A Dynamic Theory of Resource Wars

Daron Acemoglu
Michael Golosov
Aleh Tsyvinski
Pierre Yared

A Dynamic Theory of Resource Wars[*]

Daron Acemoglu[†]   Michael Golosov[‡]   Aleh Tsyvinski[§]   Pierre Yared[¶]

December 31, 2010

Abstract

We develop a dynamic theory of resource wars and study the conditions under which such wars can be prevented. The interaction between the scarcity of resources and the incentives for war in the presence of limited commitment is at the center of our theory. We show that a key parameter determining the incentives for war is the elasticity of demand. Our first result identifies a novel externality that can precipitate war: price-taking firms fail to internalize the impact of their extraction on military action. In the case of inelastic resource demand, war incentives increase over time and war may become inevitable. Our second result shows that in some situations, regulation of prices and quantities by the resource-rich country can prevent war, and when this is the case, there will also be intertemporal distortions. In particular, resource extraction will tend to be slower than that prescribed by the Hotelling rule, which is the rate of extraction in the competitive environment. Our third result is that, due to limited commitment, such regulation can also precipitate war in some circumstances in which war is avoided in the competitive environment.

Keywords: Trade, International Conflicts, War, Exhaustible Resources
JEL Classification: F10, F51, , H56, Q32.

[†]Massachusetts Institute of Technology. e-mail: daron@mit.edu.

[‡]Yale University. email: m.golosov@yale.edu.

[§]Yale University. email: a.tsyvinski@yale.edu.

[¶]Columbia University. e-mail: pyared@columbia.edu.

1 Introduction

Control over natural resources has been one of the key determinants of wars.[1] An early study of causes of modern wars during the 1878 to 1918 period by Bakeless (1921) argued that fourteen of the twenty major wars had significant economic causes, often related to conflict over resources. He emphasized

> "[t]he rise of industrialism has led to the struggle for ... raw materials."

For example, in the War of the Pacific (1879-1884), Chile fought against a defensive alliance of Bolivia and Peru for the control of guano mineral deposits. The war was precipitated by the rise in the value of the deposits due to their extensive use in agriculture. Chile's victory increased the size of its treasury by 900 percent.[2]

Westing (1986) argues that many of the wars in the twentieth century had an important resource dimension. As examples, he cites the Algerian War of Independence (1954-1962), the Six Day War (1967), and the Chaco War (1932-1935).[3] More recently, Saddam Hussein's invasion of Kuwait in 1990 was a result of the dispute over the Rumaila oil field. In *Resource Wars* (2001), Klare argues that following the end of the Cold War control of valuable natural resources has become increasingly important and these resources will become a primary motivation for wars in the future. The famous Carter Doctrine, which states that

> "Any attempt by any outside force to gain control of the Persian Gulf ... will be repelled by any means necessary, including military force,"

is just one facet of this perspective.[4]

This paper develops an economic theory of resource wars and clarifies the conditions under which such wars can be prevented. We consider the dynamic interactions between a resource-rich and a resource-poor country, which enable us to capture the effect of the increasing scarcity

---

[1]In his classic, *A Study of War*, Wright (1942) devotes a chapter to the relationship between war and resources. Another classical reference, *Statistics of Deadly Quarrels* by Richardson (1960), extensively discusses economic causes of war, including the control of "sources of essential commodities." More recently, Findlay and O'Rourke (2007) document the historical relationship between international trade and military conflict.

[2]Bakeless (1921) writes: "The desire of Chile to secure a share in the nitrate trade, of Bolivia to hold nitrate deposits [...], and of Peru to maintain its supremacy in the guano trade explain the motivation of the Nitrate War."

[3]The Algerian War of Independence was in part fought because France was reluctant to lose Algeria's rich oil deposits. An important cause of the Six Day War between Israel and Arab states was the struggle for water resources of the Jordan River and other rivers in the area. The Chaco War of 1932-1935 was a successful war by Paraguay against Bolivia to annex the Gran Chaco area that was incorrectly thought to contain significant deposits of oil.

[4]The Carter Doctrine was used in 1990 to justify the first Gulf War. Following the oil shocks in the 1970s, the Secretary of State Henry Kissinger told the editors of the *Business Week* that the United States was prepared to go to war over oil and that Washington would have no hesitation to use force "where there's some actual strangulation of the industrialized world". Klare (2001) argues that the Caspian Basin and the South China Sea are the most likely regions to witness large scale warfare over oil in the future. War over water is another pressing issue in international politics. For example, in 1980 Boutros Boutros-Ghali commented that "The next war in our region will be over the waters of the Nile, not politics."

of finite resources. Our approach combines the classic Hotelling (1931) model of exhaustible resources with a dynamic "guns and butter" model of armament and war along the lines of Powell (1993). A key friction in our model is the presence of *limited commitment*, as countries cannot commit to future policies. We use the model to ask three main questions. *First*, what is the effect of resource scarcity on the likelihood of war? *Second*, how does the threat of war affect resource extraction and prices? *Finally*, how are the realization of war and the dynamics of resource extraction affected by market structure?

In our framework, the resource-poor country (country A) exchanges a non-exhaustible good (a "consumption" good) for an exhaustible good ("oil") with the resource-rich country (country S). At any date, country A can arm and invade country S; higher armaments result in country A capturing a greater portion of the remaining resources in country S. We consider two different market structures. In the first market structure, the *competitive environment*, the stock of the exhaustible resource in country S is distributed among a set of perfectly competitive price-taking firms which supply the world market. Country A consumers purchase the resource at the world market price, unless there is a war (in which case country A captures part of the endowment and the rest of the stock is destroyed). In the second market structure, the *monopolistic environment*, the government of country S regulates the price and the level of production of the resource (for example, by setting nonlinear taxes). More specifically, following the armament decision by country A, country S makes a take-it-or-leave-it price-quantity offer to country A, where country A has the option of declaring war if this is preferable to accepting the offer. We characterize the equilibrium in Markovian strategies.

In both of these environments the elasticity of the demand for resource (or oil) plays a critical role in shaping war incentives. If the elasticity of demand for oil is below 1, the value of the outstanding stock of oil rises as the resource is depleted. This implies that the incentives for country A to arm and fight country S rise over time. In contrast, if this elasticity exceeds 1, the value of the outstanding stock of oil declines with time, as do country A's incentives to fight over these resources. For these reasons, the elasticity of demand will play a crucial role in the characterization of equilibrium dynamics. Given that empirically relevant estimates of elasticity are below 1, we focus our discussion on the implications of the model for elasticities below 1 (though we also provide the results for the converse case).[5]

Our first main result is that a novel externality emerges in the competitive environment and can precipitate war. Specifically, firms in country S do not internalize their impact on country A's war incentives. In the case with inelastic demand, firms do not take into account that their extraction decision increases country A's incentives to invade country S, since these incentives rise with resource scarcity. Moreover, if country A is militarily sufficiently powerful (i.e., it can acquire a large enough portion of the outstanding oil during war), then country A will eventually invade country S once the stock of oil has been sufficiently depleted. Firms anticipate

---

[5]Several studies estimate the short-run demand elasticity for oil to be between 0.01 and 0.1, while the long-run elasticity is found to be higher but still less than 1 (see, for example, Gately and Huntington, 2002).

this prospect of future war by increasing their extraction today, which in turn increases country A's incentives to engage in war even earlier, an effect which we refer to as the *unraveling of peace*.

Motivated by the existence of this novel externality, the rest of the paper studies whether regulation of prices and quantities by country S acting as a monopolist, the "monopolistic environment," can mitigate this externality and prevent war. Our second main result shows that in some situations, regulation by country S's government can prevent the realization of war, and that this occurs through the introduction of *intertemporal distortions*. More specifically, if demand for the resource is inelastic, then resource extraction in the monopolistic environment occurs slower than that prescribed by the Hotelling rule, which is the rate of extraction in the competitive environment. This is because under inelastic demand, country A's armaments and incentives to declare war increase as the resource is depleted. Thus, country S has an incentive to slow down the rate of extraction so as to reduce the incentives for war and reduce the cost of armaments for which it is paying indirectly, and this causes the shadow value of the outstanding stock of oil to rise at a slower pace than under the Hotelling rule.

Our final result is that, under some circumstances, regulation of prices and quantities by country S can in fact precipitate war in circumstances in which war is avoided in the competitive environment. This result emerges because of the presence of limited commitment. Specifically, because country S cannot commit to a long term contract with country A, country A must arm in every period, *even under peace*, in order to enforce such a contract and obtain favorable terms of trade. In contrast, in the competitive environment, country A only arms if it is going to war. Because in the monopolistic environment country S implicitly compensates country A for the cost of this continual armament with its offer in every single period, if this cost of continually arming is sufficiently high, then this will induce war.[6]

Despite the importance of international conflict for economic and social outcomes and the often-hypothesized links between natural resources and international conflict, there are only a handful of papers discussing these issues. Our work is related to the literature on international wars which explores how countries bargain to avoid war (e.g., Powell, 1993, 1999, Schwarz and Sonin, 2004, Skaperdas, 1992, and Yared, 2010). In particular, our work builds on Schwarz and Sonin (2004) since we consider how international transfers can serve to sustain peace in a dynamic environment. In contrast to this work, our focus is on the transfer of finite resources, and we study the two-way interaction between dynamic intertemporal resource-allocation and the threat of war.[7] In this regard, our paper is related to Hotelling's (1931) seminal work (see also Dasgupta and Heal, 1979, and more recent contributions by Pindyck, 1979, and Kremer

---

[6] This result is related to the fact that in any dynamic guns and butter type models, like Powell (1993), countries can go to war in order to avoid having to incur the cost of future armament under peace.

[7] For related work on bargaining in the shadow of conflict, see also Acemoglu and Robinson (2006), Anderlini, Gerardi, and Lagunoff (2009), Baliga and Sjöström (2004), Chassang and Padró i Miquel (2010), Dixit (1987), Esteban and Ray (2008), Fearon (1995), Garfinkel, Skaperdas, and Syropoulos (2009), Hirshleifer (1995), and Jackson and Morelli (2009).

and Morcom, 2000). Our work also contributes to the political economy of trade literature (e.g., Grossman and Helpman, 1995, Bagwell and Staiger, 1990, 2001, Maggi, 1999, Maggi and Rodriguez-Clare, 2007). Within this literature, our paper is most closely related to Antrás and Padró i Miquel (2009), who study how a dominant country can affect its trading partner's domestic politics. They show how lobbying type activities by the dominant country can be used for affecting policies and the terms of trade. In contrast to this literature, we emphasize how the ability to arm and fight wars over resources affects patterns of trade. Finally our work is related to the large literature on the political economy of natural resources (e.g., Tornell and Lane, 1999, Ross, 1999, Caselli, 2005, Robinson and Torvik, 2006, Egorov, Guriev, and Sonin, 2009). Differently from this literature, our focus is on the international dimension of conflict over resources and we abstract from domestic politics.

The paper is organized as follows. Section 2 describes the general environment. Section 3 describes the competitive environment and Section 4 describes the monopolistic environment. Section 5 considers extensions. Section 6 concludes and the Appendix includes additional proofs not included in the text.

2   Environment

Time is discrete and denoted by $t = 0, ..., \infty$. There are two countries, A and S, and two goods, an exhaustible resource, to which we refer to as "oil", and a perishable (non-resource) consumption good. Each country is inhabited by a continuum of mass 1 of identical households (or alternatively, by a representative household). We assume that the governments in both countries maximize the intertemporal utility of their citizens (of the representative household in their country). In view of this, we refer to actions by governments and countries interchangeably.

Households in country A receive the following flow utility from their consumption of the resource and the consumption good:

$$u\left(x_t^A\right) + c_t^A, \tag{1}$$

where $x_t^A \geq 0$ corresponds to their consumption of the resource and $c_t^A \gtreqless 0$ refers to the consumption good. The utility function $u\left(\cdot\right)$ is strictly increasing and concave, i.e., $u'\left(\cdot\right) > 0$ and $u''\left(\cdot\right) < 0$, and satisfies the following Inada conditions: $\lim_{x \to 0} u'\left(x\right) = \infty$ and $\lim_{x \to \infty} u'\left(x\right) = 0$. For simplicity, we assume that households in country S do not value the resource, and thus their utility is derived only from the consumption good:

$$c_t^S, \tag{2}$$

where $c_t^S \gtreqless 0$ refers to the consumption good. Households in both countries have a common discount factor $\beta \in (0, 1)$.

In each period both countries are endowed with an exogenous perishable amount of the consumption good. We normalize the endowment of this good for each country to zero (recall

4

that negative consumption is allowed). In addition, country S is endowed with $e_0 > 0$ units of the exhaustible resource (oil) in period 0. To be consumed, the resource needs to be extracted and extraction is at zero cost. We also assume that the amount extracted is non-storable and has to be consumed in the same period, which prevents country S from "selling the stock" of the resource. We denote by $x_t \geq 0$ the amount of extraction of the resource in period t. The remaining stock of the non-extracted resource in period $t+1$, e_{t+1}, follows the law of motion

$$e_{t+1} = e_t - x_t. \tag{3}$$

Country S extracts the resource and trades it for the consumption good with country A. We consider several trade environments in Sections 3 and 4.

In addition to trading, we allow country A to make two additional decisions in each period: how much to arm and whether to declare war against county S. The armament technology works as follows. At every date t, country A can choose a level of armament $m_t \in [0, \bar{m}]$ which has a per capita cost of $l(m_t)$ units of the consumption good. We assume that $l(\cdot)$ satisfies $l'(\cdot) > 0, l''(\cdot) \geq 0$, and $l(0) = 0$. The payoff from war depends on the amount of armament. If country A has armament m_t and attacks country S that has e_t units of the resource, it obtains fraction $w(m_t)$ of e_t, while the remaining fraction $1 - w(m_t)$ is destroyed.[8] We assume that $w(\cdot)$ satisfies $w'(\cdot) > 0$, $w''(\cdot) \leq 0$, $w(m) \in [0, 1]$ for all m with $w(0) = 0$ and $\lim_{m \to \bar{m}} w'(m) = 0$, which imposes sufficient diminishing returns to armaments to ensure an interior level of equilibrium armaments. In most of the analysis, we allow for $\bar{m} = \infty$, in which case, $m_t \in [0, \infty)$ and $\lim_{m \to \infty} w'(m) = 0$. We use an indicator variable $f_T = 0$ to denote that no war occurred in periods $t = 0, .., T$ and $f_T = 1$ to denote that war occurred in some period $t \leq T$.

If country A, after choosing m_t units of armament, attacks country S and the remaining endowment is e_t, the payoff to country A is $V(w(m_t)e_t) - l(m_t)$, where $l(m_t)$ is the cost of armament, incurred by the representative household in terms of the consumption good, and $V(w(m_t)e_t)$ is the continuation value of the representative household in that country starting with the ownership of the resource endowment of $w(m_t)e_t$ (since after war, the ownership of a fraction $w(m_t)$ of the remaining resource is transferred to the country A government). Since the government will use this stock to maximize the utility of its citizens, we have

$$V(w(m_t)e_t) = \max_{\{x_{t+k}, e_{t+k+1}\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} \beta^k u(x_{t+k}) \tag{4}$$

---

[8]To facilitate interpretation, we model the outcome of war as deterministic—in particular, with country A grabbing a fixed fraction of the resource. This is largely without loss of any generality. All of our results apply to an environment in which the outcome of war is stochastic, provided that after war, the two countries never interact again. For example, we can define $w(m_t)$ as the probability that country A receives a fraction λ^H of the endowment and $1 - w(m_t)$ as the probability that it receives a fraction $\lambda^L < \lambda^H$ of the endowment.

subject to the resource and nonnegativity constraints, i.e.,

$$e_{t+1+k} = e_{t+k} - x_{t+k} \text{ for } k > 0, \tag{5}$$

$$e_{t+1} = w(m_t) e_t - x_t, \text{ and} \tag{6}$$

$$x_{t+k}, e_{t+k} \geq 0 \text{ for } k \geq 0. \tag{7}$$

In the event of a war, the payoff to country S is given by $\psi < 0$. In what follows, we impose the following relatively weak condition on the utility function $u(\cdot)$ (without explicitly stating it).

Assumption 1 There exist some $(\kappa, \theta, \zeta) \in \mathbb{R}^3$ such that $u(x) \geq \kappa x^\theta + \zeta \ \forall x > 0$.

Assumption 1 is a simple sufficient condition to ensure that the value of (4) subject to (5)-(6) is finite (bounded from below) starting from any $w(m_t) e_t > 0$ and thus $V(\cdot)$ is well defined. Without any restriction on $u(\cdot)$, any feasible solution might lead to minus infinite value.[9]

For future reference, it is also useful to define $m^*(e)$ as the optimal amount of armament for country A if it attacks country S when country S has e units of resource endowment. Namely:

$$m^*(e) \equiv \arg\max_{m \geq 0} V(w(m)e) - l(m). \tag{8}$$

Given our assumptions on $u(\cdot)$, $w(\cdot)$, and $l(\cdot)$ (in particular, the Inada conditions) as well as Assumption 1, it is straightforward to see that $m^*(e)$ is well-defined, satisfies $m^*(e) > 0$ and is a continuously differentiable function of e for all $e > 0$.

One of the key variables in our analysis will be the elasticity of demand defined as $-u'(x)/(xu''(x))$. For now we prove the following useful result about the relationship between the comparative statics of m^* with respect to e and the elasticity of demand, which we will use throughout the paper.

Proposition 1 If $-u'(x)/(xu''(x)) < 1$ for all x, then $m^{*\prime}(e) < 0$. Conversely, if $-u'(x)/(xu''(x)) > 1$ for all x, then $m^{*\prime}(e) > 0$.

---

[9] To see that Assumption 1 ensures that $V(e_t)$ is bounded from below for any $e_t > 0$, consider the consumption path given by $e_{t+k+1} = \lambda e_{t+k}$ for $\lambda \in (0,1)$ with λ chosen such that $\beta \lambda^\theta < 1$, which is always feasible. Then

$$\sum_{k=0}^{\infty} \beta^k u\left((1-\lambda)\lambda^k e_t\right) \geq \sum_{k=0}^{\infty} \left(\beta\lambda^\theta\right)^k \kappa\left((1-\lambda)e_t\right)^\theta + \frac{\zeta}{1-\beta}$$

$$= \frac{\kappa\left((1-\lambda)e_t\right)^\theta}{1-\left(\beta\lambda^\theta\right)^k} + \frac{\zeta}{1-\beta} > -\infty$$

Clearly, Assumption 1 is always satisfied if $u(0) > -\infty$. Moreover, for the constant elasticity of substitution class of utility functions, introduced in (17) below, it is satisfied by choosing $\kappa = 1/(1-1/\sigma)$, $\theta = 1 - 1/\sigma$, and $\zeta = -1/(1-1/\sigma)$.

Proof. See Appendix. ∎

The elasticity of demand captures the value of resource consumption (in terms of the non-resource good) as resource consumption declines. Intuitively, as resource consumption decreases its price increases. The elasticity of demand determines which of these two effects dominates in determining the value of resource consumption. When this elasticity is less than one, the price effect dominates and thus the overall value of resource consumption rises as the quantity consumed declines. From (4), the value of the resource endowment to country A is also related to these competing effects. If the elasticity of demand is less than one, the marginal value of resource consumption, and thus the value to country A of capturing a greater stock of the resource, is greater when the resource is more scarce. From (8), this implies that country A will be willing to invest more in armaments in order to capture a larger fraction of the remaining resource endowment when there is less of it. The converse result contained in Proposition 1 has an analogous intuition.

3   Competitive Environment

We start by considering a competitive environment in which trade occurs at market clearing prices and both buyers and sellers take these prices as given. This environment allows us to highlight the key economic forces that determine incentives to fight and to illustrate the externalities in the competitive environment.

3.1   Markov Perfect Competitive Equilibrium

In the competitive environment, there is a unit measure of firms in country S. Each firm is labeled by i and owns an equal fraction of the total natural resource endowment of country S. Firm i extracts resources x_{it}^S and sells them in a competitive market at price p_t in units of the consumption good. All profits are rebated to households of country S as dividends. We next define a notion of competitive equilibrium for this environment. This definition requires some care, since producers in country S are price takers, but they must also recognize the likelihood of war, which results from the strategic choices of the government of country A. We define the notion of equilibrium in two steps. First, we impose price taking and market clearing for all relevant Arrow-Debreu commodities, i.e., for the resource at each date following any history (by Walras's law, this guarantees market clearing for the consumption good). Second, we study the problem of country A taking the relationship between the probability of war and these prices as given.

Price-taking implies that each firm i in country S chooses extraction plan $\left\{x_{it}^S\right\}_{t=0}^{\infty}$ to maximize its expected profits at time $t = 0$,

$$\max_{\left\{x_{it}^S\right\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t p_t x_{it}^S \tag{9}$$

7

subject to the constraints

$$e_{it+1} = e_{it} - x_{it}^S \text{ if } f_t = 0$$

$$x_{it}^S = 0 \text{ if } f_t = 1, \text{ and}$$

$$x_{it}^S, e_{it+1} \geq 0 \text{ for all } t \geq 0.$$

The second constraint stems from the fact that firm i loses its endowment if country A declares war. The solution to this problem implies that, when $f_t = 0$,

$$x_{it}^S \begin{cases} = 0 & \text{if } p_t < \beta p_{t+1} \Pr\{f_{t+1} = 0\} \\ \in [0, e_{it}] & \text{if } p_t = \beta p_{t+1} \Pr\{f_{t+1} = 0\} \\ = e_{it} & \text{if } p_t > \beta p_{t+1} \Pr\{f_{t+1} = 0\} \end{cases} . \tag{10}$$

Equation (10) captures the fact that firms take into account not only future prices but also the future probability of war in deciding how much to extract today.

Similarly, the representative household in country A chooses the demand for resource x_t^A as a solution to

$$\max_{x_t \geq 0} u(x_t) - p_t x_t, \tag{11}$$

which gives us the standard optimality condition

$$u'(x_t^A) = p_t. \tag{12}$$

We denote the total supply of the resource by x_t^S. Market clearing implies that the price sequence $\{p_t\}_{t=0}^{\infty}$ must be such that

$$x_t^S = x_t^A \tag{13}$$

for all t.

In addition, the country A government can impose a lump sum tax on its citizens of size $l(m_t)$ in order order to invest in armament m_t, and it can choose to attack country S at any date.

More specifically, we consider the following sequence of events. Since the game is trivial after the war has occurred, we only focus on the histories for which war has not occurred yet (i.e., on histories where $f_{t-1} = 0$).

1. Country A's government chooses a level of armament $m_t \geq 0$.

2. Firms in country S commit to extraction $x_t^S \geq 0$ and households in country A commit to consumption x_t^A at prices p_t in the event that country A does not attack country S at stage 3.

