems to be the one that associates higher persistence with a larger value of $E[\mathbf{1}_{\{|q_{t-1}|\leq c\}}]$.[6] For this reason, instead of using average of local half-lives, we use the average local speed of convergence to define the half-life-like measure of persistence. As will be seen below, our measure is closely related to the Lyapunov exponent of time series.

The largest Lyapunov exponent is a measure of stability of a dynamic system in terms of the sensitive dependence on initial conditions. For the nonlinear AR(1) model (5), the Lyapunov exponent is defined by

$$\lambda \equiv \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} \ln |Dm(q_{t-1})|. \tag{8}$$

For stationary ergodic time series, λ is known to be unique and independent of the initial value q_0 and can be replaced by $E[\ln |Dm(q_{t-1})|]$. It should be noted that it can be defined not only for the mean-reverting process but also for the non-mean-reverting case. Suppose two different initial conditions q_0 and q_0' with small difference δ ($q_0' = q_0 + \delta$). Then, λ is the average growth rate of difference between two trajectories $\{q_t\}_{t=0}^{\infty}$ and $\{q_t'\}_{t=0}^{\infty}$. The Lyapunov exponent is often used to define a chaotic system because two trajectories diverge for such a system. On the other hand, for a stable system with a steady state, the Lyapunov exponent can be interpreted as an average rate of convergence.

Recall that the both denominators in the linear half-life h and the local half-life $h(q_0)$ can be interpreted as the speed of convergence $\ln |\rho|$ or $\ln |Dm(q_0)|$. By analogy, we may construct a

[6]Such a requirement for the persistence meausure may not be shared by others. For example, Taylor (2001) defines a half-life of the TAR model by using the half-life of the linear AR model in the outer regime, regardless of the size of c or the time spent in the inner regime.

measure of persistence by replacing the denominator with the average speed of convergence for a stable nonlinear system, namely λ,

$$h^* = \frac{\ln(1/2)}{\lambda}. \tag{9}$$

As in the case of the average local half-life, h^* is identical to h under the linearity assumption, since $Dm(q_{t-1}) = \rho$ for all t and $\lambda = \ln|\rho|$. However, unlike the average local half-life, this measure is well-defined even if there is a segment of no adjustment in the model, and thus is more useful in PPP applications. As an example, let us again consider the TAR model with $\rho = 0.5$. Since the model implies $Dm(q_{t-1}) = \rho = 0.5$ outside the band and $Dm(q_{t-1}) = 1$ inside the band, λ is the average of $\ln(1/2)$ and $0(= \ln 1)$ weighted by $E[\mathbf{1}_{\{|q_{t-1}|>c\}}]$ and $E[\mathbf{1}_{\{|q_{t-1}|\leq c\}}]$ (which depends on c and σ). Then the persistence measure become $h^* = 1/E[\mathbf{1}_{\{|q_{t-1}|>c\}}] = 1/\{1 - E[\mathbf{1}_{\{|q_{t-1}|\leq c\}}]\}$. Therefore, this measure implies higher persistence when q_t spends more time in the no-adjustment regime.

3.2 Nonparametric Estimation of the New Persistence Measure

Let us now consider the estimation of h^*. If the specification of the system is completely known, as in the case of the STAR model (4), a parametric approach such as the one employed by Bask and de Luna (2002) should yield an efficient estimator of λ and thus h^*. In general, however, the nonlinear AR model (5) can be estimated by using the nonparametric regression technique without the specification of the functional form. To estimate λ from data, Nychka et al. (1992) have proposed a sample analogue estimator based on the nonparametric method. Following this idea, we estimate h^* by

$$\widehat{h}^* = \frac{\ln(1/2)}{T^{-1}\sum_{t=1}^{T} \ln\left|\widehat{Dm}(q_{t-1})\right|} \tag{10}$$

where $\widehat{Dm}(q_{t-1})$ is a nonparametric estimator of the first derivative of $m(q_{t-1})$ in (5) and T is the sample size. With a choice of a nonparametric estimator that provides a consistent estimator of λ, $\widehat{h}^*$becomes a consistent estimator of h^*. Recall that h^* is not an exact half-life for a certain δ and q_0. However, $\widehat{h}^*$ converges to a well-defined, half-life-like measure of persistence and to a exact half-life h when it is applied to the data generated from a linear model.

