Online Privacy and Price Discrimination

Liad Wagman
Illinois Institute of Technology

Vincent Conitzer
Duke University

Curtis Taylor
Duke University

July, 2010
ERID Working Paper Number 79

Economic Research Initiatives at Duke
WORKING PAPERS SERIES

Eruditio et Religio

Online Privacy and Price Discrimination*

Vincent Conitzer
Duke University

Curtis Taylor
Duke University

Liad Wagman
Illinois Inst. of Tech.

July 2010

Abstract

When a firm is able to recognize its previous customers, it may use information about their purchase histories to price discriminate. We analyze a model with a monopolist and a continuum of heterogeneous consumers, where consumers are able to maintain their anonymity and avoid being identified as past customers, possibly at an (exogenous) cost. When consumers can costlessly maintain their anonymity, they all individually choose to do so, which paradoxically results in the highest profit for the firm. Increasing the cost of anonymity can benefit consumers, but only up to a point, after which the effect is reversed.

Keywords: Privacy, anonymity, price discrimination, electronic commerce

JEL Classifications: L1, D8

1 Introduction

Perhaps the most important factor contributing to concerns about personal privacy is the potential for discrimination. In an effort to avoid differential treatment, individuals are typically reluctant to disclose sensitive personal information such as income, family status, ethnicity, race, or lifestyle. In recent years, revolutionary developments in information technology regarding collection, storage, and retrieval of personal data have brought privacy to the forefront of public awareness and debate. This paper addresses a key component of the emergent

*We have benefited from helpful discussions with session participants at the 2009 Summer Meeting of the Econometric Society, the 2009 FTC-Northwestern Microeconomics Conference, the 2009 NET Conference, the 2009 International Industrial Organization Conference, and workshops at the Duke University Social Science Research Institute and Duke University Economics Department. We especially thank Atila Abdulkadiroglu, Giuseppe Lopomo, Huseyin Yildirim, Rachel Kranton, Hanming Fang, David McAdams, Sasa Pekec, Tracy Lewis, Daniel Graham, Vasiliki Skreta, and Eric Rasmusen for their useful feedback. Conitzer's research is supported by an Alfred P. Sloan Fellowship and by NSF under award number IIS-0812113 and CAREER 0953756. Taylor's research is supported by a NET Institute grant. Wagman's research is supported by an IIT Summer Research Fellowship.

concerns regarding electronic privacy, namely, the ability of firms to track individual purchasing patterns and to use this information to practice behavior-based price discrimination (Armstrong, 2006; Fudenberg and Villas-Boas, 2006).

Records containing the sequence of web sites visited and the online purchases made by individuals provide valuable clues about their personal information, clues that can be used to target tailor-made offers to them (Chen, 2006; Wathieu, 2006; Pancras and Sudhir, 2007; Chen and Zhang, 2008). Such behavior-based advertising and price discrimination are already ubiquitous in electronic commerce (Odlyzko, 2003; Hann et al., 2007). Nevertheless, the economic impact of these practices is not fully understood. Presently, privacy practices in electronic commerce are dictated largely by voluntary compliance with industry standards, recommendations by regulatory agencies, and consumer concerns (FTC, 2007).

Although technology has allowed sellers to store and process consumers' online activities with relative ease, consumers do have some control over allowing sellers to record their individual activities. For instance, they can exert effort to understand sellers' privacy disclosures and take actions to circumvent being tracked. Such actions can include erasing or blocking browser cookies, using a temporary email address, paying with a different credit card, making payments using a gift card acquired for cash in a brick-and-mortar store, and renting a postal box. Examples of sellers discriminating based on past purchases include Comcast and Time Warner offering discounted packages that can be obtained only once per name and address, AOL offering special "new customer" accounts that can be opened only by revealing credit card numbers that have not been applied before to a similar offer, and credit report services offering once a year promotions tied to a consumer's social security number. Perhaps the most notorious example is the price discrimination fiasco of 2000 in which Amazon charged past customers higher prices for DVDs that their purchase histories suggested they would be likely to want.[1] The key to these examples is that sellers have difficulty committing to future prices *and* committing not to use information about past purchases.

The current set of guiding standards and recommendations (FTC, 2007) takes into account a large variety of concerns, but appears to have little basis in formal economic theory or empirical evidence. This paper provides a theoretical analysis of the economic impacts of privacy regulation, focusing specifically on consumer profiling and behavior-based price discrimination. We study a monopolist who is able to track (strategic) consumers' purchases from the firm. Consumers, however, are able to avoid being identified as past customers (or to "opt

[1] See, for instance, `http://www.cnn.com/2005/LAW/06/24/ramasastry.website.prices/`. It is worth noting that Amazon still price discriminates by, for example, offering past customers targeted coupons. However, to our knowledge, it no longer shows (in a direct manner) different prices to different consumers based on their past purchases.

out"), possibly at a cost. We note that in our framework, a firm is likely to charge past customers more than "new customers" because their past purchases signal a higher willingness to pay (for tractability purposes we abstract from settings where the seller may want to give discounts to past customers, for instance, due to diminishing marginal utility). We find that when consumers can costlessly opt out (and by doing so, maintain their anonymity), they all individually choose to do so, which results in the highest profit for the seller. We show that increasing the cost of obtaining anonymity can benefit consumers, but only up to a point; at that point, the effect is reversed.

The intuition for this paradoxical finding is closely related to the celebrated Coase Conjecture (Coase, 1972) and runs as follows. When the cost of maintaining anonymity is high, the seller is better able to recognize past customers and to price discriminate against them. Thus, consumers hesitate to make an initial purchase, knowing this will cause them to pay a premium on future purchases. Anticipating this reluctance by consumers, the seller is forced to offer a lower initial price, and this effect actually dominates the increase in profits arising from price discrimination in future periods. In other words, the seller would prefer to commit itself not to price discriminate based on prior purchases. When the cost of maintaining anonymity is low, consumers – in effect – give the seller this commitment power when they each rationally choose to keep their purchases private.

This paper is also related to work in the literatures on intertemporal price discrimination, consumer recognition, and online privacy. Research on intertemporal price discrimination and the "ratchet" effect, where the firm sets higher prices for consumers who signaled higher willingness to pay, dates back to the late 1970's. Stokey (1979) and Salant (1989) show that intertemporal price discrimination is never optimal for a monopolist who can commit to future prices. This is analogous to the fact, mentioned above, that in our model, the monopolist obtains its highest profit when anonymity is costless. Villas-Boas (2004) shows that committing to future prices can also help in a model with overlapping generations of consumers.

A relatively small literature on consumer recognition and online privacy has begun to develop over the past several years.[2] Early contributions by Chen (1997), Fudenberg and Tirole (1998), Fudenberg and Tirole (2000), Villas-Boas (1999), Shaffer and Zhang (2000), Taylor (2003), Chen and Zhang (2008), and Chen and Zhang (2009) introduced the notion of consumer recognition and personalized pricing into economic theory, but did not explicitly consider privacy issues in online environments. Fudenberg and Tirole (1998) explore what happens when the ability to identify consumers varies across goods. They consider a model in which con-

[2]For a general discussion of price discrimination, see Stole (2007). For an economic analysis of privacy with respect to lawful search and seizure, see Mialon and Mialon (2008).

sumers can be anonymous or "semi-anonymous," depending on the good bought. Villas-Boas (1999) and Fudenberg and Tirole (2000) analyze a duopoly model in which consumers have a choice between remaining loyal to a firm and defecting to the competitor, a phenomenon they refer to as "consumer poaching." Chen and Zhang (2008) analyze a "price for information" strategy, where firms price less aggressively in order to learn more about their customers. Chen and Zhang (2009) find that price competition can be mitigated by firms vying to distinguish their loyal customers from price sensitive shoppers.

Closest to our work is an emerging literature on optimal online privacy policies. These were first studied by Taylor (2004), Acquisti and Varian (2005), and Calzolari and Pavan (2006). Fudenberg and Villas-Boas (2006) and Esteves (2010) offer surveys of this literature. Taylor (2004) and Villas-Boas (2004) show how strategic consumers could make a firm worse off in the context of dynamic targeted pricing. Once consumers anticipate future prices, they may choose to forgo a purchase today to avoid being identified as a past customer and thus be able to purchase at a lower price targeted at new consumers. This strategic "waiting" on the part of consumers can hurt a firm both through reducing sales and diminishing the benefit of price discrimination. Acquisti and Varian (2005) show that it is never profitable for a monopolist to condition its pricing on purchase history, unless a sufficient proportion of consumers are not sophisticated enough to anticipate the firm's pricing strategy or the firm can provide enhanced services to increase consumer valuation in subsequent purchases. Acquisti and Varian (2005) also begin to study consumers' use of anonymizing technologies (so as to circumvent identification by a firm as a past customer) but do not fully study the welfare implications. Calzolari and Pavan (2006) show that in a game with two sellers selling one after the other, where buyers may have correlated valuations for their products, information trade between the first seller and the second seller will occur under some conditions and the effects of such information trade on welfare vary.

These papers provide important insights regarding the fundamental economic tensions between consumer privacy and price discrimination. This paper considers a richer environment, in which a firm's strategic customers can choose to remain anonymous at some cost. We study how this cost affects equilibrium behavior and welfare. Our model allows us to gain additional insights into the effects of tightening privacy regulation. In particular, we show that (putting aside considerations about the intrinsic value of privacy and other non-price related privacy concerns) while the firm obtains its highest profit when consumers can costlessly maintain their anonymity, consumers can, under some circumstances, be better off when maintaining anonymity is costly, but only up to a point. Beyond that point, facilitating opting out (or increasing privacy) benefits consumers and increases overall welfare.

This result is in contrast to the prior literature because it actually agrees with the common intuition that more privacy can be better (even when consumers are strategic). Even more surprising is that this welfare behavior happens in a region of opting-out costs where no consumer chooses to opt out. In other words, we find that added privacy can benefit consumers and increase overall surplus, even when no consumer decides to take advantage of it.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 gives two benchmarks: when there is no customer recognition, and when there is no opting out. The equilibrium of the game with opting out is derived in Section 4 along with comparative statics, and Section 5 concludes. Proofs that are not in the main body of the paper are relegated to an appendix.

2 The model

2.1 The consumers

There is a continuum of consumers with total mass normalized to one. All consumers are risk-neutral, possess a common discount factor $\delta \in (0, 1]$, and maximize their present expected utilities. Each consumer demands at most one unit of a non-durable, indivisible good in each of two periods. Consumer i's valuation for the good is the same in each period and is determined by the realization of a random variable v_i with support normalized to be the unit interval. Consumer valuations are independently and identically distributed according to a cumulative distribution function $F(v)$ with density $f(v)$, which is strictly positive on $(0, 1)$. Consumer i's valuation v_i is initially private information.