3. Country A's government decides whether or not to attack country S.

4. Extraction and consumption take place.

8

Note that in stage 2, firms and households trade contingent claims on the resource, where the contingency regards whether or not war is declared at stage 3.[10]

We can now define a *Markov Perfect Competitive Equilibrium* (MPCE) formally. For the same reason that the game is trivial after war has occurred, we only define strategies for dates t for which $f_{t-1} = 0$. Denote the strategy of the government of country A as φ which consists of a pair of functions φ^m and φ^f. In each period, the function φ^m assigns a probability distribution over armament decisions m_t as a function of e_t. The function φ^f assigns a probability distribution with which country S attacks as a function of $\left(e_t, m_t, p_t, x_t^S, x_t^A\right)$.[11]

Firms and households take the sequences of prices and policies by the government of country A as given. It is important to note that because we are focusing on Markov perfect equilibria, even if war is expected with probability 1 at date t, their choices do take into account the continuation strategy of the government and the future sequence of prices from $t+1$ onward in the event that war is not actually declared at t. Therefore, allocations and prices conditional on war never being declared need to be specified as part of the equilibrium. To do this, let us define a sequence $\gamma \equiv \left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$, where each element at t corresponds to the values of $\left(e_t, p_t, x_t^S, x_t^A\right)$ which would emerge if $f_{t-1} = 0$. Given such a sequence γ, one can define $U_A\left(e_t^*\right)$ as the welfare of (the representative household in) country A starting from e_t^* conditional on $f_{t-1} = 0$. Given this definition, the period t payoff to country A, starting from e_t^* under $f_{t-1} = 0$ and conditional on some choice (m_t, f_t), is

$$(1 - f_t)\left(u\left(x_t^{A*}\right) - p_t^* x_t^{A*} + \beta U_A\left(e_{t+1}^*\right)\right) + f_t V\left(w\left(m_t\right) e_t^*\right) - l\left(m_t\right). \tag{14}$$

The first term is the value in case of no war, while the second term is the continuation value following war.

Before providing a formal definition, we also note a potential source of uninteresting multiplicity in this environment. Consider a situation in which $e_t = 0$. If $u(0)$ is finite, then country A would be indifferent between choosing $f_t = 0$ on the one hand and $m_t = 0$ and $f_t = 1$ on the other. Moreover, if $u(0) = -\infty$, then country A's strategy is not well-defined. Depending on which action country A chooses at zero endowment, one can then change incentives at earlier stages and construct different equilibria. We propose a solution which deals with both of these issues simultaneously where the details are discussed in the Appendix. Specifically, we focus on a refinement of equilibria where war decisions at all e_t are optimal in the presence of an additive cost of war equal to $\upsilon > 0$ for country A. In that case, the expressions we have here correspond

---

[10]We could alternatively simplify the timing of the game by allowing country A to arm and to make its attacking decisions in the first stage, and then, if the attack did not occur, households and firms would trade in the second stage. Under our notion of equilibrium, these two setups are equivalent. We chose this setup to be consistent with the timing of the game in Section 4.

[11]Throughout the paper we focus on Markovian equilibria for two reasons. First, we believe that these capture the main commitment problems shaping economic incentives in a clean and economical manner. Second, as we explain further in subsection 5.4, even though the structure of subgame perfect equilibria appears similar, a tight characterization of the set of these equilibria is challenging.

to the limiting economy where $v \to 0$ (in the Appendix, we analyze the problem for an arbitrary $v > 0$; focusing on $v \to 0$ in the text simplifies expressions). Moreover, we impose that war decisions at $e_t = 0$ are consistent with war decisions for an arbitrarily small endowment (in the limit, zero endowment). The presence of the additive cost of war v implies that war never occurs at $e_t = 0$ if $u(0)$ is finite, though it is still the case that war may occur at $e_t = 0$ if $u(0) = -\infty$ depending on the limiting behavior of war incentives. Throughout, MPCE refers to such equilibria or "refined" MPCE (without this qualifier) as defined next.[12]

Definition 1 *A Markov Perfect Competitive Equilibrium (MPCE) consists of φ and γ such that at each t:*

1. *φ^m maximizes (14) for every $e_t^* > 0$ in γ,*

2. *φ^f maximizes (14) given m_t for every $\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right)$ with $e_t^* > 0$ in γ,*

3. *γ satisfies (3), (10), (12), and (13) with $\Pr\{f_{t+1} = 1\} = \varphi^f\left(e_{t+1}^*, m_{t+1}^*, p_{t+1}^*, x_{t+1}^{S*}, x_{t+1}^{A*}\right)$, and*

4. *If $e_t^* = 0$, then $\varphi(e_t^*) = \lim_{e \to 0} \varphi_v(e)$ where $\varphi_v(e)$ denotes the strategy for country A that maximizes (14) for some cost of war $v > 0$.*

The first three requirements are standard. They ensure that the government in country A makes its armament and fighting decisions optimally today, taking into account its future behavior and that of the private sector in the event that war is not declared today. Furthermore, firms and households behave optimally today, taking into account the future behavior of the government in the event that war is not declared today. They also impose that the continuation equilibrium in the event that war does not happen today must always be such that households and firms optimize, markets clear, and country A chooses its best response. The fourth requirement is the refinement mentioned above. It imposes that best response for country A (and in particular its war decision) at zero endowment is the limit of best responses in the perturbed economy as the endowment approaches zero. The fact that this needs to be the case for some $v > 0$ (rather than for all v or for some specific value) makes this a weaker refinement which is nonetheless sufficient for our purposes, and in the Appendix we analyze the problem for an arbitrary $v > 0$ which applies at all e_t.

3.2 Analysis

Our first result establishes the existence of MPCE in the above-described environment.

Lemma 1 *An MPCE exists.*

---

[12] Put differently, this refinement is in the spirit of "trembling hand perfection" and rules out equilibria supported by weakly dominated strategies for country A.

Proof. See Appendix. ■

We next characterize MPCEs. As a benchmark, it is useful to consider a case when country A cannot arm and declare war— i.e., focus on the case where $f_t = 0$ for all t. In that case there is no uncertainty and the first-order conditions to (10), (12), and (13) imply that the equilibrium prices p_t must satisfy

$$\beta p_{t+1} = p_t. \tag{15}$$

This is a market form of the famous Hotelling rule and requires that prices of the exhaustible resource grow at the rate of interests, which is also equal to the discount rate, $(1 - \beta)/\beta$. The intuition is straightforward: since producers are price-takers and can extract the resource at no cost, there will only be positive extraction at all dates if they make the same discounted profits by extracting at any date, which implies (15). Moreover, given the Inada conditions on the utility function and the first-order condition (12), zero extraction at any date is not consistent with equilibrium. Hence (15) must hold in any MPCE.

The connection between (15) and the Hotelling rule can be seen more explicitly by using (10), (12), and (13), which imply that the sequence of resource consumption $\{x_t\}_{t=0}^{\infty}$ must satisfy

$$\beta u'(x_{t+1}) = u'(x_t) \tag{16}$$

at all t, which is the familiar form of the Hotelling rule (with zero extraction costs).

We next turn to country A's armament and war decisions and characterize MPCE. We first consider pure-strategy equilibria (where φ^f is either 0 or 1 at each date). This gives us our first result, *the unraveling of peace*—because of the externalities that the production decisions of price-taking firms create on others, wars cannot be delayed in pure-strategy equilibria.

Proposition 2 *In any pure-strategy MPCE:*

1. *War can only occur at $t = 0$ along the equilibrium path.*

2. *The equilibrium sequence of resource extraction, x_t, satisfies (16) for all t.*

Proof. Suppose country A attacks at date $T > 0$ with probability 1. From (10), firms extract all the resource before date T, so that $e_t = 0$ for some $t \leq T$. This implies that $x_T = 0$. We now show that there is necessarily a deviation that is strictly profitable. Consider two cases. First, suppose $u(0)$ is finite. In this case, the fourth requirement of the definition of MPCE implies that country A attacking at T cannot be an equilibrium, yielding a contradiction. Second, suppose that $u(0) = -\infty$ and let the date at which the endowment is depleted be $t \geq 1$, which implies that $e_{t-1} > 0$. In this case the equilibrium payoff for country A from the viewpoint of date $t-1$ is $-\infty$. Consider the following deviation: country A chooses the level of armament $m^*(e_{t-1})$ as given by (8), and attacks country S at date $t-1$. This deviation has payoff

$$V(w(m^*(e_{t-1}))e_{t-1}) - l(m^*(e_{t-1})) > -\infty,$$

since $e_{t-1} > 0$. This implies that war at T cannot be a best response. Since this argument is true for any $T > 0$, it must be that any war can only occur at date $t = 0$. This establishes the first part of the proposition.

To derive the second part, note that if a war occurs at time 0, the first-order conditions to (4) imply that x_t must satisfy (16). If no attack occurs at $t = 0$, the first part implies that $f_t = 0$ for all t and the argument preceding the proposition establishes (16). ∎

This proposition shows that in pure-strategy equilibria, wars cannot be delayed. The intuition is simple and directly related to the externalities across firms: if there is a war at time T, price-taking firms will deplete their entire endowment before T, and this will encourage war to be declared earlier. While the fact that country S firms fail to internalize their impact on future war decisions is at the heart of Proposition 2, lack of commitment by country A also plays a role. More specifically, country A's armament and war decisions are chosen to maximize (4) at each date. Therefore, the unraveling of peace and war at date 0 occur because country A would otherwise optimally choose to go to war at some future date T. Suppose that country A's consumers are strictly better off at time $t = 0$ under permanent peace than under immediate war. In such a situation, country A's government could make its citizens better off by committing at $t = 0$ to not going to war in the future. Not only would this commitment prevent the unraveling of peace, but it would make country S households strictly better off also since they would be receiving positive payments from country A instead of receiving the payoff $\psi < 0$ from war.

Notably, Proposition 2 also implies that along the equilibrium path, consumption of the resource satisfies the Hotelling rule, (16), and that there are no intertemporal distortions. If there is no war at $t = 0$, then the equilibrium is identical to the benchmark competitive equilibrium in which war is not possible. If there is war at $t = 0$, then country A seizes a fraction $w\left(m^*\left(e_0\right)\right)$ of the initial endowment and it extracts resources according to (16) since this maximizes the welfare of households in country A.

To further characterize under which conditions wars may occur and to explore the possibility of mixed-strategy equilibria, we restrict attention to utility functions that imply a constant elasticity of demand for the resource. This is the same as the commonly used class of constant relative risk aversion (CRRA) or iso-elastic preferences:

$$u(x) = \frac{x^{1-1/\sigma} - 1}{1 - 1/\sigma} \tag{17}$$

for $\sigma > 0$. Clearly, the elasticity of demand for the exhaustible resource is constant and equal to $-u'\left(x\right)/\left(xu''\left(x\right)\right) = \sigma$. As we will see, when $\sigma < 1$, which is the empirically relevant case for oil (and perhaps also for other exhaustible resources), total spending on the exhaustible resource increases over time as its endowment is depleted because the price increase dominates the reduction in quantity. When preferences take this form, we can generalize Proposition 2 to any MPCE (i.e., also those in mixed strategies) provided that $\sigma \neq 1$.

Proposition 3 *Suppose $u(x)$ satisfies (17) and $\sigma \neq 1$. Then in any mixed-strategy MPCE:*

1. *War can only occur at $t = 0$ along the equilibrium path, and*

2. *The equilibrium sequence of resource extraction, x_t, satisfies (16) for all t.*

Proof. See Appendix. ∎

To understand the intuition for this proposition it is useful to consider how country A's incentives to declare war change over time as the endowment of the exhaustible resource declines. To do this, consider the special case where $w(\cdot)$ is a step function. In particular, if country A invests $\tilde{m} > 0$ in armament, it will receive the entire remaining endowment of the exhaustible resource, i.e., $w(\tilde{m}) = 1$. If it invests less, it will obtain none of the endowment. This functional form implies that country A is effectively choosing between zero armaments (and no war), and armaments equal to $\tilde{m}$ to obtain the entire endowment of the resource. Suppose further that country A is choosing between going to war at time t and permanent peace thereafter. Thus if it does not declare war at time t starting from some endowment e_t, the subsequent allocations are given by the standard competitive equilibrium allocations, denoted by $\{\tilde{x}_{t+k}(e_t), \tilde{p}_{t+k}(e_t), \tilde{e}_{t+k}(e_t)\}_{k=0}^{\infty}$. It is straightforward to show that $\tilde{x}_{t+k}(e_t) = (1 - \beta^\sigma) \tilde{e}_{t+k}(e_t)$, $\tilde{p}_{t+k}(e_t) = (\tilde{x}_{t+k}(e_t))^{-1/\sigma}$, and $\tilde{e}_{t+k+1}(e_t) = \tilde{e}_{t+k}(e_t) - \tilde{x}_{t+k}(e_t)$. This implies that the payoff to country A in period t from not going to war is equal to

$$
\begin{aligned}
U^C(e_t) &= \sum_{k=0}^{\infty} \beta^k u(\tilde{x}_{t+k}(e_t)) - \sum_{k=0}^{\infty} \beta^k \tilde{p}_{t+k}(e_t) \tilde{x}_{t+k}(e_t) \\
&= \sum_{k=0}^{\infty} \beta^k u(\tilde{x}_{t+k}(e_t)) - (1 - \beta^\sigma)^{-1/\sigma} e_t^{1-1/\sigma}.
\end{aligned}
$$

If country A invests $\tilde{m}$ in armament in period t and declares war, then, since $w(\tilde{m}) = 1$, its payoff is given by

$$
V(w(\tilde{m})e_t) - l(\tilde{m}) = \sum_{k=0}^{\infty} \beta^k u(\tilde{x}_{t+k}(e_t)) - l(\tilde{m}).
$$

This implies that the difference between the payoffs from war and no war is equal to

$$
V(w(\tilde{m})e_t) - l(\tilde{m}) - U^C(e_t) = (1 - \beta^\sigma)^{-1/\sigma} e_t^{1-1/\sigma} - l(\tilde{m}).
$$

Since $\{e_t\}_{t=0}^{\infty}$ is a decreasing sequence by construction, this expression monotonically decreases to zero if σ is greater than 1 and increases towards infinity if σ is less than 1. Therefore, depending on the elasticity of demand for the resource, the payoff from war either monotonically converges to zero or becomes unbounded. Which of these two cases applies depends on whether the payments that country A makes to country S in competitive equilibrium, $\sum_{k=0}^{\infty} \beta^k \tilde{p}_{t+k}(e_t) \tilde{x}_{t+k}(e_t)$, converge to zero or infinity as e_t declines. This logic allows us to show in the proof of Proposition 3 that if demand is elastic (σ is greater than one), incentives to fight must be decreasing for

country A. In particular, if it weakly prefers peace to war in any period t, it strictly prefers peace in all the subsequent periods. Alternatively, if the demand for the resource is inelastic (σ is less than one), incentives to fight must be increasing and country A eventually prefers war in which case the arguments of Proposition 2 apply directly. In particular in this case war must occur with probability 1 independently of the cost of armaments $l(\tilde{m})$ and the cost of war to country S. It is straightforward to see that the same conclusion holds if country A could, as in our model, choose to go to war at any date it wishes.

This special case illustrates the key intuition underlying Proposition 3. More generally, war has an additional cost for country A, which is that a fraction $1 - w(m^*(e))$ of the endowment is lost in war. If this cost is sufficiently high, country A may prefer not to attack country S even if its equilibrium payments $\sum_{k=0}^{\infty} \beta^k \widetilde{p}_{t+k}(e_t) \widetilde{x}_{t+k}(e_t)$ diverge to infinity. All the same, the main insights and the factors affecting the comparison between war and no war remain the same as in the case where $w(\tilde{m}) = 1$.

The next proposition contains the main result for the competitive environment. It characterizes the conditions under which equilibrium involves war.

Proposition 4 *Suppose $u(x)$ satisfies (17) and $\sigma \neq 1$.*

1. *Suppose $\sigma > 1$. Then there exists $\widehat{e} > 0$ such that if $e_0 < \widehat{e}$, then the unique MPCE has permanent peace, and if $e_0 > \widehat{e}$, then in any MPCE war occurs in period 0 with probability 1.*

2. *Suppose $\sigma < 1$. Then there exists $\widehat{w} < 1$ such that if $\lim_{m \to \bar{m}} w(m) < \widehat{w}$, then the unique MPCE has permanent peace, and if $\lim_{m \to \bar{m}} w(m) > \widehat{w}$, then in any MPCE war occurs in period 0 with probability 1.*

Proof. See Appendix. ∎

This proposition therefore shows that in the empirically more relevant case where $\sigma < 1$, provided that country A is capable of capturing most of the remaining endowment of the resource, the equilibrium will involve war at the initial date. The intuition for this result follows from Proposition 3. When $\sigma < 1$, spending on the resource and incentives to declare war increase over time. If, by spending the necessary resources, country A can capture a sufficient fraction of the remaining endowment of the resource, captured here by the condition that $\lim_{m \to \bar{m}} w(m) > \widehat{w}$, then it will necessarily find it optimal to declare war at some point. But we know from Proposition 2 that if war will occur in pure strategies, it must occur at the initial date, because anticipating war, country S producers would always deplete the entire resource and this induces country A to jump the gun and declare war at the initial date.

Notably, this conclusion is independent of the costs of war to either country (i.e., the function $l(\cdot)$ for country A, and $-\psi$ for country S). In particular, this proposition applies even if $\psi = -\infty$. In this case, of course, war is extremely costly to the citizens of country S, but under our

assumption that resource extraction takes place competitively, firms in this country can take no action to stave off a very costly war. This is one of the main motivations for our analysis of the "monopolistic" environment, where such actions will be possible. For future reference, we state this simple implication of Proposition 4 as a corollary.

Corollary 1 *If $\sigma < 1$ and if $\lim_{m \to \bar{m}} w(m)$ is sufficiently close to 1, then war will take place at date 0 even if $\psi = -\infty$.*

Proposition 4 does not cover the knife-edge case where $\sigma = 1$, which turns out to be more complicated. When $\sigma = 1$, the demand for the resource has unitary elasticity and the equilibrium payment $p_t x_t$ is constant over time (independent of e_t). In this case, when there exists a pure-strategy equilibrium with no war, there also exist mixed-strategy equilibria. In particular, country A might mix with a constant probability between war and no war at each date. When such a mixed-strategy equilibrium exists, it will involve equilibrium prices that rise at a faster rate than $(1 - \beta)/\beta$ and equilibrium allocations and prices will deviate from the Hotelling rule (16). Since such equilibria are only possible in the knife-edge case where $\sigma = 1$, we do not dwell on them.

The analysis of the competitive environment shows that the extraction patterns and prices always satisfy the Hotelling rule. We will see that this is not true in the monopolistic environment we study next.

4 Monopolistic Environment

From the point of view of country S, the competitive equilibrium is suboptimal for two reasons. The first is the standard price effect. Each producer, by extracting more, is reducing the price faced by other producers. In traditional trade models, this price effect is sometimes internalized by using "optimal" import and export taxes. The second is a novel externality resulting from the military actions of country A in response to the equilibrium path of prices. Recall the second part of Proposition 4 with $w(\tilde{m}) = 1$ and $\sigma < 1$. In this case, war is unavoidable under competitive markets and occurs immediately, even though the cost of war, $-\psi$, may be arbitrarily high for country S. War occurs because, as the price of the resource increases, payments from country A households to country S firms become arbitrarily large. Yet price-taking firms do not internalize that high resource prices increase incentives to fight for country A. If country S could somehow reduce these payments, it may be able to avoid war. The government of country S might, for example, regulate the price and quantity traded of the resource in order to prevent war or to improve the welfare of its citizens. In this section we study equilibrium allocations under such regulation. We will see that by regulating the levels of prices and production, the government of country S can indeed internalize the externalities, and that a consequence of this will be deviations of prices from the Hotelling rule. However, this type of monopolistic behavior by country S introduces a new externality due to its inability to commit to providing attractive

terms of trade to country A. Consequently, even though the monopolistic environment may be more effective at preventing war under certain conditions, it can also increase the likelihood of war and may even make country S worse off under others, despite its ability to act as the monopolist (Stackleberg leader) in its interactions with country A.[13]

4.1   Timing of Events and Markov Perfect Monopolistic Equilibrium

We consider a simple way of modeling the regulation of prices and quantities by the country S government, by allowing it to act as a "monopolist" and set prices and quantities recognizing their implications for current and future economic and military actions. In particular, suppose that the government sets nonlinear tariffs to control both the level of the price of the resource and its production. Given this resulting price-quantity pair, country A can still declare war. This environment is equivalent to one in which country S makes a take-it-or-leave-it price-quantity offer to country A. In what follows, we directly study a game in which country S makes such offers (and do not explicitly introduce the nonlinear tariffs to save on notation).

More specifically, we consider the following game. At every date t at which war has not yet occurred, country A chooses the level of armament m_t. Next, (the government of) country S makes a take-it-or-leave-it offer $z_t = \{x_t^o, c_t^o\}$ to country A, consisting of an offered delivery of x_t^o units of the resource in exchange for $-c_t^o$ units of the consumption good. Country A then accepts or rejects this offer, which is denoted by $a_t = \{0, 1\}$, with $a_t = 1$ corresponding to acceptance. Conditional on rejecting the offer, country A then chooses whether or not to declare war on country S. As in Section 3, the continuation payoff to country A following war is $V(w(m_t) e_t) - l(m_t)$, and the continuation payoff for country S is ψ.[14] If country A accepts the offer, then the flow utilities to households in countries A and S are $u(x_t^o) + c_t^o - l(m_t)$ and $-c_t^o$, respectively. If instead country A rejects the offer and does not declare war, then the flow utilities to households in country A and S are $u(0) - l(m_t)$ and 0, respectively

We formally summarize the order of events for all periods t for which $f_{t-1} = 0$ as follows:

1. Country A's government chooses a level of armament m_t.

2. Country S's government makes a take-it-or-leave-it offer z_t to country A.

3. Country A's government decides whether or not to accept the offer a_t. If $a_t = 0$, it can declare war by choosing f_t.

4. Extraction and consumption take place.

---

[13]Yet another alternative arrangement is one in which country S is restricted to set the price of the resource but cannot distort extraction decisions. Clearly, such policies are a subset of the more general set of policies we consider in this section, which allow general nonlinear tariffs and thus permit country S to choose any price-quantity combination.

[14]The additional cost of war v introduced for the refinement of MPCE in the previous section is now taken to be small or zero.

The timing of events makes it clear that this is a dynamic game between the two countries. We consider its Markov Perfect Equilibrium, which we refer to as *Markov Perfect Monopolistic Equilibrium* (MPME). This equilibrium is similar to an MPCE with the exception that firm and consumer optimality is no longer required, since country S's and country A's governments jointly determine the transfer of goods across countries. In such an equilibrium all actions depend only on payoff relevant state variables, which here include the endowment, e_t, and prior actions at the same date. As we did in the analysis of MPCE, we define strategies for dates t in which $f_{t-1} = 0$ (i.e., for histories where war has not yet occurred).

