A Paradox of Environmental Awareness Campaigns

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Abstract

We build a workable game of common-property resource extraction under rational Bayesian learning about the renewal prospects of a resource. We uncover the impact of exogenously shifting the prior beliefs of each player on the response functions of others. What we find about the role of environmental conservation campaigns is paradoxical. To the extent that such campaigns instill overly high pessimism about the potential of natural resources to reproduce, they create anti-conservation incentives: anyone having exploitation rights becomes inclined to consume more of the resource earlier, before others overexploit, and before the resource's stock is reduced to lower levels.

Keywords: renewable resources, resource exploitation, non-cooperative dynamic games, Bayesian learning, stochastic games, commons, rational learning, uncertainty, beliefs

JEL classification: D83, D84, C72, C73, O13, Q20, Q50, L70, O12

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1. Introduction

Markets invest vast amounts in renewable natural resources. Yet, it is difficult to argue that investors have rational expectations about the stochastic law of renewal of resources. In particular, a vivid debate among experts shows that important magnitudes regarding the fundamentals of natural-resource reproduction are obscure to both experts and the public.[1] So, among other goals, environmental awareness campaigns aim at providing information to the public about shifts in nature's fundamentals. "Skeptical environmentalists" such as Lomborg (2001), have expressed that awareness campaigns report unrealistically pessimistic magnitudes of environmental change. Our goal in this paper is not to examine the validity of any existing environmental campaigns. Instead, our goal is to study the impact of exogenously instilling overly high pessimism about resource renewal prospects to those who exploit common-property natural resources: it is not clear whether instilling such pessimism creates incentives for conservation.

We build a game of common-property resource exploitation. Players are unaware of nature's true parameters driving the ability of the resource to reproduce. So, players have initial beliefs about these parameters behind nature's fundamentals, and collect data in each period in order to update these beliefs, using Bayes' rule.[2] Outside this routine-type of learning, environmental awareness campaigns aim at shifting the priors of players. Whether

[1] The book by Lomborg (2001) scrutinizes this obscurity of public perceptions regarding shifts in nature's fundamentals due to a recent and drastic environmental regime switch such as "global warming". Debates about acid rain (see, for example, Lomborg (2001, pp. 178-181)), or about a large-scale biodiversity deterioration (Lomborg (2001, pp. 249-257)) are related to shifts in fundamentals regarding the natural ability of resources to reproduce. While environmental changes are not disputed among experts as facts, the magnitude of such changes is a vivid topic of disagreement among experts. For example, the article by Rörsch et al. (2005) documents the opposition against Lomborg's (2001) book by a part of the scientific community. What we keep from this debate among experts for the purposes of this study is that investor uncertainty about environmental fundamentals is a plausible working hypothesis for analyzing investments in natural resources.

[2] For example, collecting and processing data on each winter's temperature informs scientists and investors in natural-resource markets about the validity of their prior beliefs.

awareness campaigns are based on formal study or speculation is exogenous to our model. We do not model campaigns, but we build a framework where the role of priors of each player on strategies and response functions of all other players is clear.

Avoiding an ad-hoc analysis of learning and having a clear interplay between beliefs and the model's fundamentals is a key prerequisite in our analysis. To this end, we employ the concept of rational Bayesian learning. Rational-learning investors envisage the arrival of new information and anticipate learning in the future. The distinct feature of rational learning is that the mapping of priors to posteriors implied by Bayes' rule is incorporated in the recursive problem of each player together with other recursions governing laws of motion of the problem's state variables. In other words, when learning is rational prior beliefs become part of the problem's state variables.[3] Moreover, we focus on Markov-perfect Nash-equilibrium strategies. Our focus on this equilibrium concept enables us to use dynamic-programming techniques in order to tackle the problem.[4]

We combine two workable models in order to obtain robust analytical solutions. The first is the Levhari and Mirman (1980) example about strategic exploitation of renewable resources. The second is the framework by Koulovatianos, Mirman, and Santugini (2009) which combines dynamic programming with rational learning and provides analytical solutions. A key aspect of our analysis is that the stochastic structure of our model is general, i.e., our results do not depend on any specific assumptions about the densities of any random

[3] Our setup of Bayesian learners who anticipate learning in the future is similar to the case of rational learning examined by Guidolin and Timmermann (2007), Cogley and Sargent (2008), and Koulovatianos, Mirman, and Santugini (2009).

[4] Kalai and Lehrer (1993) show a key result related to our rational-learning formulation. When Bayesian updating is envisaged by each player, then Bayesian updating of collected information will lead in the long run to accurate prediction of the future play of the game and rational expectations as a limit of behavior with probability one, as time goes to infinity. This consideration on the side of players, that learning will be completed in the long run, and its impact on strategies, is a key distinctive feature of rational learning from other forms of learning. (The way to make players envisage Bayesian updating in our setting that focuses on Markov-perfect Nash strategies, is to incorporate Bayes' rule in the Bellman equation of each player.) In their survey paper, Blume and Easley (1993) explain this distinction as well.

variables in our model.

We obtain a unique and robust equilibrium in pure Markov-perfect strategies. We show that if the priors of at least one player about the renewal prospects of the resource are exogenously shifted to being more pessimistic than before, the response strategies of each player change. Making at least one player more pessimistic than before implies higher aggregate resource exploitation rates. This means that players choose to consume more of the resource earlier, before others have the chance to overexploit, and before the resource's stock is reduced to lower levels.

We also compare rational learning strategies with rational-expectations strategies under the assumption of common priors. We find that pessimism can even increase the intensity of the commons problem, i.e., the tendency to overexploit as the number of players increases. In particular, the commons problem is intensified in the case where the number of players is "small". Above all, our analysis demonstrates that, as long as those who exploit the resource are self-interested, there is one direction where incentives go in response to a campaign that shifts priors about resource renewal to higher pessimism: anti-conservation.

Our finding that pessimism leads to overexploitation incentives means environmental-awareness campaigns may not achieve their stated goals of resource conservation. Instead, shifting the priors of investors, countries, and companies with exploitation rights to more pessimism about the natural ability of resources to be renewed may even cause incentives for noncompliance with exploitation-rate quotas.

In Section 2 we present the model and its solution. In Section 3 we characterize the solution, providing results about how exploitation strategies respond to shifts in belief priors, and we discuss the paradox. In Section 4 we make concluding remarks.

2. Model

Time is discrete and the horizon is infinite ($t \in \{0, 1, ...\}$). There is a renewable resource, the stock of which is denoted by k. At time $t = 0$, the stock is $k_0 > 0$.

2.1 Nature's Law of Motion without Exploitation

When nobody exploits the resource, the law of motion of the resource's stock is,

$$k_{t+1} = k_t^{\eta_t} , \qquad t = 0, 1, ... \tag{1}$$

where parameter $\eta_t \in (0, 1)$ is a serially uncorrelated random variable with time-invariant density function $\phi(\eta|\theta^*)$ and support $\mathcal{H} \subseteq (0, 1)$, and θ^* is a vector of parameters which is constant over time and known by nature.