2.2 The firm

There is a monopolist that produces and sells the good in each period. The firm's production cost is normalized to zero, it possesses the same discount factor δ as consumers, and it maximizes its discounted expected profit. It does not observe consumer valuations directly but maintains a database containing purchasing histories. Each consumer is either *anonymous* or *identifiable*. If a consumer is anonymous, then there is no record of any prior purchases by her; i.e., she is not in the database. If she is identifiable, then in the second period the firm knows the purchasing decision that she made in the first period. We emphasize that the firm has no commitment power, i.e., the firm is unable to set and commit to second-period prices in the first period. Because there is a continuum of consumers, each of them realizes that her first-period purchasing decision alone does not affect the prices charged by the firm in the next period.[3]

2.3 The game

All aspects of the environment, including the distribution of valuations $F(v)$, are common knowledge. At the beginning of the game all consumers are anonymous. Hence, the firm offers the same first-period price p_1 to all of them. Next, each consumer decides whether to buy the good in the first period, $q_1^i = 1$, or not to buy it, $q_1^i = 0$. Consumers who elect to buy the good also decide whether to let the firm keep a record of the transaction ($r^i = 1$) or to *opt out* and maintain anonymity by deleting the record of the sale ($r^i = 0$). The cost to any consumer who opts out is $c \geq 0$,[4] and we without loss of generality assume that this cost is expended in the second period (i.e., it is discounted by δ). This cost represents the time, effort, and any monetary expense of maintaining anonymity. We also allow consumers who purchase to randomize between opting out and not opting out. A consumer who does not purchase the good continues to be anonymous and is thus pooled with the buyers who opted out, from the firm's perspective. At the beginning of period two, the firm posts a price p_2^0 to the unidentified (anonymous) consumers and a price p_2^1 to the identified ones.[5] Consumers can buy the good only at the price offered to them, $q_2^i \in \{0, 1\}$; i.e., no arbitrage is possible. Hence, a consumer i with valuation v_i who purchases in both periods has (present discounted) utility $v_i - p_1 + \delta(v_i - p_2^1)$ if he does not opt out, and utility $v_i - p_1 + \delta(v_i - p_2^0 - c)$ if he does. Figure 1 summarizes the timeline of the game. We depict the extensive form of the game in the appendix.

The solution concept we use is Perfect Bayesian Nash Equilibrium (PBE). A PBE here consists of the firm's strategy (composed of first-period price p_1 and second-period prices p_2^0 and p_2^1, corresponding to the firm's two information sets in the second period[6]); the consumers' strategies (composed of first-period purchasing decisions q_1^i and opting-out decision $r^i \in$

[3] The results hold when there is a finite number of consumers, provided that we add the following assumption: the firm cannot update its beliefs over how many consumers opted out based on an inventory count. Without this assumption, the firm could infer how many consumers opted out based on the inventory count. With a continuum of (massless) consumers, inventory expectations on the equilibrium path are confirmed even if a single agent deviates.

[4] The qualitative nature of the results still holds under certain conditions when the cost of maintaining anonymity varies across consumers, or when it is correlated with their valuations. In order to simplify the analysis, we assume that consumers incur the same cost of maintaining their anonymity. See Taylor (2004) for a model with correlated valuations, and Acquisti and Varian (2005) for a model with varying levels of consumer sophistication.

[5] If the firm sets second-period prices *before* consumers decide on whether or not to opt out, then it can be shown that no consumer would opt out in equilibrium, for any $c > 0$, which is neither realistic nor interesting.

[6] Technically, the strategy should also specify what the firm would have charged in the second period if it had made a different pricing decision in the first period, but we omit this for notational simplicity.

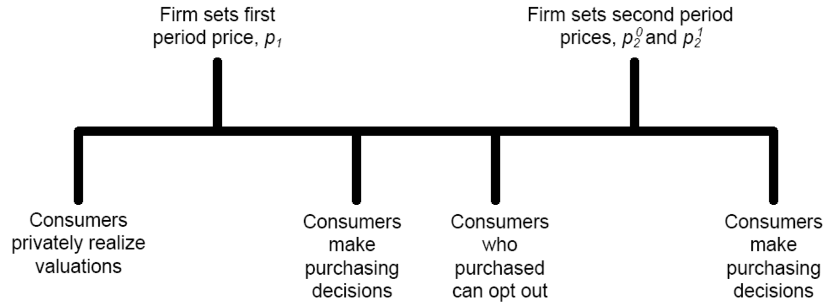


Figure 1: Timeline of the game.

$\{0, 1\}$ as a function of p_1 and v_i, and second-period purchasing decision q_2^i as a function of[7] v_i and p_2^r); and the firm's beliefs about consumers' valuations given their identification status (F^1 and F^0 for identified and anonymous consumers, respectively[8]). These constitute a PBE if all strategies are sequentially rational given the beliefs and the beliefs are consistent given the strategies.

We assume that $p(1 - F(p))$ is concave, i.e., the firm's marginal revenue in a single period game is decreasing in p (or $-2f(p) - pf'(p) \leq 0$); and that marginal revenue is concave (or $-3f'(p) - f''(p)p \leq 0$). We denote the firm's optimal price in a one-shot version of the game by $p^\star$, i.e., $p^\star = \arg\max_p p(1 - F(p))$.

3 Benchmarks

3.1 No recognition

First, consider as a benchmark the case where there is no consumer recognition, so that the firm cannot price discriminate in the second period between consumers that bought and did not buy in the first period. Since the firm does not price discriminate based on purchasing history, a consumer will buy the good in each period in which his valuation exceeds the price. Thus, the firm sets the same price in each period, $p^\star = \arg\max_p p(1-F(p))$, generating a per-period profit of $p^\star(1-F(p^\star))$. Consumer surplus in each period is given by $\int_{p^\star}^1 (v - p^\star)\, dF(v)$.

3.2 Full recognition

Consider now the opposite extreme in which the firm is able to recognize its previous customers and consumers are unable to maintain their anonymity at any cost (as in Hart and

[7]In principle, the second-period decision can also directly depend on the first-period price, but in equilibrium it will only depend on v_i and p_2^r.

[8]The firm will also have beliefs about what actions an anonymous agent took in the first period (did the agent purchase and opt out or not purchase at all), but this will not affect the analysis.

Tirole (1988), Schmidt (1993), Villas-Boas (2004), Taylor (2004), and Fudenberg and Villas-Boas (2006)). In this setting, the firm can discriminate between two different groups of consumers in the second period: identified consumers who purchased in the first period, and unidentified consumers who did not. The firm consequently sets two different prices in the second period, p_2^1 to identified consumers and p_2^0 to unidentified consumers. (We emphasize again that the firm has no commitment power.)

Proposition 1 (Fudenberg & Villas-Boas 2006)**.** *In the full-recognition equilibrium, for some $\tilde{v}$,*
(i) Consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods; consumers with valuations $v \in [p_2^0, \tilde{v}]$ purchase only in the second period. The cutoff type $\tilde{v}$ satisfies $\tilde{v} \geq p^\star$.
(ii) The firm sets $p_2^1 = \tilde{v}$, while $p_2^0 = \arg\max_p p(F(\tilde{v}) - F(p))$ is set to satisfy $F(p_2^0) + f(p_2^0)p_2^0 = F(\tilde{v})$. The cutoff type $\tilde{v}$ and the first-period price p_1 are determined from $\tilde{v} = (1 + \delta \frac{\partial p_2^0}{\partial \tilde{v}}) \frac{1-F(\tilde{v})}{f(\tilde{v})}$ and $p_1 = \tilde{v}(1-\delta) + \delta p_2^0$, respectively.

Let us consider the monopolist's pricing strategy towards identified consumers in the second period. If the cutoff type for identified consumers (those who purchase in the first period) $\tilde{v}$ satisfies $\tilde{v} \geq p^\star$, then the monopolist sets $p_2^1 = \tilde{v}$. If, on the other hand, $\tilde{v} < p^\star$, the monopolist sets $p_2^1 = p^\star$. That is, $p_2^1 = \max\{\tilde{v}, p^\star\}$. From Proposition 1, since $\tilde{v} \geq p^\star$ holds on the path of play of the full-recognition equilibrium, $p_2^1 = \max\{\tilde{v}, p^\star\} = \tilde{v}$. Hence, the marginal consumer who buys in the first period—the one with valuation $\tilde{v}$—gets no surplus in the second period. This is the *ratchet effect* of consumers who reveal their types (Freixas et al., 1985; Laffont and Tirole, 1988). The proof of Proposition 1 is in the appendix.

Paradoxically, the full-recognition case can result in higher consumer surplus than the no-recognition case, because the firm will need to set p_1 lower to attract consumers in the first period. Correspondingly, in the model with opting out, we will show that a low cost of opting out can lead to a Prisoner's Dilemma situation where consumers maintain anonymity but collectively suffer as a result, in comparison to the situation where anonymity is prohibitively costly and consumers face the ratchet effect.

By definition, a consumer with valuation $\tilde{v}$ is indifferent between purchasing in both periods and purchasing only in the second period. It follows that the indifference condition that characterizes $\tilde{v}$ is given by $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. Hence, $p_1 = \tilde{v} - \delta(\tilde{v} - p_2^0)$. Using $p_1 = \tilde{v} - \delta(\tilde{v} - p_2^0)$ and $p_2^1 = \tilde{v}$, one can simplify the firm's present discounted profit to obtain

$$\tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$$

For $\delta > 0$, since $p^\star$ uniquely maximizes $p(1 - F(p))$ and $\tilde{v} \geq p^\star$, we have

$$(1 + \delta)p^\star(1 - F(p^\star)) \geq \tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$$

We thus have the following result.

Lemma 1. *The firm's profit under full recognition is lower than its profit under no recognition.*

The intuition is that some consumers refrain from purchasing in the first period because they anticipate a lower price in the next as a result, and the firm is unable to fully recoup the loss in first-period profit by price discriminating in the second period. Hart and Tirole (1988) and Fudenberg and Villas-Boas (2006) show that if the firm is able to commit to second-period prices in the first period, it would set $p_2^1 = p_2^0 = p^\star$, a result which our Proposition 6 below extends to the general model where consumers can opt out. Hence, the firm's profit under commitment coincides with its profit in the no-recognition equilibrium.

4 Opting out and partial recognition

We now consider the setting in which consumers who purchase in the first period can opt out and preserve anonymity at a cost of c. Consumers who purchase in the first period and do not opt out are identified by the firm in the second period (the firm recognizes that they purchased at a price p_1 and did not opt out) and will be offered price p_2^1 in period 2. All other consumers are offered p_2^0 in period 2. As above, let $\tilde{v}$ denote the lowest consumer type to purchase in the first period. We note that, given that a consumer with valuation $\tilde{v}$ prefers to buy in the first period, i.e., $\tilde{v} - p_1 + \delta \max\{\tilde{v} - p_2^1, \tilde{v} - p_2^0 - c, 0\} \geq \delta \max\{\tilde{v} - p_2^0, 0\}$, then all consumers with valuations $v \geq \tilde{v}$ do as well. Denote by $\alpha(v)$ the probability that a consumer of type $v \in [\tilde{v}, 1]$ opts out after purchasing. Then the distribution of valuations among anonymous consumers is

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v}) + \int_{\tilde{v}}^1 \alpha(x) f(x)\,\mathrm{d}x} & \text{if } v \leq \tilde{v} \\ \\ \frac{F(\tilde{v}) + \int_{\tilde{v}}^v \alpha(x) f(x)\,\mathrm{d}x}{F(\tilde{v}) + \int_{\tilde{v}}^1 \alpha(x) f(x)\,\mathrm{d}x} & \text{if } v > \tilde{v} \end{cases}$$

and the distribution of valuations among identifiable consumers (for $v \geq \tilde{v}$) is given by

$$F^1(v) = \frac{\int_{\tilde{v}}^v (1 - \alpha(x)) f(x)\,\mathrm{d}x}{\int_{\tilde{v}}^1 (1 - \alpha(x)) f(x)\,\mathrm{d}x}$$

where $F^1(v) = 0$ if $v < \tilde{v}$.

4.1 Costless anonymity: equilibrium characterization

As a starting point, we first consider the case where $c = 0$. Here, we show that the equilibrium is effectively unique and corresponds to the no-recognition benchmark.

Proposition 2. *When anonymity is costless ($c = 0$), every*[9] *PBE satisfies (and a PBE exists that satisfies):*
(i) The firm sets $p_1 = p_2^0 = p^\star$ and $p_2^1 \geq p^\star$.
(ii) Consumers with valuations $v \in [p^\star, 1]$ purchase in both periods and opt out.
(iii) The no-recognition benchmark outcome is obtained.