Let country A's strategy be presented by $\phi_A = \left\{ \phi_A^m, \phi_A^a, \phi_A^f \right\}$. Here ϕ_A^m assigns an armament decision for every e_t; ϕ_A^a assigns an acceptance decision for every (e_t, m_t, x_t^o, c_t^o); and ϕ_A^f assigns a war decision for every $(e_t, m_t, x_t^o, c_t^o, a_t)$, where this decision is constrained to 0 if $a_t = 1$. Country S's strategy is denoted by ϕ_S and consists of an offer z for every (e_t, m_t). We allow mixed strategies for both countries though it will become clear later that only pure strategies are relevant for all, except for knife-edge, cases. We next provide a formal definition of equilibrium.

Definition 2 *A Markov Perfect Monopolistic Equilibrium (MPME) is a pair $\{\phi_A, \phi_S\}$ where*

1. *Given ϕ_S, ϕ_A^m maximizes the welfare of country A for every e_t, ϕ_A^a maximizes the welfare of country A for every (e_t, m_t, x_t^o, c_t^o), and ϕ_A^f maximizes the welfare of country A for every $(e_t, m_t, x_t^o, c_t^o, a_t)$ subject to $f_t = 0$ if $a_t = 1$.*

2. *Given ϕ_A, ϕ_S maximizes the welfare of country S for every (e_t, m_t) subject to (3).*

Given these strategies, we define the equilibrium continuation values $\{U_A(e_t), U_S(e_t)\}$ as the continuation values to countries A and S, respectively, at the beginning of the stage game at t conditional on no war in the past. Similar to equation (14) in the previous section, these continuation values are given by

$$
\begin{aligned}
U_A(e_t) &= (1 - f_t)(u(x_t) + c_t + \beta U_A(e_{t+1})) + f_t V(w(m_t) e_t) - l(m_t), \text{ and} \\
U_S(e_t) &= (1 - f_t)(-c_t + \beta U_S(e_{t+1})) + f_t \psi,
\end{aligned}
$$

where we have removed the "o" superscript to economize on notation.

4.2 Analysis

We next characterize the MPME. We show that unlike in the competitive environment, the time path of resource extraction is distorted away from the Hotelling rule.[15] Despite this difference in price paths, many qualitative features of equilibrium are shaped by the same forces as in

---

[15] The key reason for distortions in the monopolistic equilibrium is the armament decision of country A. To highlight how armament affects the distortion, in the Appendix we analyze the case where country A can attack country S without arming. We show that in this case wars never occur and the path of resource extraction satisfies the Hotelling rule (16).

the competitive environment, in particular, by whether the elasticity of demand is greater than or less than one, which determines whether incentives to declare war increase or decrease over time. We also show that country S may delay wars or avoid them entirely in some of the cases when wars are unavoidable under competitive markets. Nevertheless, a naive conjecture that the monopolistic environment will necessarily reduce the likelihood of war and will make country S better off since it is now acting as a Stackleberg leader and making take-it-or-leave-it offers is not correct. In fact, it is possible for war to occur in a monopolistic equilibrium in cases when war can be avoided under competitive markets, and country S can have lower utility. Both of these features are the consequence of a new source of distortion in the monopolistic environment, resulting from the fact that country S cannot commit to making attractive price-quantity offers to country A; this, in turn, induces country A to invest in armaments at each date in order to improve its terms of trade.

We first consider the optimal strategy for country S for a given level of armament m_t. Let $\tilde{U}_S(e_t; m_t)$ be the value function of country S when its makes the best offer that country A accepts, starting with endowment e_t and armament level of country A equal to m_t. This value function is given by the following recursive equation:

$$\tilde{U}_S(e_t; m_t) = \max_{x_t \geq 0, c_t} \{-c_t + \beta U_S(e_{t+1})\} \tag{18}$$

subject to the resource constraint (3), and the participation constraint of country A, given by

$$u(x_t) + c_t - l(m_t) + \beta U_A(e_{t+1}) \geq V(w(m_t)e_t) - l(m_t). \tag{19}$$

Constraint (19) requires the value of country A when it accepts the price-quantity offer (x_t, c_t) at time t to be greater than its utility if it declares war and captures a fraction $w(m_t)$ of the remaining endowment of country S. This value also needs to be greater than the continuation value from rejecting the price-quantity offer but not declaring war. But it can be easily verified that this latter option is never attractive for country A, and hence there is no need to specify it as an additional constraint in the maximization problem (18).[16]

Moreover, it is straightforward to see that constraint (19) must bind in equilibrium, since otherwise country S could make an offer with slightly greater transfers and would increase its payoff. Finally, if $\tilde{U}_S(e_t; m_t)$ is less than the payoff from war ψ, the best response for country S is to make any offer that violates (19). Thus in equilibrium, starting from (e_t, m_t), the payoff of

---

[16]In particular, this additional constraint can be written as

$$u(x_t) + c_t - l(m_t) + \beta U_A(e_{t+1}) \geq u(0) - l(m_t) + \beta U_A(e_t).$$

Suppose, to obtain a contradiction, that this constraint binds. By definition, $U_A(e_t) = u(x_t) + c_t - l(m_t) + \beta U_A(e_{t+1})$, which combined with this (binding) constraint implies that $U_A(e_t) = (u(0) - l(m_t))/(1 - \beta)$, which is necessarily less than $V(w(m_t)e_t) - l(m_t)$, showing that (19) is violated.

country A is equal to

$$V\left(w(m_t)e_t\right) - l(m_t) \tag{20}$$

regardless of whether it accepts the price-quantity offer of country S. This implies that country A's best response is to always choose a level of armament maximizing (20). We defined this level of armaments as $m^*\left(e_t\right)$ in equation (8). Therefore, the equilibrium payoffs for countries A and S can be written as:

$$U_A(e_t) = V\left(w(m^*(e_t))e_t\right) - l(m^*(e_t)) \tag{21}$$

and

$$U_S(e_t) = \max\left\{\tilde{U}_S\left(e_t; m^*(e_t)\right); \psi\right\}. \tag{22}$$

We next show that an MPME exists.

Lemma 2 *An MPME exists.*

Proof. See Appendix. ∎

We now turn to the first main result of this section.

Proposition 5 *In any MPME, if $f_{t+1} = 0$, then*

$$
\begin{aligned}
\beta u'\left(x_{t+1}\right) &> u'\left(x_t\right) \;\; \textit{if } m^{*\prime}\left(e_{t+1}\right) > 0, \textit{ and} \\
\beta u'\left(x_{t+1}\right) &< u'\left(x_t\right) \;\; \textit{if } m^{*\prime}\left(e_{t+1}\right) < 0.
\end{aligned}
\tag{23}
$$

Proof. See Appendix. ∎

The main technical difficulty in the proof of this proposition lies in the fact that the value function $U_S(e_t)$ may not be differentiable and we use perturbation arguments in the Appendix to prove this result. It is easy to verify this result heuristically if one assumes differentiability. To do this, let us substitute (21) into (18), taking into account that since $f_{t+1} = 0$, it is the case that $\tilde{U}_S\left(e_t; m_t\right) = U_S\left(e_t\right)$. Take the first-order conditions to obtain

$$u'\left(x_t\right) - \beta u'\left(x_{t+1}\right) + \beta l'\left(m^*\left(e_{t+1}\right)\right)m^{*\prime}\left(e_{t+1}\right) = 0. \tag{24}$$

Since $l'\left(\cdot\right) > 0$, equation (24) implies (23).

Proposition 5 shows that the key determinant of the growth rate of the shadow price of the resource is whether country A increases or decreases armaments as the resource stock declines. This result is driven by the inabilities of both countries to commit to future actions. If country S could commit in period 0 to a sequence of offers $\{z_t\}_{t=0}^{\infty}$, only a one-time investment in armament by country A would be necessary and this would prevent war; the shadow price of the resource would also grow at the rate of time preference, $(1 - \beta)/\beta$, as in the Hotelling rule. In our model, such commitment is not possible. Country A needs to invest in armament in each period to

obtain better terms of trade from country S. In particular, given the timing of events above, it is clear that country A will choose armaments at each date in order to maximize its continuation value $V\left(w\left(m_t\right)e_t\right) - l\left(m_t\right)$, since this will be its utility given country S's take-it-or-leave-it offer. This continuation value incorporates the sequence of future armament costs as well, and so country S will take these into account also when deciding path of extraction and prices. To develop this intuition further, let us substitute (21) into (19):

$$u\left(x_t\right) + c_t + \beta\left(V\left(w\left(m^*\left(e_{t+1}\right)\right)e_{t+1}\right) - l\left(m^*\left(e_{t+1}\right)\right)\right) \geq V\left(w\left(m^*\left(e_t\right)\right)e_t\right). \qquad (25)$$

Suppose that armaments increase as the resource stock decreases. The increase in m_t implies that constraint (25) becomes harder to satisfy over time. If country S extracts ϵ units of resources less in period t and ϵ more in period $t+1$, holding everything fixed, it changes the payoff to country A by $\left(\beta u'(x_{t+1}) - u'\left(x_t\right)\right)\epsilon$. In addition, it relaxes constraint (25) since the stock of resources is higher so that armament by country A declines, and this allows country S to decrease the offer of c_t. Therefore, as long as $\beta u'(x_{t+1}) - u'\left(x_t\right) \geq 0$, country S can be made better off postponing resource extraction to next period. Thus, it must be the case that $\beta u'(x_{t+1}) - u'\left(x_t\right) < 0$ in equilibrium. When the amount of armament is decreasing in e_t, this effect works in the opposite direction.

Proposition 1 showed that the sign of $m^{*\prime}(e)$ is determined by elasticity of demand for the resource. Using Proposition 1 we next obtain the following corollary to Proposition 5, linking the direction of deviations from the Hotelling rule to the elasticity of demand for the resource.

Corollary 2 *Suppose $u(x)$ satisfies $-u'\left(x\right)/\left(xu''\left(x\right)\right) > (<)1$ for all x. Then in any MPME, whenever $f_{t+1} = 0$, we have*

$$\beta u'(x_{t+1}) > (<)u'(x_t).$$

We saw in Section 3 that elasticity of demand played a crucial role in determining whether incentives to declare war increase or decrease as the endowment of the resource is depleted. The same effect determines the equilibrium armaments for country A in the monopolistic environment. When $\sigma < 1$, demand is inelastic and the value of the resource, $V'(e_t)e_t$, increases over time. This induces country A to invest more in armaments. Country S internalizes the effect of resource depletion on country A's incentives to arm (as it can hold country A down to its continuation value). It then counteracts the rise in country A's armament costs by reducing the rate of resource extraction. This is equivalent to a (shadow) price sequence growing at a slower rate than the rate of time preference, $\left(1 - \beta\right)/\beta$. In contrast, if $\sigma > 1$, demand is elastic and the value of the resource and country A's armaments are decreasing in the endowment. In this case, country S can further reduce country A's armament costs by raising the rate of resource depletion.

We now turn to the analysis of the conditions under which peace occurs in the monopolistic environment. A naive conjecture is that country S's ability to regulate the price and the level

of production of the resource makes wars less likely and its citizens better off relative to the competitive equilibrium. This conjecture is not correct, however, because of the commitment problem identified above, which leads to a new distortion in this monopolistic environment. Recall that at each date country S makes a price-quantity offer that gives to country A utility equal to $V(w(m_t)e_t) - l(m_t)$. It cannot commit to giving a higher utility to country A, unless the latter invests more in armaments. So country A needs to invest in armaments at each date to secure favorable terms of trades. Therefore, the monopolistic environment encourages investments in armaments at each date whereas in the competitive environment country A did not need to invest in arms in periods in which it did not declare war. Moreover, since country S needs to give country A at least utility $U_A(e_t) = V(w(m^*(e_t))e_t) - l(m^*(e_t))$, it effectively pays for country A's future costs of armaments, so country S may be made worse off by its ability to make take-it-or-leave-it offers, or by its inability to commit to future paths of prices and production. The next proposition exploits this new distortion and shows why the above-mentioned conjecture is incorrect.

Proposition 6 *Suppose $u(x)$ satisfies (17). Then in any MPME,*

1. *War is avoided when $\sigma < 1$ and*

$$-\beta l(\overline{m}) > \psi(1 - \beta),$$ (26)

2. *War can be avoided when war necessarily occurs in an MPCE,*

3. *War occurs with probability 1 along the equilibrium path if $\sigma < 1$ and*

$$-\beta l(m^*(e_0)) < \psi(1 - \beta) - (V(e_0) - V(w(m^*(e_0))e_0))(1 - \beta), \text{ and}$$ (27)

4. *War can occur with probability 1 along the equilibrium path when war is necessarily avoided in the MPCE.*

Proof. See Appendix. ∎

The first part of the proposition shows that, under some circumstances, the ability for country S to control resource extraction allows it to avoid wars in situations in which the cost of armament is bounded below by the cost of war. For instance if $\psi = -\infty$, so that war is infinitely costly to country S, then country S avoids war in any monopolistic equilibrium and this is true even though wars may be inevitable in the competitive equilibrium. Similarly, if $\bar{m} < \infty$, war does not take place in MPME for large but finite ψ. The second part of the proposition is a simple consequence of the first. When war is highly costly to country S, it still takes place, under the conditions identified in part 1 of the proposition, in the competitive environment, but not necessarily in the monopolistic environment (for example, war never takes place in MPME when

$\psi = -\infty$). Note that in this case, country S's utility will clearly be higher in the monopolistic environment.

Nonetheless, parts 3 and 4 of the proposition show that the opposites of these conclusions might also be true. In particular, if ψ is sufficiently low, offers necessary to secure peace may be very costly for country S, especially since it is implicitly paying for the costs of future armament. In this case, wars can occur along the equilibrium path. More specifically, in contrast to Section 3, country A needs to make costly investments in armament in each period, *even if war does not take place*. This is because, as we noted above, country S cannot commit to making attractive offers unless country A has an effective threat of war, and thus country A is induced to invest in armament to improve its terms of trade. But this means that war will reduce future costs of armament; consequently, to secure peace, country S must make offers that compensate country A for the costs of future armament. If these costs are increasing to infinity along the equilibrium path, then the cost to country S of such offers will eventually exceed the cost from war, $-\psi$, which means that war cannot be permanently avoided. More generally, this cost of war may be sufficiently low that country S prefers to allow immediate war in the monopolistic equilibrium even though war does not occur in the competitive equilibrium.[17]

In sum, allowing country S to control the extraction of resources introduces two new economic forces relative to the competitive environment. *First*, it implies that country S controls the externalities generated by competitive firms. *Second,* it also introduces a new strategic interaction between the two countries because country S can control the terms of trade directly but is unable to commit to making sufficiently attractive offers to country A without armaments by the latter. This lack of commitment implies that country A will have an incentive to use investments in armaments in order to enhance its terms of trade. The first force implies that war can be avoided or delayed in the monopolistic equilibrium in situations in which it is inevitable in the competitive equilibrium. The second force implies that, since country A must now invest in armament under peace, war takes place in the monopolistic equilibrium even when it can be avoided in the competitive equilibrium.

5 Extensions

In this section, we discuss several extensions that show both the robustness of the insights discussed so far and indicate new interesting effects. To simplify the discussion, we focus on the monopolistic environment (sometimes briefly mentioning how the MPCE is affected). Also to

---

[17]Though, as we have emphasized, the commitment problem facing country S is essential for the result that inefficient war can happen in MPME, as in the MPCE commitment by country A to limit its armaments in the future (say to be no more than some small $\epsilon > 0$) could also prevent war and may lead to a Pareto superior allocation. For example, a commitment by country A that in the future it will only have no or little armament implies that country S will have a high payoff from tomorrow onward and country A will have a low payoff. If country A chooses high armaments today, this would then force country S to make a large transfer today, and from tomorrow onward, oil would be traded at undistorted market prices without war. This discussion highlights that lack of commitment on the part of both countries is important for the presence of inefficient war.

simplify the exposition in this section we impose that $\psi = -\infty$ so that wars never occur in an MPME.

5.1 Inter-Country Competition

In practice, international conflict over resources can involve multiple competing resource-poor countries. In this section we consider the implications of allowing for N resource-poor countries labeled by $i = \{1, ..., N\}$ which compete over the resources from country S. The economy is identical to that of Section 4, though the resource constraint is replaced by

$$e_{t+1} = e_t - \sum_{i=1}^{N} x_{it}, \tag{28}$$

where $x_{it} \geq 0$ corresponds to the consumption of the resource by the households (each of mass 1) in country i and $c_{it} \gtrless 0$ again refers to the consumption good. The flow utility to country i from its consumption of the resource and the consumption good is equal to $u(x_{it}) + c_{it}$ and it discounts the future at the rate β. As such, country S's flow utility from the consumption good equals $\sum_{i=1}^{N} -c_{it}$ and it discounts the future at the rate β.

At any date t, country i can invest in armament $m_{it} \geq 0$ at cost $l(m_{it})$ and declare war. We assume that if any country declares war, all countries join the war, so that we have a "world war". In such a war, the fraction of the remaining endowment of oil captured by country i is assumed to be

$$w_i(m_{it}, \mathbf{m}_{-it}) = \eta \frac{h(m_{it})}{\sum_{j=1}^{N} h(m_{jt})},$$

where $\mathbf{m}_{-it} = \{m_{jt}\}_{j=1, j \neq i}^{N}$ is the vector of armaments by other countries, $\eta \in (0, 1]$, and h is increasing, continuously differentiable and concave. These assumptions imply that total amount of oil after the war is possibly less than the endowment before the war (and thus the interpretation is that each of the N resource-poor countries invades part of the territory of country S). Naturally, $w_i(m_{it}, \mathbf{m}_{-it})$ is increasing in own armament and decreasing in the armament of other countries. This specification is particularly tractable as it implies that the continuation value to country i from war is equal to $V(w_i(m_{it}, \mathbf{m}_{-it}) e_t) - l(m_{it})$ for $V(\cdot)$ defined as in (4). Given this modified environment, $f_T = 0$ now denotes that no war has been declared by any country in periods $t = 0, ..., T$, and we let $f_T = 1$ denote that war has been declared by some country in period $t \leq T$.[18]

First note that, the MPCE in this extended environment with multiple resource-poor environment is similar to Proposition 2. In particular, in the pure-strategy equilibrium, war can only take place at date $t = 0$ and the Hotelling rule applies throughout. In what follows, we focus on MPME.

---

[18] Our analysis can also be interpreted as applying to a situation in which only country i attacks country S and it seizes a fraction of the oil which is decreasing in the armament of its rivals.

At every date t, country S's government publicly makes a take-it-or-leave-it offer to each country i, $\{x_{it}^o, c_{it}^0\}$, consisting of a quantity of resource to be traded in exchange of the consumption good for each i. For simplicity, we assume that rejection of the offer by any country i automatically leads to world war.

The order of events for all periods t for which $f_{t-1} = 0$ is as follows:

1. Each country i government chooses a level of armament m_{it}.

2. Country S's government makes a take-it-or-leave-it offer $\{x_{it}^o, c_{it}^0\}$ to each i.

3. Each country i government decides whether or not to declare world war.

4. Consumption takes place.

Using this framework, we can define the MPME as in Section 4. We define $U_i(e_t)$ as the continuation value to country i conditional on e_t and $f_{t-1} = 0$ and we define $U_S(e_t)$ analogously for country S. Since $\psi = -\infty$, war is always avoided along the equilibrium path.

By the same reasoning as in Section 4, country i chooses the level of armament at each date to maximize its payoff from war in order to receive the most favorable offer from country S. More specifically, it must be that in equilibrium $m_{it} = \widetilde{m}_i^*(e_t, \mathbf{m}_{-it})$ for

$$\widetilde{m}_i^*(e_t, \mathbf{m}_{-it}) = \arg\max_{m_i \geq 0} V\left(w_i(m_i, \mathbf{m}_{-it}) e_t\right) - l(m_i), \tag{29}$$

is uniquely defined, satisfies $\widetilde{m}_i^*(e_t, \mathbf{m}_{-it}) > 0$ and is a continuously differentiable function in all of its elements. This implies an analogous equation to (21):

$$U_i(e_t) = V\left(w_i\left(\widetilde{m}_i^*(e_t, \mathbf{m}_{-it}), \mathbf{m}_{-it}\right) e_t\right) - l\left(\widetilde{m}_i^*(e_t, \mathbf{m}_{-it})\right)$$

for all i where $m_{jt} = \widetilde{m}_j^*(e_t, \mathbf{m}_{-jt})$ for all j. Note that given this formulation, $U_i(e_t)$ may not necessarily be continuously differentiable in each MPME, as it was in the case of Section 4. To simplify the discussion, let us also focus on symmetric MPME, where $m_i^*(e_t) = m^*(e_t)$ for all i and country S make the same offer to each i in each date. A symmetric MPME always exists and in such an equilibrium $U_i(e_t)$ is differentiable (from a straightforward application of the implicit function theorem). Since in a symmetric equilibrium all countries choose the same armament $m^*(e_t)$, we have that $w_i(m_{it}, \mathbf{m}_{-it}) = \eta/N$ for each i.

Proposition 7 *In any symmetric MPME,*

1. For country i, resource extraction satisfies

$$\beta u'(x_{it+1}) > u'(x_{it}) \ \text{if} \ m_i^{*\prime}(e_{t+1}) > 0 \ \text{and}$$
$$\beta u'(x_{it+1}) < u'(x_{it}) \ \text{if} \ m_i^{*\prime}(e_{t+1}) < 0.$$

2. If u satisfies

$$-u'(x)/\left(xu''(x)\right) > (<)\,1 \text{ for all } x,$$

then $m_i^{*\prime}(e_t) > (<)\,0$ for all i.

Proof. See Appendix. ∎

Proposition 7 states that the shadow value of resources in country i grows faster or slower than the rate of time preference, $(1-\beta)/\beta$, depending on whether the level of armament is rising or decreasing in the resource endowment. It is important to note that the argument leading to this result relates to how armament *for all countries* moves as the endowment declines. Thus the intuition for Proposition 7 is similar to that of Proposition 5 except that we must now take into account how future values of the endowment e_{t+1} affect the armament of all countries jointly.

It is also noteworthy that in a symmetric equilibrium countries, by definition, armament decisions of different countries will co-move as the endowment depletes. The second part of Proposition 7, which is similar to Proposition 1, states that whether armament increases or decreases as the endowment is depleted depends on the elasticity of demand. This co-movement incorporates the best responses of each resource-poor country to the armaments decisions of its neighbors.

To illustrate the complementarity in armament decisions across countries and its implications, let us consider a simple example in which we can explore the consequences of changing the number of competing countries N. Suppose that preferences satisfy (17) so that the elasticity of demand is constant. Moreover, let $w_i(\cdot)$ and $l(\cdot)$ take the following functional forms:

$$w_i\left(m_{it}, \mathbf{m}_{-it}\right) = \frac{m_{it}}{\sum_{j=1}^{N} m_{jt}} \text{ and } l\left(m_{it}\right) = m_{it}. \tag{30}$$

In this environment, it can be shown that the symmetric MPME is unique and involves:

$$m_i^*(e_t) = \left(\frac{N-1}{N}\right)\left(1-\beta^{\sigma}\right)^{-1/\sigma}\left(\frac{e_t}{N}\right)^{1-1/\sigma}. \tag{31}$$

This means that conditional on per-country endowment level e_t/N, the level of armament is increasing in military competition parameterized by $(N-1)/N$. Intuitively, if there are more resource-poor countries competing for the same total endowment, returns to arming will be higher and these returns will become more sensitive to changes in the per-country endowment. Naturally, country S takes this into account in deciding the time path of extraction. This reasoning establishes the following proposition.

