Costly Voting*

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Abstract

What are good voting rules if voting is costly? We analyse this question for the case that an electorate chooses among two alternatives. In a symmetric private value model of voting we show that majority voting with *voluntary* participation Pareto-dominates majority voting with *compulsory* participation. We also demonstrate the potential advantages of *asymmetric* voting rules. We consider three types of such rules: Rules which do not allow all individuals to vote, rules which rely on an arbitrary status quo which can only be overturned if a majority of individuals participates in the voting process, and sequential voting rules.

How should group decisions be organized when participation in the decision making process is costly? Should participation be voluntary, or should it be compulsory? Should everyone be invited to participate, or should only a small sample of those involved be invited to participate? These and related questions will be addressed in this paper.

Our analysis sheds light on the way in which companies should organize meetings and votes. How much pressure should be exerted on individuals to participate? To which extent should decisions be delegated to smaller committees? These questions are of great concern to managers who often spend a significant proportion of their working time in meetings.

Recent empirical literature has been interested in biases in collective decisions due to voluntary participation. Turner and Weninger (2001) find for a particular industry (Mid-Atlantic surf clam and ocean quahog fishery) that firms which prefer moderate policies are less likely to participate in public meetings with voluntary participation than firms that prefer extreme policies. Bulkley, Myles and Pearson (2001) have investigated the UK's House of Lords in which incidentally participation is financially rewarded. They find that members of the House of Lords are less likely to participate in votes if they are not affiliated with a party than if they are. One reason why these results are important is that selective participation in votes might allow more extreme groups to get their way more easily. Our analysis will not be rich enough to cover all the issues raised by these studies, but it will suggest an important reason for leaving participation voluntary and for not offering any financial rewards for participation.

The policy problem which we analyse arises also in national elections. While in most countries participation in such elections is voluntary, some countries (e.g. Belgium, Italy) have tried to make it compulsory. We are cautious about the relevance of our results in this context, though. Our analysis is built on game-theoretic models of voting in which participation decisions are rational, and are driven by the probability that an individual's vote is pivotal. With large electorates this probability is, under most voting rules, close to zero, yet empirically observed participation rates are often high. This is *The Paradox of Voting* (Downs (1957), Ferejohn and Fiorina (1974)). This paradox suggests that a conventional, game-theoretic analysis of costly voting is out of place if large electorates are considered. By contrast, for small electorates there seems no reason why observed voting behaviour should not be rational. This is why our paper is meant for small electorates only.

We shall assume that there are only two possible collective choices, for example two candidates. We thus avoid the complications of the well-known *Condorcet* paradox which arises if there are three or more alternatives. We postulate positive voting costs, and thus it is socially not necessarily desirable that all individuals participate in the collective decision. Full participation in some sense optimizes the "quality" of the collective decision, but it typically incurs too high costs. An optimal voting system will trade off quality of the collective decisions against participation costs.

Voting costs will be privately observed in our model. Each individual knows his or her own voting costs, but not the voting costs of any other individual. Thus, the voting system cannot be tailored for specific values of the voting costs.

We begin our analysis by considering the most common system of voting over two alternatives, simple majority voting. We ask whether participation in a majority vote should be compulsory or whether it should be voluntary. We show that in our model majority voting with voluntary participation strongly Pareto-dominates majority voting with compulsory participation. The intuition for this finding is that voting causes a negative externality: any individual's vote makes it less likely that other voters' votes are pivotal. Thus, under voluntary participation, from a social welfare point of view, the equilibrium inclination to vote is too large rather than too small. Making voting compulsory moves incentives into the wrong direction.

The finding that equilibrium participation is too high, and that a move towards a system which enforces a higher participation rate than the equilibrium rate is undesirable, might appear surprising in the context of voting. Public debate seems more concerned with too low than too high participation. Our finding becomes more intuitive if one considers analogous contexts. For example, the average length of contributions to discussions in department meetings seems excessive. Similarly, people probably spend an excessive amount of time intriguing to influence collective decisions. Our finding of excessive participation in votes is a similar instance of over-investment into political activities.

After establishing the superiority of voluntary over compulsory majority voting we go on to show that there is a variety of voting rules which sometimes improve on voluntary majority voting. These have in common that they involve some asymmetry, either between individuals or between alternatives. For example, if only a small number of individuals is called upon to vote, i.e. if the decision at hand is delegated to a committee, then there is obviously an asymmetry between individuals. If one alternative is arbitrarily declared the *status quo*, then there is an asymmetry between alternatives. Our second main point is that society may be better off under such asymmetric voting rules.

The advantage of committees arises in our model if under universal voluntary participation individuals are almost indifferent between participating and not participating. A committee then raises the probability of individual votes being pivotal, and thus offers the committee members a strict incentive to participate. If the committee membership is randomly selected, everyone is better off.

If one alternative is declared the *status quo* and non-participation is counted as a vote in favor of the status quo, then society may be better off because those who favor the status quo need not incur participation costs. We point out an important drawback of this system, though. It creates a free-riding problem among those who oppose the status quo. For example, there will always be an equilibrium in which nobody who opposes the status quo ever participates.

A third asymmetric voting system which we consider is sequential voting. Here individuals cast their votes sequentially. At least for low voting costs this system has the advantage that individuals who come last in the sequence of votes don't need to incur participation costs if an unassailable majority has already been established.

It should be emphasized that for each of these asymmetric voting rules we establish its superiority to voluntary majority voting only for some specifications of the parameters of our model, not for all. Indeed, in most cases we also give examples of parameters for which voluntary majority voting is better. By contrast, the superiority of voluntary participation over compulsory participation in majority votes is in our model completely general. It holds for all specifications of the parameters.

Our analysis is set in a private value model of voting where preferences reflect idiosynchratic individual tastes. In a common value model of voting (for example, Feddersen and Pesendorfer (1996, 1997, 1998)), where individuals have identical tastes but different information, there will be positive externalities to voting which can mitigate or outweigh the negative externality which we identify. In such a model one cannot expect as clear-cut results as we obtain here.

Our model is ex ante symmetric, both with respect to alternatives, and with respect to individuals. It seems natural to consider the design of voting rules in a setting in which there are no ex ante built-in differences between alternatives or between individuals. The symmetry assumptions are also important for the intuition behind our first main result. The negative externality of one individual's decision to vote affects those who also vote, not those who don't vote. Those who also vote face a negative externality because their vote becomes less likely to be pivotal. In a symmetric model, those who don't vote do not care whether others vote, because the vote that is being cast by others is equally likely to be in favor of either of the two alternatives. In an asymmetric model, by contrast, votes might have a positive externality on those who don't vote which will reduce the negative externality which we find.

The results of this paper will be built on a new analysis of equilibrium behaviour in majority voting with voluntary participation. Our analysis of these equilibria is closely related to work by Ledyard (1981, 1984) and Palfrey and Rosenthal (1983). Our setting is slightly different from the setting of these papers, and in an important respect more special: unlike these papers, we assume symmetry. As a consequence, we obtain a slightly stronger result: uniqueness of symmetric equilibrium.