In principle, any nonparametric estimator that satisfies the property above can be used for the derivative estimation. In this paper, we employ a class of kernel-type regression estimators called the local polynomial regression estimator. There are several advantages of local polynomial regression over the simple Nadaraya-Watson regression estimator. First, it reduces the bias of the Nadaraya-Watson estimator. Second, it adapts automatically to the boundary of design points and no boundary modification is therefore needed. Third, and most importantly for our purpose, it is superior to the Nadaraya-Watson estimator in the context of derivative estimation. In particular, the local polynomial of order two, or local quadratic smoother, is preferable for the same reasons for first derivative estimation (see Fan and Gijbels, 1996, p.77).

It is now common practice to report the confidence intervals for $\widehat{h}$ in the linear model to consider sampling variability. For example, to evaluate the precision of the half-life, Cheung and Lai (2000) reported both asymptotic and bootstrap confidence intervals for $\widehat{h}$ while Kilian and Zha (2002) used

Bayesian confidence intervals.[7] In the empirical section, we also report the confidence interval for the nonparametric measure $\widehat{h}^*$ based on the asymptotic distribution of the local quadratic estimator of the Lyapunov exponent derived in Shintani and Linton (2003). In Murray and Papell's (2002) study, they employed a median-unbiased estimator of ρ and reported that confidence intervals of $\widehat{h}$ included infinity in many cases, which implies some possibilities of a unit root. For a unit root process, the linear measure $\widehat{h}$ is consistent in the sense that half-life estimates diverge to infinity as the sample size increases. In Shintani and Linton (2003), it is shown that the Lyapunov exponent based on the local quadratic regression converges to zero when the true process is a random walk, or $m(q_{t-1}) = q_{t-1}$ with an iid error in (5). This implies that $\widehat{h}^*$ is also consistent in the sense that it diverges to infinity for a unit root case.

Finally, we report the result of a small-scale Monte Carlo simulation designed to evaluate the finite sample performance of the nonparametric estimator $\widehat{h}^*$. We generate the artificial data from (4) with $\rho = 0.5$ and with the sample sizes $T = 100$ and 200. The true measure of persistence of the model, h^*, is controlled by varying the dispersion parameter σ from 0.1 to 10.0. When σ is as large as 10.0, the probability of being in the outer regime is 97%. Therefore, our half-life like measure is 1.02 years, which is very close to the exact half-life of 1 year with a linear AR model with $\rho = 0.5$. In contrast, when σ is as small as 0.1, the probability of being in the inner regime becomes 97%. In such a case, because of the smooth transition within the inner regime, h^* becomes 4.4 years implying, the higher persistence.

Table 3 shows the mean, median, and standard deviation of $\widehat{h}^*$ using a local quadratic estimator

[7] Confidence intervals of half-lives of nearly integrated real exchange rates were also recently considered by Rossi (2004).

with the Gaussian kernel function based on 10,000 replications.[8] In addition to the nonparametric estimator $\widehat{h}^*$, we also report the result with the conventional linear half-life $\widehat{h}$ based on the OLS estimator $\widehat{\rho}$. Apparently, misspecification of a linear model generally implies inconsistency of $\widehat{h}$ as an estimator of h^*. Nevertheless, there may be some cases in which $\widehat{h}$ works well as an approximation. The results from the simulation can be summarized as follows.

First, the nonparametric estimator $\widehat{h}^*$ performs well for various values of σ. While the distribution is somewhat skewed for the case of $T = 100$, both mean and median become very close to the true h^* for the case of $T = 200$ with a smaller standard deviation. Second, the linear estimator $\widehat{h}$ performs better than $\widehat{h}^*$ when σ is 10.0. This is expected as the model becomes almost linear and h^* is very close to h of a linear AR model. However, $\widehat{h}$ is biased upward as an estimator of h^* when the role of nonlinear adjustment becomes more important with smaller σ's. This upward bias of $\widehat{h}$ becomes even larger when the sample size increases from 100 to 200. This observation supports Taylor's (2001) claim that inappropriate linear specification may result in larger half-life estimates if there is nonlinearity in the adjustment process. In the next section, we apply the nonparametric measure to the data and reexamine the persistence of PPP deviations.

