

Redistributive Taxation with Endogenous Sentiments

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Abstract

We present a model in which an individual's sentiments toward others are determined endogenously on the basis of how they perform relative to the societal average. This, in turn, affects the individual's own behavior and hence other agents' sentiments toward her. We focus on stationary patterns of utility interdependence. To demonstrate the effects of such endogeneity, we consider an example of a production economy with redistributive taxation. There are two types of stationary equilibria: one in which all agents conform to the societal norm, and a second involving social stratification on the basis of productivity into two or three groups. The main conclusion is that the tax structure, in that it affects behavior which in turn affects sentiments, plays a crucial role in determining which type of equilibrium occurs and its characteristics as well as the extent of altruism and social cohesion in society.

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1 Introduction

Clearly, peoples' feelings or sentiments toward others affect their behavior. Here, we consider the opposite, namely, that peoples' behavior affects our attitudes toward them – and hence how we act, in turn. This simple proposition, that we judge people by their behavior and that this affects our feelings for them, has far reaching implications for the analysis of economic interactions as well as for the design of economic institutions. Indeed, it profoundly changes the relationship between economic agents and their institutional environment. Typically, we study how agents behave in a given institutional setting, or perhaps, more ambitiously, we might try to explain how agents with given preferences select or determine which institutions will prevail. But to the extent that institutions affect behavior, and such behavior engenders different levels of concern among the participants, then the institutional structure can indeed affect the type of society that emerges.

In Esteban and Kranich (2002) we develop a general model of endogenous determination of individual sentiments in which each agent is assumed to have extended preferences over the welfare of others and to modify its concern on the basis of how they behave relative to a standard set forth. There, we consider alternative standards of appropriate behavior, and we examine the patterns of individual sentiments that might emerge under each. In the present paper we consider an application of this approach to the case in which income is subject to a redistributive tax. We show that the degree of redistribution can determine whether society forms a cohesive group, both in terms of agents' behavior as well as the extent of their mutual concern, or if it splinters into clusters with some agents excluded from consideration.

Specifically, we consider a production economy in which each agent contributes labor to the production of a single consumption good. The agents differ in their productivities, and their wage earnings are subject to a purely redistributive linear income tax. Individuals have private, or direct, preferences over their own consumption bundles, but they also have social, or extended, preferences which reflect their esteem or concern for others. We consider the particular example in which agents revise their esteem for others in view of the amount of labor they supply relative to the mean,¹

¹Thus, for example, if people work 40 hours per week on average, then those who work more are perceived as being industrious and those who work less are perceived as being lazy.

increasing their esteem for those who supply more than the mean level of effort and decreasing their concern for those who supply less.² We focus on the stationary outcomes of this dynamic process.

There are two types of stationary equilibria. In one all individuals conform to the standard of behavior and supply precisely the mean level of effort. Here, there is no social exclusion and altruism is inversely related to income. This equilibrium is attainable only if there is sufficient redistribution relative to the degree of inequality in individual productivities. In the other type of equilibrium society becomes stratified into two or three clusters: one group of highly productive “winners” who work more than the average number of hours and earn the full admiration of everyone, a second group of low productivity “losers” who work less than the mean and earn no esteem from others, and possibly a third group consisting of those with intermediate productivities who supply exactly the mean number of labor hours and may garner a range of esteem levels.

We can thus properly speak of social exclusion. The size of the middle class critically depends on the degree of redistribution. The lower the tax rate, the smaller will be the size of the middle class. In the limit, as the tax goes to zero a two-cluster equilibrium will occur along with the highest dispersion in labor supply and the greatest degree of exclusion. The nature of the social contract thus plays a crucial role in determining which type of equilibrium prevails as well as the equilibrium individual sentiments.

Our model should be contrasted with recent attempts to explain how otherwise similar societies adopt quite different institutions. For example, Bénabou (2000) and Alesina, et. al., (2001) ask why the United States and Western Europe, with similar structural characteristics and equally democratic political systems, should differ so significantly in their levels of social transfers. One of the main explanations proposed by the latter is based on differences in attitudes: “Americans dislike redistribution because they tend to feel that people on welfare are lazy, whereas Europeans tend to feel that people on welfare are unfortunate.”³ (Alesina, et. al., p.247.) Given

²The role of esteem in explaining social behavior has already been strongly emphasized by Homans (1961). There, he develops the view that agents reward other agents’ highly valuable actions with greater esteem.

³Bénabou (2000) rationalizes these as different steady state equilibria under capital and insurance market imperfections, one having high inequality yet low redistribution and the other the reverse. For other explanations of such a negative relationship between inequality and redistribution, see Piketty (1995a, 1995b), who focuses on individual perceptions of

such beliefs, Americans would be less inclined to favor substantial redistribution or social insurance. But how do such differences in beliefs arise? While Alesina, et. al., do not offer an explanation⁴, they do suggest that this might be related to the greater correlation between earnings and hours worked in the United States versus Europe, where labor supply appears to display a substantially lower variance.⁵ According to our model, one might observe the same correlation between attitudes and redistributive regimes, but with the causality reversed. The different regimes might encourage the adoption of such beliefs (through their effects on the incentives and thus behavior of the effected parties). Moreover, as indicated above, the model is consistent with the observation that low levels of redistribution are correlated with greater variation in hours worked.

In addition to offering a new view of the relationship between institutions and economic outcomes, our model yields further implications that accord well with observation and empirical evidence.

For example, we appear to be undergoing a profound change in the structure of society and social values. Whereas the plight of the unemployed and underemployed was once the focus of great concern and effort at alleviation, now the burden has shifted to the poor to help themselves (as evidenced by the replacement of welfare by workfare) resulting in further disenfranchisement and social exclusion. At the same time, the earnings gap between highly trained and less skilled workers is growing, resulting in greater polarization. And this is compounded by the additional trend to cut (or eliminate) estate and capital gains taxes and to reduce upper income tax rates, thereby lessening the degree of redistribution. As mentioned above, an implication of our model is that lower tax rates – hence less redistribution – are likely to entail greater polarization and exclusion, as is greater disparity in skill levels. Moreover, the advent of a disenfranchised group is accompanied by a decreased concern for its members.

The model also yields insights into the influence of group incentives on individual behavior and the role of heterogeneity. There are numerous examples of rewards that are partially or entirely based on group –

the weight of randomness in explaining earned income, and Bénabou and Ok (2001), who stress the role of expected upward mobility.

⁴“We do not know what explains these differences in beliefs.” (Alesina, et al., p.247.)

⁵In Sweden, for instance, where transfers and other social benefits are twice that of the US (as a percentage of GDP), the median worker in each income quintile works thirty-nine hours per week.

rather than individual – performance, thus involving some type of interpersonal redistribution within the team. Examples range from professional services – lawyers or architects – as examined in Kandel and Lazear (1992) to Japanese fishermen, studied by Platteau and Seki (2000). Standard theory would predict high levels of free-riding. Yet, in practice this type of reward is seen as stimulating high performance under some circumstances. Specifically, Platteau and Seki (2000, p.32) find that if the group is not too heterogeneous to begin with, a reward system based on pooling is self-reinforcing. If, on the contrary, a group is too heterogeneous initially, then it will progressively unravel. Recent empirical work by Nalbantian and Schotter (1997) is also in keeping with the view that profit sharing might have a positive role. By introducing an external effect on others, a profit-sharing mechanism induces the development of within-group altruism which in turn increases output. This, too, is consistent with our finding that the more homogenous the population (in terms of productivities), the more likely a common effort equilibrium is to occur.

The paper is organized as follows. In the next section we present the model. In Section 3 we explain the process by which sentiments change as the result of agents' behavior. Section 4 examines the existence and basic properties of stationary equilibria. In Section 5 we discuss some implications of the previous analysis for the economics of taxation and of rewards. Finally, Section 6 contains several concluding remarks and directions for future research.

2 The model

We consider an n -agent production economy in which agents have different abilities and each contributes labor to the production of a single consumption good.⁶ Let $N = \{1, \dots, n\}$ denote the set of individuals. Agents are endowed with time, and each derives *direct* utility from consumption and leisure. However, their well-being also depends on their extended or *social* utility derived from the direct utility experienced by others.