Proof: For any p_1, since it is costless to opt out, all consumers with valuations $v \geq p_1$ will purchase the good in the first period. If $p_2^1 < p_2^0$, no purchasing consumers will opt out. In this case, however, since the firm sets period 2 prices after consumers decide whether or not to maintain their anonymity, p_2^1 targets identified consumers in $[\underline{v}, 1]$, where $\underline{v} \leq p_1$ (some range of additional consumers $[\underline{v}, p_1]$ will decide to purchase in the first period in order to obtain the discount in the second period). On the other hand, p_2^0 targets the anonymous consumers in $[0, \underline{v}]$ (and there will be at least some consumers in this interval, because f is positive on $(0, 1)$). Hence, setting $p_2^1 < p_2^0$ cannot be a best response for the firm. Thus, in every equilibrium, $p_2^1 \geq p_2^0$.

Since consumers anticipate that $p_2^1 \geq p_2^0$, it is a best response for consumers who purchased in the first period to opt out. We now show that there is a PBE where all of them use this best response; moreover, we characterize all the PBEs in which this is the case. If all of them opt out, then all consumers are anonymous in the second period, and the firm sets $p_2^0 = p^\star$ to maximize period 2 profit. Moreover, given that in the second period everyone will be anonymous, only consumers with valuations $v \in [p_1, 1]$ purchase in the first period, so that $\tilde{v} = p_1$. Hence, the firm's first-period problem is to choose p_1 to maximize $(1 - F(p_1))p_1 + \delta(1 - F(p^\star))p^\star$, which results in $p_1 = p^\star$. Since no consumer is identified in period 2, the firm's beliefs about identified consumers' valuations are off equilibrium. Consistent off-equilibrium beliefs here are, for example, for the firm to believe identified consumers' valuations to be at least $\tilde{v} = p_1 = p^\star$, so that setting $p_2^1 \geq p^\star$ is a best response.

Now, suppose, for the sake of contradiction, that there are equilibria in which some consumers do *not* opt out on the path of play. For this to be the case in equilibrium, since $p_2^1 \geq p_2^0$, we must have $p_2^1 = p_2^0$, otherwise no consumer would choose to stay identified. Let $p_2^1 = p_2^0 = \tilde{p}$. First, we will show $\tilde{p} = p^\star$. For the sake of contradiction, suppose not, that is

[9] This is excluding the possibility of a PBE in which a measure zero subset of the consumers uses a different strategy.

$\tilde{p} \neq p^\star$. Then in period 2, only consumers with valuations in $[\tilde{p}, 1]$ purchase, and the firm's period 2 profit is given by $\tilde{p}(1 - F(\tilde{p}))$. However, if the firm sets $p_2^1 = p_2^0 = p^\star$ in the second period, consumers with valuations in $[p^\star, 1]$ would buy, resulting in period 2 profit of $p^\star(1 - F(p^\star))$ — which is strictly higher since $p^\star$ uniquely maximizes $p(1 - F(p))$. Hence, $p_2^1 = p_2^0 = p^\star$ holds in this case, and the firm's period 2 profit is given by $p^\star(1 - F(p^\star))$ for any p_1. Thus, it is profit maximizing for the firm to set $p_1 = p_2^0 = p_2^1 = p^\star$.

Assume now that some positive mass of consumers who purchased in period 1 stays identified. Let G denote the (un-normalized) distribution of anonymous consumers in period 2, so that $\int_0^1 \mathrm{d}G(p) < 1$. $G(p)$ coincides with $F(p)$ up to $p = p^\star$ because identified consumers can only be in $[p_1, 1] = [p^\star, 1]$. Thus, $F(p^\star) = G(p^\star)$. Let $g^-(p^\star) = \lim_{p \to p^\star} dG(p)/dp$. $g^-(p^\star)$ exists and satisfies $g^-(p^\star) = f(p^\star)$ because F is twice differentiable.[10] In period 2, the firm sets prices optimally to each group of consumers, and from our above observations, this has to result in $p_2^0 = p^\star$ in a PBE. Since $p(1 - F(p))$ is concave, the first-order condition that gives $p^\star$ is $1 - F(p^\star) - p^\star f(p^\star) = 0$. However, the first-order condition of the firm's problem in pricing towards anonymous consumers, evaluated at $p_2^0 = p^\star$, gives $\int_0^1 \mathrm{d}G(p) - G(p^\star) - p^\star g^-(p^\star) = \int_0^1 \mathrm{d}G(p) - F(p^\star) - p^\star f(p^\star) < 0$ since $\int_0^1 \mathrm{d}G(p) < 1$. Hence, for some $\epsilon > 0$, the seller is better off setting $p_2^0 = p^\star - \epsilon$ than setting $p_2^0 = p^\star$, resulting in the desired contradiction. ■

This result says that if the cost of maintaining anonymity is nil, then it is in the best interest of every individual who purchases the good in the first period to maintain her anonymity, effectively resulting in the no-recognition outcome from Subsection 3.1. However, as indicated in Subsection 3.2, this turns out to be exactly what the firm wants.

From the perspective of consumers, in Subsection 4.4, we show that this outcome is a Prisoner's Dilemma situation: individually, each consumer chooses to maintain her anonymity; as a result, however, consumer surplus ends up being lower due to there being no price discrimination. In fact, relative to the case where anonymity is costly, every consumer ends up being (weakly) worse off overall when there is no cost associated with opting out. In other words, by opting out, consumers impose a negative externality on other consumers. Below we study how firm profit and consumer surplus are affected by the cost c of maintaining anonymity.

4.2 Costly anonymity: equilibrium characterization

We now move on to the general case in which there is some cost $c \geq 0$. We will restrict our attention to PBEs in which the following holds: all consumers who purchase the good

[10] We note that if f is discontinuous, then PBEs in which some consumers stay identified do exist.

in the first period opt out with the same probability α (alternatively, α can be thought of as the proportion of consumers who opt out). This restriction is motivated by the fact that all consumers who purchased in the first period face the same tradeoff when deciding to opt out: either pay p_2^1, or pay $p_2^0 + c$. (In equilibrium, all first-period buyers will buy again in the second period.) We refer to such an equilibrium as a *pooling equilibrium.*[11]

The firm's second-period beliefs over valuations in a pooling equilibrium are given by

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v \leq \tilde{v} \\ \\ \frac{F(\tilde{v})+\alpha(F(v)-F(\tilde{v}))}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v > \tilde{v} \end{cases} \tag{1}$$

and

$$F^1(v) = \begin{cases} 0 & \text{if } v \leq \tilde{v} \\ \\ \frac{F(v)-F(\tilde{v})}{1-F(\tilde{v})} & \text{if } v > \tilde{v} \end{cases} \tag{2}$$

In the second period, the firm chooses its prices to maximize profit according to

$$\max_{p_2^r} \, (1 - F^r(p_2^r))p_2^r \;\; \text{for} \;\; r = 0, 1 \tag{3}$$

The following lemma shows that when $c > 0$, there does not exist a PBE in which all of the consumers who purchased in the first period opt out. The intuition is that when $c > 0$, it is optimal for the firm to lower the first-period price in order to attract more customers in the first period. For some of these customers, opting out is not a best response unless some other consumers purchase and stay identified.

Lemma 2. *For $c > 0$, there does not exist a PBE in which all first-period customers opt out.*

Proof: Assume on the contrary that there exists an equilibrium in which all consumers opt out. Since all consumers are anonymous in the second period, the seller sets $p_2^0 = p^\star$. Since consumers must find opting out to be a best response, we have $p_2^1 \geq p_2^0 + c = p^\star + c$; in addition, we have $\tilde{v} \geq p^\star + c$, else, a positive mass of consumers would not purchase in the

[11] The restriction to pooling equilibria can also be justified using a purification argument. Suppose that, instead of all agents facing the same cost of opting out, each agent's opt-out cost is drawn from a commonly known distribution. Furthermore suppose these costs are drawn i.i.d. across agents, and are independent of the agent's valuation. Let d_i, $i \in \mathbb{N}$, denote a sequence of continuous distributions over the opt-out cost that an individual agent faces such that $\lim_{i\to\infty} d_i$ is the degenerate distribution on c (where c is the cost in the original game G). Let G^{d_i} denote the cost-perturbed game where each consumer's cost of opting out is realized after the first-period purchasing decisions according to d_i. It can be shown that the pooling equilibrium we characterize is the unique equilibrium that results from taking the limit of the equilibria of G^{d_i} when $i \to \infty$.

second period and thus would not opt out. The resulting second-period profit for the firm is $p^\star(1 - F(p^\star))$.

The indifference condition for the cutoff type $\tilde{v}$ is thus given by $\tilde{v} - p_1 + \delta(\tilde{v} - c - p^\star) = \delta(\tilde{v} - p^\star)$, which gives $p_1 = \tilde{v} - \delta c$. Combined with $\tilde{v} \geq p^\star + c$, we have $p_1 \geq p^\star + (1 - \delta)c$.

The firm sets p_1 to maximize present-discounted profits, given by $(1 - F(p_1 + \delta c))p_1 + \delta p^\star(1 - F(p^\star))$. The first-order condition gives $p_1 = \frac{1-F(p_1+\delta c)}{f(p_1+\delta c)}$. From concavity, $\frac{1-F(p)}{f(p)}$ is decreasing in p at $p^\star$. Thus, $p_1 = \frac{1-F(p_1+\delta c)}{f(p_1+\delta c)} \leq \frac{1-F(p^\star+c)}{f(p^\star+c)} < \frac{1-F(p^\star)}{f(p^\star)} = p^\star$, a contradiction. ■

The intuition for Lemma 2 is rooted in the commitment problem of the seller. The seller is unable to directly commit not to price discriminate in the second period, but is able to influence consumers' decisions to become anonymous in the second period by raising the first-period price. However, doing so results in a loss of profit, both due to not capturing the cost consumers expend on opting out and due to lower revenues in the first period. Hence, the seller ends up choosing to set a lower first-period price and not all consumers opt out.

The next lemma provides a useful ordering of the equilibrium prices and cutoff type, and proves that anonymous consumers pay a discounted price in the second period. The key drivers for the result are the seller's inability to commit to second-period prices and consumers' strategic prediction of future prices in the first period.

Lemma 3. *For $c > 0$, if $\tilde{v} \geq p^\star$, then $p_2^0 \leq p_1 \leq \tilde{v} = p_2^1$ holds on the path of play of a pooling equilibrium.*

Proof: From Lemma 2, $\alpha < 1$, i.e., some consumers do not opt out and stay identified. From the seller's second period maximization problem it then follows that $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$. The seller's second period prices p_2^1 and p_2^0 both target the range of consumers in $[\tilde{v}, 1]$ (technically, this holds for p_2^0 only when $\alpha > 0$). However, p_2^0 also targets consumers in $[0, \tilde{v}]$ (for any α). It directly follows from the seller's maximization problem (3) that $p_2^0 \leq p_2^1$. Moreover, since consumers anticipate in the first period that $p_2^0 \leq p_2^1$, no consumer would purchase in the first period if the price p_1 exceeds her valuation. Thus, $p_1 \leq \tilde{v}$.

Now, assume on the contrary that $p_2^0 > p_1$ is part of an equilibrium; then no consumer skips purchasing in the first period to purchase in the second (in other words, there is no point to delaying a purchase in hopes of a cheaper price). Consequently, all consumers with valuations $v \geq p_1$ purchase in the first period, giving $\tilde{v} \leq p_1$. It follows that $p_2^0 > \tilde{v}$. But since $p_2^1 = \tilde{v}$, we have $p_2^0 > \tilde{v} = p_2^1$, a contradiction. ■

The following proposition characterizes the pooling equilibrium for sufficiently small values of the cost of opting out, c. (Proposition 4, which follows, gives the relevant range on

c.)