None of the papers in the previous paragraphs considers alternatives to voluntary majority voting. Ledyard (1984, Theorem 1) proves optimality of equilibrium under voluntary majority voting. His model differs from ours in that he endogenizes in a Downs (1957) type model the two alternatives voters can choose from. The optimality result applies to an equilibrium in which candidates choose identical positions which maximize voters' ex ante welfare, and in which nobody votes. Because we do not endogenize the candidates' platforms we are looking, in a sense, for a stronger optimality property than Ledyard does.[1]

Osborne, Rosenthal and Turner (2000) have a model of costly participation in a collective decision process where the set of alternatives is some convex subset of some Euclidean space. They do not model explicitly how the collective decision is arrived at, but instead work with a reduced form "compromise function" which describes the collective decision as a function of the positions of all participating individuals, for example the median. Our question how voting rules affect participation decisions corresponds to the question how different compromise function affect participation. Osborne et. al., however, do not focus on this question. Their main interest is in the features of equilibria for given compromise function. They predict that individuals with extreme positions are more likely to participate than individuals with moderate positions.[2]

Equilibria of some of the asymmetric voting procedures which we investigate have previously been analyzed by Dekel and Piccione (2000) (for Sequential Voting) and

[1] Ledyard's result is also built on an assumption of "many" voters. For reasons explained above we are reluctant to focus on this case.

[2] Bulkley, Myles and Pearson analyse a closely related model and find similar results.

Feddersen and Pesendorfer (1998) (for Voting Over a Status Quo). However, these papers are set in a common value framework. Moreover, they do not analyse endogenous participation.

This paper is organized as follows: Section I explains the setup. Section II establishes the dominance of voluntary participation over compulsory participation. In Section III we consider several asymmetric mechanisms which sometimes Pareto-dominate majority voting with voluntary participation. In Section IV we suggest that one can re-interpret of our analysis as an analysis of endogenous information acquisition and votes.

I. Setup

There are n individuals: $i = 1, 2, ..., n$. To avoid trivial case distinctions, we assume: $n \geq 3$. The individuals form a club which has to choose one of two alternatives: $a = A, B$. This is a collective choice problem. One alternative must be chosen, and this alternative will apply to all members of the club. An example would be that the club has to select either A or B as its new chairman.

The relevant characteristics of an individual are summarized in that individual's "type" $t_i = (a_i, c_i) \in \{A, B\} \times \Re_+$ where the first component, a_i, is the alternative which individual i favours[3], and the second component, c_i, indicates individual i's costs of participating in a collective decision process. If individual i is of type $t_i = (a_i, c_i)$, then i's von Neumann Morgenstern utility is highest if i's most favored alternative, a_i, is chosen, but individual i does not participate in the decision making process. In that case, individual i's utility is normalized to be equal to 1. If the alternative which i ranks second is chosen, and i does not participate, then i's utility is equal to zero. Now consider the utility of individual i if i *does* participate in the

[3] We rule out the possibility that individuals are indifferent between the two candidates. If voting is costly, such individuals will never vote. Therefore, they can safely be omitted from the analysis.

decision making process. In this case we simply subtract from the utilities described so far individual i's participation costs c_i. Hence, if a_i is chosen and individual i does vote, then her utility is $1 - c_i$, and if a_i is not chosen, and i does vote, then her utility is $-c_i$.

Note that we are assuming that the costs of participation are independent of whether individual i's participation is compulsory or voluntary. The costs are also independent of the decision making mechanism which society uses, of the strategy which individual i chooses in that mechanism, and of the alternative which society chooses[4]. These assumptions are made for simplicity.

Each individual's type t_i is a random variable. The two components of any individual's type, a_i and c_i, are stochastically independent of each other. For any individual i the alternative a_i which individual i favours is with probability $\frac{1}{2}$ equal to A, and with probability $\frac{1}{2}$ equal to B. The participation costs c_i have a distribution function F which is the same for all individuals, and which has support $[\underline{c}, \overline{c}]$ where $0 \leq \underline{c} < \overline{c}$. The distribution function has a density f which is positive on all of the support.

Note that the previous paragraph contains two distinct symmetry assumptions. Firstly, our model is symmetric with respect to alternatives. This means two things: For each individual the probability that he favours any given alternative is the same for both alternatives. Moreover, the conditional distribution of participation costs is the same for both alternatives. Secondly, our model is symmetric with respect to individuals. For each individual, the distribution of types is the same. The two symmetry assumptions together can be justified by imagining that the voting procedure is designed behind a veil of ignorance, so that nothing is known that would suggest asymmetric distributions. If we changed one of the two symmetry assumptions, and built some asymmetry into our model, then a "good" voting process would reflect the

[4]Costs would depend on the alternative which society chooses if, for example, agent i finds it more painful to participate in a majority vote and to be on the losing side, than to participate and to be on the winning side.

asymmetry. The implications of exogenous asymmetries do not seem interesting.

Next we assume that the type t_i of individual i is stochastically independent of the type t_j of individual $j \neq i$. This has two important implications. Firstly, it means that we are considering a *private* value model of voting rather than a *common* or *affiliated* value model. In a private value model of voting, types reflect purely private tastes. In a common value model, by contrast, all voters would agree on which candidate is best if they all had the same information, and differences of opinion result only from the fact that different individuals hold different pieces of information. The importance of the private value assumption to our analysis was already explained in the Introduction.

The second implication of the independence assumption for types is that different individuals' participation costs are not correlated. If they were, then an individual who found that his or her participation costs were low (for example, because the weather is bad, and therefore the opportunity costs of voting are low) would deduce that other individuals' participation costs were also low (because the weather is the same for everybody), and that therefore it would be less likely that any individual vote (or other action) matters. Such counterveiling incentives make the analysis of rational participation decisions much more complicated (Landsberger and Tsirelon (1999, 2000)).

Finally, we assume that individual i observes his own type t_i, but not the type of any other individual. This, too, should be thought of as a benchmark assumption. It would be interesting to relax this assumption, and to consider the case in which individuals know not only their own types, but have also at least some partial information about the distribution of types in the population. One issue in this case is that, in the absence of participation costs, it is known that optimal mechanisms often have very appealing properties, but appear artificial, although it is difficult to say precisely why they are rarely found in practice (Cremer and McLean (1985, 1988), McAfee and Reny (1992)).

Our comparison of different decision making mechanisms will be based on individuals' expected utilities in the equilibrium outcomes of these mechanisms. Thus we shall analyse Bayesian or sequential equilibria of the mechanisms which we propose, and then calculate each individuals' expected utility, assuming that these equilibria are played. We shall deal with the special problem of mechanisms with multiple equilibria on a case by case basis. We shall calculate individuals' expected utility on an ex ante basis. By this we mean that we calculate expected utility assuming that individuals' preferences over alternatives, and individuals' participation costs, have not yet been determined.

The symmetric nature of our model will allow us to base our comparison of different decision making mechanisms exclusively on *Pareto* comparisons. We shall say that one mechanism *strongly* Pareto-dominates another mechanism if *all* individuals' ex ante expected utility in a Bayesian equilibrium of the former mechanism is higher than it is in a Bayesian equilibrium of the latter mechanism. We shall say that one mechanism *weakly* Pareto-dominates another mechanism if *all* individuals' ex ante expected utility in a Bayesian equilibrium of the former mechanism is at least as high as it is in a Bayesian equilibrium of the latter mechanism, and if, moreover, for *some* individuals it is strictly higher.