We assume all agents have the same direct preferences represented by the utility function

⁶Our description of the economy is similar to that in Sen (1966) and Ray and Ueda (1996). However, we depart from Sen in that we endogenize the extent of individual concern for others. This issue is addressed in the following section.

$$u = u(c, L),$$

where c denotes the consumption good and L denotes labor.

In order to permit an explicit characterization of individual behavior we shall restrict our attention to the following specification:

$$u(c, L) = c - \frac{1}{2}L^2. \quad (1)$$

The social utility of individual i is given by

$$U_i = \sum_{j=1}^n \alpha_j^i u_j, \quad (2)$$

where α_j^i measures the sympathy or concern felt by individual i toward individual j . We assume $\alpha_j^i \leq \alpha_i^i$, for all i, j , and we normalize sentiments by taking $\alpha_i^i = 1$, for $i = 1, \dots, n$. We also assume $\alpha_i^j \geq 0$ for all i, j thus excluding malevolence.⁷ For notational simplicity, we write $\boldsymbol{\alpha}^i = (\alpha_1^i, \dots, \alpha_n^i)$, and we denote the entire $n \times n$ matrix of coefficients by $\boldsymbol{\alpha}$.⁸

Since individuals differ in their productivities, in the scenario we are contemplating it is possible that such differences may become a source of esteem within a community. Also, it is possible that the eventual role of productivity in engendering esteem may depend on the degree of substitutability between the various types of labor. In order to parametrize the eventual effect of substitutability on sentiments, we consider a CES production function of the form

$$Y = \left\{ \sum_{i=1}^n \beta_i L_i^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}, \quad \beta_i \geq 0 \text{ and } \sum_{i=1}^n \beta_i = 1, \quad (3)$$

where β_i measures the productivity of individual i and σ is the elasticity of substitution between the labor inputs. Without loss of generality, we will assume that the agents are ordered such that $\beta_1 \leq \beta_2 \leq \dots \leq \beta_n$.

⁷The critical assumption for our results is that α_j^i is bounded below, even if this is an arbitrary negative number. We take the bound to be zero for convenience.

⁸Generally, bold letters denote vectors.

The marginal productivity of L_j is

$$\frac{\partial Y}{\partial L_j} = \beta_j L_j^{-\sigma} \left\{ \sum_{i=1}^n \beta_i L_i^{1-\sigma} \right\}^{\frac{1}{1-\sigma}-1} = \beta_j L_j^{-\sigma} Y^\sigma. \quad (4)$$

For later reference, it will be useful to note that for the case in which $L_j = L$, for all $j = 1, \dots, n$, we have

$$Y(L) = L \quad \text{and} \quad \frac{\partial Y}{\partial L_j} \Big|_{L_i=L} = \beta_j. \quad (5)$$

One of the aims of this paper is to investigate the interplay between institutions and individual altruistic sentiments. A question we shall try to answer is whether some policies are more conducive to social cohesion while others may precipitate social clustering or fragmentation. In particular, we shall focus on the role of redistributive taxation in generating such outcomes.

Thus, we suppose labor income is taxed at a given rate $\tau \in [0, 1]$ and the proceedings are redistributed equally among all agents⁹. Hence, individual after-tax disposable income is

$$y_i = (1 - \tau)w_i L_i + \frac{\tau \sum_{h=1}^n w_h L_h}{n}, \quad (6)$$

which is entirely consumed, so that $c_i = y_i$.

Given the wage vector \mathbf{w} , the tax rate τ , and the altruism coefficients α^i , the choice problem facing individual i consists of selecting the labor supply L_i to solve:

$$L_i \max U_i(\mathbf{L}). \quad (7)$$

Substituting from (1), (2) and (6), (7) can be rewritten as

$$L_i \max \sum_{j=1}^n \alpha_j^i \left[(1 - \tau)w_j L_j + \frac{\tau \sum_{h=1}^n w_h L_h}{n} \right] - \frac{1}{2} \sum_{j=1}^n \alpha_j^i L_j^2. \quad (8)$$

⁹This is exactly equivalent to the manner in which rewards within a team depend partly on own performance and partly on the joint effort.

Since U_i is concave, an interior solution to (8) is given by

$$L_i = w_i \left[(1 - \tau) + \tau \frac{\alpha_i}{n} \right], \quad (9)$$

where $\alpha_i \equiv \sum_{j=1}^n \alpha_j^i$ is the total altruism felt by i . Equation (9) then describes the optimal behavior of an individual when facing the parameters $\langle \mathbf{w}, \tau, \boldsymbol{\alpha}^i \rangle$.

Collectively, given the technology, parametrized by $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$, as well as τ and $\boldsymbol{\alpha}$, an *equilibrium* consists of n -vectors \mathbf{L} and \mathbf{w} such that for all $i \in N$, L_i satisfies (9) and $w_i = \frac{\partial Y}{\partial L_i}$.

3 The dynamics of individual sentiments

We now turn to the issue of the endogenous determination of individual sentiments. Here, individual sentiments are reflected in the coefficient matrix $\boldsymbol{\alpha}$. As indicated above, we do not attempt to explain where those sentiments come from initially but simply how they evolve in response to the observed behavior of others, that is, how agents modify their sentiments. We then focus on stationary patterns of interdependence.

The key element of our model, and that which sets it apart, is the assumption that each individual formulates a standard of behavior for others. In our case, such a standard consists of an expected labor supply. Then if individual j 's actual labor supply exceeds i 's standard for j , then i increases its esteem for j . Conversely, if j supplies less than the standard, then i lowers its esteem. If j exactly conforms to i 's standard, then no adjustment occurs.

Two questions arise: (1) how do agents set standards for others? and (2) how do they revise their sentiments in the event other agents fail to conform to their standards?

Regarding the latter, rather than postulate a particular updating procedure, we simply require, as stated previously, that i 's esteem for j changes in accordance with the difference between j 's actual labor supply and i 's standard for j . Formally, treating time as a discrete variable, let $L_j(t)$ be j 's actual labor supply at time t , $L_j^i(t)$ be the amount of labor i thinks j should contribute at t , and $\alpha_j^i(t)$ be the esteem i feels for j at t . Then we require only that $\alpha_j^i(t+1) \geq \alpha_j^i(t)$ as $L_j(t) \geq L_j^i(t)$. Alternatively, we write

$$\alpha_j^i(t+1) = g_i(\alpha_j^i(t), L_j(t) - L_j^i(t)), \quad (10)$$

where g_i is an arbitrary function that is nondecreasing in both arguments, bounded above by 1, bounded below by 0, and $g_i(\alpha, 0) = \alpha$, and we assume g_i is given.

Then given $\langle \beta, \tau \rangle$, a *stationary equilibrium* is a triple $(\alpha, \mathbf{L}, \mathbf{w})$ such that (\mathbf{L}, \mathbf{w}) is an equilibrium with respect to $\langle \beta, \tau, \alpha \rangle$, as defined above, and $g_i(\alpha_j^i, L_j - L_j^i) = \alpha_j^i$, for all $i, j, i \neq j$.

Turning to the issue of how agents set standards, in Esteban and Kranich (2002) we explored various formulations. Here, we concentrate on a simple and plausible rule, namely, that each agent takes the mean behavior as the societal norm and judges other agents' actions accordingly.¹⁰ Thus, anyone supplying labor in excess of the mean garners additional respect, while anyone contributing less loses respect. Notice that in this case all agents revise their coefficients uniformly, that is, all revise their concern for, say, j in the same direction (with the exception of j herself). Stating this formally, let $\bar{L}(t)$ denote the mean of $\{L_1(t), \dots, L_n(t)\}$. According to the *mean standard of behavior*, $L_j^i(t) = \bar{L}(t)$, for all $i, j = 1, \dots, n, i \neq j$.

In the next section, we study the existence and properties of steady-state equilibria under the mean standard of behavior.

4 Stationary outcomes under the mean standard of behavior

In the model we have described, there may be two types of stationary solutions. One corresponds to the case in which everyone conforms to the average behavior. In this case, whatever is the matrix of coefficients necessary to support such an equilibrium, it will clearly be stationary since no agent will have reason to modify its esteem for any other agent. In the second type, society is divided into clusters: one set of individuals who conspicuously work more than the average and garner maximal esteem/concern from others, another set who work less than the mean and garner no esteem, and possibly a third set who work precisely the average. We consider each type in turn.

