Financing Speculative Booms*

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Abstract

This paper studies the financing of speculative asset-market booms in a standard framework with heterogeneous beliefs and short-sales constraints. Cash-constrained optimists use their asset holdings as collateral to raise debt financing from less optimistic creditors. Through state-contingent refinancing, short-term debt allows the optimists to reduce debt payment in upper states which they assign higher probabilities to, but at the expense of greater rollover risk if the asset fundamental deteriorates at the debt maturity. Our model identifies distinctive effects of initial and future belief dispersion in driving a short-term credit boom, and shows that it can initially fuel an asset-market boom and then exacerbate the downturn when asset fundamental deteriorates.

Keywords: Short-term credit boom, Asset bubble, Rollover risk, Debt maturity

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1 Introduction

The notion that speculation leading to both booms and crises rests on an inherent instability of credit has a long history in economics. As summarized by Kindleberger (2000), a host of classical economists including Irving Fisher, Henry Simons, and Hyman Minsky emphasized the role of debt contracted to leverage the acquisition of speculative assets for future resale and the role of debt structures in causing financial difficulties. There is also growing evidence of pronounced cycles of credit expansion and contraction that accompany the boom-and-crisis cycles of asset markets: White (1990) and Eichengreen and Mitchener (2003) for the stock market boom and crash of 1929; Adrian and Shin (2009), Brunnermeier (2009), Gorton and Metrick (2009), and Krishnamurthy (2010) for the recent credit crisis in 2007-2008; Rodrik and Velasco (1999) and Reinhart and Rogoff (2009) for the emerging-market debt crises in 1990s. In particular, there is a salient pattern in the increasing use of short-term credit in the booming periods of these boom-and-crisis episodes.

Motivated by the importance of credit in these episodes, this paper develops a dynamic model to analyze the inherent instability of credit and its role in fueling speculative booms and in driving debt crises. Our model builds on the standard asset-market framework that combines both heterogeneous beliefs and short-sales constraints, e.g., Miller (1977), Harrison and Kreps (1978), Morris (1996), Chen, Hong, and Stein (2002), and Scheinkman and Xiong (2003). In this framework, short-sales constraints cause the equilibrium asset prices to bias toward the beliefs of the optimists. However, existing models tend to ignore how optimists finance their speculative positions. In this regard, we follow Geanakoplos (2009), who had long advocated to incorporate the asset's collateral value into standard asset-market equilibrium models. His most recent paper provides a model with heterogeneous beliefs to highlight the important role played by leverage cycles in driving asset market cycles. Our model differs from Geanakoplos'—not only do we focus on the optimists' leverage choice, but also on the role of debt structure (e.g. long term vs. short term) in affecting their financing cost and leverage choice. The endogenous leverage and maturity choice jointly determine the equilibrium asset price dynamics.

Specifically, our model has two periods and a risky asset whose fundamental value is unobservable and fluctuates over time. We consider two groups of risk-neutral agents holding heterogeneous and state-contingent beliefs, which originate from their heterogeneous prior beliefs and learning processes, about the asset fundamental. If the optimists have sufficient funds, they would acquire all the asset and bid up the asset price to their optimistic valuation in settings where the optimists always hold the most optimistic belief (e.g., Miller (1977)), or to be even higher than their optimistic valuation in settings where other agents' beliefs may rise above the optimists' in the future (e.g., Harrison and Kreps (1978)). If the optimists have insufficient funds, then they have to use their asset holdings as collateral to raise debt financing from the pessimists who have excess funds. The financing cost directly affects the credit the optimists will use and the price they can offer for the asset. As a result, the asset market equilibrium is jointly determined with the credit market equilibrium. In our model, we restrict the optimists to standard non-contingent debt contracts, which are widely used in practice.[1] Despite this restriction, the optimists nevertheless face a non-trivial choice of debt structure to raise financing from the pessimistic creditors.

To tease out the financing problem, it is useful to consider the first-best allocation of the asset payoffs between the optimists and pessimists. Since the optimists assign higher probabilities to the upper states (i.e., states higher than a certain threshold) and lower probabilities to the lower states, the first-best allocation is to assign the asset payoffs in the upper states to the optimists, and those in the lower states to the pessimists. However, the first best payoff allocation is infeasible if agents only have access to non-state contingent debt financing. The standard non-contingent long-term debt, which stipulates a monotone payoff structure, is especially inefficient in this regard. More specifically, the long-term debt contract requires the optimistic borrower to make the promised payment in full in the upper states, which he values highly, but not so highly by the creditor. This makes the credit rather costly to the borrower. Indeed, our analysis shows that the financing cost indirectly pulls the equilibrium asset price toward the pessimistic creditors' beliefs and can even overturn the standard result that heterogeneous beliefs lead to an overvaluation of assets. This outcome echoes a recent paper by Simsek (2009), who also shows that financing cost can severely constrain optimists from bidding up asset prices in a static setting.

A key insight of our model is that state-contingent refinancing of short-term debt allows the optimists to structure state-contingent debt payoffs to reduce the financing cost. Suppose an optimistic borrower initially uses a short-term debt contract that matures on the interim date. If the asset fundamental improves when the debt matures, the borrower will obtain a

[1] Non-contigent debt contract is shown to be optimal in the costly state verification model of Townsend (1979), the monitoring model of Diamond (1984), and the contingent future financing model of Bolton and Scharfstein (1990). In these models, the unobservability of cash flows is important for the debt contract to be optimal.

better term in refinancing and thus be able to keep a greater fraction of the asset payoffs in the subsequent upper states to himself. This benefit, which can be termed as speculative incentive, motivates the use of short-term debt. However, there is also an opposing force. If the asset fundamental deteriorates, he will have to promise a higher debt payment to obtain refinancing or even to lose the collaterized asset in whole. As he still holds a more optimistic view about the asset fundamental, his greater promise (or the asset if forfeited) is under-valued by the creditor. Such under-valuation represents the so-called rollover risk, which has been increasingly recognized as a key trigger of short-term debt crises.[2] In our model, the optimists' initial speculative incentive and the subsequent rollover risk jointly determine whether short-term debt is desirable.

In particular, our model highlights the distinctive roles of belief dispersion at different times. A higher initial belief dispersion about the asset fundamental over the first period creates a greater speculative incentive for the optimists and thus makes short-term debt more desirable, while a higher belief dispersion after the fundamental deteriorates on the interim date increases the borrower's rollover risk and discourages the use of short-term debt. The tradeoff between the initial and future belief dispersion enriches the two-state setting considered by Geanakoplos (2009), who suggests that the higher accumulative belief dispersion over long-run (which also holds in our model) should always make short-term debt more desirable.

Our model shows that short-term debt allows the optimists to substantially reduce their financing cost over a broad region where the high cost of long-term debt financing would have constrained their capacity to bid up the asset prices. Our model thus explains the synchronization of short-term credit booms and asset price booms, a phenomenon commonly observed in various boom-and-crisis episodes (see Section 4 for three examples). Furthermore, short-term debt also acts as the bridge from booms to crises. As the asset fundamental deteriorates, the optimists will face difficulties in rolling over their short-term debt and may even be forced to turn over their asset to the pessimistic creditors at a substantial discount. Taken together, the inherent instability of short-term debt financing initially fuels the asset market boom created by the rise of heterogeneous beliefs between optimists and pessimists, and then exacerbates the downturn when the asset fundamental deteriorates.

[2] See Acharya, Gale, and Yorulmazer (2009) and He and Xiong (2009a, 2009b). In contrast to these models, the under-valuation (or the so-called firesale discount) in our model is endogenously determined by the heterogeneous beliefs between the borrowers and creditors.

The long debate on whether the commonly observed credit expansions that accompanied various asset-market booms were driven by supply shocks, such as expansionary monetary policies and external capital inflows, or by internally generated demand for credit is still unsettled.[3] The supply- and demand-driven factors have probably played different roles in different episodes. Our model helps understanding the demand-driven credit expansions, especially the expansions of short-term credit during asset-market booms. This salient phenomenon cannot be easily explained by any supply-side theory. The emphasis of our model also differs from those focusing on the tightening of credit during crises (e.g., Brunnermeier and Pedersen (2009)) and those on the shortening of debt maturity during crises (e.g., He and Xiong (2009a) and Brunnermeier and Oehmke (2009)). In particular, our model suggests that the concurrent short-term credit boom and asset-price boom are a potentially useful predictor for future crises. Our model also provides insights regarding regulating short-term leverages.

Our model is related to the literature that studies the pervasive use of short-term debt by banks and financial firms. The existing literature has emphasized several advantages of short-term debt. First, short-term debt is a natural solution to a variety of agency problems inside a firm, e.g., Calomiris and Kahn (1991) and Diamond and Rajan (2009). By choosing short-term financing, creditors keep the option to pull out if they discover that firm managers are pursuing value-destroying projects. Second, the short commitment period also makes short-term debt less information sensitive and thus less exposed to adverse-selection problems, e.g., Gorton and Pennacchi (1990). While these theories imply that firms regularly use certain amounts of short-term debt, they do not explain the increasing use of short-term debt during asset-market booms.

Finally, our model complements Garmaise (2001), who studies the security-design problem of a cash-constrained firm facing investors with heterogeneous beliefs. His model contrasts the optimal security design under heterogeneous beliefs to that under rational expectations, while our model focuses on the role played by debt structure in fueling asset-market speculation driven by heterogeneous beliefs.

The paper is organized as follows. Section 2 presents a baseline model with two groups

[3] Kindleberger (2000) provides a detailed account on different views about the driving force of credit expansions. See Eichengreen and Mitchener (2003) for an analysis of the argument that excessive supply of credit had fueled the stock market boom before the 1929 market crash, and White (1990) for the argument that the demand for credit to buy stocks had pulled funds into the market as the cost of credit had risen in sync with the use of credit.

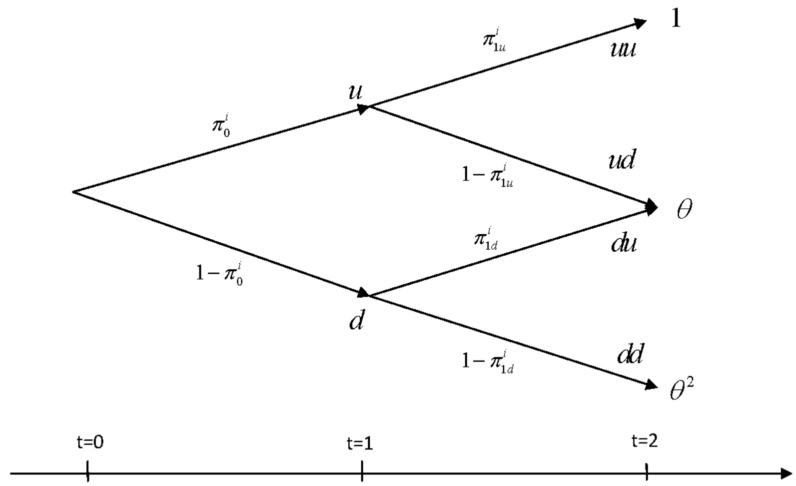

Figure 1: Timeline.

of agents holding exogenously specified beliefs. Section 3 extends the baseline model with learning and three groups of agents. We discuss the implications of the model in Section 4, and finally conclude in Section 5. All technical proofs are provided in the Appendix.

2 The Model

2.1 Asset and Agents

Consider a model with three dates and two periods. The date is indexed by $t = 0, 1, 2$. There is a long-term risky asset, which we interpret either as a house or a mortgage backed security. The asset pays a final payoff on date 2. The final payoff is determined by the final realization of a publicly observable binomial tree. Figure 1 illustrates the tree. The tree can go either up or down from $t = 0$ to $t = 1$ and from $t = 1$ to $t = 2$. The tree has four possible paths, which we denote by uu, ud, du, and dd (here, u stands for "up" and d stands for "down"), and three possible final nodes (paths ud and du lead to the same final node). We normalize the final payoff of the risky asset at the end of path uu as 1, at the end of paths ud and du as θ, and at the end of paths dd as θ^2, where $\theta \in (0, 1)$. We denote the asset payoff by $\widetilde{\theta} \in \{1, \theta, \theta^2\}$.

The probability of the tree going up in each period is unobservable. Suppose that there are two groups of risk-neutral agents, who differ in their beliefs about these probabilities.

Table 1: Asset Payoff and Agent's Belief across Different Paths

	Tree Paths			
	uu	ud	du	dd
Asset payoff	1	θ	θ	θ^2
Optimists' belief	$\pi_0^h \pi_u^h$	$\pi_0^h \left(1 - \pi_u^h\right)$	$\left(1 - \pi_0^h\right) \pi_d^h$	$\left(1 - \pi_0^h\right)\left(1 - \pi_d^h\right)$
Pessimists' belief	$\pi_0^l \pi_u^l$	$\pi_0^l \left(1 - \pi_u^l\right)$	$\left(1 - \pi_0^l\right) \pi_d^l$	$\left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right)$

In this section, we exogenously specify two sets of beliefs for the agents. Ultimately, the difference in the agents' beliefs is driven by their prior beliefs and learning processes, and we will extend the model with learning in Section 3.

There are three intermediate nodes on the tree, one on date 0 and two on date 1 (u and d depending on whether the tree goes up or down in the first period). We collect these intermediate nodes in the following set: $\{0, u, d\}$. At each of the nodes, each agent has a belief about the probability of the tree going up in the following period. We collect each agent's beliefs in the following set: $\{\pi_0^i, \pi_u^i, \pi_d^i\}$, where $i \in \{h, l\}$ indicates the agent's type. Throughout this section, we assume that the h-type agents are always more optimistic than the l-type agents across all the intermediate nodes (here, the superscript "h" and "l" stands for high and low.) That is, $\pi_n^h > \pi_n^l$ for any $n \in \{0, u, d\}$. Based on the relative order, we call the h-type agents optimists and the l-type pessimists.

In particular, we emphasize that the belief dispersion between the optimists and pessimists is not constant. Standing at $t = 0$, the difference between π_0^h and π_0^l represents the initial belief dispersion between the two groups about the asset fundamental from date 0 to 1, while the difference between π_d^h and π_d^l represents the future belief dispersion about the asset fundamental from the date-1 state d to date 2. As we will show later, these two types of belief dispersion play distinctive roles in determining the optimal debt maturity choice.