4 Empirical Results

Two different data sets are used for the analysis of persistence of PPP deviations that allows for nonlinear adjustment. The first data set is the long-horizon annual real exchange rate series

[8] We computed results with several different choices of the smoothing parameter for the nonparametric regression. The one reported in table uses 0.45 times the range.

originally constructed by Lee (1976) and later extended by Murray and Papell (2002), using the sample period 1900 to 1996. Countries under consideration are Canada, France, Italy, Japan, the Netherlands, and the U.K. All the series are WPI-based real exchange rates with the U.S. dollar used as the numeraire currency. The well-known caveat of using the long-horizon data is that it includes both fixed and float exchange rate periods. The second data set we consider consists of the real exchange rates under the current float period, and it presumably suffers less from the effect of the regime shift. We utilize the data used in Murray and Papell (2002) which consists of quarterly CPI-based real exchange rates of twenty countries from 1973:1 to 1998:2.

Before computing the nonparametric measure of persistence, we first apply a unit root test and a nonlinear specification test to the two sets of real exchange rate series. When the standard Dickey-Fuller test with a trend is used for each of six annual series, the unit root hypothesis is rejected for three countries at the 5% significance level and for five countries at the 10% level. In contrast, when the same test is applied to the quarterly series, a unit root is not rejected for all countries. This finding is consistent with previous studies (e.g., Lothian and Taylor, 1996) that found that longer span real exchange rate data reject the unit root hypothesis more frequently possibly because of the higher power compared to the case with short period data.[9] As another possibility of the failure of rejecting the unit root, Taylor et al. (2001) reported the lack of power of univariate unit root test when it is applied to the nonlinear mean-reverting processes. We then conduct Ramsey's (1969) regression specification error test (RESET) using a polynomial of order three to investigate

[9] Similar results are also obtained by applying Phillips and Perron's (1988) semiparametric unit root test that includes a trend using the QS kernel with the lag length selected by Andrews' (1991) procedure. For the annual series, a unit root is rejected for four countries at the 5% level and for five countries at the 10% level. For the quarterly series, a unit root can not be rejected for all countries.

the presence of nonlinearity in real exchange rates. When RESET is applied to the U.K., the only country that failed to reject the unit root for annual data, the null hypothesis of linear specification is rejected at the 5% level. For quarterly data, linearity is rejected for three countries at the 5% significance level and for six countries at the 10% level. While the evidence is not very strong, there are some possibilities that nonlinearity is playing a role in the adjustment of the real exchange rates.[10]

Let us now turn to the nonparametric estimation of the half-life-like measure of convergence using the real exchange rates. As in the previous section, the local quadratic regression with the Gaussian kernel is employed to obtain the first derivatives required for $\widehat{h}^*$. The smoothing parameter is selected by minimizing the residual squares criterion (RSC) given in Fan and Gijbels (1996, p.118), which is known to be an optimal selection method for the local polynomial regression. For the heteroskedasticity and autocorrelation consistent (HAC) variance estimation required for the construction of the confidence band, we employ the QS kernel with a lag window parameter selected by the optimal selection method of Andrews (1991). For the purpose of comparison, we also compute two other measures of persistence, the conventional linear half-life based on OLS, $\widehat{h}$, and the exact half-life estimates based on the nonlinear IRFs with some particular values of q_0 and δ which is denoted by $\widehat{h}(q_0, \delta)$. The nonlinear IRFs required for the latter measure are nonparametrically estimated without specifying the nonlinear functional form, as in the case for $\widehat{h}^*$.

