Federal Reserve Bank of Minneapolis
Research Department Staff Report 312

Revised November 2003

Non-Convexities in Quantitative General Equilibrium Studies of Business Cycles*

Edward C. Prescott

University of Minnesota
and Federal Reserve Bank of Minneapolis

ABSTRACT

This paper reviews the role of micro non-convexities in the study of business cycles. One important non-convexity arises because an individual can work only one workweek length in a given week. The implication of this non-convexity is that the aggregate intertemporal elasticity of labor supply is large and the principal margin of adjustment is in the number employed—not in the hours per person employed—as observed. The paper also reviews a business cycle model with an occasionally binding capacity constraint. This model better mimics business cycle fluctuations than the standard real business cycle model. Aggregation in the presence of micro non-convexities is key in the model.

*Paper prepared for the conference "Frontiers in Applied General Equilibrium Modeling," Cowles Foundation, Yale University, April 5 and 6, 2002. The author acknowledges financial support of the National Science Foundations. The author thanks Sami Alpanda, Andreas Hornstein, and Alexander Ueberfeldt for comments and discussions. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Introduction

The tool now used to study business cycles is the discipline of quantitative dynamic general equilibrium. With this discipline, given the question or issue at hand, an explicit model economy is written down and the answer to the question determined for that model economy. Theory, the question, and the available statistics dictate the choice of model economy used in the application. The pioneers in applying the discipline of quantitative general equilibrium are Herbert E. Scarf's students Shoven and Whalley (1972)[1]. They applied these tools to problems in public finance. Their models are rich in sector detail, but not truly dynamic. Subsequently Auerbach and Kotlikoff (1987), Jorgenson and Yun (1990), and others have made these public finance models dynamic.

A convenient feature of these early structures is that there is a parametric set of excess demand functions that can be easily calibrated using input-output tables and the equilibrium computed using Scarf's algorithm or other solution methods. Kydland and Prescott (1982) took a different approach in their study of business cycles. We constructed a linear-quadratic economy with the same steady state and local behavior as those of a deterministic growth model.[2] A feature of our approach is that uncertainty is easily introduced. With linear-quadratic economies, the equilibrium stochastic processes are linear, which matches well, but not perfectly, with observations.

The discipline of quantitative dynamic general equilibrium theory in conjunction with growth theory now dominates the study of business cycles and the evaluation of tax policies. Recently there have been two important additional successful applications of

[1] The works of Johansen (1960) and Harberger (1968) were very much in this tradition, but were basically static.

[2] Technically there is not a steady-state for a growing economy. The economy can be

quantitative dynamic general equilibrium methods using growth theory along with national income account statistics to address other macro problems[3]. One success is to determine what the value of the stock market should be when it is reasonable to assume agents expect current tax and regulatory policies to persist into the future,[4] and the other is to study great depressions of the twentieth century.[5] Like in business cycle theory and in public finance, almost surely, the discipline of applied general equilibrium will come to dominate the study of these fields. Of this I am certain.

The consistency of the underlying assumptions concerning preferences and technologies across these diverse applications leads to great confidence in the findings. The fact, for example, that business cycles are what this theory predicts adds confidence to the public finance findings that use the same theory. This never would have happened absent the discipline of quantitative general equilibrium. In this paper I will restrict attention to an important class of issues in business cycle theory, namely, the importance, or in some cases lack of importance, of non-convexities at the household and production unit levels for business cycle behavior.

In this paper I will abstract from money for three reasons. First, so much work has been done in this area using the discipline of quantitative general equilibrium that reviewing these developments in this paper is not feasible. Second, the findings concerning the role of monetary factors in business cycles are mostly negative with the correlations of monetary factors with real factors arising for spurious reasons (see

---

made stationary by dividing the date values of each variable by its constant growth value.

[3] Recently there is a plethora of interesting quantitative general equilibrium analyses using heterogeneous agent economies to evaluate insurance schemes and labor market policies.

[4] See McGrattan and Prescott (2000 and 2001).

[5] The volume edited by Kehoe and Prescott (2002) contains many of these studies as well

Freeman and Kydland, 2000). Third, there is not a tested theory for incorporating money in quantitative general equilibrium analysis. One, or maybe the, leading candidate for incorporating money (see Alavarez, Atkeson, and Kehoe (2002)) incorporates the Baumol-Tobin inventory theoretical role for money, which introduces a non-convexity in individual decisions. Even though a tested theory for introducing money into quantitative GE models does not now exist, almost surely in the not too distant future there will be such a theory and the tools of quantitative general equilibrium will have played a crucial role in its development.

There is a variety of interesting business cycle questions that have been addressed using this discipline of quantitative general equilibrium. The question that Kydland and Prescott (1982 and 1991) focused on is, How volatile would the U.S. economy have been in the post–Korean War period if productivity shocks had been the only shocks to the economy? The economy that Kydland and Prescott (1991) use has an important non-convexity in the stand-in household's consumption set. Workweeks of different lengths are different commodities, and a person is constrained to work one of these continuum of workweek lengths or not at all. This non-convexity turns out to be important in answering the posed question. Once this feature of reality is introduced, an implication of theory is that the principal margin of labor supply adjustment will be in the number of people working in a given week as opposed to the length of the workweek. This prediction conforms to observation. Previously Gary D. Hansen (1985) had shown that if the only margin of adjustment permitted is the number employed, then the intertemporal elasticity of labor supply is high, something that is needed if the growth model is to generate business cycles.

---

as references to earlier ones.

For many years prior to World War II, many leading economists were concerned with business cycles, namely, the recurrent fluctuations of output and employment about trend. During this period, not surprisingly, economists developed a plethora of stories attempting to explain why these fluctuations occurred. One reason for their failure to develop a successful theory of business cycles was that dynamic economic theory had not yet been sufficiently developed, much less the discipline of quantitative dynamic general equilibrium. It is true in the 1920s that Irving Fisher on this side of the Atlantic and Erik R. Lindahl on the other side recognized that static general equilibrium theory could be made dynamic by adding a date index to commodities. It is also true that in the early 1950s Kenneth J. Arrow and Gerard Debreu recognized that by indexing commodities by events, general equilibrium theory could be extended to uncertainty. But by then, the business cycle was a dormant subject.

Another reason for the failure to develop a theory of business cycles was the lack of good aggregate economic statistics. The modern U.S. quarterly system of national accounts only begins in 1947. Reasonably accurate measures the labor input were not available until about the same time. Still another reason was that modern growth theory, which was developed to account for the secular movements in aggregate outputs and inputs, had not been developed.

In fact, the view in the profession in the 1950s and 1960s was that these fluctuations were not equilibrium phenomena and therefore that general equilibrium language was not useful in their study. Even if this view were not totally dominant in the 1950s and 1960s, there were not the recursive language and computing power needed to compute the equilibrium stochastic laws of motion governing the evolution of model economies.

Equilibrium elements of business cycle model economies are stochastic processes, typically Markov with a stationary transition probability measure. This permits the comparison between the statistical properties of the model economies and the corresponding statistical properties of the actual economy. In the 1970s, the prevailing view of the profession was that changes in real factors, such as taxes and total factor productivity, gave rise to the secular movement in the aggregate data and that changes in monetary factors gave rise to business cycle fluctuations.

The use of the discipline of quantitative dynamic general equilibrium to derive the implications of growth theory surprised the profession and forced it to change its views. The result that surprised the profession, including those who first carried out the analysis, is that random persistent changes in the factors that determined the constant growth level (not the growth rate) of the growth model give rise to business cycle fluctuations of the nature observed. It turned out that Eugen Slutsky was right – business cycles are the sum of random causes and not the realization of a damped oscillatory system such as Knut Wicksell's rocking horse randomly being bumped[6].