We summarize the final asset payoffs at the end of the four possible tree paths and the optimists' and pessimists' belief about each of the paths in Table 1. Note that the optimists assign a higher probability to path uu and a lower probability to path dd. But his beliefs about the middle paths ud and du can be higher or lower than those of the pessimists, which we will specifically discuss later.

We normalize the total supply of the asset to be one unit. There are $\mu \in (0, 1)$ units of optimists, who are homogeneous. On date 0, each optimist is initially endowed with 1 unit of the risky asset and c dollars of cash. Given the optimists' optimism, it is natural for them to purchase the rest of the asset ($1-\mu$ unit) from the pessimists. Following Miller (1977), Harrison and Kreps (1978), Morris (1996), Chen, Hong, and Stein (2002), and Scheinkman and Xiong (2003), we assume that short-sales of the asset are not allowed. As a result, the pessimists cannot speculate on the asset price falling in the future and will sit on the sideline.

The focus of our analysis is on the financing of the optimists' asset purchases. Since they may not have sufficient cash, they may need to borrow from the pessimists who sit on the sideline with cash. As the pessimists' beliefs affect the cost of financing to the optimists, their beliefs can indirectly affect the equilibrium asset price.For simplicity, we assume that both the risk-free interest rate and the agents' dscount rate are zero, and that the pessimists on the sideline will always have sufficient cash. Therefore, in equilibrium they always demand zero expected return in financing the optimists.

2.2 Collaterilized Debt Financing

Like Geanakoplos (2009), we assume that the optimists use their asset holdings as collateral to obtain debt financing. We focus on non-contingent debt contracts. A non-contingent debt contract specifies a constant debt payment (face value) at maturity unless the borrower defaults. Non-contingent debt contracts are widely used in practice. Townsend (1979) explains its popularity based on the cost of verifying the state of the world. That is, non-contingent debt contracts circumvent the cost of verifying the value of the collateral as long as the borrower makes the promised payment. Diamond (1984) and Bolton and Scharfstein (1990) also derive the optimality of non-contingent debt based on unobservability of cash flows. In this model, we will restrict the optimists to use only non-contingent debt.

We will first discuss long-term debt contracts, and then short-term ones. We will restrict our attention to contracts with face values in $\left[\theta^2, \theta\right]$. We will show in Lemma 2 in Section 2.3.2 that this is without loss of generality in the equilibrium.

2.2.1 Long-term Debt

Consider a long-term debt contract, which is collateralized by one unit of the asset. The contract matures on date 2 and has a face value of $F_L \in \left[\theta^2, \theta\right]$. The debt payment is

$$\widetilde{D}_L\left(F_L\right) = \min\left(F_L, \widetilde{\theta}\right).$$

Table 2: Asset Payoff and Debt Payment across Different Paths

	Tree Path			
	uu	ud	du	dd
Asset payoff	1	θ	θ	θ^2
Long-term debt face value $F_L \in \left[\theta^2, \theta\right]$	F_L	F_L	F_L	θ^2
Short-term debt face value $F_S \in \left[\theta^2, K_d\right]$	F_S	F_S	$F_{S,1} \geq F_S$	θ^2
Short-term debt face value $F_S \in [K_d, \theta]$	F_S	F_S	θ	θ^2

Depending on the four possible paths of the tree, the asset payoff and debt payment are listed in Table 2. Given the debt payment, a pessimistic creditor is willing to provide the following credit on date 0:

$$C_L\left(F_L\right) = \mathbb{E}_0^l\left[\widetilde{D}_L\right] = \left(1 - \left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right)\right) F_L + \left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right)\theta^2, \tag{1}$$

where $\mathbb{E}_n^i[\cdot]$ denotes the conditional expectation of a type-i agent on node $n \in \{0, u, d\}$. On the other hand, from the optimistic borrower's perspective, the expected cost of using this debt contract is

$$\mathbb{E}_0^h\left[\widetilde{D}_L\right] = \left(1 - \left(1 - \pi_0^h\right)\left(1 - \pi_d^h\right)\right) F_L + \left(1 - \pi_0^h\right)\left(1 - \pi_d^h\right)\theta^2. \tag{2}$$

The difference between (1) and (2) highlights a key feature of our model—the borrower and creditor use different probabilities in assessing the cost and value of a debt contract. In particular, as the borrower is optimistic and assigns a higher probability to path uu, the promised payment F_L at the end of this path is more costly to the borrower than valued by the creditor. Thus, the first-best allocation of asset payoffs between the borrower and creditor would be to assign all of the asset payoff at the end of path uu to the borrower. However, such a non-monotonic allocation is infeasible under standard non-contingent debt contracts, which stipulates monotonic payoffs.

Interestingly, as we will show next, the standard non-contingent short-term debt—through refinancing—can generate non-monotone debt payments, which is the main advantage of short-term debt over long-term debt.

2.2.2 Short-term Debt

We now consider a short-term debt contract collateralized by one unit of asset, and with a promised payment $F_S \in \left[\theta^2, \theta\right]$ due on date 1. Different from long-term debt, short-term debt requires refinancing (or rollover) at date 1. A key insight of our model is that state-contingent refinancing of short-term debt makes it possible for the borrower to reduce debt payment at the end of path uu by trading up payments at the end of some lower paths.

Specifically, in the upper interim state u, the borrower can always get a new contract with the same face value F_S, because the asset payoff is always sufficient to pay off the debt regardless of the subsequent final state being 1 or θ. However, in the lower interim state d, the borrower will get a worse term and may even lose the asset if the initially promised payment is too large. Specifically, the face value of the new contract $F_{S,1}$ needs to ensure that the pessimistic lender's valuation of the new debt contract is sufficient for offsetting the initially promised payment F_S:

$$\mathbb{E}_d^l\left[\min\left(F_{S,1}, \widetilde{\theta}\right)\right] = F_S.$$

Since the highest possible date-2 payment the borrower can promise is θ, the maximum amount of credit the borrower can raise in this state is:

$$K_d \equiv \mathbb{E}_d^l\left[\min\left(\theta, \widetilde{\theta}\right)\right] = \mathbb{E}_d^l\left[\widetilde{\theta}\right] = \pi_d^l \theta + \left(1 - \pi_d^l\right)\theta^2 < \theta. \tag{3}$$

This implies that the borrower will fail to refinance his short-term debt in the interim state d if the initially promised date-1 debt payment F_S is higher than K_d. Therefore, we have the following two cases to consider:

1. If $F_S \in \left[\theta^2, K_d\right]$, the initial short-term debt is riskless and the borrower can obtain a credit of $C_S\left(F_S\right) = F_S$ on date 0. In the lower interim state d, the borrower can roll into a new short-term debt contract with face value $F_{S,1}$:

$$F_{S,1} = \frac{F_S}{\pi_d^l} - \frac{1 - \pi_d^l}{\pi_d^l}\theta^2 \geq F_S. \tag{4}$$

   The borrower has to promise more as the value of the collateral has deteriorated.

2. If $F_S \in \left(K_d, \theta\right]$, the initial debt contract is risky. In the lower interim state d, the payment due exceeds the maximum amount of debt the borrower can refinance from any pessimistic creditor using the asset as the collateral. The borrower thus defaults

and loses the asset to the creditor. This outcome is equivalent to the final debt payment at the end of paths du and dd being θ and θ^2, respectively. By using this debt contract, on date 0 the borrower can get a credit of:

$$C_S\left(F_S\right) = \pi_0^l F_S + \left(1 - \pi_0^l\right)\left(\pi_d^l \theta + \left(1 - \pi_d^l\right)\theta^2\right) \text{ if } F_S \in \left(K_d, \theta\right].$$

Table 2 summarizes the final debt payments across the four possible tree paths for the two cases discussed above. In both cases, the state-contingent refinancing makes the final debt payment non-monotonic with respect to the final asset payoff, i.e., the debt payment at the end of paths uu and ud is lower than at the end of path du. This rearrangement of debt payment is potentially valuable to the borrower as he assigns a higher probability to path uu than the creditor.

2.3 The Optimal Debt Contract

To study the optimal debt contract used by an optimistic buyer, we take the asset price p_0 as given. In light of Table 2, we denote a debt contract (either long-term or short-term) by a set of state-contingent debt payment $\widetilde{D}$. Furthermore, we denote $C\left(\widetilde{D}\right) \equiv \mathbb{E}_0^l\left[\widetilde{D}\right]$ as the date-0 credit that a borrower can obtain from a pessimist by using the debt contract $\widetilde{D}$.

What is the maximum unit of asset that an optimist can afford on date 0 by using the debt contract $\widetilde{D}$? He is initially endowed with c dollars of cash and 1 unit of the asset. Suppose that he purchases additional x_i units in the market. His total purchasing power is $c + (1 + x_i)\, C\left(\widetilde{D}\right)$, the sum of his cash endowment and the credit he can raise by using his asset holding ($1 + x_i$ units in total) as collateral. The budget contraint implies that

$$c + (1 + x_i)\, C\left(\widetilde{D}\right) = x_i p_0 \Rightarrow x_i = \frac{c + C\left(\widetilde{D}\right)}{p_0 - C\left(\widetilde{D}\right)}. \tag{5}$$

An implicit assumption in this calculation is that the optimist maxes out his purchasing power, a conjecture that we will verify in Propositions 5 and 6. For each unit of asset, the optimists' date-0 expectation of the date-2 cash flow after netting out the debt payment is $\mathbb{E}_0^h\left[\widetilde{\theta} - \widetilde{D}\right]$. Therefore, the optimist's date-0 value from using the contract $\widetilde{D}$ (i.e., the expectation of the final wealth) is

$$V\left(\widetilde{D}\right) = (1 + x_i)\, \mathbb{E}_0^h\left[\widetilde{\theta} - \widetilde{D}\right] = \frac{c + p_0}{p_0 - C\left(\widetilde{D}\right)}\left[\mathbb{E}_0^h\left(\widetilde{\theta}\right) - \mathbb{E}_0^h\left(\widetilde{D}\right)\right]. \tag{6}$$

This expression illustrates the tradeoff in the optimist's debt choice. On one hand, by promising a collaterized debt payment $\widetilde{D}$ on each unit of asset holding, the buyer can raise a credit of $C\left(\widetilde{D}\right)$ and thus establish a larger initial position $\frac{c+p_0}{p_0-C\left(\widetilde{D}\right)}$, which is the first part in $V\left(\widetilde{D}\right)$. This term represents a *leverage* effect. On the other hand, the debt payment reduces the asset payoff to the buyer on date 2. This *debt-cost* effect is reflected in the second part $\mathbb{E}_0^h\left(\widetilde{\theta}\right) - \mathbb{E}_0^h\left(\widetilde{D}\right)$ in $V\left(\widetilde{D}\right)$.

The debt contract contains two dimensions: *debt maturity* (long-term or short-term) and *promised payment* (i.e., the debt face value). Both are determined by the tradeoff between the leverage effect and debt-cost effect. We will first analyze the agent's maturity choice, and then the face-value choice.

2.3.1 Maturity Choice

To derive the optimal debt maturity, we consider the following question: in order to raise the same amount of credit at $t = 0$, which contract (i.e., long-term or short-term) entails the lower expected cost? Equation (6) implies that the one with the lower cost dominates the other. The following key proposition shows that the optimal maturity choice is determined by the initial and future belief dispersion between the optimists and pessimists.

Proposition 1 *Consider two debt contracts, one short-term and the other long-term. Suppose that both contracts have a face value in* $\left[\theta^2, \theta\right]$ *and give the same date-*0 *credit to an optimistic borrower. Then, from the borrower's perspective on date* 0*, the short-term contract requires a (weakly) lower expected cost if and only if*

$$\frac{\pi_0^h}{\pi_0^l} > \frac{\left(1-\pi_0^h\right)\pi_d^h}{\left(1-\pi_0^l\right)\pi_d^l}. \tag{7}$$

Proposition 1 shows that whether the short-term debt contract dominates the long-term debt contract depends on the initial belief ratio between the optimists and pessimists (π_0^h/π_0^l about the first-period fundamental), and the future belief ratio in the lower interim state d (π_d^h/π_d^l about the second-period fundamental.) The short-term contract is dominant if the initial belief ratio is sufficiently large, or if the future belief ratio is sufficiently small. Moverover, Proposition 1 holds for any given debt face value and is not restricted just to the face value used in the equilibrium. In this sense, the insight conveyed by the proposition is more general than the specific (binomial-tree) setting considered in our paper.

To understand the intuition, a debt contract not only channels the necessary financing from a creditor to an optimistic borrower to purchase the asset, but also represents an allocation of the asset payoffs between the borrower and creditor. As we have discussed earlier, a long-term contract specifies a monotonic debt payment with respsect to the asset payoff, while a short-term contract allows the borrower to reduce the debt payments at the end of paths uu and ud by trading up the payment at the end of path du (the payment at the end of path dd is maxed out.)

Is this tradeoff worthwhile? It first depends on the initial belief ratio π_0^h/π_0^l. If this ratio becomes higher, the reduced future debt payment after the upper interim state u becomes more valuable to the borrower and the increased payment after the lower interim state d becomes less important. Conversely, the creditor finds the reduced payment after u less important, while the increased payment after d more valuable. As a result, the short-term debt contract becomes more desirable to both of the borrower and creditor. This effect reflects the two parties' speculative incentives driven by their initial belief dispersion.

There is also the so-called rollover-risk effect working against short-term debt. In the lower interim state d, the borrower still holds the more optimistic belief going forward. This means that the increased future debt payment is under-valued by the creditor. In other words, the borrower gives up some future asset payoffs at a price lower than his own valuation. In the extreme case, if he cannot obtain sufficient refinancing to repay his maturing debt obligation, he has to give up the asset in whole to the creditor who does not value the asset as much as he does. This under-valuation of the increased debt payment is determined by the belief ratio between the borrower and creditor π_d^h/π_d^l in the interim state d. When this ratio becomes large, the rollover-risk effect becomes severe and thus makes it more costly for the borrower to use short-term debt.