¹⁰For example, if the norm is to work 40 hours per week, then one might measure laziness or industriousness relative to this standard.

4.1 Common effort equilibria

Let us start by studying the existence and basic properties of the first type of stationary outcome, namely, common effort equilibria. In this case $L_i = L$ for $i = 1, \dots, n$.

Proposition 1 *Let $\langle \beta, \tau \rangle$ be given. A common effort equilibrium (α, L, w) exists if and only if*

$$\frac{n}{n-1} \frac{\beta_n - \beta_1}{\beta_n} \leq \tau \leq 1. \quad (11)$$

PROOF. We shall prove existence by construction. In view of (9) we observe that the determinant of individual labor supply is total altruism, α_i (together with the wage rate), independently of how the total is distributed over the population. We begin by choosing an arbitrary common effort, L . Using this value of L in (9) and bearing in mind (5), we obtain the level of (total) altruism α that renders L the optimal individual choice for each productivity level β :

$$\alpha = -\frac{(1-\tau)n}{\tau} + \frac{nL}{\tau} \frac{1}{\beta}. \quad (12)$$

For each L , this defines a relationship between α and β which is depicted in Figure 1. Note that $\alpha \in [1, n]$ and $\beta \in [\beta_1, \beta_n]$.

Figure 1.

For each value of β , higher levels of effort require greater degrees of altruism by every individual. In order to prove the existence of an equilibrium, we need only show that there exists some L such that the corresponding values for α are indeed feasible, i.e. $\alpha_i \in [1, n]$ for all i .

Given L and β , the maximum degree of altruism corresponds to the individual with the lowest productivity β_1 , and the smallest altruism to the largest productivity β_n . Since $\alpha_i \in [1, n]$, feasibility requires that

$$\alpha_1 = -\frac{(1-\tau)n}{\tau} + \frac{nL}{\tau} \frac{1}{\beta_1} \leq n, \quad (13)$$

and

$$\alpha_n = -\frac{(1-\tau)n}{\tau} + \frac{nL}{\tau} \frac{1}{\beta_n} \geq 1. \quad (14)$$

Putting the two restrictions together, one can readily obtain (11). This proves the necessity and sufficiency of this condition. ■

According to (11), an equal effort stationary equilibrium cannot be achieved in all economies. The ratio on the left hand side is the *relative spread* of the distribution of productivities over the population. The relative spread is a (crude) measure of inequality within a distribution. Thus, the inequality in the “fundamentals” of the economy sets a lower bound on the degree of redistribution necessary to support equal effort equilibria. In the next subsection we shall examine these properties in detail.

A second point is that for the existence of an equilibrium, the larger the population, the more restrictive (11) is on τ .

From the above proof it is obvious that, generally, whenever equilibria exist, there will be many.

Proposition 2 *Suppose (11) holds. Then, for every L satisfying*

$$\beta_n \left[(1-\tau) + \frac{\tau}{n} \right] \leq L \leq \beta_1, \quad (15)$$

there exists a matrix α for which the economy is in a stationary equilibrium.

PROOF. Take conditons (13) and (14) and find the maximum and the minimum (resp.) values of L which render them equalities. ■

A diagramatic representation of the different equilibria is depicted in Figure 2. For a given level of effort, L , the range of equilibrium values of α consistent with L is determined by (12) evaluated at $\beta = \beta_1$ and $\beta = \beta_n$. The minimum level of effort, L_1 , corresponds to the case in which the most productive individual is completely egoistic, i.e. when the curve defined by (12) passes through $(\beta_n, 1)$. The maximum effort, L_n , is obtained when the least productive individual is fully altruistic, i.e. when the curve passes through (β_1, n) .

Figure 2.

Notice that this continuum of equilibria can be parametrized by the degree of altruism of any particular individual. That is, given β_i , one can derive from (12) the range of α_i consistent with each L that might occur in equilibrium. Which equilibrium will prevail will depend on the initial conditions and the assumed adjustment process for sentiments. In any case, the higher the degree of final altruism, the larger the aggregate output.

4.2 Clustering equilibria

Let us now turn to the second type of equilibria in which society is partitioned into clusters with differentiated behavior. Our first observation is that there can be at most three clusters consisting of those agents who supply above average labor, those who supply below average, and possibly a group who supply exactly the mean number of labor hours. We denote these groups respectively by A , B and M . Notice that in such an equilibrium all those individuals supplying effort above the mean will earn the (maximal) esteem of everyone in society, while those performing below will merit no concern by anyone other than themselves. Because the degree of esteem is bounded above and below, individual sentiments with respect to those in A or B will reach stationary values on the boundary. At the same time, those in M conform to the standard of behavior and might garner a range of esteem levels. Whether a middle group will exist or not will depend on the individual productivities as well as on the degree of redistribution. We start by discussing the case of a two-cluster equilibrium and then move on to consider three-cluster equilibria.

4.2.1 Two-cluster equilibria

We start by focusing on the possibility of there being two groups only: A and B . All members of group A will be admired by everyone and to the maximal extent. Members of group B will be admired by no one other than themselves. Let a and b denote the respective cardinalities of the two groups. Then, for every $i \in A$ we have that $\alpha_i = a$, and for every $i \in B$ we have $\alpha_i = a + 1$.

Writing the corresponding labor supply functions (9), and using (4), we

have¹¹

$$L_i^A = Y^{\frac{\sigma}{1+\sigma}} \left[\beta_i \left(1 - \tau + \tau \frac{a}{n} \right) \right]^{\frac{1}{1+\sigma}}, \quad (16)$$

and

$$L_i^B = Y^{\frac{\sigma}{1+\sigma}} \left[\beta_i \left(1 - \tau + \tau \frac{a+1}{n} \right) \right]^{\frac{1}{1+\sigma}}, \quad (17)$$

where L_i^A and L_i^B denote the labor supply by the i^{th} member of group A and B , respectively.

The two labor supply curves are depicted in Figure 3, where for diagrammatic convenience we use the transformation $(L_i^k)^{1+\sigma}$, $k = A, B$. The transformed labor supply is a linear function of β passing through the origin. Total altruism by the low types is higher than among the high types. Hence, the slope of the (transformed) labor supply by the B types will be steeper than for the A types. For any particular β , the B types would choose to supply strictly more effort than the A types. Therefore, to support a two-cluster equilibrium in which the B types supply less effort than the A types, we need that there exists a threshold productivity level, $\tilde{\beta}$, such that $(n - a)$ agents have productivity less than $\tilde{\beta}$ and supply effort between c and d along the type B supply curve (such as L_i^B), while a agents have productivity greater than $\tilde{\beta}$ and supply labor between e and f along the type A curve (such as L_i^A). Moreover, it must be the case that the mean labor supply (indicated by \bar{L} in the figure) vertically separate the two chords.

FIGURE 3.

It is immediate that the existence of such an equilibrium will critically depend on the shape of the distribution of productivities. Using (16) and (17) in (3), we obtain the equilibrium output for given a . That is,

$$Y = \left\{ \sum_{i=1}^{n-a} \beta_i^{\frac{2}{1+\sigma}} \left(1 - \tau + \tau \frac{a+1}{n} \right)^{\frac{1-\sigma}{1+\sigma}} + \sum_{i=n-a+1}^n \beta_i^{\frac{2}{1+\sigma}} \left(1 - \tau + \tau \frac{a}{n} \right)^{\frac{1-\sigma}{1+\sigma}} \right\}^{\frac{1+\sigma}{1-\sigma}}. \quad (18)$$

¹¹Since under the mean standard of behavior there is no need to differentiate between L_j^i 's – that is, the amount i thinks j should work – no confusion should result if we employ the notation otherwise.

