Competitive occupational choices with endogenous health risks*

Alberto Bennardo[†] and Salvatore Piccolo[‡]

July 12, 2005

Abstract

We study a general equilibrium model where agents' preferences and productivity depend on their health status, and indivisible occupational choices affect health risks. We show that efficiency typically requires agents of the same type to obtain different expected utility when assigned to different occupations. If health mainly affects production capabilities, workers with riskier jobs must get higher expected utilities under mild conditions. The same holds when health mainly affects preferences, provided that health and consumption (income) are sufficiently good substitutes (i.e. if health henancing consumption activities are sufficiently effective) ; while the converse obtains if health and consumption goods are complements (i.e. if health henancing consumption activities are *not* very effective) . As a corollary, compensating wage differentials which equalize the utilities of workers in different jobs are generally incompatible with efficiency. Competitive equilibria are first best if lottery contracts are enforceable, but typically not when agents can trade only assets with deterministic payoffs. Finally, we show that that, absent asymmetric information, there exist deterministic transfers policies which allow to achieve ex ante efficiency. By implementing cross subsidies across health insurance contracts, these policies subsidize occupations whose workers must obtain higher utility levels in the optimum.

1 Introduction

The paper studies a simple general equilibrium model where the aggregate distribution of health is determined jointly with the allocation of labor and consumption goods. The model has the following key features. First, the distribution of workers' health risks depends on their occupational choices. Second, health affects agents' productivity, and their

*We thank Pierre Andrè Chiappori, Jim Dana, David Levine, Joe Ostroy, Marco Pagano, Duncan Thomas, Bill Rogerson, Bill Zame and Lucy White for very useful discussions and comments. All remaining errors are ours.

[†]CSEF, University of Salerno and CEPR

[‡]CSEF, University of Salerno and CEPR.

preferences, namely their production and consumption capabilities. Third, occupational choices are indivisible, that is each occupation is defined by an indivisible set of tasks, and each worker can choose at most one occupation and the associated health distribution.[1] These assumptions capture some of the most significant real-life determinants and effects of individual health status. Indeed, occupational choices generally have both direct and indirect effects on health risks. They directly affect the distribution of future health states by influencing the likelihood of work-related injuries and diseases. Moreover, they may change workers' health risks indirectly by affecting their location choices, for instance by inducing them to locate in less safe areas (e.g., more crime-ridden or with poorer health facilities). Health status also affects workers' productivity, labor endowment and preferences, as largely documented by the empirical literature. Finally, an important real-world feature of most health risks associated to production activities is that they are diversifiable only to a very limited extent. This is due to the non-convexities associated to workers' specialization, which indeed lead most workers to choose a single occupation.

Our analysis encompasses both the direct and the indirect effects of occupational choices on health risks. We study the properties of efficient and equilibrium allocations in a setting where different distributions of health are associated to different occupations. Agents offer labor in a competitive labor market, produce several goods, and use financial (insurance) markets to transfer income across individual health states. At a more abstract level, we analyze a competitive set-up where agents (workers) choose among indivisible risky assets (occupations) paying either monetary or non pecuniary random returns (wages and health, respectively); and where non pecuniary contingent returns are only imperfectly transferable (health status cannot be separated from individuals and can be modified only within certain limits). Other examples of assets with these characteristics include occupations requiring minimal amounts of human capital, jobs with unpleaseant characteristics, as well as memberships to clubs and organizations.[2] Several of the results of our analysis hold in setttings where those assets are traded. For the sake of clarity, however, in this paper we will stick to the health application.

The extremely vast literature on work-related health risks and non pecuniary job attributes, that goes back to Adam Smith (see Evans and Viscusi (1993), Lucas (1972), and Rosen (1986), among many others), focuses on the determination of the equilibrium wage premia commanded by risky, or otherwise unpleasant, jobs.[3] It characterizes and estimates equilibrium wage differentials, under the "equilibrium" condition that competition equalize

[1] This assumption is imposed for simplicity. The important restriction is that a worker cannot choose an arbitrarily large number of jobs and offer a small amount of labor in each of it.

[2] Generic and sapecific human capital affect the distribution of agents' productivity as well as preferences. Similarly, different dis-amenities associated to the set tasks defining a job affect preferences as well as the productivity of the workers accomplishing that job. And, finally, the assets available to agents belonging to a club also influence both their preferences or their productivity.

[3] It formalizes the Smithian idea that "*the whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighborhood, be either perfectly equal or continually tending toward equality*".

the expected utility of workers with the same characteristics, which are assigned to different occupations in equilibrium[4]. Generally, such a condition is derived in partial equilibrium analyses of competitive labor markets, or directly imposed as part of the definition of competitive equilibrium. Moreover, a conventional wisdom has emerged within the literature on non pecuniary job attributes that *utility equalizing* wage differentials lead to market efficiency[5]. A central result of the paper, which motivate all our nalysis, is that Pareto optimality typically requires workers of the same type assigned to different occupations to get different (expected) utility levels. As a corollary, markets where wage differentials satisfy the *utility equalizing* condition in equilibrium does not, typically, implement first-best efficient allocations. Relatedly, we show that at any efficient allocation, the shadow value of the efficient consumption vector (calculated at the Pareto-optimal shadow prices) assigned to each type of worker typically differs from that of his initial endowment. Or, in other words, efficient allocations do not satisfy *budget balancing* and Pareto optimality requires transfers across occupations. These findings crucially depend on the imperfect transferability of health across agents[6], which make interdependent consumption and production decisions; precisely they rely on a basic optimality argument. Because health risks are specific to occupations, and both preferences and productivity are state-dependent, any pair of ex-ante identical workers with different occupations will generally feature different expected utility functions and budget constraints. For this reason, the equalization of marginal utilities of contingent goods across agents, which is a standard ex ante efficiency condition, will typically prevent either interim efficiency with *fair treatment* (i.e. the equalization of the expected utility of agents of the same type assigned to different jobs) or budget balancing.

The inconsistency between ex ante and interim optimality, and the *need* for Pareto efficient cross-transfers, open a number of theoretical and policy issues that we investigate in the paper. To begin with, for understanding the efficiency trade-offs between health, consumption and production choices, it becomes crucial f to characterize either the efficient utility's *wedges* across occupations, or the optimal cross-jobs transfers .

Moreover, the sub-optimality of interim efficient allocations also raises the question of what financial markets can be used to implement Pareto efficient cross-transfers in a competitive equilibrium. Finally, taking a policy point of view, the need for cross-transfers across occupations leads to study what policy interventions may result welfare beneficial when, according to a widespread belief, actual markets implement interim efficient allocations[7].

[4] Cole-Prescott (1997) who studya moral hazard model, is, however, an exception in this respect.

[5] See, for instance, the textbooks of Ehrenberg-Smith (2003) and Viscusi et al (2000).

[6] This imperfect transferability, indeed, make invalid in our setting the separability result between individual consumption and production choices which is standard in welfare analysis (see Mas Colell et al. (1995)).

[7] This is an important issue to address also because, on the one hand, real life insurance markets for work-related health risks are often heavily regulated; while, on the other hand, the rationales for policy interventions are seldom clearly expressed.

The first part of the paper provides a general characterization of Pareto optima. It shows that the properties of ex-ante efficient utility's wedges across occupations and cross-jobs transfers crucially depend both on the riskiness of health distributions and on the relative extent to which health affects agents' consumption and production capabilities. Precisely, by ordering the health risk of different occupations according to first-order stochastic dominance we obtain the following results. If health mainly affects production capabilities, workers with riskier jobs must get higher expected utilities under mild conditions on the senitivity of the individual labor supply with respect to wage and health. The same holds when health mainly affects preferences if health and consumption (income) are sufficiently good substitutes (which is the case if health henancing consumption activities that agents may undertake are sufficiently effective). The converse obtains if health and consumption goods are complements (which is the case whenever workers' health is basically determined by the Nature and the effect of health henancing consumption activities is relatively small). Moreover, we show that the shadow value of consumption of the workers obtaining relatively higher utility levels is larger than the shadow value of their produced and non produced resources.

In the second part of the paper we turn to the competitive analysis. We consider two alternative contractual regimes, one where lottery contracts are enforceable and the other where they are unenforceable. In the former, there exist competitive insurance markets to cope with all idiosyncratic risks as in Malinvaud (1973) and Cass, Chichilnisky and Wu (1996), but only financial contracts with deterministic returns are enforceable. In the latter regime, agents can "trade" also lottery contracts, i.e., contracts with random payoffs. The first regime turns out to be the natural benchmark for understanding the welfare properties of competitive markets. The analysis of the case of unenforceabilities of lotteries is warranted by several reasons. First, as we already pointed out, all the theoretical and empirical literature on non-pecuniary job attributes and compensation wage differentials has only considered contracts with deterministic payoffs. Since this literature is a natural reference point for the problem at hand, we wish to know under what conditions competition with deterministic contracts leads to efficiency. Second, on the empirical side, the use of lottery contracts (or the use of other financial instruments that may replicate allocations obtainable through random contracts) does not appear to be extremely widespread in real markets.[8] Finally, on a theoretical ground, the use of "optimal" random contracts may sometimes result severely restricted either moral hazard problems and limited liability constraints, or by the costs of verifying characteristics and outcomes of the random devices that are needed to implement them.

[8] Kehoe, Levine and Prescott (2001) show that, if there exists a sufficient number of assets paying units of numeraire in sunspot states of the world, competitive equilibria are first-best efficient. At least in our setting, however, efficient trades of financial instruments leading to random allocation are typically such that workers must take possibly big short positions in the asset markets. This is often impossible in real-life markets also because of incentive problems.

Standard arguments imply that a competitive equilibrium exists and is typically locally unique both in the high and in the low-transaction-cost regimes. In both cases "insurance" is traded at fair prices, consumption allocations differ across workers of the same type with different occupations, and equilibrium wage differentials provide a premium for health risks. However, expected wage differentials, and more generally the efficiency properties of competitive equilibria, differ markedly in our contractual environments. If lottery contracts are enforceable, competition leads to efficiency and both welfare theorems hold. Lottery contracts ensure ex-ante optimality precisely by allowing agents to make the cross-job transfers that are needed for the equalization of marginal utilities. Conversely, if lottery contracts are unenforceable, competitive equilibria satisfy a specific interim efficiency condition, guaranteeing the equal treatment of the agents of the same type assigned to different occupations; but they are typically not ex ante efficient. Indeed, in this case, by equalizing the expected utilities of workers employed in different sectors, competition creates a wedge between their marginal utilities of expected income.

The results of our welfare analysis are related with the literature on indivisibilities[9] and the general equilibrium literature with asymmetric information stemming from Prescott-Townsend (1984).[10] This literature has developed several examples where lotteries are showed to be welfare beneficial. The role of lotteries in these contexts is to convexify asymmetric information environment, where incentive constraints typically introduce natural non convexities.[11] In our our environment we prove the stronger result that random contracts are almost always necessary to achieve efficiency through the market. Moreover, differently from most of the literature on asymmetric information, our paper focus on the role that lottery contracts play as market devices in implementing cross-jobs transfers.[12]

Finally, in the last part of the paper we show that Pareto optima can be implemented through deterministic transfers' policies. These policies impose cross-subsidies among insurance policies designed for workers choosing different occupations and minimal wages aimed at ensuring a natural non-manipulability requirement of the policy scheme. The sign and the magnitude of the transfer received by each worker is then determined by the difference between the (shadow) value of his consumption and that of his production and

[9] Garrett (1995) studies lottery equilibria in economies with indivisibilities, but mainly focuses on existence issues in finite economies, and does not characterize Pareto optimal allocations.

[10] See also Allen and Gale (2003), Arnott and Stiglitz (1986), Bennardo-Chiappori (2003), Cole (1990), Kehoe, Levine and Prescott (2001), Rogerson (1988), Rustichini and Siconolfi (2003), and Bennardo (2004) among others.

[11] Our paper has in particular some close connections with that of Rustichini and Siconolfi. These authors, while mainly interested in economies with asymmetric information, also show that in economies with symmetric information, state-dependent preferences and endogenous individual risks, competitive equilibria are Pareto optimal if lottery contracts are enforceable. However, they neither characterize Pareto-optimal allocations with state-dependent preferences nor investigate under what conditions assets (contracts) with non-degenerate random payoffs are effectively welfare beneficial.

[12] See however Bennardo (2004) for a similar result on a multicommodity production economy with moral hazard.

endowment, both calculated at the optimal shadow prices. Notewhorty, Rogerson (1988) provides an example where random contracts implementing transfers across workers are welfare improving. Even if the author does not provide this intepretation, these transfers can be interpreted as a form of unemployment insurance. Specifically, in Rogerson example unemployment insurance may be welfare beneficial if labor supply choices are indivisible within each occupation (workers can choose whether to work or not but not how much to work), and a if positive fraction of workers are unemployed at equilibrium. Differently, in our model cross-jobs transfers policies are typically welfare beneficial, even if we assume perfectly divisible labor supply choices within each occupation and hence no equilibrium unemployment.

2 The Economy

Demography, consumption goods and preferences. A continuum of measure 1 of consumers-workers produce C consumption goods. There exists a finite set, $I = \{1, ..., I\}$, of agents' types; and μ_i is the total fraction (measure) of type-i agents. Agents face health risks that may affect their preferences, endowments and productivity. The set $\Theta = \{\theta_1, ..., \theta_N\}$ of possible health states is assumed to be finite, and $\theta \in \Theta$ represents a generic health state. In the economy there are $C + 1$ consumption goods, leisure and C produced goods. Type-i agents have an endowment $e_i \in \Re^C_+$ of produced goods which is the same in all individual states, and an amount L of time which is allocated between work, l, and leisure, x_L. The maximal fraction of total time that each agent can devote to work, $L(\theta)$, may depend on his health state; and $L(\theta)$ is weakly increasing in θ, so that the maximal amount of work that agents can offer depends on their health status.[13] Agents' (health) state-dependent preferences are represented by the utility function $U_i(x, \theta)$, where $x = (x_1, .., x_C, x_L) \in \Re^C_+ \times [0, L]$. $U_i(x, \theta)$ is twice continuously differentiable, weakly increasing and strictly concave, and has indifference surfaces with closure in $\Re^C_{++}$. As we wish to take into account the existence of goods such as medical treatments and health-enhancing consumption activities, we assume local non satiation and allow $D_cU_i(x, \theta) = 0$ for $(c,\theta) \in \hat{C} \times \hat{\Theta}$ with $\hat{C} \subset C$, and $\hat{\Theta} \subset \Theta$.[14] Whenever it will be convenient, will also assume the Inada conditions, $D_cU_i(x, \theta) \to +\infty$ as $x_c \to 0$, and $D_cU_i(x, \theta) \to 0$ as $x_c \to \infty$ for all $C/\hat{C}$ and θ, and for all i.

Technologies and uncertainty. Competitive firms produce goods by employing

[13] This assumption is intended to capture real-life situations where a worker can perform with an appropriate quality standard a labor activity only for a limited amount of time, that depends on his health state. This is the case, for instance, of aircraft pilots, as the amount of hours they can devote to flying activities cannot overcome a certain limit defined by safety standards. Similarly, a driver, a sportsman or a miner, who typically suffer of overuse syndromes cannot safely work more than a certain number of hours in a year.

[14] In the same spirit, Makowski-Ostroy (1995), assume that different subsets of existing commodities may enter in the utility function of agents assigned to different occupations.

workers, and labor is the only production factor. Each firm can hire a positive measure of agents, while each worker can work for at most one firm, as specialization prevents from performing different jobs. There are $T = C$ production sectors and only one type of occupation is offered in each sector. Each worker's contribution to production is measurable and may depend on his health state. Precisely, a type-i worker who is employed in sector t, and supplies l_i^t units of labor, and produces $y_i^t(\theta) = a_i^t(\theta) l_i^t$ units of commodity t when his health state is θ; where $a_i^t(.)$, is weakly increasing in θ. Moreover, each worker's distribution of health states, $\langle p_i^t, \Theta \rangle$, with $p_i^t = (p_i^t(\theta_0), ..., p_i^t(\theta_N))$, depends on his occupation. Finally, health shocks are identically and independently distributed acroos type-i workers in the same occupation, and independently distributed across sectors[15]. The endogeneity of the health distribution can be due to the direct effects of labor activities on prospective workers' health. But it can also be interpreted as the consequence of localization choices, which are determined by occupational choices.

