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Asset returns with transactions costs and uninsured individual risk*

S. Rao Aiyagari
Federal Reserve Bank of Minneapolis, Minneapolis, MN 55480, USA

Mark Gertler
New York University, New York, NY 10003, USA

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We explore whether incorporating an explicit motive for holding liquid assets within an equilibrium asset pricing model helps explain the following features of asset returns and turnover in the post-war U.S. economy: (i) the low, risk-free real interest rate, (ii) the large spread between returns on liquid assets and stocks, and (iii) the greater transaction velocity of liquid assets relative to stocks. We introduce a demand for liquid assets by adding uninsured individual risk together with differential costs of trading securities. Numerical simulations attempting to match the return data generate a ratio of liquid assets to income considerably below observed levels.

1. Introduction

The secular average annual real return on Treasury Bills is less than 1 percent. For stocks, it is about 7 percent. These two facts have stimulated a lengthy discussion in the literature, beginning with Mehra and Prescott (1985). The issue is that it is difficult to generate these kinds of numbers using the standard intertemporal model of asset pricing [Lucas (1978)]. Reasonably parameterized versions tend to predict too low a risk premium and too high a risk-free rate. These results lead Mehra and Prescott to conclude that it is *not* 'reasonable to abstract from liquidity constraints,

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transactions costs and the like and to use a frictionless Arrow–Debreu economy to explain these observations'.

A number of papers have attempted to save the frictionless framework. The strategies have included using alternative functional forms for individual preferences [Nason (1988), Constantinides (1988), Epstein and Zin (1987), Weil (1989)] and for the stochastic processes that drive dividend and consumption behavior [Reitz (1988), Labadie (1989), Cecchetti, Lam, and Mark (1989)]. While these approaches have met with some limited success, they have almost exclusively focused on only one part of the puzzle: why the equity premium is large. Largely ignored has been the other: why the risk-free rate is so low. It is unclear whether it is desirable to separate the two questions; indeed, Mehra and Prescott conclude that resolving the latter is central to resolving the former.

In this paper, we develop and numerically simulate a model aimed at providing a joint explanation of the equity premium and the risk-free rate. We follow Mehra and Prescott's suggestion and step outside the frictionless Arrow–Debreu economy. Our model relies on both incomplete securities markets and transactions costs. Individuals face idiosyncratic shocks to personal income. Markets for claims on personal income do not exist, by assumption. The absence of a complete set of contingent claims markets implies that individuals must self-insure, i.e., buy and sell assets to smooth consumption. Two kinds of securities are available, stocks and short-term government bonds (T-bills). One important distinction between the two is that, by assumption, stocks are costly to trade while T-bills are freely exchanged. One can think of T-bills as either being directly held by households or as being costlessly repackaged by an intermediary which in turn issues freely tradable securities to its depositors. A key premise is that intermediaries cannot similarly repackage stocks. In any event, regardless of whether they are directly or indirectly held by households, T-bills in our model have an edge over stocks as a vehicle for self-insurance.

Having nontraded individual income risks permits the model to generate a low risk-free rate. The equilibrium risk-free rate can potentially lie well below the rate of time preference. [See, for example, Bewley (no date), Bewley and Radner (1980), or Clarida (1990).] Introducing costs of trading stocks in conjunction with uninsured individual risks enlarges the equity premium. The need for self-insurance motivates trade in securities. Costs of trading thus become relevant to pricing a security in equilibrium. The ease of exchanging T-bills implies that stocks must pay an added premium – a transactions/liquidity premium – to be competitive with bonds.

The model is consistent with two other facts that are anomalies in the context of the standard asset pricing model. The first fact relates to trading volume. Empirically, households turn over liquid assets (assets like savings accounts and money market deposits) at a much more rapid rate than stocks.

This kind of behavior emerges in our model. Roughly speaking, individuals try to smooth income fluctuations by trading in T-bills (or assets backed by T-bills) and only use stocks as a last resort. The second fact relates to consumption behavior. Aggregate consumption is smooth in our framework, but individual consumption is highly variable due to the incompleteness in securities markets. This pattern in consumption seems consistent with the evidence.

Mankiw (1986) also appeals to uninsured individual risks to explain the equity premium. Our analysis differs in some important ways from his. First, we attempt to explain the risk-free rate puzzle as well, whereas Mankiw studies a framework where the risk-free rate is exogenous. Second, as Mankiw observes, his results rely on a very specific pattern of individual risk. Our results instead rely on costs to individuals of trading in stocks. Third, we present numerical simulations of a fully specified heterogeneous agent, dynamic equilibrium economy, as a means to judge the empirical significance of the imperfections we have introduced.

Work by Constantinides (1986) is also relevant. He studied a partial equilibrium economy with two assets, a stock and a riskless security, and with proportionate costs of trading the stock. His main conclusion was that the transactions costs had only a second-order effect on pricing the securities. In addition to being a general equilibrium analysis, our framework differs by incorporating uninsured income risks. The effect is to enlarge the trading volume which permits a potentially greater role for transactions costs. Fisher (1990) considers transactions costs in an equilibrium framework and finds a significant effect on security returns; but he does not explicitly incorporate heterogeneity and trade.

Finally, Deaton (1989) presents numerical simulations of a model where undiversified individual risk induces precautionary saving. In our model, which differs by including two assets and differential transactions costs, the uninsured risk affects portfolio choice, as well as saving. Another important difference is that security returns are endogenous in our model.

