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AN APPROACH TO COMMUNICATION EQUILIBRIA¹

By Francoise Forges

The Nash equilibrium concept may be extended gradually when the rules of the game are interpreted in a wider and wider sense, so as to allow preplay or even intraplay communication. A well-known extension of the Nash equilibrium is Aumann's correlated equilibrium, which depends only on the normal form of the game. Two other solution concepts for multistage games are proposed here: the extensive form correlated equilibrium, where the players can observe private extraneous signals at every stage and the communication equilibrium, where the players are furthermore allowed to transmit inputs to an appropriate device at every stage.

We show that the set of payoffs associated with each solution concept has a canonical representation (in the spirit of the revelation principle) and is a convex polyhedron. We also provide for each concept a "super-canonical" game such that the set of payoffs associated with the solution concept is precisely the set of Nash equilibrium payoffs of this game.

KEYWORDS: Communication, correlated equilibrium, multistage game, Nash equilibrium, noncooperative game.

1. INTRODUCTION

THE PURPOSE OF THIS PAPER is to integrate in a synthetic presentation various equilibrium concepts involving preplay or intraplay communication between the players. We will distinguish several classes of such noncooperative solution concepts associated with communication, by relating their use to the interpretation of the rules of the game. Three successive extensions of the Nash equilibrium will be considered: the "(normal form) correlated equilibrium", the "extensive form correlated equilibrium" and the "communication equilibrium." The first notion is due to Aumann (1974); the other two were first introduced for repeated games with incomplete information (in Forges (1984) and Forges (1982) respectively). Myerson (1986) also studied multistage games with communication and proposed a sequential rationality criterion in this context.

The correlated equilibrium appears as an appropriate solution concept as soon as preplay communication is taken seriously. In this case, one cannot forbid the players to use a "correlation device" selecting a vector of signals, one for every player, before the beginning of the game. So one is led to the (Nash) equilibria of the extended game including the preliminary lottery, which is a way of defining correlated equilibria. (See Aumann (1985) for a stronger foundation for this solution concept.)

Since in this first approach signals are only sent in the preplay phase, the corresponding solution concept depends only on the normal form of the game; it can thus be referred to as "normal form correlated equilibrium." If, however, the situation to be analyzed has some duration (like a multistage game), one is tempted to extend the game by adding a lottery at *every* stage (not only at the

¹ This paper is based on the introduction of my Ph.D. thesis. I am very indebted to J.-F. Mertens, my thesis advisor, for his valuable suggestions; I also wish to acknowledge helpful comments from C. d'Aspremont.

beginning), each player receiving a signal about its outcome. Or, in a more descriptive vein, it seems natural that in a multistage game the players' knowledge of the state of the world can increase over time. One is led to allow each player to observe privately extraneous signals, such as "sunspots",² at every stage of the game. A notion generalizing the correlation device can be introduced: the *autonomous device*, which selects a vector of signals, one for each player, at every stage of the game (we will assume that such a device recalls its past outputs, so that the signals of different stages may be correlated). A (Nash) equilibrium of the game extended by means of an autonomous device will be called an "*extensive form correlated equilibrium*."

Now, if the problem is to enable the players to coordinate their strategies at every stage of the game, why not go one step further and add to the game a general "communication device", selecting outputs for the players at every stage but also receiving inputs from them? Such machines could be programmed so that the signals that are sent depend on all the past inputs and outputs; this involves both preplay and intraplay communication. In the same way as above, a solution concept can be associated; it will be referred to as "communication equilibrium."

The use of communication devices acting at every stage of the game (including autonomous devices), though attractive, is hard to justify if the rules of the game are interpreted in a strict sense. In this case, there are no other intraplay communication possibilities than those consigned in the tree, so that the game is played following a scenario of the form: before the beginning of the game, the players eventually meet and communicate; next, they go into separate cubicles where all their information comes from a central machine that controls the game. If such a strict point of view is adopted, the only communication that seems legitimate is preplay communication, corresponding to the normal form correlated equilibria can be justified with an intermediate interpretation of the game tree: in the previous scenario, the cubicles would have (differently oriented) windows through which the players could observe the course of the clouds... or sunspots.)