Timing. The economy lasts two periods, $\tau = 0, 1$; at $\tau = 0$, agents trade in financial and labor markets. At $\tau = 1$, health shocks are realized; subsequently agents supply labor in production, and consumption goods are traded and consumed.The contracting space (the set of enforceable contracts) will be defined in section 4.

Throughout, we will use the following notation: $x_i^t(\theta)$ is a generic state contingent consumption vector for a type-i agent occupied in sector t, with $x_i^t = (x_i^t(\theta))_{\theta \in \Theta}$ and $x = (x_1^t(\theta), ..., x_I^t(\theta))_{\theta \in \Theta}^{t \in T}$; $l_i^t(\theta)$ is a state contingent of labor for a type-i agent occupied in sector t. Finally $\alpha_i = (\alpha_i^1, ..., \alpha_i^T)$ represents an assignment of type-i workers to production sectors.

3 Ex ante and Interim Pareto Optimality

Ex ante Pareto optimality. Let $u_i^t(x_i^t) = \sum_{\theta \in \Theta} p_i^t(\theta) U_i(x_i^t(\theta), \theta)$ and let $\bar{x}_{ic}^t = \sum_{\theta \in \Theta} x_{ic}^t(\theta) p_i^t(\theta)$ be the expected consumption of commodity c of type-i worker employed in sector t.

By the law of large numbers, a *feasible allocation* of consumption goods, labor, and workers, (x, α) must satisfy the following constraints:

$$\sum_{i \in I} \mu_i \sum_{t \in T} \alpha_i^t \bar{x}_{ic}^t \leq \sum_{i \in I} \mu_i (e_i^c + \alpha_i^c y_i^c), \ \forall \, c \in C \tag{1}$$

$$l_i^t(\theta) + x_{iL}^t(\theta) \leq L, \ \forall \, \theta \in \Theta, \ t \in T; \ \sum_{t \in T} \alpha_i^t = 1, \ \forall \, i \in I \tag{2}$$

[15] Note however that our model in the present formulation does not take into account the possibility that agents invest in prevention activities which directly affect their health distribution. The effects of prevention are considered in a conclusive section.

We discuss in the conclusive section how our results extend to economies with aggregate risk.

where $y_i^t = \sum_{\theta\in\Theta} p_i^t(\theta)\, a_i^t(\theta) l_i^t(\theta)$ for all t and i. Let F the set of feasible allocations, and let:

$$U = \left\{ \bar{u} = (\bar{u}_2, ..., \bar{u}_I) : \exists\, (x, \alpha) \in F,\ s.t. \sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i,\ \forall\, i = 2, .., I \right\}$$

A (ex ante) Pareto optimum maximizes $\sum_{t\in T} \alpha_1^t u_1^t(x_1^t)$, subject to $\sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$, $\forall\, i \in I$ for some $\bar{u} \in U$.

Note that we are not imposing that all agents of the same type get the same expected utility independently from their occupation in the optimum. Such a condition is typically satisfied in the optimum of convex economies, but there is no reason to impose it as part of the definition of first best allocations in our setting. Moreover, by maximizing $\sum_{t\in T} \alpha_1^t u_1^t(x_1^t)$, one implicitly assumes that efficient mechanisms can randomly assign agents to occupations.

Moreover, we are not taking into account the possibility that agents obtain random consumption allocations in the optimum. In our setting, this is unrestrictive, because a standard risk aversion argument implies that random consumption allocations are always suboptimal.

Interim Pareto optimality. The following definition of interim Pareto optimality will play a crucial role in the welfare analysis of competitive equilibria with contractual incompleteness and unenforceable lottery contracts.

An *interim optimal allocation with equal treatment* maximizes $\sum\limits_{t\in T} \alpha_1^t u_1^t(x_1^t)$ under (1)-(2) and the additional constraints $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$ for each $t \neq t'$ such that $\alpha_i^t > 0, \alpha_i^{t'} > 0$.

4 Competitive Equilibria

Throughout we shall postulate the existence of competitive spot markets for all goods, as well as markets for insuring *all* risks through assets with *deterministic* payoffs. We shall study either the case where only deterministic contracts (assets with random payoffs) are *enforceable* or that in which agents can also sign lottery contracts. Studying both cases is useful to fully understand either the beneficial role that random contracts may play in our economy, or the effects of a somewhat natural market friction, that may prevent their use.

4.1 Competitive equilibrium with *deterministic contracts*

Following the approach taken in several contributions of the literature on individual risks, we introduce competing, risk neutral[16] intermediaries who offer securities paying in the individual health states. Specifically, let $h_{i\theta}^t$ a security paying one unit of numeraire in the individual health state θ, to type-i agents employed in the t-th sector. Let $z_{i\theta}^t$ and $\hat{z}_{i\theta}^t$ be

[16] Intermediaries' risk neutrality is, as usual, justified by the assumption of large numbers.

the units of $h^t_{i\theta}$ purchased by type-i agents employed in sector t, and the per capita units of this security offered in the market, respectively. And denote $\phi^t_i(\theta)$ the price of $h^t_{i\theta}$.

Production firms trade state contingent labor services and consumption goods at linear prices. Let denote $w^t_i(\theta)$ the state contingent wage of type-i worker in the t-th occupation (sector), with $w^t_i = (w^t_i(\theta))_{\theta\in\Theta}$.[17] Finally, $q \in \Re^C_+$ denotes a vector of *spot prices* and q_c the c-th component of this vector[18].

Because of labor supply indivisibilities, it is expositionally convenient[19] to consider the possibility that workers choose their occupation by using mixed strategies. Denote $\varphi_i = (\varphi^1_i, .., \varphi^t_i, ..., \varphi^T_i) \in \Delta^T$, the T dimensional simplex, a generic probability vector according to which a type-i worker chooses his occupation. By the law of large numbers φ^t_i is also the fraction of type-i agents who ex post get an occupation in sector t.

A competitive (Walrasian) equilibrium with deterministic contracts is then an allocation $(x^{t*}_i, \varphi^{t*}_i)^{t\in T}_{i\in I}$, a per capita vector of securities' offers and purchases $(\widehat{z}^{t*}_i, z^{t*}_i)^{t\in T}_{i\in I}$ and a vector of state contingent prices $(q, (\phi^t_i, w^t_i))^{t\in T}_{i\in I}$ satisfying the following conditions.

(I) Type-i agents maximize

$$\left(x^{t*}_i, \varphi^{t*}_i, z^{t*}_i\right) \in \ \arg\max \sum_{t\in T} u^t_i(x^t_i)\varphi^t_i \tag{3}$$

$$s.t. \ \sum_{c\in C} q_c(x^t_{ic}(\theta) - e^c_i) = w^t_i(\theta)\left(L - x^t_{iL}(\theta)\right) + z^t_i(\theta) \ \forall\ \theta \in \Theta,\ t \in T \tag{4}$$

$$\sum_{\theta\in\Theta} z^t_i(\theta)\,\phi^t_i(\theta) \le 0,\ \forall\ t \in T \tag{5}$$

where (4) are the spot market budget constraints and (5) is the initial period budget constraint.

(II) Production firms' and intermediaries, respectively solve:

$$l^{t*}_i \in \arg\max \sum_{\theta\in\Theta} p^t_i(\theta)\left(q_t y^t_i(\theta) - w^t_i(\theta)\, l^t_i(\theta)\right) \quad \text{s.t.} \quad y^t_i(\theta) \le a^t_i(\theta)\, l^t_i(\theta),\ \forall\ \theta \tag{6}$$

$$\widehat{z}^{t*}_i \in \arg\max \sum_{\theta\in\Theta} (\phi^t_i(\theta) - p^t_i(\theta))\widehat{z}^t_i(\theta) \quad \text{s.t.} \quad \sum_{\theta\in\Theta, i\in I} \mu_i p^t_i(\theta)\,\widehat{z}^t_i(\theta) \ge 0,\ \forall\ t \in T \tag{7}$$

[17] The introduction of individual risks in a competitive settings requires that assets' payoffs which are contingent on individual shocks must also be contingent on agents' types. This has been clarified in Malinvaud (1973) and Rustichini Siconolfi (2003),

[18] In the absence of aggregate uncertainty, spot market prices are independent from the realizations of individual shocks that wash-out at the aggregate level.

[19] In our continuum setting, equilibrium mixed strategies on occupations can be interpreted as different fractions of agents choosing a pure strategy in equilibrium. In other words, given any mixed strategy equilibrium strategy profile there exists a payoff equivalent profile of pure strategies satisfying all feasibility conditions. Using mixed strategies is, however, convenient for expositional reasons.

(III) Consumption, labor and financial markets clear:

$$\sum_{i\in I}\mu_i \sum_{t\in T}\varphi_i^t \bar{x}_{ic}^{t*} \leq \sum_{i\in I}\mu_i (e_i^c + \varphi_i^c y_i^c),\ \forall\, c \in C \tag{8}$$

$$L - x_{iL}^{t*}(\theta) = l_i^{t*}(\theta),\ \text{and}\ z_i^t(\theta) = \widehat{z}_i^{t*}(\theta),\ \forall\, i \in I,\ t \in T \text{ and } \theta \in \Theta. \tag{9}$$

4.2 Competitive equilibrium with *lottery contracts*

We now introduce lottery contracts in our competitive setting, assuming that agents can buy lotteries from financial intermediaries before making any other market trade. Lotteries allow to obtain a vector of prizes (a payoff vector in units of numeraire) with positive probabilities.[20] Formally, we define a lottery contract, $\mathcal{C} = ((\gamma, G), \rho(\gamma, G))$, as: (i) a finite distribution (γ, G) with probabilities $\gamma = (\gamma^1, ..., \gamma^M) \in \Delta^M$ and payoffs $g = (g^1, ..., g^M) \in \Re^M$, with M finite, and, (ii) a price $\rho(\gamma, G) \in \Re$. The interpretation of $\mathcal{C}$ is as follows: an agent signing $\mathcal{C}$ with a financial intermediary pays him the price $\rho(\gamma, G)$, while the financial intermediary commits to deliver to the agent the payoff g^m with probability γ_m. A random devise, whose characteristics are publicly verifiable, is then used by the contracting parties. Such a device chooses an artificial state of the world by selecting a positive integer $m \in \{1, ..., M\}$ with probability γ^m. Subsequently, the intermediary pays g^m to the agent whenever the integer m turns out to be selected. The expected profit that a generic intermediary makes from signing $((\gamma, G), \rho)$ is $\rho(\gamma, G) - \sum_{m\in M}\gamma^m g^m$.

A general formulation of the competitive equilibrium in the space of random allocations would require all possible lottery contracts (an infinite set) to be priced in equilibrium (as in Rustichini-Siconolfi (2003)) and would allow agents to possibly sign many lottery contracts. In order to avoid the technicalities arising with an infinite dimensional commodity space, we will make the following unrestrictive assumptions: (i) all fair lottery contracts with a payoff support of dimension $M \leq T + 1$ are offered in the market; (ii) each agent signs at most one lottery contract with support of dimension $M = T + 1$; and (iii) an agent will offer labor in sector t if and only if he receives the t-th payoff of the lottery contract he has signed.

An arbitrage argument justifies (i). Moreover any finite distribution of net payoffs that can be obtained by means of N fair lottery contracts can also be obtained through a single fair contract [21]; and, risk aversion implies that it is individually optimal for all agents to choose a lottery contract with at most $M = T + 1$ payoffs[22] different from zero.[23]. Hence,

[20] Following Arnott-Stiglitz (1987), in the literature such randomizations have been referred to as ex ante random contracts.

[21] Such a contract is defined by probabilities and payoffs which are linear combinations of the probabilities and the payoffs of the the N fair lottery contracts

[22] The dimension is $T + 1$ and not T because an agent may also decide not to supply labor conditionally on receiving one of the possible payoffs of the contract.

[23] A risk averse agent will never find it optimal to choose a lottery contract such that: (i) he receives

(ii) is unrestrictive. Finally, once (ii) is imposed, (iii) amounts to be a merely convenient notational convention.

A competitive (Walrasian) equilibrium with lottery contracts is then an allocation $(\tilde{x}_i^t)_{i\in I}^{t\in T}$, a per capita vector of assets offers and purchases $(\widehat{z}_i^t, \tilde{z}_i^t)_{i\in I}^{t\in T}$, a vector of lottery contracts $(\mathcal{C}_i)_{i\in I}$, and a vector of prices $(\tilde{q}, \tilde{\phi}_i^t, \tilde{w}_i^t)_{i\in I}^{t\in T}$ satisfying the following conditions:

(I) Type-i agents maximize their utility by choosing:

$$(\tilde{x}_i, \tilde{z}_i^t, \mathcal{C}) \in \underset{\mathcal{C}\in\Gamma}{\arg\max} \sum_{t\in T} \gamma^t u_i^t(x_i^t) \tag{10}$$

$$s.t. \quad \sum_{c\in C} q_c(x_{ic}^t(\theta) - e_i^c) = w_i^t(\theta)\left(L - x_{iL}^t(\theta)\right) + z_i^t(\theta) + g^t - \rho(\gamma, G), \quad \forall\, \theta \in \Theta,\ t \in T \tag{11}$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\, \phi_i^t(\theta) \leq 0,\ \forall\, t \in T \tag{12}$$

where (11)-(12) are the budget constraints and: $\Gamma = \left\{(\gamma, G), \rho(\gamma, G)) : \rho(\gamma, G) = \sum_{t\in T} \gamma^t g^t\right\}$.

(II) Production firms' and intermediaries, solve the same programs (i.e., (6) -(7)) as in the competitive equilibrium with deterministic contracts :

(III) Consumption financial and labor markets clear:

$$\sum_{i\in I} \mu_i \sum_{\theta\in\Theta, t\in T} \widetilde{\gamma}^t \widetilde{x}_{ic}^t(\theta) p_i^t(\theta) = \sum_{i\in I} \mu_i (e_i^c + \widetilde{\gamma}^c \sum_{\theta\in\Theta} p_i^c(\theta)\, \widetilde{l}_i^c(\theta)\, a_i^c(\theta)), \quad \forall\, c \in C \tag{13}$$

$$L - \tilde{x}_{iL}^t(\theta) = \tilde{l}_i^t(\theta), \text{ and } \tilde{z}_i^t(\theta) = \widehat{z}_i^t(\theta),\ \forall\, i \in I,\ t \in T \text{ and } \theta \in \Theta \tag{14}$$

5 Pareto Optimal Allocations

In this section we characterize Pareto optimal allocations. Let $\lambda = (\lambda_2, ..., \lambda_I)$ and $\eta = (\eta_1, ..., \eta_C)$ be the vectors of Lagrange multipliers associated, respectively, to the utility constraints, $\sum_{t\in T} \alpha_i^t u_i(x_i^t) \geq \overline{u}_i$ for all i and to the feasibility constraints. Set $\lambda_1 = 1$, assuming interior solutions, the first order conditions with respect to $(x_i^t(\theta), x_{iL}^t(\theta), \alpha_i^t)$ of the (ex ante) Pareto program are respectively:

$$\lambda_i D_c U_i(x_i^t, \theta) - \eta_c \mu_i = 0 \text{ for all } c, t \text{ and } i \tag{15}$$

$$\lambda_i U_{ix_L}(x_i^t, \theta) - \eta_t a_i^t(\theta) \mu_i = 0 \text{ for all } t \text{ and } i \tag{16}$$

the payoffs g^m and $g^{m'}$, with $g^m \neq g^{m'}$, with positive probabilities γ^m and $\gamma^{m'}$ respectively, and (ii) he chooses to work in sector t either when he receives g^m or $g^{m'}$. This is true as by convexity there always exists another fair contract, say $\mathcal{C}'$, which pays $\gamma^m g^m + \gamma^{m'} g^{m'}$ with probability $\gamma^m + \gamma^{m'}$, which is strictly preferred to $\mathcal{C}$.

$$\lambda_i(u_i(x_i^t) - u_i(x_i^{t'})) - \mu_i(Z_i^t - Z_i^{t'}) = 0 \text{ for all } t \neq t' \text{ and } i \tag{17}$$

where for all $t = 1, .., T$,

$$Z_i^t = \sum_{c \in C, \theta \in \Theta} \left(p_i^t(\theta) x_{ic}^t(\theta) - e_i^c \right) \eta_c - \eta_t \sum_{c \in C, \theta \in \Theta} p_i^t(\theta) a_i^t(\theta) \left(L - x_{iL}^t(\theta) \right)$$

is the difference between the value of the consumption of a type-i workers employed in sector t, *as measured by the vector of shadow prices* η, and the sum of the values of its *endowments* and its production.