[10]Taylor et al. (2001) also suggest using the multivariate unit root test or cointegrating rank test for the purpose of increasing the power of the test under the alternative of nonlinear model. When the cointegrating rank test of Johansen (1991) with the null hypothesis of one unit root is applied to our real exchange rate data, test statistics are 2.40 for annual data and 4.27 for quarterly data, both of which are less than the 5% critical value of 9.24. In addition to Johansen's parametric test, we also employ a nonparametric cointegrating rank test proposed by Shintani (2001). However, the test statstics are 13.56 for annual data and 0.80 for quarterly data, with the 5% critical value of 27.51, again implying the failure of rejecting the unit root under the multivariate framework.

To be more specific, we use the local polynomial regression of order one, or local linear smoother, to estimate two conditional mean functions in the definition of nonlinear IRF (6).[11] Then $\widehat{h}(q_0, \delta)$ is obtained as the smallest h that satisfies $\widehat{IRF}_n(q_0, \delta) = \delta/2$ for all $n > h$ where $\widehat{IRF}_n(q_0, \delta)$ is the nonparametric estimator of nonlinear IRF. For the starting value q_0, we simply use the sample average $\overline{q}$. For the size of shock δ, we consider two cases, namely, a large shock 2σ and a small shock 0.1σ with σ obtained from the residual of the nonparametric regression of (5). It should be noted that all three measures considered here converge to $h^*(= h)$ when the true process is linear.

For the annual data set, the estimated results of nonparametric persistence measure $\widehat{h}^*$, the linear half-life measure $\widehat{h}$ and the half-life measure $\widehat{h}(q_0, \delta)$ based on nonparametric nonlinear IRFs are provided in Table 4. The 95 percent confidence intervals are also provided for both $\widehat{h}^*$ and $\widehat{h}$.[12] On the whole, our nonparametric estimates of persistence do not differ much from the linear half-lives except for the U.K. On one hand, quite similar values between the two measures are obtained for Canada, France, and Italy. On the other hand, somewhat shorter half-lives are obtained with a nonparametric measure for Japan and the Netherlands. It is interesting that the largest reduction is observed in the case of the U.K. The half-life based on the conventional linear measure is 4.84 years. This number is indeed very close to the 4.6 years of half-life implied by Frankel's (1986) study of the long-horizon dollar/pound real exchange rates (see Rogoff, 1996, p. 656). By employing the nonparametric measure, the number is reduced to 2.64 years with a substantially smaller confidence

[11] See Tschernig and Yang (2000), for example, on the nonparametric estimation of nonlinear IRFs. They employed the local linear regression method to estimate IRFs of the nonlinear time series process that is more general than the one considered in our paper.

[12] We compute the confidence intervals of $\widehat{h}^*$ using the symmetric confidence interval in terms of Lyapunov exponent obtained by Shintani and Linton (2003). Thus, we report a comparable confidence interval for the linear measure $\widehat{h}$ based on the limit distribution of the rate of convergence $\ln|\widehat{\rho}|$ instead of that of AR parameter $\widehat{\rho}$.

interval.

Even if there is only a moderate difference between the $\widehat{h}^*$ and $\widehat{h}$ in Table 4, it does not imply that the adjustment process is well-approximated by the linear process. This point becomes clearer if we look further at the shape of the local speed of adjustment, $\ln|\widehat{Dm}(q_{t-1})|$, and the exact half-lives based on nonlinear IRFs, $\widehat{h}(q_0, \delta)$. Figure 1 shows the estimated local speed of adjustment for six countries. Evidently, none of them are flat. More importantly, it shows faster adjustment when the deviations from the long-run level are large. The notable fact is that we have not imposed any parametric restriction to obtain a structure such as the STAR model. In addition, Table 3 shows that four out of six countries have shorter half-lives with larger shocks based on nonparametrically estimated nonlinear IRFs. These two results on faster convergence with larger deviations support the view that the presence of trade costs plays an important role as a source of nonlinearity.