Proposition 8 *Suppose that preferences and technologies satisfy* (17) *and* (30). *Then in the symmetric MPME,*

1. *There exists $\rho > 0$ such that $u'(x_{it+1}) = (1/\rho)\,u'(x_{it})$ for all t;*

2. $1/\rho > (<) 1/\beta$ *if* $\sigma > (<) 1$; *and*

3. $|\rho - \beta|$ *is increasing in* N *if* $\sigma \neq 1$.

Proof. See Appendix. ∎

This proposition states that under (17) and (30) the growth rate of the shadow value of the resource is constant and depends on the elasticity of substitution σ. Interestingly, the last part of the proposition states that the distortion in this growth rate from the Hotelling rule is *increasing* in the level of international competition. The intuition for this is that as (31) shows, when N is greater, the marginal benefit of armament is also greater, implying that global armament becomes more sensitive to changes in the resource endowment. For instance, if $\sigma < 1$ so that armament is increasing along the equilibrium path, an increase in the level of international competition (captured by a higher N) raises global armaments (because of the complementarities in armament decisions) and induces country S to further slow down oil extraction so as to mitigate the rise in armament coming from all N countries (for which it is paying indirectly through lower prices).

5.2 Armament in Defense

In practice, a defending country S can also invest in armament in order to deter an attack. In this subsection, we extend the baseline environment to allow for armaments by country S. We focus on MPME.[19] More specifically, at each t, country S can invest in armament $m_{St} \geq 0$ which costs $l(m_{St})$ whereas country A invests in armament $m_{At} \geq 0$ which costs $l(m_{At})$ as before. Country S still receives payoff ψ in the event of war, though country A's payoff now depends on both countries' armaments. In particular, it receives a fraction of the remaining endowment $w(m_{At}, m_{St})$. We assume that $w(\cdot, \cdot)$ satisfies

$$w(m_{At}, m_{St}) = \eta \frac{h(m_{At})}{h(m_{At}) + h(m_{St})},$$

where $\eta \in (0, 1]$ and h is increasing, continuously differentiable and concave.

The order of events at t if $f_{t-1} = 0$ is exactly the same as in Section 4 with the exception that in the first stage, countries A and S simultaneously choose m_{At} and m_{St}. Using this framework, we can define the MPME as in Section 4 with $U_A(e_t)$ and $U_S(e_t)$ denoting the continuation values to countries A and S, respectively, given endowment e_t.

By the same reasoning as in Section 4, at each t country A chooses the level of armament that maximizes its payoff from war in order to receive the most favorable offer from country S. More specifically, it must be that in equilibrium $m_{At} = \widetilde{m}_A^*(e_t, m_{St})$ for

$$\widetilde{m}_A^*(e_t, m_{St}) = \arg\max_{m_A \geq 0} V(w(m_A, m_{St}) e_t) - l(m_A).$$

---

[19] The analysis of MPCE is more involved in this case, though it can again be shown that given our assumptions here, war must take place at date $t = 0$ (if it will take place at all).

Given our assumptions on $u(\cdot)$, $w(\cdot)$, and $l(\cdot)$, $\widetilde{m}_A^*(e_t, m_{St}) > 0$ and is a continuously differentiable function of its arguments. Since country S's offers make country A indifferent to war and no war, a similar equation to (21) holds:

$$U_A(e_t) = V(w(\widetilde{m}_A^*(e_t, m_{St}), m_{St})e_t) - l(\widetilde{m}_A^*(e_t, m_{St})).$$

Moreover, analogous arguments to those of Section 4 imply that if $\tilde{U}_S(e_t; m_{At}; m_{St})$ corresponds to country S's welfare from its optimal offer conditional on e_t, m_{At}, and m_{St}, then it must satisfy:

$$\tilde{U}_S(e_t; m_{At}; m_{St}) = \max_{x_t \geq 0, c_t} \{-c_t - l(m_{St}) + \beta U_S(e_{t+1})\} \tag{32}$$

subject to (3), and

$$u(x_t) + c_t - l(m_{At}) + \beta U_A(e_{t+1}) \geq V(w(m_{At}, m_{St})e_t) - l(m_{At}). \tag{33}$$

Since we assumed that $\psi = -\infty$, country S always makes an offer which is accepted and $U_S(e_t) = \tilde{U}_S(e_t; m_{At}, m_{St})$. Since constraint (33) will bind in equilibrium, we can substitute (33) into (32) and obtain the value of m_{St} that maximizes $U_S(e_t)$ is given by

$$\widetilde{m}_S^*(e_t, m_{At}) = \arg\max_{m_S \geq 0} -V(w(m_{At}, m_{St})e_t) - l(m_S).$$

Clearly, when strictly positive, $m_S^*(e_t, m_{At})$ is continuously differentiable.

Note that given this formulation, and in contrast to our results in Section 4, $U_A(e_t)$ may not be differentiable. To facilitate the exposition in this subsection, we focus on a "differentiable" MPME where it is indeed differentiable. Then, the first-order conditions characterizing $\widetilde{m}_A^*(e_t, m_{St})$ and $\widetilde{m}_S^*(e_t, m_{At})$ are given by

$$V'(w(\cdot)e_t)e_t\eta\frac{h'(m_{At})h(m_{St})}{(h(m_{At}) + h(m_{St}))^2} = l'(m_{At}), \text{ and}$$

$$V'(w(\cdot)e_t)e_t\eta\frac{h'(m_{St})h(m_{At})}{(h(m_{At}) + h(m_{St}))^2} = l'(m_{St}).$$

The convexity of the l function and the concavity of the h function imply that $\widetilde{m}_A^*(e_t, m_{St}) = \widetilde{m}_S^*(e_t, m_{At})$, so that $w(\cdot)$ is always constant and equal to $\eta/2$.[20] This means that in this environment one can define $\{m_A^*(e_t), m_S^*(e_t)\}$ which represents two continuously differentiable functions corresponding to the equilibrium levels of armament for each country conditional on the endowment e_t.

Proposition 9 *In any differentiable MPME, we have that:*

---

[20] Note that $h(\cdot)$ and $l(\cdot)$ could be scaled by a player specific constant so that $w(\cdot)$ can be equal to a different constant without changing any of our results.

1. *Resource extraction satisfies*

$$\beta u' \left(x_{t+1}\right) > u' \left(x_t\right) \ \text{if} \ m_i^{*\prime} \left(e_{t+1}\right) > 0 \ \text{for} \ i \in \{A, S\}, \ \text{and}$$

$$\beta u' \left(x_{t+1}\right) < u' \left(x_t\right) \ \text{if} \ m_i^{*\prime} \left(e_{t+1}\right) < 0 \ \text{for} \ i \in \{A, S\}.$$

2. *If u satisfies*

$$-u' \left(x\right) / \left(x u'' \left(x\right)\right) > (<) \, 1 \ \text{for all} \ x,$$

then $m_A^{*\prime} \left(e_t\right) > (<) \, 0$ and $m_S^{*\prime} \left(e_t\right) > (<) \, 0$.

Proof. See Appendix. ∎

Proposition 9 states that the shadow value of the resource rises slower relative to the Hotelling rule if $m_A^{*\prime} \left(e_{t+1}\right)$ and $m_S^{*\prime} \left(e_{t+1}\right)$ rise as the resource is depleted. The intuition for this result is analogous to that of Proposition 5 except that in addition to considering the future armament of country A, country S's extraction decisions take into account how future values of the endowment will affect its own armament and through this channel also affect country A's armament (which co-moves with country S's armament).

The second part of the proposition states that if the elasticity of demand exceeds one, then the armaments of both country A and country S decline as the resource is depleted along the equilibrium path. The intuition for this result is the same as that for Proposition 1, with the exception that it takes into account how country A and country S are choosing armaments which optimally react to each other. In particular, when the elasticity of demand is less than one, the same forces as in Proposition 1 push armaments by country A to increase over time. The equilibrium response of country S then leads to increasing armaments by both countries.

5.3 Alternative Preferences

A natural question is the extent to which our conclusions depend on our assumption of quasi-linear preferences for country A. In this subsection, we focus on MPME and show that the general insights in Proposition 5 continue to hold. More specifically, consider an environment in which the flow utility to country A is equal to

$$u \left(x_t, c_t, -m_t\right),$$

where $u \left(\cdot\right)$ is increasing and globally concave in x_t, c_t, and $-m_t$. Let $\lim_{x \to 0} u_x \left(\cdot\right) = \infty$ and $\lim_{x \to \infty} u_x \left(\cdot\right) = 0$. For simplicity, we assume that $u \left(\cdot\right)$ is defined for all values of $c_t \gtrless 0$.[21]

Note that in this environment, the Hotelling rule can be written as:

$$u_x \left(x_{t+1}, c_{t+1}, -m_{t+1}\right) / u_c \left(x_{t+1}, c_{t+1}, -m_{t+1}\right) = (1/\beta) \, u_x \left(x_t, c_t, -m_t\right) / u_c \left(x_t, c_t, -m_t\right),$$

---

[21] The analysis of MPCE in this case is similar to the baseline environment since $u \left(0, 0, 0\right)$ is either finite or equal to $-\infty$. Therefore, a direct application Proposition 2 shows that in any pure-strategy equilibrium, war can only occur in the initial period.

so that the marginal rate of substitution between the resource and the consumption good is increasing in the discount rate.

Consider the order of events and define the MPME as in Section 4. In this environment, we can define:

$$\widetilde{V}(e_t) = \max_{\{x_{t+k}, e_{t+k+1}\}_{k=0}^{\infty}, m_t} u(x_t, 0, -m_t) + \sum_{k=1}^{\infty} \beta^k u(x_{t+k}, 0, 0)$$

subject to (5)-(7). Here $\widetilde{V}(e_t)$ corresponds to the highest continuation value that country A can achieve in the event of war and is the analogue of $V(w(m^*(e_t))e_t) - l(m^*(e_t))$ in the quasi-linear case. Let $m^*(e_t)$ correspond to the value of m_t associated with $\widetilde{V}(e_t)$.

Proposition 10 *In an MPME,*

$$u_x(x_{t+1}, c_{t+1}, -m_{t+1})/u_c(x_{t+1}, c_{t+1}, -m_{t+1}) > (<) (1/\beta) u_x(x_t, c_t, -m_t)/u_c(x_t, c_t, -m_t)$$

if

$$m^{*\prime}(e_{t+1}) + \frac{\widetilde{V}'(e_{t+1})}{u_m(x_{t+1}, c_{t+1}, -m_{t+1})} \left(1 - \frac{u_c(x_{t+1}, c_{t+1}, -m_{t+1})}{u_c(x_t, c_t, -m_t)}\right) > (<) 0.$$

Proof. See Appendix. ∎

Proposition 10 states that the shadow price of the resource increases faster (slower) if armament increases (decreases) in the size of the total resource endowment, which is similar to Proposition 5. Nevertheless, in relating this rate of growth to the rate of time preference, Proposition 10 differs from Proposition 5 because the rate of growth of the shadow price not only depends on $m^{*\prime}(e_{t+1})$ but also on an additional term (which was equal to zero when preferences were quasi-linear). This term emerges because even in the absence of endogenous armament, there will be distortions in the growth rate of the shadow price provided that the marginal utility of the consumption good is time varying. Intuitively, when country A's marginal utility from the consumption good is lower, it is cheaper for country S to extract payments from country A while still ensuring that country A does not declare war. Therefore, if the marginal utility of the consumption good is higher (lower) today relative to tomorrow, country S will deplete more (less) of the endowment today. Proposition 10 therefore shows that in addition to this force, the sign of $m^{*\prime}(e_{t+1})$ continues to play the same role as in the quasi-linear case.[22]

5.4 Further Extensions and Discussion

In this subsection, we discuss several alternative approaches one could adopt within the broad umbrella of the framework developed in this paper. A full analysis of these extensions is beyond the scope of the current paper, though we believe that this framework can be fruitfully developed to study several of these.

---

[22]It may be conjectured that in a richer environment with additional smoothing instruments such as bonds, this marginal utility of consumption will not vary significantly along the equilibrium path so that the dominating effect would come from $m^{*\prime}(e_{t+1})$.

A first issue is how our results would differ if we focus on subgame perfect equilibria rather than Markovian equilibria (e.g., as in MPCE or MPME). While the Markovian restriction in the MPCE is not central, we cannot give a comprehensive answer to this question for the monopolistic environment because characterizing the entire set of subgame perfect equilibria turns out to be a very challenging problem. It can be shown that subgame perfect equilibria must satisfy two incentive compatibility constraints, one ensuring that country A does not declare war (which essentially requires country A's continuation utility to be greater than (20) evaluated at $m^*(e_t)$ given by (8)) and one ensuring that country S does not deviate from the equilibrium path of offers given the current level of armament by country A (and anticipating that any sufficiently attractive offer to country A can deter it from war). This description implies that, similar to the MPME, country A will have an incentive to arm in subgame perfect equilibria. For example, suppose that preferences satisfy $-u'(x)/(xu''(x)) < 1$ for all x. Since $w(0) = 0$, it can be shown in this situation that $V(w(0) e_t) = -\infty$ for all e_t. Suppose that country A chose 0 armament at date t, then country S could extract an arbitrarily large payment from country A while still avoiding war since rejection of the offer would provide infinite disutility to country A. However, this would not be incentive compatible for country A at the armament stage since it could instead deviate to armament level $m^*(e_t)$, go to war, and make itself strictly better off. This implies that subgame perfect equilibria have much in common with MPME and suffer from the same commitment problem on the part of country S—i.e., country S will be unable to commit to offering attractive terms of trade to country A if the latter does not invest in armaments. However, a full characterization of the path of distortions requires us to first determine the "worst subgame perfect equilibrium" from the viewpoint of both countries, which turns out to be very difficult. For this reason, we have focused on Markovian equilibria, even though the argument here suggests that certain economic insights continue to hold with subgame perfect equilibria.

A second issue is whether alternative arrangements could emerge as a way of preventing war and the costs of armaments. One possibility would be a leasing agreement, where country S may sell or lease its oil fields to country A, thus reducing or eliminating future armaments. We believe that this is an interesting possibility, though it raises its own set of commitment issues. In particular, in the same way that country S can renege on any promise concerning future prices, it can renege on its lease contract and "nationalize" the oil fields. Then country A would need to arm in order to ensure that its lease contract is not violated. If we again focus on Markovian equilibria, violation of lease contracts may be attractive to country S, and may preclude leasing along the equilibrium path. On the other hand, it may well be the case that country A could protect the lease contract with lower investment in armaments than the one necessary for war. This discussion highlights that the exact implications of leasing would depend on how leasing differs from spot market transactions, particularly in regards to the type of military might that needs to be exercised to support such transactions.

A related but distinct issue is that country S may voluntarily choose to be "colonized" by

country A instead of going to war. Such colonization might be attractive relative to the payoff from war, ψ. Such an arrangement, however, raises new issues. Country A may again be forced to invest in armaments in order to protect these resources, for example, against an insurrection from its colonial subjects. Once again, exactly what types of military investments need to be made to support different types of contractual arrangements becomes central.

Yet another issue that can be studied using an extended version of this framework concerns the nature of equilibrium when country A can switch to a different technology. For example, when the resource in question is oil, country A could have access to a backstop technology in the form of nuclear power, coal or perhaps green technologies. This possibility can be analyzed using our framework, though the main results need to be modified because the Inada conditions no longer hold and the possibility of a switch to another technology affects incentives at all points in time.

Finally, our framework ignores domestic political economy issues, which are obviously critical in the context of exploitation of and conflict over natural resources. For example, most of the gains from natural resource income may accrue to an elite in country S, as they do, for example, in Saudi Arabia, Kuwait, United Arab Emirates or even Iran, while the cost of war may be borne by all citizens. Similarly, in country A there may be different constituencies in favor of different types of trade and military relationships with country S. The analysis of the interactions between domestic politics and dynamic trade of natural resources is another interesting area which can be studied by a (significant) generalization of our framework.

6 Conclusion

This paper analyzed a dynamic environment in which a resource-rich country trades an exhaustible resource with a resource-poor country. In every period, the resource-poor country can arm and attack the resource-rich country. When the resource is extracted by price-taking firms, there is a novel externality as each firm fails to internalize the impact of its extraction on military action by the resource-poor country. In the empirically relevant case where the demand for the resource is inelastic and the resource-poor country can capture most of the remaining endowment in a war, war becomes inevitable. Because the anticipation of future war encourages more rapid extraction, equilibrium war happens in the initial period.

Externalities across price-taking firms can be internalized by the government of the resource-rich country regulating the price and the level of production of the resource. This "monopolistic" environment can prevent or delay wars even when they occur immediately under competitive markets. The resource-rich country does so by making offers that leave the resource-poor country indifferent between war and peace at each date. Interestingly, this involves a deviation from the Hotelling rule because, depending on whether incentives for war are increasing or decreasing in the remaining endowment of the resource, the resource-rich country prefers to adopt a slower or more rapid rate of extraction of the resource than that implied by the Hotelling rule. In

particular, in the empirically relevant case where the demand elasticity for the resource is less than one, extraction is slower and resource prices increase more slowly than under the Hotelling rule because this enables the resource-rich country to slow down the rise in armaments, for which it is paying indirectly. Conversely, when demand is elastic, the resource-rich country can reduce armament costs by adopting a more rapid path of resource extraction than the one implied by Hotelling rule.

Nevertheless, a naive conjecture that regulation of prices and quantities by the resource-rich country will necessarily prevent war and make its citizens necessarily better off is also incorrect. The monopolistic environment, which allows for such regulation and in fact gives the resource-rich country the ability to make take-it-or-leave-it offers, leads to a different type of distortion: because the resource-rich country cannot commit to making attractive offers to the resource-poor country without the latter arming, the equilibrium path involves armaments at each date. The resource-rich country must then, implicitly, pay the future costs of armaments in order to prevent war. This might, paradoxically, make war more likely than the competitive equilibrium.

Finally, we also show that the main insights generalize to the case where there are several countries competing for resources and where the resource-rich country can also invest in armaments for defense.

We view our paper as a first step in the analysis of interactions between dynamic trade and inter-country military actions. These ideas appear particularly important in the context of natural resources since their trade is necessarily dynamic and international trade in natural resources has historically been heavily affected by military conflict or the threat thereof. Despite the simplicity of the economic environment studied here, both under competitive markets and when the resource-rich country can regulate prices and quantities, there are rich interactions between economic equilibria and international conflict. In particular, the path of prices is affected by the future probabilities of war, while simultaneously the likelihood of war is shaped by the paths of prices and quantities. We think that further study of dynamic interactions between trade, international conflict and political economy, including the several areas mentioned in subsection 5.4, is a fruitful area for future research.

7 Appendix

7.1 Proof from Section 2

Proof of Proposition 1

The first-order condition to (8) defines $m^{*}\left(e\right)$ as

$$l'\left(m\right) = V'\left(w\left(m\right)e\right)w'\left(m\right)e \text{ for all } e. \tag{34}$$

Given the solution to (4), the envelope condition implies that

$$V'\left(w\left(m_t\right)e_t\right) = \beta^k u'\left(x_{t+k}\right) \ \forall k \geq 0. \tag{35}$$

Substitution of (35) into (34) followed by implicit differentiation yields

$$\left(\frac{l''\left(m_t\right)}{\beta^k u''\left(x_{t+k}\right)w'\left(m_t\right)e_t} - \frac{u'\left(x_{t+k}\right)w''\left(m_t\right)}{u''\left(x_{t+k}\right)w'\left(m_t\right)}\right)\frac{dm_t}{de_t} = \frac{dx_{t+k}}{de_t} + \frac{u'\left(x_{t+k}\right)}{u''\left(x_{t+k}\right)e_t}. \tag{36}$$

Summing up (5) and (6) one obtains

$$\sum_{k=0}^{\infty} x_{t+k} = w\left(m_t\right)e_t \tag{37}$$

differentiation of which implies

$$\sum_{k=0}^{\infty} \frac{dx_{t+k}}{de_t} = w\left(m_t\right) + w'\left(m_t\right)e_t\frac{dm_t}{de_t}. \tag{38}$$

Taking the sum of (36) $\forall k \geq 0$ and substitution into the above equation yields

$$\frac{dm_t}{de_t} = \frac{w\left(m_t\right)\left(1 + \sum_{k=0}^{\infty} \frac{u'\left(x_{t+k}\right)}{u''\left(x_{t+k}\right)x_{t+k}}\frac{x_{t+k}}{w\left(m_t\right)e_t}\right)}{\sum_{k=0}^{\infty}\left(\frac{l''\left(m_t\right)}{\beta^k u''\left(x_{t+k}\right)w'\left(m_t\right)e_t} - \frac{u'\left(x_{t+k}\right)w''\left(m_t\right)}{u''\left(x_{t+k}\right)w'\left(m_t\right)}\right) - w'\left(m_t\right)e_t}. \tag{39}$$

Since the denominator is negative, (39) is positive if and only if the numerator is negative. If $-u'\left(x_{t+k}\right)/u''\left(x_{t+k}\right)x_{t+k} > 1 \ \forall x_{t+k}$ then the numerator is negative since from (37), $\sum_{k=0}^{\infty} \frac{x_{t+k}}{w\left(m_t\right)e_t} = 1$, and the opposite holds if $-u'\left(x_{t+k}\right)/u''\left(x_{t+k}\right)x_{t+k} < 1 \ \forall x_{t+k}$. ∎

7.2 Proofs from Section 3

Definition of Strategies at $e_t = 0$ for $u(0) = -\infty$

As noted in the text, when the endowment equals 0 and $u(0) = -\infty$, then in the unperturbed economy the payoff from war and from peace may both equal $-\infty$. We determine whether or not war occurs in this case by explicitly looking at the economy with cost of war $\upsilon > 0$ for country A as specified in Definition 1. Let

$$U^C(e) = \sum_{t=0}^{\infty} \beta^t \left(u\left(\widetilde{x}_t(e)\right) - u'\left(\widetilde{x}_t(e)\right) \widetilde{x}_t(e)\right) \tag{40}$$

$$\text{for } \{\widetilde{x}_t(e), \widetilde{e}_t(e)\}_{t=0}^{\infty} \text{ which satisfies}$$

$$u'\left(\widetilde{x}_{t+1}(e)\right) = (1/\beta) u'\left(\widetilde{x}_t(e)\right),$$

$$\widetilde{e}_{t+1}(e) = \widetilde{e}_t(e) - \widetilde{x}_t(e), \text{ and } \widetilde{e}_0(e) = e.$$

$U^C(e)$ corresponds to equilibrium welfare of country A in a permanently peaceful competitive equilibrium starting from endowment e at date 0, where $\widetilde{x}_t(e)$ and $\widetilde{e}_t(e)$ correspond to the resource consumption and resource endowment, respectively, at date t in such an equilibrium.