Because of the additive nature of preferences in our model one can decompose welfare comparisons into two components. Individuals care firstly about the quality of the collective decision, i.e. about the probability with which this decision is the alternative which they prefer. They care secondly about the cost at which the decision is reached, i.e. the expected value of their participation costs. There is a sense in which individuals agree ex ante, before types are determined, about what the collective decision rule should be. Consider any collective choice rule which assigns to profiles $(a_1, ..., a_n)$ of preferred alternatives a probability distribution over A and B. Suppose that the rule is symmetric with respect to individuals. Neglect, for the moment, the costs of decision making. Then ex ante all individuals strictly prefer the rule which always picks the alternative favored by the majority over all other rules. This is easy to see. The intuitive reason is that ex ante everybody is more likely to be a member of the majority than of the minority.

Now perfect decision making in this sense can be achieved by majority voting if everybody is forced to vote. We shall take this mechanism as our starting point in the next section. The focus of the paper is then on the extent to which one might be willing to accept a lower quality of collective decision making in return for lower costs of the collective decision process.

II. Symmetric Voting Rules

In this section we shall compare three mechanisms for collective decision making: *Compulsory Majority Voting*, *Random Decision Making*, and *Voluntary Majority Voting*. These mechanisms have in common that they treat individuals and alternatives symmetrically. In the next section we shall consider asymmetric mechanisms.

We begin with *Compulsory Majority Voting*. Under *Compulsory Majority Voting* each individual is forced to participate. Individuals have to vote for either A or B. The alternative that receives the majority of votes is selected. If the two alternatives receive exactly the same number of votes, each is selected with probability $\frac{1}{2}$.

This mechanism has multiple Bayesian equilibria. One type of equilibrium is that all individuals vote for the same alternative a, independent of their personal preferences. This is an equilibrium because, if all individuals vote for the same alternative a, then no single vote affects the majority[5], and therefore any vote is optimal. However, voting against one's true preferences is obviously a *weakly* dominated strategy. Although there are some arguments in defense of weakly dominated strategies, we shall simplify our analysis by assuming that weakly dominated strategies will not be played. We are then left with a single equilibrium of *Compulsory Majority Voting*:

[5]Recall that we have assumed $n \geq 3$.

every individual votes for his most preferred alternative. Thus, *Compulsory Majority Voting* achieves perfect decisions, albeit at the expense of maximal participation costs.

Random Decision Making is at the other extreme of symmetric mechanisms. It is the mechanism in which no individual is invited, nor indeed allowed, to participate in the decision making. Each of the two alternatives is selected with probability $\frac{1}{2}$. Thus, the quality of decision making under this mechanism is low. On the other hand, no participation costs arise.

Voluntary Majority Voting is between the two extreme mechanisms discussed so far. Under *Voluntary Majority Voting* each individual can choose whether to participate in the vote. If an individual chooses to participate, he can vote for A or B. The alternative with the larger number of votes is selected. If both alternatives get exactly the same number of votes, each is selected with probability $\frac{1}{2}$.

Under this mechanism, voting against one's true preference is a *strictly* dominated strategy. The dominating strategy is not to participate. For the analysis of Bayesian equilibria it thus suffices to consider an individual's choice between not participating, and voting for the individual's true preference. We restrict attention to symmetric Bayesian equilibria. By this we mean Bayesian equilibria which satisfy two distinct symmetry conditions. The first is that an individual's participation decision depends only on the individual's participation costs, and not on the candidate whom the individual favours. The second condition is that all individuals choose the same strategy. We are thus looking for a *voting strategy* of the form: $s : [\underline{c}, \overline{c}] \to \{0, 1\}$ which is the same for all individuals, and where $s_i(c_i) = 0$ (resp. 1) means that an individual i does not vote (resp. does vote) if her costs of voting are c_i.

All individuals choosing voting strategy s is a Bayesian equilibrium if and only if for almost all values of c_i the decision $s(c_i)$ maximizes individual i's expected utility given that all other individuals play s. When discussing equilibrium strategies, we shall ignore sets of possible cost values which are of measure zero. So, for example, we shall call an equilibrium "unique" if all equilibria are identical to this equilibrium except possibly for a set of cost values which is of measure zero. We shall adopt an analogous practice in the next section.

In equilibrium, individual i will vote if the expected benefits of voting are larger than the costs of voting c_i. The expected benefits of voting for individual i, assuming that all other individuals play voting strategy s, don't depend on the details of s. Rather, they only depend on the ex ante probability, *before* learning c_i, with which any individual votes. This probability is: $p \equiv \int_{\underline{c}}^{\overline{c}} s(c)f(c)dc$.

Consider an individual with given and fixed preference for alternative a. The expected benefits of voting to individual i are $B(p) = \frac{1}{2}\Pi(p)$. Here, $\Pi(p)$ is the probability that the difference between the votes for i's preferred alternative a, and the votes for the other alternative, is -1 or 0. In these two cases, the voter's vote makes a difference to the outcome, and voter i is *pivotal.* If i is pivotal, the effect of his vote is to increase his expected utility by exactly $\frac{1}{2}$. This is because he either turns a loss into a draw, or a draw into a win. Hence he increases the probability that his preferred alternative is chosen by $\frac{1}{2}$. Since his utility is 1 if his preferred alternative is chosen, and 0 otherwise, the expected benefit from a pivotal vote is $\frac{1}{2}$.

It remains to investigate $\Pi(p)$. Note that $\Pi(0) = 1$. Our further results regarding Π are summarized in the following remark. Observe that the properties of $\Pi(p)$ immediately carry over to $B(p)$ because $B(p) = \frac{1}{2}\Pi(p)$.

Remark 1 $\Pi(p)$ *is a differentiable function of* p*, and* $\Pi'(p) < 0$ *for all* $p \in (0,1)$.

Proof. Denote by $\widetilde{\ell}$ the number of individuals other than i who choose to vote. Thus $\widetilde{\ell}$ is a random variable with binomial distribution with parameters $n-1$ and p. The probability that $\widetilde{\ell}$ takes any particular value ℓ is given by:

$$\binom{n-1}{\ell} p^{\ell}(1-p)^{n-1-\ell}$$

Note that an increase in p leads to a rightward shift in the sense of first order stochastic dominance of the distribution of $\widetilde{\ell}$.

Conditional on the number of voters being ℓ we need to calculate the probability that voter i is pivotal. We shall denote this probability by $\pi(\ell)$. We begin by noting that $\pi(0) = 1$ and $\pi(1) = \frac{1}{2}$. The latter is true because with probability $\frac{1}{2}$ the other voter votes for the alternative which i regards as inferior, in which case i is pivotal, and with probability $\frac{1}{2}$ she votes for i's preferred alternative, in which case i is not pivotal.