The rest of this paper is organized as follows. In section 2 we present an informal discussion of the nature and magnitude of the costs of trading stocks. Section 3 describes the formal framework, a variant of the Lucas asset pricing model, where individuals face uninsured idiosyncratic risks, there are restrictions on borrowing, and trading stocks is costly. There is also an intuitive discussion of how transactions costs may impact on return spreads and of how small trading costs could generate a large spread. Section 4 describes the algorithm for computing the solution to our heterogeneous agent economy. Here we borrow insights from Imrohoroglu (1988, 1989) and Diaz-Gimenez and Prescott (1989) who studied related kinds of models. The section also discusses the parameterization of the model. In addition to the transactions costs, the unusual feature of the model is the presence of individual risk. We use panel data studies of annual earnings and hours variation to provide some guidance.

Computing a rational expectations equilibrium with both aggregate and individual risk and with a continuum of people appears to be an extremely difficult undertaking (see the discussion in section 3). We therefore restrict attention to the case of no aggregate dividend risk.[1] As a consequence of this simplification, any difference in the spread between stocks and bonds is attributable only to the frictions we have introduced, namely trading costs in conjunction with uninsured individual risk. Thus, instead of trying to reproduce the observed spread, our strategy is to determine whether the model can generate a 'transactions/liquidity' premium which is a significant fraction of the actual equity premium.

Thus, in section 5 we present results from simulations which explore the extent to which the model is capable of explaining (i) the observed low level of the riskless rate, (ii) a transactions/liquidity premium in the range of 3 percent – about half the equity premium, and (iii) the relative pattern of transactions velocities. A number of examples are studied, including ones which allow for costly borrowing. An important finding, however, is that the model predicts too low a ratio of liquid assets to income. The simulated values are between 20 and 30 percent of a rough benchmark number. At the same time, the simulated values of the stock to income ratios match the data reasonably well. We conclude that in the context of our model the equity premium puzzle can be restated as a puzzle as to why households have tended to hold such a large fraction of marketable wealth in the form of low yielding liquid assets. In section 6 we offer some suggestions as to how possible extensions of our analysis might get at this issue, in addition to some other final remarks.

2. Transactions costs for trading stocks

Statistics on trading volume are consistent with the notion that transactions costs matter. Stocks turn over much less frequently on average than, for example, do money market accounts. For stocks, the ratio of shares sold over a year to the average number of shares listed for the year is about 0.5. Further, a substantial fraction of the volume is accounted for by institutional traders which own about 50 percent of outstanding shares. Turnover by households, who own the other half, is virtually negligible. As a comparison,

[1]See Kahn (1990) for an analysis of an overlapping generations economy with aggregate dividend risk and idiosyncratic individual risk. The overlapping generations framework permits some important simplifications for calculating an equilibrium. In Kahn's framework, which does not incorporate transactions costs, the equity premium is not large, suggesting that mixing individual risk only with aggregate dividend risk is not enough. See also Weil (1990), who offers a somewhat different perspective in a two-period model.

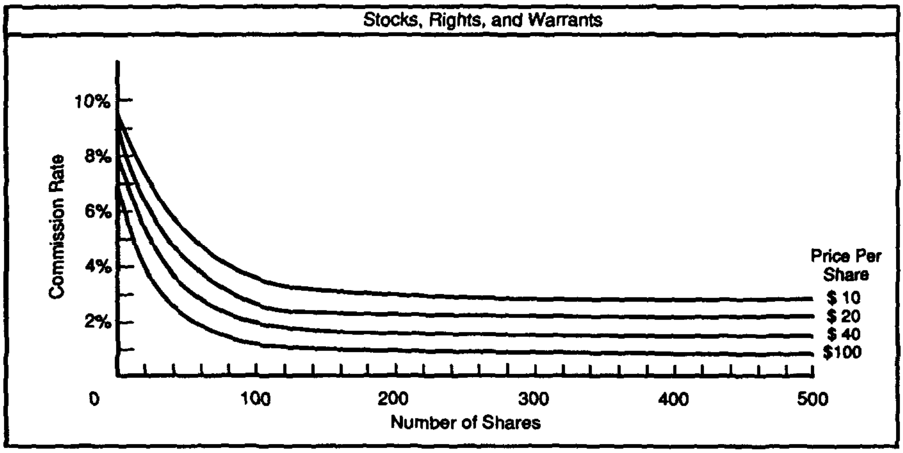

Fig. 1. Typical commission rates for selected transactions: Dollar commission as a percentage of the value of the order [from Sharpe (1985, p. 40)].

the equivalent turnover statistic for savings accounts is about 3 and that for bank money market funds is about 7, indicating a substantially higher transactions velocity.

In practice there are three basic kinds of (pre-tax) costs involved in trading stocks: (i) brokerage commission costs, (ii) buy–sell spreads, and (iii) time involved in acquiring knowledge and record keeping. At a deeper level, the existence of these costs reflects the informational fractions involved in trading heterogeneous assets like stocks. In addition, tax considerations are also likely to be a factor since capital gains levies are based on realization rather than accrual.

Brokerage costs have been declining due to deregulation, but are still consequential, particularly for small and medium-size transactions. Commission rates for retail brokers are inversely related to the quantity of shares transacted. A schedule is provided in fig. 1 [taken from Sharpe (1985, p. 40)]. For shares priced at $40 (the average share price on the NYSE varied between $33 and $39 over the past six years) commission rates decline monotonically from 8 to 2 percent as the size of the trade rises from $1 to $4000. It then remains at about 2 percent for trades up to $200,000. (There is typically also a minimum cost of $30 to $50.) Discount brokers charge 30 to 70 percent less but do not provide counseling or record keeping services. It does not appear that discount brokers are dislodging retail brokers.