For cost of war $\upsilon \geq 0$, we define

$$F_\upsilon(e) \equiv U^C(e) - \left(V\left(w\left(m^*(e)\right)e\right) - l\left(m^*(e)\right) - \upsilon\right). \tag{41}$$

$F_\upsilon(e)$ corresponds to the difference in country A's welfare between a permanently peaceful competitive equilibrium and war with optimal armament $m^*(e)$ starting from endowment e when the cost of war is equal to υ. In what follows, we will not separately give the expressions for the case where $\upsilon = 0$, which can be readily obtained from the expressions here by setting $\upsilon = 0$. Following the fourth requirement of the definition of MPCE, we will determine the behavior of country A at zero endowment (when $u(0) = -\infty$) from this function $F_\upsilon(e)$. In particular, given this function, our definition in the text implies:

Observation (Equilibrium Selection) Suppose that $f_{t-1} = 0$ and $e_t = 0$. Then $f_t = 0$ only if $\lim_{e \to 0} F_\upsilon(e) > 0$.

Note that this definition also subsumes the case for which $u(0) > -\infty$, as in this case $\lim_{e \to 0} F_\upsilon(e) = \upsilon > 0$ and thus $f_t = 0$ at $e_t = 0$. The following lemma and its corollary are useful to simplify the analysis of country A's equilibrium decisions. Because all of our results in this Appendix are true for any value of $\upsilon > 0$, we do not qualify the next lemma and other lemmas and propositions with "fix some $\upsilon > 0$".

Lemma 3 *Starting from any e_t^*, country A's payoff $U_A(e_t^*)$ must satisfy*

$$U_A(e_t^*) = \max\left\{ u\left(x_t^{A*}\right) - p_t^* x_t^{A*} + \beta U_A\left(e_{t+1}^*\right), V\left(w\left(m^*(e_t)\right)e_t^*\right) - l\left(m^*(e_t^*)\right) - \upsilon \right\} \tag{42}$$

34

Proof. By definition of MPCE, $U_A(e_t^*)$ equals (14) for some equilibrium sequence $\{e_{t+k}^*, p_{t+k}^*, x_{t+k}^{S*}, x_{t+k}^{A*}\}_{k=0}^{\infty}$ which does not depend on m_t chosen by country A. Therefore without loss of generality country A can make a joint decision over choice of (f_t, m_t) to maximize its payoff (14), which would be either setting $f_t = 1$ and $m_t = m^*(e_t)$, or $f_t = 0$ and $m_t = 0$. ∎

The immediate implication of this lemma is the following corollary.

Corollary 3 *In any MPCE, without loss of generality country A's strategies in state e can be restricted to choosing no armament and no attack with probability $\mu(e)$ and armament $m^*(e)$ and attack with probability $1 - \mu(e)$.*

Proof of Lemma 1

We prove existence of MPCE using the properties of F_v. We construct equilibria for three separate cases: (i) $\lim_{e \to 0} F_v(e) \leq 0$; (ii) $\lim_{e \to 0} F_v(e) > 0$; and there does not exist $e \leq e_0$ such that $F_v(e) < 0$; and (iii) $\lim_{e \to 0} F_v(e) > 0$ and there exists $e \leq e_0$ such that $F_v(e) < 0$. We prove each case in a separate lemma. Throughout we use the result of Corollary 3 that allows us to restrict strategies of country A to not arm and not attack with probability $\mu(e_t)$ and arm $m^*(e_t)$ and attack with probability $1 - \mu(e_t)$

Lemma 4 *If $\lim_{e \to 0} F_v(e) \leq 0$ then there exists an equilibrium in which war occurs in period 0 with probability 1.*

Proof. First, note that if $u(0)$ is finite then $\lim_{e \to 0} F_v(e) = v$. Therefore $\lim_{e \to 0} F_v(e) \leq 0$ implies that $u(0) = -\infty$.

We construct an equilibrium (γ^*, μ^*) in which war occurs with probability 1 in period 0. Let $\{e_0^*, p_0^*, x_0^{S*}, x_0^{A*}\} = \{e_0, u'(e_0), e_0, e_0\}$ and $\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\} = \{0, u'(0), 0, 0\}$ for all $t > 0$. Let $\gamma^* = \{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\}_{t=0}^{\infty}$. Let strategies of county A be $\mu^*(e_0) = 0$ and $\mu^*(0) = 0$.

To verify that this is an equilibrium we need to check that country A does not gain from deviating from strategy μ^*. The payoff of country A from choosing no armament and no war in period 0 is given by

$$u(e_0) - u'(e_0)e_0 + U_A(0) = -\infty,$$

where the equality follows from $u(0) = -\infty$. The payoff of country A from playing $\mu^*(e_0)$ is $V(w(m^*(e_0))) - l(m^*(e_0)) - v > -\infty$, therefore it is the best response for country A to play $\mu^*(e_0) = 0$. Observation 7.2 implies that $\mu(e_t) = 0$ is the best response in the states in which $e_t = 0$.

To see that γ^* is an equilibrium, note that $\mu(e_1) = 0$ implies that $\Pr\{f_1 = 0\} = 0$. Then (3), (10), (12), and (13) imply that $\{p_0^*, x_0^{S*}, x_0^{A*}\} = \{u'(e_0), e_0, e_0\}$ and $e_1^* = 0$, completing the proof. ∎

Lemma 5 *If $\lim_{e \to 0} F_v(e) > 0$ and there does not exist $e \leq e_0$ such that $F_v(e) < 0$, then there exists an equilibrium with permanent peace.*

Proof. In an equilibrium with permanent peace country A sets $\mu^*(e) = 1$ for all $e \leq e_0$, and equilibrium allocations $\gamma^* = \{\widetilde{e}_t(e_0), u'(\tilde{x}_t(e_0)), \tilde{x}_t(e_0), \tilde{x}_t(e_0)\}_{t=0}^{\infty}$ where $\{\widetilde{e}_t(e_0), \tilde{x}_t(e_0)\}_{t=0}^{\infty}$ are the competitive equilibrium allocations with permanent peace defined in (40). At every date t the payoff for country A along the equilibrium path is given by $U^C(\widetilde{e}_t(e_0))$. Since $\widetilde{e}_t(e_0) \leq e_0$ for all t,

$$
\begin{aligned}
0 \;\leq\;&\; F_v(\widetilde{e}_t(e_0)) \\
=\;&\; U^C(\widetilde{e}_t(e_0)) - (V(w(m^*(\widetilde{e}_t(e_0)))\widetilde{e}_t(e_0)) - l(m^*(\widetilde{e}_t(e_0))) - v),
\end{aligned}
$$

which implies $\mu^*(\widetilde{e}_t) = 1$ is the best response of country A. Given that country A never attacks, γ^* satisfies optimization conditions (3), (10), (12), and (13). ∎

Lemma 6 *If* $\lim_{e \to 0} F_v(e) > 0$ *and there exists* $e \leq e_0$ *such that* $F_v(e) < 0$ *then an MPCE exists.*

Proof. Define $\widehat{e} > 0$ s.t. $F_v(\widehat{e}) = 0$ and $F_v(e) > 0$ for all $e \in [0, \widehat{e})$. Such $\widehat{e}$ exists because F_v is continuous, $F_v(0) > 0$ and $F_v(e) < 0$ for some e. Let $\widehat{\widehat{e}}$ be defined implicitly by $\widetilde{e}_1(\widehat{\widehat{e}}) = \widehat{e}$. $\widehat{\widehat{e}}$ represents a value of initial endowment of oil such that in competitive equilibrium with permanent peace, remaining oil reserves in period 1 are equal to $\widehat{e}$. We construct equilibria for three different cases depending on the values of $F_v(e_0)$ and e_0 relative to $\widehat{\widehat{e}}$.

Case 1. Suppose $e_0 \leq \widehat{\widehat{e}}$ and $F_v(e_0) \geq 0$. We construct an equilibrium with permanent peace.

Define $\gamma^* = \{\widetilde{e}_t(e_0), u'(\tilde{x}_t(e_0)), \tilde{x}_t(e_0), \tilde{x}_t(e_0)\}_{t=0}^{\infty}$ and $\mu_t^*(\widetilde{e}_t(e_0)) = 1$ for all t. The proof of this case is analogous to proof of Lemma 5.

Case 2. Suppose $e_0 \leq \widehat{\widehat{e}}$ and $F_v(e_0) < 0$. We construct an equilibrium in which war occurs with probability 1 in period 0. In this case define $\gamma^* = \{\widetilde{e}_t(e_0), u'(\tilde{x}_t(e_0)), \tilde{x}_t(e_0), \tilde{x}_t(e_0)\}_{t=0}^{\infty}$ and $\mu_0^*(e_0) = 0$, $\mu_t^*(\widetilde{e}_t(e_0)) = 1$ for all $t > 0$. Given these strategies of country A, (μ^*, γ^*) is an equilibrium for the same reasons as described in the proof of Lemma 5. Since $F_v(e_0) < 0$, country A obtains higher utility under war and and thus $\mu_0^*(e_0) = 0$ is a best response in period 0. To verify that $\mu_t^*(\widetilde{e}_t(e_0)) = 1$ for all $t > 0$, note that $e_0 \leq \widehat{\widehat{e}}$ implies that $\widetilde{e}_1(e_0) \leq \widetilde{e}_1(\widehat{\widehat{e}}) = \widehat{e}$.[23] Therefore in any period $t > 0$

$$
\begin{aligned}
&\; U_A(\widetilde{e}_t(e_0)) - (V(w(m^*(\widetilde{e}_t(e_0)))\widetilde{e}_t(e_0)) - l(m^*(\widetilde{e}_t(e_0))) - v) \\
=\;&\; U^C(\widetilde{e}_t(e_0)) - (V(w(m^*(\widetilde{e}_t(e_0)))\widetilde{e}_t(e_0)) - l(m^*(\widetilde{e}_t(e_0))) - v) \\
=\;&\; F_v(\widetilde{e}_t(e_0)) \geq F_v(\widehat{e}) = 0.
\end{aligned}
$$

---

[23] This follows, for example, because the competitive equilibrium is efficient and thus equilibrium allocations $\{\tilde{e}_t\}_{t=0}^{\infty}$ can be found recursively from

$$
J(e_t) = \max_{e_{t+1}} u(e_t - e_{t+1}) + \beta J(e_{t+1}).
$$

Concavity of J implies that e_{t+1} is increasing in e_t.

Therefore peace is a dominated strategy for country A in all $t > 0$.

Case 3. Suppose $e_0 > \widehat{\widehat{e}}$. We construct an equilibrium in which oil endowment in period 1 is equal to $\widehat{e}$ followed by permanent peace from $t \geq 2$. Probabilities of war in periods 0 and 1 depend on the initial conditions.

Let

$$\left(e_0^*, p_0^*, x_0^{S*}, x_0^{A*}\right) = \left(e_0, u'\left(e_0 - \widehat{e}\right), e_0 - \widehat{e}, e_0 - \widehat{e}\right)$$

and

$$\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right) = \left(\widetilde{e}_{t-1}\left(\widehat{e}\right), u'\left(\widetilde{x}_{t-1}\left(\widehat{e}\right)\right), \widetilde{x}_{t-1}\left(\widehat{e}\right), \widetilde{x}_{t-1}\left(\widehat{e}\right)\right) \text{ for all } t \geq 1.$$

Let $\mu^*(e_1^*) = u'(e_0 - \widehat{e})/\beta u'(\widetilde{x}_0(\widehat{e}))$. Note that $\mu^*(e_1^*)$ is equal to 1 for $e_0 = \widehat{\widehat{e}}$ and monotonically converges to 0 as $e_0 \to \infty$. Therefore $\mu^*(e_1^*)$ is a well-defined probability. Set $\mu^*(e_t^*) = 1$ for all $t \geq 2$. Under this construction $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$ satisfies conditions (3), (10), (12), and (13) (since they do not depend on the probability of war in period 0, $\mu^*(e_0)$). To check that constructed strategies are also best response for country A starting from period 1, note that by construction $e_1^* = \widehat{e}$ and $e_t^* < \widehat{e}$ for all $t \geq 2$. Since $F_v(\widehat{e}) = 0$, country A is indifferent between war and peace and is weakly better off randomizing between the two outcomes with probabilities $\mu^*(e_1^*)$ and $1 - \mu^*(e_1^*)$. Since $e_t^* < \widehat{e}$ for $t \geq 2$, $F_v(e_t^*) > 0$ for $t \geq 2$, and therefore $\mu^*(e_t^*) = 1$ is a best response analogously to Case 1.

Finally we need to construct $\mu^*(e_0)$. Note that under proposed equilibrium strategies country A is indifferent between permanent peace and attack in period 1, and therefore its payoff period 1 is $U^C(\widehat{e})$. Therefore, if country A does not attack in period 0, its payoff is given by $u\left(e_0 - \widehat{e}\right) - u'\left(e_0 - \widehat{e}\right)\left(e_0 - \widehat{e}\right) + \beta U^C\left(\widehat{e}\right)$. Then we set $\mu^*(e_0) = 1$ if

$$u\left(e_0 - \widehat{e}\right) - u'\left(e_0 - \widehat{e}\right)\left(e_0 - \widehat{e}\right) + \beta U^C\left(\widehat{e}\right) \geq V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l\left(m^*\left(e_0\right)\right) - v,$$

and set $\mu^*(e_0) = 0$ otherwise. This completes construction of the equilibrium. ■

Proof of Proposition 3

First we prove a preliminary result about properties of MPCE. By Corollary 3, without loss of any generality, we can restrict attention to only two actions of country A in each period, to not arm and not attack with probability $\mu^*(e_t^*)$ and to arm $m^*(e_t^*)$ and attack with probability $1 - \mu^*(e_t)$.

Lemma 7 *Let (γ, μ) be an MPCE. Suppose that $\mu_t^* = \mu^*(e_t^*) > 0$ for all t. Then*

1. *Country A must weakly prefer permanent peace to war,*

$$\sum_{k=0}^{\infty} \beta^k \left(u\left(x_{t+k}^*\right) - p_{t+k}^* x_{t+k}^*\right) \geq V\left(w\left(m^*\left(e_t^*\right)\right)e_t^*\right) - l\left(m^*\left(e_t^*\right)\right) - v \qquad (43)$$

for all t, with strict equality if country A attacks with a positive probability (i.e. $\mu(e_t^*) < 1$).

37

2. *The payoff in the event of no war satisfies*

$$\sum_{k=0}^{\infty} \beta^k \left(u\left(x_{t+k}^*\right) - p_{t+k}^* x_{t+k}^* \right) = K_t e_t^{*1-1/\sigma} - \frac{1}{(1-\beta)(1-1/\sigma)} \tag{44}$$

where

$$K_t = \frac{1}{\sigma} \frac{1}{1-1/\sigma} \frac{\left(1 + \sum_{k=1}^{\infty} \beta^k \left(\prod_{l=1}^{k} (\beta\mu_{t+l}^*)^{\sigma} \right)^{1-1/\sigma} \right)}{\left(1 + \sum_{k=1}^{\infty} \prod_{l=1}^{k} (\beta\mu_{t+l}^*)^{\sigma} \right)^{1-1/\sigma}}. \tag{45}$$

Moreover, K_t is bounded from below, and K_t is bounded from above by

$$K^C = \frac{1}{\sigma} \frac{1}{1-1/\sigma} (1-\beta^{\sigma})^{-1/\sigma}. \tag{46}$$

3. *(x_t^*, e_t^*) for all $e_t^* > 0$ must satisfy*

$$\frac{x_t^*}{e_t^*} \geq 1 - \beta^{\sigma}. \tag{47}$$

4. *Country A's payoff in the event of war satisfies*

$$V\left(w\left(m^*\left(e_t^*\right)\right) e_t^* \right) = w\left(m^*\left(e_t^*\right)\right)^{1-1/\sigma} (1-\beta^{\sigma})^{-1/\sigma} \frac{1}{1-1/\sigma} e_t^{*1-1/\sigma} - \frac{1}{(1-\beta)(1-1/\sigma)}. \tag{48}$$

Proof. Since peace occurs with a positive probability at any $t + k \geq t$, the equilibrium payoff for country A should be equal to

$$U_A(e_t^*) = u(x_t^*) - p_t^* x_t^* + \beta U_A(e_{t+1}^*).$$

Iterating forward, this implies that

$$U_A(e_t^*) = \sum_{k=0}^{\infty} \beta^k \left(u\left(x_{t+k}^*\right) - p_{t+k}^* x_{t+k}^* \right)$$

for all $t + k \geq 0$. Substitution into (42) implies that (43) must hold, with strict equality if $\mu(e_t^*) < 1$. This establishes part (i).

Consider any $\{\mu_t^*\}_{t=0}^{\infty}$ with $\mu_t^* > 0$ for all t. Optimal extraction for firms requires that

$$\mu_{t+1}^* p_{t+1}^* = \frac{1}{\beta} p_t^*. \tag{49}$$

38

If instead $\mu_{t+1}^* p_{t+1}^* > \frac{1}{\beta} p_t^*$, then from condition (12) $x_t^{A*} > 0$ since $p_t^* < \infty$. From (10) $x_t^{S*} = 0$, but this implies that $x_t^{S*} \neq x_t^{A*}$ which violates (13). If instead $\mu_{t+1}^* p_{t+1}^* < \frac{1}{\beta} p_t^*$, then analogous arguments imply that $x_{t+1}^{A*} > 0$ and $x_{t+1}^{S*} = 0$ which violates (13). (49) together with (12) implies that

$$x_{t+1}^* = \left(\beta \mu_{t+1}^*\right)^\sigma x_t^*. \tag{50}$$

Forward substitution on (3) implies that

$$\sum_{k=0}^\infty x_{t+k}^* \leq e_t^*. \tag{51}$$

(51) must bind, since if this were not the case, a firm would be able to increase some x_{t+k}^* by $\epsilon > 0$ and increase its profits. Substitutions of (50) into (51), noting that the latter binds, yields

$$x_t^* \left(1 + \sum_{k=1}^\infty \prod_{l=1}^k \left(\beta \mu_{t+l}^*\right)^\sigma\right) = e_t^*. \tag{52}$$

Equation (52) together with the fact that $\mu_t \in (0,1] \; \forall t > 0$ implies that

$$e_t^* > 0 \text{ and } \frac{x_t^*}{e_t^*} = \frac{1}{1 + \sum_{k=1}^\infty \prod_{l=1}^k \left(\beta \mu_{t+l}^*\right)^\sigma} \geq 1 - \beta^\sigma > 0 \; \forall t. \tag{53}$$

Substitution of $p_{t+k}^* = u'\left(x_{t+k}^*\right)$ into (44) yields

$$\sum_{k=0}^\infty \beta^k \left(\frac{1}{\sigma} \frac{x_{t+k}^{*1-1/\sigma}}{1-1/\sigma}\right) - \frac{1}{(1-\beta)(1-1/\sigma)} = K_t e_t^{*1-1/\sigma} - \frac{1}{(1-\beta)(1-1/\sigma)} \tag{54}$$

where we used (50) and (52) to get (45).

We are left to show that K_t is bounded from above and below. The maximization of the left hand side of (54) subject to the resource constraint (3) implies that $x_{t+1}^* = \beta^\sigma x_t^*$ so that the maximum of the left hand side of (54) is

$$\frac{1}{\sigma} \frac{1}{1-1/\sigma} \left(1 - \beta^\sigma\right)^{-1/\sigma} e_t^{*1-1/\sigma} - \frac{1}{(1-\beta)(1-1/\sigma)}. \tag{55}$$

Since $e_t^{*1-1/\sigma} > 0$ by (53), this means that

$$K_t \leq K^C = \frac{1}{\sigma} \frac{1}{1-1/\sigma} \left(1 - \beta^\sigma\right)^{-1/\sigma}, \tag{56}$$

so that K_t is bounded from above. To see that K_t is bounded from below, note that if $\sigma > 1$, (45) implies that

$$K_t \geq \frac{1}{\sigma} \frac{1}{1 - 1/\sigma} (1 - \beta^\sigma)^{1-1/\sigma}$$

since

$$\frac{\left(1 + \sum_{k=1}^{\infty} \beta^k \left(\prod_{l=1}^{k} (\beta \mu_{t+l}^*)^\sigma\right)^{1-1/\sigma}\right)}{\left(1 + \sum_{k=1}^{\infty} \prod_{l=1}^{k} (\beta \mu_{t+l}^*)^\sigma\right)^{1-1/\sigma}} \geq \frac{1}{\left(1 + \sum_{k=1}^{\infty} \prod_{l=1}^{k} \beta^\sigma\right)^{1-1/\sigma}}$$

$$= (1 - \beta^\sigma)^{1-1/\sigma}$$

If instead $\sigma < 1$, then (43) implies that under any armament level $m > 0$,

$$K_t e_t^{*1-1/\sigma} - \frac{1}{(1-\beta)(1-1/\sigma)} \geq V(w(m) e_t^*) - l(m) - \upsilon. \tag{57}$$

The first order conditions which define (4) imply that $x_{t+1} = \beta^\sigma x_t$ which given (5) and (6) implies that

$$V(w(m) e_t^*) = w(m)^{1-1/\sigma} (1-\beta^\sigma)^{-1/\sigma} \frac{1}{1 - 1/\sigma} e_t^{*1-1/\sigma} - \frac{1}{(1-\beta)(1-1/\sigma)}. \tag{58}$$

Together with (57), this means that

$$\begin{aligned} K_t &\geq w(m)^{1-1/\sigma} (1-\beta^\sigma)^{-1/\sigma} \frac{1}{1 - 1/\sigma} - \frac{l(m) + \upsilon}{e_t^{*1-1/\sigma}} \\ &\geq w(m)^{1-1/\sigma} (1-\beta^\sigma)^{-1/\sigma} \frac{1}{1 - 1/\sigma} - \frac{l(m) + \upsilon}{e_0^{1-1/\sigma}} \end{aligned}$$

where we have used the fact that $e_t^* \leq e_0$. This means that K_t is bounded from below.

This establishes part (ii) of the lemma. Part (iii) follows from (53), and part (iv) follows by substitution of $m^*(e_t)$ in for m in (58). ∎

Now we are ready to prove Proposition 3. Here we prove a stronger version of Proposition 3 that shows that if at any node of the game (both on and off equilibrium path) war does not occur with probability 1, then permanent peace must follow after that node.

Proposition 11 *Let (γ^*, μ^*) be an MPCE. Suppose that $\mu^*(e_T^*) > 0$ for some $e_T^* > 0$. Then $\mu^*(e_t) = 1$ for all $t > T$. Moreover, $U_A(e_T^*) = U^C(e_T^*)$ where $U^C(e_T^*)$ is a payoff in permanent peace defined in equation (40) and $\{x_t^*\}_{t=T}^{\infty}$ satisfies (16).*

Proof. First, note that using the same arguments as those used in Proposition 2 we can establish that if $\mu^*(e_T^*) > 0$ for some $e_T^* > 0$ then $\mu^*(e_t) > 0$ for all $t > T$. Now substituting from Lemma

7 into equation (43), we obtain

$$K_t e_t^{*1-1/\sigma} \geq w\left(m^*\left(e_t^*\right)\right)^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_t^{*1-1/\sigma} - l\left(m^*\left(e_t^*\right)\right) - \upsilon. \qquad (59)$$

We now show that (59) cannot hold with equality which proves that there cannot equilibrium randomization by country A between war and peace. Suppose (59) holds with equality at some date $t > T$. We consider two cases separately: case 1, when there is some finite date $\hat{T}$ after which country A never attacks, and case 2, when $\mu_t^* < 1$ infinitely often.