In general, if $\ell \geq 1$ and ℓ is odd, then voter i is pivotal if his preferred alternative receives $\frac{\ell-1}{2}$ votes whereas the other alternative receives $\frac{\ell+1}{2}$ votes. This occurs with probability:

$$\pi(\ell) = \binom{\ell}{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{\frac{\ell+1}{2}} = \binom{\ell}{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{\ell} \tag{*}$$

Now suppose again that $\ell \geq 1$ and that ℓ is odd, and consider the case that $\ell + 1$ individuals vote[6]. Then voter i is pivotal if the number of votes for his preferred alternative is $\frac{\ell+1}{2}$. This occurs with probability:

$$\begin{aligned} \pi(\ell+1) &= \binom{\ell+1}{\frac{\ell+1}{2}} \left(\frac{1}{2}\right)^{\frac{\ell+1}{2}} \left(\frac{1}{2}\right)^{\frac{\ell+1}{2}} = \binom{\ell}{\frac{\ell-1}{2}} \frac{\ell+1}{\frac{\ell+1}{2}} \left(\frac{1}{2}\right)^{\ell+1} \\ &= \binom{\ell}{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{\ell} = \pi(\ell) \end{aligned} \tag{**}$$

Next suppose that, still for the same ℓ, the number of individuals who vote is $\ell + 2$.[7] Since the number of voters is then again odd, we can use formula (*) to conclude that the probability of voter i being pivotal is:

$$\begin{aligned} \pi(\ell+2) &= \binom{\ell+2}{\frac{\ell+1}{2}} \left(\frac{1}{2}\right)^{\ell+2} = \frac{\ell+2}{\ell+3} \binom{\ell}{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{\ell} \\ &= \frac{\ell+2}{\ell+3} \pi(\ell+1) \end{aligned} \tag{***}$$

Note that the formula in (***) gives a strictly smaller value than the formulas (*) and (**).

[6] Assume that $\ell \leq n - 2$.

[7] Assume that $\ell \leq n - 3$.

The three formulas (*), (**) and (***) together show how the probability of voter i being pivotal depends on ℓ. Recall that for $\ell = 0$ the probability is 1 and for $\ell = 1$ the probability is $\frac{1}{2}$. As we increase ℓ further, if we move from an odd to an adjacent even number of voters, the probability of being pivotal goes down. If we move from an even to an adjacent odd number of voters, the probability of being pivotal stays the same.

We can now prove the remark. We begin by noting that:

$$\Pi(p) = \sum_{\ell=0}^{n-1} \binom{n-1}{\ell} p^{\ell}(1-p)^{n-1-\ell}\pi(\ell)$$

Π is differentiable because it is polynomial. The easiest way to see that its derivative is strictly negative is to see that raising p leads to a right shift in first order stochastic dominance in the distribution of ℓ. Moreover, as described above, the conditional probability that voter i is pivotal, conditional on ℓ, is decreasing in ℓ where the decrease is in some instances strict. As the total probability of being pivotal is the expected value of the conditional probability, where expected values are taken over ℓ, we can conclude that Π has a strictly negative derivative for all p. ■

If all other individuals play a voting strategy s with voting probability p, then individual i's best response is to vote if $c_i < B(p)$, and not to vote if $c_i > B(p)$. An equilibrium strategy s must thus be a threshold strategy: There is some $\widehat{c}$ such that $s(c_i) = 1$ if $c_i < \widehat{c}$ and $s(c_i) = 0$ if $c_i > \widehat{c}$.

For which values of $\widehat{c}$ does the corresponding threshold strategy constitute a symmetric Bayesian equilibrium? For any $\widehat{c} \in [\underline{c},\overline{c}]$ the probability of voting as implied by a threshold strategy with threshold $\widehat{c}$ is $F(\widehat{c})$. Recall that by assumption F is differentiable, and that $F'(\widehat{c}) > 0$ for all $\widehat{c} \in (\underline{c},\overline{c})$. A value $\widehat{c}$ is the threshold for an equilibrium threshold strategy if and only if $B(F(\widehat{c})) = \widehat{c}$ (if $\widehat{c} \in (\underline{c},\overline{c})$) or $B(F(\widehat{c})) \geq \widehat{c}$ (if $\widehat{c} = \underline{c}$) or $B(F(\widehat{c})) \leq \widehat{c}$ (if $\widehat{c} = \overline{c}$). Observe that Remark 1 and the previously noted fact that $F'(\widehat{c}) > 0$ imply that $B(F(\widehat{c}))$ is differentiable and strictly decreasing in $\widehat{c}$. Thus, we are looking in a two dimensional coordinate system with $\widehat{c}$ on the horizontal axis and $B(F(\widehat{c}))$ on the vertical axis for the intersection of the graph of a differentiable, strictly decreasing function, and the 45° line. Obviously, there can only be one such point of intersection. More precisely, we have the following result, the proof of which is obvious from what has been said so far:

Proposition 2 *Voluntary Majority Voting has a unique symmetric Bayesian equilibrium.*

(i) If $\overline{c} \leq \binom{n-1}{\frac{n-1}{2}} \left(\frac{1}{2}\right)^{n-1}$ *(if n is odd) or* $\overline{c} \leq \binom{n-1}{\frac{n}{2}-1} \left(\frac{1}{2}\right)^{n-1}$ *(if n is even) then the unique equilibrium is that all individuals vote, independent of their participation costs.*

(ii) If $\underline{c} \geq \frac{1}{2}$*, then the unique equilibrium is that no individual ever votes.*

(iii) Otherwise, there is a unique threshold $c^* \in (\underline{c}, \overline{c})$ *such that an individual votes if and only if* $c_i \leq c^*$*.*

The upper boundary for $\overline{c}$ mentioned in part (i) is the value of $B(1)$. The fact that $B(1)$ has the values listed in part (i) follows from calculations in the proof of Remark 1. If $\overline{c}$ is exactly equal to the boundary, then an alternative equilibrium could be constructed in part (i) in which agents do vote if their costs are exactly equal to the boundary value. However, this would be a zero probability event. Recall that we ignore such events. This justifies the claim of uniqueness in Proposition 2 (i). Similar comments apply also to parts (ii) and (iii) of the Proposition.

The Proposition shows that if all conceivable costs of voting are below some boundary, then *Voluntary Majority Voting* is identical to *Compulsory Majority Voting* because everybody will vote voluntarily (part (i)). If all possible costs of voting are above some boundary, then *Voluntary Majority Voting* is equivalent to *Random Decision Making* because nobody will volunteer to vote (part (ii)). In intermediate cases, the unique symmetric equilibrium of *Voluntary Majority Voting* implies higher participation costs but better decisions than *Random Decision Making*, and lower participation costs but worse decisions than *Compulsory Majority Voting* (part (iii)).

Our result leaves open whether *Voluntary Majority Voting* has other, non-symmetric equilibria. The symmetric equilibrium is arguably the most prominent equilibrium of

Voluntary Majority Voting, and therefore it seems interesting to explore the properties of this equilibrium. In the following we shall assume without further mentioning that this equilibrium is played.

Although Proposition 2 is simple, we are not aware of any previous paper in which it would have been stated. However, issues related to the ones considered so far in this section have been analyzed before. The probability $\Pi(p)$ that a voter is pivotal has been studied by Beck (1975) and Chamberlain and Rothschild (1981), however, their focus is on the case that n is large. Voting games with endogenous participation similar to the one considered here have been analyzed by Ledyard (1981, 1984). These papers study a model which is more general than ours because it is possible in this model that voters are more likely to prefer one alternative than another. Ledyard focuses on equilibria in which all individuals play the same strategy, and proves existence of such equilibria. A closely related analysis is Palfrey and Rosenthal (1983).