Proposition 3 (Pooling equilibrium)**.** *For sufficiently small $c > 0$, every pooling equilibrium satisfies:*
(i) Consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods and maintain anonymity with probability α; consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ purchase only in the second period; and $\tilde{v} \geq p^{\star}$.
(ii) Prices satisfy $p_1 = \tilde{v} - \delta c$, $p_2^1 = \tilde{v}$, and $p_2^0 = \tilde{v} - c$. The firm's beliefs about anonymous and identified consumers' valuations are given by (1) and (2).
(iii) For $h(v) = F(v) + vf(v)$ and $h'(v) = 2f(v) + vf'(v)$, the cutoff type $\tilde{v}$ and opting out probability α are determined from:

$$\tilde{v} = \delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)}{f(\tilde{v})} \tag{4}$$

$$\alpha = \frac{h(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{5}$$

Excluding deviations of measure 0, Proposition 3 uniquely determines the behavior on the path of play. The resulting equilibrium, with prices $p_1 = \tilde{v} - \delta c$, $p_2^0 = \tilde{v} - c$, and $p_2^1 = \tilde{v}$, has the following properties. A consumer with valuation at least $\tilde{v}$ will purchase in the first period as well as in the second period, and be indifferent between opting out and staying identified. A consumer with valuation $\tilde{v}$ will be indifferent among only buying in the first period, only buying in the second period, and buying in both periods. A consumer with valuation at most $\tilde{v}$ will not purchase in the first period, and purchase in the second period if and only if her valuation is at least $\tilde{v} - c$. Essentially, the firm offers anonymous customers "introductory" prices in each period (though the introductory price is more attractive in the second period).

We now move on to general values of c (not necessarily small). Let $\alpha(c)$ denote the probability that a consumer who purchased in the first period maintains anonymity, when the cost of doing so is c. (In a pooling equilibrium, by definition, this probability is the same for all agents who purchase in the first period.) Also let $p_2^{1,FR}$ and $p_2^{0,FR}$ denote the second period full-recognition benchmark prices, and finally let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$.

We proceed with several lemmas that address higher level of costs, which we then integrate to give a more general characterization of equilibrium. The first lemma addresses the case where the cost of opting out is high.

Lemma 4. *For $c \geq \bar{c}$, the outcome from any pooling equilibrium coincides with the full-recognition benchmark outcome.*

The intuition for this result is the following. For high costs of opting out, the full-recognition benchmark outcome is obtainable by the firm. In fact, the firm's problem is a constrained version of its counterpart in the full-recognition benchmark, where its optimal strategy and corresponding outcome in the latter is feasible. In essence, since the cost of opting out is high, the availability of opting out does little to help the firm's profit in terms of encouraging more consumers to purchase in the first period, but more to hurt the firm's profit by failing to capture a significant part of the cost anonymizing consumers incur. Consequently, the firm sets prices that encourage consumers to stay identified in equilibrium.

The next lemma addresses the region of cost where consumers no longer opt out.

Lemma 5. *Let $\hat{c}$ denote the smallest cost such that $\alpha(\hat{c}) = 0$. Then $\hat{c} < \bar{c}$.*

The intuition for Lemma 5 relates to that of Lemma 4. In particular, the firm works to mitigate its loss of potential profit from consumers anonymizing by reducing consumers' incentive to opt out. According to Lemma 5, the firm begins to do so for costs smaller than $\bar{c}$.

The following Lemma shows that indeed no consumer opts out in equilibrium for all $c \geq \hat{c}$.

Lemma 6. *For all $c \geq \hat{c}$, $\alpha(c) = 0$.*

The intuition for this result is that the firm maintains the status quo in terms of opting out once it is able to eliminate consumers' incentives to do so at $\hat{c}$. The firm achieves this by setting a first-period price that is sufficiently low (with a corresponding cutoff type $\tilde{v}$ that is also sufficiently low).

Proposition 4 characterizes the pooling equilibrium across different values of c.

Proposition 4. *Let $p_2^{1,FR}$ and $p_2^{0,FR}$ denote the full-recognition benchmark period 2 prices, and let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$. A pooling equilibrium exists, and any pooling equilibrium satisfies the following properties on the path of play:*

1. *There exists $\hat{c} \in (0, \bar{c})$ such that $\alpha(c) > 0$ for all $c \in [0, \hat{c})$ and $\alpha(c) = 0$ for all $c \geq \hat{c}$.*

2. *For $c = 0$, the unique pooling equilibrium outcome coincides with the no-recognition equilibrium outcome and is characterized by Proposition 2.*

3. *For $c \in (0, \hat{c}]$, the unique pooling equilibrium outcome is characterized by Proposition 3.*

4. *For $c \in (\hat{c}, \bar{c})$, let $\bar{v}$ be defined by $F(\bar{v} - c) + (\bar{v} - c)f(\bar{v} - c) = F(\bar{v})$. Then $\tilde{v}$ is non-decreasing in c on this region, with $\tilde{v}(\hat{c}) < \tilde{v}(\bar{c})$, and maximizes $\tilde{v}(1 - F(\tilde{v}))(1 + \delta) + \delta p_2^0(F(\tilde{v}) - F(p_2^0))$, subject to $\tilde{v} \leq \bar{v}$, $F(p_2^0) + f(p_2^0)p_2^0 = F(\tilde{v})$, and $p_1 = (1 - \delta)\tilde{v} - \delta p_2^0$.*

5. *For $c \geq \bar{c}$, the outcome from any pooling equilibrium coincides with the full-recognition benchmark outcome.*

Figure 2 shows how the probability of opting out is affected by the cost of maintaining anonymity, c. The region $[\hat{c}, \bar{c}]$ is of particular interest: as we will show shortly, consumer and social surplus (weakly) decrease in this region, the firm's profit (weakly) increases, but the probability of opting out is fixed at 0. The various regions can be explained as follows.

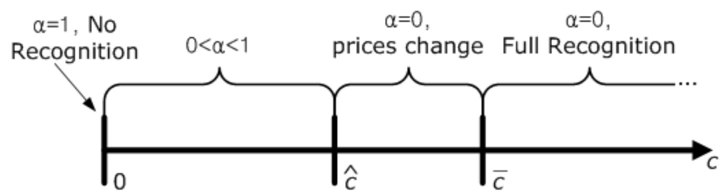


Figure 2: Equilibrium probability α of opting out as a function of the cost of opting out, c, on the path of play.

First, the firm loses profit when consumers opt out. When a consumer chooses to opt out, because the firm's second-period price for anonymous consumers is c lower than for identified consumers (to keep consumers indifferent between opting out and staying identified), this effectively costs the firm c. That is, the cost of opting out is passed on to the firm. Second, since some consumers opt out, the second-period price to anonymous consumers, p_2^0, targets both first-time buyers and repeat customers who opted out. Hence, (anonymous) repeat customers are interfering with the firm's ability to capture more first-time buyers in the second period, lowering the firm's profit. On the other hand, because consumers can opt out, more consumers decide to buy in the first period, which helps the firm's profit. This latter effect dominates when c is low, but is overcome by the former two effects as c grows larger – to the point (at $c = \hat{c}$) where it pays off for the firm to lower the first price sufficiently so that no consumer opts out. Once the cost reaches $\hat{c}$, nobody will opt out, allowing the firm to more easily price-discriminate as c increases.

The next proposition addresses the interval of opting out cost $[\hat{c}, \bar{c}]$ as it pertains to welfare. Recall that $\hat{c}$ is defined as the smallest cost such that the purchasing consumers' probability of opting out is 0 (i.e., the smallest c such that $\alpha(c) = 0$); $\bar{c}$ is defined as the difference in second-period prices in the full-recognition benchmark.

Proposition 5. *The firm's profit is non-decreasing in c over $[\hat{c}, \bar{c}]$ and is strictly higher for $c \geq \bar{c}$ than at $\hat{c}$. Consumer and social surplus are non-increasing over $[\hat{c}, \bar{c}]$ and are strictly lower at $c \geq \bar{c}$ than at $\hat{c}$.*

This result is striking for two reasons. First, it leads to the observation that the firm's profit is non-monotonic in the cost of opting out, as we illustrate in the following subsection.

Second, it shows that consumers can actually be worse off as the cost of opting out increases. In other words, facilitating opting out can actually improve consumers' welfare. This finding extends the previous literature, which finds that strategic consumers are better off under full-recognition compared to no-recognition. The above result brings forth the observation that although consumers may not opt out in some region of cost (i.e., full recognition takes place), they could still be better off when opting out is more accessible, even if they choose not to opt out. Said another way, there are different shades of "full recognition," and as far as consumers (and overall welfare) are concerned, some are better than others — namely the ones where opting out is more accessible. We further illustrate this finding in Subsection 4.4.

4.3 Firm profit

For $c \in [0, \hat{c}]$ and for $c = \bar{c}$, the firm's present-discounted profit is given by

$$(\tilde{v}(c) - \delta\alpha(c)c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0)) \tag{6}$$

By Proposition 5, the firm's profit for $c \in (\hat{c}, \bar{c})$ is lower than at $c = \bar{c}$.[12]

At $c = 0$, the firm obtains the no-recognition benchmark profit (where all consumers opt out), given by $(1 + \delta)p^\star(1 - F(p^\star))$. Lemma 1 showed that this profit exceeds the one in the full-recognition benchmark (with no opting out). Consequently, it is also greater than the firm's profit for any opting out cost $c \in (\hat{c}, \bar{c})$.

The above indicates that the firm's profit is non-monotonic in the cost of opting out: It is higher at $c = 0$ than at either $\hat{c}$ or $\bar{c}$, but lower at $\hat{c}$ than at $\bar{c}$. We also recall that in the full-recognition benchmark, if the firm is able to commit, then the outcome coincides with the no-recognition benchmark. It turns out that this remains true in the model with (costly) opting out. The following proposition proves this and summarizes the above observations.

Proposition 6 (Firm profit). *The firm's profit is highest at $c = 0$ and is non-monotonic in c. If the firm is able to commit to second-period prices in the first period, the firm would set $p_1 = p_2^0 = p_2^1 = p^\star$.*

Proof: For a given cost of opting out, c, the firm's profit cannot exceed its profit when it *collects* this c from consumers who opt out (this is a hypothetical situation). This profit is found by adding $\delta\alpha(c)c(1 - F(\tilde{v}))$ (the present-discounted amount consumers spend on opting out) to

[12]We note that for all $c \geq \bar{c}$, the firm obtains the full-recognition equilibrium profit.

the firm's profit in (6). Then, an upper bound on the firm's profit is given by:[13]

$$\tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0)) \tag{7}$$

The expression in (7) is uniquely maximized when $\tilde{v}(c) = p_2^0 = p^\star$, which gives the firm's profit in the no-recognition benchmark, also obtained, by Proposition 2, when $c = 0$. However, Proposition 4 proves that $\tilde{v} > p^\star$ for all $c > 0$. Thus, the firm's profit is higher under $c = 0$ than under any $c > 0$.

Claim: If the firm were able to commit to second-period prices, it would set p_2^0 and p_2^1 such that no positive mass of consumers opts out.

Proof: Suppose on the contrary that some mass of consumers opts out in equilibrium when the firm commits to second-period prices. For opting out to benefit some consumers, we have $p_2^1 - p_2^0 \geq c$. We now show that this leads to a contradiction:

1. First, consider the case where $p_2^1 - p_2^0 > c$. In this case, all purchasing consumers opt out. Then, the firm possesses the following profitable deviation: hold p_2^0 constant and reduce p_2^1 to anywhere in $(p_2^0, p_2^0 + c)$, because (i) no consumer would opt out, (ii) at least as many consumers purchase since prices were only reduced, and (iii) repeat consumers now pay strictly more in the second period.

