Overeagerness

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OVEREAGERNESS[*]

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Abstract

In the terminology of classical signaling models we capture the impression that high types may send lower signals than low types in order not to appear too desperate. Overeagerness of low types or conversely modesty of high types can be described by our model. In contrast to the counter-signaling literature we require only a noisy one dimensional signal, where very low signal manifestations force types to execute their outside option. The central assumption is that high types are not only more productive when working for a firm, but that they also have a higher opportunity cost of doing so. Low types are then eager not to end up with their bad outside option as a result of a low signal manifestation. High types may exploit this eagerness by using lower signals (and hence a higher risk to end up with their own better outside option) in order to be distinguishable. Type dependent signaling cost is incorporated. It allows predicting when overeagerness should occur and when the classical signaling effect should dominate.

1. Introduction

"Barking dogs don't bite"
Proverb

With his classical signaling model Spence (1973) illustrates circumstances under which rational agents should engage in a wasteful activity in order to distinguish themselves from less competitive individuals. In his terminology "high types" send a costly but non-productive "signal" that "low types" can not justify sending. Though hard to verify empirically,[1] signaling has been considered one reason for phenomena like people acquiring educational degrees, job candidates wearing a nice outfit for their interview, companies advertising their products or even evolution allowing species to develop seemingly useless features like the peacock its plumage.

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[1] See Weiss (1995) for a convincing attempt in the context of wages.

Surprisingly we sometimes observe ourselves reacting negatively to such signaling behavior. For example heavy advertisement of a product, say in the form of many friendly phone-calls, often does not lead to improving our view of its quality.

Further more there is casual evidence that “stronger” signals are not always advantageous, like a survey by “Institut der Deutschen Wirtschaft Köln” concluding that multiple degree holders are not considered favorably on German job markets (iwd Nr. 46, 1995).

If these perceptions are justified, we should find that sometimes the less competitive send stronger signals than the most competitive.

Indeed Clements (2003) points out, that high quality products often come in cheap packaging. For example premium beer does not have the more convenient screw off caps found on low-quality beer and in the US newspapers with high-quality content are sold in broadsheet format instead of the more convenient tabloid format.

Hvide (2003) cites casual evidence of the most capable of students leaving or skipping college and going straight into business in areas where education is not a formal requirement for entry. For example Stanford University is known to have lost a substantial amount of students to the high-tech sector. Also Orzach and Tauman (1996) note that in the 1996 Forbes 400 list containing the richest 400 people in the US very many do not have any academic degree.[2] Furthermore Feltovich, Harbaugh and To (2002) point out, that in the US talented students (as measured by aptitude tests such as the SAT) tend to underachieve in terms of school grades. Similarly some luxury brands, like good wineries, hardly advertise at all.[3] And being “cool” is attractive when in essence it describes individuals who seem less eager to please or less concerned about approval.

The impression we get when someone is too eager is one of desperation, while coolness or little eagerness are interpreted as strength: “The person does not need my approval” or “the company does not depend on my business” or “the applicant has alternatives to my job offer” are the kind of reactions invoked in us. They lead us to believe that the applicant is capable, the companies product of high quality or that the person has reason to be self-confident. Absence of a high signal can look like a sign of better outside options for the sender or higher opportunity costs of engaging in a relationship with the signal’s recipient.

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[2] In both examples the decision to leave school may have occurred *after* an employment option presented itself. However, if a good degree has signaling value even for successive employers, one could argue that such young professionals made the career decision to be better off without such a signal.

[3] A Cointreau slogan in Germany, rather paradoxically, reads “We are so good, we don’t need advertisement” (“Wir sind so gut, wir brauchen keine Werbung”).

These examples are only suggestive, but the notion that “overeagerness” is often interpreted as a bad sign and that the absence of a signal can be understood as a positive indicator seems a conventional wisdom. The proverb “Barking dogs don’t bite” captures the idea well. “Boasting” as opposed to modesty has a negative impact on the perception of an individual’s capabilities.

In Spence’s setup this is unexplainable. Low types should try to imitate high types as close as affordable. They will never send more of the costly signal, than high types. Of course various stories can explain some of the above. For example, returning to Spence’s terminology, education might actually augment competitiveness. Then, lower types might choose to educate more than higher types in order to compensate for their lack of ability.

This paper tries to capture the notion of overeagerness originating from desperation or bad outside options and coolness or modesty originating from attractive outside options more closely. In particular it tries to answer the question “What could lead a low type to engage in unproductive signaling more heavily than a high type?” To be able to contrast the results to the standard signaling outcomes and to make them appear in familiar terms we stick to Spence’s terminology.

First, we assume that high types have a higher opportunity cost of being employed than low types. For example high types could not only be more productive when employed, but also when self employed. Or they could have more creative and gratifying things to do with their free time. This assumption should have a lot of intuitive appeal.

Second, opportunity costs are relevant: Very low signal manifestations leave individuals facing their outside option. The content of this assumption depends on the application:

- Most directly the signal amplitude might have to exceed a “physical threshold” in order to be detectable.
- A majority of the population might be of a third type that is not interested in sending a signal due to even higher opportunity costs for being engaged by the employer. Further the employer might not be able to “afford” making an offer to the whole society (e.g. there might be some fixed cost to every offer).
- Finally it could be that, though the whole society is composed of low and high types, only the sub-group of individuals who have a specific skill is of any productive value to the company. A majority of people does not possess this skill[4]. This is convincing, if for example ability or type are thought of as exogenous or given by birth, while some basic skill has to have been acquired

[4] This is equivalent to assuming three types, low, medium and high. Because in our model only the two higher types will have an incentive to be strategic, it is more convenient to think of high and low types that have a required unobservable skill and unskilled individuals in the background that may or may not be of either type.

over time and independent of ability. Further unskilled individuals would have to face a prohibitive cost for sending a high signal.

We will phrase our model according to the last interpretation, but since unskilled individuals will have no incentive to behave strategically, the above scenarios are equivalent and our model can address all of them.

Third, there is some risk for types to actually end up with a low signal manifestation. To that end we assume that individuals have only a noisy signal available to them. This, too, seems to be an unproblematic assumption. It rules out the unrealistic extreme case of no uncertainty in sending the signal.

These three features of the model make skilled individuals of low type very eager not to appear unskilled, because they would face their unattractive outside option. So they tend to send a high signal to reduce this risk. Skilled individuals of high type on the other hand, are less concerned about being seen as unskilled, because their outside option is more attractive. If high types do accept employment, however, they would like to be distinguishable from low types so they can be rewarded according to their productivity. Hence, they may choose to send a lower signal, allowing some distinction between types, at the cost of increasing their risk to be perceived as unskilled. This is exactly the kind of behavior we hope to illustrate in some model of unproductive signaling.

Spence's assumption of a (type dependent) cost to sending the signal is incorporated. In standard signaling contexts this assumption prevents low types from perfectly imitating high types. It drives none of the features of our model, on the contrary, if signaling is cheaper to high types, then the classical signaling effect compensates overeagerness. In our setup it is not even clear which type should incur the higher loss in utility from sending a given signal. For example high types may need to spend fewer units of time on acquiring a certain educational degree, but they might also value each unit of time higher due to their better outside option.[5] In particular our model applies to situations where cheap talk is possible for both types. However, accounting for type dependent cost of sending the signal allows us to predict when overeagerness should occur, making the model more applicable.

[5] Consider the example of advertising: Clearly it is easier to highlight the positive features of a high quality product. For example consumer tests can be cited etc. So the same level of advertising should cost less money for high quality producers. However, at the same time a high quality producer could make better use of every dollar he does not invest in a public advertising campaign, say by investing in the relations with his existing customer base, the same base that constitutes his better outside option to a successful campaign. Therefore, taking opportunity costs into account, it could even be that the advertising is cheaper for a low type.

The idea that signals are not always monotonic in quality is not new. It appears in models with explicit time structure. For example Teoh and Hwang (1991) show that a firm of high type may withhold good news from investors, while a firm of low type, with a bleaker outlook on the future, would disclose the same news. Orzach, Overgaard and Tauman (2001) explain how "Modest advertising signals strength", if firms have multiple periods to sell a product and use price and advertisement expenditure as a two dimensional signal. When non-monotonic signals occur in the context of such multidimensional signals they are also referred to as counter-signaling. Clements (2004) describes how the quality of a product's packaging can be non-monotonic in the quality of the product, if price is used as a signal, as well. For schooling Araujo, Gottlieb and Maureira (2004) address the observation that wages are non-monotonic in the GED of high school dropouts. Feltovich, Harbaugh and To (2002) show that signaling can be non-monotonic, if another exogenous dimension of the signal is available.

Our work differs from those papers in sticking to a very generic one dimensional noisy signal.[6] Of course low types would like to imitate high types in our model. Thus if any separation occurs in equilibrium it has to be true that, taking everything but the wage upon employment into account, the cost a low type would incur when sending the signal high types send has to exceed the cost of the signal other low types send. This paper merely shows how the intuitive additional assumption of a positive correlation between type and outside option can reverse the role of what we think of as high and low noisy signals. We continue to refer to the signal with the lower probability of a low signal manifestation as the higher signal. When effort is costly this is also the signal bearing the higher direct cost.