Case 1. Suppose there is some $\hat{T}$ such that $\mu_{\hat{T}}^* < 1$ and $\mu_t^* = 1$ for all $t > \hat{T}$. In this case, since country A is indifferent between war and peace at $\hat{T}$ and weakly prefers peace at $\hat{T} - 1$ and $\hat{T} + 1$ to war using the same armament as at $\hat{T}$, it follows that:

$$K_{\hat{T}+1}e_{\hat{T}+1}^{*1-1/\sigma} \geq w\left(m^*\left(e_{\hat{T}}^*\right)\right)^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{\hat{T}+1}^{*1-1/\sigma} - l\left(m^*\left(e_{\hat{T}}^*\right)\right) - \upsilon \quad (60)$$

$$K_{\hat{T}}e_{\hat{T}}^{*1-1/\sigma} = w\left(m^*\left(e_{\hat{T}}^*\right)\right)^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{\hat{T}}^{*1-1/\sigma} - l\left(m^*\left(e_{\hat{T}}^*\right)\right) - \upsilon \quad (61)$$

$$K_{\hat{T}-1}e_{\hat{T}-1}^{*1-1/\sigma} \geq w\left(m^*\left(e_{\hat{T}}^*\right)\right)^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{\hat{T}-1}^{*1-1/\sigma} - l\left(m^*\left(e_{\hat{T}}^*\right)\right) - \upsilon \quad (62)$$

Since $\mu_t^* = 1 \,\forall t \geq \hat{T} + 1$, from (45), it must be the case that $K_{\hat{T}+1} = K_{\hat{T}} = K^C$ for K^C defined in (46), and since $\mu_{\hat{T}} \in (0,1)$, it must be that $K^C > K_{\hat{T}-1}$ since war is chosen with positive probability at $\hat{T}$. Moreover, it must be that

$$K^C - w\left(m^*\left(e_{\hat{T}}^*\right)\right)^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma} < 0$$

in order that (61) hold. Equations (60) − (62) therefore imply that

$$\frac{e_{\hat{T}}^{*1-1/\sigma}}{e_{\hat{T}+1}^{*1-1/\sigma}} \geq 1 \text{ and } \frac{e_{\hat{T}}^{*1-1/\sigma}}{e_{\hat{T}-1}^{*1-1/\sigma}} \geq 1.$$

If $\sigma < 1$, then by (3) this implies that $e_{\hat{T}+1}^* = e_{\hat{T}}^*$ so that $x_{\hat{T}}^* = 0$ which violates (47). If instead $\sigma > 1$, then this implies that $e_{\hat{T}}^* \geq e_{\hat{T}-1}^*$ which implies $x_{\hat{T}-1}^* = 0$, which violates (47). This establishes that it country A cannot be indifferent between attack and not attack in period T, which implies that it must choose $f_t = 0$ with probability 1.

Case 2. Suppose $\mu_t^* < 1$ infinitely often.

Consider sequence $s^1 = \{\mu_t^*, K_t\}_{t=0}^\infty$ where K_t is defined by (45). By Lemma 7, there exists some compact set S such that $(\mu_t^*, K_t) \in S$ for all t. Therefore we can select a convergent subsequence s^2 within s^1 (where K_t converges to some K^*). Consider three consecutive elements of s^2, denoted by $n - 1$, n, and $n + 1$. Weak preference for peace at $n - 1$ and $n + 1$ together

with indifference to peace at n using armament $m^*(e_n^*)$ implies:

$$K_{n+1}e_{n+1}^{*1-1/\sigma} \geq w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{n+1}^{*1-1/\sigma}-l\left(m^*\left(e_n^*\right)\right)-\upsilon \quad (63)$$

$$K_ne_n^{*1-1/\sigma} = w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_n^{*1-1/\sigma}-l\left(m^*\left(e_n^*\right)\right)-\upsilon \quad (64)$$

$$K_{n-1}e_{n-1}^{*1-1/\sigma} \geq w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{n-1}^{*1-1/\sigma}-l\left(m^*\left(e_n^*\right)\right)-\upsilon \quad (65)$$

Equations (63) and (64) imply that

$$\left(K_{n+1}-w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_{n+1}^{*1-1/\sigma} \geq \quad (66)$$
$$\left(K_n-w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_n^{*1-1/\sigma}$$

and equations (64) and (65) imply that

$$\left(K_{n-1}-w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_{n-1}^{*1-1/\sigma} \geq \quad (67)$$
$$\left(K_n-w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_n^{*1-1/\sigma}$$

Note that it cannot be that

$$\lim_{n\to\infty}\left\{K_n-w\left(m^*\left(e_n^*\right)\right)^{1-1/\sigma}\left(1-\beta^\sigma\right)^{-1/\sigma}\frac{1}{1-1/\sigma}\right\}=0, \quad (68)$$

since if this were the case, then given the indifference condition, it would violate (64) since $\upsilon > 0$. Therefore, (68) cannot hold and the left hand side of (68) must be negative for (64) to be satisfied. Then (66), (67) and the fact that K_n converges to some K^* imply that

$$\lim_{n\to\infty}\left(\frac{e_n^{*1-1/\sigma}}{e_{n+1}^{*1-1/\sigma}}\right) \geq 1 \text{ and } \lim_{n\to\infty}\left(\frac{e_n^{*1-1/\sigma}}{e_{n-1}^{*1-1/\sigma}}\right) \geq 1,$$

which given (3) implies that if either $\sigma < 1$ or $\sigma > 1$, then $\lim_{n\to\infty}e_{n+1}^*/e_n^* = 1$, but this violates (47) which requires that $e_{t+1}^*/e_t^* \leq \beta^\sigma < 1$ for all t which implies from (3) that $e_{n+1}^*/e_n^* \leq \beta^\sigma$ for all n. This establishes that it is not possible for $\mu^*(e_t^*) < 1$ for $t \geq T$ in an equilibrium in which war continues occurring forever with positive probability, and this completes the proof of the first part of the proposition.

Finally, since country A weakly prefers peace in state e_T^*, $U_A(e_T^*) = U^C(e_T^*)$ and $\{x_t^*\}_{t=T}^\infty$ must satisfy (16). ∎

Proof of Proposition 4

To prove this lemma we construct the function F_v as defined in (41) and use Lemmas 4, 5, and 6 to establish the existence of equilibrium in which either war occurs with probability 1 in period 0 or there is a permanent peace depending on the assumptions in Proposition 4. Next we use Proposition 11 to rule out other equilibria. Similarly to the proofs of all preceding lemmas, we use Corollary 3 to restrict our attention to only two strategies for country A, not arm and not attack with probability $\mu(e)$ and arm $m^*(e)$ and attack with probability $1 - \mu(e)$.

First we derive payoffs from the permanent peace $U^C(e)$ and war $V\left(w\left(m^*\left(e\right)\right)e\right)$. Set $\mu_t = 1$ for all t and use Lemma 7 to show that

$$U^C(e) = \frac{1}{\sigma} \frac{1}{1 - 1/\sigma} (1 - \beta^\sigma)^{-1/\sigma} e^{1-1/\sigma} - \frac{1}{(1 - 1/\sigma)(1 - \beta)}. \tag{69}$$

Then $F_v(e)$ is equal to

$$F_v(e) = \frac{1}{1 - 1/\sigma} (1 - \beta^\sigma)^{-1/\sigma} e^{1-1/\sigma} \left(1/\sigma - [w\left(m^*\left(e\right)\right)]^{1-1/\sigma}\right) + l(m^*(e)) + v. \tag{70}$$

Part 1: Consider the case when $\sigma > 1$. First we show that there exists a unique $\widehat{e}$ such that $F_v(e) > 0$ for all $e < \widehat{e}$ and $F_v(e) < 0$ for all $e > \widehat{e}$. Then it follows immediately from Lemma 5 that there exists an equilibrium that has no war along the equilibrium path if $e_0 < \widehat{e}$ and we show using Lemma 6 there exists an equilibrium in which war occurs with probability 1 in period 0 if $e_0 > \widehat{e}$.

Claim 1. If $\sigma > 1$ then there exists a unique $\widehat{e}$ such that $F_v(e) > 0$ for all $e < \widehat{e}$ and $F_v(e) < 0$ for all $e > \widehat{e}$.

Note that $F_v(0) = v > 0$. Differentiating F_v in (70) and use the optimality condition (34) for $m^{*\prime}(e)$ we get

$$F_v'(e) = (1 - \beta^\sigma)^{-1/\sigma} e^{-1/\sigma} \left(1/\sigma - [w\left(m^*\left(e\right)\right)]^{1-1/\sigma}\right). \tag{71}$$

If $\sigma > 1$ then from Proposition 1 $m^*(e)$ is increasing in e. Therefore $F_v(e)$ has at most one peak and it can cross zero at most once. If it crosses zero, let $\widehat{e}$ be a solution to $F_v(\widehat{e}) = 0$. If $F_v(e)$ does not cross zero we set $\widehat{e} = \infty$.

Claim 2. If $F_v(e) > 0$ for all $e \leq e_0$, then there exists no equilibrium in which war occurs with positive probability.

Claim 2 together with Claim 1 immediately imply that if $\sigma > 1$ and $e_0 < \widehat{e}$ then there exists no equilibrium in which war occurs with positive probability.

Suppose there exists an equilibrium in which war occurs with a positive probability at date 0. More formally, suppose there exists an equilibrium (γ^*, μ^*) such that $\mu^*(e_0) < 1$.

First suppose that $\mu^*(e_1^*) = 0$. In this case (10) and (3) imply that $x_0^{A*} = e_0$ and $e_1^* = 0$.

When $\sigma > 1$, then $F_v(0) > 0$, and by Observation 7.2 $\mu^*(0) = 1$. Therefore $\mu^*(e_1^*) = 1$ leading to a contradiction.

Now suppose that $\mu^*(e_1^*) > 0$. In this case from Proposition 11, $U_A(e_1^*) = U^C(e_1^*)$. Then

$$
\begin{aligned}
U_A(e_1^*) - (V\left(w\left(m^*\left(e_1^*\right)\right)e_1^*\right) - l(m^*(e_1^*)) - v) &= U^C(e_1^*) - (V\left(w\left(m^*\left(e_1^*\right)\right)e_1^*\right) - l(m^*(e_1^*)) - v) \\
&= F_v(e_1^*) > F_v(\widehat{e}) = 0,
\end{aligned}
$$

where the strict inequality follows from the definition $\widehat{e}$. This implies that peace is strictly preferred to attack and therefore $\mu^*(e_1^*) = 1$.

If $\mu^*(e_1^*) = 1$ so that peace occurs with probability 1 in period 1, then $\left(x_0^{A*}, p_0^*\right) = \left(\tilde{x}_0(e_0), u'(\tilde{x}_0(e_0))\right)$ where $\tilde{x}_0(e_0)$ is a permanent peace allocation defined in (40), and $e_1^* = \tilde{e}_1(e_0)$. Since country A attacks in period 0 with positive probability, it must be true that

$$
V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l(m^*(e_0)) - v \geq u(x_0^{A*}) - p_0^* x_0^{A*} + \beta U_A(e_1^*). \tag{72}
$$

Substitute $\left(x_0^{A*}, p_0^*\right) = \left(\tilde{x}_0(e_0), u'(\tilde{x}_0(e_0))\right)$ and $U_A(e_1^*) = U^C(e_1^*)$ into equation (72) and regroup terms to get

$$
\begin{aligned}
0 &\geq u(\tilde{x}_0(e_0)) - u'\left(\tilde{x}_0\left(e_0\right)\right)\tilde{x}_0\left(e_0\right) + \beta U^C(\tilde{e}_1(e_0)) - (V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l(m^*(e_0)) - v) \\
&= U^C(e_0) - (V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l(m^*(e_0)) - v) \\
&= F_v(e_0) > 0
\end{aligned}
$$

which is a contradiction. Therefore there cannot exist an equilibrium with $\mu^*(e_0) > 0$ and Lemma 5 establishes existence of equilibrium with $\mu^*(e_0) = 1$.

Claim 3. If $\sigma > 1$ and $e_0 > \widehat{e}$, then there exists no equilibrium in which peace occurs with positive probability in period 0.

Suppose $e_0 > \widehat{e}$ and there exists an equilibrium in which country A chooses peace with positive probability in period 0, i.e., $\mu^*(e_0) > 0$. By Proposition 11,

$$
\begin{aligned}
0 &\leq U_A(e_0) - (V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l(m^*(e_0)) - v) \\
&= U^C(e_0) - (V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l(m^*(e_0)) - v) = F_v(e_0) < 0
\end{aligned}
$$

which is a contradiction. Therefore in any MPCE $\mu^*(e_0) = 0$.

Part 2: Suppose $\sigma < 1$ and let $\widehat{w} = (1/\sigma)^{1/(1-1/\sigma)}$. By construction, $\widehat{w} \in (0, 1)$.

Claim 4. If $\sigma < 1$ and $\lim_{m \to \bar{m}} w(m) < \widehat{w}$, then there exists no equilibrium in which war occurs with positive probability.

We prove that in this case $F_v(e) > 0$ for all e, so that we can apply Claim 2 of the proof of this proposition directly to establishes this result.

In order to prove that $F_v(e) > 0$ $\forall e$, we show that $F_v'(e) < 0$ $\forall e$ and that $\lim_{e \to \infty} F(e) > 0$.

We can establish that $F'_v(e) < 0 \ \forall e$ from (71); this is true given that $w(m^*(e)) < \widehat{w} \ \forall e$. To establish that $\lim_{e\to\infty} F_v(e) > 0$, consider first the value of $\lim_{e\to\infty} m^*(e)$. Suppose that $\lim_{e\to\infty} m^*(e) = \underline{m} > 0$. Since $m^*(e)$ is the optimal armament, it must satisfy (34). The first order condition which characterizes (8) taking into account (4) and (17) implies

$$(1 - \beta^\sigma)^{-1/\sigma} e^{1-1/\sigma} = \frac{l'(m^*(e))}{[w(m^*(e))]^{-1/\sigma} w'(m^*(e))}. \tag{73}$$

If $\lim_{e\to\infty} m^*(e) = \underline{m} > 0$, then this would violate (73) since the left-hand side of (73) would converge to 0 whereas the right-hand side of (73) would converge to a positive number. Therefore, $\lim_{e\to\infty} m^*(e) = 0$ which implies that

$$\lim_{e\to\infty} (V(w(m^*(e))e) - l(m^*(e)) - v) = -\frac{1}{(1-\beta)(1-1/\sigma)} - v, \tag{74}$$

so that $\lim_{e\to\infty} F_v(e) = v > 0$.[24] This establishes that $F_v(e) > 0 \ \forall e$ and Claim 4 follows from Claim 2.

Claim 5. If $\sigma < 1$ and $\lim_{m\to\bar{m}} w(m) > \widehat{w}$, then there exists no equilibrium in which peace occurs with positive probability in period 0.

First we show that in this case $\lim_{e\to 0} F_v(e) = -\infty$. The existence of the pure-strategy equilibrium with immediate war then follows from Lemma 4 and we will use Proposition 11 to rule out existence of equilibria with a positive probability of peace in period 0.

Let us show that $\lim_{e\to 0} F_v(e) = -\infty$. Note that when $\sigma < 1$, Proposition 1 that $m^*(e)$ is decreasing in e. Suppose that $\lim_{e\to 0} m^*(e) = \overline{m}' < \overline{m}$. This would violate (73) since the left-hand side of (73) approaches ∞ as e approaches 0, whereas the right-hand side of (73) approaches $l'(\overline{m}') / \left[[w(\overline{m}')]^{-1/\sigma} w'(\overline{m}') \right] < \infty$, yielding a contradiction. Therefore $\lim_{e\to 0} m^*(e) = \overline{m}$ and $\lim_{e\to 0} w(m^*(e)) > \widehat{w}$. Now consider $\lim_{e\to 0} F_v(e)$ which satisfies:

$$\lim_{e\to 0} F_v(e) = \lim_{e\to 0} (V(w(m^*(e))e) - l(m^*(e)) - v) \left(\frac{U^C(e)}{V(w(m^*(e))e) - l(m^*(e)) - v} - 1 \right). \tag{75}$$

The first term on the right-hand side of (75) converges to $-\infty$. The limit of the second term is positive since after substituting $U^C(e)$ from (69) and $V(w(m^*(e))e)$ from (48) and applying

---

[24] (74) follows because by definition

$$V(w(m^*(e))e) - l(m^*(e)) - v \leq -\frac{1}{(1-\beta)(1-1/\sigma)} - v$$

and because optimality of $m^*(e)$ requires

$$\lim_{e\to\infty} (V(w(m^*(e))e) - l(m^*(e)) - v) \geq \lim_{e\to\infty} (V(\epsilon e) - l(\epsilon) - v) = -\frac{1}{(1-\beta)(1-1/\sigma)} - l(\epsilon) - v$$

for any $\epsilon > 0$ chosen to be arbitrarily small.

the L'Hopital's rule (together with the optimality condition (34)), we obtain

$$\lim_{e \to 0} \frac{U^C(e)}{V(w(m^*(e))e) - l(m^*(e)) - v} = \lim_{e \to 0} \frac{dU^C(e)/de}{d(V(w(m^*(e))e) - l(m^*(e)) - v)/de}$$

$$= \frac{1/\sigma}{\lim_{m \to \overline{m}} [w(m)]^{1-1/\sigma}} > 1.$$

Therefore $\lim_{e \to 0} F_v(e) = -\infty$. Since $\lim_{e \to 0} F_v(e) = -\infty$ and F_v is continuous, there exists $\widehat{e} > 0$ such that $F_v(e) < 0$ for all $e < \widehat{e}$.

Now we are ready to prove that there exist no equilibrium in which peace occurs with a positive probability in period 0. Suppose such an equilibrium (γ^*, μ^*) exists with $\mu^*(e_0) > 0$. In this case by Proposition 11, $\mu^*(e_t^*) = 1$ for all $t > 0$ and $x_t^{A*} = \tilde{x}_t(e_0)$ for all t. From the proof of Lemma 7 it follows that $e_t^* = \tilde{e}_t(e_0) = \beta^{\sigma t} e_0$. Therefore there exists some T such that $e_T^* < \widehat{e}$. Since peace is the best response for country A in state e_T^*, its payoff $U_A(e_T^*)$ should be greater then the payoff from war, so that

$$\begin{aligned}
0 &\leq U_A(e_T^*) - (V(w(m^*(e_T^*))e_T^*) - l(m^*(e_T^*)) - v) \\
&= U^C(e_T^*) - (V(w(m^*(e_T^*))e_T^*) - l(m^*(e_T^*)) - v) \\
&= F_v(e_T^*) < 0,
\end{aligned}$$

where the last inequality follows from the fact that $e_T^* < \widehat{e}$. This is a contradiction.∎

7.3   Proofs from Section 4

Proof of Lemma 2

Following the discussion in the text, the existence of an MPME is guaranteed by the existence of a function $U_S(e_t)$ which satisfies (22). Substitute (3) and (21) into (19), which holds as equality, to obtain

$$-c_t = G(e_{t+1}, e_t) \equiv u(e_t - e_{t+1}) + \beta(V(w(m^*(e_{t+1}))e_{t+1}) - l(m^*(e_{t+1}))) - V(w(m^*(e_t))e_t). \tag{76}$$

Substituting (18) and (76) into (22), we can write $U_S(e_t)$ as:

$$U_S(e_t) = \max_{f_t = \{0,1\}, e_{t+1} \in [0, e_t]} \{(1 - f_t)[G(e_{t+1}, e_t) + \beta U_S(e_{t+1})] + f_t \psi\} \tag{77}$$

To show that $U_S(e_t)$ exists and is well-defined, note that (21) and (19) imply that

$$
\begin{aligned}
U_A(e_t) &= \sum_{k=0}^{\infty} \beta^k \left(\begin{array}{c} (1 - f_{t+k})(u(e_{t+k} - e_{t+k+1}) + c_{t+k} - l(m^*(e_{t+k}))) \\ + \left(f_{t+k} \prod_{l=0}^{k-1}(1 - f_{t+l}) \right) V(w(m^*(e_{t+k}))e_{t+k}) \end{array} \right) \\
&= V(w(m^*(e_t))e_t) - l(m^*(e_t)),
\end{aligned}
$$

so that

$$
\begin{aligned}
U_S(e_t) &= \sum_{k=0}^{\infty} \beta^k \left(-(1 - f_{t+k})c_{t+k} + \left(f_{t+k} \prod_{l=0}^{k-1}(1 - f_{t+l}) \right) \psi \right) \qquad\qquad (78) \\
&= \sum_{k=0}^{\infty} \beta^k \left(\begin{array}{c} (1 - f_{t+k})(u(e_{t+k} - e_{t+k+1}) - \beta l(m^*(e_{t+k+1}))) \\ + \left(f_{t+k} \prod_{l=0}^{k-1}(1 - f_{t+l}) \right)(\psi + V(w(m^*(e_{t+k}))e_{t+k})) \end{array} \right) - V(w(m^*(e_t))e_t)
\end{aligned}
$$

for a given equilibrium sequence $\{f_{t+k}, e_{t+k+1}\}_{k=0}^{\infty}$. Consider the following problem:

$$
\begin{aligned}
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (79) \\
\widetilde{U}_S(e_t) &= \max_{\substack{\{f_{t+k}, e_{t+k+1}\}_{k=0}^{\infty} \\ f_{t+k}=\{0,1\}, \\ e_{t+k+1} \in [0, e_{t+k}]}} \left\{ \sum_{k=0}^{\infty} \beta^k \left(\begin{array}{c} (1 - f_{t+k})(u(e_{t+k} - e_{t+k+1}) - \beta l(m^*(e_{t+k+1}))) \\ + \left(f_{t+k} \prod_{l=0}^{k-1}(1 - f_{t+l}) \right)(\psi + V(w(m^*(e_{t+k}))e_{t+k})) \end{array} \right) \right\} \\
&\quad - V(w(m^*(e_t))e_t).
\end{aligned}
$$

Since $f_t = 1$ is feasible, (79) is bounded from below by ψ. Moreover, since $l(m^*(e_t)) \geq 0$, $e_{t+k} - e_{t+k+1} \leq e_{t+k} \leq e_t$, and $V(w(m^*(e_{t+k}))e_{t+k}) \leq V(e_{t+k}) \leq V(e_t)$, given $e_t > 0$, $\widetilde{U}_S(e_t)$ defined in (79) is less than

$$
\max_{\substack{\{f_{t+k}\}_{k=0}^{\infty} \\ f_{t+k}=\{0,1\}}} \left\{ \sum_{k=0}^{\infty} \beta^k \left(\begin{array}{c} (1 - f_{t+k})u(e_t) \\ + \left(f_{t+k} \prod_{l=0}^{k-1}(1 - f_{t+l}) \right)(\psi + V(e_t)) \end{array} \right) \right\} - V(w(m^*(e_t))e_t) < \infty,
$$

where the last inequality uses the facts that (i) $V(e_t)$ and $u(e_t)$ are bounded from above; (ii) in view of Assumption 1 in the text, $V(w(m^*(e_t))e_t)$ is bounded from below for $e_t > 0$ (and thus $w(m^*(e_t))e_t > 0$), ensuring that $\widetilde{U}_S(e_t)$ is also bounded from above for $e_t > 0$. Therefore, the solution to (79) exists and $\widetilde{U}_S(e_t)$ is well-defined for $e_t > 0$. This then implies that we can rewrite (79) recursively as

$$
\widetilde{U}_S(e_t) = \max_{f_t=\{0,1\}, e_{t+1} \in [0, e_t]} \left\{ (1 - f_t) \left[G(e_{t+1}, e_t) + \beta \widetilde{U}_S(e_{t+1}) \right] + f_t \psi \right\}, \qquad (80)
$$

as desired. It is also straightforward to see that $\widetilde{U}_S(e_t)$ in (79), and thus in (80), is uniquely defined. This follows simply from the observation that any MPME is given by (79) (and vice versa), and we have already established that for any $e_t > 0$, $\widetilde{U}_S(e_t)$ is bounded. ∎

Proof of Proposition 5

This is proved by a variational argument which considers a specific perturbation on the solution in which starting from e_t, the choice of e_{t+1} is increased by $\epsilon \gtrless 0$ arbitrarily small, where this increase is accommodated by a decrease in x_t by ϵ and an increase in x_{t+1} by ϵ.