Proposition 1 reflects the tradeoff between the speculative-incentive effect and the rollover-risk effect, and ties this tradeoff to the dynamics of the agents' heterogeneous beliefs. A close look at the path du makes this connection crystal clear. The state-contingent debt refinancing allows the borrower to trade up the debt payment at the end of path du for lower payments at the end of uu and ud. The borrower assigns a probability of $\left(1-\pi_0^h\right)\pi_d^h$ to the path du, while the creditor assigns $\left(1-\pi_0^l\right)\pi_d^l$. Suppose that there is only initial belief dispersion, i.e., $\pi_0^h > \pi_0^l$ and $\pi_d^h = \pi_d^l$. Then, the borrower thinks this path is less likely than the creditor. As a result, their speculative incentives make the short-term debt desirable.

However, if there is only future belief dispersion, i.e., $\pi_d^h > \pi_d^l$ and $\pi_0^h = \pi_0^l$, the borrower finds this path more likely than the creditor. As a result, the short-term debt is more costly to the borrower because of the rollover risk.[4]

As the optimists hold a more optimistic belief than the pessimists about the asset fundamental in each period, the accumulative belief dispersion between them increases with time horizon. Geanakoplos (2009) argues that this property of belief dispersion is sufficient to ensure the dominance of short-term debt over long-term debt in a setting with only two states. Proposition 1 shows that this argument may be specific to his model. In contrast, in our more general framework, the result may reverse: short-term debt is more desirable precisely when the initial disagreement about the first-period fundamental is high and future disagreement about the second-period fundamental is low.

2.3.2 Optimal Debt Face Value

To derive the optimal debt face value, we first characterize its feasible range:

Lemma 2 *If* $\mathbb{E}_0^l\left[\widetilde{\theta}\right] \leq p_0 \leq \mathbb{E}_0^h\left[\widetilde{\theta}\right]$, *then the optimal debt face value is inside* $\left[\theta^2, \theta\right]$.

Lemma 2 shows that the optimal debt face value (for either long-term or short-term contract) lies inside the interval $\left[\theta^2, \theta\right]$. The condition for this result—the date-0 asset price lies between the pessimists' and optimists' asset valuations—is innocuous because it always holds in the equilibrium throughout this section. When we extend the model to incorporate learning in the next section and allow agents' beliefs to flip on date 1, the equilibrium asset price could be higher than the optimists' asset valuation because of the asset owner's resale option. However, the feasible range of the optimal debt face value derived in Lemma 2 still holds after we modify the condition to account for the resale option.

The intuition of Lemma 2 is as follows. If the optimal debt face value is lower than θ^2, then the debt is risk free. Thus, increasing the face value by a small amount ϵ does not change the risk of the debt, and thus allows the borrower to increase his initial financing by ϵ at a cost exactly equal to ϵ. Since the asset price is lower than his asset valuation, the increased credit allows him to take a larger asset position and therefore be better off. This shows that the optimal debt face value cannot be smaller than θ^2. On the other hand, if the

[4] Condition (7) implies that even if both agents argree on the probability of the path du, π_0^h/π_0^l still plays a role. This is because when $\left(1-\pi_0^h\right)\pi_{1d}^h = \left(1-\pi_0^l\right)\pi_{1d}^l$, the borrower still gains by saving on the paths uu and ud, which he thinks are more likely to occur than the lender. This explains the appearance of the ratio π_0^h/π_0^l on the LHS of (7).

optimal debt face value is higher than θ, the borrower always defaults on the debt except at the end of the path uu. This implies that reducing the face value by a small amount ϵ allows the borrower to save debt payment at the end of path uu, which he values more than the creditor. Of course, this also cuts down his initial asset position. In the proof provided in Appendix A.2, we show that as long as the asset price is higher than the pessimistic creditor's asset valuation, the borrower is better off by reducing the debt face value. Thus, the optimal debt face value cannot be higher than θ either.

The following proposition provides the borrower's optimal long-term debt face value conditional on long-term debt being more desirable.

Proposition 3 *Suppose that the condition in (7) does not hold. Thus, it is optimal for the borrower to use long-term debt. Define*

$$P_M \equiv \left(1 - \left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right)\right) \mathbb{E}_0^h \left[\widetilde{\theta}\,|uu, ud, du\right] + \left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right) \mathbb{E}_0^l \left[\widetilde{\theta}\,|dd\right].$$

Then, the borrower's optimal debt face value is θ if $p_0 < P_M$; is either θ or θ^2 if $p_0 = P_M$; or is θ^2 if $p_0 > P_M$.

The discrete asset payoff implies that varying the long-term debt face value F_L between θ^2 and θ does not change the risk of the debt. In other words, regardless of the value of F_L in this region, the borrower will always make the promised debt payment F_L at the end of paths uu, ud, and du, and default and thus give up all the asset payoff at the end of dd. Since the borrower is risk-neutral, he will use the highest face value θ to maximize his position if the asset price is below a critical level P_M, which weighs his asset valuation and cost of financing. More precisely, P_M is a weighted average of the borrower's asset valuation in the upper (non-default) states $\{uu, ud, du\}$ and the creditor's valuation in the lower (default) state dd. If we interpret the long-term debt contract as a static contract that spans two periods, then Proposition 3 is analogous to the result of Simsek (2009).

If the borrower uses short-term debt, the default risk of the contract depends on whether the debt face value is higher or lower than K_d, where K_d is the asset's maximum debt capacity in the lower interim state d. If the face value is between θ^2 and K_d, the borrower is always able to refinance and the initial debt contract is risk free, even though the follow-up contract in the interim state d is risky as the borrower will default on date 2 at the end of path dd. If the face value is between K_d and θ, the borrower cannot get a new debt contract to pay off the initial debt in the state d, and thus default on the debt.

In the same spirit to Proposition 3, the next proposition shows that the borrower will choose to use the highest face value inside the two regions $\left[\theta^2, K_d\right]$ and $[K_d, \theta]$ if the asset price is below two thresholds P_H and P_L, respectively. These thresholds reflect the borrower's asset valuation and the cost of using debt in these regions.

Proposition 4 *Suppose that the condition in (7) holds. Thus, it is optimal for the borrower to use short-term debt. Define*

$$P_H \equiv \frac{\pi_d^l\left(\pi_0^h + \left(1-\pi_0^h\right)\pi_d^h\right)}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h\pi_d^l}\mathbb{E}_0^h\left[\widetilde{\theta}\,|uu, ud, du\right] + \frac{\left(1-\pi_0^h\right)\pi_d^h\left(1-\pi_d^l\right)}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h\pi_d^l}\mathbb{E}_0^l\left[\widetilde{\theta}\,|dd\right]$$

and

$$P_L \equiv \pi_0^l\mathbb{E}_0^h\left[\widetilde{\theta}\,|uu, ud\right] + \left(1-\pi_0^l\right)\mathbb{E}_0^l\left[\widetilde{\theta}\,|du, dd\right],$$

which satisfy

$$P_L < P_M < P_H.$$

Then, the borrower's optimal short-term debt face value is θ if $p_0 < P_L$; is either θ or K_d if $p_0 = P_L$; is K_d if $P_L < p_0 < P_H$; is either K_d or θ^2 if $p_0 = P_H$; or is θ^2 if $p_0 > P_H$.

The core of Propositions 3 and 4 is that when the asset price becomes cheaper relative to the buyer's own valuation (after adjusting for the financing cost), he will demand a greater position. To finance the greater position, he uses a higher debt face value to obtain more credit. In the next subsection, we will use these two propositions to derive the joint equilibrium of the asset and credit markets.

2.4 The Equilibrium of Asset and Credit Markets

We now derive the equilibrium on dates 0 and 1.

2.4.1 Date 0

The amount of asset purchase by an individual optimistic buyer using a debt contract $\widetilde{D}$ is given by Equation (5). Propositions 1, 3, and 4 jointly determine the optimal contract $\widetilde{D}(p_0)$ based on the buyer's and creditor's heterogeneous beliefs and the asset price p_0. The total measure of buyers in the economy is μ. Their aggregate purchase $\sum_i x_i$ should equal to the total asset endowed by the (pessimistic) sellers $1-\mu$:

$$\sum_i x_i = 1 - \mu. \tag{8}$$

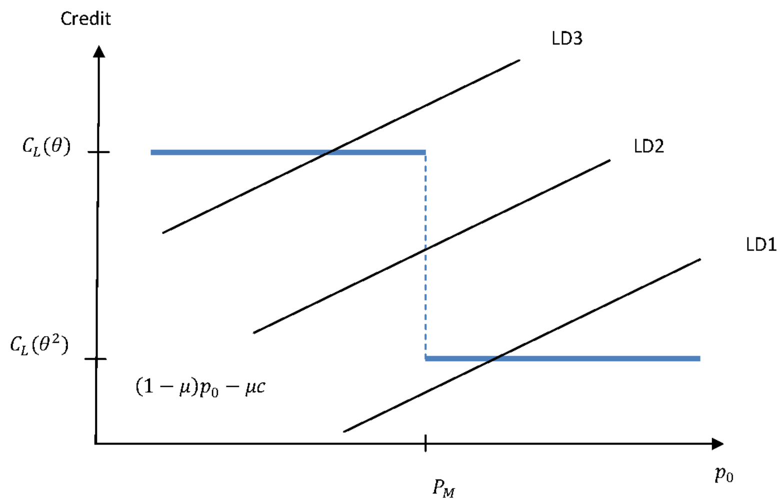

Figure 2: The equilibrium with long-term debt financing.

If all the buyers use the same debt contract $\widetilde{D}(p_0)$ to finance their purchases, the market clearing condition

$$\mu \frac{c + C\left(\widetilde{D}(p_0)\right)}{p_0 - C\left(\widetilde{D}(p_0)\right)} = 1 - \mu$$

implies that

$$C\left(\widetilde{D}(p_0)\right) = (1-\mu)\, p_0 - \mu c. \tag{9}$$

This equation illustrates the intricate interaction between the asset price and the endogenously determined amount of credit to the asset buyers. In light of Propositions 3 and 4, the buyers' optimal credit demand—the term $C\left(\widetilde{D}(p_0)\right)$ on the left hand side—decreases with the asset price p_0. On the other hand, the credit available to the buyers needs to be sufficient to support their asset purchases (market clearing condition), i.e., in equilibrium the buyers' aggregate cash shortfalls should equate their credit demand. The linearly increasing function on the right hand side gives the buyers' cash shortfall—the value of their purchases ($1-\mu$ shares multiplied by the price p_0) minus their cash endowments μc.

Long-term Debt Equilibrium Figure 2 plots the two sides of (9) when the condition in (7) fails and borrowers prefer to use long-term debt. As derived in Proposition 3, each buyer's optimal credit demand can take two possible values, $C_L\left(\theta^2\right)$ or $C_L(\theta)$. The two-piece horizontal line with a downward jump at P_M represents the credit demand $C\left(\widetilde{D}\left(p_0\right)\right)$, i.e., the left hand side of (9). The upward sloping curve represents the necessary credit needed to clear the asset market, the right hand side of (9).

As the intercept of the asset-market clearing condition ($-\mu c$) increases, we encounter three possible cases in equilibrium. First, if each buyer's cash endowment c is high, the optimal credit demand curve and the asset-market clearing condition intersect at a point, where the equilibrium asset price p_0 is higher than P_M and each buyer demands a modest amount of credit $C_L\left(\theta^2\right)$. We label this case by case LD1. In this case, the buyers' ample cash endowments allow them to bid up the asset price to a high level without using much credit. Second, if each buyer's cash endowment c is low, the two curves intersect at a point, where the equilibrium price p_0 is lower than P_M and each buyer demands a large amount of credit $C_L\left(\theta\right)$. We label this case by case LD3. In this case, the buyers' limited cash endowments constrain the price from rising high even though each buyer uses an aggresive debt contract.

The case LD2 occurs when the upward sloping asset-market clearing condition passes the middle of the two horizontal levels of the buyers' credit demand curve. In this case, the equilibrium price is exactly P_M and each buyer is indifferent between using long-term debt contracts with face values θ and θ^2 (Proposition 3). Then, the asset market clearing condition (8) is fulfilled by finding a certain mix of buyers using these two contracts. Denote by λ the fraction of buyers using the contract with face value θ. Condition (8) is equivalent to

$$\mu\lambda\frac{c+C_L\left(\theta\right)}{P_M-C_L\left(\theta\right)}+\mu\left(1-\lambda\right)\frac{c+C_L\left(\theta^2\right)}{P_M-C_L\left(\theta^2\right)}=1-\mu,$$

which implies

$$\lambda=\frac{\frac{1-\mu}{\mu}-\frac{c+C_L\left(\theta^2\right)}{P_M-C_L\left(\theta^2\right)}}{\frac{c+C_L\left(\theta\right)}{P_M-C_L\left(\theta\right)}-\frac{c+C_L\left(\theta^2\right)}{P_M-C_L\left(\theta^2\right)}}. \tag{10}$$

In the following proposition, we summarize the discussion on the joint equilibrium of the asset and credit markets in which the buyers only use long-term debt contracts. In addition, we prove that the buyers have no incentive to keep cash on date 0.

Proposition 5 *Suppose that the condition in (7) fails and the buyers use long-term debt contracts to finance their asset purchases. Then, there is no incentive for any buyer to save cash and the equilibrium can be broken down into the following three cases:*

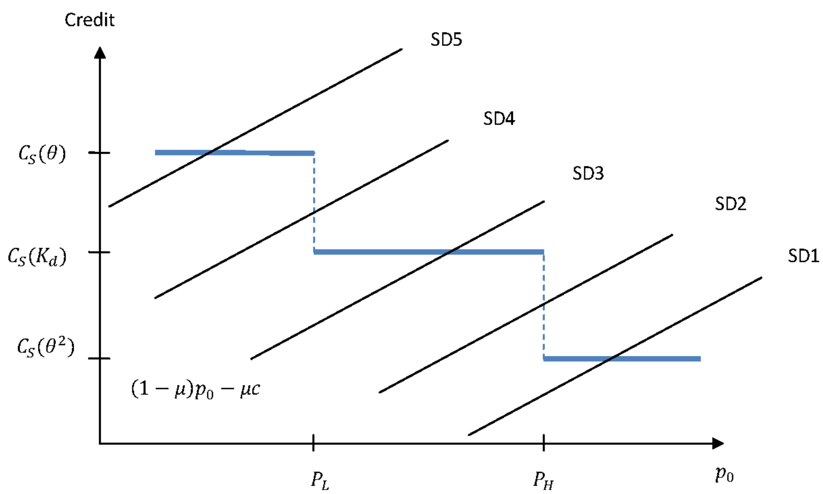

Figure 3: The equilibrium with short-term debt financing.