Another point of view can be adopted: the specified rules of the game can be interpreted as providing a "reduced form" framework within which the players must interact, yet which does not preclude them from engaging in various forms of communication. While the analysis of the tree containing all the communication moves could be very complex, the solution concepts considered here enable a manageable description. The different classes of equilibria correspond to various restrictions on the communication possibilities. Obviously, many other variants are conceivable: one could focus on memoryless communication devices (modelling telephone networks), or on "direct communication," where the players

 $^{^{2}}$ The idea of referring to extrinsic signals as "sunspots" is taken from Cass and Shell (1983). The possibility of a connection between sunspot equilibria and (extensive form) correlated equilibria was pointed out to me by J.-F. Mertens. Notice that sunspot equilibria were developed in a context completely different than the present one and from here the players may have private observations on the sunspots.

are restricted to public announcements, heard exactly as they are made (see Farrell (1984)), and so on.

In the next sections, we formalize the solution concepts and show, for each of them, that the set of corresponding equilibrium payoffs has a canonical representation (in the spirit of the revelation principle) and is a convex polyhedron. We also provide for each concept a "super-canonical" game such that the set of payoffs associated with the solution concept is precisely the set of standard Nash equilibrium payoffs of this game.

2. BASIC DEFINITIONS

We concentrate on a multistage game G with perfect recall, played by N players (indexed by n = 1, 2, ..., N) during T periods (indexed by t = 1, 2, ..., T). Period t of G begins with a move of nature; then every player n gets additional information, which concerns the past moves, including those of nature; finally, the players move simultaneously to conclude period t. Let S_t^n be the (finite) set of possible additional information of player n at period t; the set of information of player n at period t is thus $H_t^n = \prod_{t=1}^t S_t^n$; let M_t^n be the (finite) set of possible moves for player n at period t. We use Σ_t^n to denote the set of all the pure behaviors available to player n at time t, i.e. the set of all mappings from H_t^n to M_t^n . The description of the game is completed by real payoff functions defined on the space of all histories (i.e., sequences of moves of all players, including nature). This is very close to von Neumann's definition of an extensive form; we discuss extensions to the more general extensive form of Kuhn in the concluding remarks.

To refer to events occurring before the beginning of the game G, it will be convenient (but in fact not necessary (see Proposition 1)) to add a preliminary stage (denoted "stage 0") to the description above. At stage 0, no information is given and no move is made; we take thus the convention that S_0^n and M_0^n are singletons for every n = 1, ..., N.

DEFINITION 1: A communication device d for G is a collection $\{I_t^n, O_t^n, P_t: t = 0, ..., T; n = 1, ..., N\}$ where I_t^n (resp., O_t^n) is a set of inputs (resp., outputs) for player n in period t and P_t is a transition probability that chooses the outputs (in $\prod_n O_t^n$) as a function of the past and present inputs (in $\prod_{r=0}^t \prod_n I_r^n$) and the past outputs (in $\prod_{r=0}^{t-1} \prod_n O_t^n$).

A communication device is called *autonomous* if it does not involve any set of inputs (that is, I_t^n is a singleton set, for every *n* and *t*).

A correlation device is an autonomous device where all outputs precede the beginning of the game: it is completely described by sets O^n of outputs for every player n(n = 1, ..., N) together with a probability distribution P on $\prod_n O^n$ (i.e., $O^n = O_0^n$; O_t^n , $t \ge 1$, and I_t^n , $t \ge 0$, are singleton sets).

Given a communication device d, one can define the extension G_d of G as the new game with perfect recall obtained by adding d to G. For every t = 0, ..., T, period t of G_d can be described as follows: All players n = 1, ..., N get simultaneously their new information in S_t^n . They transmit simultaneously an input in I_t^n to the device d, which then selects a vector of outputs in $\prod_n O_t^n$, one for every player n = 1, ..., N, using P_t . The players n = 1, ..., N make their move in M_t^n .