Average labor supply is then

$$\bar{L} = \frac{1}{n} Y^{\frac{\sigma}{1+\sigma}} \left\{ \sum_{i=1}^{n-a} \beta_i^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{a+1}{n} \right)^{\frac{1}{1+\sigma}} + \sum_{i=n-a+1}^n \beta_i^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{a}{n} \right)^{\frac{1}{1+\sigma}} \right\}. \quad (19)$$

Therefore, a two-cluster equilibrium will exist if there is an a such that

$$L_{n-a}^B < \bar{L} < L_{n-a+1}^A. \quad (20)$$

This condition will be satisfied if and only if there exists a such that,

$$(\beta_{n-a} T(a, \tau))^{\frac{1}{1+\sigma}} < \frac{\sum_{i=1}^{n-a} (\beta_i T(a, \tau))^{\frac{1}{1+\sigma}} + \sum_{i=n-a+1}^n \beta_i^{\frac{1}{1+\sigma}}}{n} < \beta_{n-a+1}^{\frac{1}{1+\sigma}}, \quad (21)$$

where

$$T(a, \tau) \equiv \left(\frac{1 - \tau + \tau \frac{a+1}{n}}{1 - \tau + \tau \frac{a}{n}} \right) > 1, \text{ for } \tau > 0. \quad (22)$$

We thus have demonstrated the following proposition.

Proposition 3 *A two-cluster equilibrium exists if there is an integer a , $1 \leq a \leq n-1$, such that (21) is satisfied.*

In the same fashion as for the common effort equilibrium, the existence of equilibrium jointly depends on τ and the shape of the distribution of β . Unfortunately, we cannot give here an explicit, closed form characterization of the entire class of economies containing two-cluster equilibria. Yet, we can prove the following result concerning the existence of taxes for given distribution of productivities, and the existence of a vector β for given taxes, such that a two-cluster equilibrium exists.

Proposition 4 *For any (unequal) distribution of productivities β there is always τ small enough for which a two-cluster equilibrium exists. Further, for any $\tau \in [0, 1)$ there exists a distribution of productivities such that a two-cluster equilibrium exists.*

PROOF. To prove that for any given unequal distribution of productivities there always exists τ small enough for which a two-cluster equilibrium exists, observe that $T(a, \tau) \rightarrow 1$ as $\tau \rightarrow 0$. Therefore, as $\tau \rightarrow 0$, (21) becomes

$$\beta_{n-a}^{\frac{1}{1+\sigma}} \leq \sum_{i=1}^n \frac{1}{n} \beta_i^{\frac{1}{1+\sigma}} \leq \beta_{n-a+1}^{\frac{1}{1+\sigma}}.$$

It is immediate that as long as $\beta_i \neq \beta_{i+1}$ for some i , then such an a always exists.

Let us now consider an arbitrary $\tau > 0$. It is clear that condition (21) may or may not be satisfied in general, depending on τ and the distribution of β . However, consider a perfectly bi-polarized distribution with $\frac{n}{2}$ individuals with productivity β_A and $\frac{n}{2}$ with productivity β_B , with $\beta_A > \beta_B$.¹² Then it can be readily computed that if

$$\frac{\beta_A - \beta_B}{\beta_B} \geq \frac{2\tau}{n(2 - \tau)}, \quad (23)$$

then equilibrium condition (21) is satisfied. ■

Two remarks are in order. The first concerns individual behavior in a two-cluster equilibrium. In spite of the fact that the population is clustered into two classes, this is generally compatible with a rich heterogeneity of individual behavior within each class. Indeed, in this equilibrium the work time is strictly increasing with individual productivity.

Our second remark is that condition (23) depends on n . The larger is n , the less restrictive is the inequality and hence the more likely is a given (bi-polar) β to satisfy the restriction. In the (lower) limit case, when $n = 2$, condition (23) requires that,

$$\tau \leq 2 \frac{\beta_A - \beta_B}{\beta_A}. \quad (24)$$

Inequality (24) is complementary to condition (11) for the existence of a common effort equilibrium. For $n = 2$, we either have a common effort equilibrium or a two-cluster equilibrium depending on the degree of redistribution.

¹²Here, we implicitly assume n is even. If n were odd, then it would be sufficient to assign $\frac{n-1}{2}$ agents productivity β_A and $\frac{n-1}{2} + 1$ productivity β_B .

4.2.2 Three-cluster equilibria

Finally, let us examine the case of three-cluster equilibria. Again, individuals are classified into three groups, A , B and M , depending on the quantity of their labor supply relative to the societal average. We again use a and b to denote the cardinalities of the first two groups, and we let m denote the cardinality of the third group, which we refer to as the “middle class.” Recall that each member of M supplies precisely the average labor hours.

Individual choice of labor effort is given by (9), where what matters is the total esteem felt, α_i , and not how this esteem is distributed. Because of the nature of this equilibrium, the esteem earned by each member of groups A and B from every other member of the economy is 1 and 0, respectively. Only the members of the middle group may experience varying degrees of esteem. We shall denote the total esteem by i to all the members of M by α_i^M . Notice that if $i \in M$, $\alpha_i^M \geq 1$. The values α_i can attain depend on whether i belongs to A , B or M . The possible values are:

$$\begin{aligned} \alpha_i &= a + \alpha_i^M, \text{ with } 0 \leq \alpha_i^M \leq m, \text{ for all } i \in A, \\ \alpha_i &= a + \alpha_i^M + 1, \text{ with } 0 \leq \alpha_i^M \leq m, \text{ for all } i \in B, \\ \alpha_i &= a + \alpha_i^M, \text{ with } 1 \leq \alpha_i^M \leq m, \text{ for all } i \in M. \end{aligned} \quad (25)$$

Consider now a specific equilibrium with aggregate output Y and average labor supply L . Using the labor supply function (9) as well as (4), we have that

$$\begin{aligned} L_i^A &= \left[\beta_i Y^\sigma \left(1 - \tau + \frac{\tau(a + \alpha_i^M)}{n} \right) \right]^{\frac{1}{1+\sigma}}, \quad 0 \leq \alpha_i^M \leq m, \text{ for all } i \in A, \\ L_i^B &= \left[\beta_i Y^\sigma \left(1 - \tau + \frac{\tau(a + \alpha_i^M + 1)}{n} \right) \right]^{\frac{1}{1+\sigma}}, \quad 0 \leq \alpha_i^M \leq m, \text{ for all } i \in B, \\ L_i^M &= L = \left[\beta_i Y^\sigma \left(1 - \tau + \frac{\tau(a + \alpha_i^M)}{n} \right) \right]^{\frac{1}{1+\sigma}}, \quad 1 \leq \alpha_i^M \leq m, \text{ for all } i \in M. \end{aligned} \quad (26)$$

The relation between labor supply and β depends on the individual-specific value of α_i^M – agent i ’s esteem for the members of the middle group. In Figure 4 we represent labor supply as a function of β for the minimum and maximum values of α_i^M . To facilitate the diagrammatic representation, we again use the transformation $(L)^{1+\sigma}$. Note that for the agents in M , their

(exogenous) productivities, β_i , and (endogenous) altruism expressions, α_i^M , must exactly off-set (as in Figure 1) in order to support the same choice of L . Unlike the case of a two-cluster equilibrium the agents in A or B can now differ in their aggregate degree of concern, thus generating distinct labor supply curves.

FIGURE 4.

As before, let us solve for total output. Using (3) and (26) we have,

$$Y = \left\{ \sum_{i \in B} \beta_i^{\frac{2}{1+\sigma}} \left(1 - \tau + \tau \frac{(1+a+\alpha_i^M)}{n} \right)^{\frac{1-\sigma}{1+\sigma}} + \sum_{i \in M} \beta_i^{\frac{2}{1+\sigma}} \left(1 - \tau + \tau \frac{(1+a+\alpha_i^M)}{n} \right)^{\frac{1-\sigma}{1+\sigma}} + \sum_{i \in A} \beta_i^{\frac{2}{1+\sigma}} \left(1 - \tau + \tau \frac{(a+\alpha_i^M)}{n} \right)^{\frac{1-\sigma}{1+\sigma}} \right\}^{\frac{1+\sigma}{1-\sigma}} \quad (27)$$

Here, output varies with the composition of the sets A , B and M , as well as with the values of α_i^M . Also, for the members of M the values of α_i^M must satisfy (26).