-LD1: If $C_L\left(\theta^2\right) > (1-\mu)\,P_M - \mu c$, *then* $p_0 = \frac{\mu c + C_L\left(\theta^2\right)}{1-\mu}$ *and all the buyers use the same long-term debt contract with face value* θ^2*;*

-LD2: If $C_L\left(\theta^2\right) \leq (1-\mu)\,P_M - \mu c \leq C_L(\theta)$, *then* $p_0 = P_M$ *and each buyer is indifferent between the long-term debt contracts with face values of* θ *and* θ^2, *with the fraction of buyers using the former contract given in (10);*

-LD3: If $C_L(\theta) < (1-\mu)\,P_M - \mu c$, *then* $p_0 = \frac{\mu c + C_L(\theta)}{1-\mu}$ *and all the buyers use the same long-term debt contract with face value* θ.

Short-term Debt Equilibrium If the condition in (7) holds, the buyers would prefer short-term debt. Figure 3 shows five possible cases for the equilibrium. Proposition 6 lists these cases. The logic for these cases is similar to that for Proposition 5.

Proposition 6 *Suppose that the condition in (7) holds and the buyers use short-term debt contracts to finance their asset purchases. Then, there is no incentive for any buyer to save cash and the equilibrium can be broken down into the following five cases:*

-SD1: If $C_S\left(\theta^2\right) > (1-\mu)\,P_H - \mu c$, *then* $p_0 = \frac{\mu c + C_S\left(\theta^2\right)}{1-\mu}$ *and all the buyers use the same short-term debt contract with face value* θ^2*;*

-*SD2: If* $C_S(\theta^2) \leq (1-\mu)P_H - \mu c \leq C_S(K_d)$, *then* $p_0 = P_H$ *and each buyer is indifferent between the short-term debt contracts with face values of* K_d *and* θ^2, *with the fraction of buyers using the former contract as* $\frac{\frac{1-\mu}{\mu} - \frac{c+C_S(\theta^2)}{P_H - C_S(\theta^2)}}{\frac{c+C_S(K_d)}{P_H - C_S(K_d)} - \frac{c+C_S(\theta^2)}{P_H - C_S(\theta^2)}}$;

-*SD3: If* $(1-\mu)P_L - \mu c < C_S(K_d) < (1-\mu)P_H - \mu c$, *then* $p_0 = \frac{\mu c + C_S(K_d)}{1-\mu}$ *and all the buyers use the same short-term debt contract with face value* K_d*;*

-*SD4: If* $C_S(K_d) \leq (1-\mu)P_L - \mu c \leq C_S(\theta)$, *then* $p_0 = P_L$ *and each buyer is indifferent between the short-term debt contracts with face values of* θ *and* K_d, *with the fraction of buyers using the former contract as* $\frac{\frac{1-\mu}{\mu} - \frac{c+C_S(K_d)}{P_L - C_S(K_d)}}{\frac{c+C_S(\theta)}{P_L - C_S(\theta)} - \frac{c+C_S(K_d)}{P_L - C_S(K_d)}}$;

-*SD5: If* $C_S(\theta) < (1-\mu)P_L - \mu c$, *then* $p_0 = \frac{\mu c + C_S(\theta)}{1-\mu}$ *and all the buyers use the same short-term debt contract with face value* θ.

2.4.2 Date 1

On date 1, there are two states: u and d. In the upper state u, each asset holder's financial condition is strong. Regardless of the possible equilibrium debt contract he has taken on date 0, he faces no default risk going forward. If he has used a short-term debt contract, he can always refinance the maturing debt with a new one. Thus, any potential buyer of the asset has to pay a price equal to the optimistic asset holder's valuation:

$$p_u = \mathbb{E}_u^h\left[\widetilde{\theta}\right] = \pi_u^h + \left(1 - \pi_u^h\right)\theta. \tag{11}$$

In the lower state d, the equilibrium depends on the debt contracts used by the asset holders on date 0. As shown by Propositions 5 and 6, the optimists do not save any cash on date 0. As a result, if any asset holder runs into financial distress after the asset fundamental deteriorates in state d, the price of the asset is determined by the pessimists, instead of the optimists. We determine the equilibrium asset price in this state by the shadow cost of a potential buyer, who needs to buy out the stake of the current asset holder in the asset as well as the stake of his creditor.

The following proposition summarizes the equilibrium on date 1.

Proposition 7 *On date 1, in the upper state* u, *each asset holder faces no default risk going forward and the asset price is given in (11). In the lower state* d, *the equilibrium depends*

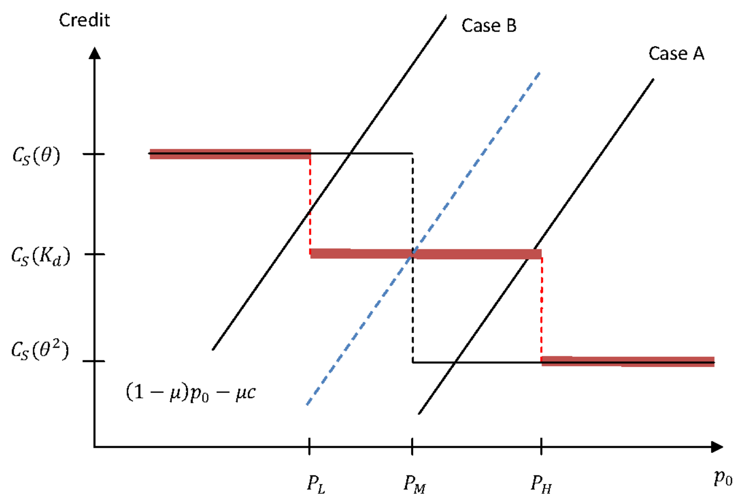

Figure 4: An illustration of the equilibrium effect of using short-term debt.

on the debt contracts used by the asset holders, which are given by different cases derived in Propositions 5 and 6. In cases LD1 and SD1, all the asset holders use debt contracts with face value θ^2, and face no default risk going forward. As a result, the equilibrium asset price is determined by their valuation:

$$\mathbb{E}_d^h\left[\widetilde{\theta}\right] = \pi_d^h\theta + \left(1-\pi_d^h\right)\theta^2.$$

In the other cases, at least some of the asset holders used aggresive debt contracts with face values of K_d or θ, and these asset holders are now in distress. As a result, the equilibrium asset price is determined by the pessimistic creditors' valuation:

$$\mathbb{E}_d^l\left[\widetilde{\theta}\right] = \pi_d^l\theta + \left(1-\pi_d^l\right)\theta^2.$$

2.5 The Role of Short-term Debt

We now analyze the equilibrium effect of short-term debt in financing optimistic buyers' asset purchases. To facilitate our analysis, suppose that short-term debt is initially not available and asset buyers have access only to long-term debt. Figure 4 illustrates the changes in the equilibrium after the asset buyers have access to short-term debt.

Let us first focus on Case A shown in the figure. In the absence of short-term debt, the asset buyers manage to purchase the asset at a price higher than P_M by using risk-free long-term debt. As we have derived in Proposition 5, this situation occurs only when the asset buyers' initial cash endowment is reasonably high so that a modest amount of leverage is enough for the buyers to clear the asset market. The price level is also high so that each buyer is discouraged from using the more aggresive long-term debt contract with face value θ to take a greater position. However, if the buyers are allowed to use short-term debt contracts, their credit demand curve shifts up when the asset price p_0 is between the two critical levels P_M and P_H. This is because short-term debt allows the buyers to obtain the same credit at a lower cost under the condition in Proposition 1. This shift causes buyers to use more credit and results in a higher equilibrium asset price. Figure 4 allows us to precisely identify the condition for this situation to occur and the exact changes in equilirbium price and credit usage.

Proposition 8 *Suppose that the condition in (7) holds, and that*

$$(1-\mu) P_M - \mu c < C_S (K_d) .$$

Then, after the introduction of short-term debt, the equilibrium asset price on date 0 *increases, and the price increase is supported by the use of the short-term debt contract with face value* K_d *by at least some of the asset buyers.*

To some extent the situation analyzed in Proposition 8 is analogous to the US housing market in the pre-ARM (adjustable rate mortgage) era. Before the ARMs became popular in the last decade, home buyers had predominantly relied on 30-year fixed rate mortgages to finance their home purchases. Historically speaking, these long-term mortgage loans had low default rates. According to Mayer, Pence, and Sherlund (2009), the share of US mortgage loans that were "seriously deliquent" (90 days or more past due or in the process of foreclosure) averaged only 1.7 percent from 1979 to 2006. These fixed-rate mortgages thus resemble the risk-free long-term debt contracts derived in case A of Figure 4.

The ARMs are effectively short-term loans because they typically have a fixed "teaser" rate for 2 or 3 years, which is scheduled to rise by two or more percentage points after the initial period ends. The rate hike often forces the home owners to refinance their mortgages at the market rate, in the same spirit of rolling over the short-term debt on the interim date of our model. If the housing price rises when the rate is reset, a home owner faces no difficulty in refinancing his loan at the market rate. However, if the housing price falls, the home owner will not be able to refinance his loan and will be strained by the largely increased mortgage payment. As a result, he may be forced to turn over the home to his creditor.

Mayer, Pence, and Sherlund (2009) document that by mid-2008, serious delinquencies on ARMs had risen to over 29 percent, while the similar rate for fixed-rate mortgages rose only to 9 percent.[5]

Financial innovations, such as the securitization of subprime loans, were at least partially responsible for the popularity of ARMs in the last decade. Thus the synchronous growth of ARMs and housing prices during the recent boom, together with the large spike in the delinquency rate of ARMs during the bust, match squarely with the situation described in Proposition 8.[6]

Figure 4 also illustrates another possiblity, case B, in which each asset buyer has scarce cash endowment. As a result, in the absence of short-term debt the equilibrium asset price remains below P_M despite the buyers' aggresive use of long-term debt contract with face value θ. Then, the introduction of short-term debt allows the buyers to use a more modest short-term debt contract with face value K_d. As a result, the equilibrium asset price drops, accompanied by a reduction in credit used by the buyers.

Proposition 9 *Suppose that the condition in (7) holds and that*

$$(1-\mu) P_M - \mu c > C_S (K_d) .$$

Then, after the introduction of short-term debt, the equilibrium asset price on date 0 *decreases and the price decrease is accompanied by the shift to the more modest short-term debt contract with face value* K_d *by at least some of the asset buyers.*

Taken together, Propositions 8 and 9 demonstrate that introducing short-term debt could either increase or decrease the credit used by optimists in the equilibrium, depending on the severity of their cash constraints. If they are so severely constrained that the asset prices remain low even after they have used extremely aggressive long-term debt, then introducing short-term debt would actually allow them to reduce their leverages. This result is intriguing from a theoretical perspective. However, this situation is probably less relevant to the recent

[5] The ARMs were primarily used to finance the home purchases of sub-prime and near-prime households, who were constrained from qualifying for the regular fixed-rate mortgages. Mian and Sufi (2009) find that from 2002 to 2005, areas with disproportionately large share of subprime borrowers in 1996 had experienced an unprecedented growth in subprime credit, despite the sharply declining relative income growth in these areas. Interestingly, these areas also had significantly higher mortgage delinquency rates in 2007 when the housing prices across US started to decline.

[6] One could also attribute the synchronous growth of ARMs and housing prices to a growing divergence of beliefs among agents about the housing-market fundamentals. This argument is also consistent with our model, which we will analyze in the next subsection.

housing boom and other historical episodes. Thus, we will focus on the situation identified in Proposition 8 in the following analysis.

2.6 Heterogeneous Beliefs and Asset Price Cycles

In this subsection, we analyze the effects of agents' heterogeneous beliefs in driving the boom-and-bust cycle of asset prices. The standard result of Miller (1977) suggests that heterogeneous beliefs cause asset overvaluation in the presence of short-sales constraints. Our analysis will focus on the interaction between heterogeneous beliefs and debt financing of optimistic buyers. For illustration, we use the following baseline parameter values:

$$\mu = 0.3, c = 0.5, \theta = 0.4, \pi_0^h = 0.7, \pi_0^l = 0.3, \pi_u^h = 0.6, \pi_u^l = 0.4, \pi_d^h = 0.6, \pi_d^l = 0.4. \tag{12}$$

These numbers imply the following: Optimists consist of 30% of the population and each is endowed with 0.5 dollar in cash. The final asset payoff can be 1, 0.4, or 0.16. We let the objective probability of the tree going up each period be 0.5 and the optimists and pessimists' beleifs be equally spread around the objective probability. As learning is likely to cause belief dispersion to decrease over time, we make the beliefs of the optimists and pessimists on date 0 to be 0.7 and 0.3, and on date 1 in both of the u and d states to be 0.6 and 0.4.

In the illustration, we examine the equilibrium asset price dynamics when 1) only LD (long-term debt) is available, and 2) both LD and SD (long-term and short-term debt) are available. The difference between these two cases highlights the role of short-term debt. We also discuss the debt contracts used by the asset buyers to finance their purchases.