2.2 Players, Payoff Functions, and Resource Exploitation

Let a fixed set of $N \geq 1$ players, each having equal rights to exploit the stock k, as the resource is perfectly rivalrous and nonexcludable. Each player $i \in \{1, ..., N\}$ consumes $c_{i,t}$ units of the stock k in period $t \in \{0, 1, ...\}$, before nature reveals its realization η_t within period t.[5] So, after exploitation the law of motion becomes,

$$k_{t+1} = \left(k_t - \sum_{i=1}^{N} c_{i,t}\right)^{\eta_t} , \qquad t = 0, 1, ... \tag{2}$$

All players are infinitely-lived and have the same objective function, maximizing expected life-time utility from consumption. For player $i \in \{1, ..., N\}$ lifetime utility given by,

$$E_0 \left[\sum_{t=0}^{\infty} \delta^t \ln(c_{i,t})\right] ,$$

[5] We use a player's index $i \in \{1, ..., N\}$ as subscript for variables and as superscripts for functions, and we drop it whenever it is redundant.

where $\delta \in (0,1)$ is the discount factor. All players know the density function $\phi(\eta|\cdot)$, but do not know the true parameter vector θ^*. In period 0 each player $i \in \{1,...,N\}$ has a prior distribution function of beliefs about vector θ given by ξ_0^i. All players are Bayesian learners, so given ξ_0^i, for any sequence of realizations of the shock η, $\{\eta_t\}_{t=0}^{\infty}$, the sequence $\left\{\xi_t^i\right\}_{t=0}^{\infty}$ is generated by,

$$\xi_{t+1}^i(\theta|\eta_t) = \frac{\phi(\eta_t|\theta)\,\xi_t^i(\theta_t)}{\int_\Theta \phi(\eta|x)\,\xi_t^i(x)\,\mathrm{d}x}\ ,\ \ t \in \{0,1,...\} \text{ and } i \in \{1,...,N\}\ .$$

2.3 Assumptions about Players' Information

For notational simplicity let $\Xi_t \equiv \bigcup_{i\in\{1,\ldots,N\}} \xi_t^i$. A key assumption we make is that, *in period $t = 0$, all players know Ξ_0*. This assumption, that each player knows the priors of all other players, is both reasonable for our application, and it helps in clarifying the role of beliefs on strategic behavior in our analysis. For example, if each player is a country that wishes to manage its fishing industry, it is reasonable to assume that each country is accurately informed about collective beliefs in other countries. Such information can be collected through reading the press or public opinion poll findings about environmental issues in foreign countries.

Another key assumption is that players fully observe η_t in all periods $t \in \{0,1,...\}$. Assuming that additional noise is implicit in an observed signal does not add insights to understanding the effect of beliefs on players' strategies in our application.

2.4 Equilibrium Concept with Rational Learning

Definition 1 gives the core equilibrium concept we use throughout the paper.

Definition 1 *A Rational-Learning Markov-Perfect Nash Equilibrium (RLMPNE) is a set of strategies of the form $\{c_i = C^i(k;\Xi)\}_{i=1}^N$ such that for all $i \in \{1,...,N\}$,*

$c_i = C^i(k;\Xi)$ solves

$$V^i(k;\Xi) = \max_{c_i \geq 0} \left\{ \ln(c_i) + \right.$$

$$\left. +\delta \int_{\mathcal{H}} V^i\left(\left[k - c_i - \sum_{j \neq i} C^j(k;\Xi) \right]^{\eta} ; \hat{\Xi} \right) \left[\int_{\Theta} \phi(\eta|\theta)\, \xi^i(\theta)\, \mathrm{d}\theta \right] \mathrm{d}\eta \right\} \qquad (3)$$

subject to,

$$\hat{\xi}^i(\theta|\eta) = \frac{\phi(\eta|\theta)\, \xi^i(\theta)}{\int_{\Theta} \phi(\eta|x)\, \xi^i(x)\, \mathrm{d}x}\ , \quad i = 1, ..., N\ ,$$

where Θ is the support of θ, and Ξ is common knowledge to all players $i \in \{1, ..., N\}$.

Notice that V^i is the value function of player i, and it is distinguished from any other player's value function since players differ in that initial beliefs, ξ_0^i, generally differ across players. Notice also that we have used a hat for denoting variables one period ahead.

2.5 Rational-Learning Equilibrium

Proposition 1 states formally the solution to the model under rational learning, which is the key result of this paper. For notational compactness we let function μ be,

$$\mu(\theta) \equiv \int_{\mathcal{H}} \eta \phi(\eta|\theta)\, \mathrm{d}\eta\ , \qquad (4)$$

which captures nature's efficiency in natural resource reproduction in each period.

Proposition 1 *There is a unique pure-strategy RLMPNE solving the problem described by Definition 1, where $\{c_i = C^i(k;\Xi)\}_{i=1}^N$ is such that all players exhibit constant exploitation rates at all times, i.e. $C^i(k;\Xi)$ is of the form $C^i(k;\Xi) = c^i(\Xi) \cdot k$ with*

$$c^i(\Xi) = \frac{\left(\int_{\Theta} \frac{\delta\mu(\theta)}{1-\delta\mu(\theta)} \xi^i(\theta)\, \mathrm{d}\theta \right)^{-1}}{1\ +\ \sum_{j=1}^{N} \left(\int_{\Theta} \frac{\delta\mu(\theta)}{1-\delta\mu(\theta)} \xi^j(\theta)\, \mathrm{d}\theta \right)^{-1}}\ , \quad i = 1, ...N, \qquad (5)$$

while the aggregate resource exploitation rate by all players is,

$$\sum_{j=1}^{N} c^j(\Xi) = \frac{\sum_{j=1}^{N} \left(\int_\Theta \frac{\delta\mu(\theta)}{1-\delta\mu(\theta)} \xi^j(\theta)\, \mathrm{d}\theta \right)^{-1}}{1 + \sum_{j=1}^{N} \left(\int_\Theta \frac{\delta\mu(\theta)}{1-\delta\mu(\theta)} \xi^j(\theta)\, \mathrm{d}\theta \right)^{-1}} . \tag{6}$$

Proof See the Appendix. □

An attractive feature of the solution described in Proposition 1 is that there is only one pure-strategy RLMPNE. In addition, the closed form of equations (5) and (6) makes the impact of beliefs of other players on player i explicit. We exploit this tractability in a later section where we examine the shifting in players' priors through environmental conservation campaigns. Before moving to characterizing the equilibrium we briefly derive the rational-expectations equilibrium.

2.6 Benchmark Rational-Expectations Equilibrium

Learning is passive, and as $t \to \infty$, all players learn the true parameter with certainty. So, as $t \to \infty$, the game described in Definition 1 converges to rational expectations. *A Rational-Expectations Markov-Perfect Nash Equilibrium (REMPNE) is a set of strategies of the form* $\left\{c_i = C^{RE,i}(k)\right\}_{i=1}^{N}$ *such that for all* $i \in \{1, ..., N\}$, $c_i = C^{RE,i}(k)$ *solves the problem given in Definition 1 after substituting the generic rational expectations distribution for* ξ *(denote by* ξ^{RE}*), given by,* $\xi^{RE}(\theta) = 1$, *if* $\theta = \theta^*$, *and* $\xi^{RE}(\theta) = 0$, *if* $\theta \neq \theta^*$, *with* $\xi_i = \xi^{RE}$ *for all* $i \in \{1, ..., N\}$. The derivation of strategies $\left\{c_i = C^{RE,i}(k)\right\}_{i=1}^{N}$ is immediate from equations (5) and (6), after substituting the generic rational expectations distribution ξ^{RE} for all players. Given this immediate proof, Corollary 1 only states the unique Markov-perfect equilibrium under rational expectations.