We can now compute the average labor supply. This is equal to the average supply among the members of A and B , i.e.,

$$\bar{L} = \frac{1}{a+b} Y^{\frac{\sigma}{1+\sigma}} \left\{ \sum_{i \in B} \beta_i^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(1+a+\alpha_i^M)}{n} \right)^{\frac{1}{1+\sigma}} + \sum_{i \in A} \beta_i^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(a+\alpha_i^M)}{n} \right)^{\frac{1}{1+\sigma}} \right\}. \quad (28)$$

The equilibrium conditions are that

$$\beta_i^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(1+a+\alpha_i^M)}{n} \right)^{\frac{1}{1+\sigma}} \leq \frac{1}{a+b} \left\{ \sum_{h \in B} \beta_h^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(1+a+\alpha_h^M)}{n} \right)^{\frac{1}{1+\sigma}} + \sum_{h \in A} \beta_h^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(a+\alpha_h^M)}{n} \right)^{\frac{1}{1+\sigma}} \right\} \quad (29)$$

for all $i \in B$,

$$\beta_i^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(a+\alpha_i^M)}{n} \right)^{\frac{1}{1+\sigma}} \geq \frac{1}{a+b} \left\{ \sum_{h \in B} \beta_h^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(1+a+\alpha_h^M)}{n} \right)^{\frac{1}{1+\sigma}} + \sum_{h \in A} \beta_h^{\frac{1}{1+\sigma}} \left(1 - \tau + \tau \frac{(a+\alpha_h^M)}{n} \right)^{\frac{1}{1+\sigma}} \right\} \quad (30)$$

for all $i \in A$,

and that (26) is satisfied for all $i \in M$.

The pairs (β_i, α_i^M) compatible with a three-cluster equilibrium are depicted in Figure 5. The members of M are those with pairs (β_i, α_i^M) lying on the outer curve, for $\alpha_i^M \in [1, m]$. This curve corresponds to the labor supply of the M types in (26) for the fixed average labor supply. Group A consists of those with (β_i, α_i^M) lying strictly above this curve with $\alpha_i^M \in [0, m]$. Finally, the agents in B are those with pairs strictly below the inner curve with $\alpha_i^M \in [0, m]$. The inner curve corresponds to the labor supply curve of the B types in (26), again evaluated at the average labor supply.

FIGURE 5.

Conditions (25) to (30) fully characterize a three-cluster equilibrium. We have thus proven the following proposition.

Proposition 5 *A three-cluster equilibrium exists if and only if there are sets of individuals A , B and M and a vector $(\alpha_i^M)_{i \in N}$ such that (25) through (30) are satisfied.*

The case of three-cluster equilibrium bridges the gap between the common effort equilibrium and the two-cluster equilibrium. In fact, the pooling (common effort) equilibrium can be seen as the limit when the middle class encompasses the entire society and the two-cluster equilibrium as the case when the middle class disappears.

The relative size of the middle class has obvious implications for the heterogeneity of observed behavior. In the pooling equilibrium, everyone contributes the same amount of effort. As the middle class shrinks, the set of individuals conforming to a common pattern of labor supply shrinks too. All those who deviate supply different amounts of labor according to their individual productivities, some supplying more and some less than the middle class pattern. In the limit, as the middle class vanishes all individuals deviate and the hours worked will be as disperse as the individual productivities.

We now proceed to explore the relationship between the size of the middle class and the degree of redistribution in society. Let the average labor supply be \bar{L} , and recall that agents are assumed to be ordered by their productivities. Let us consider the least and the most productive members of the middle class with productivities β_{b+1} and β_{b+m} , respectively. We

know that for the least productive member of the middle class, their total esteem cannot exceed $a + m$. Therefore, since $a + b + m = n$, $\alpha_{b+1} \leq a + m = n - b$. Likewise, for the most productive member of the middle class, their total esteem is at least $a + 1$, hence, $\alpha_{b+m} \geq n + 1 - b - m$.

Using (26), these inequalities become

$$\begin{aligned}\bar{L} &\leq \left[\beta_{b+1} Y^\sigma \left(1 - \tau + \frac{\tau(n-b)}{n} \right) \right]^{\frac{1}{1+\sigma}}, \text{ and} \\ \bar{L} &\geq \left[\beta_{b+m} Y^\sigma \left(1 - \tau + \frac{\tau(n+1-b-m)}{n} \right) \right]^{\frac{1}{1+\sigma}}.\end{aligned}$$

We have thus obtained the following necessary condition for a three-cluster equilibrium to obtain.

Proposition 6 *A necessary condition for the existence of a three-cluster equilibrium is that*

$$\frac{\beta_{b+m} - \beta_{b+1}}{\beta_{b+m}} \leq \frac{(m-1)\tau}{n - \tau b}. \quad (31)$$

Notice that (31) generalizes restriction (11), which is necessary and sufficient for the existence of a common effort equilibrium. Indeed, this corresponds to the case in which $m = n$ and $b = 0$. Also, (31) sets an implicit upper bound on the size of m . Let us start by observing that as $\tau \rightarrow 0$ the *RHS* $\rightarrow 0$ and hence the support of the productivities of the eventual members of the middle class vanishes. Our second observation is that, since $m \leq n - 2$ and $b \geq 1$, we have

$$\frac{(m-1)\tau}{n - \tau b} \leq \frac{(n-3)\tau}{n - \tau}. \quad (32)$$

Therefore, a *necessary condition for the existence* of a three-cluster equilibrium is that

$$\frac{\beta_{b+m} - \beta_{b+1}}{\beta_{b+m}} \leq \frac{(n-3)\tau}{n - \tau}. \quad (33)$$

Clearly, the lower is τ , the smaller the permissible support for the distribution of productivities of the middle class and hence the smaller that class will be.

5 The role of redistribution

In most of the literature on (endogenous) income taxation, taxes rightly depend on individual preferences. The more altruistic the agents, the more redistribution we would expect. As described in the Introduction, our argument is that the relationship goes the other way as well. Different redistributive regimes elicit different behavior which, in turn, engender different levels of concern among the participants. Thus, the institutional structure affects the type of society that develops. Let us discuss in some detail the implications of the present model for the theory of taxation.

In the previous section we have established that the economy can have two types of equilibria. In one type individuals conform to a common labor effort and develop levels of altruism that do not entail social exclusion of any particular subset of individuals. In the second type society is clustered into two or three groups with different patterns of behavior. Highly productive individuals work above the mean and earn the admiration of everyone. Low productive individuals perform below the mean and elicit no concern from the rest of society, including the other members of their own cluster. Finally, the “middle class” is composed of individuals who each supply the mean level of effort. This cluster is composed of individuals with intermediate productivities (and hence income). Notice that in this case we face an economy with clearly polarized sentiments based on personal characteristics. The high performing fraction of society attracts the esteem of everyone, while no one cares about the fate of the low productivity individuals other than themselves. The latter thus constitute a socially excluded group.

The first point we wish to make pertains to the relationship between taxes and the type of equilibrium that prevails.

Property 1 *The degree of redistribution is critical in generating either type of equilibrium. Economies with a low level of redistribution settle in the clustered equilibrium with social exclusion. For economies with high levels of redistribution, equilibria entail equal effort by all individuals and no exclusion. In the latter equilibrium, individual altruism can be distributed over the population without having to be correlated with individual characteristics such as productivity or income.*

The above property clearly follows from our Propositions 1, 4 and 6.

This result seems in keeping with Alesina, et. al., (2001) who observe different attitudes with respect to the poor in the US and in Europe.

Our next proposition follows from our analysis of three-cluster equilibria.

Property 2 *When the economy is in a three-cluster equilibrium, the degree of redistribution is critical in determining the “size” of the middle class. In the limit, when the tax becomes nil, there is no middle class in equilibrium, i.e., the range of productivities supplying the average labor time collapses to a point.*

This result also conforms with the finding in Alesina, et. al., (2001) that there is a much higher variance in labor supply in the US than in Europe and even less so in a highly redistributive country such as Sweden.

The previous properties focus on the interdependence between the fiscal system, the type of equilibrium sentiments and the possibility of social clustering. However, the distribution of individual productivities among the population also plays an essential role.

Property 3 *Basic inequality in productivities plays a role similar to that of taxes. Given the tax schedule, one or the other type of equilibrium is realized depending on the degree of inequality in β . Further, if*

$$\beta_n > n\beta_1, \tag{34}$$

there is no tax supporting a common effort equilibrium.