Kydland and Prescott (1982) determined how big the variance of the persistent component of technology shock has to be to generate fluctuations of the magnitude observed in the United States in the 1954-1980 period. Subsequent estimates of this variance (Prescott, 1986) found that the variance was of this magnitude. This is a success

[6] Adelman and Adelman (1959), at the suggestion of Arrow, found that time series models, namely, the Klein-Goldberger Model, displayed damped oscillation as the dominant eigenvalue of the model was 0.74. This empirical result is consistent with the sum-of-random-causes

for the discipline of quantitative general equilibrium and for growth theory, a theory which was developed to account for the secular movements in the aggregate time series and not to account for business cycles. Quantitative dynamic general equilibrium methods are needed to show that growth theory implies business cycle fluctuations. This is not something that one can derive without the use of quantitative general equilibrium analysis.

Kydland and Prescott (1982) found that the growth model displays business cycle fluctuations if and only if the aggregate intertemporal elasticity of labor supply is high, a fact that was not then accepted by most labor economists.[7] The labor economists ignored the consequences of aggregation in the face of non-convexities in coming to their incorrect conclusion that the aggregate elasticity of labor supply is small. Non-convexities at the household level imply high intertemporal elasticity of labor supply even if the intertemporal elasticity of labor supply of the households being aggregated is small.

This paper considers non-convexities in quantitative GE business cycle analyses. Non-convexities at the micro level abound and can be measured. Consistency between micro observations and macro theory is crucial. Only with this consistency can economists evaluate public policies with any confidence. One notable success of theory was the recognition that an aggregation result underlies the stand-in household in the aggregate theory. This result is analogous to the aggregation result that justifies the concave, constant-returns-to-scale, aggregate production function. In spite of non-

construct and not with the damped oscillation construct.

[7] Lucas and Rapping (1969) estimated the intertemporal elasticity of labor supply and found it large.

convexities at the firm or household levels, the aggregate economy is convex if the micro units are infinitesimal. A very important implication of this aggregation is that the substitution elasticities of the stand-in household or stand-in firm are very different from the elasticities of the micro units being aggregated.

There is a fundamental and important non-convexity associated with the workweek length. Rosen (1986) pointed out that workweeks of different lengths are different commodities and that these commodities are indivisible. Rogerson (1988) formalized this concept in a static setting where people either worked a standard workweek in the market or did not work in the market sector. Hansen (1985) introduced this feature into business cycle theory and found it resulted in a much higher intertemporal elasticity of labor supply for the stand-in household than for individual households and therefore larger fluctuations in output and employment resulting from any set of shocks.

On the technology side, Herbert E. Scarf's fixed cost associated with lumpiness of investment, which leads to an (S,s) policy, has little consequence for aggregate behavior in the economies of Fisher and Hornstein (2000) and Julia Thomas (2002). These economies are calibrated so that the amounts of micro and aggregate fluctuations are in line with observations.

The findings are dramatic. The paper that makes this so clear is Thomas (2002). As she points out, the lumpiness of investment at the plant level is a well-established fact. She carries out an applied general equilibrium analysis with non-convex adjustment costs at the plant level and (S,s) adjustment rules as equilibrium behavior. In contrast to conclusions based on partial equilibrium analyses, such as Abel and Eberly (1996), Bertola and Caballero (1994), Caballero and Engel (1991, 1999), and Cooper, Haltiwanger, and Power (1999), she finds that the aggregate effects of these micro non-convexities have negligible consequences for aggregate behavior. Partial equilibrium reasoning for addressing an inherently general equilibrium question cannot be trusted.

An exception to micro non-convexities not mattering for business cycle fluctuations is Hansen and Prescott (forthcoming). Hansen and I find that capacity constraints lead to non-linearities of the type observed in the aggregate time series. This analysis is reviewed in this paper. The resulting aggregate production function is not Cobb-Douglas, yet for secular growth its implications are the same as the Cobb-Douglas production function. The second exception is Kahn and Thomas (2002), who introduce non-convex capital adjustment. In both cases, the consequences of the non-convexities are small.

There are a number of other interesting quantitative business cycle analyses with non-convexities. Fitzgerald (1998) endogenizes the workweek length with skilled and unskilled labor being required to operate a production unit in order to evaluate laws that restrict workweek length. He finds that the high-paid skilled workers benefit from these laws and the low-paid unskilled workers lose. In his economy, at a given plant in a given period both the skilled and the unskilled must work the same workweek length. Another innovative analysis is Hornstein (2002), who introduces the option of varying the number of shifts. His objective was to come up with a better definition of capacity utilization. He was not that successful in achieving this objective, but did show that existing measures of capacity utilization are seriously flawed.

This paper is organized as follows. Section 1 briefly reviews what business cycles are, why they are puzzling, and what the principal findings are to date. Section 2 presents the class of economies used in most of business cycle research. These economies have a finite number of household types, where the number is typically one, and each type has convex preferences. The aggregate technology is a convex cone, typically with a single composite output good that can be used for consumption or investment purposes. This technology is typically represented by an aggregate production function with all the standard properties. Justifications based on aggregation theory are provided for these assumptions in Section 3. Section 4 presents the case where the workweek length is endogenous. Here there is not an aggregate production function with capital and labor services as the factor inputs. Following Alpanda and Ueberfeldt's (2002) generalization and simplification of Hornstein and Prescott (1993), it is shown that the margin of labor adjustment used is the number employed up to the point where all are employed. The workweek length margin is not used unless all are employed. Section 5 presents an economy with a sometimes binding capacity constraint. This micro non-convexity in technology leads to an interesting non-linearity in the equilibrium process governing output and employment.

Section 1: Business Cycles

Robert E. Lucas, Jr. (1977) defines business cycles as being recurrent fluctuations of output and employment about trend with the key regularities being the statistical properties of the comovements of the time series. An issue is, What is the trend? Robert J. Hodrick (1997) and I concluded that theory fails to provide a concept of trend and that it was necessary to come up with an operational definition that mimics the smooth curve that students of business cycles draw through the data. Our particular representation turned out to be a useful way to decompose the data into a trend and a cyclical component. There was a lot of theory behind the representation, which made clear some puzzling behavior of the time series from the perspective of production and utility maximization theory.

Why were business cycles puzzling?

On the household side the puzzling feature of the behavior of the cyclical components was that consumption and the labor input moved strongly procyclical, yet the real wage moved little. Here the real wage is defined to be aggregate labor compensation divided by aggregate market hours. This is puzzling because it requires the intertemporal elasticity of substitution to be high, far higher than what labor economists had estimated at the time.

On the production side, two-thirds of the variation in cyclical output is accounted for by variation in the labor input and the remainder by total factor productivity, while contemporaneously, the capital stock is orthogonal to output. Labor productivity and hours are positively correlated with output, but they are roughly orthogonal to each other. Increases in the labor input holding the capital input steady should lead to declines in labor productivity and a negative association between output and labor productivity by standard production theory.

Section 2: Convex Economies

In this paper I will be using the language of Arrow-Debreu-McKenzie and will be dealing with economies that have the following properties. The aggregate technology set is a convex cone. An implication of this is that payments to the factors of production exhaust product. There is a finite number of household types with an atomless measure of each type. A consequence of this is that all agents are small, and the no market power assumption is literally true in the model economies studied. Preferences of households are not convex, but preferences of the stand-in household for each type will turn out to be convex. Similarly, technologies of individual production units are not convex. Given the assumptions, however, the aggregate technology set will be a convex cone.