It will be convenient in the following to denote $\mathcal{F}(.) = 0$ the system (15)-(17). As standard, (15) and (16) imply that marginal rates of substitution between all pairs of state contingent commodities, including leisure, are the same for all types. The first-order conditions with respect to α (17) are less standard, and play a crucial role in our analysis. Precisely, they indicate that differences in utilities $\Delta u_i(t,t') = u_i(x_i^t) - u_i(x_i^{t'})$ are proportional to $\Delta Z^i(t,t') = Z_i^t - Z_i^{t'}$. Only if $\Delta Z^i(t,t') = 0$, the type-i workers assigned to the $t-th$ and $t'-th$ occupations will get the same utility, and ex ante and interim optima coincide. This is, indeed, one of the distinguishing feature of our setting.

Next proposition will show that, typically, $\Delta u_i(t,t') \neq 0$, at $\mathcal{F}(.) = 0$, implying that interim efficiency is generally incompatible with ex ante efficiency. In order to prove this result, we need to introduce some notation. Let $t_i = \left(\left\langle p_i^t, \Theta \right\rangle, A_i^t \right)_{t \in T}$, with $A_i^t = \left\{ a_i^t(\theta_1), ..., a_i^t(\theta_N) \right\}$, the sector t technology available to type-i workers. Finally, let $\varepsilon = \langle e, \mathbf{t}, U \rangle$ represent a specific economy with aggregate endowment $e \in \Re_{++}^C$, a vector of production technologies $\mathbf{t} = (t_1, ..., t_I)$ and a profile of utility functions $U = (U^i, ..., U^I)$. The set of possible economies is then defined as $\mathcal{E} = \Re_{++}^C \times T \times \mathcal{U}$, where $T = \Re_{++}^{I \times C \times \#\Theta} \times \Delta^\Theta$, $\mathcal{U} = \prod_{i=1}^I \mathcal{U}^i$; and where $\mathcal{U}^i$ is the set of type-i admissible utility functions' profiles which will be precisely defined in the proof of the next proposition.

Proposition 1 *For each vector of reservation utilities $\bar{u}$, the Pareto optimum is unique. Moreover, the subset $\mathcal{S} \subset \mathcal{E}$ where ex ante and interim Pareto optima are different is generic if the number of produced goods is larger than the number of agents' types.*

The proof, which is provided in the appendix, applies a transversality argument; it relies on the following argument. As the Pareto optimum is unique for each $\bar{u}$ because of strict convexity, it is defined by $\mathcal{F}(.) = 0$. Moreover, by definition, interim efficiency imposes $\Delta u_i(t,t') = 0$ for all α^{it} and $\alpha^{it'}$ strictly positive. Then either the solution of $\mathcal{F}(.) = 0$ satisfies the additional conditions $\Delta u_i(t,t') = 0$, or such conditions do not hold at $\mathcal{F}(.) = 0$, and ex ante and interim efficiency are incompatible. The proof demonstrates that the former case is exceptional[24].

[24] It is worthwhile to stress that under standard regularity conditions ex ante and interim efficient allocations typically coincide in economies with exogenous distributions of individual risks. This remains true also in the presence of occupational indivisibilities.

A consequence

Proposition 1 has two important corollaries. First, ex-ante efficiency typically requires transfers of resources across workers assigned to different occupations and a random allocation of workers across occupations. Second, compensating wage differentials equating (expected) utilities of workers assigned to different sectors are typically incompatible with first-best efficiency. In light of these results, it is then natural to ask whether, at least, occupations entailing riskier health distributions command higher contingent wages in the optimum. As the Pareto shadow wages take the form $\eta_t a_i^t(\theta)$, this is equivalent to ask whether, the Lagrangean multipliers associated to the feasibility constraints of the goods produced with riskier technologies are relatively larger in the optimum.

Next proposition shows that this is indeed the case whenever health distributions can be ordered according to the First Order Stocastic Dominance (FOSD) criterion. Throughout, we shall use the following standard definition of FOSD. For any pair of health distributions, $\left\langle p^t, \Theta \right\rangle$ and $\left\langle p^{t'}, \Theta \right\rangle$, $\left\langle p^t, \Theta \right\rangle$ FOSD $\left\langle p^{t'}, \Theta \right\rangle$ if $\sum_{\theta \leq \theta_n} p^t(\theta) \leq \sum_{\theta \leq \theta_n} p^{t'}(\theta), \forall\, \theta_n \in \Theta$ with at least one strict inequality.

Proposition 2 *If* $\left\langle p^t, \Theta \right\rangle$ *FOSD* $\left\langle p^{t'}, \Theta \right\rangle$ *then* $\eta^{tP} < \eta^{t'P}$ *at any solution of the Pareto program where* $\alpha_i^{tP} > 0$ *and* $\alpha_i^{tP} > 0$ *for some* i*.*

The results of Propositions 1 and 2 motivate the analysis of next section that investigates the determinants of wage differentials and Pareto optimal transfers across occupations.

6 Ex ante Optimality and Pareto optimal Transfers

This section characterizes efficient allocations. We shall consider economies where occupations differ for their *degree of health riskiness* and assume that the health distributions associated to different occupations can be ordered according to the FOSD criterion. We then study how the effects of health on preferences, endowments, and productivity contribute to determine either the sign of the optimal trasfers across jobs or the differences in utilities across workers assigned to different allocations.

For expositional purposes only, we shall study a simplified setting where two goods are produced by a representative agent[25], and assume that $\left\langle p^1, \Theta \right\rangle$ FOSD $\left\langle p^2, \Theta \right\rangle$. Consistently with the notation used before, define $x = (x_1, x_2, x_L)$ and $\hat{x} = (x_1, x_2)$. In order to distinguish the effect of health status on the utility of produced consumption from that on the disutility of labor, we shall use the following certainty utility representation: $U(x, \theta) = f(x, \theta) - \psi(l, \hat{x}, \theta)$; where $U(x, \theta)$ is assumed to satisfy all the assumptions stated in Section 2, and where $f(x, \theta)$ and $\psi(\hat{x}, l, \theta)$ represent the utility of consumption commodities and

[25] It should be clear in the following that none of the results of the analysis depends on the assumptions on the number of agents and the number of consumption goods.

the disutility of labor, $l = L - x_L$, respectively. According to this representation, both $f(.)$ and $\psi(.)$ may possibly depend on θ. By introducing $\hat{x}$ among the arguments of $\psi(.)$, we take into account the possibility that consumption activities affect the workers' disutility of labor.[26]

In this section we will also impose the following assumptions on preferences:

A1. $U(.,\theta)$ is supermodular in (x, x_L), i.e., $U_{cx_L}(.,\theta) = f_{cx_L}(.,\theta) - \psi_{cx_L}(.,\theta) > 0$ for all θ and $c = 1, 2$.

A2. All derivatives of U are bounded above.

A3. $\psi_{l\theta}(.) \leq 0$ for all (x, θ).

Supermodularity, is a simplifying assumption[27]. **A2** is basically unrestrictive since even if all the derivatives must be finite, they can take any finite value. **A3** imposes that marginal disutility of labor is decreasing with health. As health is typically an input for production, this is a very weak assumption that, neverthless, will play an important role in determining optimal transfers across occupations. As we will show, optimal consumption choices across health states, and hence optimal transfers are also affected in a crucial way by the sign of the cross derivatives, $U_{c\theta}$. In contrast to **A3**, though, we shall not impose any restriction on the sign of the components of the vector $U_{c\theta} = (U_{1\theta}, U_{2\theta})$. This is appropriate given the specificities of health services. First, indeed, health is an input for most consumption activities. As a consequence, better health status will generally increase the marginal utilities of consumption goods. Were this the only channel through which health affects consumption decisions, one should assume $U_{c\theta} > 0$ for all c. However, health has also a second important effect on consumption choices. Such an effect arises as an agent who has *received* a certain health state θ from the Nature, can generally devote some resources to improve his health conditions. Indeed, agents can either consume medical treatments or engage in health-enhancing consumption activities in at least a subset of health states. This possibility is implicitly taken into account within our setting. To see it more explicitly, one may represent agents' health status, $\hat{\theta}$, by a real valued function $\hat{\theta} = \rho(x, \theta)$, satisfying $\rho_\theta(.) > 0$ $\rho_c(.) \geq 0$ for all (x, θ). If health-enhancing consumption activities involving good c are marginally more productive in relative worse health states, which is generally the case, at lest in many common real-life situations, it follows that $\rho_{c\theta}(x, \theta) < 0$. Preferences can then be represented by the utility function: $U(x, \theta) = \hat{U}(x, \rho(x, \theta)) = \hat{f}(x, \rho(x, \theta)) - \hat{\psi}(l, \rho(x, \theta))$, where $\hat{f}(.)$ and $\hat{\psi}(.)$ must satisfy the usual assumptions.

Differentiating U one gets $\hat{U}_{c\theta} = \hat{U}_{c\rho}\rho_\theta + \hat{U}_\rho \rho_{c\theta} + \hat{U}_{\rho\rho}\rho_c\rho_\theta$; the first term of this sum, which is positive, represents the effect of θ on the marginal utility of consumption activities; the second term is negative and represents the effect of health-enhancing consumption

[26] In the real-world, one can easily found either instances where a larger consumption of consumption goods (such as food and housing, drugs, kindergarden services, etc...) reduces the disutility of labor, and instances where a larger consumption of certain goods (alcohol etc.) increases the disutility of labor.

[27] This assumption can be easily relaxed. In order to derive our characterization results we will only need U_{cx_L} *not too negative.*

activities on $\hat{\theta}$; while the third captures a second order effect which reinforces that of health-enhancing consumption activities. Of course, which effects prevail will depend on how *productive* are health-enhancing activities. Whenever they are sufficiently effective the sign of $\hat{U}_{c\theta}$ may well be negative[28].

In the rest of the section, we shall study within the setting described by the assumptions stated in section 2 and **A1-A3** how the direction of transfers among workers using technologies with different degree of riskiness depends on: (I) *direct effects of health status on agents well being*, measured by U_θ. (II) *health effects on consumption choices*, measured by the vector $(U_{1\theta}, U_{2\theta})$; and (III) *health effects on agents' production choices* which depend by the health effects on the disutility of labor, represented by $\psi_{l\theta}$, on labor endowment, and on labor productivity. For the sake of clarity, throughout we consider each of these effects in isolation.

It is finally worthwhile to point out that the distinction between health effects on consumption and production choices introduced above can be helpful to understand what type of transfers across real-world occupations we should observe in efficient, possibly regulated, competitive markets. Indeed, in the real-world it is possible to distinguish, at least roughly, among occupations for which phisical and or menthal health are more important or fundamental prerequisites for productive activities and other jobs which require only a minimal level of health to be performed satisfactorily. Health effects on production choices should determine the sign of cross transfers for jobs of the former type, while health effects on consumption choices should be more important otherwise.

6.1 Health Effects on Consumption Choices

We begin by studying how the effects of health status on the utility of consumption influence the properties of Pareto optima. To this end, we assume here that workers supply labor inelastically ($l^t(\theta) = L$ for $t = 1, 2$ and for all θ), and that either workers' productivity or labor endowments are independent from health status: $a^t(\theta) = a(\theta)^{29} = a$ and $L(\theta) = L$ for all t and θ.

Arguably, these asssumptions describe an agent owing a relatively low amount of human capital whose productivity is only marginally affected by his phisical, mental or psycophisical health. For such an agent, health should play a minor role in determining production decisions. It still remains true, however, that consumption activities improving health conditions may well be effective in reducing the disutility of labor. For instance, if one

[28] For simplicity in all the above discussion we only considered the case where $\rho_x \geq 0$. In words we did not consider the case in which the consumption of some commodities, such as alcohol, smoking, pollution etc. worsens agents' health conditions. These situations can be readily taken into account. For instance, if health reducing consumption activities, c, have a relatively larger impact on the health conditions, ρ, of healthier agents (smoking reduces relatively more sportsman respirations' capacities) then it is easy to verify that $\rho_{c\theta} < 0$:

[29] Assuming that the support of the distribution of $a^t(\theta)$ invariant across sectors is basically unrestrictive normalization whenever $a^t(\theta_0) = 0$ for all t.

considers *low income* agents who spend a large fraction of their income for nutritional and housing needs, it is sensible to assume that: (i) higher levels of consumption decrease the disutility of labor; and that (ii) this consumption effect gets stronger for lower health states, as additional consumption and additional health are substitute in reducing the disutility of labor. This is equivalent to impose $\psi_{c\theta} < 0$[30].

Next proposition shows how the direct health effect on well being, measured by $U_\theta(x, \theta)$, and the effect of health on the (marginal) utility of consumption, mesured by $U_{c\theta}(x, \theta)$, contribute to determine either optimal cross transfers or the sign of $\Delta u^P = u^1(x^{1P}) - u^2(x^{2P}) = \sum_{\theta \in \Theta} p^1(\theta) U(x^{1P}(\theta), \theta) - \sum_{\theta \in \Theta} p^2(\theta) U(x^{2P}(\theta), \theta)$.

As we show, the direct health effect on utility has a positive impact on Δu^P and affects positively the transfer from sector-2 to sector-1 (i.e. from the safer to the riskier sector). Differently, health effects on consumption choices influence positively Δu^P and the transfer from sector-2 to sector-1 if health and consumption goods are complements, while it has a negative effec on Δu^P and the transfer from sector-1 to sector-2 if health and consumption goods are substitutes. Finally, in the intermediate case where $U_{c\theta} > 0$ and $U_{c'\theta} < 0$ the impact on Δu^P depends on the relative magnitude of $U_{c\theta}$ and $U_{c'\theta}$.

Let $\Delta Z^P = Z^{1P} - Z^{2P}$, since from the first order conditions with respect to α^P of the Pareto program, we have $\Delta u^P \gtreqqless 0$ if and only if $\Delta Z^P \gtreqqless 0$, (i.e. the sign of the optimal transfer is the same as that of the utility differential) from hereafter, we shall only study the sign of Δu^P.