This property is consistent with Platteau and Seki’s (2000) findings that when inequality in abilities is too large there is no reward scheme sustaining the cohesiveness of the team. Further, it implies that a technological shock that increases the spread of individual productivities may precipitate the economy to depart from a common effort equilibrium and move towards a clustered equilibrium with social exclusion.

The following properties pertain to equal-effort equilibria.

Property 4 *In a common effort equilibrium altruism is inversely related to income.*

This follows immediately from the equilibrium condition applied to the labor supply function (11). High income individuals are less concerned

about others than poor people. This result also seems to conform with casual evidence.

The intuition for this result is quite straightforward. Individuals choose their effort by equating the marginal cost of effort to its marginal benefit. The latter consists of the benefit to oneself as well as the external benefit to others. Since the return to effort is high for high productivity agents, such agents must have lower levels of altruism in order to sustain the same choice of L across the entire population.

Property 5 *In a common effort equilibrium, output is increasing in the collective level of altruism.*

To see this, observe that for a given level of output, Y , the equilibrium degree of altruism, as pointed out in Property 4, is decreasing in productivity. Since β_n is the highest productivity, using (11) we have that

$$Y = \beta_n \left[1 - \tau + \tau \frac{\alpha_n}{n} \right]. \quad (35)$$

We can thus parametrize the level of output by the degree of altruism of the most productive, highest income individual. In view of (35) it is plain that higher altruism produces higher output.

In Proposition 2 we have characterized the range of output levels that can be supported in a common effort equilibrium. From (5) this range of output levels is

$$Y_m = \beta_n \left[1 - \tau + \frac{\tau}{n} \right] \leq Y \leq Y_M = \beta_1. \quad (36)$$

Notice that the maximum level of output attainable is independent of the tax rate, while the minimum depends inversely on τ . Increases in the tax rate widen the range of possible equilibrium levels of output. Further, in view of (35), the effect on output of an increase in taxation depends on the general degree of altruism. In the limit, when individuals are completely altruistic, $\alpha_i = n$ for $i = 1, \dots, n$, the tax rate has no effect on output. Thus we have the following property.

Property 6 *In a common effort equilibrium, the effect of an increase in the tax rate is to widen the range of values of potential equilibrium output levels. Further, the greater the overall level of altruism, the less the effect of a tax increase. Under perfect altruism the tax rate has no effect on output.*

Our final observation concerns the role of labor substitutability in the development of sentiments of concern for others. When introducing the model, we allowed that the elasticity of substitution between types of labor might be an important factor. Our results show that σ plays no significant role in our qualitative results.

6 Concluding remarks

Rather than treat agents' preferences as given, we have presented a model in which they are susceptible to endogenous influences; specifically, other agents' behavior affects our feelings for them. This allows scope for the institutional setting to affect the economic outcome by influencing behavior and hence agents' sentiments. To demonstrate the pathway by which such influence occurs, we have considered an example involving redistributive taxation. Our main conclusion is that the extent of altruism and social cohesion in a society is (at least partly) shaped by the degree of redistribution exhibited by the fiscal system. This seems to provide a fresh view on the nature of the "social contract" in different societies studied by Bénabou (2000) and Alesina, et. al., (2001).

One immediate direction for future research would be to wed this approach with a politico-economic model of voting over taxes. That is, in the context of redistributive taxation, the traditional political economy model takes agents' preferences as given and studies the emergence of a tax structure. Here, we have taken the tax structure as given and studied the resulting stationary patterns of utility interdependence, or extended preferences. A full equilibrium analysis would attempt to do both; i.e., it would focus on determining a fixed point of this two-way process.

Another, or subsidiary, line of research is the following. Since a change in the structure of individual sentiments will surely affect each individual's valuation of the effect of tax reform, this suggests that enriching our model to include voting over taxes seems likely to produce multiple equilibria. This would provide an alternative answer to the question raised by Bénabou (2000) and Alesina, et. al., (2001) concerning the co-existence of very different social contracts in otherwise similar societies. Like Bénabou, this would attribute such differences to distinct equilibria.

In the Introduction we have mentioned recent social trends toward greater disparity in earnings, less redistribution, more social exclusion and

less concern for the poor. One could also argue that with the recent boom in the technology sector and in the financial sector before that, highly productive, successful workers are becoming the new social elite, admired by all (consider Bill Gates).

How does our model account for these facts? Suppose we were in a cohesive society with no social clustering in which everyone worked, say, forty hours per week and where there was an unprecedented amount of redistribution. Our model suggests that if this economy were to undergo a technological shock which resulted in a substantial widening of the productivity gap within the population, then the previous scenario would no longer be possible. The economy would precipitate into a clustering equilibrium with a group of highly productive, high paid workers who are held in great esteem, and a group of low skilled workers for whom society has little regard. The prediction of the model seems to fit well with the reported social trends.

Also in the Introduction we referred to the issue of reward systems in teams. From the literature we have seen, there are a number of findings over which there seems to be little controversy. The first is that in very different scenarios (ranging from US law firms to Japanese fishermen), reward schemes involve a substantial amount of pooling. The second is that such pooling is considered essential for team spirit, or loyalty, to develop. Third, the higher the mutual esteem among team members, the higher is output.

Our model accords nicely with these facts. We obtain the result that without a sufficient commonality of interests, the economy settles into a clustering equilibrium in which one faction is completely disregarded by the rest. Further, in the equal effort equilibrium, output increases with the degree of esteem among the team members. Thus, another line for future research would be to formalize and to focus on the dynamics of “team spirit.”

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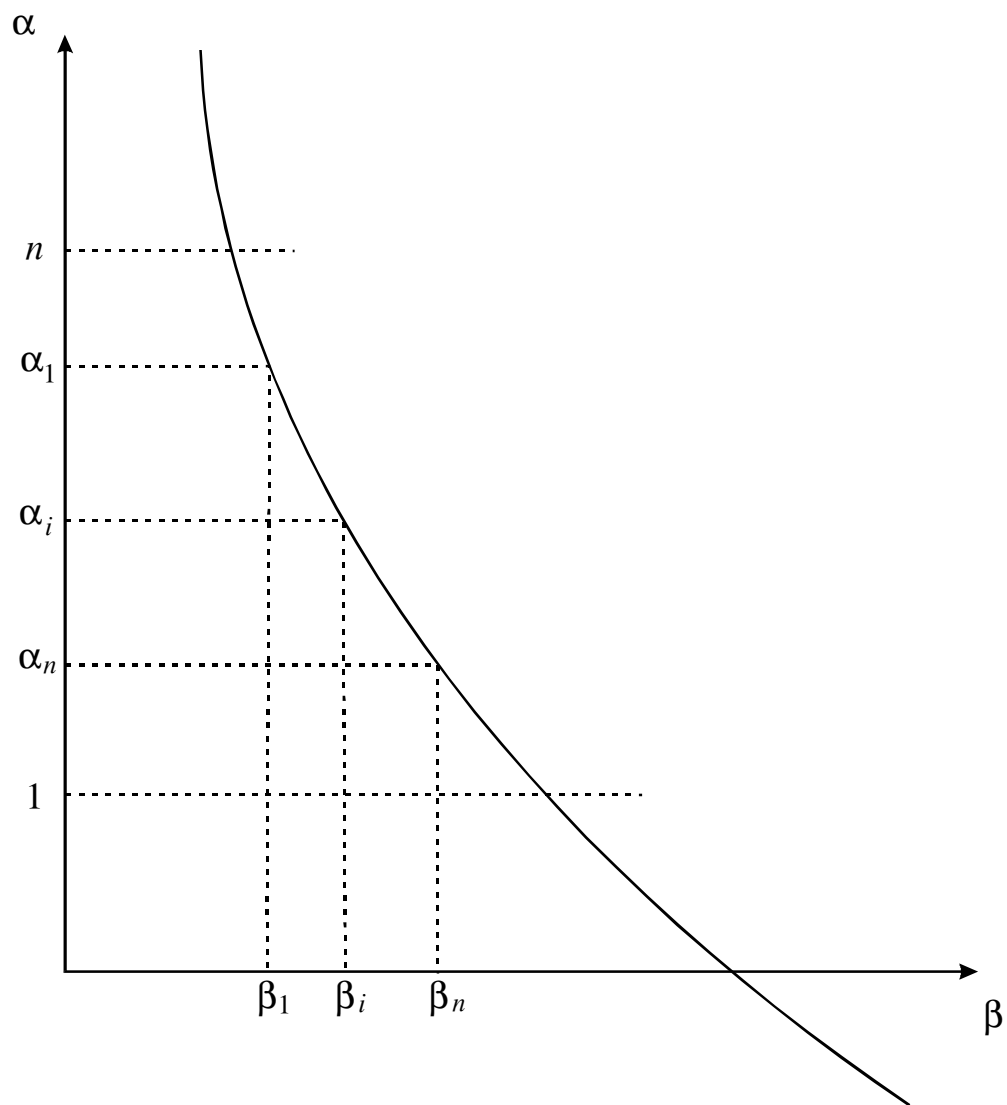


Figure 1. α, β -tradeoff for given L .