We assume preferences are such that households maximize expected utility and the utility function is continuous. The expected utility assumption is standard in applied analyses and has survived many efforts to replace it with something better. With expected utility maximization and an appropriate commodity vector, preferences of the stand-in households for the types are convex if randomization is permitted.[8] De facto, the model economies are convex and have a finite number of households.

Preferences of the type i stand-in households are ordered by

$$u_i(x) = \max_z \int U_i(c) z(dc)$$

subject to $\int c\, z(dc) \le x$, $z \ge 0$, and $\int z(dc) = 1$.

In the above maximization problem, the probability measure z is defined on the Borel σ-algebra of the underlying consumption set of a type i, which is denoted by C_i. I emphasize that the problem facing an individual of type i is not convex, or there would be no need for a stand-in household. Thus either U_i is not concave or C_i is not convex or both. The set C_i is a compact separable metric space and function U_i is continuous. Given these conditions the program has a solution for a given x provided the constraint set is

[8] Prescott and Townsend (1984a, 1984b) introduce lotteries into the Arrow-Debreu-

nonempty. The set X_i is the set for x for which the constraint set is nonempty. This set is convex given that the program's constraint set is jointly convex is $\{x,z\}$. The set X_i is the *consumption set* of the type i stand-in household. The function $u_i{:}X_i \to \Re$ is continuous and concave given the linearity of the constraint correspondence and the linearity of the objective function. The function $u_i{:}X_i \to \Re$ is that *utility function* of the type i stand-in household.

Thus preferences of the stand-in households are convex. The advantage of introducing a stand-in household in applied analysis is that the traded commodities are the ones reported in the accounts. This facilitates the interaction between theory and measurement that is central in applied general equilibrium analysis. This is in contrast to the Prescott and Townsend (1984a, 1984b) approach that treated commodities as probabilities from the perspective of the household.[9]

The commodity space is normed linear space S. An economy is specified by the set of elements $\{\{\lambda_i, X_i \subset S, u_i\}_{i=1,\ldots,I}, Y \subset S\}$. Here $\lambda_i > 0$ is the measure of type i. X_i is the type i stand-in consumption set, and the utility functions $u_i : X_i \to \Re$ are continuous and concave. The aggregate technology set Y is a convex cone.

An allocation $\{\{x_i\}_{i\in I}, y\}$ is feasible if $x_i \in X_i$ for all i , $y \in Y$, and the resource balance constraint

$$\sum_i \lambda_i \ x_i = y$$

McKenzie general equilibrium framework. They were needed to fully realize all the gains from trade and had the consequence of making preferences convex.

[9] Here I am following Hansen (1985) and Kehoe, Levine, and Prescott (2002).

is satisfied. A competitive equilibrium is a feasible allocation and continuous linear function on S such that the stand-in households maximize utility subject to their budget constraint and operators of technologies maximize value given their technology.

As shown by Debreu and Scarf (1963) in their core equivalence paper, with convex preferences, restricting attention to type-identical allocations is not an important restriction in the following sense. If a non-type-identical equilibrium exists, a type-identical equilibrium exists with the same equilibrium price systems, the same commodity vector for the aggregate technology, the same type-average consumption vector, and the same utilities.

In *theoretical* general equilibrium theory, the household sector demands the commodities and the business sector supplies the commodities. A disadvantage of this approach is that it results in the household sector demanding negative quantities of factors of productions rather than the household supplying factors of production such as labor services and capital services. In *applied* general equilibrium, the household sector supplies factors of production and demands other commodities subject to its budget constraint, where the budget constraint constrains expenditures to be less than or equal to income. Income is the value of the factors of production that the household supplies. The firm maximizes profit, that is revenue less costs. Costs are the value of inputs while revenue is the value of output. In theoretical work, the concepts of income, revenue, expenditures, and costs are not needed, but these accounting concepts are useful in applied work. There is no concept of gross national income and product within the more parsimonious theoretical general equilibrium language. When discussing applications, I will use the applied general equilibrium language.

Section 3: The Aggregate Production Function

The aggregate production function is used to characterize the aggregate production set. Here I briefly review the aggregation theory underlying aggregate production functions and why they are continuous, increasing and concave and why they display constant returns to scale. This aggregation theory will prove useful when endogenizing the workweek length, something that is central in business cycle theory.

The plant technologies underlying the aggregate production function are the following (note that *x* and *y* now denote different things then they did in Section 2):

(i) There are *n* factor inputs and a composite output good[10].

(ii) The vector of inputs is $x \in \Re_{+}^{n}$ and the output good is *y*.

(iii) A plant technology is indexed by $x \in T$ with $f(x)$ being the output of a plant of type *x*.

(iv) $X \in \Re_{+}^{n}$ is the vector of aggregate inputs, and *Y* is aggregate output.

Definition: An aggregate production function *F*(*X*) is the maximum output that can be produced given the input vector *X*.

Assumption 1: Any measure of technologies of type $x \in T$ can be operated.

Assumption 2: $T \subset \Re_{++}^{n}$ and T is compact.

Assumption 3: $f : T \to \Re$ is continuous.

---

[10] In this exposition the number of factors is finite. There are important business cycle applications where there is a continuum of factors and the input vector is a measure on the Borel σ-algebra of a subset of a Euclidean space.

The aggregate production function is the solution to the following program, where $M_{+}(T)$ is the set of measures on the Borel σ-algebra of T:

$$F(X) = \max_{z \in M_{+}(T)} \int f(x)\, z(dx)$$

$$\text{subject to } \int_T x_i z(dx) \le X_i \quad i = 1, 2, \ldots, n.$$

Proposition 1: $F(X)$ exists and is weakly increasing, continuous, weakly concave, and homogenous of degree one.

Proof. Given the assumptions, the constraint set is compact and non-empty and the objective function is continuous in the weak-star topology. Therefore the program has a solution. The function being increasing is immediate given larger X increases the constraint set. Continuity follows from the Theorem of the Maximum. Concavity follows from the convexity of the constraint set and concavity of the objective in (X,z). Because scaling z and X by a common factor is feasible and scales the objective function by the same factor, the function F must be homogenous of degree one. □

The function F summarizes the relevant aspects of the aggregate technology set and therefore is the element about which empirical knowledge can be organized. Multi-industry generalizations with intermediate goods are straightforward. However, in macro analyses the single sector version almost always suffices and is therefore used.

An Example:

The Cobb-Douglas production function has come to dominate in aggregate quantitative GE analysis. The reason is that both over time and across countries, labor's share of product is surprisingly constant at a little below 70 percent.[11] The Cobb-Douglas production function, with its unit elasticity of substitution, is the only aggregate production function with the property that factor cost shares are the same for all relative factor prices.

An example of an underlying set of plant technologies for the Cobb-Douglas production function is the following one. Suppose that the factor inputs to a production unit are k units of capital and e workers and that the plant technologies are $g(e)k^{\theta}$, where $0 < \theta < 1$. In addition, the function g is such that the function $g(e)\, e^{\theta - 1}$ has a unique maximum. This maximum is denoted by *A* and the maximizing *e* by *e**.

Proposition 2: For this example, the aggregate production function is

$$F(K,E) = A\, K^{\theta} E^{1-\theta},$$

where *E* is aggregate employment and *K* aggregate capital.