Proposition 3 *(i) If U has increasing differences in (x, θ), then $\Delta u^P > 0$; (ii) if U has decreasing differences in (x, θ), then $\Delta u^P > 0$ whenever $U_{c\theta}/U_\theta < k$ for all c, with $k \in \Re$ and sufficiently small; while $\Delta u^P < 0$ whenever $U_{c\theta}/U_\theta > K$ for at least one good, c, and $K \in \Re$ sufficiently large; (iii) if $U_{1\theta} > 0$ and $U_{2\theta} < 0$ then $\Delta u^P \leq (>) 0$ whenever $U_{1\theta}/|U_{2\theta}|$ sufficiently small (resp. large) .*

These results can be explained as follows. Pareto optimality requires risk-averse workers assigned to different occupations to get the same consumption in each individual health state (i.e., $x^1(\theta) = x^2(\theta) = x^P(\theta)$ for all θ). If consumption goods and health are complements, i.e.if $U_{c\theta}(x, \theta) > 0$, optimality requires agents' consumption to be larger in better health states, hence $U\left(x^P(\theta), \theta\right)$ increases in θ. Furthermore, since workers using safer technologies experience better health states with larger probabilities they will obtain larger utility levels with higher probabilities as well; so that their expected utility will be larger. Conversely, if consumption goods and health are substitutes, i.e., if $U_{c\theta}(x, \theta) < 0$, $x^P(\theta)$ will be smaller in better health states. If this substitution effect is sufficiently large

[30] As an example, consider the case of an unskilled worker, who live in a low-income African country with higher diffusion rates of contagious diseases (such as malaria or AIDS), and spend a large fraction of his income in buying food and housing services. Contracting the disease generally increases the worker's disutility of labor (i.e $\psi_\theta < 0$) by impairing his phisical working aptitudes. In addition, it is completely sensible that the more adequately this worker can satisfy his basic consumption needs the smaller will be the effects of the disease on his labor disutility. Making this assumption is equivalent to impose $\psi_{c\theta} < 0$.

to compensate the direct effect of health on utility, $U\left(x^P(\theta),\theta\right)$ will be decreasing in θ. Thus, by using the same line of reasoning as above, one concludes that the expected utility of workers using riskier technologies is larger at the optimum.

Proposition 3 does not cover the case in which one good is substitute with health, the other is complement and none of these effects is negligible relatively to the other[31]. The direction of the optimal transfer in this case, depends not only on the magnitude of the second cross derivatives but also on the vector of marginal utilities (which, in turn, are affected by the aggregate endowments). The main issue then becomes whether one can find a synthetic measure, possibly one with empirical correlates, which allows to determine which of the two effect prevails. Next proposition provides, indeed, the appropriate measure.

Define $V(q,I(q),\theta) \equiv \max_{x\in\Re^+}\{U(x,\theta)\ s.t.\ qx \leq I(q)\}$ the certainty indirect utility associated to the vector of prices q and total wealth $I(q)$. We will show that the sign and the magnitude of $V_{I\theta}(q,I,\theta)$[32], determines the sign of Δu^P.

Proposition 4 *Assume $V_{I\theta}$ has constant sign for all q. Then: (i) if $V_{I\theta} > -k$ with k positive and sufficiently small, $\Delta u^P > 0$; (ii) if $V_{I\theta} < -K$ with K positive and sufficiently large, $\Delta u^P < 0$.*

The proof is left to the reader[33]. It simply consists in verifying that $U\left(x^P(\theta),\theta\right)$ is increasing (resp. decreasing) whenever health and income are sufficiently good complements (resp. substitutes). The slope $U\left(x^P(\theta),\theta\right)$ in turn implies, by FOSD, the sign of Δu^P.

6.2 Health Effects on Production Choices

We turn now to the study of health effects on production choices and labor supply. We begin by considering the effects of health status on the labor endowment. Besides being the simplest to analyze, this case, provides basic insights for understanding the effects of health on the workers' disutility of labor and on their productivity. We shall assume in the following that health has no effect of consumption choices. Again this extreme assumption is for the sake of clarity. Our aim is to describe optimal transfers for the case in which health effects on consumption are relatively less important with respect to those on production choices.

6.2.1 Health Effects on Labor Endowment

To focus on the effects of health risks on labor endowment, in this section we assume that health has no direct effect on utility $U_\theta(x,\theta) = 0$ nor on the marginal utility of consumption

[31] This may be for instance the case when one of the two good, say good 1 is a particularly effective medical treatment that agent can consume in bad health states, while good 2 is a commodity whose aadditional consumptions yields higher utility's increases in good health states.

[32] $V_{I\theta}(q,I,\theta)$ has a positive (resp. negative) sign if health and income are complements (resp. substitute)

[33] Note that $U_{c\theta}(x,\theta) > 0$ (resp. < 0) for all c implies $V_{I\theta} > 0$ (resp. > 0), hence Proposition 4 generalizes the result in Proposition 3.

and labor $U_{c\theta}(x, \theta) = 0$ for all $x = (x_1, x_2, x_L)$. Moreover we set $a(\theta) = a$ for all $\theta \in \Theta$.

Once more for the sake of simplicity, it is convenient $l^t(\theta) = L(\theta)$ for all θ and $t = 1, 2$. In fact, the assumption that labor supply is inelastic does not change qualitatively the result of the analysis of this section. This is true provided that the agents' labor supply is maximal in at least one health state. Assuming $l^t(\theta) = L(\theta)$ only magnifies the effects of health shocks on labor supply. Indeed, it imposes that health entirely determines the amount of labor offered in production, which, is then completely unaffected by state contingent shadow prices and wages. Next sections will analyze the more complex cases in which contingent wages and health status jointly determine labor supply.

Next proposition shows that the effects of health on labor endowment is such that agents employed in risky sectors obtain an higher utility at the optimum.

Proposition 5 *If $L(\theta_n) > L(\theta_{n-1})$, for some θ_n, then $\Delta u^P < 0$.*

Either the proof, which is left to the reader, or the intuition for the result of Proposition 5 rely on the same arguments developed for Proposition 3.

6.2.2 Health Effects on the Disutility of Labor

This section analyzes the effects of health risks on the disutility of labor Consistently we assume $f_{x\theta}(x, \theta) = 0$ for all $x \in X$ and $\theta \in \Theta$, $a(\theta) = a$ and $L(\theta) = L$ for all $\theta \in \Theta$. As we want to focus on the effects of health on labor supply, we *will not* anymore assume that labor supply is completely inelastic.

Next proposition shows that whenever the health effect on the marginal disutility of labor (i.e., on production capabilities) is relatively more important than the direct health effect on well being (utility), and the marginal disutility of labor is "sufficiently increasing", then agents working in the riskier sector will get an higher expected utility in the optimum, i.e. $\Delta u^P < 0$. Otherwise the converse obtains. Let $\sigma_{x_L} = u_{x_L x_L}/u_{x_L}$

Proposition 6 *Assume $\sigma_{x_L} > k$, with k sufficiently large for all x_L, then (i) If U_θ sufficiently large, $\Delta u^P > 0$; (ii) if there exists a positive δ such that $|U_{x_L\theta}/\sigma_{x_L}| > \delta > 0$, $\Delta u^P < 0$ whenever U_θ is sufficiently small.*

Either the formal arguments to prove these results or their intuitions are more easily provided by considering first the case of separability between labor and consumption goods, where the utility function has the form $U(x, L - l, \theta) = f(x) - \psi(l, \theta)$. Analogously to the previous sections, in this case, the direct effect of health on utility, captured by U_θ, goes in the direction of increasing Δu^P. This is simply before for given consumption allocations health accidents affects more utility, in expected terms, in the sector where they are more likely. Moreover, now Pareto optimality imposes compensating wage differentials in favor of riskier occupation. Since the individual labor schedule is increasing in the shadow wage, compensating differentials imply that workers assigned to the riskier occupation must exert more labor in each individual health state. For this reason, also the wage effect has a positive impact on Δu^P. On the other hand, though, optimality requires agents to work more in good health states, as in these states the disutility of labor is lower. Since workers using riskier technologies will enjoy *less often* good health status, this effect reduces Δu^P. If U_θ is relatively small and the convexity of preferences makes $\Delta l^P(\theta) = l^{1P}(\theta) - l^{2P}(\theta)$ sufficiently small, the last effect prevails and Δu^P is negative; otherwise Δu^P has a positive sign. Summarizing, workers in the riskier sector receive a positive (negative) transfer and a positive (negative) utility differential whenever health is sufficiently more (less) important than wages in determining the optimal labor supply.

In the more general case in which preferences are not separable one has must also consider the effects of substitutability between labor and consumption. The presence of these effects does not change the conclusions just discussed, as the convexity of preferences with respect to labor implies that $\Delta l^P(\theta) = l^{1P}(\theta) - l^{2P}(\theta)$ sufficiently small for σ_{x_L} sufficiently large; and an appropriate continuity argument, formally developed in the proof, allow to show that the difference between the consumption vectors consumed by the workers in the two sectors are also small; so that the arguments of the separability case can be extended.

An important assumption in the previous proposition is that imposing σ_{x_L} large. This is a simplifying sufficient condition. As we show below, however, at least under the assumption of separability in labor and consumption goods, this assumptions can be replaced by a more common restriction on the third derivative of ψ, which are customarily used in a large part of the applied literature. Precisely, next proposition shows that agents using riskier technologies must obtain a higher utility whenever ψ_θ is relatively small and $\psi_{lll} > 0$. As can be easily verified from the Pareto program, the latter condition is necessary and sufficient for the labor supply schedule to be concave in the (shadow) wage. This restrictions appears to be quite realistic in most applications, as it implies that marginal wage increases raise more individual labour supply for relatively smaller wages.

Proposition 7 *Assume* $U(x,\theta) = f(x) - \psi(l,\theta)$*, if* $\psi_{lll}(l,\theta) > 0$ *for all* (l,θ) *and* $|\psi_{l\theta}|/\sigma_\psi \geq 2|\psi_\theta|$ *then* $\Delta u^P \leq 0$.

Since labour supply is concave in the wage, the difference between labor supply schedules across sectors cannot become too large as the shadow wages increase. This, in turn, implies that the effects of the shadow wages on Δu^P, described above, cannot overcome the health effect on the marginal disutility of labor.

6.2.3 Health Effects on Productivity

This section concludes the characterization of Pareto optima by considering the case where health risks only affect individual production. Precisely, we will now assume that agents' individual productivity, $a(\theta)$, is increasing in the health status, θ and neglect the effects of health on preferences and labor endowment by assuming: $U_\theta(x,\theta) = 0$ and $L(\theta) = L$ for all

x and θ. We begin by stating the following proposition, whose interpretation is analogous to that provided for Proposition 6

Proposition 8 *Assume σ_ψ has strictly positive upper and lower bounds, if σ_ψ and $\partial a(\theta)/\partial\theta$ are sufficiently large for all θ, then $\Delta u^P < 0$.*

The proof of this claim uses exactly the same argument developed in the proof of Proposition 6, thus it will be omitted.

Differently from Proposition 6, however, Proposition 8 does not present results for the case of a small $\sigma_\psi(l)$. Indeed, it can be showed that, in the absence of this assumption, signing Δu^P, generally requires specific assumptions on health distributions. Nevertheless, below we characterize optima for the particular but important case where one of the two technologies allows to obtain the highest health state with certainty (i.e., is completely safe). Let $l(\eta a(\theta), \theta)$ be the contingent labor supply schedule implicitly defined by the optimality conditions. Then $\zeta_{l,w} = dl(w_\theta, \theta)/dw_\theta)/(l(w_\theta, \theta)/w_\theta)$, with $w_\theta = \eta a(\theta)$, represents a measure of sensitivity of the equilibrium labor schedule with respect to the the shadow wage w_θ. As a preliminary result we state a lemma providing conditions on preferences that allow to sign $\partial\zeta_{l,w}/\partial w$. This in turn requires some notation. Let $h(l) = \psi'(l)l$, and let $\sigma_h(l) = h''(l)/h'(l)$;

Lemma 9 *$\partial\zeta_{l,w}/\partial w \gtreqless 0$ for all (l, θ) if and only if $\sigma_\psi(l) \gtreqless \sigma_h(l)$.*

The proof follows from straightforward algebraic manipulations of the FOCs of the Pareto program, and is omitted.

The lemma shows that efficiency requires agents using riskier technologies to get an higher utility and a positive subsidy in the optimum whenever $\zeta_{l,w}$ is non decreasing in the shadow wage. This assumption is in line with the empirical findings; its interpretation is that agents who are already "working a lot" are less reactive to wage increases. Next proposition characterizes optimal utility wedges and cross transfers across jobs.

Proposition 10 *Assume $p^1(\theta) = 1$ for $\theta = \theta_{\bar{N}}$. If $\partial\zeta_{l,w}/\partial w \gtreqless 0$ then $\Delta u^P \gtreqless 0$.*

7 Characterization of Competitive Equilibria

In this section we characterize competitive equilibria for both economies with deterministic and lottery contracts. We begin by proving the existence of a competitive equilibrium. The proof exploits the convexifying effect of large numbers.

Proposition 11 *An equilibrium always exists either in both economies with deterministic and lottery contracts.*

Next proposition states the first welfare theorem for an economy where lottery contracts are enforceable.

Proposition 12 *Competitive equilibria in economies with lottery contracts are first-best allocations.*

The argument of the proof is standard and it is omitted. The logic of the first welfare theorem can also be used to prove that competitive equilibria in economies where only deterministic contracts are enforceable, are interim efficient allocations with fair treatment.

Proposition 13 *Competitive equilibria in economies with deterministic contracts are interim efficient allocations with fair treatment.*

Given the results proved in characterizing of Pareto optimal allocations, Proposition 13 has the following immediate but important corollaries

Corollary 14 *Competitive equilibria with deterministic contracts are typically not first best allocations.*

This simply follows by the results that competitive equilibria with deterministic contracts are interim efficient and that ex-ante and interim efficient allocations are typically different.

We conclude the equilibrium analysis by considering the properties of equilibrium prices. Next proposition shows that in both classes of economies we are studying agents trade individual securities at fair prices, and that state-contingent wages are equal to the value of state-contingent labor productivity for each type of worker. Moreover, occupations associated with riskier health distributions in the sense of first order stochastic dominance command relatively higher expected wages. Finally, if lotteries are enforceable, equilibrium prices (wages) are such that value of consumption of agents of the same type assigned to different occupations typically differs from the sum of the values of its endowment and its production. By using lottery contracts agents transfer wealth across occupations in such a way that agents with the higher (respectively lower utility) expected utility consume get a positive (negative) transfer. Or, saying it differently, lottery contracts allow to make transfers across occupations. Let $\tilde{Z}_i^t = \sum_{c\in C,\theta\in\Theta}(q_c(p_i^t(\theta)x_{ic}^t(\theta) - e_i^c) - q_t p_i^t(\theta)a_i^t(\theta)(L - x_{iL}^t(\theta))$.

Proposition 15 *In all competitive equilibria with either deterministic or lottery contracts the following properties hold: (i) securities prices are fair, i.e.,* $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ *for some* $g_i^t \in \Re_+$ *for all* $i \in I$, $t \in T$ *and* $\theta \in \Theta$*; (ii)* $w_i^t(\theta) = b_i a_i^t(\theta) q_t$ *for some* $b_i^t \in \Re_+$ *for all* $i \in I, t \in T$ *and* $\theta \in \Theta$*; (iii) if* $\langle p_i^t, \Theta\rangle$ *first order stochastically dominates* $\left\langle p_i^{t'}, \Theta\right\rangle$ *and strictly positive measures of type-i agents are assigned to both sector* t *and* t' *then* $w_i^t(\theta) < w_i^{t'}(\theta)$*; (iv) In any equilibrium with lottery contracts such that positive measures of type-i agents are employed in sector* t *and in sector* t'*, then* $u_i^t(x_i^t) - u_i^{t'}(x_i^{t'}) \gtreqless 0$ *if and only if* $\tilde{Z}_i^t - \tilde{Z}_i^{t'} \gtreqless 0$

Part (i) and (ii) of Proposition 15 follows respectively from the linearity of the intermediaries and the production firms programs. (iii) expresses the *compensating wage differentials principle.* (iv) follows as a corollary from the optimality analysis.

8 Second Welfare Theorem and Decentralization

In this section we will show that if agents' types are public information, the Pareto optima can be implemented as competitive equilibria with deterministic cross transfers. We will restrict attention to the case of unenforceable lottery contracts[34]. In fact, in the real world transfers across agents with different degree of health riskiness can be implemented in several possible ways. Two widespread policiy schemes are systems of cross transfers across health insurance contracts designed for occupations, and subisidies to health henancing activities such as medical treatments, hospitals' services etc. For the sake of brevity, in this section we formally study policies based on cross subsidies across contracts, and discuss only briefly and informally the welfare effect of health services' subsidization.