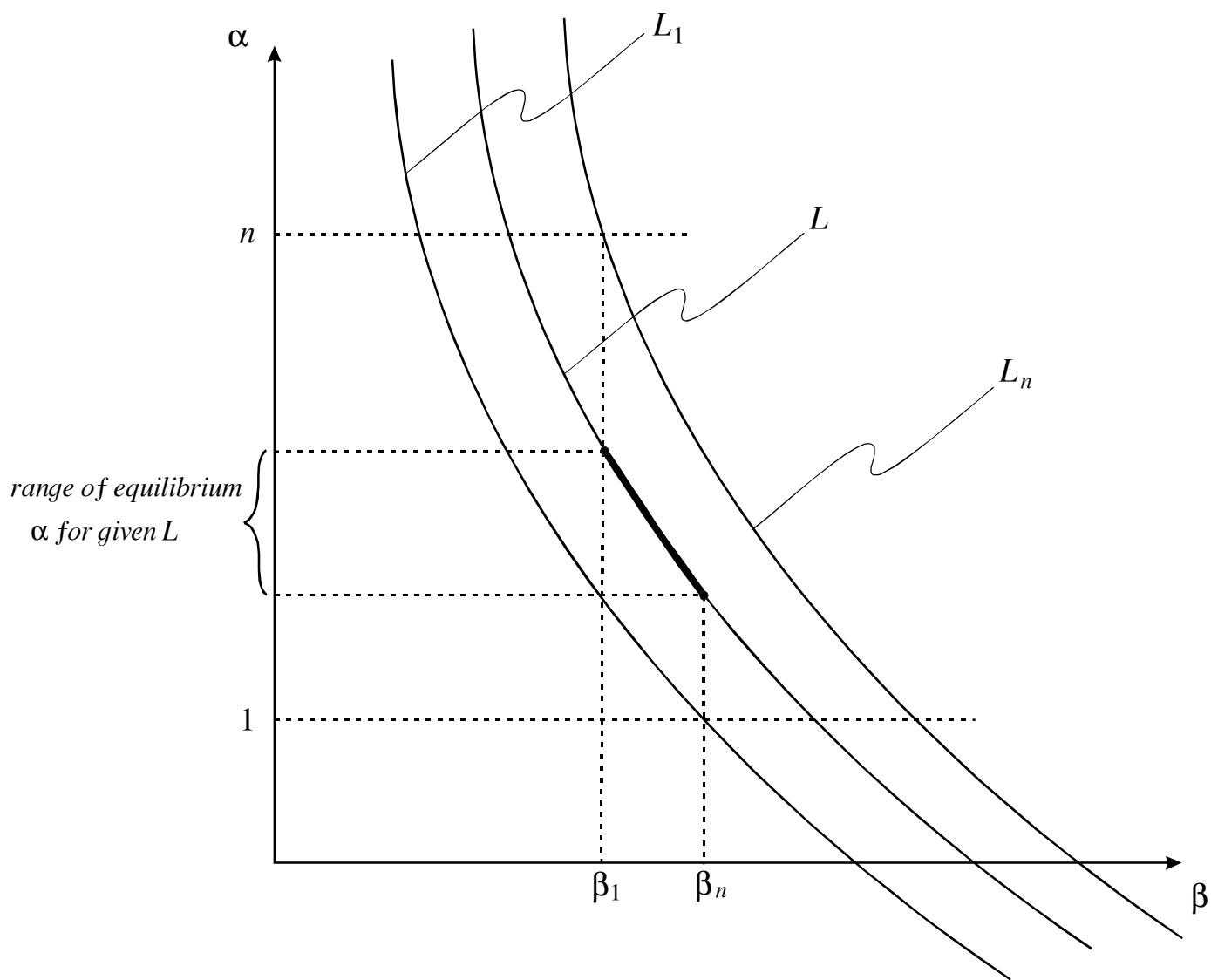


Figure 2. Range of equilibria for each L .

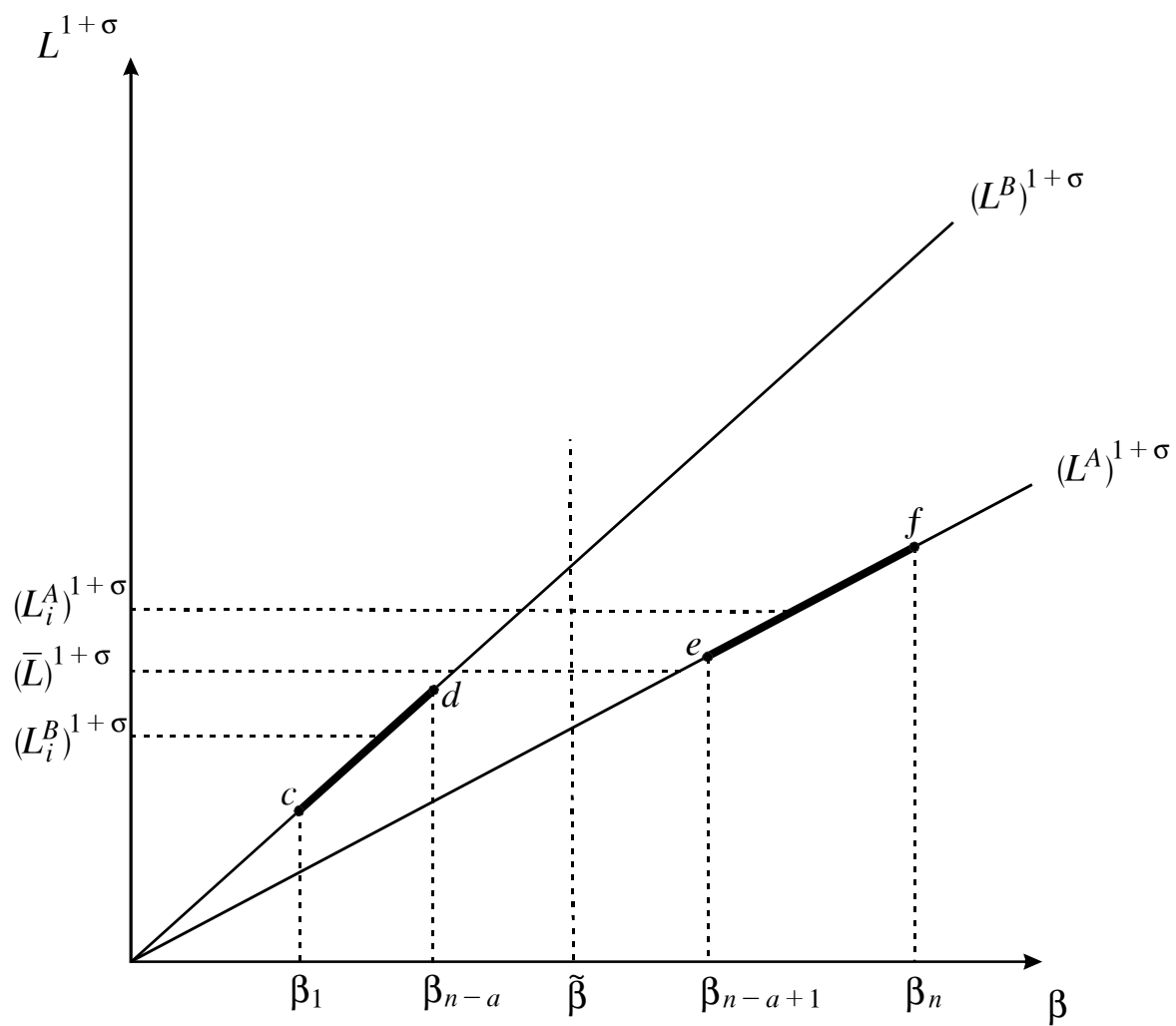


Figure 3. Labor supply in a two-cluster equilibrium.