Proof. The linear program has two constraints. Therefore, there is an optimum that places mass on at most two points. Let (e_i, k_i) be one of these points and (E_i, K_i) be the aggregate quantities of the inputs allocated to this point. As much or more output is produced by (E_i, K_i) if they are allocated to E_i / e^* production units of type $(e^*, K_i / (E_i / e^*))$. Thus, all operated production units have the same number of workers. All operated units have the same quantity of capital as well, because this is necessary to equate marginal products of capital across these units given that employment

[11] See Gollin (2002) for the cross-country numbers. He uses the Kravis (1959) economy-wide assumption for assigning proprietor’s income and indirect business taxes to capital and labor.

e is equated across operated units. This implies that it is optimal to assign e^* workers and $k^* = K/(E/e^*)$ to E/e^* operated production units.

Section 4: Labor Indivisibility

Richard Rogerson in his dissertation (1984) analyzed an artificial economy where people are confronted with the choice of either working or not working. On first blush this appears to be a non-convexity. If a point in the commodity space specifies the quantity of the consumption good and the measure of workweek lengths, the economy becomes convex. From the perspective of the aggregate stand-in firm, the measure of workweek lengths specifies the number of people employed that work $h \in B$ for any Borel measurable $B \subseteq H$, where H is the set of possible workweek lengths. From the perspective of a household, the measure of workweek lengths is a probability measure of workweek lengths that the household must supply.

By an appropriate law of large numbers, the total measure of workweek length supplied is the measure of people times the probability that each works. There are many ways that the firm can pick the set of identical, but not independent, 0-1 random variables specifying whether or not each person works. This is the Prescott-Townsend (1984a, 1984b) lottery equilibrium approach. Another equivalent approach is to construct a stand-in household with all the randomization being done within the group of type-identical individuals. This is the Hansen (1985) approach that has been generalized by Kehoe, Levine and Prescott (forthcoming).

The Rogerson economy has measure one of type-identical people. They all maximize expected utility and have identical utility functions. Their utility is $u(c) - v(\bar{h})$ if they work and $u(c)$ if they do not. The function $u : \Re_+ \to \Re$ is continuous, strictly increasing, and concave. The number $v(\bar{h})$ is positive, indicating that people prefer not working to working.

Here we take the stand-in household approach. Let e b*e* the fraction or measure of the group that work. Maximizing the expected utility of group members, the stand-in household's utility function, $U:\Re_+ \times [0,1] \rightarrow \Re$, is

(1) $$U(C,E) = u(C) - E\, v(\bar{h}) .$$

As shown by Hansen, a simple unemployment insurance scheme works in this environment, where those that do not work receive benefits. Alternatively, having members of the group enter into wealth gambles is another way to support this within group allocation. Still another way is to index individual allocations by some random variable with a continuous density, which Shell and Wright (1993) call the sunspot approach. The advantage of using lotteries to this sunspot approach is that the economy is convex and all standard general equilibrium theory is easily applied.

The principle is to deal with the simplest commodity space for which preferences are convex. This is sufficient to insure that there are no gains from introducing randomness. Given that the technology exists for gambling, ruling out trades that are feasible and mutually beneficial is inconsistent with equilibrium. To summarize, once a group exploits all gains from randomization, a type-identical group has a stand-in that behaves as if it is maximizing (1).

Section 5: Why a Fixed Workweek Length

Hansen (1985) established that the intertemporal elasticity of labor supply is infinite up to the point that all are working if the workweek length is fixed. This fits well with observation as the principal margin of adjustment is the number employed and not hours that offices and factories are operated. A question, however, is why the number working is the principal margin of adjustment and not the length of the workweek. In this section this question is addressed. The model economy used is as follows. There is measure one of identical individuals. Each household preferences are ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

for $c_t \in C = \Re_+$ and $h_t \in H = [0,1]$. The maximum amount of time that a given individual can physically work is 1. The utility function is strictly increasing in both its arguments and strictly concave as well as being continuously differentiable. Each person has $\bar{k} > 0$ units of capital at the beginning of period zero.

The technology is described by the plant production functions

$$c + i \leq g(h)k^{\theta}.$$

Here consumption is c and investment *i*. A technology is described by the four-tuple $s = (c, i, h, k)$. The set of *s* satisfying the plant technology set is *S*. A firm's production plan *a* is a measure on the Borel σ-algebra of *S*.

Capital depreciates at rate δ so $k_{t+1} = (1-\delta)k_t + i_t$. The function $g(h)$ is concave and increasing. If an individual works h and uses k units of capital, his output is $z \leq A\, g(h)^{1-\theta} k^{\theta}$. Andreas Hornstein and Prescott (1993) dealt with the special case that $g(h)^{1/(1-\theta)} = h.$ Osuna and Rios-Rull (2001) dealt with the generalization $g(h)^{1/(1-\theta)} = h^{\zeta}$ where $\zeta > 1-\theta.$ The argument followed here is due to Alpanda and Ueberfeldt (2002). Their argument is more general and simpler than the one Hornstein and I developed.

The period commodity space is $L = M(\Re^2 \times H \times K)$. A point in this space is a measurable set of (c, i, h, k) vectors. Here *M* denotes a space of signed measures on the Borel σ-algebra of the space in question. The interpretation of *h* is the amount of workweeks of length h.

The period consumption set is

$X = \{x \in L \mid$ x is a probability measure and $k \leq \bar{k}\}$.

The period utility function is

$$u(x) = \int U(c,h)\,dx .$$

Proposition 3: Preferences are convex.

This result is immediate given that *u* is linear and *X* convex. The convexity of preferences permits attention to be restricted to type-identical allocations. Further, the utility function is continuous.

The period aggregate production set is

$Y = \{y \in L_+ \mid \exists$ measure of production units such that

(i) $\int (c+i)\,dy - \int A\,g(h)k^{\theta}\,da \leq 0$

(ii) for all measurable $B \subseteq H$, $y(z \mid h \in B) = a(s \mid h \in B)$

(iii) $-\int k\;dy + \int k\,da \leq 0\,\}$.

Constraint (i) is that enough is produced to supply the quantity of output specified by commodity vector *y*. Constraints (ii) are that enough of the types of workweeks are acquired by the firm to carry out its production plan. Constraint (iii) is that the firm acquires a sufficient quantity of capital services to carry out its plan.

Proposition 4: The set *Y* is convex.

Proposition 5: A type-identical optimum exists.

The existence of a type-identical competitive equilibrium is straightforward even if there is uncertainty for this economy. See Stokey and Lucas (1989, ch. 15).

I now show that the workweek is constant up to the point that all are employed if preferences and technology are consistent with constant growth. I deal first with technology. The aggregate production set is characterized by an aggregate production function $F(K,x)$, where x is a measure on the Borel σ-algebra of *H*. This function has all the standard properties of an aggregate production function, but is difficult to deal with given that x is a signed measure. For this reason, here I restrict the technology in a non-binding way to one in which only one type of plant being operated. This greatly simplifies notation. With this restriction, the aggregate production function is

$$c + i \le A h^{\zeta} k^{\theta} e^{1-\theta} \text{ , where } \zeta > 1-\theta .$$

Here employment *e* is the measure of people working a workweek length *h*.

The utility function is

$$U(c,h) = \frac{[c^{\gamma}(1-h)^{1-\gamma}]^{\varepsilon} - 1}{\varepsilon},$$

where $\varepsilon < 0$. Here we deal only with the case where there is more curvature than the log. The argument simplifies in the case in which the utility function is $U(c,h) = \gamma \log c + (1-\gamma)\log(1-h)$.

Proposition 6: In this class of economies, $e < 1$ and $h = \bar{h}$ or $e = 1$ and $h \ge \bar{h}$.