To present the main results of the section, we introduce a class of policy schemes based on deterministic transfers across health insurance contracts and minimal wages.In order to simplify the formal description of the policy instruments, we shall now assume that each agent trades with only one intermediary. This unrestrictive assumption allows to reinterpret each agent's vector of assets' trades as an insurance contract. Let s_i^t the monetary transfer[35] received by a type-i agent who signs a health insurance policy designed for sector-t workers, also denote f_i^t the (possibly negative) monetary transfer received by a sector-t firm for each type-i worker employed. Finally let $\hat{w}_i^t(\theta)$ the minimal state contingent wage for type-i workers employed in sector t under the health state θ.

A transfers' policy, $\wp = (s, f, \hat{w})$, is then defined as: a vector, $s = (s_i^t)_{i\in I}^{t\in T}$, of subsidies to the workers; a vector $f = (f_i^t)_i^{t\in T}$ of transfers to the firms and a vector $w = (\hat{w}_i^t(\theta))_{i\in I,\ \theta\in\Theta}^{t\in T}$ of state contingent minimal wages. Feasible policies must be budget-balancing. Formally, a budget balancing policy is an element of: $P = \left\{\wp : \sum_{t\in T, i\in I} \mu_i \alpha_i^t (s_i^t + f_i^t) = 0\right\}$ where α_i^t is the measure of type-i workers who are effectively assigned to sector-t in an equilibrium with transfers[36].

As minimal wages may induce rationing, market clearing rules must be carefully specified. We assume that in any equilibrium with transfers all commodity as well as asset markets clear without rationing at "walrasian" prices (i.e., exactly as in the absence of transfers), and that firms' labor demand is not rationed. Differently, as transfers and

[34] The proof that the second welfare theorem hold when random transfers are implementable follows standadrd argument.

[35] We will use monetary transfer as a synonimus of "transfer in units of numeraire".

[36] As the focus of this section is mostly on the decentralizability of Pareto optimal allocations and workers' assignments, we will not introduce further notation to indicate the workers' assignment of a generic equilibrium with transfers. For simplicity, we prefer to denote by α a workers' assignment, as in the definition of (ex-ante) efficient allocations.

minimal wages may well make some occupations more attractive than others, one need to specify the rule through which workers are assigned to occupations yielding different utilities. We shall assume that whenever sector-t workers receive a higher utility than sector-t' workers, for some $t' \neq t$ in an equilibrium with transfers, the probability that a worker is assigned to sector-t is equal to α_i^t, which is also the measure of type-i workers assigned to sector-t.

The motivation for the clearing rules in consumption and assets markets is usual one: at any non walrasian price vector, rationed firms and/or agents would have an incentive to manipulate prevailing prices[37]. The same argument justifies our assumption that labor demand is never rationed in the equilibrium. Finally, our workers' assignment rule can be taught as the result of a *decentralized* job search process where workers simultaneously apply for several occupations in a first stage, subsequently applications are randomly selected whenever the number of workers applying for a job is larger than the number of vacancies posted; while in a final stage workers, whose applications may have been selected by several firms, choose an occupation within the set of offers. Notewhorty, while this type of assignment mechanism introduces a randomization on agents' labor supply, the transfers policies we consider are completely deterministic, and hence do not need any random device for their implementation.

A *rational expectation equilibrium with transfers* $\left\{\varphi_i^t, x, \alpha, z, p, w, \phi, \wp\right\}$ is now formally defined by the following conditions: (i) consumers' choose $\left(x_i^t, \alpha_i^t, z_i^t(\theta)\right)_{\theta\in\Theta}$ by maximizing $\sum_t u_i^t(x_i^t)\varphi_i^t$ subject to the budget constraints

$$\sum_{c\neq L} q_c(x_{ic}^t(\theta) - e_i^c) = w_i^t(\theta)\left(L - x_{iL}^t(\theta)\right) + z_i^t(\theta) + s_i^t,\ \forall\ (\theta, t)$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \leq 0\ \forall\ t,$$

and to a set of *rationing* constraint of the type

$$\varphi_i^t \leq \alpha_i^t$$

implying that a type-i agent who offers labor in sector-t will be assigned to that sector with probability α_i^t, equal to the measure of type-i workers who are effectively assigned to sector-t in the equilibrium; (ii) production firms' labor demand, l_i^t, and intermediaries assets' supply, $\widehat{z}_i^t$, satisfy the same conditions as in the competitive equilibrium with deterministic contracts (i.e. conditions (6) and (7)) except that, because of the presence of transfers, the firms' objective function is now $\sum_{\theta\in\Theta} p_i^t(\theta)\, l_i^t(\theta)\left(q_t y_i^t(\theta) - w_i^t(\theta)\right) + f_i^t$; (iii) the minimal wages' constraints, $w_i^t(\theta) \geq \hat{w}_i^t(\theta)$, are satisfied; and (iv) all feasibility conditions hold.

Next proposition shows that Pareto optimal allocations can be implemented as equilibria with transfers. We show that optimal policy schemes generally hinge on state and

[37] See for instance Mas Colell and others (pp. 315, 1995).

sector contingent minimal wages. We also prove that, in the case of inelastic labor supply, uniform minimal wages suffice to implement Pareto optima. A continuity argument then implies that, whenever the elasticity of labor supply is sufficiently small, there exists Pareto improving policy schemes imposing uniform minimal wages[38].

Proposition 16 *Any Pareto optimal allocation can be implemented as an equilibrium with transfers and state contingent minimal wages. Moreover, if workers' labor supply is completely inelastic for any positive wage, all Pareto optima are implementable through policies imposing uniform minimal wages.*

Finally , the same type of decentralization result stated in the previous proposition can be proved by considering policy schemes based on type and occupation contingent non linear subsidies to health-henancing consumption activities. The logic of the proof remains the same as (possibly negative) non linear subsidies to the purchase of health services turns out to be formally equivalent to cross subsidies[39]. Noticeably, however, it can be easily showed that the non linearity of consumption subsidies is a crucial property for the implementation of Pareto optima. Indeed, *linear* consumption subsidies introduce distortions in the individual consumption choices that prevent the equalization of marginal rates of substitution to relative prices. And for this reason, they cannot implement the first best.

Similarly, policies that do not discriminate across types (either cross subisidies on insurance or subsidies to health services purchaces), generally do not allow to equalize, for all possible types, the marginal utility of expected wealth of agents assigned to different occupations.

Finally, even if the implementation of optimal policies for second best economies is out of the scope of this paper, it is worthwhile to mention that one can construct robust examples where, in the absence of lottery contracts, simple cross-transfers insurance policies that do not discriminate across types can allow to improve upon competitive allocation (see Bennardo Piccolo 2005). These policies can be based on (i) a uniform, public or regulated insurance scheme (which implement cross transfers) and on (ii) an opt-out clause that allows the agents who prefer to buy insurance at market rates to opt-out from the regulated scheme.

9 Extensions

In the analysis of the previous setting we made two main simplifying assumptions: we imposed the absence of aggregate uncertainty, and assumed away the possibility that workers carry out prevention activities. As we now argue, the results of the paper can be generalized by relaxing both these assumptions.

[38] Uniform minimal wages and sector dependent minimal wages are both observed in developed countries.

[39] The formal proof of this claim are available upon request .

Aggregate uncertainty Introducing aggregate uncertainty does not involve any analytical complication. All the results of the paper, and the formal arguments upon which their proofs relay, extendto economies where individual endowments or productivities differ in different aggregate states.

Prevention activities Introducing prevention technologies requires some carefulness. One can formally think of prevention activities as workers' investments which allow to obtain, at a positive cost, a first-order stochastic shift of the health distributions associated to different occupations. More specifically, in a setting where two occupation specific health distributions are ordered according to first-order stochastic dominance, prevention activities may determine three possible scenarios. In the *first*, the costs and the effectiveness of prevention activities are such that the ordering of the two health distributions according to the FOSD criterion is preserved. This is, for instance, the case when prevention activities are relatively too costly or when the effects of prevention activities are sufficiently symmetric. In the *second case*, the ordering of the two health distributions is reversed This may happen when prevention activities are unexpensive and sufficiently more effective for the agents using the riskier technology. Finally, one has also to take into account a (*third*) case where, after prevention activities have been undertaken, health distributions cannot be anymore ordered by the FOSD criterion. As for the *first* case introducing prevention leaves unaltered the results derived in the previous sections. In the second case, all the analysis of the previous sections still applies but must be appropriately reinterpreted. Precisely, once prevention is introduced, the ordering of the riskiness of health distributions determining optimal cross transfers and utility differentials is the ex post one (i.e. the one emerging in equilibrium as a result of prevention activities), and not that holding ex ante. Finally, in the third case our characterization does not anymore apply as it relies on the FOSD criterion.[40]

10 Conclusive Remarks

The endogeneity of the individual health distributions' generates specific "cost-benefit trade-offs" involving agents' marginal utilities of health and consumption goods and their occupational choices. We have studied how these trade-offs determine the shape of the Pareto frontier of the economy, and the agents' competitive behavior. We showed that the he relative magnitude of health effects on production and consumption choices determines either the sign of Pareto optimal utility differentials across workers who use different technologies or the sign of cross-jobs Pareto. Both have been proved to be generally different from zero. Competitive equilibria are ex ante efficient if lottery contracts are enforceable, but not otherwise. Finally we showed that the specific form of contract incompleteness, associated to lotteries' unenforceability, may justify the introduction of policy schemes

[40] All the results mentioned in this section are formally proved in a more extended version of this paper. Their proofs are available on request.

implementing cross-transfers across occupations.

All these results have been assuming away all asymmetric information problems which may affect either the amount of labor supplied by workers or their insurance schemes. Moreover, a recent literature several contributions (Cole-Prescott (1997), Ellickson-Grodal-Scothcmer-Zame (1999), Makowski-Ostroy (2003)) have developed general equilibrium analyses focusing on the basic and complex issues related to the pricing of institutions and firms, and to the implicit prices emerging at the equilibrium within firms. Our conjecture based on the analysis of this paper is that the result of generic inconsistency between ex-ante and interim optimality continues to hold in most of the settings studied in the clubs literature and in the asymmetric information literature. Finally, a result in this spirit is obtained by Bennardo (2005) in a moral hazard economy where health effects are not considered but occupational choices affect the agents' indirect utility via incentive constraints.

References

Arnott, R., and Stiglitz, J. E. "Randomization with Asymmetric Information", RAND Journal of Economics,19: pp. 344-362, 1987.

Bennardo A., "Competitive Equilibria and Second Best Allocations of Multicommodity Economies with Moral Hazard: The Case of Perfect Verifiability of Trades", DELTA, Paris, 1997, revised version 2005.

Bennardo A., and Chiappori, P.A. "Bertrand and Walras Equilibria with Moral Hazard", Journal of Political Economy, 2003.

Bennardo A., and Piccolo S. "Pareto optima and second best allocations under imperfectly transferable production returns", mimeo, 2005.

Cass, D., Chichilnisky G., and Wu, H. M. "Individual Risks and Mutual Insurance", Econometrica, 64: pp. 333-341, 1996.

Citanna, A., Kajii, A., and Villanacci A. "Constrained Suboptimality in Incomplete Markets: a General Approach and two Applications", Economic Theory, 90: pp. 222-253, 2000.

Cole, H. L. "Comment: General Competitive Analysis in an Economy with Private Information", International Economic Review, 30: pp. 249-252, 1989.

Cole, H. L., and Prescott, E. C. "Valuation Equilibria with Clubs", Journal of Economic Theory, 74: pp.19-39, 1997.

Ehrenberg R. G., and Smith R.S., "Modern Labor Economics: Theory and Public Policy", 2003, (8th edition), Addison Wesley.

Ellickson, B.,Grodal B., Scothcmer S., Zame W.R., "Clubs and the Market" Econometrica, 67: pp. 1185-1217, 1999.

Evans, W. N., and Viscusi, W. K. "Utility Functions that Depend on Health Status Estimates and Economic Implications", American Economic Review, 80: pp. 353-374, 1990.

Evans, W. N., and Viscusi, W. K. "Income Effects and the Value of Health", Journal of Human Resources, 28: pp. 497-518, 1993.

Garratt, R. J. "Decentralizing Lottery Allocations in Markets with Indivisible Commodities", Economic Theory, 5: pp. 295-314, 1995.

Grossman, M. "On the Concept of Health Capital and the Demand for Health", Journal of Political Economy, 80: pp. 223-255, 1972.

Hansen, G. D., "Indivisible Labor and the Business Cycle", Journal of Monetery Economics, 16: pp. 309-327, 1985.

Levine, D., Kehoe, T.J., and Prescott, E.C. "Lottery, Sunspots and Incentive Constraints", Journal of Economic Theory, 2001.

Lucas, R. E. B. "The Distribution of Job Characteristics", Review of Economics and Statistics, 1974.

Makowski, L., and Ostroy, J. "Appropriation and Efficiency: A Revision of the First Theorem of Welfare Economics", American Economic Review, 85: pp. 808-827, 1995.

Malinvaud, E. "Markets for an Exchange Economy with Individual Risks",Econometrica, 41: pp. 383-410, 1973.

Prescott, E. C. , and Townsend, R. M. "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard", Econometrica 52: pp. 21-45, 1984.

Rogerson, R. "Indivisible Labor, Lotteries, and Equilibrium", Journal of Monetary Economics", 21: pp. 3-16, 1988.

Rosen, S."The Theory of Equalizing Differences" Handbook of Labor Economics Edided by Orley Ashenfelter and Richard Layard, North Holland, pp. 641-692, (1986).

Rustichini, A., and Siconolfi, P. "Economies with Asymmetric Information and Individual Risks", mimeo, 2003.

Viscusi, K. "The Value of Risks to Life and Health" Journal of Economic Literature, 31: pp. 1912-1946, 1993.

Viscusi K. W., Harrington J. E., Vernon J.M. "Economics of Regulation and Antitrust", MIT press, 2000.

11 Appendix

Proof of Proposition 1

In order to prove the genericity result, we need to formally define $\mathcal{U}^i$. Following the literature[41] assume that, beyond satisfying the assumptions previously stated, agents' preferences satisfy the following property: a sequence $U_{ik}(x_i,\theta)$ in $\mathcal{U}^i$ converges to $U_i(x_i,\theta)\in\mathcal{U}^i$ if and only if $U_{ik}(x_i,\theta)$, $DU_{ik}(x_i,\theta)$ and $D^2U_{ik}(x_i,\theta)$ uniformly converge to $U_i(x_i,\theta)$, $DU_i(x_i,\theta)$ and $D^2U_i(x_i,\theta)$, respectively, for all θ, on any compact subset of $\Re^C_{++}\times[0,L]$.[42]

[41] See A. Citanna, et al (1994) for a detailed discussion.

[42] In other words, we assume that $\mathcal{U}_i$ is endowed with the subspace topology of the C^2 uniform convergence topology on compact sets. Notice also that $\mathcal{U}=\prod_{i=1}^I\mathcal{U}_i$ is endowed with product topology.