The results for the quarterly data set are reported in Table 5. For the conventional linear measures, slightly shorter half-lives are obtained than those based on the long-horizon data. The only exception is Canada with fairly long half-life point estimates. The median half-life based on the linear measure is 2.52 years compared to 3.01 years obtained from the long-horizon annual data. At the same time, the wide confidence intervals of linear measure show the uncertainty of the point estimates. Indeed, infinity is included for seventeen out of twenty countries, which implies the difficulties of excluding the possibility of a unit root. These observations are consistent with the former findings in the literature as well as with the result of the unit root test in this paper.

Similar to the result with a linear measure, somewhat less persistent results compared to those based on the long-horizon data are also observed with the nonparametric measure. However, the most notable finding is that the nonparametric method provides less persistent estimates than the corresponding linear estimates for all the countries except Canada. The median of the nonparametric half-life-like measure is 1.44 years and the median of the difference between the nonparametric and linear measure is 0.99 years (the average values and difference become 1.53 and 1.08 years, respectively, when Canada is excluded). On average, about a 40 percent reduction in persistence is observed by introducing nonlinearity into the adjustment process. With respect to the precision of the point estimates, the confidence intervals of nonparametric measures are considerably shorter than those for the linear half-lives. In some cases, 95 percent upper bounds for the nonparametric measures are indeed lower than corresponding point estimates based on the linear measure. In contrast to the linear measure, infinity is excluded from all the confidence intervals of nonparametric measure, again with the exception of Canada. The considerable difference between the $\widehat{h}^*$ and $\widehat{h}$ with quarterly data can be interpreted as an indication of the significant role of nonlinearity in the persistence of deviation from PPP. This conjecture is also supported by $\widehat{h}(q_0, \delta)$ reported in the same table, that shows the shorter half-lives with larger shocks for seventeen countries among the total of twenty countries.

5 Conclusion

This paper introduced a nonparametric persistence measure of PPP deviations which allows for general nonlinear real exchange rate adjustment. The measure utilizes the notion of the Lyapunov exponent of the nonlinear time series and is simple in computation with no requirement of estimating the nonlinear IRFs. It can be interpreted as a half-life of locally linearized process, which is convenient for comparison with the linear half-life measure of persistence often used in the PPP literature. If the nonlinearity in the adjustment process is a possible pitfall in understanding the PPP puzzle as discussed in Taylor (2001), our nonparametric measure seems to be a useful alternative for evaluating the speed of adjustment.

The finite sample properties of our measure is found to be satisfactory, while the results are based on a very limited experiment. An interesting empirical finding is obtained when the proposed measure is applied to two different real exchange rates data sets. When the annual historical data is used, the nonparametric method yields more than two years of reduction in the persistence of U.K./U.S. real exchange rates compared to the linear half-life estimate of 4.84 years. When the current float data is used, a one-year reduction from the linear estimates is observed on average in twenty countries. On the whole, the empirical results suggest a faster speed of mean-reversion compared to the findings in previous studies that used linear assumption. Furthermore, the nonparametric measure yields a shorter confidence interval than that of linear measure. In case of the former, infinite half-lives are excluded from the intervals for almost all cases. The lower persistence results obtained in this paper compared to the previous studies in the PPP literature may lessen the problem of the PPP puzzle to some degree. While our nonparametric methods are not capable of identifying the source of nonlinearity, the presence of trade costs seems to be a reasonable candidate.

We would like to conclude the paper by pointing out two possible directions of further analysis. First, developing a similar nonlinear measure that can be applied to the panel analysis seems to be useful since recent PPP studies often utilize the panel data (e.g., Frankel and Rose, 1996). Second, instead of using the aggregated price index, applying the nonlinear persistence measure to the good-by-good international price differentials would be interesting given the fact several micro studies reveal faster convergence at the individual price level (e.g., Crucini and Shintani, 2002, and Goldberg and Verboven, 2004).