Mutual funds provide an alternative to directly managing a portfolio, but still involve trading costs. There are two basic kinds of funds, load and no-load. Load funds, which are by far the most prevalent, charge an up-front commission (typically) of 5 to 8 percent. The rate tends to vary positively with the riskiness of the portfolio. While the up-front charge is steep, there is usually no extra charge for liquidation. No-load funds do not charge an initial fee, but typically place restrictions on the speed at which the account can be liquidated.[2] One form this restriction may take, for example, is a steep charge (up to 8 percent) for early withdrawal.

The bottom line is that whether individuals hold stocks directly or via mutual funds, they can lose considerably by frequently moving in and out of the market. Conventional wisdom dictates not to 'churn'.

Bid–ask spreads add to the cost of trading. For actively traded stocks of large companies, which constitute about 50 percent of the market, the ratio of the spread to the price averages 0.52 percent. This ratio rises as company size declines. It averages around 1 percent for the rest of the market, reaching as high as 6.55 percent for a typical firm with assets under ten million dollars.

Finally, actively trading stocks requires time and expertise. Not much thought is required for exchanging safe, homogeneous securities like money-market deposits. Knowing which stocks to trade is a much more complex decision. Also, record keeping requirements are considerable. Survey data indicates that only about 25 percent of households own stocks. [See Mankiw and Zeldes (1989).] This is consistent with the notion that managing a stock portfolio is neither costless nor effortless.

One last consideration is the frequency of the need to exchange the security. That is, even if the costs of a single transaction are small, the need to trade often can make the costs over a given time period large. This consideration then will have a bearing on what kinds of securities to hold at the margin.

3. The basic model

We consider a stationary, infinite horizon, pure exchange economy with no aggregate uncertainty. Time is discrete and is denoted by t which takes values $0, 1, 2, \ldots$. One kind of good exists, a nonstorable consumption good. There is a continuum of people of measure unity. Each person i has preferences over consumption given by

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty} \beta^t U(c_t^i)\right\}, \qquad 0 < \beta < 1, \tag{3.1}$$

[2]There are, however, some no-load funds which appear to have minimal costs or restrictions on trading. It is puzzling that these kinds of funds aren't more popular, and more generally, that the size of assets in no-load funds is so small as compared to load funds. It is noteworthy, though, that these funds typically do not provide counseling services.

where c_t^i is consumption by i in period t, β is the subjective discount factor, and $\mathrm{E}_0\{\cdot\}$ is the mathematical expectation conditioned on information at time zero.

Each period, supplies of the perishable consumption good arrive from two kinds of sources. The first source is 'capital'. There exist $\bar{s}$ capital machines which costlessly produce output each period. The proceeds are distributed as dividends to shareholders who own the machines. There are $\bar{s}$ equity claims which are tradable and perfectly divisible. One claim entitles the owner to $1/\bar{s}$ percent of the total output from all the machines each period. We assume that the output per machine, d, is constant over time. The second kind of income is 'labor'. Each period, individual i receives an endowment of the consumption good, y_t^i, which obeys a stationary Markov chain. Further, fluctuations in labor income are independent across individuals. Thus per capita labor income is smooth, while individual labor income is highly variable. Moreover, while a market exists for claims on capital income, the same is not true for claims on labor income. Thus the variation in y_t^i reflects uninsured individual risk. Later we demonstrate that the model can be easily reformulated so that this variation incorporates taste shocks as well as idiosyncratic income fluctuations.

There is a government sector which consumes g units per capita each period. It finances this activity with a per capita lump sum tax, τ, and by issuing T-bills. The government budget constraint is given by

$$g + \bar{b}_t = \tau + \bar{b}_{t+1}/(1 + r_t), \tag{3.2}$$

where $\bar{b}_t$ is the per capita quantity of T-bills at the beginning of period t in terms of market value and r_t is the riskless interest rate from t to $t+1$.

Each period, an individual decides how much to consume and the amounts of stock and T-bills to acquire. We assume that there are costs of trading stock that are proportionate to the value of the trade.[3] Let α_b be the per unit of value buying cost and α_s the per unit of value selling cost. An individual i's momentary budget constraint is then given by

$$\begin{aligned} &c_t^i + p_t(s_{t+1}^i - s_t^i) + b_{t+1}^i/(1 + r_t) \\ &= y_t^i + s_t^i d + b_t^i - \tau - \max\{\alpha_b p_t(s_{t+1}^i - s_t^i),\, \alpha_s p_t(s_t^i - s_{t+1}^i)\}, \end{aligned} \tag{3.3}$$

where p_t is the period t price of equity.

[3]In view of the discussion in section 2, proportionate trading costs are a plausible approximation. It is not difficult to allow for fixed costs or decreasing marginal costs as depicted in fig. 1. In fact, we consider the implications of fixed costs in section 5.

Short sales of stock and borrowing are disallowed (later we relax the constraint on borrowing). The following restrictions thus apply:

$$s_t^i \geq 0, \tag{3.4a}$$

$$b_t^i \geq 0. \tag{3.4b}$$

We restrict attention to steady states. Let $F(s, b, y)$ be the joint cross-section distribution of stock holdings at the beginning of t, bond holdings at the beginning of t, and labor income realization at t. That is,

$$F(s,b,y) = \text{fraction of people at the beginning of } t$$
$$\text{for whom:} \quad (s_t, b_t, y_t) \leq (s, b, y). \tag{3.5}$$

The Markov process describing the evolution of individual labor incomes is given by the following:

$$Y(y', y) = \text{prob}[\, y_{t+1} \leq y' | y_t = y\,]. \tag{3.6}$$

Since there is no aggregate uncertainty, a steady state consists of a constant over time stock price p, a constant interest rate on bonds r, a constant per capita quantity of bonds $\bar{b}$, and a cumulative distribution function $F(s, b, y)$ which are consistent with individual optimization, the government budget constraint (3.2), and market clearing at each date.