Let $\xi = (x, \alpha, \eta, \lambda)$ define the vector of variables in the Pareto program. We consider first the case where the solution of the Pareto program is internal. A Pareto optimum then solves:

$$\mathcal{F}(\xi, \varepsilon, \bar{u}) = \left(\begin{array}{c} \lambda_i D_c U_i(x_i^t, \theta) - \eta\mu_i \\ -\lambda_i U_{ix_L}(x_i^t, \theta) + \eta_t a_i^t(\theta)\mu_i \\ \lambda_i(u_i^t(x_i^t) - u_i^T(x_i^T)) - \mu_i(Z_i^t - Z_i^T) \text{ for all } t \neq T \\ \sum_{i\in I} \mu_i (\overline{x}_i - e_i) - \sum_{i\in I} \mu_i y_i \\ \sum_{t\in T} \alpha_i^t u_i^t(x_i^t) - \overline{u}_i \;\; for\ all\ i \neq 1 \end{array} \right)_{i\in I, t\in T, \theta\in\Theta} = \mathbf{0}$$

for some vector of weights, $\bar{u} = (\bar{u}_i)_{i\neq 1}$. Interim efficiency imposes $u_i^t(x^{it}) = u_i^{t'}(x^{it'})$ for all t, t' such that $\alpha_i^t > 0$ and $\alpha_i^{t'} > 0$. Given an arbitrary $\varepsilon \in \mathcal{E}$, let $\alpha_1^t \in (0,1)$ for $t = 1$ and $t = T$ and define the following extended system of equations $\mathcal{G}(\xi, \varepsilon, \bar{u}) = (\mathcal{F}(\xi, \varepsilon, \bar{u}), (u_1^1(x_1^1) - u_1^T(x_1^T))) = 0$. Finally let $\mathcal{S}_{\bar{u}} = \{\varepsilon \in \mathcal{E} : \mathcal{G}(\xi, \varepsilon, \bar{u}) = 0\}$ be the subset of economies where a solution, $\xi(\varepsilon, \bar{u})$ of $\mathcal{G}(.)$ exists for any $\bar{u}$. We will show that ex ante and interim Pareto optima are generically different, by proving the equivalent statement that the complement of $\mathcal{S}_{\bar{u}}$ is open and dense.

(i) *Density*

The space, $\mathcal{E}$, of economies is infinite dimensional. However, as *density* is a local property, in proving it, one may restrict attention to a properly defined finite subset of $\mathcal{E}$. Specifically, we will consider the linear subspace of $\mathcal{U}$ defined as follows. Fix arbitrarily an economy $\bar{\varepsilon} \in \mathcal{E}$ and a vector $\bar{u}$, and let $x_{\bar{\varepsilon}}^{\bar{u}P}$ be the Pareto optimal allocation associated to $\bar{\varepsilon}$, and to a particular vector of Pareto weights, $\bar{u}$. Given an utility profile $\hat{U} \in \mathcal{U}$ of $\bar{\varepsilon}$, consider the perturbed utility functions $U_i(x_i, \theta) = \hat{U}_i(x_i, \theta) + \kappa_i(\theta) + \beta_i(\theta)(x_i - x_{i\bar{\varepsilon}}^{\bar{u}P}(\theta))$ where $\kappa_i(\theta)$ is a scalar and $\beta_i(\theta)$ denotes a $(C+1)$ dimensional vector for all (θ, i). Assume $|(\kappa_i(\theta)|$ and $\|\beta_i(\theta)\|$ sufficiently small for all (θ, i). The set of certainty utility functions, $\widehat{\mathcal{U}}$, defined by all possible perturbations obtained in this way is a finite linear subspace of $\mathcal{U}$. Let $\widehat{\mathcal{E}} = E \times T \times \widehat{\mathcal{U}}$, density will be proved on $\widehat{\mathcal{E}}$. Specifically, define $\widehat{\mathcal{S}}_{\bar{u}} = \left\{\varepsilon \in \widehat{\mathcal{E}} : \mathcal{G}(\xi, \varepsilon, \bar{u}) = 0\right\}$ and let $(\xi^{\bar{u}P}, \varepsilon^{\bar{u}P})$ a generic point such that $\mathcal{G}(.) = 0$. We now show that the complement of $\widehat{\mathcal{S}}_{\bar{u}}$ is dense by proving that $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^{\bar{u}P}, \varepsilon^{\bar{u}P})$, the matrix associated to Jacobian of $\mathcal{G}(.)$ evaluated at $(\xi^{\bar{u}P}, \varepsilon^{\bar{u}P})$, has full row rank, i.e., that $\mathcal{G}(.)$ is transversal to zero. Let $e^c = \sum_{i\in I} \mu_i e_i^c$ for all c and assume, without loss of generality, that $\alpha_i^T \in (0,1)$ for all i. Moreover, let $e \in \Re^C$, $a = (a_1(\theta), ..., a_I(\theta))$ with $a \in \Re^{(T-1)\times I}$ for some $\theta \in \Theta$ and $a_i(\theta) = (a_i^1(\theta), .., a_i^{T-1}(\theta)) \in \Re^{T-1}$, $\kappa_i^t = \sum_{\theta\in\Theta} p_i^t(\theta)\kappa_i(\theta)$ for all pairs (t, i) with $\kappa^T = (\kappa_2^T, .., \kappa_I^T) \in \Re^{I-1}$, and $\beta = (\beta_{1L}(\theta), .., \beta_{IL}(\theta)) \in \Re^I$. Straightforward elementary operations imply that the rank of $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^{\bar{u}P},\varepsilon^{\bar{u}P})$ is equal to the rank of[43]:

$$\mathbf{A} = \begin{array}{rc} & \begin{array}{cccccc} x & e & a & \kappa^T & \kappa_1^1 & \beta \end{array} \\ \begin{array}{r} \text{FOCs}(x) \\ \text{FEAs} \\ \text{FOCs}(\alpha) \\ \text{UT.CON.} \\ u_1^1(x_1^1) - u_1^T(x_1^T) = 0 \end{array} & \begin{pmatrix} \mathbf{H} & \mathbf{0} & \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{E} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & \mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \end{pmatrix} \end{array}$$

where $\mathbf{H}$, the submatrix of the Hessians, has full rank since preferences are strictly convex; $\mathbf{I}$ is a C dimensional identity matrix; all entries in $\mathbf{B}$ are equal to zero except for the ones in correspondence of the first-order conditions (FOCs) with respect to $x_{iL}^t(\theta)$ which take the value $\eta_t\mu_i$ for all (i,t); $\mathbf{C}$ is a $(T-1)\times I$ dimensional square matrix with all null entries except the ones of the principal diagonal which take the value $p_i^t(\theta)l_i^t(\theta)$ for all (i,t); and $\mathbf{E}$ has all null entries except for the elements corresponding to FOCs with respect to $x_{iL}^t(\theta)$, which are equal to 1 for all (i,t). $\mathbf{A}$, indeed, is simply obtained by summing the columns corresponding to e (multiplied by appropriate scalars) to the ones corrseponding to a and by summing the rows corresponding to the utility constraints to the ones corresponding to the FOCs with respect to α. Consider now the matrices:

$$\mathbf{M}_i = \begin{array}{rc} & \begin{array}{ccc} x_i & a_i(\theta) & \beta_i \end{array} \\ \begin{array}{r} \text{FOCs}(x_i) \\ \text{FOCs}(\alpha_i) \end{array} & \begin{pmatrix} \mathbf{H}_i & \mathbf{B}_i & \mathbf{E}_i \\ * & \mathbf{C}_i & \mathbf{0} \end{pmatrix} \end{array}, \text{ for all } i = 1, .., I$$

where "$*$" indicates generic submatrices. As $\mathbf{C}$ and $\mathbf{I}$ are non-singular matrices, and all the Hessians submatrices $\mathbf{H}_i$ are also non-singular, straightforward elementary operations imply that $\mathbf{A}$ has full row rank if $\mathbf{M}_i$ has full row rank for $i = 1, .., I$. By summing the columns of $\mathbf{M}_i$ corresponding to β_i (multiplied by appropriate scalars) to the ones corresponding to x_i and $a_i(\theta)$, repectively, one obtains

$$\hat{\mathbf{M}}_i = \begin{array}{rc} & \begin{array}{ccc} x_i & a_i(\theta) & \beta_i \end{array} \\ \begin{array}{r} \text{FOCs}(x_i) \\ \text{FOCs}(\alpha_i) \end{array} & \begin{pmatrix} \hat{\mathbf{H}}_i & \mathbf{0} & \mathbf{E}_i \\ * & \mathbf{I}_i & \mathbf{0} \end{pmatrix} \end{array} \quad \text{where} \quad \hat{\mathbf{H}}_i = \begin{pmatrix} \mathbf{I}.. & ..\mathbf{0}.. & ..\mathbf{0}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ *.. & ..\mathbf{I}.. & ..\mathbf{0}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}.. & ..*.. & ..\mathbf{I}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}.. & ..\mathbf{0}.. & ..*.. & ..\mathbf{I} \end{pmatrix}$$

It is immediate to verify that $\hat{\mathbf{M}}_i$ has full rank and so does $\mathbf{M}_i$, it follows that $\mathbf{A}$ must have full rank. Thus $\mathcal{G}(.)$ is transversal to zero and $\widehat{\mathcal{S}}_{\bar{u}}$ is dense when the Pareto program has an internal solution. Finally, the proof extends to the case where $\mathcal{F}(.)$ has a corner solution

[43] This can be easily verified by using the condition $u_1^1(x_1^1) - u_1^T(x_1^T) = 0$ to rewrite the FOC with respect to α_1^1 in $\mathcal{G}$ as $Z_1^1 - Z_1^T = 0$.

such that $\alpha_i^t \in \{0,1\}$, say $\alpha_i^t = 0$, for some pairs (i,t). Indeed, in this case it suffices to replace $\mathcal{G}(.) = 0$ by an equivalent system $\mathcal{G}'(.)$ which differs from $\mathcal{G}(.) = 0$ only because α_i^t is fixed to zero in all the equations and the first order condition with respect α_i^t does not appear anymore. □

(ii) *Openness*

Let $\mathcal{P}_{\bar{u}} = \{(\xi, \varepsilon) : \mathcal{F}(\xi, \varepsilon, \bar{u}) = 0\}$ denote the Pareto optimal manifold for $u = \bar{u}$, and consider the natural projection of $\mathcal{P}_{\bar{u}}$: $\pi : \mathcal{P}_{\bar{u}} \rightarrow \mathcal{E}$, $\pi(\xi, \varepsilon, \bar{u}) = \varepsilon$. As proper mappings take closed sets into closed set, $\mathcal{S}_{\bar{u}}$ is open if the natural projection is *proper.* Hence we need to prove that for any sequence $(\xi_k^{\bar{u}P}, \varepsilon_k)_{k=1}^{\infty}$ such that $F(\xi_k^{\bar{u}P}, \varepsilon_k, \bar{u}) = 0$ for all k, and $\varepsilon_k \rightarrow \varepsilon$, as $k \rightarrow \infty$, there exists a converging subsequence of $(\xi_k^{\bar{u}P})_{k=1}^{\infty}$ with limit $\xi^{\bar{u}P}$ such that $F(\xi^{\bar{u}P}, \varepsilon, \bar{u}) = 0$.

In order to do this, note first that $\left\{\alpha_k^{\bar{u}P}\right\}_{k=1}^{\infty}$ must converge, say to α, as it belongs to the compact set $[0,1]^{T \times I}$. Moreover boundary conditions imply $\left\{x_k^{i\bar{u}P}\right\}_{k=1}^{\infty} \gg 0$ for all i, while Inada conditions imply there exists a positive vector G such that $x_k^{i\bar{u}P} < G$, hence $\left\{x_k^{\bar{u}P}\right\}_{k=1}^{\infty}$ must converge, say to x. Given the assumptions on $U(.)$, $U_k^i(x^i, \theta) \rightarrow U^i(x^i, \theta)$ implies $DU_k^i(x^i, \theta) \rightarrow DU^i(x^i, \theta)$ uniformly on compact sets for all (x^i, θ), then $DU_k^i(x_k^{i\bar{u}P}, \theta) \rightarrow DU^i(x^{i\bar{u}P}, \theta)$ for all (i, θ). Finally, from (15)-(17) one gets $(\lambda_k^{\bar{u}P}, \eta_k^{\bar{u}P}) \rightarrow (\lambda^{\bar{u}P}, \eta^{\bar{u}P})$. This completes the proof. □

Proof of Proposition 2

Let $\tilde{\pi}$ be an $N \times N$ matrix, $\tilde{\pi}(x^{2P}(\theta_n), \theta_m)$ a generic element of $\tilde{\pi}$, and assume: $\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = 0$ for all pairs (n,m) with $n > m$; $\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = p^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l), \theta_m)$ for all n and m with $n = m$, and:

$$\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = \min \left\{ [p^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l), \theta_m)],\ [p^2(\theta_n) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_n), \theta_k)] \right\}$$

for all pairs (n,m) with $n < m$.

By construction, $0 \leq \tilde{\pi}(x^{2P}(\theta_n), \theta_m) \leq 1$ for all m and n. Moreover, $\tilde{\pi}$ satisfies (i) as the definition of $\tilde{\pi}(x^{2P}(\theta_n), \theta_{m=n})$ directly implies $\sum_m \tilde{\pi}(x^{2P}(\theta_n), \theta_m) = p^1(\theta_n)$.

Property (ii) is proved by induction. Let first show that $\tilde{\pi}$ satisfies (ii) for $n = 1$ (i.e. (ii) holds for all the elements of the first row of $\tilde{\pi}$).

To begin with, for $n = 1$, $\sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l), \theta_k) = 0$, and hence there must exist a positive $m \leq N$ such that $p^1(\theta_m) > p^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_1), \theta_k)]$.

This is true as otherwise f it should be $p^1(\theta_m) \leq p^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_1), \theta_k) = \max \left\{0, p^2(\theta_1) - \sum_{k=1}^{m-1} p^1(\theta_k)\right\}$, or all $m = 1, ..., N+1$, which is impossible since

$\max \left\{0, p^2(\theta_1) - \sum_{k=1}^{m-1} p^1(\theta_k)\right\} = 0$ for some m sufficiently large but lower than $N+1$.

As a consequence $\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = [p^2(\theta_n) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l), \theta_m)]$ for some $n \leq N$ and this implies that (i) holds for $n = 1$. Moreover as $\langle p^1, \Theta \rangle$ first order stocastically dominates $\langle p^2, \Theta \rangle$ one can show by using the same type of argument that if (ii) holds for all $n \leq n'$ it also holds for all $n \leq n' + 1$. Hence (ii) must holds for all n.

Finally, as $\pi(x^{2P}(\theta_n),\theta_m)=0$ for all (n,m) with $n>m$, for all n, we have $\sum_m \pi(x^{2P}(\theta_n),\theta_m)U(x^{2P}(\theta_n),\theta_m) > \sum_m \pi(x^{2P}(\theta_n),\theta_m)U(x^{2P}(\theta_n),\theta_n) = p^2(\theta_n)U(x^{2P}(\theta_n),\theta_n)$, by summing over n one obtains (iii).

We can now show that $\eta_1^P < \eta_2^P$. Assume first $\eta_1^P > \eta_2^P$. Consider now an allocation such that (i) a fraction (measure) $\alpha^1 = \alpha^{1P} + d\alpha$ of the workers in the economy is assigned to sector 1 and a measure $\alpha^2 = \alpha^{2P} - d\alpha$ is assigned to sector 2; (ii) a fraction $(\alpha^{1P} - d\alpha)/\alpha^1$ of sector 1 workers and all sector 2 workers obtain the the Pareto optimal allocations, x^{1P} and x^{2P}, respectively ; (iii) a fraction $d\alpha/\alpha^1$ of sector 1 workers obtain $\tilde{x}^1 = (..., \tilde{x}^1(\theta_n), ...)$ with $\tilde{x}^1(\theta_m) = \sum_n \pi(x^{2P}(\theta_n),\theta_m)x^{2P}(\theta_n)$; and, finally, a fraction $d\alpha/\alpha^1$ of sector-1 workers obtainthe allocation $x^{1P}+\varepsilon = (...,x^{1P}(\theta_n)+\varepsilon(\theta_n),...)$ where, for all θ_n, $\varepsilon(\theta_n) = (\varepsilon, -\varepsilon)$ with $\varepsilon > 0$ and sufficiently small. By construction, the new allocation is feasible and both $\tilde{c}^1$ and $c^{1P}+\varepsilon$ are strictly preferred to c^{2P} and c^{1P} respectively. And this contraddicts the optimality of $(\alpha^P, x^{1P}, x^{2P})$. A standard continuity argument finally allows to extend the the proof to the case $\eta_1^P = \eta_2^P$.

Proof of Proposition 3

Preliminarily to the characterization of Pareto optima, we state, without proof, the following well know lemma which turns out to be a useful tool our analysis.