Let e_{t+1}^* denote the optimal choice of e_{t+1} starting from e_t. Since $f_{t+1} = 0$, then $f_t = 0$. Use that for equation (77) to obtain

$$U_S(e_t) = u\left(e_t - e_{t+1}^*\right) + \beta\left[V\left(w\left(m^*\left(e_{t+1}^*\right)\right)e_{t+1}^*\right) - l\left(m^*\left(e_{t+1}^*\right)\right)\right] - V\left(w\left(m^*\left(e_t\right)\right)e_t\right) + \beta U_S\left(e_{t+1}^*\right).$$
(81)

Since $f_{t+1} = 0$, (81) also holds replacing e_t with e_{t+1}^* and e_{t+1}^* with e_{t+2}^*, where e_{t+2}^* denotes the optimal choice of e_{t+2} starting from e_{t+1}^*.

Optimality requires that the solution at e_t weakly dominates the choice of $e_{t+1}^* + \epsilon$ for $\epsilon \gtrless 0$. Let $x_t^* = e_t - e_{t+1}^*$ and let $x_{t+1}^* = e_{t+1}^* - e_{t+2}^*$. Optimality of the choice of e_{t+1}^* implies

$$u\left(x_t^*\right) + \beta\left[V\left(w\left(m^*\left(e_{t+1}^*\right)\right)e_{t+1}^*\right) - l\left(m^*\left(e_{t+1}^*\right)\right)\right] + \beta U_S\left(e_{t+1}^*\right) \geq$$
(82)
$$u\left(x_t^* - \epsilon\right) + \beta\left[V\left(w\left(m^*\left(e_{t+1}^* + \epsilon\right)\right)\left(e_{t+1}^* + \epsilon\right)\right) - l\left(m^*\left(e_{t+1}^* + \epsilon\right)\right)\right] + \beta U_S\left(e_{t+1}^* + \epsilon\right).$$

Since starting from $e_{t+1}^* + \epsilon$ country S can always choose policy e_{t+2}^* associated with e_{t+1}^* together with $f_t = 0$, this implies that

$$\begin{aligned} U_S\left(e_{t+1}^* + \epsilon\right) &\geq U_S\left(e_{t+1}^*\right) + u\left(x_{t+1}^* + \epsilon\right) - u\left(x_{t+1}^*\right) \\ &\quad + V\left(w\left(m^*\left(e_{t+1}^*\right)\right)e_{t+1}^*\right) - V\left(w\left(m^*\left(e_{t+1}^* + \epsilon\right)\right)\left(e_{t+1}^* + \epsilon\right)\right). \end{aligned}$$
(83)

Combining (82) with (83) we achieve:

$$\begin{aligned} &\left[u\left(x_t^*\right) - u\left(x_t^* - \epsilon\right)\right] - \beta\left[u\left(x_{t+1}^* + \epsilon\right) - u\left(x_{t+1}^*\right)\right] \\ &+ \beta\left[l\left(m^*\left(e_{t+1}^* + \epsilon\right)\right) - l\left(m^*\left(e_{t+1}^*\right)\right)\right] \geq 0. \end{aligned}$$
(84)

Divide both sides of (84) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0. This yields:

$$u'\left(x_t\right) - \beta u'\left(x_{t+1}\right) + \beta l'\left(m^*\left(e_{t+1}\right)\right)m^{*\prime}\left(e_{t+1}\right) = 0.$$
(85)

Since $l'(\cdot) > 0$, (85) implies that $u'\left(x_{t+1}\right) > (<)\left(1/\beta\right)u'\left(x_t\right)$ if $m^{*\prime}\left(e_{t+1}\right) > (<)0$. ∎

Proof of Proposition 6

Part 1. Suppose that (26) holds. We can prove by contradiction that the equilibrium cannot involve war for any e_t. Suppose there exists an MPME in which war occurs for some e_t. Consider an offer by country S in state e_t that satisfies $x_t^o = (1 - \beta^\sigma) w (m^* (e_t)) e_t$ and

$$-c_t^o = u(x_t^o) + \beta \left(V \left(w \left(m^* \left(e_t - x_t^o \right) \right) \left(e_t - x_t^o \right) \right) - l \left(m^* \left(\left(e_t - x_t^o \right) \right) \right) \right) - V \left(w \left(m^* \left(e_t \right) \right) e_t \right). \quad (86)$$

This offer makes country A indifferent between accepting it, and rejecting it and declaring war. We show next that the payoff for country S from making this offer strictly exceed the payoff from war ψ, which implies that there exists a strategy for country S that gives it a higher payoff than the payoff from war.

Payoff for country S from offer (x_t^o, c_t^o) is

$$
\begin{aligned}
& u(x_t^o) + \beta \left(V \left(w \left(m^* \left(e_t - x_t^o \right) \right) \left(e_t - x_t^o \right) \right) - l \left(m^* \left(e_t - x_t^o \right) \right) \right) - V \left(w \left(m^* \left(e_t \right) \right) e_t \right) \quad (87) \\
& + \beta U_S(e_t - x_t^o) \\
\geq \; & u(x_t^o) + \beta \left(V \left(w \left(m^* \left(e_t - x_t^o \right) \right) \left(e_t - x_t^o \right) \right) - l \left(\bar{m} \right) \right) - V \left(w \left(m^* \left(e_t \right) \right) e_t \right) + \beta \psi \\
\geq \; & u(x_t^o) + \beta \left(V \left(w \left(m^* \left(e_t - x_t^o \right) \right) e_t - x_t^o \right) - l \left(\bar{m} \right) \right) - V \left(w \left(m^* \left(e_t \right) \right) e_t \right) + \beta \psi \\
\geq \; & u(x_t^o) + \beta \left(V \left(w \left(m^* \left(e_t \right) \right) e_t - x_t^o \right) - l \left(\bar{m} \right) \right) - V \left(w \left(m^* \left(e_t \right) \right) e_t \right) + \beta \psi
\end{aligned}
$$

The first inequality follows from (22) and $-l \left(m^* \left(\left(e_t - x_t^o \right) \right) \right) \geq -l \left(\bar{m} \right)$. The second inequality holds because $w \left(m^* \left(e_t - x_t^o \right) \right) \leq 1$. The third inequality holds because Proposition 1 and $\sigma < 1$ imply that $w \left(m^* \left(e_t - x_t^o \right) \right) \geq w \left(m^* \left(e_t \right) \right)$.

Note that x_t^o was chosen so that it is the optimal amount of oil extraction for country A when it owns $w \left(m^* \left(e_t \right) \right) e_t$ of oil (i.e. it is the optimal x_t in the maximization problem (4)). Therefore

$$u(x_t^o) + \beta V \left(w \left(m^* \left(e_t \right) \right) e_t - x_t^o \right) = V \left(w \left(m^* \left(e_t \right) \right) e_t \right). \quad (88)$$

Substitute (88) into the right-hand side of (87) to show that payoff from offer (x_t^o, c_t^o) for country S is bounded from below by $-\beta l \left(\bar{m} \right) + \beta \psi$, which exceeds ψ if (26) holds. Therefore war cannot occur for any e_t.

Part 2. Suppose preferences satisfy (17) for $\sigma < 1$ and $w \left(\bar{m} \right) > (1/\sigma)^{1/(1-1/\sigma)}$, then war occurs with probability 1 in the MPCE by Proposition 4. Suppose that (26) also holds. Then war is avoided in the MPME by part 1. To show that this is possible, suppose that $l(m) = m$ and $w(m) = 2m - m^2$ for $\bar{m} = 1$. Then the condition that $w \left(\bar{m} \right) > (1/\sigma)^{1/(1-1/\sigma)}$ is satisfied and any value of $\psi < -\beta / (1 - \beta)$ satisfies (26).

Part 3. Suppose $\sigma < 1$ and (27) holds. Suppose that war never occurs along the equilibrium path. Using the fact that constraint (19) must hold with equality to substitute for c_t, and using

(3), $(21$, and (22), the optimality of a permanently peace equilibrium implies that for all $e_t \leq e_0$:

$$U_S(e_t) = \max_{e_{t+1}} \left\{ \begin{array}{c} u\left(e_t - e_{t+1}\right) + \beta V\left(w\left(m^*\left(e_{t+1}\right)\right)e_{t+1}\right) - \beta l\left(m^*\left(e_{t+1}\right)\right) \\ -V\left(w\left(m^*\left(e_t\right)\right)e_t\right) + \beta U_S(e_{t+1}) \end{array} \right\} \geq \psi \qquad (89)$$

Forward iteration on (89) implies that the equilibrium sequence $\{x_t^*, e_t^*\}_{t=0}^{\infty}$ must satisfy

$$
\begin{aligned}
U_S\left(e_0\right) &= \sum_{t=0}^{\infty} \beta^t \left(u\left(x_t^*\right) - l\left(m^*\left(e_t^*\right)\right)\right) - \left(V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l\left(m^*\left(e_0\right)\right)\right) \qquad (90) \\
&\leq \sum_{t=0}^{\infty} \beta^t u\left(x_t^*\right) - \frac{\beta l\left(m^*\left(e_0^*\right)\right)}{1-\beta} - V\left(w\left(m^*\left(e_0\right)\right)e_0\right) \\
&\leq V\left(e_0\right) - \frac{\beta l\left(m^*\left(e_0^*\right)\right)}{1-\beta} - V\left(w\left(m^*\left(e_0\right)\right)e_0\right).
\end{aligned}
$$

The first inequality in (90) follows from the fact that $e_{t+1} \leq e_t$ from (3) and from Proposition 1 which establishes that $m^{*\prime}(e) < 0$ so that $l\left(m^*\left(e_{t+1}\right)\right) \geq l\left(m^*\left(e_t\right)\right)$ for all e_t. The second inequality in (90) follows from the fact that the maximization of $\sum_{t=0}^{\infty} \beta^t u\left(x_t\right)$ s.t. (3) yields $V\left(e_0\right)$. Given (27), the last inequality implies that $U_S\left(e_0\right) < \psi$ which means that the best response for country S at $t = 0$ is to make any offer that violates (19) and leads to war. Therefore, war must occur along the equilibrium path.

Part 4. Suppose $\sigma < 1$, $w\left(\overline{m}\right) < (1/\sigma)^{1/(1-1/\sigma)}$, and (27) is satisfied. By part 3, war occurs in the MPME. In the MPCE, by Proposition 4 war does not occur. To show that it is possible for $w\left(\overline{m}\right) < (1/\sigma)^{1/(1-1/\sigma)}$ and (27) to be satisfied, suppose that

$$l\left(m\right) = m \text{ and } w\left(m\right) = \eta m/\left(m + \delta\right)$$

for $\delta > 0$. Let $\overline{m} = \infty$ so that $w\left(\overline{m}\right) = \eta$. Suppose that η satisfies

$$\eta < (1/\sigma)^{1/(1-1/\sigma)},$$

which is always feasible for η sufficiently low. Suppose that

$$\frac{1-\beta}{1/\sigma - 1} < \beta \qquad (91)$$

which is always feasible for σ sufficiently low. Finally, suppose that ψ and e_0 satisfy

$$\psi\left(1 - \beta\right) > e_0^{1-1/\sigma}\left(1 - \beta^{\sigma}\right)^{-1/\sigma} \times \qquad (92)$$
$$\left(-\frac{1-\beta}{1/\sigma - 1} + w\left(m^*\left(e_0\right)\right)^{1-1/\sigma}\left(\frac{1-\beta}{1/\sigma - 1} - \beta\frac{\delta}{m^*\left(e_0\right) + \delta}\right)\right).$$

This is possible because ψ can be chosen to be arbitrarily close to zero from below and because the right hand side of (92) becomes negative for sufficiently high e_0. This is because $m^*\left(e_0\right)$

declines towards 0 as e_0 rises by the arguments in claim 4 in the proof of part 2 of Proposition 4 which means given (91) that the second term on the right hand side of (92) becomes negative for high e_0. In this situation, the first-order condition which characterizes $m^*(e)$ given (8) implies

$$1 = w(m^*(e))^{-1/\sigma} w'(m^*(e)) (1 - \beta^\sigma)^{-1/\sigma} e^{1-1/\sigma},$$

which by some algebraic manipulation yields

$$l(m^*(e)) = m^*(e) = w(m^*(e))^{1-1/\sigma} \frac{\delta}{m^*(e) + \delta} (1 - \beta^\sigma)^{-1/\sigma} e^{1-1/\sigma}.$$

which means that

$$(V(e_0) - V(w(m^*(e_0))e_0))(1 - \beta) - \beta l(m^*(e_0))$$

equals the right hand side of (92) so that (27) is satisfied.∎

7.4 Proofs from Section 5

Proof of Proposition 7

Part 1. Define

$$\widetilde{V}_i(e_t) = V\left(w\left(m_i^*(e_t), \{m_j^*(e_t)\}_{j=1,j\neq i}^N\right) e_t\right)$$

Given the discussion in the text, country S's program can be written as:

$$U_S(e_t) = \max_{\{x_{it} \geq 0, c_{it}\}_{i=1}^N} \left\{ -\sum_{i=1}^N c_{it} + \beta U_S(e_{t+1}) \right\} \text{ s.t. (28) and} \tag{93}$$

$$u(x_{it}) + c_{it} + \beta\left(\widetilde{V}_i(e_{t+1}) - l(m_i^*(e_{t+1}))\right) = \widetilde{V}_i(e_t) \ \forall i \tag{94}$$

Now consider the solution given that $f_t = f_{t+1} = 0$. Let x_{it}^* and e_{t+1}^* denotes the implied optimal choice of e_{t+1} starting from e_t so that

$$U_S(e_t) = \sum_{i=1}^N \left(u(x_{it}^*) + \beta\left[\widetilde{V}_i(e_{t+1}^*) - l(m^*(e_{t+1}^*))\right] - \widetilde{V}_i(e_t) \right) + \beta U_S(e_{t+1}^*). \tag{95}$$

Since $f_{t+1} = 0$, (95) also holds replacing e_t with e_{t+1}^* and e_{t+1}^* with e_{t+2}^*, where e_{t+2}^* denotes the optimal choice of e_{t+2} starting from e_{t+1}^*. Optimality requires that the solution at e_t weakly dominates the choice of $e_{t+1}^* + \epsilon$ for $\epsilon \gtrless 0$ where this is achieved by reducing x_{it}^* by ϵ. Optimality

of the choice of e_{t+1}^* implies

$$u\left(x_{it}^*\right) + \beta \sum_{j=1}^{N} \left[\tilde{V}_j\left(e_{t+1}^*\right) - l\left(m_j^*\left(e_{t+1}^*\right)\right)\right] + \beta U_S\left(e_{t+1}^*\right) \geq \tag{96}$$

$$u\left(x_{it}^* - \epsilon\right) + \beta \sum_{j=1}^{N} \left[\tilde{V}_j\left(e_{t+1}^* + \epsilon\right) - l\left(m_j^*\left(e_{t+1}^* + \epsilon\right)\right)\right] + \beta U_S\left(e_{t+1}^* + \epsilon\right).$$

Since starting from $e_{t+1}^* + \epsilon$ country S can always choose policy e_{t+2}^* associated with e_{t+1}^* so that x_{it+1}^* is increased by ϵ this implies that

$$U_S\left(e_{t+1}^* + \epsilon\right) \geq U_S\left(e_{t+1}^*\right) + u\left(x_{it+1}^* + \epsilon\right) - u\left(x_{it+1}^*\right) \tag{97}$$
$$+ \sum_{j=1}^{N} \left[\tilde{V}_j\left(e_{t+1}^*\right) - \tilde{V}_j\left(e_{t+1}^* + \epsilon\right)\right].$$

Combining (96) with (97) we achieve:

$$\left[u\left(x_{it}^*\right) - u\left(x_{it}^* - \epsilon\right)\right] - \beta\left[u\left(x_{it+1}^* + \epsilon\right) - u\left(x_{it+1}^*\right)\right] \tag{98}$$
$$+ \sum_{j=1}^{N} \beta\left[l\left(m_j^*\left(e_{t+1}^* + \epsilon\right)\right) - l\left(m_j^*\left(e_{t+1}^*\right)\right)\right] \geq 0.$$

Divide both sides of (98) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0. This yields:

$$u'\left(x_{it}\right) - \beta u'\left(x_{it+1}\right) + \sum_{j=1}^{N} \beta l'\left(m_j^*\left(e_{t+1}\right)\right) m_j^{*\prime}\left(e_{t+1}\right) = 0. \tag{99}$$

Since $l'\left(\cdot\right) > 0$, (99) implies that $u'\left(x_{it+1}\right) > (<)\left(1/\beta\right)u'\left(x_{it}\right)$ if $m_j^{*\prime}\left(e_{t+1}\right) > (<) 0 \; \forall j$. Since $m_i^{*\prime}\left(e_t\right) = m_j^{*\prime}\left(e_{t+1}\right)$ for all j, this implies that this depends only on the sign of $m_i^{*\prime}\left(e_t\right)$.∎

Part 2. At each t, given e_t, equilibrium profile of armaments $\mathbf{m}_t^*$ is such that m_{it} is the same for all countries, which implies that $w_i\left(m_i, \mathbf{m}_{-it}\right) = \eta/N$ and that

$$w_{i_{m_{it}}}\left(m_{it}, \mathbf{m}_{-it}\right) = \eta h'\left(m_i\right) \frac{\sum_{j \neq i} h\left(m_j\right)}{\left[\sum_j h\left(m_j\right)\right]^2}$$
$$= \frac{h'\left(m_{it}\right)}{h\left(m_{it}\right)} w_i\left(m_{it}, \mathbf{m}_{-it}\right)\left(1 - \frac{1}{\eta}w_i\left(m_{it}, \mathbf{m}_{-it}\right)\right) = \eta \frac{h'\left(m_{it}\right)}{h\left(m_{it}\right)} \frac{N-1}{N^2}.$$

This implies that the first-order condition which characterizes equilibrium armament $m_i^*\left(e_t\right)$ is uniquely defined by

$$V'\left(\eta e_t/N\right) \eta e_t \frac{N-1}{N^2} \frac{h'\left(m_i^*\left(e_t\right)\right)}{h\left(m_i^*\left(e_t\right)\right)} = l'\left(m_i^*\left(e_t\right)\right). \tag{100}$$

Given the solution to (4), the envelope condition implies that

$$V'\left(\eta e_t/N\right) = \beta^k u'\left(x_{it+k}\right) \ \forall k \geq 0. \tag{101}$$

Substitution of (101) into (100) followed by implicit differentiation yields

$$\frac{u'\left(x_{it+k}\right)}{u''\left(x_{it+k}\right)} \left(\frac{l''\left(m_i^*\left(e_t\right)\right)}{l'\left(m_i^*\left(e_t\right)\right)} + \left[\frac{h'\left(m_i^*\left(e_t\right)\right)}{h\left(m_i^*\left(e_t\right)\right)} - \frac{h''\left(m_i^*\left(e_t\right)\right)}{h'\left(m_i^*\left(e_t\right)\right)}\right]\right) \frac{dm_{it}}{de_t} = \frac{dx_{it+k}}{de_t} + \frac{u'\left(x_{it+k}\right)}{u''\left(x_{it+k}\right)e_t} \tag{102}$$

Summing up (5) and (6) one obtains

$$\sum_{k=0}^{\infty} x_{it+k} = \eta e_t/N \tag{103}$$

differentiation of which implies

$$\sum_{k=0}^{\infty} \frac{dx_{it+k}}{de_t} = \eta/N.$$

Taking the sum of (102) $\forall k \geq 0$ and substitution into the above equation yields

$$\frac{dm_{it}}{de_t} = \frac{\dfrac{\eta}{N}\left(1 + \sum_{k=0}^{\infty} \dfrac{u'\left(x_{it+k}\right)}{u''\left(x_{it+k}\right)x_{it+k}} \dfrac{x_{it+k}}{\eta e_t/N}\right)}{\left(\dfrac{l''\left(m_i^*\left(e_t\right)\right)}{l'\left(m_i^*\left(e_t\right)\right)} + \left[\dfrac{h'\left(m_i^*\left(e_t\right)\right)}{h\left(m_i^*\left(e_t\right)\right)} - \dfrac{h''\left(m_i^*\left(e_t\right)\right)}{h'\left(m_i^*\left(e_t\right)\right)}\right]\right)\sum_{k=0}^{\infty} \dfrac{u'\left(x_{it+k}\right)}{u''\left(x_{it+k}\right)}} \tag{104}$$

Since the denominator is negative, (104) is positive if and only if the numerator is negative. If $-u'\left(x_{it+k}\right)/u''\left(x_{it+k}\right)x_{it+k} > 1 \ \forall x_{it+k}$ then the numerator is negative since from (103), $\sum_{k=0}^{\infty} \frac{x_{t+k}}{(\eta e_t/N)} = 1$, and the opposite holds if $-u'\left(x_{it+k}\right)/u''\left(x_{it+k}\right)x_{it+k} < 1 \ \forall x_{it+k}$. ∎