Proof. I denote the supply reservation price schedule for workweeks of different lengths in units of the consumption good by $w(h)$ for a particular period t given the event-history $(A_1, A_2, \ldots, A_t)$. Throughout this proof the event history argument will be implicit because it plays no role in the argument. Similarly, r is the rental price of capital. These prices are in terms of the period t consumption good.

First I show that if all work, all work the same number of hours. Next I show that if $e < 1$, then some work $h = \bar{h}$ and others work $h = 0$. This $\bar{h}$ depends only on the parameters of preferences and technology and not on the event history or the initial capital stock. Finally I show that if $e = 1$, then $h \geq \bar{h}$.

The first step in showing that if all work, they work the same length workweek is to show that the supply reservation wage is strictly convex in h. This is immediate because

(2) $$w(h) = B\, h^{\zeta/(1-\theta)} = \max_k \{A h^{\zeta} k^{\theta} - r\, k\}$$

given $\zeta > 1-\theta$. B is a constant that depends on k and A, which are event history dependent. In the case $\zeta = 1-\theta$, function $w(h)$ is proportional to h.

The period problem facing a household is

$$\max_{x \geq 0} \int U(c, 1-h)\, dx(dc \times dh)$$

$$\text{s.t.} \quad \int dx = 1$$

$$\text{s.t.} \quad \int c\, dx - \int w(h)\, dx = \int c\, dx - \int B\, h^{\zeta/(1-\theta)}\, dx \leq R\, B\,.$$

Here R is a constant that depends upon the event history and the initial capital stock. The first-order conditions for this linear program are

(3) $$U(c,h)-\lambda c+\lambda w(h)+\phi \le 0\,.$$

Here ϕ and λ are the Lagrange multipliers associated with the two constraints. Multiplier λ is the marginal utility of consumption and is strictly positive.

Equating the marginal utility of consumption to λ yields

(4) $$c(h,\lambda)=\left(\frac{\gamma}{\lambda}\right)^{1/(1-\gamma\varepsilon)}(1-h)^{(1-\gamma)\varepsilon/(1-\gamma\varepsilon)}\,.$$

Using (1) and (3) to substitute for *c* and *w*(*h*), the first-order conditions (2) can be written as a function of *h* and the Lagrange multipliers only,

$$f\,(h,\lambda,\phi)\le 0\,.$$

Equality must hold at *h* in the support of the marginal measure on *h* of the optimal measure.

Function *f* has a single inflection point, $f_1(h,\lambda,\phi)<0,$ and $f_1(1,\lambda,\phi)=-\infty$. This implies that the shape of the function is as in Figure 1 or Figure 2. Thus the optimum either puts all its measure on a single point or splits the measure between $h=0$ and some other point. This establishes the first part of the proof.

Consider now the case in which measure is placed on *h* = 0. In this case the program facing the household can be written as

$$\max_{e,c_0,c_1,h_1}\{eU(c_1,h_1)+(1-e)\,u(c_0,0)\}$$
$$s.t.\quad e\,c_1+(1-e)\,c_2-e\,w(h_1)\le R\,B.$$

The first-order conditions for this program are

$$U_1(c_1,h_1)=\lambda$$
$$U_1(c_0,0)=\lambda$$
$$U(c_1,h_1)-u(c_0,0)+\lambda\,w(h_1)-\lambda(c_1-c_0)=0$$
$$U_2(c_1,h)+\lambda\,w'(h_1)=0.$$

Using (2), (4), and the fact that

$$(5) \qquad \frac{c\,u_1(c,h)}{\gamma\varepsilon} = \frac{c\lambda}{\gamma\varepsilon} = U(c,h)\,,$$

an implication of these first-order conditions is that

$$(6) \qquad (\varepsilon^{-1}-\gamma)(\gamma-1)^{-1}(1-h-(1-h)^{\frac{1-\varepsilon}{1-\varepsilon\gamma}}) = \frac{w(h)}{w'(h)} = \frac{(1-\theta)}{\zeta}\,h\,.$$

The important result is that equation (6) is a function of h and parameters of the model. Let this solution to (6) be $\bar{h}$. What differs if *B* and *R* are different is *e* and not *h*, unless, of course, the change is so large that $e=1$. This completes the second part of the proof.

The best *h* if all work is a decreasing function of *R* as shown in Figure 3. When the function decreases to $\bar{h}$, it is optimal to shift to the *e* margin of adjustment. At this point, the optimal *e*(*R*) becomes strictly decreasing and optimal consumptions remain constant. All increase in “wealth” is taken in the form of a lower fraction of the population that work in the market sector. This completes the proof. □

Section 6: Capacity Constraints and Non-Linearities

A problem with the Cobb-Douglas production function is that it implies constant factor shares for both the smooth secular movements in output as well as the non-smooth business cycle fluctuations. Cyclically, capital share is procyclical, being particularly high at cyclical peaks. Another problem is that there is non-symmetry in the economic time series. Business cycle peaks are smaller than troughs and are flatter. This suggests that an alternative aggregate production function is needed to better model business cycle fluctuations. In particular, the abstraction must capture the fact that the economy is hitting capacity constraints at many production units when the economy is at the peak.

The problem is to develop an alternative production technology that is tractable and captures these features. This technology must generate both the growth facts and the business cycle facts. Hansen and Prescott (forthcoming) developed such a technology. We started at the micro level and did the aggregation. The micro foundations are no more realistic than those for the Cobb-Douglas production function, but are important because they led us to this alternative aggregate production function for the study of business cycle fluctuations.

The economy Hansen and I studied is a one-sector stochastic growth model in which output is produced from three factors of production, labor and two types of capital. One type of capital, identified with long-run capacity, for want of a better term, will be referred to as the location at which production can potentially take place. Examples include office buildings, factories, and large ships. The second type of capital is called equipment. It can be assigned, along with labor, to a location to form an operating plant.[12] The production function of a plant is given by

(7) $$y = \begin{cases} zk^{\theta}n^{\phi} & \text{if } n \geq \bar{n} \\ 0 & \text{otherwise.} \end{cases}$$

In this expression, k is the quantity of equipment and n is the quantity of labor employed at the plant in a given period. The variable z, where $z \in \{z_1, ..., z_{n_z}\}$, is the realization of an aggregate technology shock that follows an n_z state Markov chain with transition probabilities $\pi_{z,z'}$. We assume eventual decreasing returns to scale at the plant level, so $\theta + \phi < 1$. This assumption guarantees that it is profitable to operate many small plants rather than one large one and that all operating plants will employ the same amount of

[12] We will refer to this second type of capital as equipment for lack of a better term. The distinction between the two types of capital does not correspond to the distinction between structures and equipment used by the U.S. Department of Commerce. For example, a Boeing 747 is a "location at which production can potentially take place," and is therefore long run capacity in our model. Similarly, a storage shed used by a manufacturing firm is formally a structure, but is not a location where production takes place and should probably be classified as the second type of capital.

equipment and labor. In addition, the requirement $n \geq \bar{n}$, along with a limited population of potential workers, implies an upper bound on the total number of plants that can be operated.

In any period there is a fixed number, *M*, of available locations that can be potentially operated. Equipment and labor (*k* and *n*) can be costlessly moved across locations, so these factor inputs will only be placed at operating plants. This assumption, along with the minimum labor requirement ($n \geq \bar{n}$), implies that there may be idle locations in some states, although equipment will never be left idle.