Lemma 17 *For any map $g : \Theta \to \Re^+$, $\theta \to g(\theta)$, with $dg(\theta_{n+1}) = g(\theta_{n+1}) - g(\theta_n)$, the following identity holds:*

$$\sum_{\theta\in\Theta}\left(p^1(\theta) - p^2(\theta)\right)g(\theta) := \sum_{n=0}^{N}\left(P^2(\theta_n) - P^1(\theta_n)\right)dg(\theta_{n+1}) \ \forall\ n = 0, ...N.$$

We can now prove the statement of the proposition. The first order conditions with respect to x^{tP} of the Pareto program, together with strict concavity of $U(x,\theta)$ imply $x^{1P}(\theta) = x^{2P}(\theta) = x^P(\theta)$ for all θ. Let $x^P : \Theta \to \Re^2_+$, $\theta \to x^P(\theta)$, be the map which associates the optimal consumption vector $x^P(\theta)$ to each $\theta \in \Theta$. Define $d\theta = \theta_{n+1} - \theta_n$ for all n, let $d\theta$ sufficiently small, one has:

$$dU\left(x^P(\theta_n),\theta_n\right) = U\left(x^P(\theta_{n+1}),\theta_{n+1}\right) - U\left(x^P(\theta_n),\theta_n\right) \approx \sum_{c=1,2} dx_c^P(\theta_n)\eta_c^P + U_\theta\left(x^P(\theta_n),\theta_n\right)d\theta \tag{18}$$

By lemma (17), $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $dU\left(x^P(\theta),\theta\right) \gtreqless 0$. Moreover, by (18) $\sum_{c=1,2} dx^P(\theta_n)\eta_c^P + U_\theta\left(x^P(\theta_n),\theta_n\right)d\theta \gtreqless 0$ implies $u^1(x^{1P}) \gtreqless u^2(x^{2P})$. For $d\theta$ small $dx_c^P(\theta_n)$ can be approximated as: $dx_1^P(\theta_n) \approx (U_{1\theta}|U_{22}| + U_{2\theta}U_{21})/\Lambda)d\theta$ and $dx_2^P(\theta_n) \approx (U_{2\theta}|U_{11}| + U_{1\theta}U_{12})/\Lambda)d\theta$ where $\Lambda = U_{11}U_{22} - (U_{12})^2 > 0$. Summing up, we obtain:

$$\sum_{c=1,2} dx_c^P(\theta_n)\eta_c \approx U_1\frac{U_{1\theta}|U_{22}| + U_{2\theta}U_{12}}{\Lambda}d\theta + U_2\frac{U_{2\theta}|U_{11}| + U_{1\theta}U_{12}}{\Lambda}d\theta \tag{19}$$

(18) and (19) then imply that $dU\left(x^P(\theta_n),\theta_n\right) \gtreqless 0$ if:

$$\frac{U_{1\theta}}{\Lambda}\left(U_1|U_{22}|+U_2U_{12}\right)+\frac{U_{2\theta}}{\Lambda}(U_2|U_{11}|+U_1U_{12})+U_\theta \gtreqless 0 \tag{20}$$

Under increasing differences in (x,θ), $U_{c\theta}\geq 0$ for all (x,θ) and $c=1,2$, while $U_{12}\geq 0$ for all θ by supermodularity in x for any given θ. Since $U_\theta>0$, (i) follows from (20). Conversely, under decreasing differences $U_{c\theta}\leq 0$ for all (x,θ) and $c=1,2$, and (ii) again follows from (20). Finally (iii) follows, by the same logic, from simple algebraic manipulations of (20). □

Proof of Proposition 6

We first show that the claim is true under separablity, i.e., when $U(x,L-l,\theta)=f(x)-\psi(l,\theta)$, with $\psi_\theta(.)\leq 0$, $\psi_{\theta l}(.)\leq 0$ for all (θ,l).

Part (i). Let $\Delta\tilde{u}^P=\sum_{\theta\in\Theta}p_1(\theta)\psi(l^{1P}(\theta),\theta)-\sum_{\theta\in\Theta}p_2(\theta)\psi(l^{2P}(\theta),\theta)$ and define $\sigma_\psi(l,\theta)\equiv\psi_{ll}(l,\theta)/\psi_l(l,\theta)$ for all (l,θ). Summing by parts, $\Delta\tilde{u}^P$ can be written as

$$\Delta\tilde{u}^P=-(\sum_{\theta\in\Theta}(p_1(\theta)-p_2(\theta))\psi(l^{1P}(\theta),\theta)+\sum_{\theta\in\Theta}p_2(\theta)(\psi(l^{2P}(\theta),\theta)-\psi(l^{1P}(\theta),\theta))$$

Now, denote η_t^P the lagrange multiplier associated to the t-th feasibility constraint. Define $l_1(\theta)$ the function defined by the first-order condition $\psi_l(l(\theta),\theta)=\eta_1^P$; and $l(\theta,\eta)$ that defined by $\psi_l(l(\theta),\theta)=\eta$. We then have:

$$\Delta\tilde{u}^P=\sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_n+\Delta\theta}(d\psi(l_1(\theta),\theta)/d\theta)d\theta+\sum_{\theta\in\Theta}p^2(\theta)\int_{\eta_1^P}^{\eta_2^P}(\sigma_\psi(l(\theta,\eta),\theta))^{-1}d\eta$$

where, for each $\theta_n\in\Theta$, $\Delta P(\theta_n)=(P^1(\theta_n)-P^2(\theta_n))$ Finally, by using the definitions of $l_1(\theta)$ and $l(\theta,\eta)$ one obtains:

$$\Delta\tilde{u}^P=\sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_n+\Delta\theta}\left(\frac{|\psi_{l\theta}(l_1(\theta),\theta)|}{\sigma_\psi(l_1(\theta),\theta)}+\psi_\theta(l_1(\theta),\theta)\right)d\theta+\sum_{\theta\in\Theta}p^2(\theta)\int_{\eta_1^P}^{\eta_2^P}\left(\sigma_\psi(l(\theta,\eta),\theta)\right)^{-1}d\eta \tag{21}$$

Since $\eta_2^P>\eta_1^P$ by Proposition 2 and $\sigma_\psi(l,\theta)\geq 0$ for all (l,θ), the second addendum in (21) is positive. First order stochastic dominance implies that the first addendum is also positive for $\psi_\theta(l,\theta)$ sufficiently small. Hence this condition implies $\Delta\tilde{u}^P>0$.

We can now extend the proof to the non-separability case where $U_{cx_L}\geq 0$ for $c=1,2$. To begin, we define an auxiliary program which maximizes the expected utility under the feasibility constraints and the additional constraints $x^1=x^2=x^{1P}$, where x^{1P} is part of the solution of the Pareto program. Let $(\alpha^F,x_L^{1F},x_L^{2F})$ be the solution of this program and define $\sigma_{x_L}(x,x_L,\theta)=-U_{x_Lx_L}(x,x_L,\theta)/U_{x_L}(x,x_L,\theta)>0$ for all (x,x_L,θ).Moreover, define $\Delta u^F=\sum p^1(\theta)U(x^{1P}(\theta),x_L^{1F}(\theta),\theta)-\sum p^2(\theta)U(x^{1P}(\theta),x_L^{2F}(\theta),\theta)$.

As an intermediate result, we show that, for any ε, there exists $\sigma_{x_L}(.)$ sufficiently large such that $|x_L^F - x_L^P| < \varepsilon$ and $|\Delta u^P - \Delta u^F| < \varepsilon$, with ε arbitrarily small. In order to prove this result, define $\pi(\theta) = \alpha^P p^1(\theta)/(\alpha^P p^1(\theta) + (1-\alpha^P)p^2(\theta))$ for all θ, and consider the allocation $(\alpha^P, \hat{x})$ such that $\hat{x}_L(\theta) = \pi(\theta)\hat{x}_L^{1P}(\theta) + (1-\pi(\theta))\hat{x}_L^{2P}(\theta)$, and $\hat{x}_c^1 = \hat{x}_c^2 = \hat{x}_c$ for $c = 1, 2$, where $\hat{x}_c$ is chosen to be sufficiently small so as to satisfy the feasibility constraints. $\Delta EU = EU(\alpha^P, x^P) - EU(\alpha^P, \hat{x})$ can be rewritten as:

$$\Delta EU = \alpha^P \sum_{\theta\in\Theta} p^1(\theta)A(\theta) + (1-\alpha^P)\sum_{\theta\in\Theta} p^2(\theta)B(\theta) + \sum_{\theta\in\Theta}(\alpha^P p^1(\theta) + (1-\alpha^P)p^2(\theta))C(\theta)$$

where $A(\theta) = U(x^{1P}(\theta), x_L^{1P}(\theta), \theta) - U(\hat{x}(\theta), x_L^{1P}(\theta), \theta)$, $B(\theta) = U(x^{2P}(\theta), x_L^{2P}(\theta), \theta) - U(\hat{x}(\theta), x_L^{2P}(\theta), \theta)$, and $C(\theta) = \pi(\theta)U(\hat{x}(\theta), x_L^{1P}(\theta), \theta) + (1-\pi(\theta))U(\hat{x}(\theta), x_L^{2P}(\theta), \theta) - U(\hat{x}(\theta), \hat{x}_L(\theta), \theta)$ for all θ;

and where $C(\theta)$ is proportional to $\sigma_{x_L}(x, x_L\theta)$; and for all θ both $A(\theta)$ and $B(\theta)$ are bounded below because the aggregate endowment is positive and Inada conditions hold, and is bounded above because the aggregate endowment is finite. Hence, ΔEU becomes negative for $\sigma_{x_L}(.)$ sufficiently large. AS a consequence, an optimality argument implies that for any $\varepsilon > 0$ there exists $\sigma_{x_L}(.)$ is sufficiently large such that $\left|x_L^F - x_L^P\right| < \varepsilon$. Then, the signs of Δu^P and Δu^Fmust coincide for $\sigma_{x_L}(.)$ sufficiently large. We now use the above result to prove part (i) by showing that $\Delta u^F > 0$ if $\Delta\tilde{u}^P > 0$. Summing by parts, after some algebraic manipulations one obtains $\Delta u^F = \Delta\tilde{u}^P - G$, where the extra term G is[44]:

$$G = -\sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_n+\Delta\theta}[\sum_{c=1,2}(U_c^F + \frac{U_{cx_L}^F}{\sigma_{x_L}^F}) \times \frac{U_{x_L\theta}^P(U_{12}^P U_{c'x_L}^P + |U_{c'c'}^P| U_{cx_L}^P)}{|\Lambda^P|}]d\theta$$

The claim of part (i) then follows as G is negative for all (x, θ) and $c = 1, 2$.□

Part (ii) Again, we begin with the separability case. Observe that $|\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)| > 0$ implies that $\sigma_\psi < \infty$, which in turn, entails $\psi_l > g > 0$ for all $(l^P(\theta), \theta)$. This together with Lemma 17, and the continuity of preferences, imply the existence of $d \in \Re_{++}$ such that if $dl^P(\theta) = l_2^P(\theta) - l_1^P(\theta) \leq d$ for all θ, $\Delta\tilde{u}^P < 0$. Thus, it remains to prove that $\Delta\tilde{u}^P < 0$ whenever $dl(\theta) > d$ for some θ. From (21) we have $\sum_{n\in N}\Delta P(\theta)\int_{\theta_n}^{\theta_n+\Delta\theta}(d\psi(l_1(\theta), \theta)/d\theta)d\theta < 0$ for $\psi_\theta(l,\theta)$ sufficiently small, since $\Delta P(\theta_n) < 0$ for all $n \in N$ and $-\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)$ positive for all θ. Then $\Delta\tilde{u}^P < 0$ if $\sigma_\psi(l,\theta)$ is sufficiently large, and $\Delta\eta^P = \eta_2^P - \eta_1^P < h$ for some strictly positive h since $|\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)| > k$. In the following, we use an optimality argument to prove the existence of this upper bound on $\Delta\eta^P$. By definition, for all feasible (x', α'), $EU(x^P, \alpha^P) \geq EU(x', \alpha')$. In particular, consider the consumption allocation $\hat{x}$ such that $\hat{x}_c^t(\theta) = x_c^{tP}(\theta)$ for $c = 1, 2$; $\hat{l} = \beta l^{1P} + (1-\beta)l^{2P}$, with $\beta \in (0,1)$. Since $l^{1P} < l^{2P}$, a continuity argument implies that for any β sufficiently small there exists a real number

[44] In order to simplify the notation, hereafter the superscripts F and P will indicate that a function is evaluated at $x^F$and x^P, respectively.

k such that $\hat{\alpha} = \alpha^P + k < 1$, and $(\hat{x}, \hat{\alpha})$ satisfies the feasibility constraints (possibly as inequality).

Let $\Delta EU = EU(x^P, \alpha^P) - EU(\hat{x}, \hat{\alpha}) \geq 0$. By adding and subtracting $EU(\hat{x}, \alpha^P)$ to ΔEU, and then using the first order conditions of the Pareto program one gets $\Delta EU = \tilde{A}^P + \tilde{B}^P$ where $\tilde{A}^P = \sum_{t\in T,\theta\in\Theta} \alpha_t^P p^t(\theta)\Delta\psi(l^t(\theta))$, with $\Delta\psi(l^t(\theta)) = (\psi(\hat{l}(\theta),\theta) - \psi(l^{tP}(\theta),\theta))$, and where:

$$\tilde{B}^P = \Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} (\psi_l(\hat{l}(\theta),\theta)[\beta\frac{|\psi_{l\theta}(l_1(\theta),\theta)|}{\psi_{ll}(l_1(\theta),\theta)} + (1-\beta)\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\psi_{ll}(l_2(\theta),\theta)}] + \psi_\theta(\hat{l}(\theta),\theta))d\theta$$

with $\Delta\alpha = (\hat{\alpha} - \alpha^P)$. For β sufficiently close to 0,

$$\tilde{B}^P \approx \Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} (\psi_l(\hat{l}(\theta),\theta)\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\psi_{ll}(l_2(\theta),\theta)} + \psi_\theta(\hat{l}(\theta),\theta))d\theta <$$

$$\Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} [\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + \psi_\theta(\hat{l}(\theta),\theta)]d\theta$$

Moreover, from the first order conditions of the Pareto program and the convexity of $\psi(.)$ it follows $\tilde{A}^P < A' = \sum_{t\in T} \alpha_t^P \eta_t^P \sum_{\theta\in\Theta} p^t(\theta)\Delta l^t(\theta) > 0$ where $\Delta l^t(\theta) = (\hat{l}(\theta) - l^{tP}(\theta))$. Using the definition of $\hat{l}(\theta)$ we then get:

$$A' = \alpha_1^P\eta_1^P(1-\beta)\sum_{\theta\in\Theta} p^1(\theta)(l^{2P}(\theta) - l^{1P}(\theta)) - \alpha_2^P\eta_2^P\beta\sum_{\theta\in\Theta} p^2(\theta)(l^{2P}(\theta) - l^{1P}(\theta))$$

As $(l^{2P}(\theta) - l^{1P}(\theta)) > d$, the above expression implies $\tilde{A}^P \to -\infty$ as $\eta_2^P - \eta_1^P \to +\infty$. In turn, since $|\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)|$ is bounded above, $\tilde{B}^P$ is bounded above. We can conclude that $\Delta EU = \tilde{A}^P + \tilde{B}^P \geq 0$ implies $\eta_2^P - \eta_1^P < h$ for some positive h. This finally implies that $\Delta\tilde{u}^P < 0$ for $\sigma_\psi(l,\theta)$ sufficiently large. Indeed $\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} (d\psi(l_1(\theta),\theta)/d\theta)d\theta$ is strictly positive and bounded below; while $\sum_{\theta\in\Theta} p^2(\theta) \int_{\eta_1^P}^{\eta_2^P} 1/\sigma_\psi(l(\theta,\eta),\theta)d\eta$ becomes arbitrarily small for $\sigma_\psi(l,\theta) \to +\infty$. $\square$

As in the previous case, we now extend the proof to the cse of non separability. Let $\hat{x} = (x_1^{1P}, x_2^{1P}, \hat{x}_L^F)$, where $\hat{x}_L^F = \beta x_L^{1F} + (1-\beta)x_L^{2F}$ with $\beta \in (0,1)$. Since $x_L^{1F} \geq x_L^2$ by Proposition 2, for β sufficiently small there musr exist by continuity a real number k such that $\hat{\alpha} = \alpha^F + k < 1$, and $(\hat{x}, \hat{\alpha})$ satisfies feasibility constraints. Let $\Delta EU^F = EU(x^F, \alpha^F) - EU(\hat{\alpha}, \hat{x})$. By adding and subtracting $EU(\hat{x}, \alpha^F)$, one gets $\Delta EU^F = A^F + B^F$ where $A^F = \sum_{t\in T,\theta\in\Theta} \alpha^F p^t(\theta)\Delta U(x_L^{tF}(\theta),\theta)$ with $\Delta U(x_L^t(\theta),\theta) = U(x^{1P}(\theta), x_L^{tF}(\theta),\theta) - U(x^{1P}(\theta),\hat{x}_L(\theta),\theta)$ for all t, and $B^F = \tilde{B}^P + S$, with: $\tilde{B}^P = \Delta\alpha^F \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} [\frac{U_{x_L\theta}^F}{\sigma_{x_L}^F} + U_\theta^F]d\theta$, $\Delta\alpha^F$ $= (\hat{\alpha} - \alpha^F)$ and:

$S = \Delta\alpha^F \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} [\sum_{c=1,2} (U_c^F + \frac{U_{cx_L}^F}{\sigma_{x_L}^F}) \times \frac{U_{x_L\theta}^P (U_{12}^P U_{c'x_L}^P + |U_{c'c'}^P| U_{cx_L}^P)}{|\Lambda^P|}] d\theta$. Since **A3** implies that S is bounded above, the rest of the proof follows exactly the same steps proved in the case of separability.