2. Let us now direct attention to the case where $p_2^1 - p_2^0 = c$. Without loss of generality, suppose there is a mass m of repeat consumers, a proportion $\alpha > 0$ of which decides to opt out. Consider the following deviation: decrease p_2^1 by ϵ. By doing so, the firm's profit gains $\alpha m(c - \epsilon)$ and loses $(1 - \alpha)m\epsilon$. As $\epsilon \to 0$, this deviation is profitable. □

Any outcome that the firm can achieve with prices where no consumer opts out is of course also feasible in the commitment version of the full-recognition game. Thus, the firm's commitment problem in the partial-recognition game is a constrained version of the firm's commitment problem in the full-recognition game (with $p_2^1 - p_2^0 \leq c$ as an added constraint). Moreover, the firm's optimal strategy and corresponding outcome in the unconstrained problem (setting $p_1 = p_2^0 = p_2^1 = p^\star$, with consumers in $[p^\star, 1]$ purchasing in both periods without opting out) is feasible in the constrained problem. The result follows. ■

[13] For $c \in [0, \hat{c}]$, this profit is given by (7). For $c \in (\hat{c}, \bar{c})$, this profit is bounded above by the profit under $c = \bar{c}$, which is given by (7). For $c > \bar{c}$, profit is the same as under $c = \bar{c}$. Hence, the profit in (7) provides an upper bound on profit for any given c.

4.4 Do consumers benefit from more privacy?

We now turn to consumer surplus. Proposition 6 shows that the firm obtains its highest profit when consumers can costlessly maintain their anonymity. However, this does not immediately imply that consumer surplus is at its lowest in this case, because the total surplus may vary depending on the cost of opting out. Specifically, since there is no cost to production, the efficient outcome in this model – the first best – would be for every consumer to obtain the good in each period. Hence, the efficient outcome is not obtained for any $c \geq 0$, since some consumers do not purchase.

For $c \in [0, \hat{c}]$ and $c \geq \bar{c}$, consumer surplus is given by[14]

$$\underbrace{\int_{\tilde{v}}^{1} v f(v) dv - (1 - F(\tilde{v}))(\tilde{v} - \delta c)}_{(\star)} + \underbrace{\delta\Big(\int_{\tilde{v}-c}^{1} v f(v) dv - (1 - F(\tilde{v} - c))\tilde{v} + (F(\tilde{v}) - F(\tilde{v} - c))c\Big)}_{(\star\star)} \tag{8}$$

In (8), $(\star)$ is consumer surplus from first period transactions: consumers with valuations $v \in [\tilde{v}, 1]$ purchase the good and pay a price $p_1 = \tilde{v} - \delta c$; $(\star\star)$ is consumer surplus from period 2 transactions: consumers with valuations $v \in [\tilde{v}, 1]$ are repeat customers and end up expending $\tilde{v}$ (factoring in the cost of opting out), and consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ are first-time customers who receive a price discount of c.

When valuations are uniformly distributed, Proposition 4 can be used to derive equilibrium firm profit and consumer surplus as a function of the cost of opting out c . In particular, we have $\hat{c} = \frac{1+\delta}{4+3\delta}$, and $\bar{c} = \frac{2+\delta}{8+2\delta}$; consumers with $v \in [\tilde{v}, 1]$ purchase in the first period and opt out with probability α, where

$$\tilde{v} = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} + \delta c & \text{if} \quad c \leq \hat{c} \\ \min\{2c, 2\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{9}$$

$$\alpha = \begin{cases} \frac{1+\delta-(4+3\delta)c}{1+\delta(1-c)} & \text{if} \quad c \leq \hat{c} \\ 0 & \text{if} \quad c > \hat{c}, \end{cases} \tag{10}$$

The firm sets the first-period price

$$p_1 = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} & \text{if} \quad c \leq \hat{c} \\ \min\{(2-\delta)c, (2-\delta)\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{11}$$

[14] The expression in (8), evaluated at $c = \bar{c}$, gives a lower bound on consumer surplus for $c \in (\hat{c}, \bar{c})$, by Proposition 5. Consumer surplus for $c \geq \bar{c}$ equals consumer surplus at $c = \bar{c}$, since the equilibrium outcome is unchanged.

Second-period prices satisfy $p_2^0 = \frac{1}{2}(\tilde{v} + \alpha(1 - \tilde{v}))$, and $p_2^1 = \tilde{v}$. The firm's beliefs about anonymous and identified consumers' valuations are given by (1) and (2).

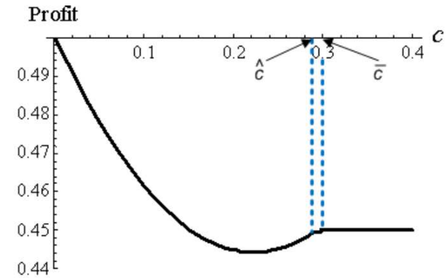
Figures 3(a)-(d) show comparative statics for the case of $\delta = 1$. In Proposition 7 below as well as in Figure 3, we also study social surplus, which can be interpreted in two different ways, depending on whether the cost of opting out is deadweight loss, or collected as a fee by a third party (for example, one can rent an anonymous postal box for a fee). In the former case, social surplus equals firm profit plus consumer surplus; in the latter, social surplus is higher than this sum if consumers opt out at positive costs.

Proposition 7. *With uniformly distributed valuations, the following properties are satisfied:*

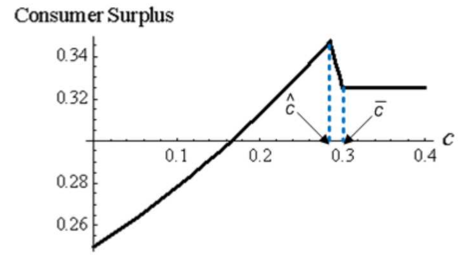
- *Consumer surplus is non-monotonic in c, increasing in c over $[0, \hat{c})$, decreasing over $[\hat{c}, \bar{c})$, higher for $c \geq \hat{c}$ than at $c = 0$, and highest at $\hat{c}$.*
- *The firm's profit is non-monotonic in c, decreasing in c over $[0, \frac{1+\delta}{4+5\delta})$, increasing over $[\frac{1+\delta}{4+5\delta}, \bar{c})$, and highest at $c = 0$.*
- *When the cost of opting out is deadweight loss, social surplus is non-monotonic in c, decreasing in c over $[0, \frac{1+\delta}{12+11\delta})$, increasing over $[\frac{1+\delta}{12+11\delta}, \hat{c})$, decreasing over $[\hat{c}, \bar{c})$, higher for $c \geq \hat{c}$ (where no consumer opts out) than at $c = 0$ (all purchasing consumers opt out), and highest at $\hat{c}$.*
- *When the cost of opting out is not wasted, social surplus is non-monotonic in c, increasing in c over $[0, \hat{c})$, decreasing over $[\hat{c}, \bar{c})$, higher for $c \geq \hat{c}$ than at $c = 0$, and highest at $\hat{c}$.*

The intuition is as follows: when $c = 0$, all consumers who purchase in the first period choose to maintain their anonymity. As c begins to rise, consumers must pay a non-trivial cost in order to opt out. The firm resorts to reducing the first-period price to counteract the negative effect on consumers' buying incentives: consumers are reluctant to purchase in the first period because of the cost they will incur in the second period either from opting out or from paying a high price. This results in a lower profit for the firm than it would have had with a lower c, but in higher consumer surplus. In the case where c is deadweight loss, this also results in lower social surplus because many consumers opt out in this region of cost.

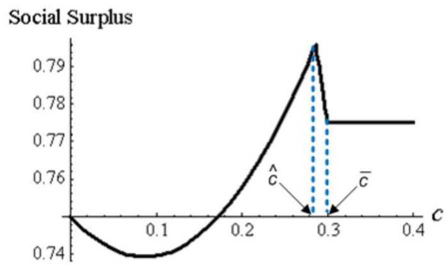
As c approaches $\hat{c}$, fewer consumers opt out. This gives the firm more flexibility in setting second-period prices, allowing it to better price discriminate which leads to a slight increase in profit. The firm continues to depress the first-period price over this range; additionally, better price discrimination allows the firm to target more low valuation consumers in the second period. This results in higher consumer surplus. Hence, as c approaches $\hat{c}$, both profit



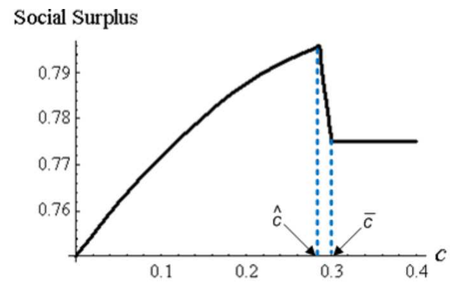
(a) Firm profit as a function of c



(b) Consumer surplus as a function of c



(c) Social surplus, c is deadweight loss



(d) Social surplus, c is a fee

Figure 3: Comparative statics when valuations are uniformly distributed and $\delta = 1$.

and consumer surplus are increasing so that social surplus is increasing when the cost c is wasted. When it is not wasted, social surplus is increasing but not as steeply because there is no surplus recovered directly from fewer consumers anonymizing (we recall that in this case, social surplus equals the sum of profit, consumer surplus, and the total cost of opting out). When c is in $[\hat{c}, \bar{c}]$, no consumer opts out. The firm increases prices in this range in order to better price discriminate in the second period, which results in fewer consumers purchasing and lower surplus overall.

The following proposition addresses the surplus of an individual consumer when valuations are uniformly distributed, after the consumer learns her valuation for the good.

Proposition 8. *With uniformly distributed valuations, each consumer is individually (weakly) better off under $c > 0$ than at $c = 0$.*

Proof: When $c = 0$, only consumers with types in $[p^\star, 1]$ (or in $[\frac{1}{2}, 1]$, using $p^\star = \frac{1}{2}$) purchase in both periods. We proceed in several steps.

(i) We first study the welfare of a consumer i with valuation $v_i \in [\tilde{v}(c), 1]$ for some $c > 0$. When opting out is costless, i's present-discounted consumer surplus is given by $(1+\delta)(\tilde{v} - \frac{1}{2})$. Under costly anonymity, i obtains $v_i - p_1 + \delta(v_i - \tilde{v})$. Note that we can rewrite i's consumer surplus as $v_i - \tilde{v} + \tilde{v} - p_1 + \delta(v_i - \tilde{v})$, which, using $p_1 = \tilde{v} - \delta c$, is equal to $(1+\delta)(v_i - \tilde{v}) + \delta c$. Since $v_i \in [\tilde{v}, 1]$, i's consumer surplus is bounded below by δc. Thus, consumer i is always better off under costly anonymity if $\delta c > (1+\delta)(\tilde{v} - \frac{1}{2})$.

When $c \in (0, \hat{c})$, we can simplify using $\tilde{v} = \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} + \delta c$. The inequality reduces to $0 > -\delta c(\frac{1}{2})$, hence i is better off with $c > 0$ in this region. When $c \in [\hat{c}, \bar{c}]$, we can simplify using $\tilde{v} = 2c$. The inequality reduces to $0 > c(2+\delta) - \frac{1}{2}(1+\delta)$. Since $c \leq \bar{c} = \frac{2+\delta}{8+2\delta}$, the RHS is bounded above by $-\frac{-\delta}{8+\delta}$, so that the inequality is indeed satisfied. Thus, i is better off under $c > 0$ relative to $c = 0$ (we note that consumers have the same welfare under $c > \bar{c}$ as at $\bar{c}$, thus considering the above two regions is sufficient).

(ii) We now consider a consumer i with valuation $v_i \in [p^\star, \tilde{v}]$. For a given $c > 0$, i will only consider purchasing in the second period. When doing so, her surplus is given by $\delta(v_i - p_2^0)$. At $c = 0$, i obtains $(1+\delta)(v_i - \frac{1}{2})$ as before. Hence, i is better off under $c > 0$ if $\delta(v_i - p_2^0) > (1+\delta)(v_i - \frac{1}{2})$, or if $0 > v_i - \frac{1+\delta}{2} + \delta p_2^0$. Using $p_2^0 = \tilde{v} - c$, this holds if $0 > v_i - \frac{1+\delta}{2} + \delta(\tilde{v} - c)$ for all $v_i \in [p^\star, \tilde{v}]$. The expression on the RHS is bounded above by $(1+\delta)(\tilde{v} - \frac{1}{2}) - \delta c$, which, as shown in (i), is indeed negative.