Benabou and Tirole (2004) construct a similar reversal of high and a low signals in the context of pro-social behavior. To do so, they consider type dependent preferences: if signaling brings a direct monetary reward, then the most altruistic individuals may find it more costly to signal altruism, than individuals who are greedy.

Section two of this paper introduces and illustrates our model of overeagerness and contains the general result that low types will engage more heavily in signaling than high types in any

[6] The story most similar to ours is that of Orzach, Overgaard and Tauman (2001). They propose firms whose production cost is correlated with their type. They assume given demand for good and bad quality and show that high type firms may advertise less. Their setup is different however: Firms set the price as a second dimension of their signal. Instead, our model accounts for noisy signals. Hence there can be partially separating equilibria, allowing us to meet the incentive constraint with a one dimensional signal.

equilibrium where high types do not send the highest possible signal. Section three considers a binomial distribution of signals as an instructive example. Section four concludes.

2. A Model of Overeagerness

We consider a sender-receiver game. The senders will also be called applicants, the receivers firms.

Some applicants have a specific skill which is essential to be of productive value to the firm, those without this skill have zero productivity.

There are two types of individuals in society: High types and low types. If they possess the required skill, high types have productivity $\theta_H \in \mathbb{R}_+^*$ and low types have $\theta_L \in \mathbb{R}_+^*$ when employed by the firm, where $\theta_H > \theta_L$. High types have opportunity cost v_H for being employed by the firm, low types have opportunity cost v_L, where $v_H > v_L > 0$ and $\theta_L > v_L$. These parameters are common knowledge.

There is a continuum of potential applicants and common knowledge about the fraction r of the population that possesses the required skill. The distribution of applicant types in the skilled fraction of the population is commonly known, too: The ratio of high types over low types is n.[7]

Applicants have a noisy and public signal with support in $[0,1]$ available to them. We assume for now that it is cost free.

Applicants without the required skill send signals from a distribution with support $[0, s^*]$, where $s^* < 1$. Let $\tilde{f}_u(s)$ denote the according pdf.

Applicants with the required skill can choose the effort $e \in [0,1]$ they put into sending the signal, which will then be a random draw according to the density function $f_e(s)$, where $f_e(s)$ has full support for all e. Let $f_e(s)$ satisfy the Monotone Likelihood Ratio Property (MLRP): $\forall s_1 > s_2$ the ratio $\frac{f_e(s_1)}{f_e(s_2)}$ is increasing in e. Also let $f_e(s)$ be continuous in e for all s.

For $i \in \{L, H\}$ the utility of a representative applicant is

[7] This is consistent with a possible correlation of “skill” and “type”, as the ratio of high over low types in the unskilled fraction of the population can differ from *n*. Note again that this is equivalent to assuming there to be a third type of “very low” ability.

$$u_i\left(w \middle| accept\ offer\right) = w$$

from accepting employment at a certain wage w or

$$u_i\left(w \middle| do\ not\ accept\ offer\right) = v_i$$

from executing their outside option.

On the firm's side there is perfect competition. Consequently senders of a specific signal have to be offered a wage according to the expected productivity of those accepting the offer. If indifferent between accepting and declining an offer applicants are assumed to accept. The wage schedule offered by firms is $w(s)$, $w:[0,1]\to[0,\theta_H]$.

Applicants are concerned with their expected utility

$$U_i(w,e) = \int_0^1 \max\left(w(s), v_i\right) f_e(s)\, ds\ .$$

A strategy is an effort choice e. A pure strategy Nash equilibrium then consists of effort choices (e_L, e_H) and the resulting wages for $s \in \left(s^*, 1\right]$ are

$$w(s) = \frac{nf_{e_H}(s)\theta_H + f_{e_L}(s)\theta_L}{nf_{e_H}(s) + f_{e_L}(s)}$$

if high types accept such an offer and $w(s) = \theta_L$ if they would decline it.[8]

For $s \in \left[0, s^*\right]$ wages are specified accordingly, taking into account the presence of unskilled applicants and whether or not the two types accept such an offer. The fraction $1-r$ of unskilled applicants is assumed to be large in the sense that any justifiable offer would be unacceptable to

[8] Formally firms could be considered as players, too. But due to the full support of the signal distribution and zero profit they can be understood as a mechanism that rewards applicants. There is no out of equilibrium signal value.

both types. So $w(s) < v_L < v_H$ for $s \in [0, s^*]$ has to hold in equilibrium.[9] Expected utility then becomes

$$U_i(w,e) = F_e(s^*)v_i + \int_{s^*}^{1} \max(w(s), v_i)\, f_e(s)\, ds ,$$

where $F_e(s)$ denotes the cdf corresponding to $f_e(s)$.

Theorem 1 *In any pure strategy Nash equilibrium of the model,* $1 \ge e_L > e_H \ge 0$ *or* $e_L = e_H = 1$.

The intuition for this result was described in the introduction: skilled individuals of low type are very eager not to appear unskilled, because they would face their unattractive outside option. So they will tend to send a high signal to reduce this risk. Skilled individuals of high type on the other hand, are less concerned about being seen as unskilled, because their outside option is more attractive. If high types do accept employment, however, they would like to be distinguishable from low types so they can be rewarded according to their productivity. Hence they may choose to send a lower signal, allowing some distinction between types, at the cost of increasing their risk to be perceived unskilled.

Proof of Theorem 1: Consider two cases:

i) Assume $e_H > e_L$. The wage schedule can not be weakly increasing everywhere, since then low types would deviate by playing $e_L = 1 \ge e_H$. So there are $s_1 > s_2$ with $w(s_1) < w(s_2)$. This holds if and only if the wage justified by both types accepting at signal s_1 is smaller than the one justified by both types accepting at s_2:

[9] In specific it has to be true that high types want to deviate from accepting a wage justified by both types accepting and low types want to deviate from accepting a wage justified by low types only accepting it. If m denotes the ratio of high over low types in the unskilled fraction of the population, then

$$\frac{1-r}{r} > \max\left(\frac{\left(nf_{e_H}(s)\left(\theta_H/v_H - 1\right) + f_{e_L}(s)\left(\theta_L/v_H - 1\right)\right)}{(n+1)\,\tilde{f}_u(s)}, \frac{(m+1)\, f_{e_L}(s)\left(\theta_L/v_L - 1\right)}{(n+1)\,\tilde{f}_u(s)} \right)$$

for all $e_H, e_L \in [0,1]$ and $s \in [0, s^*]$ is a sufficient constraint on r to guarantee this.

$$\frac{nf_{e_H}(s_1)\theta_H + f_{e_L}(s_1)\theta_L}{nf_{e_H}(s_1)+f_{e_L}(s_1)} < \frac{nf_{e_H}(s_2)\theta_H + f_{e_L}(s_2)\theta_L}{nf_{e_H}(s_2)+f_{e_L}(s_2)},$$

which is equivalent to

$$\frac{f_{e_H}(s_1)}{f_{e_L}(s_1)} < \frac{f_{e_H}(s_2)}{f_{e_L}(s_2)} \quad \Leftrightarrow \quad \frac{f_{e_H}(s_1)}{f_{e_H}(s_2)} < \frac{f_{e_L}(s_1)}{f_{e_L}(s_2)} \quad \overset{MLRP}{\Rightarrow} \quad e_L > e_H .$$

This is a contradiction to the assumption. Hence there is no equilibrium with $e_H > e_L$.

ii) Now assume $e_H = e_L$. Then for $s \in (s^*, 1]$ the average productivity $\overline{\theta} := \frac{n\theta_H + \theta_L}{n+1}$ has to be offered because of perfect competition among the firms: $w(s) = \overline{\theta}$. Hence low types choose $e_L = 1$. Then $e_H = 1$ by assumption. So $e_H = e_L = 1$ is an equilibrium for $\overline{\theta} \geq v_H$. The wage schedule then satisfies $w(s) = \overline{\theta}$ for $s > s^*$ and $w(s) < v_L$ otherwise. □

Proposition 1 *In every equilibrium with* $e_H < 1$ *there is* $s^{**} \geq s^*$ *such that*

$$w(s) = \begin{cases} < v_L & s \leq s^* \\ \dfrac{nf_{e_H}(s)\theta_H + f_{e_L}(s)\theta_L}{nf_{e_H}(s)+f_{e_L}(s)} & s^* < s \leq s^{**} \\ \theta_L & s > s^{**} \end{cases} .$$

Proof: According to the theorem, $e_L > e_H$ in any such equilibrium. Then by MLRP for all $s_1 > s_2 > s^*$:

$$\frac{f_{e_H}(s_1)}{f_{e_H}(s_2)} < \frac{f_{e_L}(s_1)}{f_{e_L}(s_2)} \quad \Leftrightarrow \quad \frac{f_{e_H}(s_1)}{f_{e_L}(s_1)} < \frac{f_{e_H}(s_2)}{f_{e_L}(s_2)} \quad \Rightarrow w(s_1) < w(s_2).$$

Remember $w(s) = 0$ for all $s \leq s^*$. Define $h(s)$ as the wage that would be justified, if both types accepted at signal s:

$$h(s) := \frac{nf_{e_H}(s)\theta_H + f_{e_L}(s)\theta_L}{nf_{e_H}(s)+f_{e_L}(s)}.$$

Since $f_e(s)$ is continuous in $s \in (0,1)$ for all e, so is $h(s)$. Due to the MLRP it is decreasing in s. Therefore $h^{-1}(.)$ is well defined. Then

$$s^{**} = \begin{cases} 1 & h(1) \geq v_H \\ h^{-1}(v_H) & h(1) < v_H; h(s^* + \varepsilon) > v_H \text{ for some } \varepsilon. \square \\ s^* & h(s) < v_H \ \forall s \end{cases}$$

Proposition 2 *An equilibrium always exists.* $e_L = e_H = 1$ *is an equilibrium if and only if* $\bar{\theta} \geq v_H$.