We now turn to the question which of the three procedures discussed so far resolves best the trade-off between quality of decisions and participation costs. The following proposition shows that the answer is unambiguous. This proposition is the main result of this section.

Proposition 3 *Voluntary Majority Voting weakly Pareto-dominates Compulsory Majority Voting and Random Decision Making. Moreover, whenever the collective decision induced by Voluntary Majority Voting differs with positive probability from the collective decision induced by one of the other voting rules, then Voluntary Majority Voting strongly Pareto-dominates that voting rule.*

Proof. *Step 1: Comparison between Voluntary Majority Voting and Compulsory Majority Voting*

Suppose that in *Voluntary Majority Voting* all individuals vote if and only if their participation costs are below some common threshold $\widehat{c} \in [\underline{c},\overline{c}]$. As Proposition 2 shows there is only one value of $\widehat{c}$ for which this is an equilibrium. However, we can investigate the welfare implications of such behaviour independent of whether it is equilibrium behaviour or not.

Denote by $U(\widehat{c})$ the expected utility of any individual if all individuals adopt the strategy just described. Observe that the expected utility from the unique symmetric equilibrium of *Voluntary Majority Voting* is: $U(c^*)$, and the expected social welfare from *Compulsory Majority Voting* is $U(\overline{c})$. Our task is to prove that $U(c^*) \geq U(\overline{c})$ and $U(c^*) > U(\overline{c})$ if $c^* < \overline{c}$. We shall do so by showing that U is differentiable, and that $U'(\widehat{c}) < 0$ if $\widehat{c} \in (c^*, \overline{c})$.

We can write $U(\widehat{c})$ as follows:

$$U(\widehat{c}) = \int_{\underline{c}}^{\widehat{c}} \left(\frac{1}{2} + B(F(\widehat{c})) - c \right) f(c)dc + \int_{\widehat{c}}^{\overline{c}} \frac{1}{2} f(c)dc$$

The first integral represents an individual's expected payoff in case that c_i is sufficiently low so that the individual votes. The second integral represents the expected payoff for the case that c_i is so high that the individual does not vote. Conditional on not voting, the expected payoff is $\frac{1}{2}$, since each alternative is equally likely to be chosen. To obtain the expected payoff conditional on voting, we have to add to $\frac{1}{2}$ the benefits from voting, $B(F(\widehat{c}))$, but have to subtract the costs of voting, c. Observe that we can re-write expected utility as follows:

$$U(\widehat{c}) = \frac{1}{2} + \int_{\underline{c}}^{\widehat{c}} (B(F(\widehat{c})) - c) f(c)dc$$

By elementary results of calculus, the function U is differentiable for all $\widehat{c} \in (c^*, \overline{c})$ and its derivative is:

$$\begin{aligned} U'(\widehat{c}) &= \int_{\underline{c}}^{\widehat{c}} B'(F(\widehat{c}))F'(\widehat{c})f(c)dc \\ &\quad +(B(F(\widehat{c})) - \widehat{c})f(\widehat{c}) \\ &= B'(F(\widehat{c}))F'(\widehat{c})F(\widehat{c}) \\ &\quad +(B(F(\widehat{c})) - \widehat{c})f(\widehat{c}) \end{aligned}$$

The first term in this sum is negative because $B'(F(\widehat{c})) < 0$, as argued in Remark 1, and, as mentioned before, $F'(\widehat{c}) > 0$. Moreover, for $\widehat{c} > c^*$ the second term is negative, too. Thus, for $\widehat{c} > c^*$, we have: $U'(\widehat{c}) < 0$.

Step 2: Comparison between Voluntary Majority Voting and Random Decision Making

In the notation of *Step 1*, we need to show that $U(c^*) \geq \frac{1}{2}$, and that $U(c^*) > \frac{1}{2}$ whenever $c^* > \underline{c}$. Consider the difference: $U(c^*) - \frac{1}{2}$. From the calculations in *Step 1* we know:

$$U(c^*) - \frac{1}{2} = \int_{\underline{c}}^{c^*} (B(F(c^*)) - c) f(c) dc$$

If $c^* = \underline{c}$ the right hand side is evidently zero. Otherwise, $B(F(c)) - c > 0$ for all $c \in (\underline{c}, c^*)$, and the right hand side is strictly positive. ■

The comparison between *Voluntary Majority Voting* and *Random Decision Making* is relatively obvious. If under *Voluntary Majority Voting* an individual does not vote, and if the other individuals play a symmetric equilibrium, then from this individual's perspective the probability of each alternative being selected is exactly $\frac{1}{2}$. Thus, it is the same as under *Random Decision Making.* If an individual chooses to vote, it must be that he expects that voting will yield a utility larger than $\frac{1}{2}$. Therefore, under *Voluntary Majority Voting*, if individuals choose to vote voluntarily, each individual's expected utility is larger than under *Random Decision Making.*

To understand the comparison between *Voluntary Majority Voting* and *Compulsory Majority Voting* suppose all individuals vote if and only if their costs are below some threshold $\widehat{c}$. Imagine that we raise $\widehat{c}$ for all individuals. There will be two effects for the expected utility of some individual i. Firstly, the *direct* effect reflect the change in i's expected utility due to the change in i's own voting behaviour. This effect is given by $B(F(\widehat{c})) - \widehat{c}$, and is positive if $\widehat{c}$ is below the equilibrium value c^* and negative otherwise. The second effect is the voting externality. Raising $\widehat{c}$ means that all individuals other than i become more likely to vote. For those cost types of individual i which don't vote, this doesn't matter. From their perspective the probability that each alternative is chosen is independent of the other individual's voting probabilities, and is $\frac{1}{2}$. However, for those types of individual i which do vote, there is a *negative* externality because the probability that individual i is pivotal decreases as the voting probability increases. Thus, for $\widehat{c} > c^*$, both effects are negative, and hence their sum is negative. As this is true for all individuals, raising $\widehat{c}$ from c^* (*Voluntary Majority Voting*) to $\overline{c}$ (*Compulsory Majority Voting*) makes all individuals worse off, and is thus a Pareto worsening.

III. Asymmetric Voting Rules

Is it possible to improve on *Voluntary Majority Voting*? In this section, we shall show that the answer is affirmative. We shall present a sequence of examples of voting rules which under some conditions perform better than *Voluntary Majority Voting*. These voting rules are based on *Voluntary Majority Voting*, but they are asymmetric, either with respect to individuals, or with respect to alternatives. We shall show that this is potentially advantageous despite of the symmetry of our model.

In our first example the asymmetry concerns individuals; not all individuals are called to vote. In the second voting rule, an asymmetry with respect to alternatives is introduced by declaring one alternative arbitrarily to be the *status quo*. Finally, in our last example, we return to asymmetries among individuals, and assume that individuals vote sequentially in an exogenously given order.

Although asymmetries are the key to the voting rules in this section, we shall formally define these voting rules so that they are, in fact, symmetric. In the context of the first voting rule we shall do this by assuming that who is allowed to vote is determined by a random device where all individuals have the same chance of being selected. Similarly, we shall assume that a random device decides which alternative is the status quo, and that both alternatives have a probability 0.5 of being chosen. Finally, when considering sequential voting, we shall assume that the order is determined randomly, and that all possible orders have the same probability.