2.6.1 Initial Belief Dispersion on Date 0

We first examine the effect of the initial belief dispersion on date 0. We let the values of π_0^h and π_0^l to deviate from their baseline value and instead take the following values:

$$\pi_0^h = 0.5 + \delta_0 \text{ and } \pi_0^l = 0.5 - \delta_0$$

where δ_0 changes from 0 to 0.45 and drives the initial belief dispersion between the optimists and pessimists. Figure 5 illustrates the asset market and credit market equilibrium. Panel A plots the date-0 asset price p_0 with respect to δ_0. The horizontal dotted line at the 0.49 level represents the asset's fundamental valued by the objective probabilities. The dotted upward sloping line represents the asset price in the Miller setting, where optimists always

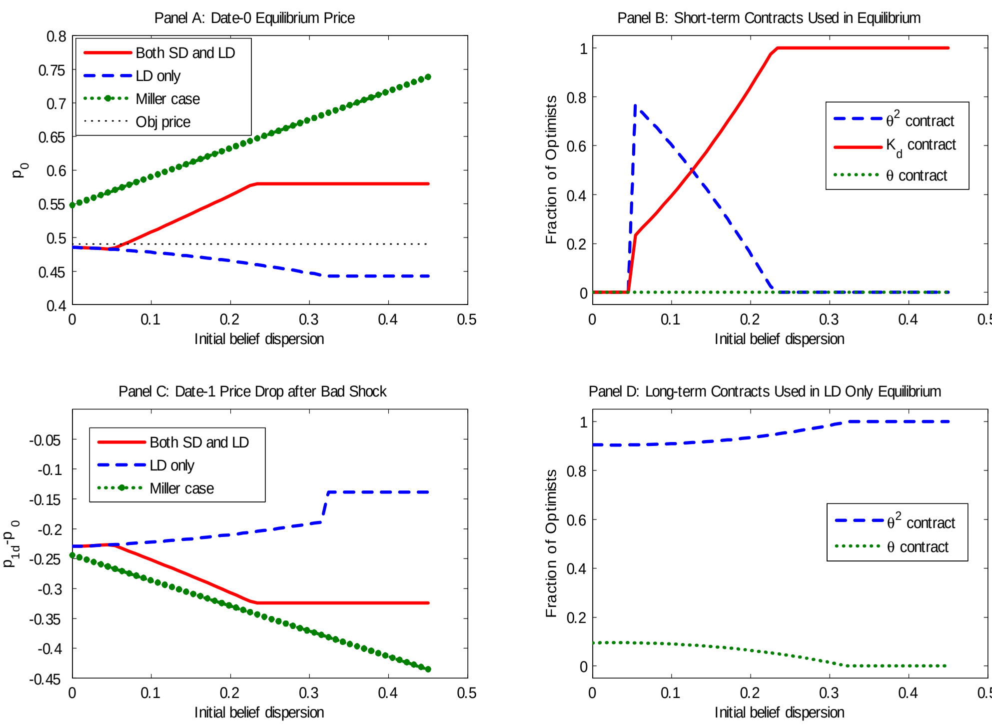

Figure 5: The equilibrium effects of initial belief dispersion on asset and credit markets.

have sufficient funds to execute their purchases. As δ_0 increases from 0 to 0.45, the optimists become more optimistic and p_0 increases from 0.548 to 0.74 (p_0 is higher than 0.49 at $\delta_0 = 0$ because of the belief dispersion on date 1).

LD-only equilibrium The dashed line plots p_0 when the optimists have access only to long-term debt to finance their asset purchases. Interestingly, this price decreases from 0.485 to 0.443 as δ_0 increases from 0 to 0.32 and then stays flat as δ_0 increases further. The dramatic difference between this line and the price under the Miller setting origins from the optimists' financing cost. The increase in belief dispersion not only makes the optimists more optimistic, but also makes the pessimists more pessimistic. As a result, optimists face more costly financing from the pessimists even though their own valuation of the asset is higher. The financing cost gives an indirect channel for the pessimists to affect the equilibrium price despite the short-sales constraints. In the illustrated LD only equilibrium, the financing-cost effect can even overturn the standard Miller result and cause the equilibrium price to decrease with agents' belief dispersion.

Panel D provides a breakdown of the long-term debt contracts used by the optimists in the LD only equilibrium. Initially, when δ_0 increases from 0 to 0.32, there is a mix of optimists using long-term contracts with face values of θ and θ^2 (e.g., the LD2 case in Proposition 5) and the fraction of optimists using the risk-free contract with face value θ^2 rises from 0.9 to 1. This fraction stays at 1 as δ_0 increases further (e.g., the LD1 case in Proposition 5).

Equilibrium with both SD and LD available The solid line in Panel A plots p_0 when the optimists can choose between long-term and short-term debt. p_0 slightly decreases from its initial value at 0.485 as δ_0 increases from 0 to 0.05, then monotonically increases to 0.58 as δ_0 increases further to 0.23, and finally stays flat at 0.58 as δ_0 continues to rise. This pattern is dramatically different from that of the asset price in the LD only equilibrium. This difference highlights the role of short-term debt in reducing the optimists' financing cost.

Panel B provides a breakdown of the short-term debt contracts used by the optimists. Over the region $\delta_0 \in [0, 0.05]$, the optimists do not use any short-term debt. The reason is Proposition 1: Short-term debt is advantageous to long-term debt only when the specualtive incentives caused by both parties' initial belief dispersion dominates the rollover-risk effect due to their future belief dispersion. Once δ_0 rises above 0.05, the optimists start to use a mix of short-term debt contracts with face values θ^2 and K_d. The fraction of optimistis using the more aggresive contract with face value K_d rises monotonically from 0 to 1 as δ_0 rises from 0.05 to 0.23, and stays at 1 as δ_0 rises further. This panel shows that the increase of the equilibrium price with δ_0 in the region $\delta_0 \in [0.05, 0.23]$ is financed by the optimists' increasing reliance on the short-term debt with face value K_d. Taken together, even though the asset price in the collateral equilibrium is substantially lower than that in the standard Miller setting, short-term debt allows the optimists to manage their financing cost more effectively and thus to preserve the standard Miller result on the equilibrium price increasing with agents' belief dispersion.

Date-1 crash Panel C of Figure 5 plots the price drop on date 1 when the lower state d is realized, i.e., $p_d - p_0$, under different settings. After the realization of the negative shock, the asset price drops and the optimistic asset holders suffer losses on their positions. In the Miller setting, as the date-0 price p_0 monotonically increases with the initial belief dispersion δ_0, the price drop in state d also increases with δ_0. However, in the LD only equilibrium, the large financing cost constrains optimists from fully bidding up the asset price based on their belief on date 0. As a result, the price drop in state d becomes decreasing with δ_0. In contrast, the price drop in the setting with both LD and SD available is generally increasing with δ_0. In fact, the slope of the price drop with respect to δ_0 in the LD-and-SD equilibrium is even steeper than that in the Miller setting in the middle region. This is because of the rollover risk effect. When the optimists are financed by short-term debt, they are forced to turn over their asset to the pessimistic creditor after the negative shock. This causes the price to drop more than that in the Miller setting, where the optimists are always the marginal investor. This effect shows that not only can short-term debt fuel the asset over-valuation on date 0, but can also exacerbate the downturn after a negative shock.

2.6.2 Future Belief Dispersion on Date 1

Next, we examine the effects of the belief dispersion between the optimists and pessimists on date 1. We will focus on the dispersion in the lower state d. Proposition 1 suggests that the belief dispersion in this state introduces rollover risk, which discourages optimists from using short-term debt. Specifically, we deviate from the baseline parameters in (12) by specifying the following beliefs for the optimists and pessimists:

$$\pi_d^h = 0.5 + \delta_d, \text{ and } \pi_d^l = 0.5 - \delta_d$$

where δ_d changes from 0 to 0.45. Figure 6 illustrates the impact on the asset and credit markets.

Panel A of Figure 6 plots p_0 with respect to δ_d. In the Miller setting, p_0 is again increasing with δ_d for the same reason as before—an increase in δ_d makes the optimists more optimistic about the asset fundamental. In contrast, p_0 decreases with δ_d in both the equilibria with LD and SD and with LD only. This pattern is different from the illustration in Figure 5—while p_0 in the LD only equilibrium is decreasing with δ_0, p_0 tends to increase with δ_0 when both LD and SD are available. Taken together, Figures 5 and 6 suggest that when only long-term debt is avaliable, an increase in either δ_0 or δ_d causes the equilibrium price to drop because the increases in belief dispersion increases the optimists' finacing cost and this effect can dominate the increase in the optimists' asset valuation. But when short-term debt is available, these two types of belief dispersion can have different impacts. In particular, an increase in δ_d unambiguously discourages the use of short-term debt because of the increased

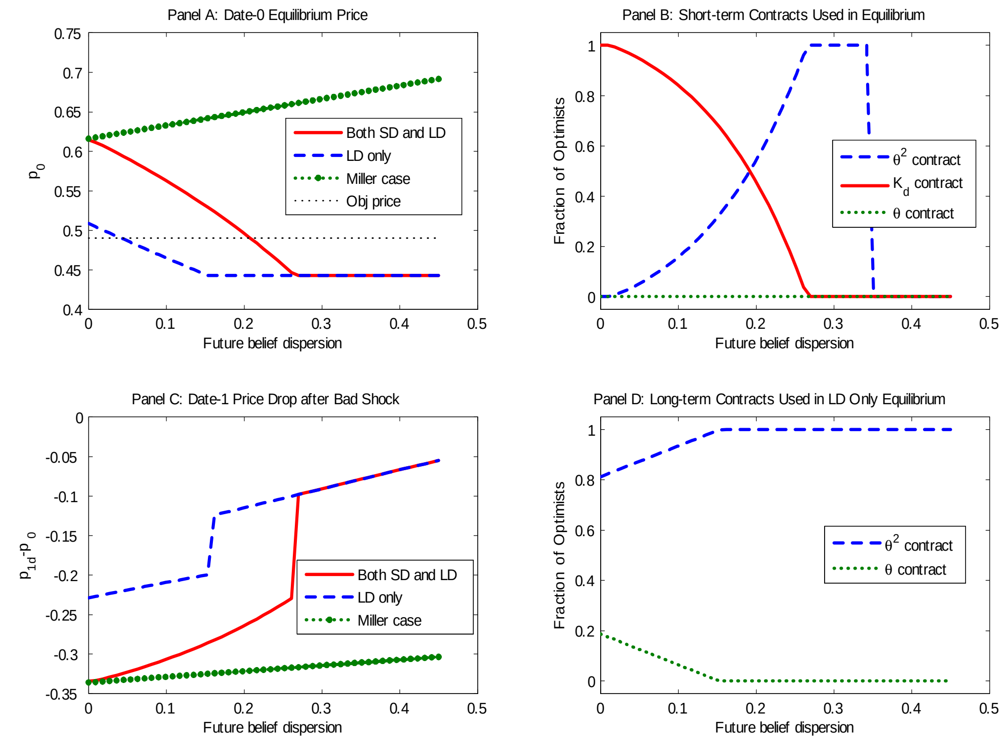

Figure 6: The equilibrium effects of long-run belief dispersion in state d on the asset and credit markets.

rollover risk. However, optimists may use short-term debt to take advantage of the trading opportunity presented by the increase in δ_0, which leads the asset price to rise with δ_0.

In general, we can prove the following proposition regarding the effects of these two types of belief dispersion on the date-0 equilibrium price.

Proposition 10 *Based on the conditions listed in Proposition 8, the date-*0 *asset price* p_0 *increases with the belief dispersion between the optimists and pessimists on date* 0 *and decreases with their belief dispersion in the lower interim state* d*.*

3 An Extended Model with Learning

In this section, we extend the baseline model with learning. We allow each agent to update his belief about the asset fundamental on date 1 based on the realized fundamental shock. Such learning justifies the state-contingent beliefs specified in the baseline model. Learning can also lead to the flips of beliefs across agents, which in turn intensifies the speculative incentives of agents through asset holders' resale options, a la Harrison and Kreps (1978). The learning technology we adopt is analogous to that used by Morris (1996).

3.1 The Model Setting

Suppose that the fundamental move on the tree is independently and identically distributed in each period. Let π be the unobservable probability of an upward jump. On date 0, each agent has a prior about the distribution of π. There are three groups of risk-neutral agents, who differ in their priors about the distribution of π. We label these groups by A, B, and C. Suppose that the prior of a group-i agent ($i \in \{A, B, C\}$) has a beta distribution with parameters $\left(\alpha^i, \beta^i\right)$.[7] We denote the mean of this distribution as the agent's prior belief:

$$\pi_0^i \equiv \frac{\alpha^i}{\gamma^i}$$

where $\gamma^i \equiv \alpha^i + \beta^i$ represents the agent's confidence about his prior belief. This confidence determines how much the agent reacts to new information on date 1.

On date 1, if the tree moves up, each agent will update his belief in response to the positive shock. The agent's posterior still has a beta distribution, but with parameter $\left(\alpha^i + 1, \beta^i\right)$, which has an increased confidence $\gamma^i + 1$ and a posterior belief of

$$\pi_u^i = \frac{\gamma^i}{\gamma^i + 1}\pi_0^i + \frac{1}{\gamma^i + 1}.$$

One can intuitively interpret the posterior belief as a weighted average of the prior belief π_0^i and the new information shock 1, where the weights depend on the confidence of the prior γ^i and the precision of the information 1. If the agent is more confident about his prior, he puts more weight on the prior but less weight on the information shock.

If the tree moves down, the agent's posterior is a beta distribution with parameters $\left(\alpha^i, \beta^i + 1\right)$, which also has an increased confidence $\gamma^i + 1$ and a posterior belief of

$$\pi_d^i = \frac{\gamma^i}{\gamma^i + 1}\pi_0^i.$$

[7] A beta distribution with parameters (α, β) with $\alpha > 0$ and $\beta > 0$ is defined on the interval $(0, 1)$, and has density function:

$$f(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}.$$

The mean of the distribution is $\frac{\alpha}{\alpha + \beta}$.

To facilitate our analysis, we let the group-A agents be the optimists on date 0 and the group-B and group-C agents share the same pessimistic prior belief:

$$\pi^h \equiv \pi_0^A > \pi^l \equiv \pi_0^B = \pi_0^C.$$

Furthermore, we assume that the two groups of pessimists differ in the confidence of their priors—$\gamma^B > \gamma^A > \gamma^C$—so that the group-$C$ agents will react most strongly to the positive shock in the interim state u and can become buyers of the group-A agents' asset on date 1, while group-B agents have the least reaction to the shocks, including the negative shock in state d, and thus become the natural creditor to finance the group-A agents' asset purchases on date 0. For simplicity, we assume that the group-B and group-C agents have sufficient cash endowments to fulfill any trade or lending agreement they desire.