Proof: See Appendix.

This implies that for $\bar{\theta} < v_H$ an equilibrium with $e_L > e_H$ exists.

For illustration, suppose that $U_i(w,e)$ was differentiable with respect to e. If, given $w(s)$,

$$\frac{\partial U_H}{\partial e} > \frac{\partial U_L}{\partial e}$$

for all e, then $e_H > e_L$ would have to hold in any equilibrium. This is the way the Mirrlees-Spence Single Crossing Property usually applies to signaling models.

The equilibrium we find means this condition must be violated. In our model

$$\frac{\partial U_L}{\partial e} = \frac{\partial U_H}{\partial e} - \frac{\partial F_e(s^*)}{\partial e}(v_H - v_L) + \frac{\partial F_e(s^{**})}{\partial e}(v_H - \theta_L).$$

For e small enough,

$$\left(-\frac{\partial F_e(s^*)}{\partial e}\right) > \left(-\frac{\partial F_e(s^{**})}{\partial e}\right)\frac{(v_H - \theta_L)}{(v_H - v_L)}.$$

Therefore, for e small enough,

$$\frac{\partial U_L}{\partial e} > \frac{\partial U_H}{\partial e}.$$

2.1 An Example: Advertising

Think of a bank trying to acquire new customers by sending out a brochure about their current extremely competitive credit offer. The informative part of the brochure has a table of numbers explaining the offer. Due to the many offers of such kind clients will only notice the offer, if the brochure is sufficiently appealing in a combination of aspects ($s > s^*$). Imagine one possibility to make the brochure appealing is to announce giving away mobile phones to some readers. The bank does not know for sure, whether the brochure is already appealing enough to catch readers attention without the announcement.

Quality of a bank θ is not determined by the credit offer only, but also by its service etc. Think of a high quality bank as one that can thrive even without such effort to acquire new clients, because word of mouth is another way for them to reach new customers (their outside option v_H). However, they would still like to increase the number of their clients faster.

A low quality ($\theta_H > \theta_L$) bank does not have word of mouth working for them ($v_H > v_L$). Their only option to thrive is to succeed in their marketing effort.

In this scenario it seems reasonable that low quality banks would add the mobile-phone-give-away to their brochure to make sure it is as appealing as possible: they want to make sure to receive attention (they expect to send a high signal s). Then low frequency borrowers will most likely take the time to read the good offer and take it, as they have no alternative nearly as good.

Naturally high frequency borrowers have established contact with another bank that may give them decent if not quite as competitive offers regularly. Thus high quality banks may distinguish themselves by not including a similar give-away on their brochure (they expect to send a lower signal). Even though the brochure with the give-away engages their attention, high frequency borrowers may rightfully interpret such a brochure as a signal of a low quality bank and borrow money from their alternative source instead.

Thus a high type bank may run the risk of sending out an unappealing brochure that does not attract customers, but if it manages to create an appealing brochure without a phone-give-away, it attracts low and high frequency borrowers (corresponding to a high wage in the setup of the model). A low type bank may choose to make sure they reach the low frequency borrowers with bigger certainty and pay the price of not acquiring high frequency borrowers (a lower wage in the setup of the model).

This story is intuitive. If an offer is too little about the product it does not seem serious, especially in banking. Yet the existence of all those offers in our mailboxes indicates that some people do accept them.

More importantly the above scenario has close resemblance to a feature of a rare field experiment conducted by Bertrand, Karlan, Mullainathan, Shafir and Zinman (2005) in South Africa. Without claiming that overeagerness explains their finding or that the description above does the complexity of their experiment justice, it is worth noting the parallels: They do find a negative impact of such a phone-give-away on the take-up rate among high frequency borrowers, while for low frequency borrowers they do not. So it does seem as if the give-away is interpreted as a signal of low quality among high frequency borrowers. They write that *"… when we break up the sample into borrowing categories, we see that this effect* [of the phone-give-away] *is very large and statistically significant among the more frequent borrowers. For this group of customers, introducing this promotional feature* […] *in fact reduces the likelihood of loan take-up. The nonnegative effect among the lower frequency borrowers may indicate that this negative choice effect of the promotional lottery may be offset in this case by an attention-getting-effect …"* This matches the behavior we suggested for the respective customers very well.

2.2 Type Dependent Cost of Effort

Until now cheap talk was possible in our setup and the results can be interpreted as boasting versus modesty. Clearly a more general model would allow for a type dependent cost of the effort required for sending a certain signal, for example preparation for an exam.

Signalling models typically assume the cost of sending a specific signal to be negatively correlated with productivity θ. And indeed, it seems reasonable to assume that high types would need less time for preparing a certain exam or in general send a certain signal. In our model, however, they also have a better outside option. Hence, the opportunity cost for spending a certain amount of time is higher for them than for low types. In our model both types draw their signal from the same distribution when exerting the same effort, so effort is measured as "output" in terms of signal send, not "input" in terms of time spend sending it. Hence opportunity cost of the time needed to exert a certain effort is the relevant cost. Therefore it is unclear which type incurs the higher cost of effort.[10]

If the cost of effort is higher for low types we expect the classical signalling effect to compete with overeagerness. In principle incorporating cost into the setup makes the model more applicable and testable. It allows one to predict, whether or not to expect overeagerness.

To incorporate cost of effort, assume that there is a function $c(e), c:[0,1]\rightarrow[0,\infty]$ with low type's cost $c_L(e)=c(e)$ and high type's cost $c_H(e)=ac(e)$ where $a\in(0,\infty)$. Let c be twice continuously differentiable, $\frac{\partial c(e)}{\partial e}>0$ and $\frac{\partial^2 c(e)}{\partial e^2}>0$ for all e and $\lim_{e\rightarrow 1} c(e)=\infty$.

Whenever $e_L > e_H$ in any equilibrium we shall say overeagerness dominates. For $e_L < e_H$ the classical signalling effect dominates. Whenever $e_L = e_H$, we say there is perfect pooling.

Theorem 2 *In the model specified above there is* $\overline{a}<1$ *sufficiently large such that for* $a>\overline{a}$ *overeagerness dominates. Furthermore if costs are significant enough in the sense that* $c'(0)>\underline{c}'$ *for an appropriate* $\underline{c}'$*, then there is* $\underline{a}>0$ *sufficiently small such that for* $a<\underline{a}$ *the classical signalling effect dominates.*

Perfect pooling occurs, if and only if $a=a^*:=\frac{\overline{\theta}-v_H}{\overline{\theta}-v_L}<1$ *and* $\overline{\theta}\geq v_H$.

Proof: See Appendix.

Theorem 2 immediately implies $\underline{a}\leq\overline{a}$ with strict inequality for $\overline{\theta}\geq v_H$.

3. Binomially Distributed Signals

As an instructive example that allows us to solve for equilibria, we consider a sender-receiver game as in section 2. Everything shall be as defined there, but we specify the uncertainty:

Applicants have a noisy, public and (for now) cost free signal available to them. It will be called a test result. For simplicity suppose the test consists of an infinite number of questions. Each can be answered right or wrong. After the test is completed, two questions are chosen at

[10] See footnote 5 to the Introduction for a more explicit example in the context of advertisment.

random for evaluation. So the test result $s \in \{0,1,2\}$ counts the correct answers to the two questions considered. For applicants of either type that do not have the required skill, none of the questions can ever be answered correctly. Both types of the skilled variant of applicants can choose the rate at which they answer question correctly (say by putting marginal effort into studying for the test). Let $e_L, e_H \in [0,1]$ be the efforts chose by the respective types. Test signals are then distributed binomially for skilled applicants:

$p_0 = (1-e)^2$	is the probability not to get any question right ($s=0$)
$p_1 = 2(1-e)e$	is the probability to get one of the questions right ($s=1$)
$p_2 = e^2$	is the probability to get both questions right ($s=2$).

For unskilled applicants the probability of $s=0$ is $p_0 = 1$.

The wage schedule can condition on the value of s. $w = \begin{pmatrix} w_0 & w_1 & w_2 \end{pmatrix}$ is the vector of conditional wage offers.