Corollary 1 *There is a unique pure-strategy REMPNE solving the problem described by Definition 1 with* $\xi_i = \xi^{RE}$ *for all* $i \in \{1, ..., N\}$*, where* $\left\{c_i = C^{RE,i}(k)\right\}_{i=1}^{N}$ *is such that all players exhibit constant exploitation rates at all times, i.e.* $C^{RE,i}(k)$ *is of the form* $C^{RE,i}(k) = c^{RE,i} \cdot k$ *with*

$$c^{RE,i} = \frac{(\delta\mu(\theta^*))^{-1} - 1}{1 \ + \ N \cdot \left[(\delta\mu(\theta^*))^{-1} - 1\right]} \ , \quad i = 1, ...N, \tag{7}$$

while the aggregate resource exploitation rate by all players is,

$$\sum_{j=1}^{N} c^{RE,j} = \frac{N \cdot \left[(\delta\mu(\theta^*))^{-1} - 1\right]}{1 \ + \ N \cdot \left[(\delta\mu(\theta^*))^{-1} - 1\right]} \ . \tag{8}$$

One of the features of REMPNE is that under rational expectations all players have common priors. In addition, these priors under rational expectations reflect confidence that the correct parameter of nature is known by all players.

3. Belief Bias, Resource Exploitation Rates, and Conservation Campaigns

In this section we explore the impact of shifting priors towards more optimism/pessimism on exploitation rates of players. First, we look at the case where some players' priors are shifted within our RLMPNE concept. Then we examine how RLMPNE with common priors compares with rational expectations. In particular, we show how optimism/pessimism affects the intensity of the tragedy of the commons.

3.1 Increasing Optimism/Pessimism

Our goal is to apply our analysis to problems where priors at time 0 can be exogenously altered. An example of exogenously altering Ξ can be found in the case of environmental campaigns where novel expert information about laws of renewal of natural resources aims at shifting the priors of players. In particular, we are interested in understanding how bias of shifted priors towards optimism vs. pessimism alters exploitation strategies. To this end, in this section we focus on prior distributions that are comparable in terms of the direction of belief bias they entail. In particular, assume that ξ, $\underline{\xi}$, and $\bar{\xi}$ are related through a *strict first-order stochastic dominance* (FOSD) relationship, where

$$\underline{\xi} \prec_{FOSD} \xi \prec_{FOSD} \bar{\xi} \ .$$

Moreover, assume, for simplicity, that θ is not a parameter vector, but a single parameter. By the definition of strict FOSD,[6]

$$\underline{\xi} \prec_{FOSD} \xi \prec_{FOSD} \bar{\xi} \Leftrightarrow \int_{\Theta} h(\theta)\, \underline{\xi}(\theta)\, \mathrm{d}\theta < \int_{\Theta} h(\theta)\, \xi(\theta)\, \mathrm{d}\theta < \int_{\Theta} h(\theta)\, \bar{\xi}(\theta)\, \mathrm{d}\theta \ ,$$

for all strictly increasing functions h on Θ. So, if we assume that priors are initially ξ, and let $\mu' > 0$, then $\underline{\xi}$ represents a pessimistic shift in priors and $\bar{\xi}$ represents an optimistic shift in priors, since

$$\int_{\Theta} \mu(\theta)\, \underline{\xi}(\theta)\, \mathrm{d}\theta < \int_{\Theta} \mu(\theta)\, \xi(\theta)\, \mathrm{d}\theta < \int_{\Theta} \mu(\theta)\, \bar{\xi}(\theta)\, \mathrm{d}\theta \ .$$

Proposition 2 reveals the impact of shifts in any player's priors on the strategies of other players and on the aggregate exploitation rate.

Proposition 2 *Let $\mu' > 0$, and $\underline{\xi}^i \prec_{FOSD} \xi^i \prec_{FOSD} \bar{\xi}^i$ for some $i \in \{1, ..., N\}$. Then,*

$$\begin{array}{llcll}
\text{(i)} & c^i(\underline{\Xi}) & > & c^i(\Xi) > c^i(\bar{\Xi}) & \\
\text{(ii)} & c^j(\underline{\Xi}) & < & c^j(\Xi) < c^j(\bar{\Xi}) & \text{for all } j \in \{1, ...N\} \quad \text{with } j \neq i \\
\text{(iii)} & \sum_{j=1}^{N} c^j(\underline{\Xi}) & > & \sum_{j=1}^{N} c^j(\Xi) > \sum_{j=1}^{N} c^j(\bar{\Xi}) &
\end{array}$$

[6] For the definition of strict FOSD, see, for example, Jackson and Rogers (2007, p. 6).

where

$$\underline{\Xi} \equiv \bigcup_{\substack{j=1 \\ j\neq i}}^{N} \xi^j \cup \underline{\xi}^i \qquad \text{and} \qquad \bar{\Xi} \equiv \bigcup_{\substack{j=1 \\ j\neq i}}^{N} \xi^j \cup \bar{\xi}^i \; .$$

Proof Since $\mu' > 0$, $\int_\Theta \mu(\theta) / [1 - \delta\mu(\theta)] \, \underline{\xi}(\theta)\mathrm{d}\theta < \int_\Theta \mu(\theta) / [1 - \delta\mu(\theta)] \, \xi(\theta)\mathrm{d}\theta < \int_\Theta \mu(\theta) / [1 - \delta\mu(\theta)] \, \bar{\xi}(\theta)\mathrm{d}\theta$. So, inequalities (i) and (ii) are derived immediately from equation (5), whereas inequality (iii) is an immediate consequence of (6). □

Inequality (i) of Proposition 2 states that if a player i becomes pessimistic ($\underline{\xi}^i \prec_{FOSD} \xi^i$), then her exploitation rate rises. This is a similar result to the case of a single controller who plays a game alone with nature ($N = 1$). When $N = 1$, the single player is not encouraged to conserve the resource if she perceives that the resource's renewal rate is less productive: this creates an incentive to crop gains now instead of waiting for consuming later (so, $c^i(\underline{\Xi}) > c^i(\Xi)$).[7] Inequality (i) of Proposition 2 states that this logic dominates even in the presence of other players ($N > 1$). Yet, inequality (ii) of Proposition 2 states that all other players $j \neq i$ decrease their exploitation rate in response to the rising pessimism of player i ($\underline{\xi}^i \prec_{FOSD} \xi^i$). This happens because players $j \neq i$ see fewer resources left available from player i, since player i consumes at a higher rate, both currently and in the future. But overall, the aggregate exploitation rate drops when at least one player becomes pessimistic, i.e., the anti-conservation incentive dominates on aggregate (inequality (iii) of Proposition 2). By virtue of inequality (ii) of Proposition 2, if, say, an environmental campaign manages to shift the priors of some players towards more pessimism, the incentives will be to invest less in conserving the resource and to exploit more of it on aggregate.