A typical individual's dynamic optimization problem can be described in terms of usual Bellman's equation of dynamic programming. The individual state vector is denoted z_t^i and consists of (s_t^i, b_t^i, y_t^i). We will use variables without primes to denote date t values and variables with primes to denote date $t+1$ values. Let $V(z^i)$ be the optimal value function for an individual. This must satisfy the Bellman equation,

$$V(z^i) = \max[U(c^i)] + \beta \mathrm{E}\{V((z^i)')|z^i\}, \tag{3.7}$$

subject to (3.3) and (3.6).

The solution consists of decision rules for $s^{i\prime}$ and $b^{i\prime}$,

$$s^{i\prime} = \sigma_s(z^i), \tag{3.8a}$$

$$b^{i\prime} = \sigma_b(z^i). \tag{3.8b}$$

The above decision rules can be aggregated using $F(.)$ to obtain the aggregate demand for stocks and bonds at the beginning of $t+1$. The aggregate supply of stocks is $\bar{s}$ and the aggregate supply of bonds is found from (3.2). The first requirement for a steady state is that the markets for stocks and bonds clear. The second requirement is that the c.d.f. $F(.)$ be consistent with individual optimization and market clearing. That is, when we use (3.8) and (3.6) together with $F(.)$ to compute the distribution of (s', b', y'), the new distribution should coincide with $F(.)$. This completes the description of the steady state.

We have abstracted from aggregate uncertainty because the general computational problem is quite formidable if, for example, dividends are stochastic. Asset prices will depend on the dividend shock as well as the beginning of period distribution of asset holdings. The distribution of asset holdings itself will be changing stochastically over time in response to dividend shocks. For the same reason, we have also assumed that government expenditures and per capita bonds and taxes are constant over time. This enables us to look for a stationary equilibrium in which the interest rate r, the stock price p, and the cross-section distribution of asset holdings and income $F(.)$ are all constant over time. Note that the government budget constraint (3.2) simplifies to the following:

$$g + r\bar{b}/(1+r) = \tau. \tag{3.9}$$

Another advantage of fixing dividends is that we can isolate the impact of the frictions we have introduced. Since there is no dividend risk, any spread between the returns on stocks and bonds is due only to the transactions costs operating in conjunction with the uninsured individual income risk.

Some Intuition on Return Spreads. Here we provide some intuition for the role that transactions costs play in generating a spread between the returns to equity and government bonds. We begin by considering an individual's decision whether to buy or sell stocks. The transactions costs introduce a wedge between the buying price $(1+\alpha_b)p$ and the selling price $(1-\alpha_s)p$. As a consequence, there will be two levels of income denoted $-y_{\text{buy}}(s,b)$ and $y_{\text{sell}}(s,b)$, with $0 < y_{\text{sell}}(s,b) < y_{\text{buy}}(s,b)$ such that whenever income is below y_{sell}, the individual sells stocks; when it is between y_{sell} and y_{buy} he holds; and when it is above y_{buy}, he buys.[4] Notice that these regions will depend on the individual's initial holdings of stocks and bonds.

Arbitrage requires that any individual buying both stocks and bonds at time t must be indifferent between acquiring either kind of asset at the margin. Therefore, for each person i in this position at t, the following Euler conditions must hold (where the i superscripts for agents are dropped for

[4]Examples of how transactions costs introduce bands of inaction can be found in Bertola and Caballero (1990) and Dixit (1989).

convenience):

$$U_c(c) = \beta(1+r)\mathrm{E}\{U_c(c')\}, \tag{3.10a}$$

$$\begin{aligned} p(1+\alpha_b)U_c(c) = \beta\{&\pi^b[d+p(1+\alpha_b)]\mathrm{E}_b\{U_c(c')\} \\ &+\pi^h[d+p(1+\lambda_h)]\mathrm{E}_h\{U_c(c')\} \\ &+\pi^s[d+p(1-\alpha_s)]\mathrm{E}_s\{U_c(c')\}\}, \end{aligned} \tag{3.10b}$$

where π^b, π^s, and π^h are the probabilities the individual will be buying, selling or holding the stock next period; where E_b, E_s, and E_h are the expectations conditional on buying, selling, or holding stocks next period;[5] and where the number λ_h satisfies

$$\alpha_b > \lambda_h > -\alpha_s. \tag{3.11}$$

The left side of eq. (3.10b) is the cost of buying a stock and the right side is the expected marginal gain, after factoring in transactions costs. Importantly, the marginal gain depends on whether and how the individual expects to be adjusting his stock holdings in the subsequent period. The marginal value of a stock equals $(1+\alpha_b)p$ for someone who is buying stocks, and $(1-\alpha_s)p$ for someone who is selling. For someone holding, it lies between the buying and selling price, at $(1+\lambda_h)p$.[6] Everything else equal, the larger π^s, the smaller the expected marginal benefit from purchasing stock. The unattractive aspect of having to turn around and sell the stock in the subsequent period is having to incur the transactions cost.