(iii) It remains to consider a consumer i with valuation $v_i \leq p^\star$. Such a consumer is trivially (weakly) better off under $c > 0$ than at $c = 0$, since i obtains a payoff of zero at $c = 0$. ■

Proposition 8 clearly illustrates the Prisoner's Dilemma nature of the outcome: when $c = 0$, individually, each consumer chooses to maintain her anonymity; however, every consumer ends up being (weakly) worse off as as a result of there being no price discrimination. The reason is that by opting out, consumers impose a negative externality on other consumers; a non-zero cost of opting out serves to at least partially alleviate this effect by diminishing the incentive to opt out.

5 Conclusions and extensions

We studied a model in which a firm is able to recognize and price discriminate against its previous customers, while consumers can maintain their anonymity at some cost. We showed that the firm obtains its highest profit when consumers can costlessly maintain their anonymity, but consumers can be better off when maintaining anonymity is costly, though only up to a point.

This paper suggests that certain aspects of online consumer privacy may be misjudged by policymakers and consumer advocacy groups. In particular, facilitating opting out can decrease consumer and social surplus when the cost of opting out is already low, although the opposite (or a neutral) effect takes place at higher costs. Of course, in practice, many other considerations need to be taken into account that are not in the scope of our model, such as the intrinsic value of privacy, as well as the (possibly accidental) release of sensitive information and corresponding spillover effects—for example, the release of an individual's medical records to her employer.

Our model can, in principle, be extended to take certain other economic considerations into account. An obvious direction is to study a setting with competition. Indeed, we have some initial findings in a similar model with two firms selling differentiated products; these findings suggest that phenomena similar to those identified above continue to occur. Another direction for further study is to make the cost of opting out endogenous. For example, one can consider a setting where a third party—a privacy gatekeeper—can provide anonymity to consumers for a fee.[15] One of our preliminary findings in this extension of the model is that the privacy gatekeeper would prefer to bargain with the firm and actually set the cost to consumers of opting out to zero. One can also consider settings where consumers obtain some benefit from being identified, such as smaller search costs or better technical support. Our preliminary finding here is that the results from the basic model carry through, except the relevant range of costs grows larger.[16] Another variant is to consider an opt-*in* policy. For instance, the firm could pay consumers to be identified (as in the case of membership programs that offer discounts). Another direction is to increase consumer heterogeneity by, for instance, allowing each consumer to have a different cost for opting out. One can also enrich the model by studying consumers with diminishing marginal utility for future units of the product. Finally, it would be interesting to study the steady-state equilibrium of an infinite-horizon version of our model with overlapping generations of consumers.

There are, in fact, a multitude of questions concerning issues of online privacy that are both interesting and potentially important. A primary message of this paper is that the answers to such questions may not be as obvious as they first appear. Indeed — as we have illustrated — it is often necessary to parse carefully the underlying economic forces at work before one can make correct policy recommendations.

References

Acquisti, A. and H. Varian (2005): "Conditioning Prices on Purchase History," *Marketing Science*, 24, 367–381.

[15]For instance, a consumer could rent a postal box from UPS so as not to disclose her home address. However, UPS could make such an "anonymizing service" much simpler: it could provide customers with individualized one-time codes. Consumers would give these codes to sellers instead of their home addresses. When sellers ship purchases via UPS, they would print the corresponding code on the label, and UPS would use this code to determine the customer's address.

[16]Notably, in such environments, consumers could benefit from having multiple accounts: one account to use for obtaining these benefits, and another account to use for potential access to lower prices.

Armstrong, M. (2006): “Recent Developments in the Economics of Price Discrimination,” in *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress of the Econometric Society*, ed. by N. Blundell and Persson, Cambridge University Press.

Calzolari, G. and A. Pavan (2006): “On the Optimality of Privacy in Sequential Contracting,” *Journal of Economic Theory*, 130, 168–204.

Chen, Y. (1997): “Paying Customers to Switch,” *Journal of Economics and Management Strategy*, 6, 877–897.

——— (2006): “Marketing Innovation,” *Journal of Economics and Management Strategy*, 15, 101–123.

Chen, Y. and J. Zhang (2009): “Dynamic targeted pricing with strategic consumers,” *International Journal of Industrial Organization*, 27, 43–50.

Chen, Y. and Z. J. Zhang (2008): “Price-for-Information Effect and Benefit of Behavior-Based Targeted Pricing,” Working paper, Wharton School of Business, Philadelphia, PA.

Coase, R. H. (1972): “Durability and Monopoly,” *Journal of Law and Economics*, 15, 143–149.

Esteves, R. B. (2010): “A Survey on the Economics of Behaviour-Based Price Discrimination,” Working paper, University of Minho, Portugal.

Freixas, X., R. Guesnerie, and J. Tirole (1985): “Planning under Incomplete Information and the Ratchet Effect,” *Review of Economic Studies*, 52, 173–191.

FTC (2007): “US Federal Trade Commission, Protecting the Consumers in the Next Tech-ade,” Http://ftc.gov/os/2007/12/P859900stmt.pdf.

Fudenberg, D. and J. Tirole (1998): “Upgrades, Trade-ins, and Buybacks,” *RAND Journal of Economics*, 29, 238–258.

——— (2000): “Customer Poaching and Brand Switching,” *RAND Journal of Economics*, 31, 634–657.

Fudenberg, D. and J. M. Villas-Boas (2006): “Behavior-Based Price Discrimination and Customer Recognition,” in *Handbook on Economics and Information Systems*, ed. by T. Hendershott, Amsterdam, North-Holland.

Hann, I.-H., K.-L. Hui, S.-Y. T. Lee, and I. P. Png (2007): “Overcoming Information Privacy Concerns: An Information Processing Theory Approach,” *Journal of Management Information Systems*, 24, 13–42.

Hart, O. and J. Tirole (1988): "Contract Renegotiation and Coasian Dynamics," *Review of Economic Studies*, 55, 509–540.

Laffont, J.-J. and J. Tirole (1988): "The Dynamics of Incentive Contracts," *Econometrica*, 56, 1153–1175.

Mialon, H. M. and S. H. Mialon (2008): "The Effects of the Fourth Amendment: An Economic Analysis," *Journal of Law, Economics, and Organization*, 24, forthcoming.

Odlyzko, A. (2003): "Privacy, Economics, and Price Discrimination on the Internet," in *ICEC2003: Fifth International Conference on Electronic Commerce*, 355–366.

Pancras, J. and K. Sudhir (2007): "Optimal Marketing Strategies for a Customer Data Intermediary," *Journal of Marketing Research*, 44, 560–578.

Salant, S. W. (1989): "When is Inducing Self-Selection Suboptimal for a Monopolist?" *Quarterly Journal of Economics*, 104, 391–397.

Schmidt, K. (1993): "Commitment through Incomplete Information in a Simple Repeated Bargaining Game," *Journal of Economic Theory*, 60, 114–139.

Shaffer, G. and Z. J. Zhang (2000): "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs," *Journal of Economics and Management Strategy*, 9, 397–424.

Stokey, N. (1979): "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, 93, 355–371.

Stole, L. (2007): "Price Discrimination and Competition," in *Handbook of Industrial Organization*, ed. by M. Armstrong and R. Porter, Amsterdam, North-Holland.

Taylor, C. (2003): "Supplier Surfing: Competition and Consumer Behavior in Subscription Markets," *RAND Journal of Economics*, 34, 223–246.

——— (2004): "Consumer Privacy and the Market for Customer Information," *RAND Journal of Economics*, 35, 631–650.

Villas-Boas, J. M. (1999): "Dynamic Competition with Customer Recognition," *RAND Journal of Economics*, 30, 604–631.

——— (2004): "Price Cycles in Markets with Customer Recognition," *RAND Journal of Economics*, 35, 486–501.

Wathieu, L. (2006): "Marketing and Privacy Concerns," Working paper, Harvard Business School, Boston, MA.

Appendix

A Extensive Form

The game we analyze is isomorphic to one between a firm and a single consumer whose type is distributed according to F, with one additional assumption: in the second period, the firm cannot distinguish between the situation where the consumer did not purchase in the first period, and the situation where the consumer did purchase in the first period and opted out. (For example, the firm cannot check its inventory to find out if a purchase was made. This assumption is unnecessary when there is a continuum of consumers because the firm's information would not change if a single, massless consumer deviated.) Since the firm sets the first-period price when it only knows the prior distribution F, the game is unaffected if consumers realize their types (according to the same F) after the firm sets p_1; it will be conceptually easier to think of p_1 being set first. This results in the extensive-form game shown in Figure 4.

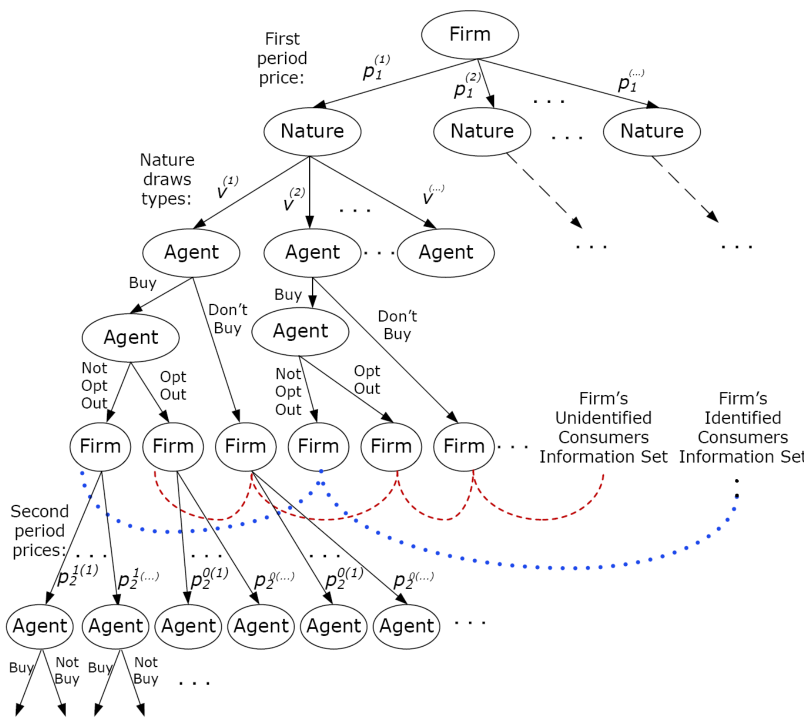


Figure 4: Sketch of the extensive form of the isomorphic game.

B Omitted Proofs

Proof of Proposition 1

Proof: Let $\tilde{v}$ denote the cutoff type of the marginal consumer who purchases in the first period, and assume that $\tilde{v} \geq p^\star$. We will show that this is indeed satisfied on the path of play in equilibrium.

The firm's second-period problems towards identified and anonymous consumers are given by $p_2^1 = \arg\max_{p \geq \tilde{v}} p(1-F(p))$ and $p_2^0 = \arg\max_p p(F(\tilde{v})-F(p))$, respectively. From concavity of $v(1-F(v))$, it follows that $p_2^1 = \max\{\tilde{v}, p^\star\}$ and p_2^0 satisfies $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$. Since we assume $\tilde{v} \geq p^\star$, we have $p_2^1 = \tilde{v}$. Thus, a consumer who purchases in the first period obtains zero payoff in the second.