For $i \in \{L, H\}$ remember that θ_i was the productivity and v_i the outside option of type i.

Let there be sufficiently many unskilled applicants of low type such that $w_0 < v_L$ has to hold as in section 2. Expected utility now is

$$U_i(w,e) = (1-e)^2 v_i + 2e(1-e)\max\{w_1, v_i\} + e^2 \max\{w_2, v_i\}.$$

A strategy consists of an effort choice e. A pure strategy Nash equilibrium then consists of effort choices (e_L, e_H) and the resulting wages are $w = (w_0, w_1, w_2)$. Due to perfect competition on the firm's side:

$$w_1(e_H, e_L) = \frac{ne_H(1-e_H)\theta_H + e_L(1-e_L)\theta_L}{ne_H(1-e_H) + e_L(1-e_L)}, \quad w_2(e_H, e_L) = \frac{ne_H^2\theta_H + e_L^2\theta_L}{ne_H^2 + e_L^2}$$

are the wages justified by the average productivity of applicants with one or two correct answers ($s=1$ or $s=2$) respectively, if the offer is accepted by both types. The formulas make it clear, that equilibria could be only partially separating again: The signal might be informative about the type it was send from, but not sufficient to determine the type for sure.[11]

In the case types of productivity θ_i accept neither w_1 nor w_2, we assume $e_i = 0$ for tie-breaking.[12] In case they accept only one wage offer w_s and are indifferent to declining it we assume that they maximize the probability of corresponding outcome s. So only equilibria in which all players of one type act the same are possible: In case of indifference between strategies their behavior is unambiguous. Therefore mixed strategy equilibria are ruled out: Mixing between $e_i = 0$ and $e_i = 1$ is precluded. Our assumptions also rule out mixing between accepting and declining. Then zero profit wages are uniquely determined by the ratio of accepting high types over accepting low types. Consequently optimal efforts e_H and e_L are also uniquely determined by utility maximization, as the expected utility is concave in the individual effort choice. Hence no type can be mixing over effort choices $e_i \in (0,1)$, $i \in \{L, H\}$. So we can concentrate on finding equilibria in pure strategies.

For this specific setup Theorem 1 implies

Theorem 1' *In any pure strategy Nash equilibrium of the model described above* $1 \geq e_L > e_H \geq 0$ *or* $e_H = e_L = 1$.

The intuition for this result was described in section 2.1.

The case $e_H = e_L = 1$ is equivalent to the equilibrium we would get for a binary signal $s \in \{0,1\}$ where skilled individuals among both types are not distinguishable. Then high types will work for $\bar{\theta} := \dfrac{n\theta_H + \theta_L}{n+1} \geq v_H$. This will be called "benchmark case" in the discussion below.

Since we gave up the assumption of full support for the distribution of this discrete signal (in $e \in \{0,1\}$ we have $s = 0$ or $s = 1$ respectively) we convince ourselves in the appendix, that the proof runs through as for Theorem 1. Proposition 1 concerning the wage schedule has its analog in

[11] Out of equilibrium believes on the firm's side can now be relevant only for boundary cases with $e_H, e_L \in \{0,1\}$, where signals do not have full support. We will specify these believes in the proof of the fact below, which deals with these cases.
[12] A very small cost of effort would make this the only equilibrium choice.

Proposition 1' *In every equilibrium, where* $0 < e_H < 1$, $w_1 > w_2$ *has to hold.*

Proof: In such equilibrium we have $e_L > e_H$ according to the theorem and high types accept at least one of the offers. Then zero profit dictates $w_1 > w_2$. □

In section 2 we showed for a very general signal distribution that in any interior equilibrium low types exert higher effort than high types. For the specific binomial signal distribution we can go further and ask for which values of $\theta_H, \theta_L, v_H, v_L$ the possible boundary and interior equilibria exist.

Normalize $\theta_L \equiv 1$, $v_L \equiv 0$.[13] Clearly there can be multiple equilibria for most parameter combinations, if we consider boundary cases:

Fact *There are two possible boundary equilibria: The benchmark equilibrium* $e_H = e_L = 1$ *exists for* $v_H \leq \overline{\theta}$ *and* $e_H = 0, e_L = 1$ *exists for* $v_H > \theta_L$.

Proof: See Appendix.

Consider briefly how robust the different equilibria are under small symmetric (with respect to type and direction) trembles (small uncertainties in the effort actually chosen):

- For $v_H \leq \overline{\theta}$ the tremble will eliminate the equilibrium with $e_L = 1, e_H = 0$, because now the wage w_1 becomes determined at $w_1 = \overline{\theta}$ and high types will react with choosing $e_H > 0$.
- The benchmark case survives for $v_H \leq \overline{\theta}$, as $w_1 = w_2 = \overline{\theta}$.
- Interior equilibria survive, because applicants maximize expected utility. In the limit "small" symmetric trembles do not change the expected utility for the respective types.

Therefore we are mostly interested in interior equilibria. They are partially separating. The questions to answer are: How can those equilibria be categorized? Which parameter combinations allow the different types of equilibria? Are equilibria unique (apart from boundary equilibria)? Can we explicitly calculate those equilibria for specific parameters?

[13] Formally we have only one degree of freedom left for normalization, as we already set the productivity of unskilled types to zero. Note well, however, that setting it to any negative value would not change any result. So the suggested normalization is valid.

The answer to the first question is obvious: Low types always work for $w_1, w_2 \geq 0$. For $e_H \neq 1$ we established that $w_1 > w_2$. So high types either do not work at all or they accept to work only when offered w_1 or they are willing to work for both, w_1 and w_2. By assumption $\theta_H > \theta_L = 1$, thus consider $(v_H, \theta_H) \in [0,\infty) \times (1,\infty)$. For ease of notation also fix the ratio of high over low types to be $n = 1$; other values give analogous results. Call equilibria where high types never work "No participation" (N), those where they accept w_1 and w_2 "Full participation" (F) and the ones where they accept only w_1 call "Selective participation" (S).

Note that we can rule out fully separating equilibria, where the signals and the acceptance of a certain offer are a sufficient statistic for the type of the applicants and in which at least some high types accept employment. If they existed, high types would have to decline offer w_2. Hence $e_H = 1/2$, $w_2 = \theta_L$. Also low types would have to choose $e_L = 1$ in equilibrium, requiring $\left.\frac{\partial U_L}{\partial e_L}\right|_{e_L=1} \geq 0$ where $U_L(e_L) = e_L^2 \theta_L + 2e_L(1-e_L)\theta_H$ is the utility of an individual applicant of low type. So $\theta_L - \theta_H \geq 0$ would be necessary, which is precluded by assumption.

Proposition 2' *For n=1 and $\theta_H > 1$ a partially separating equilibrium exists for $v_H < \bar{v}(\theta_H)$. In specific, with parameter dependent boundaries as given in the proof, the possible equilibria are:*

"No participation" (N)

High types never work and $e_L = 1, e_H = 0, w_1 \leq \theta_L, w_2 = \theta_L = 1$ is the unique equilibrium for $v_H \in (\bar{v}(\theta_H), \infty)$. It exists for all $v_H > \theta_L = 1$.

"Selective Participation" (S)

High types accept only w_1 and $e_L = e_L(\theta_H), e_H = 1/2, w_1 = w_1(e_L, \theta_H), w_2 = \theta_L = 1$ exists as an equilibrium for $v_H \in (\underline{v}(\theta_H), \bar{v}(\theta_H)]$.

"Full Participation" (F)

High types accept both, w_1 and w_2 with $1 > e_L > e_H \geq 1/2$ and $w_1 > w_2 > \theta_L = 1$ exists for $v_H \in (0, \underline{v}(\theta_H)]$ and possibly also for $v_H \in (\underline{v}(\theta_H), \bar{\theta}]$.

The benchmark equilibrium exists for $v_H \in (0, \bar{\theta}]$.

Proof: See Appendix.

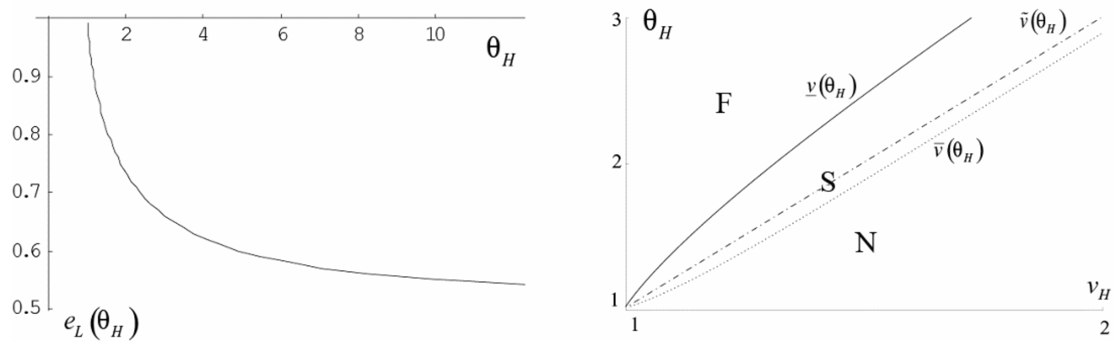

Figure: a) $e_L(\theta_H)$ for S-equilibria; b) Types of equilibria.