Combining the Euler equations for bonds (3.10a) and for stocks (3.10b) yields

$$\begin{aligned} d/p - r &= (1+r)\alpha_b - [\pi^b\alpha_b\mathrm{E}_b\{U_c(c')\} + \pi^h\lambda_h\mathrm{E}_h\{U_c(c')\} \\ &\quad - \pi^s\alpha_s\mathrm{E}_s\{U_c(c')\}]/\mathrm{E}\{U_c(c')\} \\ &> [1-(\pi^b\mathrm{E}_b\{U_c(c')\} + \pi^h\mathrm{E}_h\{U_c(c')\})/\mathrm{E}\{U_c(c')\}]\alpha_b \\ &\quad + \pi^s[\mathrm{E}_s\{U_c(c')\}/\mathrm{E}\{U_c(c')\}]\alpha_s \\ &\geq \pi_s(\alpha_b+\alpha_s)\mathrm{E}_s\{U_c(c')\}/\mathrm{E}\{U_c(c')\}. \end{aligned} \tag{3.12}$$

[5]Note that the Euler conditions for agents who are selling stocks or who are borrowing constrained and/or short sale constrained will be different from (3.10).

[6]In general, λ_h will depend on whether the individual expects to be buying or selling down the road.

Quite clearly, the transactions costs are responsible for the spread between the returns to stocks and bonds. The spread is increasing in α_b, α_s, and π^s. Further, it is likely to be larger, the more risk-averse the individual; this is because sales of stocks are likely when consumption is low, which makes the utility measure of the transactions costs of selling (relatively) high.

The lower bound for the spread equals $\pi^s(\alpha_s + \alpha_b)$, the probability of selling times the roundtrip transaction cost.[7] This value arises (approximately) when individuals are risk-neutral and when the shadow value of stock for someone holding is arbitrarily close to its upper bound, $(1 + \alpha_b)p$. As the discussion of the magnitude of transactions costs in section 2 indicates, the number $\pi_s(\alpha_b + \alpha_s)$ may be significant. For example, if the period is a quarter, the roundtrip transaction cost is 4 percent, and π^s is 15 percent, then the lower bound for the spread is 0.6 percent per quarter or 2.4 percent per year.[8] Further, this calculation does not take into account any aggregate riskiness in dividends or the impact of risk aversion.

In summary, the existence of trading costs for stocks in conjunction with the need to trade securities to smooth consumption can introduce a spread between stocks and bonds. Further, the incompleteness of markets for insurance implies a 'low' riskless rate of interest in equilibrium. We verify these conjectures in section 5 where we present some measures of the kinds of magnitudes involved. Before presenting some results we need to discuss how the stationary equilibrium is computed and the model is parameterized. We do this in the next section.

4. Computation and model parameterization

Generally speaking, the computational procedure involves first specifying values for asset returns and taxes, and then finding values for asset stocks and government purchases which support these returns in equilibrium. Thus, in addition to choosing numbers for preference and technology parameters, we also pick values for r_s, r, and τ. How successful the model is in explaining a particular configuration of asset returns then depends on how well the computed asset/income ratios and relative transaction velocities match with observed data.

We assume that the Markov process for income given by (3.6) is a finite Markov chain. We also assume that an agent can buy or sell each asset in discrete units only, where the unit is a small fraction of average income and

[7]The argument presumes that $\mathrm{E}_s\{U(c')\} \geq \mathrm{E}\{U(c')\}$; that is, marginal utility conditional on selling is higher than the unconditional marginal utility, based on the idea that sales occur when consumption is low.

[8]Note that π^s is the probability of next period selling the marginal unit of stock purchased this period, as opposed to all the stock purchased this period. Note also that π^s will vary across individuals as a function of the individual state vector $z^i = \{b^i, s^i, y^i\}$.

that there is an upper bound to the quantity of stocks and bonds that can be held.[9] These assumptions make the space of state vectors (s, b, y) for the agent's dynamic programming problem into a finite space. The agent's decision rules (3.8) together with (3.6) define a Markov chain on the finite space of vectors (s, b, y). The stationary cross section distribution $F(.)$ can be obtained from the stationary probability distribution corresponding to the above Markov chain.

The transaction velocities are then computed as follows

$$TVS = \tfrac{1}{2}\mathrm{E}\{|\sigma_s - s|\}/\bar{s},$$

$$TVB = \tfrac{1}{2}\mathrm{E}\{|\sigma_b - b|\}/\bar{b},$$

where σ_s and σ_b are given by (3.8) and where the expectation is taken with respect to F.

We next turn to parameterization. The values of some of the parameters depend on the period length. In what follows we report all parameter values as if the period is one year but in fact the values are adjusted in the appropriate fashion to reflect the period length chosen. We use a period length of one quarter. This seems to be a short enough time to allow for liquidity trading but long enough to permit some temporal aggregation in preferences.

We chose parameter values in the following fashion:

Preferences.

$$\beta = 0.96 \text{ (annual)}, \tag{4.1}$$

$$U(c) = -(c^{-1} - 1). \tag{4.2}$$

Income Process. We assume a three-state Markov chain where the states are denoted u, l, h (and ordered the same way) and stand for unemployment, low employment, and high employment, respectively. The low and high employment states are treated symmetrically so that the probability transition matrix is of the form:

$$\pi^y = \begin{array}{c} \\ (u) \\ (l) \\ (h) \end{array} \begin{array}{c} \begin{array}{ccc} (u) & (l) & (h) \end{array} \\ \begin{bmatrix} \pi_u & (1-\pi_u)/2 & (1-\pi_u)/2 \\ 1-\pi_e & \pi_e/2 & \pi_e/2 \\ 1-\pi_e & \pi_e/2 & \pi_e/2 \end{bmatrix} \end{array}. \tag{4.3}$$

[9] We assume that a unit equals 10 percent of quarterly income. Also, the upper bounds are adjusted to ensure that they do not bind in equilibrium.