By postulating such random moves, we transform asymmetric voting rules into symmetric ones. However, the driving force behind our analysis is that at the stage of decision making the outcome of the random move is common knowledge among individuals. It is for this reason that we emphasize the asymmetric nature of the voting rules in this section.

For the determination of individuals' expected utility in Bayesian equilibria of alternative voting rules we have two different approaches available to us. We can consider expected utility *before* or *after* the initial random move has taken place. We call the result of the former calculation the *ex ante* expected utility, and the result of the latter calculation the *interim* expected utility. Observe that interim expected utility is calculated after the initial random move, but before individuals have learned their type. Thus, our usage of the terminology of interim and ex ante utility differs from that of Holmström and Myerson (1983).

Whenever we can prove strong or weak Pareto-dominance of the Bayesian equilibrium of one mechanism over another in terms of interim expected utility, we shall speak of interim strong or weak Pareto-dominance. Similarly, if we can prove strong or weak Pareto-dominance in terms of ex ante expected utility, we shall speak of ex ante strong or weak Pareto-dominance. Note that strong (resp. weak) interim Pareto-dominance implies strong (resp. weak) ex ante Pareto-dominance, but not vice versa.

In all examples, we shall show only that the proposed voting rules perform better than *Voluntary Majority Voting* for *some* distributions F of the participation costs. We shall not show this for all such distributions. This is a crucial difference between the results in this section and Proposition 3 of the previous section. Indeed for the first two procedures we give counterexamples of distributions for which *Voluntary Majority Voting* would clearly be the superior procedure.

For the sake of brevity all formal proofs are omitted from this section. They are available from the author upon request.

A. *Voting in a Committee*

We begin by considering a voting rule in which only a subset of all individuals, a *committee*, is allowed to vote. We shall call this voting rule *Voting in a Committee.* Let m be the number of members of the committee: $1 \leq m \leq n$.[8] The members of the committee are randomly selected. Each subset of m individuals has the same probability. The committee decides by *Voluntary Majority Voting*, as described in the previous section. We shall assume that within the committee the unique symmetric equilibrium of *Voluntary Majority Voting,* as described in Proposition 2, is played. Individuals who are not on the committee are not allowed to participate.

Proposition 4 *For every m where $1 \leq m \leq n-1$ there is an open set*[9] *of distributions F such that Voting in a Committee of size m strongly ex ante Pareto-dominates Voluntary Majority Voting.*

To see why Proposition 4 is true consider a distribution F which has very small support (i.e. for which $\overline{c}-\underline{c}$ is small) and such that under *Voluntary Majority Voting* agents are with probability one almost indifferent between voting and not voting. Then the introduction of a committee of a size $m < n$ will make all agents strictly better off. This is because the benefits of voting increase as the committee size is reduced. Committee members will thus strictly prefer voting over not voting. If committee membership is decided by an initial random move, all individuals are better off.

[8] Allowing $m = n$, and thus making *Voluntary Majority Voting* a special case of *Voting in a Committee* simplifies some of our terminology below.

[9] For the purposes of this proposition, we endow the set of all distributions F which satisfy the assumptions of this paper with the relative topology derived from the topology of weak convergence.

Next we will - roughly speaking - show that Proposition 4 is not true for *all* distributions F but that there are distributions F for which *Voluntary Majority Voting* among all voters strongly ex ante Pareto-dominates *Voting in a Committee* with a committee size less than n. The distributions for which we shall establish our result will be those for which participation costs are so low that even under *Voluntary Majority Voting* all individuals always vote. As we mentioned in Section I, if participation costs are negligible, all individuals prefer ex ante that the will of the true majority is implemented. Intuitively, this is why committees of size less than n are not advantageous in this case.

Our formal result will not be quite as simple as we have just suggested. If the number n of individuals is even, it will in fact be advantageous to form a committee of $n-1$ members. The reason for this is as follows. If participation costs are so small that all individuals vote, then majority voting among all n (where n is even) individuals implements the same decision rule as majority voting among a randomly formed committee of $n-1$ individuals but the committee of $n-1$ members incurs lower participation costs. The second part of this argument is obvious. To see why the first part is true, note that if there is a draw among the n individuals, then randomly removing one will have the same effect as resolving the tie at random. On the other hand, if there is a majority among the n individuals in favor of one of the two alternatives, then the majority will be at least of size two, and therefore randomly removing one individual will not affect it. These considerations lead to the following result:

Proposition 5 *(i) If n is odd, then there is some $\widetilde{c} > 0$ such that $\overline{c} < \widetilde{c}$ implies that the unique optimal committee size is n.*

(ii) If n is even, then there is some $\widetilde{c} > 0$ such that $\overline{c} < \widetilde{c}$ implies that the unique optimal committee size is $n-1$.

B. *Voting Over a Status Quo*

Voting Over a Status Quo makes one alternative, say A, arbitrarily the *status quo*. Individuals are free not to participate, but if they do participate, then they have only one option: to vote against A. If a majority of individuals participates and hence votes against A, then B is adopted. If exactly the same number of individuals votes against A as do not participate, then each alternative is chosen with probability $\frac{1}{2}$. If a majority of individuals does not participate, then A is adopted. We make the mechanism symmetric by postulating an initial random move which selects each of the alternatives with equal probability as the status quo. The idea of this mechanism is to reduce participation costs by making non-participation a meaningful signal. Our analysis below confirms this intuition but also indicates that the introduction of a status quo may exacerbate the free riding problem in costly voting.

Suppose that the random move has determined the status quo, say A. We begin by analyzing rational behaviour in the subsequent voting game. Note first that it is a strictly dominant strategy for an individual who favours A not to participate. We thus restrict attention to strategies according to which an individual votes only if he favours B. We define individual i's *voting strategy* to be a function $s_i : [\underline{c}, \overline{c}] \to \{0, 1\}$ where $s_i(c_i) = 1$ (resp. 0) means that individual i votes (resp. does not vote) if her costs of voting are c_i *and* she prefers B.

We shall again restrict attention to symmetric Bayesian equilibria. Symmetry means that all individuals choose the same voting strategy s. All individuals choosing voting strategy s is a Bayesian equilibrium if and only if for almost every value of c_i the decision $s(c_i)$ maximizes individual i's expected utility given that all other individuals play strategy s, conditional on the participation costs being c_i.

Consider some individual i, and assume that i prefers alternative B. In a Bayesian equilibrium, individual i will vote if and only if the benefits of voting are at least as large as the costs c_i of voting. The benefits of voting only depend on the probability which an outsider who doesn't know the participation costs attaches to the event that an individual votes. This probability, conditional on an individual preferring alternative B, is: $q \equiv \int_{\underline{c}}^{\overline{c}} s(c)f(c)dc$. The expected benefits of voting to individual i are now, as before, the probability that this individual's vote is pivotal, times the benefits derived from casting a pivotal vote. We wish to calculate these two factors. Their value depends on whether the number of individuals, n, is odd or even.