Proof of Proposition 7

Let define the function $T(\eta,\theta) \equiv \sigma_\psi^{-1}(l(\eta,\theta),\theta) - l(\eta,\theta)$, Inada conditions imply $T(0,\theta) = 0$ for all θ. Moreover, one can readily verify that $\psi_{lll}(l,\theta) \geq 0$ for all (l,θ) implies $\partial T(\eta,\theta)/\partial\eta \leq 0$ for all (η,θ). It follows that $l(\eta,\theta) \geq \sigma_\psi^{-1}(l(\eta,\theta),\theta)$ for all (η,θ). This inequality together with (21) imply

$$\Delta u^P \leq \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + \psi_\theta(l_2(\theta),\theta) \right) d\theta + \sum_{\theta\in\Theta} p^1(\theta) \left(\int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta) d\eta \right) \tag{22}$$

Let $h(l,\theta) \equiv \psi_l(l,\theta) l$ for all (l,θ), substituting $\psi_l(l^t(\theta),\theta) = \eta_t^P$ in (17), and adding and substracting $\sum_{\theta\in\Theta} p^1(\theta)\psi(l^2(\theta),\theta)$ to the left hand side, and $\sum_{\theta\in\Theta} p^1(\theta) h(l^2(\theta),\theta)$ to the right hand side of (17), respectively, one gets: $\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \psi_\theta(l_2(\theta),\theta) d\theta = \sum_{\theta\in\Theta} p^1(\theta) \left(\int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta) d\eta \right)$. This equality together with (22) imply:

$$\Delta u^P \leq \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + 2\psi_\theta(l_2(\theta),\theta) \right) d\theta$$

Then $\Delta u^P \leq 0$ since $\Delta P(\theta) < 0$ for all θ by FOSD and the integrand function is positive. $\square$

Proof of Proposition 10

As a preliminar result we prove that if $\sigma_\psi(l) \gtreqless \sigma_h(l)$ then $\Delta u^P = 0$ implies $\Delta h \lesseqgtr 0$ whenever $p^1(\theta) = 1$ for $\theta = \theta_N$. Setting $\Delta u^P = 0$, one gets $\psi\left(l^1(\theta_N)\right) = \sum_{\theta\in\Theta} p^2(\theta)\psi\left(l^2(\theta)\right)$ as $x^{1P} = x^{2P}$. $l^1(\theta_N)$ is then the certainty equivalent of $\sum p^2(\theta)\psi\left(l^2(\theta)\right)$. Let $l^2(h)$ denote the certainty equivalent for $\sum_{\theta\in\Theta} p^2(\theta) h\left(l^2(\theta)\right)$, we have $\Delta h = h(l^2(h)) - h(l^1(\theta_N))$. Since $h(l) = \psi'(l) l$ is an increasing function and $\sigma_\psi(l) \gtreqless \sigma_h(l)$ implies $l^2(\psi) \gtreqless l^2(h)$, it follows that $\Delta h \lesseqgtr 0$ if $\sigma_\psi(l) \gtreqless \sigma_h(l)$. $\square$

We now turn to the proof of the claim. We begin by showing that $\sigma_\psi(l) < \sigma_h(l)$ implies $u^1(x^{1P}) < u^2(x^{2P})$. Let introduce an auxiliary maximization program that maximizes the expected utility of the representative agent under the feasibility constraints and the additional constraint

$$\Delta u = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \sum_{\theta\in\Theta} p^1(\theta)\psi(l^1(\theta)) \leq 0 \tag{23}$$

The FOCs with respect to $l^t(\theta)$, $t = 1,2$, and α of this auxiliary program are:

$$\psi'(l^1(\theta_N)) = \eta_1 a(\theta_N) + \frac{\varkappa}{\alpha}\psi'(l^1(\theta_N)) \tag{24}$$

$$\psi'(l^2(\theta)) = \eta_2 a(\theta) - \frac{\varkappa}{1-\alpha}\psi'(l^2(\theta)) \ \ \forall\, \theta \in \Theta \tag{25}$$

$$\sum p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) = \eta_2 \sum p^2(\theta)a(\theta)l^2(\theta) - \eta_1 a(\theta_N)l^1(\theta_N) \tag{26}$$

where $\varkappa \geq 0$ is the multiplier associated with (23). Substituting (24) and (25) into (26) one gets:

$$\Delta u = \Delta h + \varkappa\left(\frac{h^2}{1-\alpha} + \frac{h^1}{\alpha}\right) \tag{27}$$

We begin by showing that if $\sigma_\psi(l) < \sigma_h(l)$ the solution of the auxiliary program solves the Pareto program. This is equivalent to prove that $\sigma_\psi(l) < \sigma_h(l)$, implies either $\varkappa = 0$ or $\Delta u \leq 0$ at the Pareto optimum. Suppose, on the contrary, that $\varkappa > 0$, then $\Delta u = 0$, and by equation (27) it follows $\Delta h = -\varkappa\left(\frac{h^2}{1-\alpha} + \frac{h^1}{\alpha}\right) < 0$. But this is a contradiction since above we have proved that $\sigma_\psi(l) < \sigma_h(l)$ implies $\Delta h > 0$ whenever $\Delta u = 0$.

It now remains to show that $\sigma_\psi(l) < \sigma_h(l)$ implies $\Delta u < 0$. This is true as $\Delta u = 0$ implies $\Delta h > 0$ whenever $\sigma_\psi(l) < \sigma_h(l)$ so that (27) cannot be satisfied. Proving that $\sigma_\psi(l) > \sigma_h(l)$ implies $u^1(x^{1P}) > u^2(x^{2P})$ and that $\sigma_\psi(l) = \sigma_h(l)$ implies $u^1(x^{1P}) = u^2(x^{2P})$, requires the same type of argument developed above.

Proof of Proposition 11

We begin with the case where lottery contracts are unenforceable. Consider the auxiliary program which maximizes $\sum_{t\in T} u_i^t(x_i^t)\varphi_i^t$ within the compact set defined by the agents' budget constraints and the additional constraints $\varphi_i^t \geq \varepsilon$ for all t and i, and $x_i^t \in \bar{X}$,with $\bar{X}$ finite. Under Inada conditions, the constraints, $\varphi_i^t \geq \varepsilon$ are not binding for ε sufficiently small. Moreover, $x_i^t \in int\bar{X}$ for $\bar{X}$ sufficiently large since the endowment of the economy is finite. Hence, the set of the equilibrium solutions and the set of the equilibrium solutions of the auxiliary program coincide for ε sufficiently small and $\bar{X}$ sufficiently large. As both production and intermediation technologies are linear, equilibrium prices satisfy: $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ for some $g_i^t \in \Re_+$ for all $i \in I$ and $t \in T$ and $\theta \in \Theta$; (ii) $w_i^t(\theta) = p_i^t(\theta) b_i^t a_i^t(\theta)$. Using these conditions and normalizing prices appropriately, the budget correspondence can be rewritten as:

$$B_i^t(q) = \sum_{\theta\in\Theta} p_i^t(\theta)\,(\sum_{c\in C} q_c(x_{ic}^t(\theta) - e_i^c - a_i^t(\theta)q_t(L - x_{iL}^t(\theta)) \leq 0\ \forall\, t$$

$B_i^t(q)$ is continuous for all $q \gg 0$. As a consequence,

$$(\zeta_i^t(p), \varphi_i^t(q)) = \left\{(x_i^t, \varphi_i^t) : (x_i^t, \varphi_i^t) \in \arg\max \sum_{t\in T} u_i^t(x_i^t)\varphi_i^t \text{ s.t. } (x_i^t, \varphi_i^t) \in B^i(q)\right\}$$

is upper hemicontinuous. While $\varphi_i^t(q)$ is also convex valued, however, $\zeta_i^t(q)$ has not a convex graph. By construction, however, the per capita demand correspondence $\xi_i^t(p) = \sum_{t\in T}\varphi_i^t(q)\zeta_i^t(q)$ is upperhemicontinuous and convex valued. Hence, a standard application of the Kakutani fixed point theorem, in the space $\bar{X} \times \Delta^{C-1}$ implies the existence result. The existence proof of an equilibrium for the case where lottery contracts are enforceable follows exactly the same argument developed above.

Proof of Proposition 13

Competitive equilibria satisfy the fair treatment condition. Indeed, if $u_i^t(x_i^t) > u_i^{t'}(x_i^{t'})$ for some pair (t, t') i $\varphi_i^t = 0$ would be optimal, contradicting $(\varphi_i^t, \varphi_i^{t'}) > 0$. Now let $(x^*, \varphi^*, q^*, z^*, \phi^*)$ a competitive equilibrium and assume it is not interim efficient. Assume that there exists a feasible allocation $(\widehat{x}, \widehat{\varphi}) \neq (x^*, \varphi^*)$ such that $u_i^t(\widehat{x}_i^t) = u_i^{t'}(\widehat{x}_i^{t'})$ for all i, t and t' with $(\widehat{\varphi}_i^t, \widehat{\varphi}_i^{t'}) > 0$, and $(\widehat{x}_i, \widehat{\varphi}_i) \succeq_i (x_i^*, \varphi_i^*)$ with $(\widehat{x}_i, \widehat{\varphi}_i) \succ_i (x_i^*, \varphi_i^*)$ for at least one i. Then:

$$\sum_{t\in T} \widehat{\varphi}_i^t \sum_{\theta\in\Theta} p_i^t(\theta) \sum_{c\in C} q_c^*(\widehat{x}_{ic}^t(\theta) - e_i^c) \geq \sum_{t\in T} \varphi_i^t \sum_{\theta\in\Theta} p_i^t(\theta) q_t^* a_i^t(\theta)(L - \widehat{x}_{Li}^t(\theta)), \quad \forall\, i \in I$$

where the inequality must be strict for at least one i. Multiplying both sides of type-i budget constraint by μ_i and adding up, one obtains:

$$\sum_{c\in C} q_c^* \left(\sum_{i\in I} \mu_i \left(\sum_{t\in T, \theta\in\Theta} \widehat{\varphi}_i^t p_i^t(\theta) \sum_{c\in C} \widehat{x}_{ic}^t(\theta) - e_i^c - \sum_{t\in T, \theta\in\Theta} \varphi_i^t p_i^t(\theta) a_i^t(\theta)(L - \widehat{x}_{Li}^t(\theta)) \right) \right) > 0$$

which implies that $(\widehat{x}_i, \widehat{\varphi}_i)$ violates feasibility.

Proof of Proposition 15

(i) and *(ii)* result from a standard application of the Separation Theorem**;** *(iii)* follows immediately by Proposition 2; *(iv)* is obtained by using the first-order conditions with respect to γ_i^t of the agents maximization program and the ex-ante budget constraint.

Proof of Proposition 16

Consider a solution, (α^P, x^P), of the Pareto program associated to a feasible vector of reservation utilities, $\overline{u}$. There exists an equilibrium with transfers such that:

$$\wp = (s_i^t = \sum_{c\in C, \theta\in\Theta} p_i^t(\theta)(\eta_c^P(x_{ic}^{tP}(\theta) - e_i^c) - \eta_t^P a_i^t(\theta) l_i^{tP}(\theta));\ \hat{w}_i^t(\theta) = \eta_t^P a_i^t(\theta);\ f_i^t(\theta) = 0, \forall\, \theta \in \Theta)$$

$\varphi_i^t = \alpha_t^{iP}$, $x^i = x^{iP}$, $q_c/q_1 = \eta_c^P/\eta_1^P$, $\phi_i^t(\theta) = p_i^t(\theta)$, and $w_i^t(\theta) = \eta_t^P a_i^t(\theta)$ for all $\theta \in \Theta, c \in C$, $t \in T$ and $i \in I$.

Indeed, for $\phi_i^t(\theta) = p_i^t(\theta)$, $q_c/q_1 = \eta_c^P/\eta_1^P$, by using the budget constraints defining the agent program one obtains: $\sum_{\theta\in\Theta} p_i^t(\theta) \left(\sum_{c\in C} \eta_c^P(x_{ic}^{tP}(\theta) - e_i^c) - \eta_t^P a_i^t(\theta) l_i^{tP}(\theta) - s_i^t\right) \leq 0$, where $l_i^{tP}(\theta) = (L - x_{iL}^{tP}(\theta))$. The vector $(x^i = x^{iP}, \varphi_i^t = \alpha_i^{tP})$ then solves the type-i agents maximization program for $q_c/q_1 = \eta_c^P/\eta_1^P\ \forall\, c$, and $s_i^t = \sum_{c\in C, \theta\in\Theta} p_i^t(\theta)(\eta_c^P(x_{ic}^{tP}(\theta) - e_i^c) - \eta_t^P a_i^t(\theta) l_i^{tP}(\theta))$. Moreover, by construction the transfers' policy is budget balancing. Finally, all the market clearing conditions are satisfied at $\phi_i^t(\theta) = p_i^t(\theta)$ and $w_i^t(\theta) = \hat{w}_i^t(\theta)$ for all $t \in T$, $\theta \in \Theta$. Indeed, at these prices $l_i^{tP}(\theta)$ as well as the supply of all state contingent assets are indeterminate, and all the feasibility conditions are satisfied. Assume now

$\hat{x}_{iL}^{P}(\theta) = L - L(\theta)$ for all $w_i^t(\theta) > 0$, and take a generic Pareto optimum, (α^P, x^P). Define a transfers' policy such that: $s_i^t = \sum_{c \in C, \theta \in \Theta} p_i^t(\theta)(\eta_c^P(x_{ic}^{tP}(\theta) - e_i^c) - \hat{w}_i^t L(\theta))$, $\hat{w}_i^t(\theta) = \hat{w}_i = \max_{t \in T}\left\{\eta_t^P a_i^t(\theta_N)\right\}\ \forall\ \theta \in \Theta$; and $f_i^t = -\eta_t^P \sum_{\theta \in \Theta} p_i^t(\theta) a_i^t(\theta) L(\theta) + \hat{w}_i \sum_{\theta \in \Theta} p_i^t(\theta) L(\theta)$. By following the argument developed above, one verifies that all feasibility constraints are satisfied.