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Table 1
Half-Lives of the STAR Model Using
Nonlinear Impulse Response Functions

	$\delta = k\sigma$							
	$\sigma = 0.1$							$\sigma \to 0$
	k =5.0	4.0	3.0	2.0	1.0	0.5	0.1	k =1.0
q_0 =0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.4	1.0	1.0	1.0	1.0	1.1	1.2	1.4	1.3
0.3	1.2	1.2	1.3	1.5	1.9	2.3	2.6	2.4
0.2	1.5	1.7	2.0	2.8	3.9	4.7	5.3	5.6
0.1	2.3	2.9	4.0	5.5	7.1	7.8	8.3	23.0
0	4.1	5.4	6.8	8.2	9.2	9.5	9.6	∞
-0.1	6.2	7.5	8.5	9.2	9.2	8.9	8.6	23.0
-0.2	7.4	8.2	8.5	8.2	7.1	6.3	5.7	5.6
-0.3	7.4	7.5	6.8	5.5	3.9	3.2	2.8	2.4
-0.4	6.2	5.4	4.0	2.8	1.9	1.7	1.5	1.3
-0.5	4.1	2.9	2.0	1.5	1.1	1.0	1.0	1.0

Note: q_0 is the initial value. δ is the size of a shock. $c = 0.4426$.

Table 2
Half-Lives of the TAR Model Using
Nonlinear Impulse Response Functions

	$\delta = k\sigma$							
	$\sigma = 0.1$							$\sigma \to 0$
	k =5.0	4.0	3.0	2.0	1.0	0.5	0.1	k =1.0
q_0 =0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.4	1.2	1.3	1.4	1.6	2.4	4.4	6.0	∞
0.3	1.8	2.2	3.0	5.1	9.6	11.6	13.2	∞
0.2	3.5	5.2	8.5	13.3	16.9	18.3	19.3	∞
0.1	8.1	11.9	16.2	19.2	21.3	22.1	22.6	∞
0	14.6	18.2	20.7	22.4	23.4	23.7	23.7	∞
-0.1	19.3	21.4	22.7	23.4	23.4	23.2	22.8	∞
-0.2	21.4	22.4	22.7	22.4	21.3	20.5	19.7	∞
-0.3	21.4	21.4	20.7	19.2	16.9	15.3	13.9	∞
-0.4	19.3	18.2	16.2	13.3	9.6	7.8	6.5	∞
-0.5	14.6	11.9	8.5	5.1	2.4	1.0	1.0	1.0

Note: q_0 is the initial value. δ is the size of a shock. $c = 0.4426$.

Table 3
Finite Sample Performance of Persistence Estimators of the STAR Model

σ		10.00	1.00	0.50	0.30	0.25	0.20	0.15	0.10
$E\left[\mathbf{1}_{\{q_{t-1}>\lvert c\rvert\}}\right]$		0.97	0.72	0.52	0.34	0.27	0.20	0.11	0.03
h^*		1.02	1.24	1.51	1.91	2.12	2.45	3.04	4.40
(1) $T = 100$									
Nonparametric	Mean	0.92	1.14	1.40	1.77	1.96	2.26	2.76	3.74
Persistence	Median	0.89	1.11	1.36	1.73	1.91	2.20	2.69	3.62
Measure $(\widehat{h}^*)$	Std.	0.25	0.31	0.37	0.47	0.53	0.62	0.80	1.23
Half-life Using	Mean	1.00	1.27	1.61	2.13	2.41	2.85	3.60	5.11
Linear AR Model	Median	0.97	1.23	1.57	2.07	2.34	2.76	3.48	4.89
$(\widehat{h})$	Std.	0.25	0.32	0.40	0.54	0.63	0.78	1.09	1.89
(2) $T = 200$									
Nonparametric	Mean	0.97	1.21	1.51	1.93	2.16	2.51	3.12	4.38
Persistence	Median	0.96	1.20	1.49	1.91	2.14	2.48	3.09	4.33
Measure $(\widehat{h}^*)$	Std.	0.18	0.22	0.27	0.34	0.38	0.45	0.59	0.93
Half-life Using	Mean	1.01	1.29	1.66	2.21	2.51	2.99	3.83	5.60
Linear AR Model	Median	1.00	1.28	1.63	2.18	2.47	2.94	3.78	5.50
$(\widehat{h})$	Std.	0.18	0.22	0.28	0.38	0.44	0.55	0.76	1.32

Note: The smoothing parameter for the nonparametric estimator is $0.45\times$range. 10,000 replications.