Suppose that in a given period, there are *K* units of equipment and *M* locations. In addition, suppose that *N* units of labor are employed. The aggregate production function is defined by the following expression, where z is the measure plant types (k, n) that are operated. The measure *z* is defined on the Borel σ-algebra of the set $\Re_+ \times [\bar{n}, \infty]$.

$$
\begin{aligned}
F(K,N,M) &\equiv \max_{x \geq 0} \int a k^{\theta} n^{\phi} dx \\
\text{subject to } &\int k\, dx \leq K \\
&\int n\, dx \leq N \\
&\int dx \leq M.
\end{aligned}
\tag{8}
$$

A solution to this problem will equate marginal products across operating plants. It can be shown that there will be just one type of plant operated in any particular period, $z_{\hat{k}\hat{n}}$. That is, all operating plants employ the same quantity of equipment and labor. If $m \leq M$ is the number of locations operated, then $\hat{k} = K/m$, $\hat{n} = N/m$, and $m = x_{\hat{k}\hat{n}}$. With this change of variables, equation (8) can be rewritten as

$$
F(K,N,M) = \max_{m \leq \min\left\{M, \frac{N}{\bar{n}}\right\}} z\left(\frac{K}{m}\right)^{\theta}\left(\frac{N}{m}\right)^{\phi} m. \tag{9}
$$

The constraint $m \leq N/\bar{n}$ in this problem follows from the requirement that the amount of labor employed at each plant, N/m, must be greater than $\bar{n}$.

The assumption that $\theta + \phi < 1$ implies that the constraint $m \leq \min\{M, N/\bar{n}\}$ will always bind in problem (9). Hence, two possibilities can arise: $M < N/\bar{n}$, in which case $m = M$ in equation (9), or $M > N/\bar{n}$, in which case $m = N/\bar{n}$. Hence, solving problem (9), we obtain

$$F(K,N,M) = \begin{cases} zK^{\theta}N^{\phi}M^{1-\theta-\phi} & \text{if } N > M\bar{n} \\ zK^{\theta}N^{1-\theta}\bar{n}^{\theta+\phi-1} & \text{if } N < M\bar{n}. \end{cases} \tag{10}$$

The aggregate production function in equation (10) can be understood as follows. In the first case, all *M* locations are assigned equipment and at least $\bar{n}$ units of labor. Hence, the economy is operating at "full capacity" in that all locations are operated and the shadow value of additional locations is positive. As a result, in a decentralized version of this economy with competitive markets, locations earn a share of total income equal to $1-\theta-\phi$. In the second case, an insufficient amount of labor is employed to operate all *M* locations, so the economy is operating at less than full capacity. Location capital, since it is not a scarce input, earns no rent. Instead, in this "excess capacity" case, labor earns a larger share, $1-\theta$, of income. Notice that labor's share under full capacity can be as large as ϕ, which is smaller than $1-\theta$ given our assumption that $\theta + \phi < 1$.

Resource Constraint and the Evolution of Capital

Output can be used to provide a perishable consumption good C_t, to provide an investment good X_t, and to establish new locations $M_{t+1} - M_t$.

The evolution over time of the equipment component of the capital stock is standard. One unit of investment today produces one unit of equipment, K_{t+1}, available for use in the following period. The depreciation rate is denoted by δ, where $0 < \delta < 1$, so the law of motion of the stock of equipment is given by

$$K_{t+1} = (1-\delta)K_t + X_t. \tag{11}$$

In comparison, each additional unit of location capital $M_{t+1} - M_t$, which also requires one period to produce, requires that ω units of output be invested today.[13] Location capital does not depreciate, and location investments are irreversible. Hence, the resource constraint can be written

$$C_t + X_t + \omega(M_{t+1} - M_t) \leq z_t F(K_t, N_t, M_t), \tag{12}$$

where $M_{t+1} \geq M_t$.

Preferences

The economy has a measure one continuum of identical individuals each endowed with one unit of time each period. Preferences are ordered by the expected value of $\Sigma_{t=0}^{\infty} \beta^t [\log c_t + v(l_t)]$, where v is an increasing function of leisure. Labor is indivisible, meaning that individuals work a given workweek length or not at all. In addition, given a lottery mechanism for allocating time use, a stand-in household exists with preferences ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t (\log C_t - \gamma N_t), \quad 0 < \beta < 1, \; \gamma > 0, \tag{13}$$

where N_t is the fraction of available household time employed in market production.

Computing Equilibrium Allocations

Given that there are no distortions in this economy, equilibrium allocations are equivalent to those that would be chosen by a social planner who maximizes (13) subject to (10)–(12). This problem has the property that, once a sufficient amount of location capital has been accumulated, no further investments will be made in *M*. This follows

[13] A reasonable assumption would be that more time is required to produce location capital than equipment. Although this is likely to be true in actual economies, we have chosen to make the minimum number of assumptions to guarantee that location capital is not varied over the business cycle in the invariant distribution implied by our theory.

from the fact that *M* does not depreciate, that the technology shock *z* has bounded support, and the fact that we have abstracted from population growth. We suppose that this economy has been operating for a long time, so we restrict ourselves to computing equilibrium allocations that are relevant once this sufficient quantity of *M* has been accumulated.

The result that, in the limit, investment in location capital is zero in all states generalizes to a constant growth version of this economy with exogenous technological progress. In particular, if we were to replace equation (7) with the same technology premultiplied by $\rho^{(1-\theta)t}$, where $\rho > 1$, the balanced growth path would involve output, C_t, X_t, and K_t all growing at the rate $\rho - 1$. The variables M_t and N_t are constant along this balanced growth path. Intuitively, N_t is constant because the population is fixed and M only earns rents if $N_t > M_t\bar{n}$, where $\bar{n}$ is a constant. Hence, *M* cannot, in the limit, grow at a rate higher than the *N*. Of course, if there is population growth, *M* does grow and ongoing investment in location capital would be undertaken.

This can be done in two steps. First, optimal decision rules for the social planner's problem given an arbitrary fixed value of M are computed. Second, given these decision rules, one can compute the constant value of *M* that would hold in a stationary solution to the planner's problem.[14]

The following is the dynamic program solved by a social planner given a fixed value of *M*:

[14] Alternatively, we can back out the fixed cost ω that would induce the value of *M* used when computing the decision rules.

$$v(z,K;M) = \max_{N,K'} \left\{ \log C - \gamma N + \beta \sum_{z'} \pi_{z,z'} v(z',K';M) \right\}$$

(14) subject to

$$C + K' = (1-\delta)K + \begin{cases} zK^{\theta} N^{\phi} M^{1-\theta-\phi} & \text{if } N > M\overline{n} \\ zK^{\theta} N^{1-\theta} \overline{n}^{\theta+\phi-1} & \text{if } N < M\overline{n} \end{cases}$$

$$0 \le N \le 1.$$

The solution to this problem is a set of decision rules of the form $N = \mathrm{N}(z,K;M)$, $K' = \mathrm{G}(z,K;M)$, and $C = \mathrm{C}(z,K;M)$.