[7] We elaborate on this point below, in our discussion of conservation incentives by self-interested utilitarian exploiters.

3.2 Belief Bias and the Intensity of the Commons Problem

Let's assume common priors, i.e. that $\xi_i = \xi$ for all $i \in \{1, ..., N\}$. Then (6) is of the form

$$\sum_{j=1}^{N} c^j(\xi) = \frac{N}{f(\xi) + N} \ , \tag{9}$$

where the functional $f(\xi)$ is given by,

$$f(\xi) = \int_{\Theta} \frac{\delta\mu(\theta)}{1 - \delta\mu(\theta)} \xi(\theta)\, d\theta \ .$$

Equation (9) implies that

$$\frac{\partial^2 \left[\sum_{j=1}^{N} c^j(\xi) \right]}{\partial N \partial f(\xi)} = [f(\xi) + N]^{-3} \cdot [N - f(\xi)] \ . \tag{10}$$

Equation (10) captures the intensity of the tragedy of the commons, as it determines whether a shift in priors captured by ξ intensifies or mitigates the tendency of the aggregate exploitation rate to rise after adding more players. Based on (10), Proposition 3 analyzes how a shift in common priors affects the intensity of the tragedy of the commons.

Proposition 3 *Let* $\mu' > 0$*, let all players have common priors, and* $\underline{\xi} \prec_{FOSD} \xi \prec_{FOSD} \bar{\xi}$*. If*

$$f(z) < (>) \ N \ , \quad \text{for all } z \in \left\{ \underline{\xi}, \xi, \bar{\xi} \right\} \ , \tag{11}$$

then pessimism mitigates (intensifies) the intensity of the tragedy of the commons while optimism intensifies (mitigates) it.

Proof Immediate from (10) since $f\left(\underline{\xi}\right) < f(\xi) < f\left(\bar{\xi}\right)$. This statement holds for exogenous shifts in priors. In addition, it holds for pessimism or optimism related to nature's true parameter θ^*. In particular, equation (8) coincides with equation (9) in the special case where $f\left(\xi^{RE}\right)$ is substituted in equation (9). Moreover, setting setting $\xi = \xi^{RE}$ above, which is to assume $\underline{\xi} \prec_{FOSD} \xi^{RE} \prec_{FOSD} \bar{\xi}$, also implies $f\left(\underline{\xi}\right) < f\left(\xi^{RE}\right) < f\left(\bar{\xi}\right)$. $\square$

What we learn from Proposition 3 is that there is a possibility for pessimism to intensify the tragedy of the commons: this happens when the number of players is small. Nevertheless, Proposition 3 says that, if the number of players is large, the impact of more pessimism is to mitigate the tendency for overexploitation as more players are added.

Our framework can also accommodate the study of second-order stochastic dominance of belief priors, focusing on mean-preserving spreads. While such an analysis is both interesting and easy to conduct, for the purpose of understanding the role of conservation campaigns, we wish to focus on the belief bias aspect alone. Certainly, an aspect of environmental awareness campaigns may be that campaigns can instill additional parameter uncertainty to players, thus increasing perceived risk. This exercise is immediate, following results on second-order stochastic dominance of belief priors shown in Koulovatianos, Mirman and Santugini (2009). It would not offer sufficient additional insight to elaborate on mean preserving spreads for the purposes of this paper.

3.3 Utilitarianism and the Paradox

Players in this game are self interested, but not myopic. Each player is infinitely-lived. Thinking of each player as a dynasty, each generation within dynasty $i \in \{1, ...N\}$ cares about the well being of its offsprings. In this sense, there is no lack of altruism towards future generations, apart from the fact that the discount factor, δ, places more weight on the current period compared to future periods. Yet, the noncooperative solution may reveal lack of altruism towards the community of players, so, below, we compare the noncooperative with the cooperative solution. Moreover, we discuss the fact that those who exploit the resource are utilitarian: they derive utility only from harvesting the resource and do not derive utility *directly/structurally* by the stock of the resource itself. We discuss the possibility that breaking the assumption of utilitarianism may lead to a resolution of the paradox.

3.3.1 Cooperative vs. Noncooperative solution and Beliefs

In order to investigate the possible role of altruism towards the community of players, we focus on the special case where $N = 1$, which can be seen as the cooperative solution. The comparison between equilibrium choices with $N = 1$ and equilibrium strategies with $N > 1$ can help in understanding the role of pessimism for anti-conservation after removing any strategic interaction among players.

In the special case where $N = 1$, equation (5) implies that

$$c(\Xi) = c(\xi) = \left(\int_{\Theta} \frac{1}{1 - \delta\mu(\theta)} \xi(\theta)\, d\theta\right)^{-1} , \tag{12}$$

which is the decision rule reported in Koulovatianos, Mirman, and Santugini (2009). Moreover, equation (12) immediately implies that

$$c\left(\xi^{RE}\right) = 1 - \delta\mu(\theta^*) \ . \tag{13}$$

Recall that $\mu(\theta^*)$ captures nature's efficiency in natural resource reproduction. Equation (13) reveals that whenever nature is less efficient in reproduction ($\mu(\theta^*)$ is low), then a self-interested utilitarian coalition has weaker conservation incentives, and chooses to consume the resource at higher rates. In the absence of rational expectations, equation (12) says that if the coalition does not have rational expectations and thinks that nature is less efficient in reproduction than it really is (say, $\xi \prec_{FOSD} \xi^{RE}$), then the coalition finds it optimal to have a higher exploitation rate than this of the coalition with rational expectations, as long as $\mu' > 0$ ($c(\xi) > c\left(\xi^{RE}\right)$). In addition, if an environmental awareness campaign instills extra pessimism to the coalition (i.e., $\underline{\xi} \prec_{FOSD} \xi$), then anti-conservation incentives will be strengthened further ($c\left(\underline{\xi}\right) > c\left(\xi\right)$).

Propositions 2 and 3 reveal that a utilitarian noncooperative group of players with $N > 1$ will exploit even more on aggregate. This is the well-known commons problem, as noncooperative players try to exploit the resource before others do so and reduce the stock of the resource in the future. Yet, what Propositions 2 and 3 add in this paper, is that pessimism does not alter the anti-conservation incentives that we observe in the cooperative case when lack of cooperation is considered. In fact, Proposition 3 reveals that in the case where N is sufficiently small, anti-conservation incentives generated by exogenously instilled pessimism to (all) players with common priors, are strengthened further by the fact that the harvesting solution is not cooperative.

3.3.2 Direct (Structural) Care about the Resource

Players in the game we have examined care about conservation of the resource. Yet, their conservation concerns are *indirect*, and are captured by the dependence of each player's *indirect utility function* on the resource stock, k. The direct utility of players is derived solely by the amounts they harvest and consume, as their momentary utility function is of the form $u\left(c_t\right)$.