Figure a) visualizes this result. *Figure b)* also shows the boundary $\tilde{v}(\theta_H) = \overline{\theta}$. The benchmark equilibrium exists for $v_H \leq \tilde{v}(\theta_H)$. Recall that in this equilibrium all low and all high types with the required skill work and receive the same wage. It is equivalent to the only equilibrium with working high types for a binary signal. For parameters $\tilde{v}(\theta_H) < v_H \leq \overline{v}(\theta_H)$ high types would never work with only a binary signal, while in the S-equilibrium half the high types will work for w_1. This is a clear Pareto improvement since low types also gain on average. F-equilibria can not be determined analytically.[14]

It remains to introduce a specific type-dependent cost to complete this example. Let $c(e) = -k\ln(1-e)$ and as before $c_L(e) = c(e)$ and $c_H(e) = ac(e)$. Then

$$\frac{\partial c}{\partial e} = \frac{k}{1-e} > 0; \quad \frac{\partial^2 c}{\partial e^2} = \frac{k}{(1-e)^2} > 0; \quad \lim_{e \to 1} c = \infty$$

as required. Hence the model predicts, that classical signalling dominates for $a < \underline{a}$ where $\underline{a} = 1 - \frac{2}{k}(v_H - v_L)$, as established in Theorem 2 above. Note that $\underline{a} > 0$ for $k > 2(v_H - v_L)$.

[14] For example in $\theta_H = 3, v_H = 1.3$ a numerically calculated equilibrium is $e_H \approx 0.683, e_L \approx 0.838, w_1 \approx 2.229, w_2 \approx 1.799$.

4. Conclusion

The model presented here shows that high types may choose to send lower signals than low types in a framework very close to classical signaling models. Our predictions are driven by a correlation between an individual's outside options or opportunity costs and its productivity for a firm and by the assumption that signals are noisy. Firms correctly interpret very high signals as overeagerness of low types, who are desperate not to end up with their bad outside option. Only high types can afford the risk of a very low signal for the benefit of not being seen as overeager, because if they do end up with a very low signal they have to execute their outside option, which is more attractive.

Outcome and assumptions capture the common perception of overeagerness in different situations, so they certainly have intuitive appeal. However, the predictions, like those of Spence's original model, will be hard to test, mainly because the effect is difficult to isolate, because there are few completely unproductive signals etc. In principle including type dependent cost of effort allows to predict when overeagerness should occur, adding testability.

Consider the policy implications of the model for the context of workers applying for employment. For example if being educated is actually productive, then we may want to prevent high types from choosing less education or from performing below their potential. So we may want to prevent the kind of effect predicted by the model. This is a possible argument in favor of formal degree or grade requirements to entry into a certain profession: A prohibition of rewarding low signals as a sign of good outside options.

From a social welfare point of view, high types taking the risk of executing their outside option have negative externalities for low types, who are paid less on average. If high types gain little from their behavior, this may be an argument for a society to prevent behavior as in the model, even if the signal is nonproductive. Cleary, if high types are more productive employed than self-employed ($\theta_H > v_H$), society as a whole gains from high types always accepting employment. So if high types are willing to work for the wage justified by average productivity ($\overline{\theta} \geq v_H$), then redistribution would allow for a Pareto improvement over the overeagerness outcome, if high types always worked. This might be another argument for formal degree or grade requirements or even for uniform wages.

On the other hand, in the example of binomially distributed signals, rationally rewarding low signals may allow signal specific wage offers to induce a proportion of capable individuals to work, who would decline employment at a uniform wage ($\bar{\theta} < v_H$). In this case giving up such formal requirements can lead to a Pareto improvement, as high types benefit and low types are paid more on average, so they also benefit.

5. Appendix

Proof of Proposition 2: Define $X = \left\{ \left(e_L, e_H \right) \middle| e_L \geq e_H \right\}$ and consider the correspondence $k : X \to [0,1] \times [0,1]$ with $k\left(e_L, e_H\right) = \left(e_L^*\left(e_L, e_H\right), e_H^*\left(e_L, e_H\right)\right)$, where e_L^* and e_H^* are the optimal effort choices for individual low and high types given that everybody else chooses e_L or e_H according to their type. Note that e_L and e_H determine $s^{**}\left(e_L, e_H\right)$ and that the wage schedule $w_{e_L, e_H}\left(s\right)$ dictated by e_L and e_H is decreasing on $\left[s^*, s^{**}\right]$.

First show that $k : X \to [0,1] \times [0,1]$ is upper hemicontinuous. To see this recall that $f_e\left(s\right)$ is continuous in e for all s. Hence $s^{**}\left(e_L, e_H\right)$ and $w_{e_L, e_H}\left(s\right)$ are continuous in e_L and e_H for all $s < s^{**}\left(e_L, e_H\right)$. Therefore

$$
\begin{aligned}
U_i\left(w_{e_L, e_H}, e\right) &= F_e\left(s^*\right) v_i + \int_{s^*}^{1} \max\left(w_{e_L, e_H}\left(s\right), v_i\right) f_e\left(s\right) ds \\
&= F_e\left(s^*\right) v_i + \int_{s^*}^{s^{**}\left(e_L, e_H\right)} w_{e_L, e_H}\left(s\right) f_e\left(s\right) ds + \left(1 - F_e\left(s^{**}\right)\right) \max\left(\theta_L, v_i\right)
\end{aligned}
$$

is continuous in e, e_L and e_H for $i \in \left\{L, H\right\}$. Consequently $e_i^*\left(e_L, e_H\right) = \arg\max_{e \in [0,1]} U_i\left(w, e\right)\Big|_{e_L, e_H}$ is uhc according to the Theorem of the Maximum.

Next we need to show that $e_L^* \geq e_H^*$, which implies $k : X \to X$. Then k has a fixed point in X by Kakutani's fixed point theorem, which establishes the claim.

Assume to the contrary, that one of the following holds:

Case 1) $e_L > e_H$ and $e_H^* > e_L^*$. Then $s^{**} > s^*$ has to hold, because otherwise $e_H^* = 0$. Define $g_e\left(s\right) := \dfrac{f_e\left(s\right)}{F_e\left(s^{**}\right) - F_e\left(s^*\right)}$. Note that MLRP for f_e on $[0,1]$ implies MLRP for g_e on $\left[s^*, s^{**}\right]$. Then the following statements are equivalent consequences of high types behaving optimally:

$$
U_H\left(e_H^*\right) \geq U_H\left(e_L^*\right) \Leftrightarrow \int_{s^*}^{s^{**}} \left(w\left(s\right) - v_H\right) f_{e_H^*}\left(s\right) ds \geq \int_{s^*}^{s^{**}} \left(w\left(s\right) - v_H\right) f_{e_L^*}\left(s\right) ds \Leftrightarrow
$$

$$
(*) \qquad \left(F_{e_H^*}\left(s^{**}\right) - F_{e_H^*}\left(s^*\right)\right) \int_{s^*}^{s^{**}} \left(w\left(s\right) - v_H\right) g_{e_H^*}\left(s\right) ds \geq \left(F_{e_L^*}\left(s^{**}\right) - F_{e_L^*}\left(s^*\right)\right) \int_{s^*}^{s^{**}} \left(w\left(s\right) - v_H\right) g_{e_L^*}\left(s\right) ds
$$

Because $w(s)$ is decreasing and because g_e satisfies MLRP it must be that $\int_{s^*}^{s^{**}} (w(s)-v_H) g_{e_H^*}(s) ds < \int_{s^*}^{s^{**}} (w(s)-v_H) g_{e_L^*}(s) ds$ and then from (*) we conclude, that $\left(F_{e_H^*}(s^{**})-F_{e_H^*}(s^*)\right) > \left(F_{e_L^*}(s^{**})-F_{e_L^*}(s^*)\right)$. Further note that $1-F_{e_H^*}(s^{**}) > 1-F_{e_L^*}(s^{**})$ due to MLRP. Remember $v_H > v_L$ and $\theta_L > v_L$.

With all this in mind consider low type's utility from playing e_H^* and e_L^* respectively:

$$
\begin{aligned}
U_L(e_H^*)-v_L &= \int_{s^*}^{s^{**}} (w(s)-v_H) f_{e_H^*}(s) ds + \left(F_{e_H^*}(s^{**})-F_{e_H^*}(s^*)\right)(v_H-v_L) + \left(1-F_{e_H^*}(s^{**})\right)(\theta_L-v_L) \\
&> \int_{s^*}^{s^{**}} (w(s)-v_H) f_{e_L^*}(s) ds + \left(F_{e_L^*}(s^{**})-F_{e_L^*}(s^*)\right)(v_H-v_L) + \left(1-F_{e_L^*}(s^{**})\right)(\theta_L-v_L) = U_L(e_L^*)-v_L
\end{aligned}
$$

Since e_H^* gives strictly higher utility to low types than e_L^*, (e_H^*, e_L^*) can not be optimal responses. This contradicts the assumption.