The value of M in a stationary solution to the planner's problem is determined by setting the maximal marginal value of an additional location across all possible states equal to the cost of establishing the location, ω. The marginal value of an additional location given the current state, $v_M(z,K,M)$, is the present discounted marginal product of the location over its infinite lifetime. This can be found by solving the following functional equation:

$$v_M(z,K,M) = \sum_{z'} \pi_{z,z'} [z' F_3(K', N(z',K';M), M) + v_M(z',K',M)] .$$

In this expression, $K' = G(z,K;M)$ and F_3 is the partial derivative with respect to *M* of the function *F* in equation(10). The stochastic discount factor employed by the social planner, *Q*, is given by

$$Q(z,z') = \beta\, \pi_{z,z'} \frac{\mathrm{C}(z,K;M)}{\mathrm{C}(z',\mathrm{G}(z,K;M);M)} .$$

The value of *M* in a stationary solution to the planner's problem is determined as follows, where E_M is the ergodic subset of the state space implied by the solution to problem (14):

(15) $$\omega = \sup_{\{z,K\}\in E_M} v_M(z,K,M) .$$

Computing Equilibrium Factor Shares

Although most of the variables we are interested in are quantities, we are also interested in computing factor shares for this economy. This requires that we compute factor prices. In a decentralized growth model, the wage rate is normally equal to the marginal product of labor evaluated at the values for capital and labor that solve the planner's problem. In this model, however, the presence of a kink in the aggregate production function (at $N = M\bar{n}$) means that the marginal product of labor is not uniquely defined at this point. Hence, given that N will often equal $M\bar{n}$ in our simulations, the wage cannot be computed from the first-order conditions of the firm's problem as is usually done.

This does not mean that the wage is not uniquely determined at the kink point, but instead implies that we must compute it from the first-order conditions of the household's problem rather than the firm's problem. The first-order condition associated with the labor supply decision of the stand-in household, given a period utility function $U(C,N)$, implies that $w = U_2(C,N)/U_1(C,N)$. Given our choice of preferences, this implies that $w = \gamma C$, and, hence, labor's share is equal to $\gamma CN/Y$ where Y is aggregate output.

Solution Method

To solve the planner's problem (14), we use a variation on value iteration to compute piecewise linear approximations to the optimal decision rules.[15] In particular, a set of values for the stock of equipment with n_K elements is chosen, and we let Ω be the set $\{z_1,...,z_{n_z}\}\times\{K_1,...,K_{n_K}\}$[16]. We then chose initial guesses for the values of the decision rules, $N_0(z,K)$ and $G_0(z,K)$ at each point in Ω, that satisfy the constraints in problem

[15] Our solution procedure is similar to the Howard improvement algorithm described in Ljungqvist and Sargent (2000).

[16] We experiment to insure that the upper and lower bounds of the capital stock grid are chosen so that the interval $[K_1, K_{n_K}]$ includes all points that have positive probability in the invariant distribution implied by the solution to the dynamic program.

(14). We also chose a function $v_0(z,K)$ that assigns a real number to each element of Ω. Setting $\tilde{v}_0(z,K) = v_0(z,K)$, we iterate on the following, mapping a large number (100) of times:

(16) $$\tilde{v}_{i+1}(z,K) = \log C - \gamma N + \beta \sum_{z'} \pi_{z,z'} \tilde{v}_i(z',K'), \text{ for all } (z,K) \in \Omega,$$

where $K' = G_0(z,K)$, $N = N_0(z,K)$, $C = z\,F(K,N,M) + (1-\delta)K - K'$, and M is taken as a parameter.

The next step is to compute functions $N_1(z,K)$ and $G_1(z,K)$, for each $(z,K) \in \Omega$, as follows:

(17) $$\begin{aligned}\{N_1(z,K), G_1(z,K)\} = \arg\max_{N,K'} \{\log(z\,F(K,N,M) \\ +(1-\delta)K - K') - \gamma N + \beta \sum_{z'} \pi_{zz'} \tilde{v}_N(z',K')\}.\end{aligned}$$

We use linear interpolation to evaluate $\tilde{v}_N$ at values of K' not in Ω. In addition, we define $v_1(z,K)$ to be the maximized value of the function on the right side of (17).

Using the functions N_1, G_1 and v_1 in place of N_0, G_0 and v_0, these steps are repeated to obtain N_2, G_2 and v_2. We continue in this manner until successive iterations converge. For each $z \in \{z_1, ..., z_{n_z}\}$, we form piecewise linear decision rules by linearly interpolating between points on the grid $\{K_1, ..., K_{n_K}\}$.

Section 7: Concluding Comments

The discipline of applied general equilibrium has provided an understanding of business cycles. Partial equilibrium reasoning led to a conclusion that could stand the test of applied general equilibrium discipline. In this paper, I focused on aggregation when there are non-convexities at the micro level. Non-convexities at the firm level give rise to lumpy investment at the production level, but not at the aggregate level. Non-convexities at the household level give rise to high intertemporal elasticity of supply. The analyses reviewed here use the classical competitive equilibrium theory of Arrow-Debreu-McKenzie, a theory that abstracts from financial factors. The aggregate economy is convex. This aggregation is important in making connections between the micro observations and the stand-in firm(s) and the stand-in household(s) used in business cycle and other aggregate analyses.

REFERENCES

ABEL, A. B., and J. C. EBERLY (1996): “Optimal Investment with Costly Reversibility,” Review of Economic Studies, 63, 581-593.

ADELMAN, I., and F. L. ADELMAN (1959): “The Dynamic Properties of the Klein-Goldberger Model,” Econometrica, 27, 596-625.

ALPANDA, S. and A. UEBERFELDT (2002): “A Note on Workweek Variation in Real Business Cycle Models,” Mimeo, University of Minnesota.

ALVAREZ, F., A. ATKESON, and P. J. KEHOE (2002): “Money, Interest Rates, and Exchange Rates in Endogenously Segmented Markets,” Journal of Political Economy, 110, 73-112.

AUERBACH, A. J., and L. KOTLIKOFF (1987): Dynamic Fiscal Policy, Cambridge: Cambridge University Press.

BERTOLA, G., and R. J. CABALLERO (1994): “Irreversibility and Aggregate Investment,” Review of Economic Studies, 61, 223-246.

CABALLERO, R. J., and E. ENGEL (1991): “Dynamic (S,s) Economies,” Econometrica, 59, 1659-1686.

__________ (1999): “Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach,” Econometrica, 67, 783-826.

COOPER, R. W., J. C. HALTIWANGER, and L. POWER (1999): “Machine Replacement and the Business Cycle: Lumps and Bumps,” American Economic Review, 89, 921-946.

DEBREU, G., and H. SCARF (1963): “A Limit Theorem on the Core of an Economy,” International Economic Review, 4, 235-246.

FISHER, J. D. M., and A. HORNSTEIN (2000): "(S,s) Inventory Policies in General Equilibrium," Review of Economic Studies, 67, 117-145.

FITZGERALD, T. J. (1998): "Work Schedules, Wages, and Employment in a General Equilibrium Model with Team Production," Journal of Economic Dynamics and Control, 1, 809-830.

FREEMAN, S., and F. E. KYDLAND (2000): "Monetary Aggregates and Output," American Economic Review, 90, 1125-1135.

GOLLIN, D. (2002): "Getting Income Shares Right," Journal of Political Economy, 110, 458-474.

HANSEN, G. D. (1985): "Indivisible Labor and the Business Cycle," Journal of Monetary Economics 16, 309-28.

HANSEN, G. D., and E. C. PRESCOTT (forthcoming): "Capacity Constraints, Asymmetries, and the Business Cycle," Review of Economic Dynamics.

HARBERGER, A. C. (1968): "A Landmark in the Annals of Taxation," Canadian Journal of Economics, 1 Supplement, 183-194.

HODRICK, R. J. and E. C. PRESCOTT (1997): "Post-War U.S. Business Cycles: A Descriptive Empirical Investigation," Journal of Money, Credit, and Banking, 29, 1-16.

HORNSTEIN, A. (2002): "Towards a Theory of Capacity Utilization: Shiftwork and the Workweek of Capital," Federal Reserve Bank of Richmond Economic Quarterly, 88, 65-86.