More specifically, in the interim state u, the group-C agents become more optimistic than group-A agents if γ^C is sufficiently small so that

$$\pi_u^C = \frac{\gamma^C}{\gamma^C + 1}\pi^l + \frac{1}{\gamma^C + 1} > \pi_u^A = \frac{\gamma^A}{\gamma^A + 1}\pi^h + \frac{1}{\gamma^A + 1}.$$

If so, group-A agents will sell their asset to group-C agents at a price equal to the group-C agents' valuation. Note that in state u, the group-B agents' belief is always lower than that of group-A agents because of their pessimistic prior belief and higher confidence about the prior. Thus,

$$p_u = \max\left\{\pi_u^A + \left(1 - \pi_u^A\right)\theta,\ \pi_u^C + \left(1 - \pi_u^C\right)\theta\right\}.$$

The option to resell the asset to the group-C agents at a higher price is valuable to the group-A agents, and motivates them to pay a price on date 0 that is higher than their buy-and-hold valuation, even though they already hold the most optimistic valuation, e.g., Harrison and Kreps (1978). This speculative component in asset price has been widely used to study asset-price bubbles, e.g., Scheinkman and Xiong (2003).

3.2 The Financing-Cost Effects on Price Bubble

In this section, we focus on analyzing how financing cost affects speculation by the optimists and the equilibrium price bubble. We adopt the same assumptions from the baseline model on the agents' initial cash and asset endowments. In light of our analysis in the previous section, their financing cost is determined by the creditor's beliefs about the likelihood of the default states. Since the group-B agents are less responsive to the negative shock in the

Table 3: Asset Payoff and Debt Payment in the Extended Model

	Tree Path			
	uu	ud	du	dd
Asset payoff	1	θ	θ	θ^2
Payoff to the initial buyers	p_u	p_u	θ	θ^2
Long-term debt face value $F_L \in \left[\theta^2, \theta\right]$	F_L	F_L	F_L	θ^2
Short-term debt face value $F_S \in \left[\theta^2, K_d\right]$	F_S	F_S	$F_{S,1} \geq F_S$	θ^2
Short-term debt face value $F_S \in [K_d, \theta]$	F_S	F_S	θ	θ^2

lower interim state d, credit provided by them is cheaper than that by the group-C agents. We also assume that γ^B is not too large so that the belief of the group-B agents is always lower than that of the group-A agents.

We can easily extend our derivation of the baseline model to cover the extended model. The group-A agents correspond to the optimistic buyers in the baseline model, and the group-B agents correspond to the pessimistic creditors. Their state-dependent beliefs are now determined by their priors on date 0 and learning processes on date 1. The presence of group-C agents provides group-A agents the resale option on date 1 in the upper state u. We summarize the asset payoff to the initial buyers and their debt payments from using different contracts in Table 3, which differs from Table 2 in the asset payoff only on paths uu and ud due to the resale option.

The changes in the asset payoff to the initial buyers do not affect the payments of the equilibrium-relevant debt contracts, and thus do not affect the buyers' optimal debt maturity choice given in Proposition 1. Propositions 3, 4, 5, and 6 also remain the same, except that we have to modify the expressions for P_M, P_L, and P_H to account for the changes in the asset payoff on the uu and ud paths:

$$P_M = \left(\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l\right) \frac{\pi_0^h p_u + \left(1 - \pi_0^h\right) \pi_d^h \theta}{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h} + \left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right)\theta^2,$$

$$P_H = \frac{\pi_d^l \left[\pi_0^h p_u + \left(1 - \pi_0^h\right) \pi_d^h \theta\right] + \left(1 - \pi_0^h\right) \pi_d^h \left(1 - \pi_d^l\right) \theta^2}{\pi_0^h \pi_d^l + \left(1 - \pi_0^h\right) \pi_d^h},$$

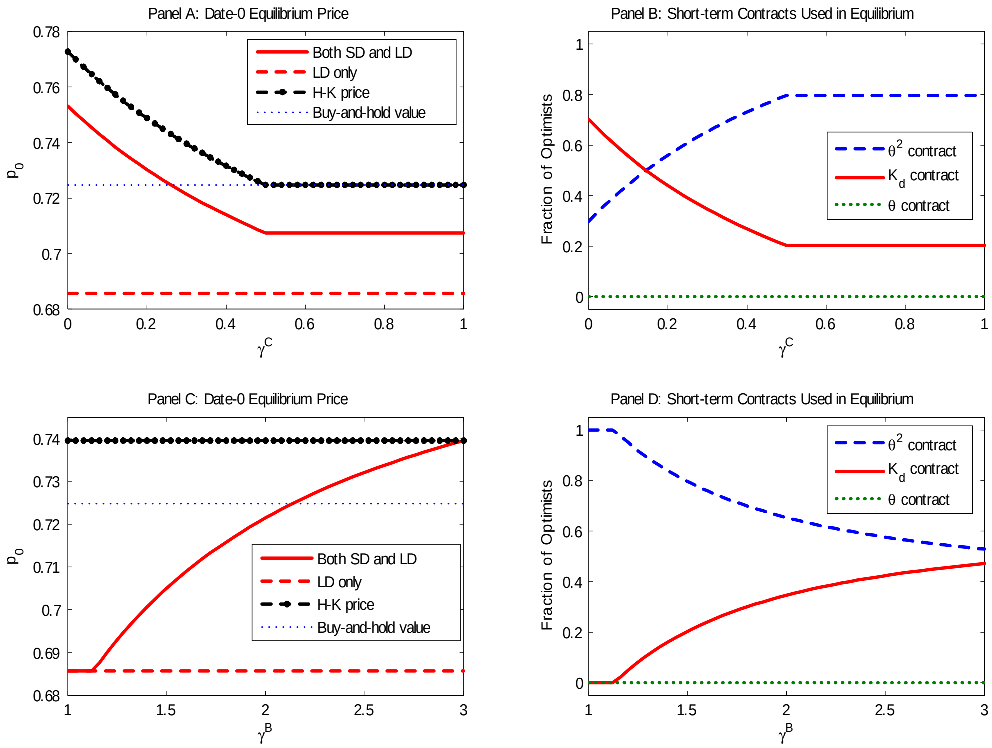

Figure 7: The equilibrium effects of learning.

and

$$P_L = \pi_0^l p_u + \left(1 - \pi_0^l\right)\left[\pi_d^l \theta + \left(1 - \pi_d^l\right)\theta^2\right].$$

To illustrate the effects of financing cost on the asset price bubble, we use a set of numerical examples, based on the following baseline parameters:

$$\mu = 0.3, c = 0.5, \theta = 0.4, \pi^h = 0.6, \pi^l = 0.4, \gamma^A = 1, \gamma^B = 0.3, \gamma^C = 2.$$

We focus on varying the values of γ^B and γ^C, which control the learning intensities of the creditors to the initial asset buyers and the potential new buyers of the asset, respectively.

Figure 7 illustrates the equilibrium effects of varying γ^C from 0 to 1 and γ^B from 1 to 3. As γ^C decreases, the belief of the group-C agents (the potential asset buyers in the upper interim state u) becomes more responsive to the information shock on date 1 and thus increases the initial buyers' resale option value in the upper state u. Panel A plots the equilibrium price p_0 with respect to γ^C. The flat dotted line at 0.725 provides the group-A agents' buy-and-hold value on date 0, while the dashed line with big dots provides their valuation in the Harrison-Kreps setting, which takes into account the resale option under the assumption that they always have sufficient funds for their asset purchases. The Harrison-Kreps price starts to rise above the buy-and-hold value as γ^C drops below a critical level around 0.48, below which the group-C agents' belief in the u state becomes higher than that of the group-A agents.

The flat dashed line at 0.686 represents the equilibrium price when the buyers have access to only long-term debt. As this line is substantially below the group-A agents' buy-and-hold valuation on date 0, it suggests that the cost of using long-term debt financing severely constrains the optimists from biding up the asset price. Interestingly, the solid line shows that once the optimists are allowed to use short-term debt, the equilibrium price is always above the price level in the LD only equilibrium, and starts to rise when γ^C drops below 0.48 in parallel with the Harrison-Kreps price. In fact, the price eventually passes the optimists' buy-and-hold valuation when γ^C drops below 0.26. This suggests that the cheap financing provided by short-term debt makes it possible for the optimists to bid up the price to levels closer to their speculative valuations without financing cost. Panel B also plots the types of short-term debt contracts used by the initial buyers. The plot shows that the price increase is financed by their increasing use of the more aggressive debt contract with face value K_d.

As γ^B increases, the belief of the group-B agents (the creditor to the initial buyers) becomes more stable in the lower interim state d and thus reduces the buyers' rollover risk. Panel C of Figure 7 plots the equilibrium price p_0 with respect to γ^B. The plot gives the two benchmark price levels, the group-A agents' buy-and-hold valuation and the Harrison-Kreps price by the two horizontal lines at 0.725 and 0.74, respectively. If group-A agents have access to only long-term debt, the financing cost constrains them to bid up the price only to 0.686, which is substantially below the two benchmark levels. When short-term debt is available, the financing cost becomes lower, especially when the rollover risk is low. Panel C shows that as γ^B increases, the reduced financing cost allows the optimists to bid up the price closer to the Harrison-Kreps price. In fact, the two prices coincide when $\gamma^B = 3$, at which point $\pi_d^A = \pi_d^B$ (i.e., there is no rollover risk.)

4 Discussion

In this section, we relate our model to several bubble-and-crisis episodes and discuss various model implications.

The credit crisis of 2007-2008 This crisis followed a dramatic housing price boom across many areas of US from late 1990s to 2007. Different from many previous regional housing booms, which were typically stimulated by excitements about the economic and/or population growth of the involved regions, this national housing boom was at least partially stimulated by the financial innovation of securitizing non-standard sub-prime mortgage loans, which had held the promise of broadening home ownership to many low-income households. Like many technology innovations in the past, this innovation had stimulated widespread housing speculation across the country. As we have discussed in Section 2.5, this housing boom was accompanied by a rapid growth of ARMs, a type of short-term mortgage loans, which allowed many subprime or near-prime households to finance their home purchases with low down payments. Interestingly, these ARMs also experienced much higher default rates when the housing prices started to decline after 2007 and thus exacerbated the housing market downturn.

Like the households, investment banks also experienced a concurrent leverage cycle. Adrian and Shin (2008) show that during the housing market boom before 2007, investment banks had aggressively expanded their holdings of mortgage-backed securities and other assets by using repurchase agreements (repos), a form of short-term debt collaterized by liquid financial assets. Interestingly, not only had investment banks used higher leverages, the maturity of their repo financing had also been significantly shortened. According Brunnermeier (2009), the fraction of total investment bank assets financed by overnight repos roughly doubled from 2000 to 2007, while fraction by term repos with a maturity of up to three months have stayed roughly constant. In other words, investment banks had gradually increased their reliance on overnight financing during the housing booming. The increased reliance on overnight repos later contributed to the collapses of Bear Stearns and Lehman Brothers in 2008 because both had difficulties in rolling over their repos, e.g., Brunnermeier (2009), Gorton and Metrick (2009), and Krishnamurthy (2010).

The 1929 crash The 1920s was a decade of expansion, propelled by new information technologies like radio and new processes like motor vehicle production using assembly-line methods. These new technologies had stimulated intensive speculation in the stock market, and leverages were widely used by firms, trusts and individuals to finance their speculation. White (1990) provides detailed information on the New York Stock Exchange's brokers' loans, which allowed investors to borrow on margin from their brokers to finance their stock purchases. He shows that the volume of brokers' loans had risen and fallen in sync with the stock market index throughout the boom-and-bust period between 1926 and 1930. Like the modern repos, there were two types of brokers' loans, call loans (demand loans) and time loans with common maturities of 60 and 90 days. Rappoport and White (1993) document evidence of maturity shortening for brokers' loans before the stock market crash in October 1929: "In 1926 and 1927, time loans accounted for between 21 and 32 percent of all brokers' loans, but after mid-1928, they declined to under 10 percent."

The Emerging-Market Debt Crisis in 1990s Short-term debt had also been heavily used by many emerging countries to finance their economic booms in the 1990s. According to Rodrik and Velasco (1999), the outstanding stock of debt of emerging-market economy roughly doubled between 1988 and 1997, from \$1 trillion to \$2 trillion. While medium- and long-term debt grew rapidly as well, it was short-term debt that rose particularly rapid during this period. They further show that in a data sample covering 32 emerging-market economies over the period 1988-1998, the ratio of short-term debt to foreign reserve is a robust predictor of financial crises triggered by reversals of capital flows. Reinhart and Rogoff (2009) provide similar evidence in a larger sample of emerging-market economies over a longer period.

Demand-driven credit expansions One common feature across the aforementioned three financial market boom-and-bust episodes, which occurred in different time periods and in different countries, is that the booms were all heavily financed by short-term debt. It is important to differentiate the shortening of debt maturity during the booming periods from another common phenomenon—the shortening of debt maturity during the crises. When a crisis disrupts, creditors tend to become more averse to uncertainty and illiquidity. As a result, they are less willing to lend and especially less willing to lend for longer terms. Such a contraction from the supply side can easily explain the shortening of debt maturity during crises. However, it is not so obvious for any supply-driven credit expansion theory to explain the shortening of debt maturity during booms.

To the extent that short-term debt reduces the borrowers' financial stability, the increasing short-term leverages during the aforementioned market booms reflected the borrowers' optimism about the future market perspectives, as well as the creditors' concerns about the borrowers' ability to repay their debt in the long term. Thus, these short-term credit booms were at least partically driven by demands of borrowers to speculate in the asset markets. Of course, neither can one deny the importance of the ready availability of credit from the supply side. The global savings glut and low interest rate environment in the early 2000s had made abundant credit available to the housing speculators and optimistic investment banks. The ample gold reserves accumulated by the US during World War I also made credit readily available for stock speculators during the stock market boom before 1929, e.g., Eichengreen and Mitchener (2003).