Case 2) $e_L = e_H$ and $e_H^* > e_L^*$. This implies $w(s)$ is constant for all $s > s^*$. If $w(s) < v_H$, high types will never accept the offer and choose $e_H = 0$. But then $w(s) = \theta_L > v_L$ and low types choose $e_L = 1$, hence $e_L^* > e_H^*$. Else $w(s) \geq v_H$ and high types accept all wage offers for $s > s^*$. So $w(s) = \overline{\theta} \geq v_H$ will have both types choose $e_L = e_H = 1$. This is an equilibrium if and only if $\overline{\theta} \geq v_H$.

From Cases 1 and 2 we conclude that indeed $k: X \to X$ and by Kakutani's fixed point theorem, an equilibrium with $e_L^* \geq e_H^*$ always exists. The argument under Case 2 establishes that an equilibrium with $e_L^* = e_H^*$ exists if and only if $\overline{\theta} \geq v_H$. □

Proof of Theorem 2: Define $h(s)$ as the wage offer justified if both types accept at signal $s > s^*$:

$$
h(s) := \frac{n f_{e_H}(s)\theta_H + f_{e_L}(s)\theta_L}{n f_{e_H}(s) + f_{e_L}(s)}.
$$

Claim: Perfect pooling occurs, if and only if $a = a^* := \frac{\overline{\theta}-v_H}{\overline{\theta}-v_L} < 1$ and $\overline{\theta} \geq v_H$.

Proof: Consider two cases:

Case 1) $\overline{\theta} \geq v_H$. For perfect pooling $e_H = e_L = e^*$ for some e^* and consequently $w(s) = h(s) = \overline{\theta} \geq v_H$ for $s > s^*$. So high types always accept employment for $s > s^*$. This is an equilibrium if and only if

$$
\begin{aligned}
e_L &= e^* = \arg\max_e \left(F_e(s^*) v_L + \left(1-F_e(s^*)\right)\overline{\theta} - c(e)\right) \\
e_H &= e^* = \arg\max_e \left(F_e(s^*) v_H + \left(1-F_e(s^*)\right)\overline{\theta} - ac(e)\right)
\end{aligned}.
$$

First order conditions are

$$\left.\frac{\partial F_e\left(s^*\right)}{\partial e}\right|_{e=e^*}\left(v_L-\bar{\theta}\right)=c'\left(e^*\right)$$
$$\left.\frac{\partial F_e\left(s^*\right)}{\partial e}\right|_{e=e^*}\left(v_H-\bar{\theta}\right)=ac'\left(e^*\right)$$

determining $a^*=\dfrac{\bar{\theta}-v_H}{\bar{\theta}-v_L}<1$ as we claimed.

Case 2) $\bar{\theta}<v_H$. Then given $e_H=e_L=e^*$ high types will never accept employment. Hence $e_H=0$. But $e_L>0$ always holds. So no perfect pooling is possible. $\|$

Claim: There is $\bar{a}<1$ sufficiently large, such that for $a>\bar{a}$ overeagerness dominates.

Proof: Assume to the contrary, that for all a there is an equilibrium with $e_H\geq e_L$. In such equilibrium the wage schedule is monotonic. As can be shown analogous to the proof of Proposition 1 there is s^{**} such that

$$w(s)=\begin{cases}0 & s\leq s^*\\ \theta_L & s^*<s\leq s^{**}\\ h(s) & s>s^{**}\end{cases}.$$

Define $Y:=\left\{\left(e_L,e_H\right)\middle| e_L\leq e_H\right\}$, the uhc correspondence $\tilde{k}:Y\rightarrow[0,1]\times[0,1]$ analogous to $k:X\rightarrow[0,1]\times[0,1]$ in the proof of Proposition 2 and $\left(e_L^*,e_H^*\right):=\tilde{k}\left(e_L,e_H\right)$. We are searching for $\bar{a}$ such that for all $a>\bar{a}$ and for all $\left(e_L,e_H\right)\in Y$ we have $e_L^*>e_H^*$. This would rule out equilibria where $e_H\geq e_L$.

In equilibrium individual low and high types maximize respectively:

$$U_H(e)=F_e\left(s^{**}\right)v_H+\left(1-F_e\left(s^{**}\right)\right)\int_{s^{**}}^{1}\left(w(s)-v_H\right)g_e(s)\,ds-ac(e)$$
$$U_L(e)=U_H(e)+\left\{\left(1-F_e\left(s^*\right)\right)\left(v_H-v_L\right)+\left(F_e\left(s^{**}\right)-F_e\left(s^*\right)\right)\left(\theta_L-v_L\right)-(1-a)c(e)\right\}$$

Now we will find bounds on e_H^* and s^{**}:

- Low types will at least get θ_L for $s>s^*$. Therefore $e_L^*\geq\underline{e}_L:=\arg\max_e\left\{\left(1-F_e\left(s^*\right)\right)\left(\theta_L-v_L\right)-c(e)\right\}>0$. We set out under the assumption $e_H^*>e_L^*$.
- High types will never get more than θ_H for $s>s^*$. Therefore $e_H^*\leq\bar{e}_H$ where $\theta_H\left(1-F_{\bar{e}_H}\left(s^*\right)\right)=ac\left(\bar{e}_H\right)$.
- High types need to gain at least the cost of their lowest effort. Therefore $\left(1-F_e\left(s^{**}\right)\right)\left(\theta_H-v_H\right)\geq ac\left(\underline{e}_H\right)>ac\left(\underline{e}_L\right)$. This implies $s^{**}\leq\tilde{s}$ where $\left(1-F_{\bar{e}_H}\left(\tilde{s}\right)\right)\left(\theta_H-v_H\right)=ac\left(\underline{e}_L\right)$.

So we have bounded $e_H^* \in [\underline{e}_L, \overline{e}_H]$ and $s^{**} \in [s^*, \tilde{s}]$. Further $f_e(s)$ has full support for all e, so MLRP implies $\frac{\partial F_e(s)}{\partial e} < 0$ for all $s \in (0,1)$.

Now consider

$$\frac{\partial(U_L(e) - U_H(e))}{\partial e} = -\frac{\partial F_e(s^{**})}{\partial e}(v_H - \theta_L) - \frac{\partial F_e(s^*)}{\partial e}(\theta_L - v_L) - (1-a)c'(e)$$

With the definitions $\overline{c}' := \max_{e \in [\underline{e}_L, \overline{e}_H]} c'(e) = c'(\overline{e}_H)$ and $\phi(s) := \min_{e \in [\underline{e}_L, \overline{e}_H]} \left(-\frac{\partial F_e(s)}{\partial e} \right) > 0$ and $\underline{\phi} := \min_{s \in [s^*, \tilde{s}]} \phi(s) > 0$ we find

$$\frac{\partial(U_L(e) - U_H(e))}{\partial e} \geq \underline{\phi}(v_H - \theta_L) + \phi(s^*)(\theta_L - v_L) - (1-a)\overline{c}' \geq \underline{\phi}(v_H - v_L) - (1-a)\overline{c}'$$

for all $e \in [\underline{e}_L, \overline{e}_H]$. Therefore $\frac{\partial(U_L(e) - U_H(e))}{\partial e} > 0$ for all $e \in [\underline{e}_L, \overline{e}_H]$ for $a > \overline{a} := 1 - \frac{\underline{\phi}(v_H - v_L)}{\overline{c}'}$. Clearly $\overline{a} < 1$.

We also know that $U_H(e)$ is maximized in e_H^*. Therefore $U_L(e_H^*) \geq U_L(e)$ for all $e \leq e_H^*$ and $\left.\frac{\partial(U_L(e) - U_H(e))}{\partial e}\right|_{e=e_H^*} > 0$, which implies $e_L^* > e_H^*$. This contradicts the initial assumption.

Hence for all $a > \overline{a}$ there is no equilibrium with $e_H^* \geq e_L^*$. Overeagerness dominates. ||

Claim: If costs are significant enough in the sense that $c'(0) > \underline{c}'$ for an appropriate $\underline{c}'$, then there is $\underline{a} < \overline{a}$ sufficiently small such that for $a < \underline{a}$ the classical signalling effect dominates.

Proof: Define

$$\underline{c}' := \max_{e \in [0,1], s \in [s^*,1]} \left\{ -\frac{\partial F_e}{\partial e}(s^*)(v_H - v_L) + \frac{\partial F_e}{\partial e}(s)(v_H - \theta_L) \right\} = \max_{e \in [0,1]} \left\{ -\frac{\partial F_e}{\partial e}(s^*)(v_H - v_L) \right\}.$$

Assume contrary to the claim, that for all a there is an equilibrium with $e_H \leq e_L$. In such equilibrium the wage schedule is as in Proposition 1: There is s^{**} such that

$$w(s) = \begin{cases} 0 & s \leq s^* \\ h(s) & s^* < s \leq s^{**} \\ \theta_L & s > s^{**} \end{cases}.$$

Define X and the uhc correspondence $k : X \to [0,1] \times [0,1]$ as in the proof of Proposition 2. We are searching for $\underline{a}$ such that for all $a < \underline{a}$ and $c'(0) > \underline{c}'$ we have $k : X \to \{(e_L, e_H) | e_L < e_H\}$. This would rule out equilibria where $e_H \leq e_L$.