The numbers π_u and π_e are determined from the following considerations. Let θ_u be the fraction of people in the unemployment state in a stationary equilibrium and let D_u be the duration of unemployment. It is easy to calculate that these are given by

$$\theta_u = (1-\pi_e)/[(1-\pi_e)+(1-\pi_u)], \tag{4.4a}$$

$$D_u = 1/(1-\pi_u). \tag{4.4b}$$

We assume the following values for θ_u and D_u, chosen to roughly match the actual numbers, and use these in (4.4) to solve for the π's.

$$\theta_u = 0.05, \tag{4.5a}$$

$$D_u = 1.5 \text{ quarters}. \tag{4.5b}$$

These restrictions imply the following income probability transition matrix:

$$\pi^y = \begin{pmatrix} 0.34 & 0.33 & 0.33 \\ 0.035 & 0.4825 & 0.4825 \\ 0.035 & 0.4825 & 0.4825 \end{pmatrix}. \tag{4.6}$$

The (quarterly) income levels corresponding to the three states are chosen as follows. Let θ_e be the fraction of people in employment state l, also equal to the fraction of people in employment state h. (Thus, $\theta_e = (1-\theta_u)/2$.) Let $\bar{y}$ be the average income and y_e be the average income conditional on being employed. We normalize $\bar{y}$ to unity. We assume that income while employed can fluctuate up or down (relative to y_e) by 30 percent. In addition we assume that income in the unemployed state is 30 percent of average income while employed. Thus, we have

$$\bar{y} = \theta_u y_u + \theta_e(y_l + y_h) = 1, \tag{4.7a}$$

$$y_u = 0.3y_e, \quad y_l = 0.7y_e, \quad y_h = 1.3y_e. \tag{4.7b}$$

The above equations can be solved to obtain incomes in each state. The solutions follow:

$$y_u = 0.3100, \quad y_1 = 0.7254, \quad y_h = 1.3470. \tag{4.8}$$

Our choices for the representation of the income process are based on the following considerations. We follow Diaz-Gimenez and Prescott (1989) by assuming that income while unemployed is equal to one third of mean income while employed, based on the fact that the ratio of the average manufacturing wage to the minimum wage is about three to one. (The argument presumes that the unemployed always have the option of working at minimum wage jobs.) In addition, we have chosen to divide the employment state into two employment states to allow for variation in income while employed.

Our income process implies a standard deviation of earnings relative to trend of slightly more than 30 percent for quarterly income and slightly more than 15 percent for annual income. The latter falls within the ballpark of estimates for the variation of annual earnings. (We have been unable to locate measures of the variation in quarterly earnings.) Kydland (1984) calculates that the standard deviation of annual hours worked for employed prime-age males from the Panel Study on Income Dynamics (PSID) is about 15 percent. Since wages are mildly procyclical, variations in income while employed would be at least that much. Abowd and Card (1987), using data from the PSID as well as the National Longitudinal Survey (NLS) of men 45–49, report that the standard deviations of percent changes in real earnings and annual hours are about 40 percent and 35 percent, respectively. If deviations of real earnings from trend are serially uncorrelated, then the above figures suggest that the standard deviation of real earnings relative to trend for employed prime-age males is about 28 percent. However, deviations of real earnings from trend are likely to be positively serially correlated which would result in an even larger figure for the standard deviation of real earnings relative to trend. If the serial correlation coefficient exceeds one half, then real earnings relative to trend will be even more variable than real earnings growth.[10]

We feel, therefore, that our income process matches up to conservative estimates of the variation in annual earnings. Unfortunately we could not find analogous numbers to match up the quarterly variation. We chose to make the quarterly percentage variation about twice the annual percentage variation by postulating that quarterly fluctuations of income about trend while employed are i.i.d. This assumption may be a reasonable way to approximate the quarterly idiosyncratic risk faced by individuals, since this risk includes factors in addition to income variation from which we have abstracted. These other factors consist primarily of taste shocks and uninsured components of accidents. It is worth noting that we can easily modify our model to incorporate taste shocks. Under this reformulation, the idiosyncratic risk is interpretable as arising from income as well as preference shocks. For example, the utility function in our model can be respecified as

[10]Let W be real earnings, $\sigma(w)$ be the standard deviation of real earnings relative to trend and $\sigma(g_w)$ be the standard deviation of real earnings growth. Suppose W_t can be represented as $(\text{Trend})_t(1+\varepsilon_t)$ where $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$, and u_t is i.i.d. with mean zero and standard deviation σ_u. Then it is easy to calculate that $\sigma(w) = \sigma_u/\sqrt{(1-\rho^2)}$ and $\sigma(g_w) = \sigma_u\sqrt{2/(1+\rho)}$. If ρ exceeds 1/2, then $\sigma(w)$ will exceed $\sigma(g_w)$.

$U(c^* + \varepsilon)$ where ε represents taste shocks and c^* is consumption. As can be seen from the budget constraint (3.3), this formulation is equivalent to one where consumption is taken to be $c = c^* + \varepsilon$ and income is taken to be $y + \varepsilon$. Thus, fluctuations in effective income are partly due to taste shocks and hence larger.