Our model provides an analytical framework based on the heterogeneous beliefs between the optimistic borrowers and not so optimistic creditors to help understand credit expansions from the demand side. In particular, our model provides a sharp prediction about the type of debt contracts used in the expansions: Greater initial disagreement about asset fundamentals makes short-term debt more preferably; while greater future disagreement makes long-term debt more preferably.

Predicting future crises There have been many financial market booms. But not every boom ended with a financial crisis. What is so special about the three episodes discussed above? The high leverages and, more importantly, the high short-term leverages used by the optimistic speculators are certainly crucial. Across all these episodes, short-term debt created the speculators' financing difficulties during the downturns. In the absence of short-term debt, the speculators would have suffered large losses when the fundamental shocks went against them, but the markets may not have experienced the severe crises. The experience of the Internet bubble in the late 1990s is a good example. While the day traders of the Internet stocks had lost substantial wealth when the bubble burst in early 2000, not any major bank or financial institution had failed, very different from the situation after the decline of the housing market in 2007. In junction with our model, we have the following prediction: The combination of a short-term credit boom and an asset price boom tend to predict a higher probability of future financial crisis.

Consistent with this prediction, Borio and Drehmann (2009) show that unusually strong increases of both credit and asset prices have out-of-sample predicting power for banking crises by using data from 1980 to 2008 and across a set of countries.

Regulating short-term leverages Should the policy makers regulate short-term leverages? Our model suggests that this issue is subtle as Propositions 8 and 9 indicate opposite effects of short-term debt on the equilibrium depending on the market condition. When the optimistic asset buyers' initial cash endowment is above a certain threshold, introducing short-term debt induces them to take on higher leverages and push the asset price even higher. The opposite can happen if their cash endowment is below the threshold. In this situation the buyers are so cash-constrained that they use high long-term leverages to finance their asset purchases, then introducing short-term debt allows them to replace the high long-term leverages with lower short-term leverages and thus to reduce the asset price.

5 Conclusion

Appendix A Proofs for Propositions

A.1 Proof of Proposition 1

There are two cases depending on whether the short-term debt is risky. Suppose that the short-term debt is riskless, i.e., its face value $F_S \in \left[\theta^2, K_d\right]$. Then, on date 0 the optimistic borrower can raise $D_S(F_S) = F_S$ from the debt contract. His expected debt payment is (recall (4) and Table II)

$$\mathbb{E}_0^h\left[\widetilde{D}_S\right] = \pi_0^h F_S + \left(1-\pi_0^h\right)\pi_d^h\left(\frac{F_S}{\pi_d^l} - \frac{1-\pi_d^l}{\pi_d^l}\theta^2\right) + \left(1-\pi_0^h\right)\left(1-\pi_d^h\right)\theta^2.$$

On the other hand, the long-term debt contract that delivers the same initial credit as F_S requires a face value of F_L such that

$$C_L(F_L) = \left(1-\left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\right)F_L + \left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\theta^2 = F_S.$$

This implies that

$$F_L = \frac{F_S - \left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\theta^2}{1-\left(1-\pi_0^l\right)\left(1-\pi_d^l\right)}.$$

Then, the borrower's expected payment by using the long-term contract is

$$\mathbb{E}_0^h\left[\widetilde{D}_L\right] = \left(1-\left(1-\pi_0^h\right)\left(1-\pi_d^h\right)\right)F_L + \left(1-\pi_0^h\right)\left(1-\pi_d^h\right)\theta^2.$$

Therefore, the difference between the costs of the short-term and long-term debt contracts is

$$\mathbb{E}_0^h\left[\widetilde{D}_L\right] - \mathbb{E}_0^h\left[\widetilde{D}_S\right] = \frac{\pi_0^h\left(1-\pi_0^l\right)\pi_d^l - \pi_0^l\left(1-\pi_0^h\right)\pi_d^h}{\pi_d^l\left(\pi_0^l+\pi_d^l-\pi_0^l\pi_d^l\right)}\left(1-\pi_d^l\right)\left(F_S-\theta^2\right).$$

The short-term debt contract is less costly if and only if (7) is satisfied.

We follow a similar procedure for the case that $F_S \in (K_d, \theta]$. The borrower's expected debt payment by using a short-term debt contract with face value F_S is

$$\mathbb{E}_0^h\left[\widetilde{D}_S\right] = \pi_0^h F_S + \left(1-\pi_0^h\right)\left(\pi_d^h \theta + \left(1-\pi_d^h\right)\theta^2\right),$$

and the date-0 credit that the borrower receives is

$$D_S\left(F_S\right) = \pi_0^l F_S + \left(1-\pi_0^l\right)\left(\pi_d^l \theta + \left(1-\pi_d^l\right)\theta^2\right).$$

For a long-term debt contract to deliver the same initial credit, its face value F_L has to satisfy

$$\left(1-\left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\right)F_L + \left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\theta^2 = \pi_0^l F_S + \left(1-\pi_0^l\right)\left(\pi_d^l \theta + \left(1-\pi_d^l\right)\theta^2\right).$$

This implies that

$$F_L = \frac{\pi_0^l F_S + \left(1-\pi_0^l\right)\pi_d^l \theta}{1-\left(1-\pi_0^l\right)\left(1-\pi_d^l\right)}.$$

Thus, the borrower's expected debt payment is

$$\mathbb{E}_0^h\left[\widetilde{D}_L\right] = \left(1-\left(1-\pi_0^h\right)\left(1-\pi_d^h\right)\right)\frac{\pi_0^l F_S + \left(1-\pi_0^l\right)\pi_d^l \theta}{1-\left(1-\pi_0^l\right)\left(1-\pi_d^l\right)} + \left(1-\pi_0^h\right)\left(1-\pi_d^h\right)\theta^2.$$

Direct algebra gives the difference between the costs of the short-term and long-term contracts:

$$\mathbb{E}_0^h\left[\widetilde{D}_L\right] - \mathbb{E}_0^h\left[\widetilde{D}_S\right] = \frac{\pi_0^h\left(1-\pi_0^l\right)\pi_d^l - \pi_0^l\left(1-\pi_0^h\right)\pi_d^h}{\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l}\left(\theta - F_S\right).$$

Again, the short-term debt is less costly if and only if (7) holds.

A.2 Proof of Lemma 2

Suppose that the borrower's optimal face value F is lower than θ^2. The contract could be long-term or short-term. Since the face value is lower than θ^2, the debt contract is risk free across all the four possible paths, i.e., $\widetilde{D} = F$. As a result, the expected debt payment to the borrower is F and the date-0 credit the borrower gets is also F. Then, according to equation (6), the borrower's expected value is

$$\frac{c+p_0}{p_0-F}\left[\mathbb{E}_0^h\left(\widetilde{\theta}\right) - F\right].$$

Now, consider increasing the debt face value by a tiny amount ϵ. The debt contract is still risk free, and the borrower's expected value becomes

$$\frac{c+p_0}{p_0-F-\epsilon}\left[\mathbb{E}_0^h\left(\widetilde{\theta}\right) - F - \epsilon\right].$$

Since $p_0 \leq \mathbb{E}_0^h\left(\widetilde{\theta}\right)$, this expression is increasing with ϵ. In other words, the borrower is better off by borrowing more. This contradicts with F being the optimal debt face value. Thus, the optimal debt face value cannot be lower than θ^2.

Next, suppose that the borrower's optimal face value F is higher than θ. The contract could be long-term or short-term. We denote the debt payment on date 2 as $\widetilde{D}_0$. Since the face value is higher than θ, the borrower always default on the debt contract except at the end of the path uu. That is, $\widetilde{\theta} - \widetilde{D}_0$ equals $1 - F$ at the end of the path uu, and 0 at the end of the other paths. Then, according to equation (6), the borrower's expected value is

$$\frac{c + p_0}{p_0 - \mathbb{E}_0^l\left(\widetilde{D}_0\right)} \mathbb{E}_0^h\left(\widetilde{\theta} - \widetilde{D}_0\right).$$

Consider reducing the debt face value by a tiny amount ϵ. We denote the debt payment of the new contract by $\widetilde{D}_1$. Note that $\widetilde{D}_1$ differs from $\widetilde{D}_0$ only by $-\epsilon$ at the end of the path uu. The borrower's expected value is now

$$\frac{c + p_0}{p_0 - \mathbb{E}_0^l\left(\widetilde{D}_1\right)} \mathbb{E}_0^h\left(\widetilde{\theta} - \widetilde{D}_1\right) = \frac{c + p_0}{p_0 - \mathbb{E}_0^l\left(\widetilde{D}_0\right) + \pi_0^l \pi_u^l \epsilon} \left[\mathbb{E}_0^h\left(\widetilde{\theta} - \widetilde{D}_0\right) + \pi_0^h \pi_u^h \epsilon\right].$$

This expression is increasing with ϵ if

$$\frac{\mathbb{E}_0^h\left(\widetilde{\theta} - \widetilde{D}_0\right)}{p_0 - \mathbb{E}_0^l\left(\widetilde{D}_0\right)} \leq \frac{\pi_0^h \pi_u^h}{\pi_0^l \pi_u^l}.$$

Note that since $p_0 \geq \mathbb{E}_0^l\left(\widetilde{\theta}\right)$,

$$\frac{\mathbb{E}_0^h\left(\widetilde{\theta} - \widetilde{D}_0\right)}{p_0 - \mathbb{E}_0^l\left(\widetilde{D}_0\right)} \leq \frac{\mathbb{E}_0^h\left(\widetilde{\theta} - \widetilde{D}_0\right)}{\mathbb{E}_0^l\left(\widetilde{\theta} - \widetilde{D}_0\right)} = \frac{\pi_0^h \pi_u^h}{\pi_0^l \pi_u^l}.$$

Thus, the borrower's expected value increases with ϵ, which contradicts with F being the optimal debt face value. This suggests that the optimal debt face value cannot be higher than θ.

A.3 Proof of Proposition 3

On date 0, the borrower's expected value is given in (6). Based on the asset payoff and debt payment listed in Table 2, we have

$$\mathbb{E}_0^h\left[\widetilde{\theta}\right] - \mathbb{E}_0^h\left[\widetilde{D}_L\right] = \left[\pi_0^h \pi_u^h (1 - F_L) + \left(\pi_0^h\left(1 - \pi_u^h\right) + \left(1 - \pi_0^h\right)\pi_d^h\right)(\theta - F_L)\right].$$

By substituting this and $C_L(F_L)$ in (1) into (6), we derive the borrower's date-0 expected value as

$$V_L(F_L) = (c + p_0) \frac{\pi_0^h \pi_u^h + \left(\pi_0^h\left(1 - \pi_u^h\right) + \left(1 - \pi_0^h\right)\pi_d^h\right)\theta - \left(\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h\right) F_L}{p_0 - \left(1 - \pi_0^l\right)\left(1 - \pi_d^l\right)\theta^2 - \left(\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l\right) F_L}.$$

Direct algebra shows that $V_L(F_L)$ is increasing with F_L if and only if $p_0 < P_M$ where

$$P_M = \left(\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l\right) \frac{\pi_0^h \pi_u^h + \left(\pi_0^h \left(1-\pi_u^h\right) + \left(1-\pi_0^h\right)\pi_d^h\right)\theta}{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h} + \left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\theta^2. \quad (13)$$

As a result, the borrower's optimal long-term debt leverage is θ if $p_0 < P_M$, is θ^2 if $p_0 > P_M$, and is either θ or θ^2 if $p_0 = P_M$.

A.4 Proof of Proposition 4

We first consider the case in which the face value of the short-term debt $F_S \in \left[\theta^2, K_d\right]$. On date 0, the borrower's expected value $V_S(F_S)$ is given in (6). Note that in this case $C_S(F_S) = F_S$. By substituting in the expression of $\widetilde{D}_S$ in Table 2 and $F_{S,1}$ in (4), we have

$$V_S(F_S) = (c+p_0)\frac{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h \pi_d^l}{\pi_d^l} \frac{\frac{\pi_d^l\left[\pi_0^h\left(\pi_u^h + \left(1-\pi_u^h\right)\theta\right) + \left(1-\pi_0^h\right)\pi_d^h \theta\right] + \left(1-\pi_0^h\right)\pi_d^h\left(1-\pi_d^l\right)\theta^2}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h \pi_d^l} - F_S}{p_0 - F_S}. \quad (14)$$

This immediately implies that $V_S(F_S)$ is increasing in F_S if and only if

$$p_0 < \frac{\pi_d^l\left[\pi_0^h\left(\pi_u^h + \left(1-\pi_u^h\right)\theta\right) + \left(1-\pi_0^h\right)\pi_d^h \theta\right] + \left(1-\pi_0^h\right)\pi_d^h\left(1-\pi_d^l\right)\theta^2}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h \pi_d^l} = P_H \quad (15)$$

Next, we consider $F_S \in [K_d, \theta]$. Similarly, by substituting the expression of $\widetilde{D}_S$ in Table 2 into (6), we obtain

$$\begin{aligned} V_S(F_S) &= (c+p_0)\frac{\pi_0^h\left(\pi_u^h + \left(1-\pi_u^h\right)\theta - F_S\right)}{p_0 - D_S(F_S)} \\ &= (c+p_0)\frac{\pi_0^h}{\pi_0^l}\frac{\pi_0^l \mathbb{E}_0^h\left[\widetilde{\theta}\,|uu, ud\right] - \pi_0^l F_S}{p_0 - \left(1-\pi_0^l\right)\mathbb{E}_0^l\left[\widetilde{\theta}\,|du, dd\right] - \pi_0^l F_S} \end{aligned} \quad (16)$$

where $\mathbb{E}_0^h\left[\widetilde{\theta}\,|uu, ud\right] = \mathbb{E}_u^h\left[\widetilde{\theta}\right] = \pi_u^h + \left(1-\pi_u^h\right)\theta$ and $\mathbb{E}_0^l\left[\widetilde{\theta}\,|du, dd\right] = \mathbb{E}_d^l\left[\widetilde{\theta}\right] = K_d$. It is easy to show that $V_S(F_S)$ is increasing in F_S if and only if

$$p_0 < \pi_0^l \mathbb{E}_0^h\left[\widetilde{\theta}\,|uu, ud\right] + \left(1-\pi_0^l\right)\mathbb{E}_0^l\left[\widetilde{\theta}\,|du, dd\right] = P_L. \quad (17)$$

By combining the properties of $V_S(F_S)$ across the intervals of $\left[\theta^2, K_d\right]$ and $[K_d, \theta]$, it is direct to verify the borrower's optimal short-term debt face value given in Proposition 4.