In equilibrium individual low and high types maximize respectively:

$$U_L(e) = \left(F_e(s^{**}) - F_e(s^*)\right)\int_{s^*}^{s^{**}} \left(w(s) - v_L\right) g_e(s)\,ds + \left(1 - F_e(s^{**})\right)\left(\theta_L - v_L\right) - c(e)$$

$$U_H(e) = U_L(e) + \left\{\left(1 - F_e(s^{**})\right)\left(v_H - \theta_L\right) + F_e(s^*)\left(v_H - v_L\right) + (1-a)c(e)\right\}$$

Consider

$$\frac{\partial\left(U_H(e) - U_L(e)\right)}{\partial e} = -\frac{\partial F_e(s^{**})}{\partial e}\left(v_H - \theta_L\right) - \frac{\partial F_e(s^*)}{\partial e}\left(v_H - v_L\right) + (1-a)c'(e) > -\underline{c}' + (1-a)c'(e).$$

So $\frac{\partial\left(U_H(e) - U_L(e)\right)}{\partial e} > 0$ for all $e \in \left[\underline{e}_L, \overline{e}_H\right]$ for $a < \underline{a} := 1 - \frac{\underline{c}'}{c'(0)} > 0$.

We also know that $U_L(e)$ is maximized in e_L^*. Therefore $U_H(e_L) \geq U_H(e)$ for all $e \leq e_L^*$ and $\left.\frac{\partial\left(U_H(e) - U_L(e)\right)}{\partial e}\right|_{e=e_L^*} > 0$, which implies $e_H^* > e_L^*$. This contradicts the initial assumption. ||

Hence for all $a < \underline{a}$ there is no equilibrium with $e_H^* \leq e_L^*$. Classic signaling dominates. □

Proof of Theorem 1' Consider two cases:

Case 1) $e_H > e_L$. High types have to accept at least w_1 or w_2, else we had $e_H = 0$. But then zero profit implies, that $w_2 > w_1$ has to be offered by the firms and w_2 is surely accepted by both types. This implies low types maximize expected utility by choosing $e_L = 1$. Since effort is bounded above by 1 we have $e_H \leq e_L$, which is a contradiction to the assumption.

Case 2) $e_H = e_L$. We know $e_L \neq 0$, thus high types have to accept at least one offer to justify $e_H \neq 0$. Then $w_1 = w_2$ has to be offered for $e_L \neq 1$ because of perfect competition among the firms. Hence low types choose $e_L = 1$. To satisfy the assumption we need $e_H = 1$. So $e_H = e_L = 1$ and $w_2 = \frac{n\theta_H + \theta_L}{n+1}$ is an equilibrium for $w_2 \geq v_H$. Out of equilibrium believes of the firms have to be such that $w_1 \leq w_2$ is offered. □

Proof of Fact There are four potential kind of equilibria where either $e_L \in \{0,1\}$ or $e_H \in \{0,1\}$. With $z \in [0,1]$:

i) $e_L = z, e_H = 1$. This is the first equilibrium in the proposition for $z = 1$ and $v_H \leq \overline{\theta}$: The wage w_1 is only relevant outside the equilibrium, since nobody ever ends up with $s = 1$. It can have any value $w_1 \leq w_2 = \overline{\theta}$ to support this equilibrium. So firms can believe that types are "equally likely" to send the out of equilibrium signal $s = 1$. If they thought it was more likely to come from a high type, both types would deviate from $e = 1$. For $v_H > \overline{\theta}$ high types will deviate. For $z \neq 1$ low types will deviate by choosing $e_L = 1$.

ii) $e_L = z, e_H = 0$ is the other boundary equilibrium in the proposition for $z = 1$ and $v_H > \theta_L = 1$: $w_2 = \theta_L = 1$ will not induce high types to deviate and w_1 is not determined, since nobody ever ends up with $s = 1$. It can have any value $w_1 \leq w_2 = 1$ to support this equilibrium. So here firms have to believe $s = 1$ to come from a low type out of equilibrium. For $v_H < \theta_L = 1$ high types will deviate. For $z \neq 1$ low types deviate by choosing $e_L = 1$. For $v_H > \theta_H$ this equilibrium exists for any out of equilibrium believes.

iii) $e_L = 0, e_H = z$ has both types deviate by choosing $e = 1$.

iv) $e_L = 1, e_H = z$ can be the equilibrium described in i for $z = 1$ and the one in ii for $z = 0$. For $z \in \left]0, 1/2\right[$ high types will deviate to $e_H = 0$ if $w_1 < v_H$ and to $e_H \geq 1/2$ for $w_1 \geq v_H$. $z \geq 1/2$ implies $w_1 > w_2$, which makes low types deviate to benefit from w_1, since the first order condition from their utility function is $e_L = \dfrac{w_1}{2w_1 - w_2}$. □

Proof of Proposition 2'

S) For S-equilibria high types are to accept w_1 and not w_2, thus $w_1 > w_2 = \theta_L$ has to hold. Clearly $\theta_L < v_H$ and $\theta_H > v_H$ are necessary and $e_H = 1/2$ will result. To find all $v_H(\theta_H)$ that allow for S-equilibria, express θ_H and v_H as functions of e_L: An S-equilibrium exists, if and only if the wage offers satisfy

$$(1) \qquad w_1 = \frac{2\left(\frac{1}{4}\theta_H + e_L(1-e_L)\right)}{2\left(\frac{1}{4} + e_L(1-e_L)\right)} \geq v_H$$

to make high types accept w_1 and

$$(2) \qquad w_2 = \frac{\frac{1}{4}\theta_H + e_L^2}{\frac{1}{4} + e_L^2} < v_H$$

to prevent high types from accepting w_2, because otherwise a firm could offer this wage and all high types would accept. Equivalently:

$$(1') \qquad \tfrac{1}{4}(\theta_H - v_H) \overset{!}{\geq} e_L(1-e_L)(v_H - 1)$$

$$(2') \qquad \tfrac{1}{4}(\theta_H - v_H) \overset{!}{<} e_L^2(v_H - 1).$$

For $v_H > \bar{\theta} = \dfrac{\theta_H + 1}{2}$ the benchmark equilibrium exists. Then $(2')$ holds trivially and for $e_L \geq e_L^{(1)}$ with

$$e_L^{(1)} = \frac{1}{2}\left(1 + \sqrt{1 - \frac{\theta_H - v_H}{v_H - 1}}\right)$$

the constraint $(1')$ holds, too. Similarly $(1')$ holds trivially for $v_H \leq \bar{\theta}$ and for $e_L > e_L^{(2)}$ with

$$e_L^{(2)} = \frac{1}{2}\sqrt{\frac{\theta_H - v_H}{v_H - 1}}$$

$(2')$ holds as well. The expected utility of low types is $U_L = 2e_L(1-e_L)w_1 + e_L^2$ where

$$w_1(e_L, \theta_H) = \frac{\theta_H/4 + e_L(1-e_L)}{\frac{1}{4} + e_L(1-e_L)}$$

as used above and consequently low types choose e_L according to the first order condition

$$\frac{\partial U_L}{\partial e_L} = (2-4e_L)w_1 + 2e_L = (2-4e_L)\frac{\theta_H/4 + e_L(1-e_L)}{1/4 + e_L(1-e_L)} + 2e_L \overset{!}{=} 0 .$$

From this condition we uniquely determine θ_H as a function of e_L :

$$\theta_H(e_L) = \frac{4e_L^3 - 8e_L^2 + 5e_L}{2e_L - 1} .$$

Now the definitions of $e_L^{(1)}$ and $e_L^{(2)}$ yield an upper limit $\overline{v}(e_L)$ and a lower limit $\underline{v}(e_L)$ for values of v_H respectively, between which an S-equilibrium exists. The expressions are

$$\overline{v}(e_L) = \frac{1+\theta_H(e_L) - (2e_L - 1)^2}{2-(2e_L-1)^2},\ \underline{v}(e_L) = \frac{\theta_H(e_L) - 4e_L^{\ 2}}{1-4e_L^{\ 2}} .$$

$\theta_H(e_L)$ is continuous and monotonous on $e_L \in \left]1/2, 1\right]$ with values $\theta_H \in [1,\infty[$, so the inverse $e_L(\theta_H)$ exists. Hence for any $\theta_H \in [1,\infty[$ and $v_H \in \left]\underline{v}(\theta_H), \overline{v}(\theta_H)\right]$ one interior equilibrium has high types accept only w_1 and choose $e_H = 1/2$ and low types choose $e_L(\theta_H)$. The resulting wages are $w_1(e_L, \theta_H)$ and $w_2 = \theta_L = 1$ as zero profit dictates.