By definition, a consumer with valuation $\tilde{v}$ is indifferent between purchasing now and later. Thus, $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$, and $p_1 = \tilde{v}(1-\delta) + \delta p_2^0$. Implicitly differentiating this expression with respect to p_1, we obtain $\partial\tilde{v}/\partial p_1 - 1 = \delta(\partial\tilde{v}/\partial p_1 - p_2^{0'}\partial\tilde{v}/\partial p_1)$. Thus, $\partial\tilde{v}/\partial p_1 = (1-\delta(1-p_2^{0'}))^{-1}$ and equivalently, $\partial p_1/\partial\tilde{v} = (1-\delta(1-p_2^{0'}))$

The firm's first-period problem is given by choosing p_1 to maximize

$$p_1(1-F(\tilde{v})) + \delta p_2^1(1-F(p_2^1)) + \delta p_2^0(F(\tilde{v}) - F(p_2^0)) \tag{12}$$

where $\tilde{v}$ is a function of p_1, p_2^1 and p_2^0 are functions of $\tilde{v}$ (and thus of p_1). An alternative approach that is technically simpler is for the firm's to choose $\tilde{v}$ to maximize (12), treating p_1 as a function of $\tilde{v}$ instead. Simplifying (12) using $p_2^1 = \tilde{v}$, the firm's problem is then given by setting $\tilde{v}$ to maximize

$$p_1(1-F(\tilde{v})) + \delta\tilde{v}(1-F(\tilde{v})) + \delta p_2^0(F(\tilde{v}) - F(p_2^0))$$

The first-order condition gives

$$1 - F(\tilde{v}) - p_1 f(\tilde{v}) - \delta\tilde{v}f(\tilde{v}) + \delta p_2^{0'}(1-F(p_2^0)) + \delta p_2^0(f(\tilde{v}) - f(p_2^0)p_2^{0'}) = 0$$

Simplifying using $p_1 = \tilde{v}(1-\delta) + \delta p_2^0$, we have

$$1 - F(\tilde{v}) - \tilde{v}f(\tilde{v}) + \delta p_2^{0'}(1 - F(p_2^0) - p_2^0 f(p_2^0)) = 0$$

Simplifying using $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$, we have

$$\tilde{v} = (1 + \delta p_2^{0'}) \frac{1 - F(\tilde{v})}{f(\tilde{v})} \tag{13}$$

Assume on the contrary that $\tilde{v} \le p^\star$. Since $p^\star = \frac{1-F(p^\star)}{f(p^\star)}$ and $\tilde{v} < \frac{1-F(\tilde{v})}{f(\tilde{v})}$ for $\tilde{v} < p^\star$, it follows from (13) that $p_2^{0'} \le 0$. From $\tilde{v} = p^\star < 1$, concavity of $p(1 - F(p))$, and from the fact that p_2^0 satisfies $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$, it follows that $p_2^{0'} > 0$, a contradiction. Thus, $\tilde{v} > p^\star$. ■

Proof of Proposition 3

Proof: Assume $\tilde{v} \ge p^\star$ holds on the path of play. By Lemma 2, $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$. (We show below that $\tilde{v} \ge p^\star$ is indeed satisfied.) From Lemma 3, the solution to the firm's period 2 problem for anonymous consumers satisfies the first-order condition of (3):

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})(1 - \alpha) + \alpha \tag{14}$$

If $\alpha > 0$, in order for consumers who mix between opting out and not opting out to be indifferent, $p_2^0 + c = p_2^1$ must hold. Since $p_2^1 = \tilde{v}$, we therefore have $p_2^0 = \tilde{v} - c$. Combining this observation with (14), we obtain:

$$\alpha = \frac{(\tilde{v} - c)f(\tilde{v} - c) + F(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{15}$$

Substituting using $h(v) = F(v) + vf(v)$ in the above, we immediately obtain (5).

From $p_2^1 = p_2^0 + c = \tilde{v}$, it follows that a consumer with valuation $\tilde{v}$ obtains a payoff of zero in period 2. Moreover, since a consumer with valuation $\tilde{v}$ is indifferent between purchasing now and possibly later and purchasing only later, she is overall indifferent between purchasing only now, purchasing only later, and purchasing now and later (with and without opting out). Since this consumer receives zero payoff in period 2, the following equality holds: $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$, i.e., the consumer is indifferent between purchasing only in the first period and only in the second period. Substituting using $p_2^0 = \tilde{v} - c$, we obtain $\tilde{v} = p_1 + \delta c$. Hence, if $\tilde{v} \ge p^\star$, the firm's first-period problem of choosing p_1 is equivalent to choosing $\tilde{v}$ such that $p_1 = \tilde{v} - \delta c$ and $p_2^0 = \tilde{v} - c$. The firm's first-period problem is to choose p_1 to maximize its present-discounted profit:

$$\max_{p_1} (1 - F(\tilde{v}(p_1)))p_1 + \delta\big((1 - \alpha)(1 - F(\tilde{v}(p_1)))\tilde{v}(p_1) +$$

$$+ (F(\tilde{v}(p_1)) + \alpha(1 - F(\tilde{v}(p_1))) - F(p_2^0(\tilde{v}(p_1))))p_2^0(\tilde{v}(p_1))\big)$$

Using the above observations, the firm's first-period problem is reduced to

$$\max_{\tilde{v}} \, (\tilde{v} - \alpha\delta c)(1 - F(\tilde{v})) + \delta(\tilde{v} - c)(1 - F(\tilde{v} - c)) \tag{16}$$

Substituting for α using (15) in the above, we obtain

$$\max_{\tilde{v}} \, \tilde{v}(1 - F(\tilde{v})) + \delta(\tilde{v} - c)(1 - F(\tilde{v} - c))) - \delta c(F(\tilde{v} - c) + (\tilde{v} - c)f(\tilde{v} - c) - F(\tilde{v}))$$

Letting $h(v) = F(v) + vf(v)$ and $h'(v) = \partial h(v)/\partial v = 2f(v) + vf'(v)$, the first-order condition is

$$1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + \delta(1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)) + \delta c f(\tilde{v}) = 0 \tag{17}$$

Rearranging (17), we obtain

$$\delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)}{f(\tilde{v})} - \tilde{v} = 0 \tag{18}$$

Given a sufficiently small c, (18) can be used to solve for $\tilde{v}$ (we recall $\delta > 0$ is assumed throughout).

To show that $\tilde{v} \geq p^\star$ is indeed satisfied, substitute $\tilde{v} = p^\star$ into (17). Using the fact that $1 - F(p^\star) - p^\star f(p^\star) = 0$ and dividing by δ gives $1 - h(p^\star - c) - c(h'(p^\star - c) - f(p^\star))$. First, it is easy to see that this expression equals 0 when $c = 0$, since $h(p^\star) = 1$. Differentiating this expression with respect to c gives $f(p^\star) + ch''(p^\star - c)$, or alternatively, $f(p^\star) + c(3f'(p^\star - c) + (p^\star - c)f''(p^\star - c))$. Since $3f'(v) + (v)f''(v)$ is assumed non-negative for $v \in [0, 1]$ and $f(p^\star) > 0$, this derivative is positive. Hence, the first-order condition in (18) evaluated at $\tilde{v} = p^\star$ is nonnegative and strictly positive for $c > 0$. It follows that $\tilde{v} \geq p^\star$.

Rearranging (18) gives

$$1 + \delta(1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)) = (\tilde{v} - \delta c)f(\tilde{v}) + F(\tilde{v}) \tag{19}$$

By assumption, $v(1 - F(v))$ and its derivative are concave. It follows that the left-hand side of (19) is decreasing in $\tilde{v}$ while the right-hand side is increasing. Hence, a unique $\tilde{v}$ satisfies the first-order condition, giving a unique behavior in a pooling equilibrium on the path of play.

The firm's problem is defined over a compact interval, where profit is not maximized at neither boundary $\tilde{v} = 0$ nor $\tilde{v} = 1$. Moreover, the firm's first and second-period problems are well defined given $\tilde{v} \geq p^\star$ and the solution to the firm's problem indeed satisfies $\tilde{v} \geq p^\star$. It follows that a pooling equilibrium exists.

We note that equilibrium behavior off the path of play has not been specified. In the first period, for instance, if the firm sets $p_1 = 0$, there exists an equilibrium of the continuation game in which all consumers buy. If no consumer opts out, the firm does not learn anything about identified consumers. In this case, the firm sets $p_2^1 = p^\star$, while any $p_2^0 \geq p^\star$ can be sustained since beliefs about anonymous consumers are off path. Similarly, if p_1 is set sufficiently high, as characterized in the proof of Lemma 2, it can be an equilibrium of the off-path continuation game for all consumers to opt out.

In the second period, per Lemma 2, for all $c > 0$, there is a positive mass of both identified and anonymous consumers on the path of play. Aside for the case of $c = 0$, addressed in Proposition 2, and the case of $p_1 \geq p^\star + (1-\delta)c$, addressed in Lemma 2, there are no other non-trivial[17] off-equilibrium beliefs on the continuation game that follows the first period. ■

Proof of Lemma 4

Proof: We have shown in Proposition 2 that the pooling equilibrium outcome coincides with the no-recognition benchmark outcome when $c = 0$. Let p_1^{FR}, $p_2^{0,FR}$, and $p_2^{1,FR}$ denote the first and second-period prices in the full-recognition benchmark outcome, respectively. Let $\hat{c}$ denote the smallest $c > 0$ such that $\alpha(\hat{c}) = 0$, and let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$.

It is straightforward to see that in the model with opting out, given opting out cost $\bar{c}$, the full-recognition outcome is obtainable by the seller. Namely, if the seller sets $p_1 = p_1^{FR}$, then indeed $p_2^1 = p_2^{1,FR}$, $p_2^0 = p_2^{0,FR}$, and $\alpha(\bar{c}) = 0$ are part of a pooling equilibrium.

Assume on the contrary the firm possesses a profitable deviation by setting a different first-period price, which would lead to a different pooling equilibrium and give it higher profit. If the deviation maintains $\alpha = 0$, it would have been possible in the full-recognition benchmark — a contradiction. Thus, $\alpha > 0$ must occur in this deviation. Since $\alpha > 0$, we have $p_2^1 - p_2^0 = \bar{c}$ to satisfy indifference between opting out and not. However, then the same deviation is possible in the full-recognition benchmark, only with no opting out, and thus a higher profit, contradicting the firm setting prices optimally.

An analogous argument can be made for any $c > \bar{c}$. It follows that for any $c \geq \bar{c}$, the pooling equilibrium outcome coincides with the full-recognition benchmark outcome. ■

[17] There are off-path situations in which consumers behave in a non-utility maximizing way, but such individual behavior does not affect the prices offered by the firm nor the firm's beliefs. Similarly, off-path situations following $p_1 = 1$ (no consumer purchases in the first period) or $p_1 = 0$ (all consumers purchase in the first period) are obvious. These prices cannot be sustained in equilibrium: the firm possesses profitable deviations in the first period by setting $p_1 \in (0, 1)$, strictly increasing its first-period profit, while it is still able to obtain the same second-period profit by setting $p_2^0 = p_2^1 = p^\star$.

Proof of Lemma 5

Proof: From Lemma 4, it immediately follows that there exists $\hat{c} \in (0, \bar{c}]$ such that $\alpha(\hat{c}) = 0$. Since $\hat{c}$ is the smallest cost at which no consumer opts out, the first-order condition (henceforth FOC) of the firm's first-period problem (4) is satisfied at $\hat{c}$. We can then apply Proposition 3 to obtain $p_2^1 - p_2^0 = \tilde{v} - p_2^0 = \hat{c}$.