It is, perhaps, the case that some environmentalists have some momentary utility function of the form $v\left(c_t, k_t\right)$ with $\partial v / \partial k_t > 0$, $t = 0, 1....$ This function v assumes a *direct/structural* derivation of utility about the stock of the resource, and may imply a direct conservation concern. Having $\partial v / \partial k_t > 0$ for some players means that players enjoy a walk in a forest, or a trip in a clean ocean, or that they care about conserving a part of the natural resource that they do not plan to consume.

Perhaps the rationale behind some awareness campaigns is to shift the fundamentals of players by strengthening the derivative $\partial v/\partial k_t > 0$ and, as a consequence, to potentially create direct conservation incentives. Whether this rationale is valid, is an open question, both theoretically and empirically. Theoretically, future work may examine whether the relative strength of the derivative $\partial v/\partial k_t > 0$ interacts with pessimism about nature's resource-renewal efficiency in a way that such an interaction leads to conservation incentives. Theoretically validating the role of such an interaction could be a resolution to the paradox stressed by our model in this paper. Empirically, it may be possible to uncover whether a structural derivative $\partial v/\partial k_t > 0$ really exists among the public.[8] Such an empirical investigation may be conducted through well-designed opinion poll surveys.

4. Conclusions

The stated goal of environmental campaigns is the conservation of natural resources. The goal of such campaigns is to shift the priors of investors, companies with exploitation rights, countries, even consumers, by presenting novel evidence irrelevant to the everyday process of learning by observation. Ideally, such campaigns would create *incentives* to investors for shifting towards a strategy of lower exploitation rate that conserves the resource. We have shown that, to the extent that such campaigns create pessimism, they lead to creating the opposite incentives. Instead of incentives to conserve, players have an incentive to increase their exploitation rates and to lower the stock of the renewable resource faster. They increase their exploitation rates because they find it suboptimal to invest in resource conservation, and they also do so in response to the strategies of other players who feel urged to crop gains from the resource as early as possible. As our workable example has shown, pessimism can even make the commons problem more intense, at least in cases where the number of players is relatively small. Since incentives are important for guaranteeing that even conservation

[8] Other environmental paradoxes seeking empirical validation are presented by Sinn (2008).

policies as direct as quotas can be implemented successfully, our study recommends caution to those who use environmental campaigns as a tool that may contribute to environmental conservation: incentives may go towards the opposite direction.

We have discussed that the reason behind the paradox might be that we have employed a fully utilitarian approach to environmental conservation. By utilitarianism we mean that those who exploit the resource derive utility only from harvesting the resource and do not derive utility *directly/structurally* by the stock of the resource itself. We think that our working hypothesis of restricted utilitarian self interest may be plausible for the majority of the public. Perhaps environmentalists who initiate conservation campaigns have the stock of the resource itself in their structural utility function and hope to instill the same structural concern to the public through overly pessimistic campaigns. Whether this is the key rationale and objective of some environmentalist groups that lead particular conservation campaigns and whether this rationale can overcome the anti-conservation paradox that our utilitarian framework revealed in this study, is a key question for future research. Such research may rely on carefully designed public opinion polls capable of measuring structural vs. indirect flows of utility from natural resource stocks to the public (i.e., whether the public cares about seeing a sufficiently conserved forest and clean oceans vs. caring only about future harvest gains implied by better conserved forests and oceans).

Regarding our framework of analysis, our study has assumed that each player knows the prior beliefs of all other players. If, for example, players are countries, then the media may have the beliefs of each country fully revealed, validating our assumption. Nevertheless, our study has left unexplored games where players are uninformed about the beliefs of other players, perhaps more suitable for different applications. This case of second-order learning (with each player updating her beliefs about the beliefs of other players) is an extension for future research that our framework may accommodate.

5. Appendix – Proofs

The proof of Proposition 1 relies on Lemmata 1 and 2, which are separate results. We state and prove Lemma 1 below, while the proof of Lemma 2 appears in the literature, so we state Lemma 2 and provide an appropriate citation for its proof.

Lemma 1 *Let $h : \mathbb{R} \to \mathbb{R}$ and a given prior distribution ξ_0. Then for any $t \in \{1, 2, ...\}$ and $\{\xi_s\}_{s=0}^{t-1}$ generated through*

$$\xi_{\tau+1}(\theta|\eta) = \frac{\phi(\eta|\theta)\,\xi_\tau(\theta)}{\int_\Theta \phi(\eta|x)\,\xi_\tau(x)\,\mathrm{d}x}\ , \quad \tau = 0, 1, \dots\ , \tag{14}$$

from any sequence $\{\eta_s\}_{s=0}^{t-1}$ of independent draws of the shock η, the conditional expectation $E_0\left(h\left(\eta_{t-1}\right)|\xi_0\right)$ where,

$$E_0\left(h\left(\eta_{t-1}\right)|\xi_0\right) \equiv \int_{\mathcal{H}}\int_\Theta \cdots \int_{\mathcal{H}}\int_\Theta\int_{\mathcal{H}}\int_\Theta \prod_{s=0}^{t-1} h(\eta_s)\,\phi\left(\eta_{t-1}|\theta_{t-1}\right)\xi_{t-1}(\theta_{t-1})\,\mathrm{d}\theta_{t-1}\mathrm{d}\eta_{t-1}\times$$

$$\times\phi\left(\eta_{t-2}|\theta_{t-2}\right)\xi_{t-2}(\theta_{t-2})\,\mathrm{d}\theta_{t-2}\mathrm{d}\eta_{t-2}\times\cdots\times\phi(\eta_0|\theta_0)\,\xi_0(\theta_0)\,\mathrm{d}\theta_0\mathrm{d}\eta_0\ ,$$

is given by,

$$E_0\left(h\left(\eta_{t-1}\right)|\xi_0\right) = \int_\Theta\left[\int_{\mathcal{H}} h(\eta)\,\phi(\eta|\theta)\,\mathrm{d}\eta\right]^t \xi_0(\theta)\,\mathrm{d}\theta\ . \tag{15}$$

Proof of Lemma 1

We express ξ_{t-1} as a function of ξ_{t-2}, according to the Bayesian update of beliefs given by (14), and we substitute it into the LHS of (15),

$$\int_{\mathcal{H}}\int_\Theta \cdots \int_{\mathcal{H}}\int_\Theta\int_{\mathcal{H}}\int_\Theta \prod_{s=0}^{t-1} h(\eta_s)\,\phi\left(\eta_{t-1}|\theta_{t-1}\right)\frac{\phi\left(\eta_{t-2}|\theta_{t-1}\right)\xi_{t-2}(\theta_{t-1})}{\int_\Theta \phi\left(\eta_{t-2}|x\right)\xi_{t-2}(x)\,\mathrm{d}x}\times$$

$$\times\mathrm{d}\theta_{t-1}\mathrm{d}\eta_{t-1}\phi\left(\eta_{t-2}|\theta_{t-2}\right)\xi_{t-2}(\theta_{t-2})\,\mathrm{d}\theta_{t-2}\mathrm{d}\eta_{t-2}\times\cdots\times\phi(\eta_0|\theta_0)\,\xi_0(\theta_0)\,\mathrm{d}\theta_0\mathrm{d}\eta_0 =$$