Table 4
Persistence of PPP Deviations
(Annual Data: 1900-1996)

Country	Nonparametric Persistence Measure ($\widehat{h}^*$)			Half-life Using Linear AR Model ($\widehat{h}$)			Half-life Using Nonlinear IRFs ($\widehat{h}(\overline{q}, \delta)$)	
		95% CI			95% CI		$\delta = 2\sigma$	0.1σ
1. Canada	3.10	[2.09,	5.98]	3.03	[1.81,	9.37]	1.80	0.81
2. France	1.41	[1.08,	2.05]	1.36	[0.89,	2.87]	0.78	0.95
3. Italy	2.53	[1.53,	7.40]	2.47	[1.52,	6.61]	3.96	6.30
4. Japan	6.14	[3.78,	16.28]	6.50	[3.28,	426.50]	9.75	35.57
5. Netherlands	2.21	[1.50,	4.18]	2.99	[1.77,	9.62]	1.87	2.66
6. United Kingdom	2.64	[1.88,	4.40]	4.84	[2.58,	39.47]	2.67	1.95

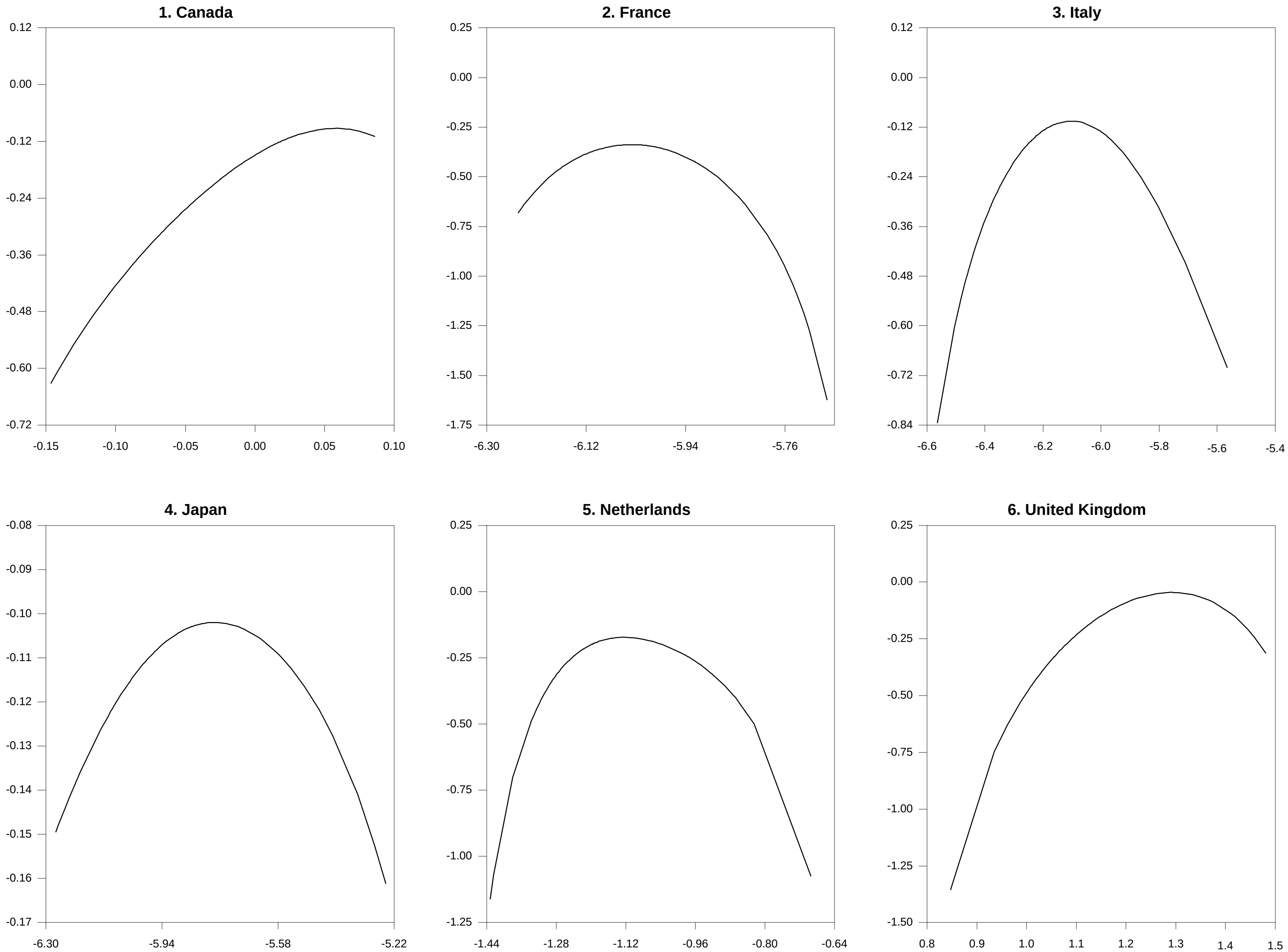
Note: QS kernel with optimal lag window (Andrews,1991) is used to construct confidence intervals for the nonparametric measure.