HORNSTEIN, A., and E. C. PRESCOTT (1993): "The Firm and the Plant in General Equilibrium Theory," in General Equilibrium, Growth, and Trade II: The Legacy of Lionel McKenzie, ed. by R. Becker, M. Boldrin, R. Jones, and W. Thomson. San Diego: Academic Press, 393-410.

JOHANSEN, L. (1960): A Multi-sector Study of Economic Growth, Amsterdam: North Holland.

JORGENSON, D. and K. Y. Yun (1990): "Tax Reform and U.S. Economic Growth," Journal of Political Economy, 98, 151-193.

KAHN, A., and J. K. THOMAS (2002): "Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?" Journal of Monetary Economics, forthcoming.

KEHOE, T. J., D. K. LEVINE, AND E. C. PRESCOTT (2002): "Lotteries, Sunspots, and Incentive Constraints," Journal of Economic Theory, 107, 36-39.

KEHOE, T. J., and E. C. PRESCOTT (2002): "Great Depressions of the Twentieth Century," Review of Economic Dynamics, 5, 1-18.

KRAVIS, I. (1959): "Relative Income Shares in Fact and Theory," American Economic Review, 49, 917-949.

KYDLAND, F. E., and E. C. PRESCOTT (1982): "Time to Build and Aggregate Fluctuations," Econometrica, 50, 1345-1370.

__________ (1991): "Hours and Employment Variation in Business Cycle Theory," Economic Theory 1, 63-82.

LJUNGQVIST, L., and T. SARGENT (2000): Recursive Macroeconomic Theory. Cambridge: MIT Press.

LUCAS, R. E., Jr. (1977): " Understanding Business Cycles," in Stabilization of the Domestic and International Economy, ed. by K. Brunner, and A.H. Meltzer. New York: North-Holland.

LUCAS, R. E., Jr., and L. A. RAPPING (1969): "Real Wages, Employment, and Inflation," Journal of Political Economy, 77, 721-754.

MCGRATTAN, E. R., and E. C. PRESCOTT (2001): “Taxes, Regulation, and Asset Prices,” Federal Reserve Bank of Minneapolis Working Paper, 610.

___________ (2000): “Is the Stock Market Overvalued?” Federal Reserve Bank of Minneapolis Quarterly Review, Fall, 20-40.

OSUNA, V., and J. V. Rios-Rull (2001): “Implementing the 35 Hour Workweek by Means of Overtime Taxation,” Mimeo, University of Pennsylvania.

PRESCOTT, E. C. (1986): “Theory Ahead of Business Cycle Measurement,” Federal Reserve Bank of Minneapolis Quarterly Review, Fall, 9-22.

PRESCOTT, E. C., and R. TOWNSEND (1984a): “General Competitive Analysis in an Economy with Private Information,” International Economic Review, 25, 1-20.

___________. (1984b): “Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard,” Econometrica, 52, 21-45.

ROGERSON, R. (1988): “Indivisible Labor, Lotteries, and Equilibrium,” Journal of Monetary Economics, 21, 3-16.

_________ (1984): Topics in the Theory of Labor Markets. Ph.D. Thesis, University of Minnesota.

ROSEN, S. (1986): “The Supply of Work Schedules and Employment,” in Work Time and Employment, ed. by O. Ashenfelter and R. Layard. Amsterdam: Elsevier Science Publishers BV, 641-692.

SHELL, K., and R. WRIGHT (1993): “Indivisibilities, Lotteries, and Sunspot Equilibria,” Economic Theory, 3, 1-17.

SHOVEN, J. B., and J. WHALLEY (1972): “A General Equilibrium Calculation of the Effects of Differential Taxation of Income and Capital in the U.S.,” Journal of Public Economics, 1, 281-321.

STOKEY, N. L. and R. E. LUCAS, JR. (1989): Recursive Methods in Economic Dynamics, Cambridge: Harvard University Press.

THOMAS, J. K. (2002): "Is Lumpy Investment Relevant for the Business Cycle?" Journal of Political Economy, 110, 508-534.

Figure 1: The not all work case

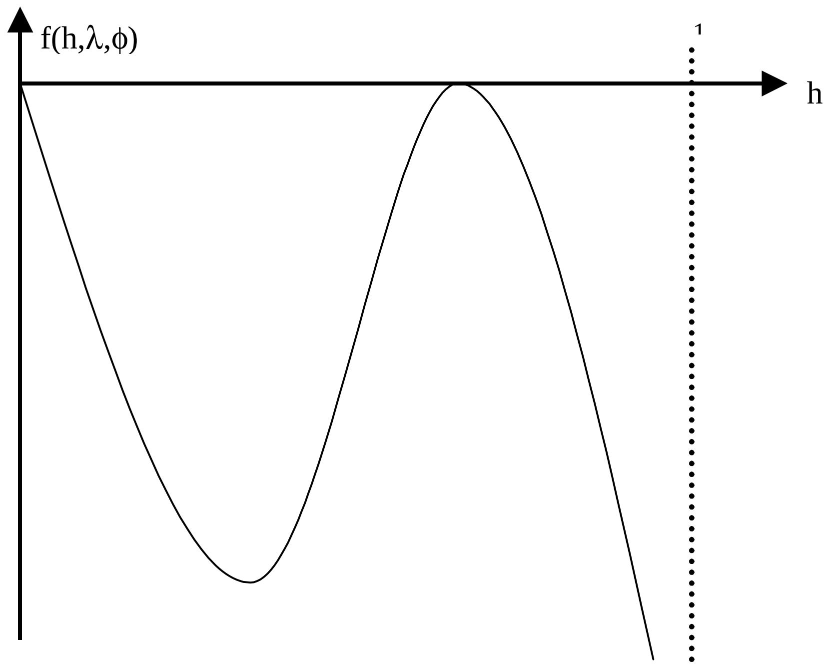

Figure 2: The all work case

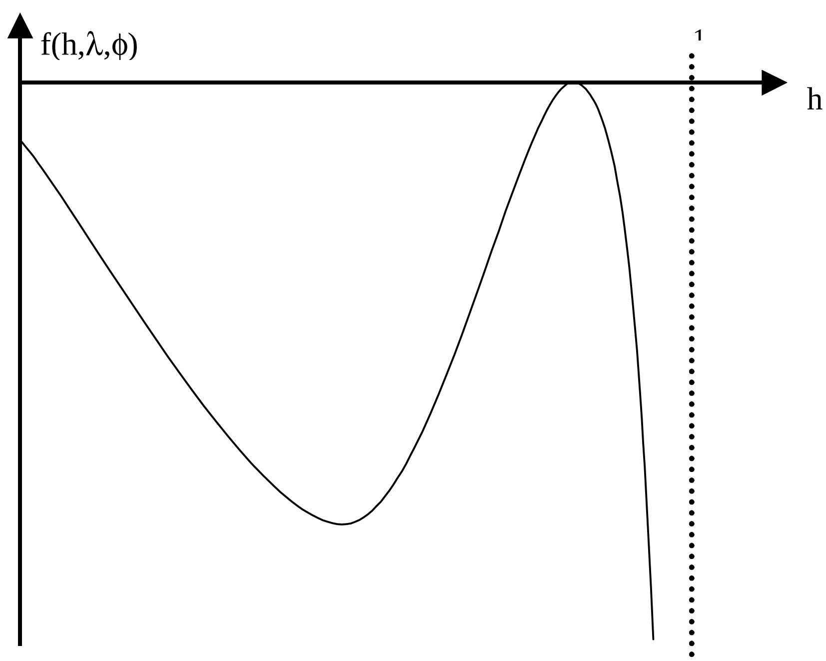

Figure 3: Behavior of employment and workweek as R varies