Proof of Proposition 8

We proceed first by proving that $U_S\left(e_t\right)$ is uniquely defined in the symmetric MPME, and then we guess and verify a function for $U_S\left(e_t\right)$ in order to prove the properties of the equilibrium allocations described in the proposition. Given the symmetry of the equilibrium $\widetilde{V}_i\left(e_t\right)$ and $m_i^*\left(e_t\right)$ are the same across countries, so that they can be denoted by $\widetilde{V}\left(e_t\right)$ and $m^*\left(e_t\right)$, respectively, and all countries receive the same resource consumption equal to $\left(e_t - e_{t+1}\right)/N$. Define

$$G\left(e_{t+1}, e_t\right) = N\left(u\left(\frac{1}{N}\left(e_t - e_{t+1}\right)\right) + \beta\left(\widetilde{V}\left(e_{t+1}\right) - l\left(m^*\left(e_{t+1}\right)\right)\right) - \widetilde{V}\left(e_t\right)\right).$$

Given (17) and (30), (100) implies (31). Therefore, $G\left(e_{t+1}, e_{t}\right)$ can be rewritten as:

$$G\left(e_{t+1}, e_{t}\right) = N \left(\begin{array}{c} \dfrac{\left(\frac{1}{N}\left(e_{t}-e_{t+1}\right)\right)^{1-1/\sigma}}{1-1/\sigma} \\ +\beta\left(\dfrac{\left(1-\beta^{\sigma}\right)^{-1/\sigma}}{1-1/\sigma}\left(\dfrac{e_{t+1}}{N}\right)^{1-1/\sigma}-\left(\dfrac{N-1}{N}\right)\left(1-\beta^{\sigma}\right)^{-1/\sigma}\left(\dfrac{e_{t+1}}{N}\right)^{1-1/\sigma}\right) \\ -\dfrac{\left(1-\beta^{\sigma}\right)^{-1/\sigma}}{1-1/\sigma}\left(\dfrac{e_{t}}{N}\right)^{1-1/\sigma} \end{array} \right)$$
(105)

Substitution of (94) into (93) implies that country S's optimal offer satisfies

$$U_{S}\left(e_{t}\right) = \max_{e_{t+1}\in[0,e_{t}]} G\left(e_{t+1}, e_{t}\right) + \beta U_{S}\left(e_{t+1}\right)$$
(106)

By analogous arguments to those of Lemma 2, there is a unique $U_{S}\left(e_{t}\right)$. Let us guess and verify that $U_{S}\left(e_{t}\right)$ satisfies

$$U_{S}\left(e_{t}\right) = Q\frac{e_{t}^{1-1/\sigma}}{1-1/\sigma}$$
(107)

for some constant $Q > 0$. It is straightforward to see that under this assumption, and given that the second line of (105) is increasing and concave in e_{t+1}, the program defined by (106) is strictly concave and yields a unique solution characterized by first order conditions. The first order conditions and the envelope condition for the program defined in (106) yield:

$$\left(\frac{1}{N}\right)^{-1/\sigma}\left(\begin{array}{c} -\left(e_{t}-e_{t+1}\right)^{-1/\sigma} \\ +\beta\left(\left(1-\beta^{\sigma}\right)^{-1/\sigma}\left(1-\left(1-1/\sigma\right)\left(\frac{N-1}{N}\right)\right)\right)e_{t+1}^{-1/\sigma} \end{array}\right) = -\beta Q e_{t+1}^{-1/\sigma} \quad (108)$$

$$\left(\frac{1}{N}\right)^{-1/\sigma}\left(\left(e_{t}-e_{t+1}\right)^{-1/\sigma}-\left(1-\beta^{\sigma}\right)^{-1/\sigma}e_{t}^{-1/\sigma}\right) = Q e_{t}^{-1/\sigma}. \quad (109)$$

Define $\rho \in (0,1)$ such that the e_{t+1} which satisfies (108) and (109) also satisfies $e_{t+1} = \rho^{\sigma} e_{t}$. Substitution of $e_{t+1} = \rho^{\sigma} e_{t}$ into (108) and (109) allows us to combine both equations to cancel out for Q, so that ρ satisfies

$$\left(1-1/\sigma\right)\left(\frac{N-1}{N}\right)\left(1-\beta^{\sigma}\right)^{-1/\sigma} = \left(1-\frac{\rho}{\beta}\right)\left(1-\rho^{\sigma}\right)^{-1/\sigma},$$
(110)

which implies that ρ is independent of e_{t} and Q. Given (109), this means that Q must satisfy

$$Q = \left(\frac{1}{N}\right)^{-1/\sigma}\left(\left(1-\rho^{\sigma}\right)^{-1/\sigma}-\left(1-\beta^{\sigma}\right)^{-1/\sigma}\right)$$
(111)

for ρ defined in (110). To complete the proof, we can substitute in for e_{t+1} and Q on the right hand side of (106) using the fact that $e_{t+1} = \rho^{\sigma} e_{t}$ and that Q is defined by (111) for ρ defined

in (110), and this confirms that the original guess in (107) is correct.

To prove the first part of the proposition, note that since $e_{t+1} = \rho^\sigma e_t$, then this implies that $x_{it} = (e_t - e_{t+1})/N = (1 - \rho^\sigma) e_t/N$. Therefore,

$$u'(x_{it+1}) = [(1 - \rho^\sigma) e_{t+1}/N]^{-1/\sigma} = (1/\rho)[(1 - \rho^\sigma) e_t/N]^{-1/\sigma} = (1/\rho) u'(x_{it}).$$

The second part of the proposition follows from the fact that the left hand side of (110) is positive (negative) if $\sigma > (<) 1$. Therefore, if $\sigma > (<) 1$, then for the right hand side of (110) to be positive (negative) it must be the case that $\rho < (>) \beta$. To prove the third part of the proposition note that the derivative of the right-hand side of (110) with respect to ρ has the same sign as:

$$-\frac{1}{\beta} + \left(\frac{1}{\rho} - \frac{1}{\beta}\right)\frac{\rho^\sigma}{1 - \rho^\sigma} \tag{112}$$

which must be negative. This is because if $\sigma < 1$, then $\rho > \beta$ so that (112) is negative and if $\sigma > 1$, then $\rho < \beta$ and (112) cannot be greater than

$$-\frac{1}{\beta} + \left(\frac{1}{\rho} - \frac{1}{\beta}\right)\frac{\rho}{1 - \rho} = \frac{1}{1 - \rho}\left(-\frac{1}{\beta} + 1\right) < 0.$$

Therefore, ρ is uniquely defined. It follows that if $\sigma < 1$, the left-hand side of (110) declines as N rises, so that ρ rises as N rises. Alternatively, if $\sigma > 1$, the the left-hand side of (110) rises as N rises, so that ρ declines as N rises, which completes the argument.∎

Proof of Proposition 9

Part 1. Given the discussion in the text, country S's program can be written as:

$$U_S(e_t) = \max_{x_t \geq 0, c_t} \{-c_t - l(m_S^*(e_t)) + \beta U_S(e_{t+1})\} \text{ s.t. (3) and}$$

$$u(x_t) + c_t + \beta[V(w(m_A^*(e_{t+1}), m_S^*(e_{t+1})) e_{t+1}) - l(m_A^*(e_{t+1}))] = V(w(m_A^*(e_t), m_S^*(e_t)) e_t).$$

Now consider the solution given that $f_t = f_{t+1} = 0$. Let e_{t+1}^* denotes the implied optimal choice of e_{t+1} starting from e_t so that

$$
\begin{aligned}
U_S(e_t) = {} & u(e_t - e_{t+1}^*) - l(m_S^*(e_t)) + \beta[V(w(m_A^*(e_{t+1}), m_S^*(e_{t+1})) e_{t+1}) - l(m_A^*(e_{t+1}))] \tag{113}\\
& - V(w(m_A^*(e_t), m_S^*(e_t)) e_t) + \beta U_S(e_{t+1}^*).
\end{aligned}
$$

Follow the same perturbation arguments as in the proof of Proposition 5. This yields:

$$[u(x_t^*) - u(x_t^* - \epsilon)] - \beta[u(x_{t+1}^* + \epsilon) - u(x_{t+1}^*)] \tag{114}$$
$$+\beta[l(m_A^*(e_{t+1}^* + \epsilon)) - l(m_A^*(e_{t+1}^*)) + l(m_S^*(e_{t+1}^* + \epsilon)) - l(m_S^*(e_{t+1}^*))] \geq 0.$$

Divide both sides of (114) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0. This yields:

$$u'(x_t) - \beta u'(x_{t+1}) + \beta l'(m_A^*(e_{t+1})) m_A^{*\prime}(e_{t+1}) + \beta l'(m_S^*(e_{t+1})) m_S^{*\prime}(e_{t+1}) = 0. \tag{115}$$

Since $l'(\cdot) > 0$, (115) implies that $u'(x_{t+1}) > (<)(1/\beta) u'(x_t)$ if $m_A^{*\prime}(e_{t+1}) > (<)0$ and $m_S^{*\prime}(e_{t+1}) > (<)0$.

Part 2. Analogous arguments to those of part 2 of Proposition 7 imply that $m_A^*(e_t)$ and $m_S^*(e_t)$ increase (decrease) in e_t if $-u'(x)/(xu''(x)) > (<)1$ for all x. ∎

Proof of Proposition 10

Analogous arguments as in the proof of Proposition 5 imply that $m_t = m^*(e_t)$, that

$$U_A(e_t) = \widetilde{V}(e_t),$$

and that country S's optimal offer must satisfy:

$$U_S(e_t) = \max_{x_t \geq 0, c_t} \{-c_t + \beta U_S(e_{t+1})\} \text{ s.t. (3) and}$$

$$u(x_t, c_t, -m^*(e_t)) + \beta \widetilde{V}(e_{t+1}) = \widetilde{V}(e_t).$$

Let e_{t+1}^* denote the implied optimal value of e_{t+1} starting from e_t, and let e_{t+2}^* denote the implied optimal value of e_{t+2} starting from e_{t+1}^*. Let $\widetilde{c}_t(\epsilon)$ and $\widetilde{c}_{t+1}(\epsilon)$, respectively, solve:

$$u\left(e_t - e_{t+1}^* - \epsilon, \widetilde{c}_t(\epsilon), -m^*(e_t)\right) + \beta \widetilde{V}\left(e_{t+1}^* + \epsilon\right) = \widetilde{V}(e_t) \text{ and} \tag{116}$$

$$u\left(e_{t+1}^* - e_{t+2}^* + \epsilon, \widetilde{c}_{t+1}(\epsilon), -m^*\left(e_{t+1}^* + \epsilon\right)\right) + \beta \widetilde{V}\left(e_{t+2}^*\right) = \widetilde{V}\left(e_{t+1}^* + \epsilon\right) \tag{117}$$

for $\epsilon \gtrless 0$. Note that by implicit differentiation:

$$\widetilde{c}_t'(0) = \frac{u_x(x_t, c_t, -m_t) - \beta \widetilde{V}'(e_{t+1})}{u_c(x_t, c_t, -m_t)}$$

$$\widetilde{c}_{t+1}'(0) = \frac{-u_x(x_{t+1}, c_{t+1}, -m_{t+1}) + u_m(x_{t+1}, c_{t+1}, -m_{t+1}) m^{*\prime}(e_{t+1}) + \widetilde{V}'(e_{t+1})}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})}$$

Optimality requires that

$$-\widetilde{c}_t(0) + \beta U_S\left(e_{t+1}^*\right) \geq -\widetilde{c}_t(\epsilon) + \beta U_S\left(e_{t+1}^* + \epsilon\right)$$

$$\geq -\widetilde{c}_t(\epsilon) + \beta\left(-\widetilde{c}_{t+1}(\epsilon) + \widetilde{c}_{t+1}(0) + U_S\left(e_{t+1}^*\right)\right)$$

which implies that

$$\widetilde{c}_t(0) - \widetilde{c}_t(\epsilon) \leq \beta\left(\widetilde{c}_{t+1}(\epsilon) - \widetilde{c}_{t+1}(0)\right). \tag{118}$$

Divide both sides of (118) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0 so as to achieve:

$$-\tilde{c}'_t(0) = \beta \tilde{c}'_{t+1}(\epsilon),$$

which by substitution yields:

$$\frac{u_x(x_{t+1}, c_{t+1}, -m_{t+1})}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})} = \frac{1}{\beta} \frac{u_x(x_t, c_t, -m_t)}{u_c(x_t, c_t, -m_t)} + \frac{u_m(x_{t+1}, c_{t+1}, -m_{t+1})}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})} m^{*\prime}(e_{t+1})$$
$$+ \tilde{V}'(e_{t+1}) \left(\frac{1}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})} - \frac{1}{u_c(x_t, c_t, -m_t)} \right),$$

which completes the proof since $u_c(\cdot), u_m(\cdot) > 0$. ∎

7.5 Monopolistic Environment without Armament

Here we briefly consider the implications of allowing country A to engage in war without the possibility for armament. In particular, suppose that

$$w(m) = \bar{w} \in (0, 1] \text{ for all } m, \tag{119}$$

which implies that country A never invests in armament in equilibrium.

It is then straightforward to see that wars do not occur in any period. This is because country S can always structure offers to country A so as to replicate the outcome of war while making itself better off by avoiding war which costs it ψ.

Formally, if country A attacks country S over any stock of the resource e_t, country A's payoff is $V(\bar{w}e_t)$ and its path of extraction of the resource following the war $\{\tilde{x}_{t+k}(\bar{w}e_t)\}_{k=0}^{\infty}$ is a solution to (4) when $w(m) = \bar{w}$. Note that it satisfies

$$V(\bar{w}e_t) = u(\tilde{x}_t(\bar{w}e_t)) + \beta V(\bar{w}e_t - \tilde{x}_t(\bar{w}e_t)). \tag{120}$$

It is feasible for country S to make offers in equilibrium that replicate the payoff of country A in the event of war. In fact, we can show a stronger statement that country S in any period can make an offer that makes both countries strictly better off than having a war. Consider an offer $\tilde{z}_t = \{\tilde{x}_t(\bar{w}e_t), \epsilon\}$ where $\epsilon \in (0, -(1-\beta)\psi)$. Since the payoff of country A in period $t+1$ is bounded by the payoff from attacking country S, $V(\bar{w}(e_t - \tilde{x}_t(\bar{w}e_t)))$, its payoff in period t from accepting offer $\tilde{z}_t$ satisfies

$$u(\tilde{x}_t(\bar{w}e_t)) + \epsilon + \beta U_A(e_t - \tilde{x}_t(\bar{w}e_t)) > u(\tilde{x}_t(\bar{w}e_t)) + \beta V(\bar{w}e_t - \tilde{x}_t(\bar{w}e_t))$$
$$= V(\bar{w}e_t)$$

where the last line uses (120). This means country A is made strictly better off accepting this alternative offer.

Similarly, the payoff of country S in period $t+1$ is bounded by the payoff from being attacked ψ, since country S can always make an offer which is rejected.[25] Therefore, country S's payoff following the acceptance of the offer is

$$-\epsilon + \beta U_S(e_t - \tilde{x}_t(\bar{w}e_t)) \geq -\epsilon + \beta\psi.$$

Since $-\epsilon + \beta\psi > \psi$, country S is made strictly better off so that war cannot be an equilibrium with any endowment e_t.

Since wars are never an equilibrium, country S makes an offer z_t to extract the maximum surplus from country A subject to avoiding war. We can then show that such an offer always satisfies the Hotelling rule. Formally, country S's maximization problem is

$$U_S(e_t) = \max_{x_t \geq 0, c_t} \{-c_t + \beta U_S(e_{t+1})\} \tag{122}$$

subject to (3),

$$u(x_t) + c_t + \beta U_A(e_{t+1}) \geq V(\bar{w}e_t). \tag{123}$$

With the same argument as in the text, the participation constraint is given by (123) and this constraint must bind; if it did not, country S could strictly improve its payoff by offering a lower value of c_t to country A. Therefore, in this case, $U_A(e_t) = V(\bar{w}e_t)$ for all e_t so that country A is indifferent between attacking and not attacking country S in every period. Therefore, the maximization problem of country S can be written as a maximization of (122) subject to (3), and

$$u(x_t) + c_t + \beta V(\bar{w}e_{t+1}) \geq V(\bar{w}e_t).$$

The first-order conditions to this problem establishes that x_t must satisfy Hotelling rule (16).[26]

It is optimal for country S to equalize country S's marginal rate of substitution over x to the marginal rate of transformation since this is the most efficient means of extracting payments from country A. As an illustration of this intuition, suppose that $\beta u'(x_{t+1}) > u'(x_t)$. If country S extracts ϵ units of resources less in period t and $\epsilon > 0$ more in period $t+1$, holding everything fixed, it changes payoff of country A by $(\beta u'(x_{t+1}) - u'(x_t))\epsilon > 0$, which relaxes constraint (123). This allows country S to reduce c_t and hence increase the payments it receives from country A. If instead $\beta u'(x_{t+1}) < u'(x_t)$, then analogous arguments imply that country S could improve its payoff by extracting $\epsilon > 0$ units of resources more in period t and ϵ less in

---

[25]Formally, starting from any e_t, country S can offer $\{0, 0\}$, which yields a payoff $\beta U_S(e_t)$ if it does not lead to war and ψ if it leads to war. This implies that

$$U_S(e_t) \geq \min\{\beta U_S(e_t), \psi\} = \psi, \tag{121}$$

where we have used the fact that if it were the case that $\beta U_S(e_t) < \psi < 0$, (121) would imply $U_S(e_t) \geq 0$, yielding a contradiction.

[26]To take the first-order condition one needs to assume that $U_S(e)$ is differentiable. One can prove the same result without assuming differentiability by following the same steps as in the proof of Proposition 5.

period $t + 1$.

We summarize the results of this section in the following proposition:

Proposition 12 *Suppose $w\left(\cdot\right)$ satisfies (119). Then in any MPME:*

1. *War never occurs.*

2. *The equilibrium sequence of resource extraction, x_t, satisfies (16) for all t.*

References

Acemoglu, Daron and James A. Robinson (2006), *Economic Origins of Dictatorship and Democracy*, Cambridge University Press; New York.

Anderlini, Luca, Dino Gerardi, and Roger Lagunoff (2009) "Social Memory, Evidence, and Conflict," forthcoming in *Review of Economic Dynamics*.

Antrás, Pol and and Gerard Padró i Miquel (2009) "Foreign Influence and Welfare," Working Paper.

Bagwell, Kyle and Robert W. Staiger (1990) "A Theory of Managed Trade" *American Economic Review*, 80, 779-795.

Bagwell, Kyle and Robert W. Staiger (2001) "Domestic Policies, National Sovereignty and International Economic Institutions," *Quarterly Journal of Economics*, 116, 519-562.

Bakeless, John Edwin (1921) *The Economic Causes of Modern War: A Study of the Period: 1878-1918*, Mofatt, Yard, and Company, New York.

Baliga, Sandeep and Tomas Sjöström (2004) "Arms Races and Negotiations," *Review of Economic Studies*, 71, 351-369.

Caselli, Francesco (2005) "Power Struggles and the Natural Resource Curse," Working Paper.

Chassang, Sylvain and Gerard Padró i Miquel (2010) "Conflict and Deterrence under Strategic Risk," *Quarterly Journal of Economics*, forthcoming.

Dasgupta, Partha and Geoffrey M. Heal (1979) *Economic Theory and Exhaustible Resources*, Cambridge University Press, Cambridge.

Dixit, Avinash (1987) "Strategic Behavior in Contests," *American Economic Review*, 77, 891-898.

Egorov, Georgy, Sergei Guriev, and Konstantin Sonin (2009) "Why Resource-Poor Dictators Allow Freer Media: A Theory and Evidence from Panel Data," *American Political Science Review*, forthcoming.

Esteban, Joan and Debraj Ray (2008) "On the Salience of Ethnic Conflict," forthcoming, *American Economic Review*, 98, 2185–2202.

Fearon, James D. (1995) "Rationalist Explanations for War," *International Organization*, 49, 379-414.

Findlay, Ronald and Kevin H. O'Rourke (2007) *Power and Plenty: Trade, War, and the World Economy in the Second Millennium*, Princeton University Press, Princeton.

Garfinkel, Michelle R., Stergios Skaperdas, and Constantinos Syropoulos (2009) "International Trade and Transnational Insecurity: How Comparative Advantage and Power are Jointly Determined," Working Paper.

Gately, Dermot and Hillard Huntington (2002) "The Asymmetric Effects of Changes in Price and Income on Energy and Oil Demand," *Energy Journal*, 23, 19-37.

Grossman, Gene, and Elhanan Helpman (1995) "Trade Wars and Trade Talks," *Journal of Political Economy*, 103, 675-708.

Hirshleifer, Jack (1995) "Anarchy and its Breakdown," *Journal of Political Economy*, 103, 26-52.

Hotelling, Harold (1931) "The Economics of Exhaustible Resources," Journal of Political Economy, 39, 137-175.

Jackson, Matthew O. and Massimo Morelli (2009) "Strategic Militarization, Deterrence, and Wars," *Quarterly Journal of Political Science*, 4, 279-313.

Klare, Michael T. (2001) *Resource Wars: The New Landscape of Global Conflict*, Henry Holt and Company, New York.

Kremer, Michael and Charles Morcom (2000) "Elephants," *American Economic Review*, 90, 212-234.

Maggi, Giovanni (1999) "The Role of Multinational Organizations in International Trade Cooperation," *American Economic Review*, 89, 190-214.

Maggi, Giovanni and Andres Rodriguez-Clare (2007) "A Political-Economy Theory of Trade Agreements," *American Economic Review*, 97, 1374-1406.

Pindyck, Robert S. (1978) "The Optimal Exploration and Production of Nonrenewable Resources," *Journal of Political Economy*, 86, 841-861.

Powell, Robert (1993) "Guns, Butter, and Anarchy," *American Political Science Review*, 87, 115-132.

Powell, Robert (1999) *In the Shadow of Power: States and Strategies in International Politics*, Princeton University Press; Princeton.

Richardson, Lewis F. (1960) *Statistics of Deadly Quarrels*, ed.Quincy Wright and C. C. Lienau, Quadrangle Books, Chicago.

Robinson, James A. and Ragnar Torvik (2006) "Political Foundations of the Resource Curse" *Journal of Development Economics*, 79, 447-468.

Ross, Michael L. (1999) "The Political Economy of the Resource Curse" *World Politics*, 52, 297-322.

Schwarz, Michael and Konstantin Sonin (2004) "A Theory of Brinksmanship, Conflicts, and Commitments," *Journal of Law, Economics, and Organization*, forthcoming.

Skaperdas, Stergios (1992) "Cooperation, Conflict, and Power in the Absence of Property Rights", *American Economic Review*, 82, 720-739.

Tornell Aaron and Philip R. Lane (1999) "The Voracity Effect," *American Economic Review*, 89, 22-46.

Westing, Arthur H. (1986) *Global Resources and International Conflict: Environmental Factors in Strategic Policy and Action*, Oxford University Press, Oxford.

Wright, Quincy (1942) *A Study of War*, University of Chicago Press, Chicago.

Yared, Pierre (2010) "A Dynamic Theory of War and Peace," *Journal of Economic Theory*, 145, 1921-1950.