If the number of individuals is odd, then an individual i is pivotal if and only if exactly $\frac{n-1}{2}$ of the other $n-1$ individuals vote against A. In that case individual i's vote resolves a tie in favor of B. Any individual votes against A with probability $\frac{1}{2}q$. Thus, we obtain as the probability of individual i being pivotal:

$$\Upsilon(q) = \binom{n-1}{\frac{n-1}{2}} (\frac{1}{2}q)^{\frac{n-1}{2}} (1 - \frac{1}{2}q)^{\frac{n-1}{2}}.$$

The benefits derived from casting a pivotal vote are $\frac{1}{2}$ Thus, the expected benefits from voting equal $\frac{1}{2}\Upsilon(q)$.

If the number of individuals is even, then individual i is pivotal if and only if exactly $\frac{n}{2} - 1$ of the other $n-1$ individuals vote against A. In that case, individual i's vote determines whether alternative A is implemented, or whether there is a draw between A and B. The probability of this event occurring is:

$$\Upsilon(q) = \binom{n-1}{\frac{n}{2}-1} (\frac{1}{2}q)^{\frac{n}{2}-1} (1 - \frac{1}{2}q)^{\frac{n}{2}}.$$

The expected benefit from voting in this case is again $\frac{1}{2}\Upsilon(q)$.

In the following we shall write $\Psi(q)$ for the benefits of voting, conditional on preferring B. Hence $\Psi(q) = \frac{1}{2}\Upsilon(q)$. A simple calculation yields:

Remark 6 *$\Upsilon(q)$ is a differentiable function of q, and $\Upsilon'(q) > 0$ for all $q \in (0,1)$. The same is true for $\Psi(q)$.*

If all other individuals play a voting strategy which induces the voting probability q, then individual i's best response is to vote if $c_i < \Psi(q)$, and not to vote if $c_i > \Psi(q)$. This implies that an equilibrium strategy s must be a threshold strategy: There must be some $\widehat{c}$ such that $s(c_i) = 1$ if $c_i < \widehat{c}$ and $s(c_i) = 0$ if $c_i > \widehat{c}$.

For which values of $\widehat{c}$ does the corresponding threshold strategy constitute a symmetric Bayesian equilibrium? For any $\widehat{c} \in [\underline{c},\overline{c}]$ the probability of voting, conditional on preferring B, is $F(\widehat{c})$ if individuals follow a threshold strategy with threshold $\widehat{c}$. Observe that F is differentiable, and that $F'(\widehat{c}) > 0$ for all $\widehat{c} \in (\underline{c},\overline{c})$. A value $\widehat{c}$ is the threshold for an equilibrium threshold strategy if and only if $\Psi(F(\widehat{c})) = \widehat{c}$ (if $\widehat{c} \in (\underline{c},\overline{c})$) or $\Psi(F(\widehat{c})) \geq c^*$ (if $\widehat{c} = \underline{c}$) or $\Psi(F(\widehat{c})) \leq \widehat{c}$ (if $c^* = \widehat{c}$). Observe that Remark 6 and the previously noted fact that $F'(\widehat{c}) > 0$ imply that $\Psi(F(\widehat{c}))$ is differentiable and strictly increasing in $\widehat{c}$. Thus, in the two dimensional non-negative orthant with $\widehat{c}$ on the horizontal axis and $\Psi(F(\widehat{c}))$ on the vertical axis, we are looking for the intersection of the graph of a differentiable, strictly increasing function, and the 45° line.

Unlike in Section II, there can be many such points of intersection, because $\Psi(F(\widehat{c}))$ increases. Consequently multiplicity of equilibria poses a severe problem for the analysis. The intuitive reason for this is that if individuals vote over a status quo, individuals' voting decisions are strategic complements whereas in *Voluntary Majority Voting*, individuals' voting decisions are strategic substitutes.

We shall not completely characterize all equilibria of *Voting Over a Status Quo*. For our purposes, the following very simple result will be enough. The result follows immediately from what has been said so far.

Proposition 7 *(i) Voting Over a Status Quo always has a Bayesian equilibrium in which no individual ever votes.*

(ii) If $\overline{c} \leq \binom{n-1}{\frac{n-1}{2}}\left(\frac{1}{2}\right)^{n-1}$ *(if n is odd) or if* $\overline{c} \leq \binom{n-1}{\frac{n}{2}-1}\left(\frac{1}{2}\right)^{n-1}$ *(if n is even)*[10]*, then Voting Over a Status Quo has a Bayesian equilibrium in which every individual who favours B always votes.*

(iii) If $\underline{c} \geq \binom{n-1}{\frac{n-1}{2}}\left(\frac{1}{2}\right)^{n-1}$ *(if n is odd) or if* $\underline{c} \geq \binom{n-1}{\frac{n}{2}-1}\left(\frac{1}{2}\right)^{n-1}$ *(if n is even), then the only Bayesian equilibrium of Voting Over a Status Quo is that no individual ever votes.*

[10] The upper boundary for $\overline{c}$ is the value of $\Psi(1)$.

Because we haven't characterized *all* equilibria of *Voting Over a Status Quo*, we cannot provide a complete welfare analysis of this mechanism. It is clear that the mechanism may do badly. If no individual ever votes, which is a symmetric equilibrium of this mechanism by part (i) of Proposition 7, the mechanism performs not better than *Random Decision Making.* This failure of the mechanism is due to the strong externality which it creates. However, the mechanism also has the potential to perform very well. This is shown by the following proposition. The proposition refers to interim Pareto-dominance. Recall that interim Pareto-dominance implies ex ante Pareto-dominance.

Proposition 8 *Suppose* $\overline{c} \leq \binom{n-1}{\frac{n-1}{2}} \left(\frac{1}{2}\right)^{n-1}$ *(if* n *is odd) or* $\overline{c} \leq \binom{n-1}{\frac{n}{2}-1} \left(\frac{1}{2}\right)^{n-1}$*, and suppose that in Voting Over a Status Quo the symmetric Bayesian equilibrium in which all individuals vote with probability one, provided that they favor* B*, is played. Then Voting Over a Status Quo strongly interim Pareto-dominates Voluntary Majority Voting.*

To see why this is true note that if $\overline{c}$ is not larger than the upper boundaries indicated in the proposition, Proposition 2 implies that the unique symmetric Bayesian equilibrium of *Voluntary Majority Voting* is that all individuals always vote. Part (iii) of Proposition 7 says that *Voting Over a Status Quo* also has an equilibrium in which all individuals always vote, provided that they oppose A. By assumption this equilibrium is being played. As a consequence, both equilibria will lead to the same outcomes. However, all individuals will have lower voting costs under *Voting Over a Status Quo* because they will incur voting costs only if they oppose A. Thus, all individuals' expected utility is higher under *Voting Over a Status Quo* than under *Voluntary Majority Voting.*

Proposition 8 thus formalizes the obvious intuition that *Voting Over a Status Quo* economizes on the participation costs of supporters of the status quo. Recall, however, that even under the assumptions of Proposition 8 *Voting Over a Status Quo* is not unambiguously good. The good equilibrium co-exists with the equilibrium in which social decisions are random. For other distributions of costs, *Voting Over a Status Quo* is, in fact, unambiguously inferior to *Voluntary Majority Voting:*

Proposition 9 *Suppose* $\binom{n-1}{\frac{n-1}{2}}\left(\frac{1}{2}\right)^{n-1} < \underline{c} < \frac{1}{2}$ *(if* n *is odd) or* $\binom{n-1}{\frac{n}{2}-1}\left(\frac{1}{2}\right)^{n-1} < \underline{c} < \frac{1}{2}$ *(if* n *is even). Then Voluntary Majority Voting strongly interim Pareto-dominates Voting Over a Status Quo. The set of values* $\underline{c}$ *to which this result applies is non-empty.*

The argument for this is as follows: Part (iii) of Proposition 2 shows that under the assumptions of Proposition 9 the unique symmetric Bayesian equilibrium of *Voluntary Majority Voting* has an interior threshold $c^* \in (\underline{c}, \overline{c})$ such that individuals vote if and only if their participation costs are below c^*. By contrast, according to part (iii) of Proposition 7, under *Voting Over a Status Quo* no individual will ever vote. Thus, under *Voting Over a Status Quo,* all individuals' expected utility is $\frac{1}{2}$, whereas under *Voluntary Majority Voting* it is more than $\frac{1}{2}$ because each individual expects with positive probability to be pivotal.