Finally we show that $P_L < P_M < P_H$, where three objects are defined in (17), (13), and (15). To show that $P_M < P_H$, we only need to show that

$$\left(1-\pi_0^l\right)\left(1-\pi_d^l\right) > \frac{\left(1-\pi_0^h\right)\pi_d^h\left(1-\pi_d^l\right)}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h \pi_d^l}.$$

Simple calculation shows that it is equivalent to (7). Now we show that $P_L < P_M$. The term involving θ^2 has common coefficient $\left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\theta^2$ which cancels out. Because the sum of coefficients of 1, θ, and θ^2 is one for P_L and P_M, it suffices to show that P_L has a smaller coefficient for 1:

$$\pi_0^l \pi_u^h < \left(\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l\right) \frac{\pi_0^h \pi_u^h}{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h}.$$

Simplifying, we find that this is equivalent to (7).

A.5 Proof of Proposition 5

We only need to prove that there is no incentive for any optimistic buyer to save cash on date 0. The only reason to save cash is that on date 1 the asset might be under-valued in the lower state d if some other asset holders run into distress and causes the asset to be under-valued. According to Proposition 7, this situation arises in cases LD2 and LD3, in which at least some of the asset holders use debt contract with face value θ. Thus, we will focus on these cases.

First, suppose that the equilibrium falls in either case LD2 or LD3. What is the marginal value for an optimistic buyer to save a small amount ε? Going forward, if the state moves into d, he has cash to buy under-valued asset, which is priced at $p_d = \mathbb{E}_d^l\left[\widetilde{\theta}\right] = \pi_d^l \theta + \left(1-\pi_d^l\right)\theta^2$ (Proposition 7). He can further lever up by using a debt contract with face value θ^2 (one can show that he has no incentive to use a different face value.) Thus, he can use his cash to a position of size $\frac{\varepsilon}{p_d - \theta^2}$ and thus has an expected value of

$$\frac{\varepsilon}{p_d - \theta^2} \pi_d^h \left(\theta - \theta^2\right) = \varepsilon \frac{\pi_d^h}{\pi_d^l}.$$

This in turn implies that his expected value on date 0 is

$$\pi_0^h \varepsilon + \left(1-\pi_0^h\right) \varepsilon \frac{\pi_d^h}{\pi_d^l} = \varepsilon \left[\pi_0^h + \left(1-\pi_0^h\right) \frac{\pi_d^h}{\pi_d^l}\right].$$

Thus, the marginal value of saving cash on date 0 is $\pi_0^h + \left(1-\pi_0^h\right)\frac{\pi_d^h}{\pi_d^l}$.

Next, we show that the marginal value of establishing an asset position on date 0 is greater than that of saving cash. If the buyer uses the cash ε to increase his date-0 position, equation (6) implies that the marginal value is

$$\frac{\mathbb{E}_0^h\left[\widetilde{\theta}\right] - \mathbb{E}_0^h\left[\widetilde{D}_L\right]}{p_0 - C_L\left(F_L\right)}.$$

In the equilibrium cases LD2 and LD3, the face value of the buyer's optimal debt contract F_L is either θ or θ^2. Regardless of the exact value, we can expand the marginal value as

$$\frac{\pi_0^h \pi_u^h + \left(\pi_0^h\left(1-\pi_u^h\right) + \left(1-\pi_0^h\right)\pi_d^h\right)\theta - \left(\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h\right) F_L}{p_0 - \left(1-\pi_0^l\right)\left(1-\pi_d^l\right)\theta^2 - \left(\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l\right) F_L}.$$

Note that $p_0 \leq P_M$ (Figure 2). Thus, the marginal value is higher than

$$\frac{\pi_0^h \pi_u^h + \left(\pi_0^h \left(1-\pi_u^h\right) + \left(1-\pi_0^h\right) \pi_d^h\right) \theta - \left(\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h\right) F_L}{P_M - \left(1-\pi_0^l\right)\left(1-\pi_d^l\right) \theta^2 - \left(\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l\right) F_L} = \frac{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h}{\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l}.$$

Also note that for the long-term debt to be used in equilibrium, the condition in (7) needs to fail. This directly implies that

$$\frac{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h}{\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l} > \frac{\pi_d^h}{\pi_d^l},$$

where the right-hand side is further higher than $\pi_0^h + \left(1-\pi_0^h\right) \frac{\pi_d^h}{\pi_d^l}$, the marginal value of saving cash on date 0. Thus, there is no incentive for any buyer to save cash on date 0.

A.6 Proof of Proposition 6

The only non-trivial part of the proposition is that the buyers have no incentive to save cash on date 0. To prove this, we again compare the marginal value of establishing an asset position and saving cash on date 0. The marginal value of saving cash is higher than 1 in the equilibrium cases SD2, SD3, SD4, and SD5. In these cases, at least some of the buyers use debt contracts with face values K_d or θ, and thus will run into distress on date 1 in the lower state d. Following the proof of Proposition 5, in these cases the marginal value of saving cash on date 0 is $\pi_0^h + \left(1-\pi_0^h\right) \frac{\pi_d^h}{\pi_d^l}$.

First, we consider the marginal value of establishing a larger asset position in cases SD2 and SD3. According to equation (14), the marginal value is

$$\frac{\left(1-\pi_0^h\right) \pi_d^h + \pi_0^h \pi_d^l}{\pi_d^l} \frac{P_H - F_S}{p_0 - F_S}$$

where the debt face value F_S could be either K_d or θ^2. Since $p_0 \leq P_H$ in these cases (Figure 3), the marginal value is higher than

$$\frac{\left(1-\pi_0^h\right) \pi_d^h + \pi_0^h \pi_d^l}{\pi_d^l} = \pi_0^h + \left(1-\pi_0^h\right) \frac{\pi_d^h}{\pi_d^l},$$

which is the marginal value of saving cash on date 0.

Next, we consider the cases SD4 and SD5. According to equation (16), the marginal value of establishing a larger asset position on date 0 is

$$\frac{\pi_0^h}{\pi_0^l} \frac{\pi_0^l \mathbb{E}_0^h \left[\widetilde{\theta}\,|uu, ud\right] - \pi_0^l F_S}{p_0 - \left(1-\pi_0^l\right) \mathbb{E}_0^l \left[\widetilde{\theta}\,|du, dd\right] - \pi_0^l F_S}.$$

Since $p_0 \leq P_L$ in these cases (Figure 3), the marginal value is higher than $\frac{\pi_0^h}{\pi_0^l}$. Note that for the short-term debt to be desirable in equilibrium, the condition in (7) needs to hold. This condition directly implies that

$$\frac{\pi_0^h}{\pi_0^l} > \pi_0^h + \left(1 - \pi_0^h\right) \frac{\pi_d^h}{\pi_d^l}.$$

Thus, the marginal value of establishing a larger asset position on date 0 is higher than that of saving cash.

In summary of all the cases considered above, there is no incentive for any buyer to save cash on date 0.

A.7 Proof of Proposition 7

Suppose that the optimistic asset holders acquired their asset holdings on date 0 using collateralized long-term debt contracts with face value F_L. Then, if a new buyer wants to purchase the asset on date 1, how much does he need to pay? Note that not only does the new buyer has to buy out the stake of the original owner, which is $\mathbb{E}_1^h\left[\max\left(\widetilde{\theta} - F_L, 0\right)\right]$, but also the stake of the creditor, which is $\mathbb{E}_1^l\left[\min\left(\widetilde{\theta}, F_L\right)\right]$. Therefore, the asset price on date 1 is[8]

$$p_1 = \mathbb{E}_1^h\left[\max\left(\widetilde{\theta} - F_L, 0\right)\right] + \mathbb{E}_1^l\left[\min\left(\widetilde{\theta}, F_L\right)\right].$$

In the upper state u, it is easy to see that regardless of the debt face value, the debt is risk free going forward. As a result, $p_u = \mathbb{E}_u^h\left[\widetilde{\theta}\right]$.

In the lower state d, the debt can be risky if the face value is θ and risk free if it is θ^2. Thus, in case LD1, the equilibrium asset price is still determined by the optimistic asset holder's valuation $\mathbb{E}_d^h\left[\widetilde{\theta}\right]$. However, in cases LD2 and LD3, at least some of the asset holders used debt contracts with face value θ. Now, they will surely default on their debt on date 2 and the shaddow value of their asset is

$$\mathbb{E}_d^h\left[\max\left(\widetilde{\theta} - \theta, 0\right)\right] + \mathbb{E}_d^l\left[\min\left(\widetilde{\theta}, \theta\right)\right] = \mathbb{E}_d^l\left[\widetilde{\theta}\right].$$

The date-1 equilibrium for cases SD1, SD2, SD3, SD4, and SD5 can be derived in a similar way.

A.8 Proof of Proposition 10

According to Figure 4, the equilibrium identified by the conditions of Proposition 8 can be either in the SD2 or SD3 case of Proposition 6. We consider these cases separately.

In the SD3 case, the date-0 asset price is given by

$$p_0 = \frac{\mu c + C_S\left(K_d\right)}{1-\mu} = \frac{\mu c + \pi_d^l \theta + \left(1 - \pi_d^l\right)\theta^2}{1-\mu},$$

[8]The new buyer can still use leverage (possibly provided by pessimists) to purchase the asset at $t = 1$; but this does not affect how much he needs to pay the original buyer and creditor.

which is indifferent to δ_0 and decreases with δ_d.

In the SD2 case, the date-0 price is given by

$$p_0 = P_H = \frac{\pi_d^l \left[\pi_0^h \left(\pi_u^h + \left(1-\pi_u^h\right)\theta\right) + \left(1-\pi_0^h\right)\pi_d^h\theta\right] + \left(1-\pi_0^h\right)\pi_d^h\left(1-\pi_d^l\right)\theta^2}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h\pi_d^l}.$$

First, we consider the comparative static with respect to δ_0. Note that p_0 depends on only $\pi_0^h = 0.5+\delta_0$, but not on $\pi_0^l = 0.5-\delta_0$. Let

$$X = \pi_d^h\left[\pi_d^l\theta + \left(1-\pi_d^l\right)\theta^2\right], \text{ and } Y = \pi_d^l\left[\pi_u^h + \left(1-\pi_u^h\right)\theta\right].$$

Then,

$$p_0 = \frac{\left(1-\pi_0^h\right)X + \pi_0^h Y}{\left(1-\pi_0^h\right)\pi_d^h + \pi_0^h\pi_d^l} = \frac{X}{\pi_d^h}\frac{\left(1-\pi_0^h\right)+\pi_0^h\frac{Y}{X}}{\left(1-\pi_0^h\right)+\pi_0^h\frac{\pi_d^l}{\pi_d^h}}.$$

It is easy to see that p_0 is increasing with δ_0 if and only if $\frac{Y}{X} > \frac{\pi_d^l}{\pi_d^h}$, which is equivalent to $\frac{\pi_u^h+\left(1-\pi_u^h\right)\theta}{\pi_d^l\theta+\left(1-\pi_d^l\right)\theta^2} > 1$. Since $\pi_u^h + \left(1-\pi_u^h\right)\theta > \theta > \pi_d^l\theta + \left(1-\pi_d^l\right)\theta^2$, this inequality holds. Thus, p_0 increases with δ_0.

We now consider the comparative static with respect to δ_d, which affects p_0 through $\pi_d^h = 0.5+\delta_d$ and $\pi_d^l = 0.5-\delta_d$.

$$\begin{aligned}
\frac{dp_0}{d\delta_d} &\propto \left\{\begin{array}{c} -\left[\pi_0^h\left(\pi_u^h+\left(1-\pi_u^h\right)\theta\right)+\left(1-\pi_0^h\right)(0.5+\delta_d)\theta\right] \\ +(0.5-\delta_d)\left(1-\pi_0^h\right)\theta+\left(1-\pi_0^h\right)(1+2\delta_d)\theta^2\end{array}\right\}\left[\begin{array}{c}\left(1-\pi_0^h\right)(0.5+\delta_d) \\ +\pi_0^h(0.5-\delta_d)\end{array}\right] \\
&\quad -\left\{\begin{array}{c}(0.5-\delta_d)\left[\pi_0^h\left(\pi_u^h+\left(1-\pi_u^h\right)\theta\right)+\left(1-\pi_0^h\right)(0.5+\delta_d)\theta\right] \\ +\left(1-\pi_0^h\right)(0.5+\delta_d)^2\theta^2\end{array}\right\}\left(1-2\pi_0^h\right) \\
&\propto \left\{(0.5-\delta_d)\theta+(1+2\delta_d)\theta^2\right\}\left[\left(1-\pi_0^h\right)(0.5+\delta_d)+\pi_0^h(0.5-\delta_d)\right] \\
&\quad +(0.5+\delta_d)^2\theta^2\left(2\pi_0^h-1\right)-\left[\pi_0^h\left(\pi_u^h+\left(1-\pi_u^h\right)\theta\right)+\left(1-\pi_0^h\right)(0.5+\delta_d)\theta\right] \\
&< \left\{(0.5-\delta_d)\theta+(1+2\delta_d)\theta\right\}\left[\left(1-\pi_0^h\right)(0.5+\delta_d)+\pi_0^h(0.5-\delta_d)\right] \\
&\quad +(0.5+\delta_d)\left(1-\pi_d^l\right)\theta\left(2\pi_0^h-1\right)-\left[\pi_0^h+\left(1-\pi_0^h\right)(0.5+\delta_d)\right]\theta \\
&\propto \left\{1.5+\delta_d\right\}\left[\left(1-\pi_0^h\right)(0.5+\delta_d)+\pi_0^h(0.5-\delta_d)\right]+(0.5+\delta_d)^2\left(2\pi_0^h-1\right) \\
&\quad -\left[\pi_0^h+\left(1-\pi_0^h\right)(0.5+\delta_d)\right] \\
&= 0
\end{aligned}$$

which proves that p_0 decreases with δ_d.

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