Transactions Costs. Based on the discussion in section 2 we experiment with several different values for the transactions costs parameters. We set the buying and selling costs the same and we denote the common value by α,

$$\alpha_b = \alpha_s = \alpha \in \{0.02, 0.035, 0.05\}. \tag{4.9}$$

Asset Returns and Asset / Income Ratios. We pick the following values for asset returns and taxes:

$$r = 0, \tag{4.10a}$$

$$r_s = d/p = 0.03 \text{ (annual)}, \tag{4.10b}$$

$$\tau = 0. \tag{4.10c}$$

Following Labadie (1989, p. 289), we calculate the average annual real return on 90-day government Treasury bills from 1949 to 1978 to be about zero. Her figure for the average annual real return on the S&P 500 over the same period is 7.7 percent (standard deviation = 7.03 percent). Since we certainly do not wish to claim that transactions costs are the sole explanation for the observed return differential, we set ourselves the more modest goal of explaining a 3 percent return differential.[11] This explains our choice of r_s in (5.8b). Finally, we set taxes at zero. This allows for some simplification, since at $r = 0$, the implied value of g is also zero, regardless of $\bar{b}$.

Finally, as suggested earlier, an important consideration for judging the model is ascertaining how well the computed asset/income ratios match up with observed values. We use the following numbers as benchmarks for the latter:

$$\bar{s}/y = 0.65, \tag{4.11a}$$

$$\bar{b}/y = 0.35. \tag{4.11b}$$

[11] Labadie (1989) argues that by using a continuous state space generalization of the Mehra and Prescott (1985) model, it is possible to obtain an equity premium close to 3 percent (though the risk-free real rate that she obtains is over $3\frac{1}{2}$ percent). While we do not mean to imply that one can simply add what we get to her premium, the results are suggestive that factoring in transactions costs can close the gap. Further, Weil (1990), in a two-period setting, defines circumstances under which aggregate and individual risk interact to magnify the equity premium.

The first number is the average of the ratio of household ownership of tradable equity (including both direct holdings and mutual funds) to national income over the period 1964–80.[12] It is probably worth adding that this number varied considerably over the time period, ranging from about 0.4 to 1.1. Also the ratio tended to decline steadily over the period. The second number is a rough measure of the ratio of household liquid assets to national income over the same period. We included in the numerator household holdings of liquid securities which bore approximately the same return as T-bills: specifically, the sum of savings accounts at depository institutions, time deposits with a maturity of a year or less, money market accounts, and direct holdings of marketable government securities. (Recall that a working hypothesis of our model is that T-bills which households do not hold are held by intermediaries which in turn issue liquid claims to households.) The liquid asset/income ratio did vary over the period, but not nearly as much as the stock/income ratio.

5. Results

In this section we describe the results of computations based on our model. As discussed earlier, we consider a period length of one quarter. However, the numbers we report are converted to annual values (when relevant).

Example 1.

		$r_s = 0.03$		
	$\alpha =$	0.02	0.035	0.05
solutions:	$\bar{s}/y =$	0.69	0.65	0.60
	$\bar{b}/y =$	0.07	0.10	0.12
	$TVS =$	0.08	0.07	0.06
	$TVB =$	1.44	1.15	0.99

Several features of the example are of interest. First, while the ratio of stocks to income matches up reasonably well, the ratio of liquid assets to income is way too low. It appears to be off by a factor of between 3 and 5, depending on the transactions cost. The relative transaction velocities, however, seem reasonable. This particular example leads to liquid assets circulating about 16 times more rapidly than stocks. It is true that the absolute transaction velocities are too low. However, this is probably in large part due

[12] Think of pension fund holdings as entering the nontradable component of individual income.

to the fact that the period length is so long. Clearly, if a year is divided into N periods, then the transaction velocity can never exceed N/year. Also we have abstracted from other reasons for trading securities besides liquidity trading.

In this example we considered a 3 percent spread between stocks and bonds. The next example considers the sensitivity of the results to small adjustments in the spread: first down to 2.6 percent, then up to 3.4 percent.

Example 2. Except for the return on stocks, the rest of the parameters are the same as in example 1. The results are as follows:

$r_s = 2.6$

	$\alpha =$	0.02	0.035	0.05
solutions:	$\bar{s}/y =$	0.60	0.54	0.49
	$\bar{b}/y =$	0.07	0.11	0.14
	$TVS =$	0.09	0.07	0.06
	$TVB =$	1.40	1.08	0.91

and

$r_s = 3.4$

	$\alpha =$	0.02	0.035	0.05
solutions:	$\bar{s}/y =$	0.83	0.79	0.75
	$\bar{b}/y =$	0.04	0.08	0.11
	$TVS =$	0.10	0.07	0.05
	$TVB =$	2.00	1.37	1.08

Even at the 2.6 percent spread the average quantity of liquid assets is too low.

In the next example we allow for costly borrowing.

Example 3 (costly borrowing): We assume that the parameters are the same as in example 1, except that we also allow for negative values in the grid for bonds so that individuals are permitted to borrow. However, there is a transactions cost associated with borrowing (but not with lending) which is a fixed percentage of the amount borrowed. This percentage borrowing cost is chosen to be 0.02. This number implies an annual spread between the consumer loan rate and the risk-free rate of 8 percent, which is reasonable given historical data on consumer loan rates (the historical difference between the credit card rate and the risk-free rate is larger than 8 percent).[13] In addition we impose a credit limit on consumer loans equal to 40 percent of quarterly income, so that the lower support of the grid on bonds now extends to -0.4. The results follow.