REMARK 1: For the sake of simplicity and brevity, the sets I_t^n and O_t^n will be assumed to be finite. Using the results of Aumann (1964), the same analysis could be done for probability spaces (which could be required in a Bayesian approach). In particular, Proposition 1 holds in the general context, which justifies a posteriori the finiteness assumption. (Further details can be found in Forges (1985b).)

REMARK 2: If d is an autonomous device, the outputs of stage 0 (before the players get their information in S_1^n) can as well be sent at stage 1 (after they get their information in S_1^n) because the players cannot make any input in between. But it may be easier to think in terms of the events "preceding the beginning of the game."

Now we turn to the associated solution concepts.

DEFINITION 2: A communication equilibrium (resp., extensive form correlated equilibrium; resp., normal form correlated equilibrium) in G is a Nash equilibrium in the extended game G_d obtained by adding a communication (resp., autonomous; resp., correlation) device d to the game G.

A set of equilibrium payoffs can be associated with every class of devices; we denote by D (resp., D_0 ; resp., C) the set of payoffs from communication equilibria (resp., extensive form correlated equilibria; resp., (normal form) correlated equilibria). Obviously, C is a subset of D_0 which is itself included in D; these inclusions may be strict as the following examples show.

EXAMPLE 1: D is not included in D_0 .

Nature selects at random one of the two following payoff matrices shown in Figure 1, and informs player 1 of its choice. At the first stage, player 2 chooses L or R without knowing the true matrix. The pair of payoffs (0, 1) is not in D_0 but it is in D where player 1 can reveal the choice of nature to a device which suggests to player 2 to choose L if T, R if B. This example illustrates that general

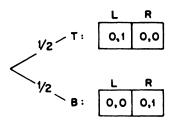
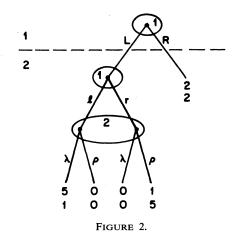


FIGURE 1.



communication devices (with inputs) are not legitimate when the rules of the game are interpreted in a strict sense: in the original game tree, player 1 is a dummy player; the situation changes completely if a communication device is added.

EXAMPLE 2 (Myerson, 1986): D_0 is not included in C.

At stage 1, player 1 has two possible actions, L and R; if player 1 chooses L, the two players must move simultaneously at stage 2. The payoffs are as shown in Figure 2. Here, the pair of payoffs (3, 3) is in D_0 but not in C; indeed, if the game is described in normal form, (L, r) is strictly dominated by R so that in any correlated equilibrium, player 2's payoff cannot exceed 2. But (3, 3) can be achieved by means of a device choosing (l, λ) or (r, ρ) at random just before stage 2.

3. A CANONICAL REPRESENTATION

We will now establish that every set of equilibrium payoffs has a *canonical representation*. This shows in particular that the "descriptive approach" to autonomous devices (where the players can observe extraneous signals from their outside environment) is fully equivalent to the "normative approach" (where devices are used for strategic coordination, i.e., recommendations are given to the players). In the case of normal form correlated equilibria, the canonical representation was obtained by Aumann (1974, 1985). Various other particular cases are known under the name "revelation principle" (see, for instance, Myerson, 1982). To get a precise statement, we need some terminology.

DEFINITION 3: A communication device is called canonical if the set of inputs of every player n in period t is a copy of his set of additional information at that time $(I_t^n = S_t^n)$ and his set of outputs is a copy of the corresponding set of moves $(O_t^n = M_t^n)$. An autonomous device is called canonical if the output to every player n in period t consists of a pure strategy (in G) for stage t $(O_t^n = \Sigma_t^n)$.

A correlation device is called canonical if the output to every player n consists of a pure strategy in $G(O^n = \prod_i \Sigma_i^n)$.