$$= \int_{\mathcal{H}} \int_{\Theta} \cdots \int_{\mathcal{H}} \int_{\mathcal{H}} \int_{\Theta} \prod_{s=0}^{t-1} h(\eta_s)\, \phi\left(\eta_{t-1} | \theta_{t-1}\right) \phi\left(\eta_{t-2} | \theta_{t-1}\right) \xi_{t-2}(\theta_{t-1})\, \mathrm{d}\theta_{t-1} \times$$

$$\times \mathrm{d}\eta_{t-1} \mathrm{d}\eta_{t-2} \phi\left(\eta_{t-3} | \theta_{t-3}\right) \xi_{t-3}(\theta_{t-3})\, \mathrm{d}\theta_{t-3} \mathrm{d}\eta_{t-3} \times \cdots \times \phi(\eta_0 | \theta_0)\, \xi_0(\theta_0)\, \mathrm{d}\theta_0 \mathrm{d}\eta_0 \ ,$$

i.e., ξ_{t-1} has been cancelled from the expression. Continuing in this way up to period 0, the LHS of (15) becomes

$$\int_{\mathcal{H}} \cdots \int_{\mathcal{H}} \int_{\mathcal{H}} \int_{\Theta} \prod_{s=0}^{t-1} h(\eta_s)\, \phi(\eta_s | \theta_{t-1})\, \xi_0(\theta_{t-1})\, \mathrm{d}\theta_{t-1} \mathrm{d}\eta_{t-1} \mathrm{d}\eta_{t-2} \times \cdots \times \mathrm{d}\eta_0 \ ,$$

and as η's are independent over time, this last expression is equal to the RHS of (15). □

Lemma 2 (matrix determinant lemma) *Let A be an $N \times N$ nonsingular matrix, and x, y be any $N \times 1$ vectors. Then,*

$$\det\left(A + x \cdot y^T\right) = \left(1 + y^T \cdot A^{-1} \cdot x\right) \cdot \det(A) \ .$$

Proof of Lemma 2

See Harville (1997, p. 416, Theorem 18.1.1 and Corollaries 18.1.2 and 18.1.3). □

Proof of Proposition 1

Our solution approach follows Levhari and Mirman (1980). We start from deriving RLMPNE in the finite-horizon setting. Then we use the finite-horizon RLMPNE results in order to generalize them to the infinite-horizon case. The approach of Levhari and Mirman (1980) for proving the result helps in exhibiting the informational structure of the problem.

The static problem (0-period-horizon problem)

The Nash-equilibrium solution is not unique in this case, but without loss of generality we can set,

$$c_0^i = \kappa_i^{(0)} k \ , \tag{16}$$

where $\kappa_i^{(0)}$ is some constant with $\kappa_i^{(0)} \in [0,1]$ for all $i \in \{1,...,N\}$, with $\Sigma_{i=1}^N \kappa_i^{(0)} = 1$. (We are solving this problem recursively, so we denote the n-th iteration by a superscript "(n)" wherever this is applicable.) In order to keep each player's problem well-defined in next iteration, without loss of generality we focus on a solution where $\kappa_i^{(0)} \in (0,1)$ for all $i \in \{1,...,N\}$, with with $\Sigma_{i=1}^N \kappa_i^{(0)} = 1$. So, the value function of the agent in the static problem is,

$$V^{i,(0)} (k; \Xi_0) = \ln(k) + \ln\left(\kappa_i^{(0)}\right) \ , \tag{17}$$

which is the only case where the value function does not depend on Ξ_0.

The 1-period-horizon problem

The decision of the player i is determined by the Bellman equation,

$$V^{i,(1)} (k; \Xi_0) = \max_{c_i \geq 0} \Bigg\{ \ln(c_i) + \\ + \delta \int_{\mathcal{H}} V^{i,(0)} \left(\left[k - c_i - \sum_{j \neq i} C^{j,(1)} (k; \Xi_0) \right]^{\eta} ; \Xi_1 \right) \left[\int_{\Theta} \phi(\eta|\theta) \, \xi_0^i(\theta) \, \mathrm{d}\theta \right] \mathrm{d}\eta \Bigg\}$$

so, using (17),

$$V^{i,(1)} (k; \Xi_0) = \max_{c_i \geq 0} \Bigg\{ \ln(c_i) + \\ + \delta \int_{\mathcal{H}} \eta \ln \left[k - c_i - \sum_{j \neq i} C^{j,(1)} (k; \Xi_0) \right] \left[\int_{\Theta} \phi(\eta|\theta) \, \xi_0^i(\theta) \, \mathrm{d}\theta \right] \mathrm{d}\eta + \delta \ln\left(\kappa_i^{(0)}\right) \Bigg\} \tag{18}$$

with first-order condition,

$$\frac{1}{c_i} = \frac{\delta \int_{\mathcal{H}} \eta \left[\int_{\Theta} \phi\left(\eta|\theta\right) \xi_0^i\left(\theta\right) \mathrm{d}\theta \right] \mathrm{d}\eta}{k - c_i - \sum\limits_{j \neq i} C^{j,(1)}\left(k; \Xi_0\right)} \ ,$$

which implies that, for all $i \in \{1, ..., N\}$, $C^{i,(1)}\left(k; \Xi_0\right)$ is of the multiplicatively-separable form,

$$C^{i,(1)}\left(k; \Xi_0\right) = c^{i,(1)}\left(\Xi_0\right) \cdot k \ , \tag{19}$$

where $\left\{c^{i,(1)}\left(\Xi_0\right)\right\}_{i=1}^{N}$ is the unique solution to the linear system,

$$\begin{bmatrix} 1 + \delta E_0\left(\eta|\xi_0^1\right) & 1 & \cdots & 1 \\ 1 & 1 + \delta E_0\left(\eta|\xi_0^2\right) & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 + \delta E_0\left(\eta|\xi_0^N\right) \end{bmatrix} \begin{bmatrix} c^{1,(1)}\left(\Xi_0\right) \\ c^{2,(1)}\left(\Xi_0\right) \\ \vdots \\ c^{N,(1)}\left(\Xi_0\right) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \ , \tag{20}$$

with

$$E_0\left(\eta|\xi_0^i\right) \equiv \int_{\mathcal{H}} \eta \left[\int_{\Theta} \phi\left(\eta|\theta\right) \xi_0^i\left(\theta\right) \mathrm{d}\theta \right] \mathrm{d}\eta \ , \quad i = 1, ..., N \ . \tag{21}$$

The linear system given by (20) has a unique solution with $c^{i,(1)}\left(\Xi_0\right) \in (0, 1)$, and $\Sigma_i c^{i,(1)}\left(\Xi_0\right) \in (0, 1)$. Substituting $\left\{C^{i,(1)}\left(\Xi_0\right)\right\}_{i=1}^{N}$ of the form given by (19) into the Bellman equation given by (18) leads to a value function of the form,

$$V^{i,(1)}\left(k; \Xi_0\right) = \left\{1 + \delta \int_{\mathcal{H}} \eta \left[\int_{\Theta} \phi\left(\eta|\theta\right) \xi_0^i\left(\theta\right) \mathrm{d}\theta \right] \mathrm{d}\eta \right\} \ln\left(k\right) + \kappa^{i,(1)}\left(\Xi_0\right) \ , \tag{22}$$

for all $i \in \{1, ..., N\}$, where $\kappa^{i,(1)}\left(\Xi_0\right)$ is a constant that does not affect optimization in future steps. Unlike before, $V^{i,(1)}\left(k; \Xi_0\right)$ depends on Ξ_0, but we have an explicit form for the way this function depends on Ξ_0. Most interestingly, in equation (22) the coefficient of $\ln\left(k\right)$ depends on ξ_0^i only, and not on the beliefs of other individuals.