To show that the set of values $\underline{c}$ to which the proposition applies is non-empty, we need to show that $\binom{n-1}{\frac{n-1}{2}}\left(\frac{1}{2}\right)^{n-1} < \frac{1}{2}$ (if n is odd) or $\binom{n-1}{\frac{n}{2}-1}\left(\frac{1}{2}\right)^{n-1} < \frac{1}{2}$ (if n is even). Consider the case that n is odd. We prove the assertion by induction over n. The assertion can be verified through calculation for the case $n = 3$. If we raise n to $n + 2$, the term on the left hand side gets multiplied by $\frac{n(n+1)}{\frac{n+1}{2}\frac{n+1}{2}}\left(\frac{1}{2}\right)^2 = \frac{n}{n+1}$. Thus, it decreases. Therefore, the assertion is true for all even n. A similar argument proves that the assertion is true for all even n.

C. *Sequential Voting*

Sequential Voting is like *Voluntary Majority Voting*, except that individuals make their decisions sequentially rather than simultaneously. Each individual observes all the moves of the individuals moving before him or her. The order in which individuals move is randomly selected. Every possible order has the same probability.

The idea of this mechanism is that it reduces strategic uncertainty, and thus allows individuals to avoid the participation costs if they know that their vote is not pivotal. However, note that we are assuming that *observation* of the collective decision procedure is costless, and that it is only the actual *participation* which is costly. This is not always true, but an example where this is plausible is when members of the US Congress vote sequentially. They can follow the progress of the vote in their offices on television, and need not leave their offices until shortly before they are called to vote. Thus, they can delay the decision whether to participate until the last minute.

We now analyse rational behaviour under *Sequential Voting.* Without loss of generality we assume that the initial random move has determined that individual 1 moves first, then individual 2 moves, until, finally, individual n makes her decision.

A complete analysis of the sequential equilibria of *Sequential Voting* would require many case distinctions. For our purposes, the following simple result, which is intuitively obivous, will be enough.

Proposition 10 *There is some* $\widetilde{c} > 0$ *such that* $\overline{c} < \widetilde{c}$ *implies that Sequential Voting has a unique sequential equilibrium. In this equilibrium, an individual* i *who prefers an alternative* a *votes in favor of that alternative unless the preceding individuals' votes have established a majority which can not be overturned. If such a majority exists individual* i *does not participate.*

More specifically, let n_a *be the number of votes cast in favor of alternative* a *when it is* i*'s turn to vote. Let* $n_{a'}$ *be the number of votes cast in favor of alternative* $a' \neq a$*. Individual* i *does not participate if either* $n_a - n_{a'} > n - i$ *or if* $n_{a'} - n_a > n - i + 1$*. Otherwise, individual* i *participates, and votes in favor of* a*.*

If this equilibrium is played, then the outcome will be the same as under Voluntary Majority Voting in the case in which all individuals always vote.

Now suppose that $\overline{c}$ is so low that Proposition 10 applies, and also part (iii) of Proposition 2. Then *Sequential Voting* and *Voluntary Majority Voting* will lead to the same collective decisions, but *Sequential Voting* will achieve this with lower participation costs, because individuals who know that their vote cannot be pivotal will not participate. Thus we have shown the following result.

Proposition 11 *There is some $c' > 0$ such that $\overline{c} < c'$ implies that Sequential Voting weakly interim Pareto-dominates Voluntary Majority Voting and it strictly ex ante Pareto-dominates Voluntary Majority Voting.*

This proposition claims only weak interim Pareto-dominance because the individuals who move initially are indifferent between *Sequential Voting* and *Voluntary Majority Voting.* Only the individuals who move later strictly prefer *Sequential Voting.* If the order in which individuals vote is determined randomly, then *Sequential Voting* strongly Pareto-dominates *Voluntary Majority Voting* because each individual has a chance of being sufficiently late in the sequence to be able to economize on voting costs.

Next, we wish to show that Proposition 11 is true only for some, not for all distributions of participation costs. We can show this only for that part of Proposition 12 which refers to interim Pareto-dominance, not for the part which refers to ex ante Pareto dominance. Our argument is based on the following example.

Example 12 *There are three individuals: $n = 3$, and participation costs are uniformly distributed on the interval $[0; \frac{1}{2}]$. Individuals vote in the order $1, 2, 3$.*

The unique sequential equilibrium of Sequential Voting is that individuals 1 and 2 vote whenever $c_1 < \frac{1}{4}$ resp. $c_2 < \frac{1}{4}$ and individual 3 votes whenever he is pivotal, independent of his costs. Individual 1 and 2's interim expected payoff is $\frac{9}{16}(= 0.5625)$. Individual 3's interim expected payoff is $\frac{43}{64}(= 0.6718)$. All individuals' ex ante expected payoff is $\frac{114}{192}(\approx 0.5990)$.

The unique symmetric equilibrium of Voluntary Majority Voting is that individual i votes for his or her preferred alternative whenever the participation costs satisfy $c_i <$

$1 - \sqrt{0.5} (\approx 0.2929)$. *All individuals' expected utility in this equilibrium is* $2(1 - \sqrt{0.5})$ (≈ 0.5858).

In this example individuals 1 and 2 prefer *Voluntary Majority Voting* over *Sequential Voting,* whereas individual 3 prefers *Sequential Voting.* However, note that ex ante all individuals prefer *Sequential Voting.* We have not found any example for which this were not true.

IV. Conclusion

To conclude, we suggest a re-interpretation of our model as a model of endogenous information acquisition in voting. Specifically, suppose that individuals need to make an effort to find out which of the two alternatives they prefer. Suppose the costs of that effort to individual i are c_i, and assume that an individual i who hasn't found out which alternative she prefers does not vote. Then the analysis of this paper indicates how different voting rules affect the process of endogenous information acquisition game.

This question has previously attracted interest in a common value setting (Persico (1999)). Our contribution is to point out that this issue is also in a private value setting interesting. Even in a private value setting some voting procedures provide better incentives for information acquisition than others. Note that if the participation decision is interpreted as an information acquisition decision, then *compulsory* participation does not seem enforceable. Thus, our results in Section III about the potential advantages of asymmetric voting rules seem more relevant than our results about the drawbacks of compulsory participation in Section II.

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