A communication (resp., extensive form correlated; resp., normal form correlated) equilibrium is called canonical if it uses a canonical communication (resp., autonomous; resp., correlation) device and if every player reveals truthfully his knowledge (in the nonautonomous case) and follows the recommendation of the device.

PROPOSITION 1: D (resp., D_0 ; resp., C) is the set of canonical communication (resp., extensive form correlated; resp., normal form correlated) equilibrium payoffs.

PROOF: The proof follows the scheme of the standard proof of the revelation principle. For example, in a communication equilibrium using an arbitrary communication device d, the strategy of player n can be used to program a device d^n which first receives as input from player n his private information, next evaluates the input player n would originally have sent to d and transmits it to d, then receives the output d would originally have sent to player n, and finally evaluates the move to be made by player n, constituting its output to player n. The communication device d' formed by d and the d^n 's considered as a whole is clearly canonical. The N-tuple of strategies where every player reports truthfully his knowledge and plays the suggested move is an equilibrium in $G_{d'}$; indeed, the players have less information, and thus less possible deviations, in $G_{d'}$ than in G_d . For extensive and normal form correlated equilibria, a similar construction goes through (or, in the latter case, see Aumann, 1985). Q.E.D.

Proposition 1 provides several corollaries. The first one³ states that to realize an equilibrium payoff in D_0 , the players do not need to observe private signals at every stage of the game: provided that they receive a private signal before the beginning of the game, one can restrict to public lotteries at the next stages. This strengthens the analogy with sunspot equilibria, sunspots being usually thought of as publicly observable.

COROLLARY 1: Every equilibrium payoff in D_0 can be achieved using an autonomous device where all outputs from stage 1 on are public (private outputs being sent at stage 0).

PROOF: Let $x \in D_0$; x can be achieved by means of a canonical device d. Let us modify it into a new autonomous device d'. At stage 0, d' selects independently for every n and $t \ge 1$ a random permutation $\prod_{i=1}^{n}$ of $\sum_{i=1}^{n}$; the sequence $(\prod_{i=1}^{n})_{i\ge 1}$ is only transmitted to player n. At every stage $t \ge 1$, d' selects outputs σ_t^n in $\sum_{i=1}^{n}$ as d but announces publicly $\prod_{i=1}^{n} (\sigma_t^n)$, n = 1, ..., N. It is easily checked that d' satisfies our requirements. Q.E.D.

³ Corollary 1 is a corollary of Proposition 1 in the sense that it uses the result that the sets of outputs of an autonomous device can be assumed to be finite (see Remark 1 after Definition 1).

4. The structure of D, D_0 , and C, and a super-canonical representation

We will now show that, from a computational point of view, the concepts introduced here are more tractable than the Nash equilibrium. The sets of equilibrium payoffs considered here have indeed a very simple structure. For C, this property was established by Aumann (1974, 1985).

COROLLARY 2: The sets D, D_0 , and C are (compact) convex polyhedra.

PROOF: The proof is given for D; it is similar for D_0 and C (see also Aumann, 1985). Let us add to G an N+1st player with zero payoff on every history and pure strategies $(\sigma_t^{N+1})_{t\geq 1}$ where

$$\sigma_t^{N+1}: \left(\prod_{n=1}^N H_t^n\right) \times \left(\prod_{n=1}^N \prod_{r=1}^{t-1} M_r^n\right) \to \prod_{n=1}^N M_t^n.$$

Every canonical communication device for G can be described by a mixed strategy P of player N+1. Indeed, σ_t^{N+1} is the typical mapping that would be used by a deterministic canonical device at stage t and even if Definition 1 was rather in terms of behavioral strategies, Kuhn's theorem (see Kuhn, 1953) is applicable (since the devices have perfect recall).