Table 5
Persistence of PPP Deviations under the Current Float
(Quarterly Data: 1973:1-1998:2)

Country	Nonparametric Persistence Measure $(\widehat{h}^*)$	95% CI	Half-life Using Linear AR Model $(\widehat{h})$	95% CI	Half-life Using Nonlinear IRFs $(\widehat{h}(\overline{q},\delta))$ $\delta = 2\sigma$	0.1σ
1. Australia	2.64	[1.52, 9.74]	3.42	[1.38, ∞]	2.53	0.13
2. Austria	1.19	[0.80, 2.29]	2.35	[1.16, ∞]	1.67	2.55
3. Belgium	2.16	[1.31, 6.03]	3.12	[1.40, ∞]	1.58	1.86
4. Canada	32.22	[5.99, ∞]	20.00	[3.16, ∞]	2.09	0.08
5. Denmark	0.98	[0.70, 1.61]	2.59	[1.23, ∞]	1.45	2.44
6. Finland	2.27	[1.33, 7.64]	2.84	[1.30, ∞]	1.11	1.22
7. France	0.94	[0.67, 1.54]	2.47	[1.17, ∞]	0.94	2.32
8. Germany	1.08	[0.70, 2.32]	2.36	[1.13, ∞]	1.52	1.95
9. Greece	1.28	[0.90, 2.16]	2.56	[1.22, ∞]	2.37	2.98
10. Ireland	0.91	[0.64, 1.61]	1.60	[0.85, 13.61]	1.23	2.30
11. Italy	1.75	[1.08, 4.57]	2.37	[1.14, ∞]	1.63	2.02
12. Japan	2.76	[1.78, 6.12]	3.78	[1.73, ∞]	2.55	3.48
13. Netherlands	1.53	[0.93, 4.45]	2.22	[1.09, ∞]	1.42	1.87
14. New Zealand	2.09	[1.34, 4.78]	2.25	[1.09, ∞]	1.11	0.37
15. Norway	0.44	[0.30, 0.83]	1.87	[0.95, 89.15]	0.91	2.78
16. Portugal	2.25	[1.42, 5.37]	3.85	[1.65, ∞]	3.07	3.54
17. Spain	2.54	[1.58, 6.41]	3.65	[1.63, ∞]	1.53	2.12
18. Sweden	0.43	[0.26, 1.21]	3.27	[1.43, ∞]	1.78	4.14
19. Switzerland	0.63	[0.47, 0.92]	1.19	[0.67, 4.94]	1.00	1.92
20. United Kingdom	1.35	[0.87, 3.00]	2.06	[1.02, ∞]	1.21	1.99

Note: See note of Table 4.

Figure 1. Local Speed of Convergence



Note: Estimated local speed of convergence ($\ln|Dm(q_{t\text{-}1})|$) versus the level of real exchange rate ($q_{t\text{-}1}$).