The 2-period-horizon problem

The decision of player i is now determined by the Bellman equation,

$$V^{i,(2)}(k;\Xi_0)=\max_{c_i\geq 0}\left\{\ln(c_i)+\right.$$

$$\left.+\delta\int_{\mathcal{H}}V^{i,(1)}\left(\left[k-c_i-\sum_{j\neq i}C^{j,(2)}(k;\Xi_0)\right]^{\eta};\Xi_1\right)\left[\int_{\Theta}\phi(\eta|\theta)\,\xi_0^i(\theta)\,\mathrm{d}\theta\right]\mathrm{d}\eta\right\}$$

so, using (22),

$$V^{i,(2)}(k;\Xi_0)=\max_{c_i\geq 0}\left\{\ln(c_i)+\right.$$

$$+\delta\int_{\mathcal{H}}\eta_0\left\{1+\delta\int_{\mathcal{H}}\eta_1\left[\int_{\Theta}\phi(\eta_1|\theta_1)\,\xi_1^i(\theta_1|\eta_0)\,\mathrm{d}\theta_1\right]\mathrm{d}\eta_1\right\}\ln\left[k-c_i-\sum_{j\neq i}C^{j,(2)}(k;\Xi_0)\right]\times$$

$$\left.\times\left[\int_{\Theta}\phi(\eta_0|\theta)\,\xi_0^i(\theta)\,\mathrm{d}\theta\right]\mathrm{d}\eta_0+\delta\kappa^{i,(1)}(\Xi_1)\right\}\qquad(23)$$

subject to,

$$\xi_1^i(\theta|\eta)=\frac{\phi(\eta|\theta)\,\xi_0^i(\theta)}{\int_{\Theta}\phi(\eta|x)\,\xi_0^i(x)\,\mathrm{d}x}\ ,\qquad i=1,...,N\ .\qquad(24)$$

What is crucial to observe here is the notation about the timing of shocks. In the problem expressed by (23), each player is deciding upon a strategy in period 0, expecting both a shock η_0 in period 0, after the decision has been made, and a shock η_1 in period 1. Yet, it is the shock η_0 which will determine how the prior distribution ξ_0^i will evolve to ξ_1^i, which is an element that the analytic form of (23) allows us to see explicitly. So, in this case where the horizon is expanded, we can see how prior beliefs determine what type of information is expected to arrive and also how this information is expected to be exploited.

To simplify notation, we can re-write (23) as,

$$V^{i,(2)}(k;\Xi_0)=\max_{c_i\geq 0}\left\{\ln(c_i)+\right.$$

$$+\delta\left[E_0\left(\eta_0|\xi_0^i\right)+\delta E_0\left(\eta_1|\xi_0^i\right)\right]\ln\left[k-c_i-\sum_{j\neq i}C^{j,(2)}\left(k;\Xi_0\right)\right]+\delta\kappa^{i,(1)}\left(\Xi_1\right)\Bigg\}\ , \tag{25}$$

with

$$E_0\left(\eta_0|\xi_0^i\right)\equiv\int_{\mathcal{H}}\eta_0\left[\int_\Theta\phi\left(\eta_0|\theta\right)\xi_0^i\left(\theta\right)\mathrm{d}\theta\right]\mathrm{d}\eta_0\ ,$$

as in equation (21) above, and,

$$E_0\left(\eta_1|\xi_0^i\right)\equiv\int_{\mathcal{H}}\eta_0\int_{\mathcal{H}}\eta_1\left[\int_\Theta\phi\left(\eta_1|\theta_1\right)\xi_1^i\left(\theta_1|\eta_0\right)\mathrm{d}\theta_1\right]\mathrm{d}\eta_1\left[\int_\Theta\phi\left(\eta_0|\theta\right)\xi_0^i\left(\theta\right)\mathrm{d}\theta\right]\mathrm{d}\eta_0\ ,$$

where $\xi_1^i\left(\theta_1|\eta_0\right)$ is given from (24). The first-order conditions of (25) are given by,

$$\frac{1}{c_i}=\delta\left[E_0\left(\eta_0|\xi_0^i\right)+\delta E_0\left(\eta_1|\xi_0^i\right)\right]\frac{1}{k-c_i-\sum_{j\neq i}C^{j,(2)}\left(k;\Xi_0\right)}\ ,$$

which implies that, $C^{i,(1)}\left(k;\Xi_0\right)$ is of the multiplicatively-separable form,

$$C^{i,(2)}\left(k;\Xi_0\right)=c^{i,(2)}\left(\Xi_0\right)\cdot k\ , \tag{26}$$

for all $i\in\{1,...,N\}$, where $\left\{c^{i,(2)}\left(\Xi_0\right)\right\}_{i=1}^N$ is the unique solution to the linear system,

$$\begin{bmatrix} A^{(2)}\left(\xi_0^1\right) & 1 & \cdots & 1 \\ 1 & A^{(2)}\left(\xi_0^2\right) & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & A^{(2)}\left(\xi_0^N\right) \end{bmatrix}\cdot\begin{bmatrix} c^{1,(2)}\left(\Xi_0\right) \\ c^{2,(2)}\left(\Xi_0\right) \\ \vdots \\ c^{N,(2)}\left(\Xi_0\right) \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}\ , \tag{27}$$

where

$$A^{(2)}\left(\xi_0^i\right)\equiv 1+\delta\left[E_0\left(\eta_0|\xi_0^i\right)+\delta E_0\left(\eta_1|\xi_0^i\right)\right]\ ,\ \ i\in\{1,...,N\}\,.$$

Again, (27) has a unique solution with $c^{i,(2)}\left(\Xi_0\right)\in(0,1)$, and $\Sigma_i c^{i,(2)}\left(\Xi_0\right)\in(0,1)$, while substitution of $\left\{C^{i,(2)}\left(\Xi_0\right)\right\}_{i=1}^N$ as given by (26) into the Bellman equation given by (25) gives a value function of the form,

$$V^{i,(2)}\left(k;\Xi_0\right)=A^{(2)}\left(\xi_0^i\right)\ln\left(k\right)+\kappa^{i,(2)}\left(\Xi_0\right)\ ,$$

where $\kappa^{i,(2)}(\Xi_0)$ is a constant that does not affect optimization in any future step. At this point we have seen enough of the problem's structure to be able to deduce the formulas of the n-period horizon problem.