Let $\sigma_0 = (\sigma_0^n)$ be the N-tuple of pure strategies of players $1, \ldots, N$ consisting of reporting the truth and playing the suggested move at every stage. The set of canonical communication equilibria in G can be represented as the set of all mixed strategies P of player N+1 such that (σ_0, P) is an equilibrium in the N+1 person game. This set is a convex polyhedron since it is described by finitely many linear inequalities (expressing that for every n, σ_0^n is preferable to any other pure strategy). The same property holds for the set of associated payoffs, this being the image of the polyhedron by a linear mapping, and hence for D. Q.E.D.

This characterization enables us to construct, for each given game G, a single communication (resp., autonomous; resp., correlation) device d such that D (resp., D_0 ; resp., C) is precisely the set of Nash equilibrium payoffs of G_d . To see why such a "super-canonical form" may be useful, observe that each canonical device constructed in Proposition 1 is designed with a particular communication (resp., autonomous; resp., correlation) equilibrium in mind. Suppose that the players have to negotiate over and agree upon the design of the communication (resp., autonomous; resp., correlation) device. Then one might worry that these negotiations would lead to a leaking of private information (see Holmström and Myerson (1983) for a discussion of this problem). With the super-canonical form we will construct, the players do not have to bargain about the device to be used: one single extension of the underlying game G serves for all equilibrium payoffs in D (resp., D_0 ; resp., C).

To construct a super-canonical communication device d, let x_1, \ldots, x_k denote the extreme points of D; they can be achieved by canonical communication

devices d_1, \ldots, d_k respectively. d corresponds to the following: before the first stage, every player transmits to d the weights w_1, \ldots, w_k corresponding to the payoff $\sum_i w_i x_i$ to be achieved; d chooses then among d_1, \ldots, d_k using the probability distribution w_1, \ldots, w_k . If all the players do not report the same weights, d chooses according to the majority rule; this works for $N \ge 3$; if N = 2and the two vectors of weights are not identical, d sends a specific output to both players. The equilibrium strategy of every player (in the extended game) may then include applying a punishment strategy (minmax in the original game) when the specific output is sent, preventing the opponent from reporting "wrong weights."

For D_0 and C, consider the following autonomous device d: d selects a (k+1)-tuple of signals, independently of each other; for the ℓ th component $(1 \le \ell \le k)$, d proceeds as d_ℓ ; the last component consists of a random variable x uniformly distributed on [0, 1] and is transmitted to every player at the first stage of the game; every player receives in addition his k-tuple of signals selected above (for C, all the signals are sent at the first stage; for D_0 , the procedure goes on at every stage). The players can thus decide to use the ℓ th $(1 \le \ell \le k)$ component of their information if

$$\sum_{i=0}^{\ell-1} w_i \leq x < \sum_{i=0}^{\ell} w_i$$

where we set $w_0 = 0$. Using the result of Blackwell (1953), the uniform distribution on [0, 1] can be replaced by an appropriate distribution on the positive integers so that d uses only countably many outputs.

REMARK: The latter construction cannot be used for D because then outputs are selected as a function of inputs from the players and a vector of outputs associated with different canonical devices d_1, \ldots, d_k can reveal much more information than a single output from one of the d_i 's (think of the nonrevealing equilibrium and the completely revealing equilibrium in Example 1).

5. CONCLUDING REMARKS

REMARK 1: We have seen that in general D is strictly larger than D_0 and D_0 is strictly larger than C. There are however classes of games where C = D. This has some importance since the computations needed for C are conceptually less difficult. Also, if C = D, one can get the effect of a general communication device without violating the rules of the game. Such an equivalence result was first established for a class of repeated games with incomplete information (Forges, 1982, 1985a). Similar arguments can be applied to show that C = D in (one-shot) games of information transmission, a model studied for instance by Green and Stokey (1980) and Crawford and Sobel (1982), where one player has private information and sends a signal to a second player who then takes an action.⁴ (See also Forges, 1985b.)

⁴ This result requires the use of costless signals; therefore, it does not directly pertain to signalling games where signals are costly.