We recall from (5) that

$$\alpha(c) = \frac{F(\tilde{v}(c) - c) + (\tilde{v}(c) - c)f(\tilde{v}(c) - c) - F(\tilde{v}(c))}{1 - F(\tilde{v}(c))} \tag{20}$$

From $\alpha(\hat{c}) = 0$, we have (where we henceforth use $\tilde{v}$ in leu of $\tilde{v}(c)$ to simplify expressions)

$$F(\tilde{v} - \hat{c}) + (\tilde{v} - \hat{c})f(\tilde{v} - \hat{c}) = F(\tilde{v}) \tag{21}$$

Deriving (20) with respect to $\tilde{v}$ and simplifying using (21), we have:

$$\alpha'(\hat{c}) = \frac{\partial \alpha}{\partial \tilde{v}}\bigg|_{c=\hat{c}} = \frac{2f(\tilde{v} - \hat{c}) + (\tilde{v} - \hat{c})f'(\tilde{v} - \hat{c}) - f(\tilde{v})}{1 - F(\tilde{v})} \tag{22}$$

We recall from (18) the (expanded) FOC of the firm's first-period problem:

$$\delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - F(\tilde{v} - c) - (\tilde{v} - c)f(\tilde{v} - c) - 2cf(\tilde{v} - c) - c(\tilde{v} - c)f'(\tilde{v} - c)}{f(\tilde{v})} - \tilde{v} = 0$$

Using (21) and simplifying, the above evaluated at $\hat{c}$ reduces to:

$$\delta\hat{c} + \frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta) - \hat{c}\delta \frac{2f(\tilde{v} - \hat{c}) + (\tilde{v} - \hat{c})f'(\tilde{v} - \hat{c}) - f(\tilde{v}) + f(\tilde{v})}{f(\tilde{v})} - \tilde{v} = 0$$

where $-f(\tilde{v}) + f(\tilde{v})$ was added as a wash. We can now simplify further using (22) to obtain

$$\tilde{v} = \frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1 - \alpha'(\hat{c})\hat{c})) \tag{23}$$

We now compare (23) with the corresponding FOC in the full-recognition benchmark to illustrate the relationship between $\hat{c}$ and $\bar{c}$. The firm's FOC in the full-recognition benchmark gives

$$\tilde{v}^{FR} = \frac{1 - F(\tilde{v}^{FR})}{f(\tilde{v}^{FR})}(1 + \delta p_2^{0',FR}) \tag{24}$$

where

$$p_2^{0',FR} = \frac{f(\tilde{v}^{FR})}{2f(\tilde{v}^{FR} - \bar{c}) + (\tilde{v}^{FR} - \bar{c})f'(\tilde{v}^{FR} - \bar{c})} \tag{25}$$

Since the price $p_2^{0,FR}$ only targets consumers in $[0, \tilde{v}(\bar{c})]$ and is set to maximize $p(F(\tilde{v}(\bar{c})) -$

$F(p))$, it follows that $p_2^{0,FR}$ satisfies $F(\tilde{v}(\bar{c})) = F(p_2^{0,FR}) + p_2^{0,FR} f(p_2^{0,FR})$. From concavity of $p(1 - F(p))$, we then have $p_2^{0',FR} < 1$.

For the sake of contradiction, assume that $\hat{c} = \bar{c}$. Then $\tilde{v}(\hat{c}) = \tilde{v}^{FR}$ follows from Lemma 4. For (23) and (24) to yield the same solution, it is required that $p_2^{0',FR} = 1 - \alpha'(\hat{c})\hat{c} = 1 - \alpha'(\bar{c})\bar{c}$. This holds if

$$\frac{f(\tilde{v})}{2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c})} = 1 - \bar{c}\frac{2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c}) - f(\tilde{v})}{1 - F(\tilde{v})} \tag{26}$$

Let $h'(v - c) = 2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c})$. Then the above expression reduces to:

$$\frac{f(\tilde{v})}{h'(\tilde{v}-\bar{c})} = 1 - \bar{c}\frac{h'(\tilde{v}-\bar{c}) - f(\tilde{v})}{1 - F(\tilde{v})}$$

which further reduces to:

$$f(\tilde{v}) = h'(\tilde{v}-\bar{c})$$

In other words, $\hat{c} = \bar{c}$ requires $f(\tilde{v}) = 2f(\tilde{v}-\bar{c}) + (\tilde{v}-\bar{c})f'(\tilde{v}-\bar{c})$. If we substitute this back in (25) we obtain $p_2^{0',FR} = 1$, a contradiction. ■

Proof of Lemma 6

Proof: From (23), the firm's FOC at $\hat{c}$ gives

$$\tilde{v} = \frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1 - \alpha'(\hat{c})\hat{c}))$$

where $\alpha' = \partial\alpha/\partial\tilde{v}$. From (22), we also have

$$\alpha'(\hat{c}) = \frac{2f(\tilde{v}-\hat{c}) + f'(\tilde{v}-\hat{c})(\tilde{v}-\hat{c})}{1 - F(\tilde{v})} - \frac{f(\tilde{v})}{1 - F(\tilde{v})}$$

Substituting α' into the above FOC and simplifying, we have

$$\frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1 - \frac{2f(\tilde{v}-\hat{c}) + f'(\tilde{v}-\hat{c})(\tilde{v}-\hat{c})}{1 - F(\tilde{v})}\hat{c})) - (\tilde{v} - \delta\hat{c}) = 0 \tag{27}$$

For the sake of contradiction, assume that there exists some $k' \in (\hat{c}, \bar{c})$ such that $\alpha(k') > 0$. Since $\alpha(\bar{c}) = 0$, there must exist $k > k'$ such that $\alpha(k) = 0$ *and* the first-order condition (27) is satisfied at $c = k$ (i.e., α turns 0 at k and Proposition 3 can be applied). From Proposition 3, we also have $p_2^0(k) = \tilde{v}(k) - k$. Substituting this into the firm's second-period problem, we obtain

$$(\tilde{v}(k) - k)f(\tilde{v}(k) - k) + F(\tilde{v}(k) - k) = F(\tilde{v}(k)) \tag{28}$$

Due to concavity of $p(1 - F(p))$, the latter expression together with (28) can both hold only if $\tilde{v}(k) - \tilde{v}(\hat{c}) > k - \hat{c}$. We thus have $\tilde{v}(k) - \tilde{v}(\hat{c}) > k - \hat{c} \geq \delta(k - \hat{c})$, so that $\tilde{v}(k) - \delta k > \tilde{v}(\hat{c}) - \delta\hat{c}$. Since $\alpha(k) = 0$, the first-order condition at k similarly gives:

$$\underbrace{\frac{1 - F(\tilde{v}(k))}{f(\tilde{v}(k))}(1 + \delta(\overbrace{\frac{1 - F(\tilde{v}(k)) - k\big(2f(\tilde{v}(k) - k) + f'(\tilde{v}(k) - k)(\tilde{v}(k) - k)\big)}{1 - F(\tilde{v}(k))}}^{(\star\star)}))}_{(\star\star\star)} \underbrace{-(\tilde{v}(k) - \delta k)}_{(\star)} \tag{29}$$

To see that this expression is negative, we first note that $(\star)$ is lower (more negative) than its corresponding term in (27). In addition, the hazard rate $\frac{1-F(v)}{v}$ is decreasing, and since $\tilde{v}(k) > \tilde{v}(\hat{c})$, $1 - F(\tilde{v}(k)) < 1 - F(\tilde{v}(\hat{c}))$.

It remains to show that $(\star\star)$ is smaller, but this follows directly from concavity of $p(1 - F(p))$ and its derivative. Hence, the FOC is violated at $c = k$, a contradiction. It follows that for all $c \in [\hat{c}, \bar{c}]$, $\alpha(c) = 0$. ■

Proof of Proposition 4

Proof: Lemmas 4-6 prove parts (1) and (5). Proposition 2 proves part (2) and Proposition 3 proves part (3). It remains to prove part (4).

By Lemma 6, for $c \in [\hat{c}, \bar{c})$, the equilibrium first-period price p_1 and corresponding cutoff type $\tilde{v}$ are low enough to satisfy $\alpha(c) = 0$. From the firm's second-period problem, when $\alpha = 0$, p_2^0 is derived from

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v}) \tag{30}$$

Let $\bar{v}(c)$ satisfy

$$F(\bar{v} - c) + f(\bar{v} - c)(\bar{v} - c) = F(\bar{v}) \tag{31}$$

From (5), we have

$$\alpha(c) = \frac{F(\tilde{v}(c) - c) + (\tilde{v}(c) - c)f(\tilde{v}(c) - c) - F(\tilde{v}(c))}{1 - F(\tilde{v}(c))}$$

Since $\alpha(c) = 0$ holds on $[\hat{c}, \bar{c}]$, $\bar{v}(c)$ denotes the highest cutoff type such that $\alpha = 0$ in this region. The firm's first-period problem is given by

$$\max_{\tilde{v}(c)} \tilde{v}(c)(1 - F(\tilde{v}(c)))(1+\delta) + \delta p_2^0(\tilde{v}(c))(F(\tilde{v}(c)) - F(p_2^0(\tilde{v}(c))) \tag{32}$$

subject to $\tilde{v}(c) \leq \bar{v}(c)$, i.e., subject to $\alpha(c) = 0$. Since $\partial \bar{v}(c)/\partial c > 0$, the firm is less constrained as c increases, thus profit is non-decreasing on $[\hat{c}, \bar{c}]$.

Assume on the contrary that $\tilde{v}'(c) < 0$ holds at some $c_1 \in (\hat{c}, \bar{c})$ so that $\bar{v}(c_1) - \tilde{v}(c_1) = \epsilon$. Let $k < c_1$ satisfy $\bar{v}(c_1) - \bar{v}(k) \leq \epsilon$. Then the same strategy is feasible for the firm for all $c \in [k, c_1]$. Since the firm is less constrained as c increases in this region, and an optimal strategy and corresponding outcome in a less constrained problem is feasible in a more constrained problem, $\tilde{v}(c_1)$ would indeed be optimal for $c \in [k, c_1]$. However, one can make a similar argument for any $c \in (c_1, \bar{c})$. In particular, one can make a similar argument for c slightly above c_1. But then $\tilde{v}'(c_1) \geq 0$, a contradiction. Thus, $\tilde{v}'(c) \geq 0$.

From the fact that both $\hat{c}$ and $\bar{c}$ satisfy $F(\tilde{v} - c) + f(\tilde{v} - c)(\tilde{v} - c) = F(\tilde{v})$ and $\hat{c} < \bar{c}$, we have $\tilde{v}(\hat{c}) < \tilde{v}(\bar{c})$. Thus, $\tilde{v}'(c) > 0$ holds for some $c \in (\hat{c}, \bar{c})$. ∎

Proof of Proposition 5

Proof: Following the proof of part (4) of Proposition 4, the firm's constraint in this region of cost is $\tilde{v} \leq \bar{v}$. Because $\partial \bar{v}(c)/\partial c > 0$, the firm is less constrained as c increases (with the firm's objective and other constraints remaining the same). Thus, profit is non-decreasing.

Since no consumer opts out for $c \in [\hat{c}, \bar{c}]$, social surplus is simply a function of how many consumers purchase in equilibrium. From part (4) of Proposition 4, $\tilde{v}(c)$ is non-decreasing on $[\hat{c}, \bar{c}]$ and is strictly increasing for some c in this region. Combining this result with $p_2^1 = \tilde{v}$ and the fact that p_2^0 satisfies $F(p_2^0) + p_2^0 f(p_2^0) = F(\tilde{v})$, where $p(1 - F(p))$ is concave, it follows that in this region, p_2^0 and p_2^1 are non-decreasing in c and strictly increasing for some c. From the indifference condition of type $\tilde{v}$, we also have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$, or $p_1 = \tilde{v}(1-\delta) + \delta p_2^0$; hence, p_1 exhibits the same behavior as p_2^0 and p_2^1. Therefore, both consumer and social surplus are non-increasing in c on this range and are strictly lower at $\bar{c}$ than at $\hat{c}$. ∎