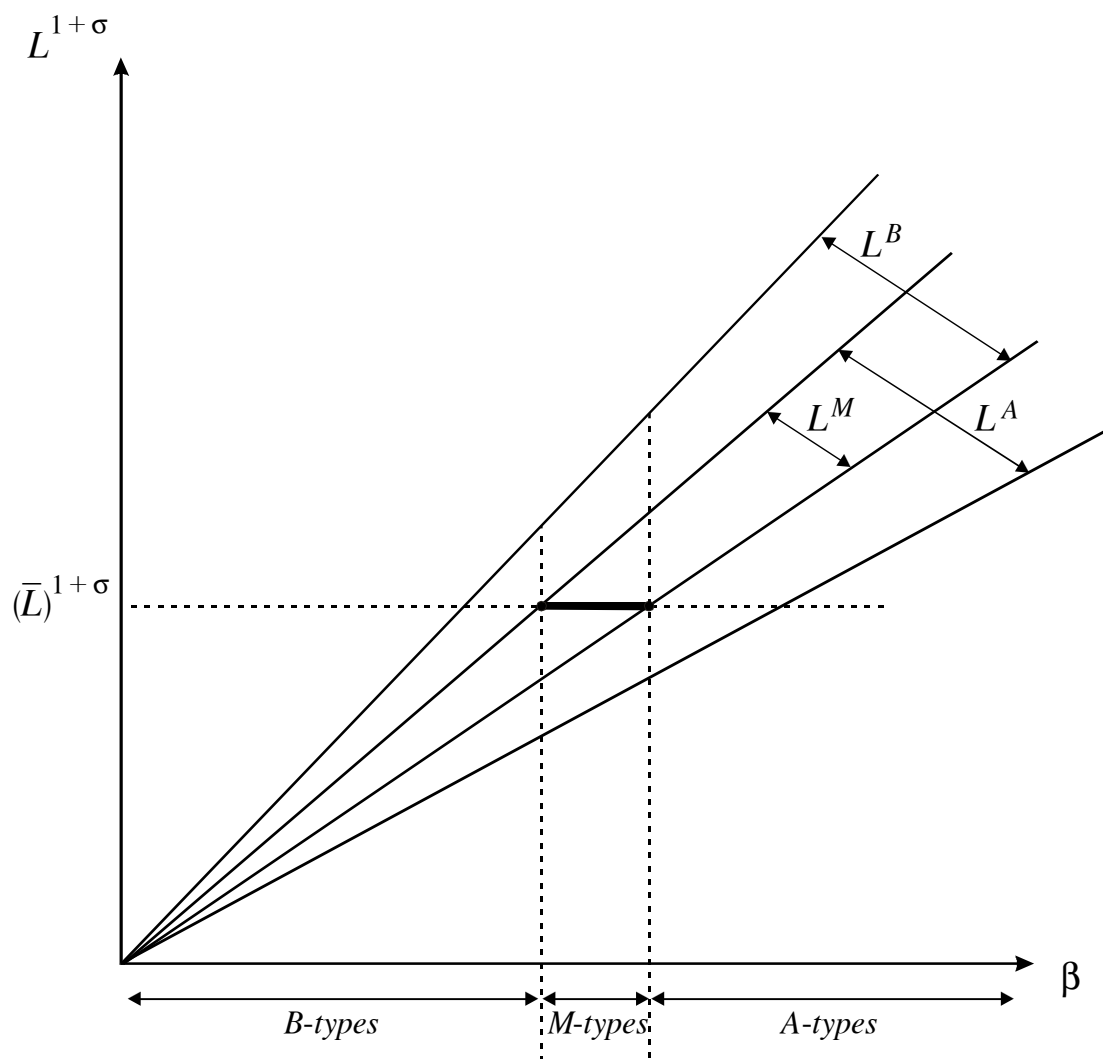


Figure 4. Labor supply in a three-cluster equilibrium.

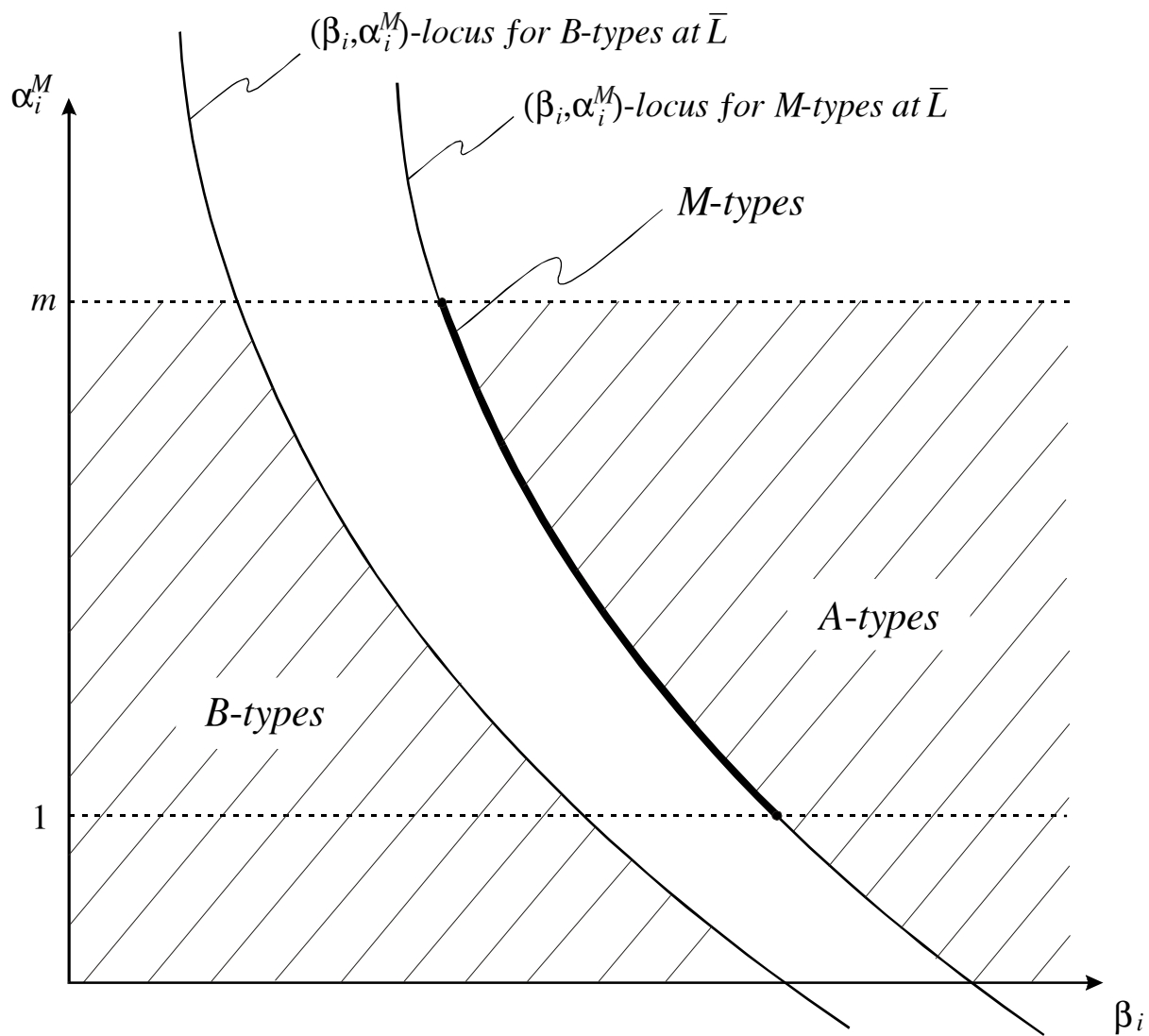


Figure 5. Type characteristics in a three-cluster equilibrium.