REMARK 2: One may wonder whether the communication equilibrium is a strictly noncooperative solution concept. Obviously, it is defined as a noncooperative solution (Nash equilibrium) of an extension of the game. On the other hand, a communication device is a kind of outside enforcement mechanism, requiring some commitment of the players, in the sense that they are asked to make inputs. Now, one can always give every player the option of not sending any input, provided that the device has a default message procedure, consisting of acting as if the player had sent some specified input. In any case, the communication devices are not directly connected to the original game; the only connection is through the players. In particular, if the device makes recommendations, the move remains the choice of the player. Our extensions of Aumann's correlated equilibrium are thus completely different from the one proposed by Moulin and Vial (1978) and Gérard-Varet and Moulin (1978): there, explicit commitments are required because the devices may play for the players. Finally, let us recall that the cooperative aspect of the mechanism design phase can be avoided by using the "super-canonical form" of Section 4.

REMARK 3: A last remark concerns general extensive games. We focused on games with a time structure, which is not the most general model for games in extensive form (unlike von Neumann's extensive games, Kuhn's extensive games are not endowed with a chronological order: see von Neumann and Morgenstern (1953, pp. 73-76), and the discussion in Kuhn (1953)). To extend the analysis to arbitrary extensive games, one is tempted to allow the output of the devices at any node to depend on the information set containing that node (the devices would be "attached" to the information sets as before to the stages; this amounts to working with the agent normal form of the game, which is obtained by giving the running of every information set to a different agent). But even for games with the simple temporal structure used here, this extension raises grave questions. To see this, let us consider an example. At the beginning of the game, nature chooses one of two payoff matrices, T or B, with probability 1/2 and informs player 1 of its choice. Player 1 then sends a message to his uninformed opponent, who has to take one of two possible actions, L or R. Here, we want signalling to model the idea that player 1 can "talk" to player 2: we allow thus a large set M of messages, say M = [0, 1] to simplify the analysis (we will come back to this later on), and we assume that signalling is costless. The payoffs are as shown in Figure 3.It is easily checked that $D = \{(0, 1)\}$, every communication equilibrium being necessarily nonrevealing: independently of his type, player 1 wants to induce action L with the highest probability; hence player 2 can just maximize his expected payoff, leading him to play R (this holds for every set M).

Now, let us describe an autonomous device attached to the information sets and analyze its effects. As before, the device is not connected with the game, in the sense that it does not have access to the information of the players; in particular, it does not know whether T or B has been chosen by nature. Being autonomous, the device can only send outputs, it cannot get any information from the players. But the device consists of "connected branches," each branch

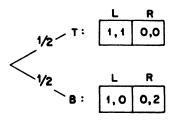


FIGURE 3.

acting at an information set, so that it can send a different output to player 1 at the information set "T" and at the information set "B."

Let us show that this enables achievement of a completely revealing equilibrium, where player 2 plays L if T and R if B. For this, let the autonomous device select a message μ uniformly in M and transmit it as an output to player 1 at his information set T and to player 2 at any information set. No output is sent to player 1 at the information set B. The equilibrium strategies are: for player 1, at T, send the message μ received from the device; at B, send an arbitrary message in M; for player 2, play L on μ , R on all the other messages. The associated payoff is (0.5, 1.5). Here, one uses that μ is chosen uniformly in [0, 1], so that player 1 at B cannot "guess" μ (the probability that a message m coincides with μ is zero). But the same analysis can be done with a large finite set M, in which case the equilibrium payoff has the form $(0.5 + \varepsilon/2, 1.5 - \varepsilon)$.

Notice that this scenario is equivalent to the one corresponding to the "agent normal form" of the game where nature chooses between two individuals 1_T and 1_B at the beginning. The autonomous device would not know whether player 1_T or 1_B has been selected to play the game but could transmit different outputs to each of them, to be used by the agent if he is active.

This example illustrates that generalized devices are too powerful. If one applies to a multistage game the generalization of the autonomous device designed to deal with arbitrary extensive games, one can get a set of equilibrium payoffs not only larger than D_0 but larger than D. What is then the appropriate device for general extensive games? This question is left for future research.

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