N) If $v_H > \overline{v}(e_L)$, then high types are not willing to work for $w_1(e_L,\theta_H)$ as specified above. Because $e_H = e_L = 1$ is not an equilibrium for $\overline{\theta} < v_H$, the unique equilibrium implied by the assumptions is $e_L = 1, w_2 = \theta_L = 1, w_1 \le \theta_L, e_H = 0$. We called it N-equilibrium.

F) If $v_H \le \underline{v}_H$, then high types would deviate from an S-equilibrium by accepting the w_2 that would be justified if all high types accepted it.

Does a F-equilibrium exist for $v_H \le \underline{v}(\theta_H)$? If it does it involves $e_L > e_H \ge 1/2$. Note that $\underline{v}(\theta_H) < \overline{\theta}$ for $e_L > 1/2$. This guarantees $w_1 > v_H$. Define the optimal effort choices of an individual in reaction to everybody else playing according to e_L, e_H as $e_L^*(e_L, e_H)$ and $e_H^*(e_L, e_H)$ for low and high types respectively. Further define

$$\hat{e}_L(e_H) = \left\{e_L \middle| \left(e_L = e_L^*(e_L, e_H)\right)\right\}$$
$$\hat{e}_H(e_L) = \left\{e_H \middle| \left(e_H = e_H^*(e_L, e_H)\right)\right\}.$$

Clearly in equilibrium $e_L \in \hat{e}_L(e_H)$ and $e_H \in \hat{e}_H(e_L)$ have to hold. With these assumptions and definitions the following lemmata hold.

Lemma 1 *For $v_H \le \underline{v}(\theta_H)$ the set $\hat{e}_L(e_H)$ is a function (it contains one element) that is increasing in e_H .*

Proof: The first order condition corresponding to low type's utility is

$$e_L^*(e_L, e_H) = \frac{w_1(e_L, e_H)}{2w_1(e_L, e_H) - w_2(e_L, e_H)} .$$

Note that, slightly abusing notation,

$$w_2\left(e_L, e_H\right) = I_{w_2 \geq v_H} \frac{e_H^2 \theta_H + e_L^2}{e_H^2 + e_L^2} + I_{w_2 < v_H} = \frac{e_H^2 \theta_H + e_L^2}{e_H^2 + e_L^2}$$

as the wage w_2 depends on whether high types accept it or not, which they do due to $v_H \leq \underline{v}\left(\theta_H\right)$. Also $w_1 > v_H$ is guaranteed by the assumptions. Since for $e_L > e_H \geq 1/2$ we have $w_1 > w_2$, e_L^* has a solution in $\left(1/2, 1\right]$. It is continuous in e_L, because w_1 and w_2 are. With $\frac{\partial w_1}{\partial e_L} > 0,\ \frac{\partial w_2}{\partial e_L} < 0$, we conclude

$$\frac{\partial e_L^*}{\partial e_L} = \frac{w_1 \partial w_2 / \partial e_L - w_2 \partial w_1 / \partial e_L}{\left(2w_1 - w_2\right)^2} < 0.$$

Hence $\left|\hat{e}_L\left(e_H\right)\right| = 1$ (since obviously $identitiy\left(e_L\right)$ is continuous, strictly increasing and takes all values in $\left[1/2, 1\right]$).

From $\frac{\partial w_1}{\partial e_H} < 0,\ \frac{\partial w_2}{\partial e_H} > 0$ conclude

$$\frac{\partial e_L^*}{\partial e_H} = \frac{w_1 \partial w_2 / \partial e_H - w_2 \partial w_1 / \partial e_H}{\left(2w_1 - w_2\right)^2} > 0,$$

establishing that $\hat{e}_L\left(e_H\right)$ must indeed increase in e_H. ||

Lemma 2

i) In the above situation $f\left(e_L, e_H\right) := \left(e_L^*\left(e_L, e_H\right), e_H^*\left(e_L, e_H\right)\right)$ *is continuous for* $v_H \leq \underline{v}\left(\theta_H\right)$.

ii) For the set $X_\varepsilon \in \mathbb{R}^2$

$$X_\varepsilon := \left[\frac{1}{2}, \frac{\theta_H}{\theta_H + \varepsilon}\right] \times \left[\frac{1}{2}, 1\right] \cap \left\{\left(e_L, e_H\right) \middle| w_1\left(e_L, e_H\right) \geq w_2\left(e_L, e_H\right) + \varepsilon\right\}$$

there is ξ *small enough such that* $f : X_\varepsilon \to X_\varepsilon$ *for all* $\varepsilon \leq \xi$ *for* $v_H \leq \overline{\theta}$.

Proof:

i) For $v_H \leq \underline{v}$ we know $w_2 \geq v_H$ and then f is obviously continuous, since $e_L^*\left(e_L, e_H\right)$ and $e_H^*\left(e_L, e_H\right)$ are continuous in both arguments.

ii) In $w_1 \geq w_2 + \varepsilon$ we have $e_L^* \leq \frac{w_1}{w_1 + \varepsilon}$ and with $w_1 \leq \theta_H$ this implies $e_L^* \leq \frac{\theta_H}{\theta_H + \varepsilon}$. Remember that $e_L^* \geq 1/2$ and $e_H^* \geq 1/2$ are guaranteed by the restrictions prior to the lemma. Choose δ such that $w_2 = w_1 - \delta$ with $\delta > 0$. Then optimal effort choices are

$$e_L = \frac{w_1}{w_1 + \delta}$$

for low types and

$$e_H = \begin{cases} \dfrac{w_1 - v_H}{w_1 + \delta - v_H} & \text{for } w_2 \geq v_H \\ \dfrac{1}{2} & \text{else} \end{cases}$$

for high types, since they may not accept w_2. Then zero profit determines wages to become

- For $w_2 \geq v_H$:

$$\tilde{w}_1 - \tilde{w}_2 = \frac{(e_L - e_H)(\theta_H - 1)}{\frac{e_H^2(1-e_H)}{e_L} + \frac{e_L^2(1-e_L)}{e_H} + (e_H + e_L - 2e_H e_L)}.$$

Collecting terms

$$e_L - e_H = \frac{v_H \delta}{(w_1 + \delta)(w_1 + \delta - v_H)}$$
$$1 - e_H = \frac{\delta}{w_1 + \delta - v_H}$$
$$1 - e_L = \frac{\delta}{w_1 + \delta}$$
$$e_H + e_L - 2e_H e_L = \frac{(2w_1 - v_H)\delta}{(w_1 + \delta)(w_1 + \delta - v_H)}$$

gives

$$\begin{aligned} \tilde{w}_1 - \tilde{w}_2 &= \frac{v_H(\theta_H - 1)}{(2w_1 - v_H) + \frac{(w_1 - v_H)^2}{(w_1 + \delta - v_H)^4 w_1} + \frac{w_1^2}{(w_1 + \delta)^4 (w_1 - v_H)}} \\ &> \frac{v_H(\theta_H - 1)}{(2w_1 - v_H) + \frac{1}{(w_1 - v_H)^2 w_1} + \frac{1}{(w_1)^2 (w_1 - v_H)}} =: \xi_1 \end{aligned}$$

for all $\delta > 0$, because all the contributing terms are positive due to the restrictions.

- For $w_2 < v_H$ and hence high types rejecting the offer w_2:

$$\tilde{w}_1 - \tilde{w}_2 = \frac{(\theta_H - 1)}{1 + \frac{e_L(1-e_L)}{4}} =: \xi_2 > 0$$

Define $\xi := \min\{\xi_1, \xi_2\}$. Then $\xi > 0$. As a sanity check observe that due to $w_1 > v_H$ the wages that result from individuals adjusting their effort to a given pair of wage offers differ less than $\theta_H - \theta_L = \theta_H - 1$. Choose $\varepsilon < \xi$ to establish that indeed $f : X_\varepsilon \to X_\varepsilon$ with X_ε defined as in the Lemma. ||

Corollary *In the situation in which both lemmas apply, there is at least one equilibrium* (e_H, e_L) *such that* $e_H = e_H^*(e_L, e_H)$ *and* $e_L = e_L^*(e_L, e_H)$. *If* $(\tilde{e}_H, \tilde{e}_L)$ *is another such equilibrium, then* $\frac{e_L - \tilde{e}_L}{e_H - \tilde{e}_H} \geq 0$.

Proof: Lemma 2 guarantees the existence of a fixed point of f in X_ε for ε small enough and $v_H \leq \underline{v}(\theta_H)$ due to Brouwer's fixed point theorem. Any such fixed point is an equilibrium as specified in the corollary. According to Lemma 1 $\hat{e}_L(.)$ is an increasing function in e_H and any equilibrium has to be a value of this function. This establishes the second claim of the corollary. □

Now notice that X_ε does not contain either boundary equilibrium. Also for $v_H \leq \underline{v}(\theta_H)$ equilibria according to the corollary involve $w_2 \geq v_H$, as established under B). Therefore $e_H > 1/2$ must hold for high types optimal effort choice, where they accept both offers, w_1 and w_2. Hence an F-equilibrium exists in addition to the benchmark case. □

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