$r_s = 0.03$

$\alpha =$	0.02	0.035	0.05
$\bar{s}/y =$	0.61	0.61	0.56
$\bar{b}/y =$	0.05	0.04	0.07
$TVS =$	0.08	0.06	0.05
$TVB =$	*	*	*
$LA/y =$	0.06	0.06	0.09
$TVLA =$	1.48	1.32	1.10

In the above table, *LA* and *TVLA* stand for the quantity of liquid assets and the transaction velocity of liquid assets, respectively. Note that the supply of liquid assets now consists of the sum of government bonds and consumer loans, i.e., nonnegative holdings of private bonds. (As mentioned earlier, think of private intermediaries as holding these securities as assets and issuing liquid liabilities to consumers.) It is interesting to note that the stock/income ratio now becomes less sensitive to the transactions cost. This occurs since individuals have borrowing as an alternative to smoothing consumption, making the need for a distress sale of stocks less likely. As with the other examples, however, the ratio of liquid assets to income is too low.[14] It is worth noting that the possibility of borrowing to smooth consumption induces individuals to hold fewer liquid deposits at intermediaries. Thus, allowing for borrowing only tends to reduce the ratio of liquid assets to income.

In the next example we consider the impact of fixed costs.

Example 4 (fixed cost): We assume that the parameters are the same as in example 3 (which includes costly borrowing) except that we allow for a fixed

[13]The costs of borrowing are somewhat lower for individuals who own large amounts of stock and can pledge the stock as collateral. For example, the spread between the loan rate and the risk-free rate is about $5\frac{1}{2}$ percent for a collateralized loan under ten thousand dollars and declines to about $2\frac{1}{2}$ percent for a collateralized loan over one hundred thousand dollars. There are also minimum income and margin requirements which add to the effective costs. Thus, except on very large loans, it seems that even wealthy stockholders face a nontrivial gap between borrowing and lending rates.

[14]See Huggett (1989) for a related analysis with only inside lending and borrowing, and where borrowing is costless (i.e., individuals can borrow at the riskless rate). With costless borrowing and a large credit limit, the risk-free rate gets close to the rate of time preference, which suggests that some kind of frictions in borrowing may be needed to explain a low riskless rate. See also Mehrling (1989).

cost of transacting in stocks in addition to the constant marginal cost represented by α. Fig. 1 clearly implies that fixed costs are relevant. The fixed cost is assumed to be 1 percent of average quarterly income which is consistent with the schedule depicted in fig. 1.

	$r_s = 0.03$		
$\alpha =$	0.02	0.035	0.05
$\bar{s}/y =$	0.58	0.53	0.49
$\bar{b}/y =$	0.07	0.09	0.12
$TVS =$	0.07	0.05	0.05
$TVB =$	*	*	*
$LA/y =$	0.09	0.11	0.13
$TVLA =$	1.42	1.16	0.99

In comparison with example 3, we find that the fixed cost increases the ratio of liquid assets to income and reduces the ratio of stocks to income and the transaction velocities of stocks as well as liquid assets. The relative transaction velocity of liquid assets to stocks is not much affected. More importantly, the ratio of liquid assets to income still falls too short of the target.

6. Conclusion

Our goal was to explore whether allowing for an explicit demand for liquidity could help resolve the risk-free rate and equity premium puzzles. We motivated a household demand for liquid assets by introducing uninsured individual risks in conjunction with costs of trading equity. While the simulated model did well on some grounds – explaining the relative transaction velocities of stocks and liquid assets and the ratio of stocks to income – it predicted too low a ratio of liquid assets to income. In our view the asset return puzzles should be thought of in this way: why is it that household demand for low yielding liquid assets has been historically so high?

Closer inspection of the data indicates that a substantial fraction of liquid assets are held by a group of households who own relatively little stock and, relatedly, that the ownership of stock is heavily concentrated. For example, Avery, Elliehausen, and Kennickell (1988) estimate that in 1963 the bottom 90 percent of the wealth distribution held 53 percent of the total quantity of liquid assets, but only 9 percent of the equity. Conversely, the top 1 percent held over 60 percent of the equity, but only 10 percent of liquid assets. These figures suggest that one possible way of adjusting our model to resolve the 'liquid assets' puzzle is to allow for additional heterogeneity, in the form of stockholders versus nonstockholders.[15] Our general approach suggests one possible reason for this segmentation: Since average costs of trading stock decline with size and since borrowing costs vary inversely with wealth, stocks may be viewed as having relatively greater liquidity for wealthier individuals. It is conceivable that allowing for differential costs of trading securities and differential borrowing costs across wealth groups could generate a good fraction of the observed heterogeneity in individual stockholding.

Another possible factor is that the only motive for holding liquid assets in our framework involves precautionary considerations. We ignore transactions motives. Certainly a component of household holdings of savings and money market accounts stems from transactions needs. Subject to computational considerations and some of the usual issues in introducing money, one could modify our framework to allow for transactions demands. (We would also need to introduce a small cost of trading liquid assets other than money.)

There are some other extensions of our analysis which would be desirable. On the theoretical side, our model does not endogenize the absence of insurance markets, limited nature of financial markets, limitations on borrowing, and short selling or, for that matter, government policy. Endogenizing limitations on insurance and borrowing along the lines of Phelan and Townsend (1989) is one possibility. It seems more difficult to endogenize costs of trading equity. Finally, we would like to allow for aggregate dividend risk, but this task appears to be quite formidable.

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[15] Mankiw and Zeldes (1989) also emphasize the importance of distinguishing between individuals who regularly hold stock and those not inclined to do so, though for somewhat different reasons.

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