The n-period-horizon problem

The strategy of player i is determined by the Bellman equation,

$$V^{i,(n)}(k;\Xi_0) = \max_{c_i \geq 0} \left\{ \ln(c_i) + \right.$$

$$\left. +\delta \int_{\mathcal{H}} V^{i,(n-1)} \left(\left[k - c_i - \sum_{j \neq i} C^{j,(n)}(k;\Xi_0) \right]^{\eta} ; \Xi_1 \right) \left[\int_{\Theta} \phi(\eta|\theta)\, \xi_0^i(\theta)\, \mathrm{d}\theta \right] \mathrm{d}\eta \right\}$$

with $V^{i,(n)}(k;\Xi_0)$ being of the form,

$$V^{i,(n)}(k;\Xi_0) = \left[1 + \delta \sum_{t=0}^{n-1} \delta^t E_0\left(\eta_t|\xi_0^i\right) \right] \ln(k) + \kappa^{i,(n)}(\Xi_0) \ , \tag{28}$$

where $\kappa^{i,(n)}(\Xi_0)$ is a constant, and

$$E_0\left(\eta_t|\xi_0^i\right) \equiv \int_{\mathcal{H}} \int_{\Theta} \cdots \int_{\mathcal{H}} \int_{\Theta} \int_{\mathcal{H}} \int_{\Theta} \prod_{s=0}^{t} \eta_s \phi(\eta_t|\theta_t)\, \xi_t^i(\theta_t)\, \mathrm{d}\theta_t \mathrm{d}\eta_t \times$$

$$\times \phi\left(\eta_{t-1}|\theta_{t-1}\right) \xi_{t-1}^i(\theta_{t-1})\, \mathrm{d}\theta_{t-1}\mathrm{d}\eta_{t-1} \times \cdots \times \phi(\eta_0|\theta_0)\, \xi_0^i(\theta_0)\, \mathrm{d}\theta_0 \mathrm{d}\eta_0 \ .$$

Moreover, players' strategies are of the form

$$C^{i,(n)}(\Xi_0) = c^{i,(n)}(\Xi_0) \cdot k \ , \quad i = 1, ..., N \ , \tag{29}$$

where $\left\{c^{i,(n)}(\Xi_0)\right\}_{i=1}^{N}$ is the unique solution to the linear system,

$$\begin{bmatrix} A^{(n)}\left(\xi_0^1\right) & 1 & \cdots & 1 \\ 1 & A^{(n)}\left(\xi_0^2\right) & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & A^{(n)}\left(\xi_0^N\right) \end{bmatrix} \cdot \begin{bmatrix} c^{1,(n)}(\Xi_0) \\ c^{2,(n)}(\Xi_0) \\ \vdots \\ c^{N,(n)}(\Xi_0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \ , \tag{30}$$

where

$$A^{(n)}\left(\xi_0^i\right) \equiv 1+\delta\sum_{t=0}^{n-1}\delta^t E_0\left(\eta_t|\xi_0^i\right) \ , \ i \in \{1,...,N\} \ .$$

To calculate $E_0\left(\eta_t|\xi_0^i\right)$ we rely on Lemma 1. From equation (15) of Lemma 1, after setting $h(\eta)=\eta$, the identity function, we obtain,

$$E_0\left(\eta_t|\xi_0^i\right) = \int_\Theta \left[\int_{\mathcal{H}} \eta\phi\left(\eta|\theta\right) \mathrm{d}\eta\right]^{t+1} \xi_0^i\left(\theta\right) \mathrm{d}\theta \ ,$$

and from (4) it is,

$$E_0\left(\eta_t|\xi_0^i\right) = \int_\Theta \left[\mu\left(\theta\right)\right]^{t+1} \xi_0^i\left(\theta\right) \mathrm{d}\theta \ . \tag{31}$$

Substituting (31) into (28) we obtain,

$$V^{i,(n)}\left(k;\Xi_0\right) = \left[1+\delta\int_\Theta \sum_{t=0}^{n-1}\delta^t \left[\mu\left(\theta\right)\right]^{t+1} \xi_0^i\left(\theta\right) \mathrm{d}\theta\right] \ln\left(k\right) + \kappa^{i,(n)}\left(\Xi_0\right) \ . \tag{32}$$

The infinite-horizon problem

After taking the limit when $n \to \infty$, (32) gives,

$$V^{i,(n)}\left(k;\Xi_0\right) = V^i\left(k;\Xi_0\right) = \left[1+\delta\int_\Theta \sum_{t=0}^{\infty}\delta^t \left[\mu\left(\theta\right)\right]^{t+1} \xi_0^i\left(\theta\right) \mathrm{d}\theta\right] \ln\left(k\right) + \kappa^{(\infty)}\left(\Xi_0\right) \ ,$$

or,

$$V^i\left(k;\Xi\right) = \int_\Theta \frac{1}{1-\delta\mu\left(\theta\right)} \xi^i\left(\theta\right) \mathrm{d}\theta \ln\left(k\right) + \kappa^{(\infty)}\left(\Xi\right) \ .$$

(Subscript "0" of Ξ_0 has appeared in order to remind that Ξ_0 denotes prior beliefs in period 0. In the infinite-horizon setup this timing does not matter any more, so subscript "0" can be dropped. So, we drop it throughout the rest of the proof.) Moreover, the solution is again of the multiplicatively separable form

$$C^{i,(\infty)}\left(k;\Xi\right) = C^i\left(k;\Xi\right) = c^i\left(\Xi\right) \cdot k \ ,$$

and (30) is generalized to,

$$\mathbf{A} \cdot \begin{bmatrix} c^1(\Xi) \\ c^2(\Xi) \\ \vdots \\ c^N(\Xi) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} , \tag{33}$$

where

$$\mathbf{A} \equiv \begin{bmatrix} \int_\Theta \frac{1}{1-\delta\mu(\theta)} \xi^1(\theta)\, \mathrm{d}\theta & 1 & \cdots & 1 \\ 1 & \int_\Theta \frac{1}{1-\delta\mu(\theta)} \xi^2(\theta)\, \mathrm{d}\theta & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & \int_\Theta \frac{1}{1-\delta\mu(\theta)} \xi^N(\theta)\, \mathrm{d}\theta \end{bmatrix} .$$

Notice that since $\mathcal{H} \subseteq (0,1)$, $\mu(\theta) \in (0,1)$, which guarantees that $V^i(k;\Xi)$ is well-defined, and that

$$\int_\Theta \frac{1}{1-\delta\mu(\theta)} \xi^i(\theta)\, d\theta > 1 , \quad i \in \{1, .., N\} . \tag{34}$$

Inequality (34) guarantees that $\mathbf{A}$ is nonsingular, implying that the solution to (33)

$$\begin{bmatrix} c^1(\Xi) \\ c^2(\Xi) \\ \vdots \\ c^N(\Xi) \end{bmatrix} = \mathbf{A}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{35}$$

exists and it is unique.

The compact solution form given by (35) hides the role of other player's beliefs on the beliefs of each particular player. Yet, equation (35) can lead to a closed-form solution through the aid of the *matrix determinant lemma (Lemma 2)*. In particular, let any $i \in \{1, .., N\}$, and notice that (35) implies,

$$c^i(\Xi) = \mathbf{0}_{\mathbf{1}_i}^T \cdot \mathbf{A}^{-1} \cdot \mathbf{1}_N , \tag{36}$$

where $\mathbf{1}_N$ is an $N \times 1$ vector of ones, and $\mathbf{0}_{\mathbf{1}_i}$ is an $N \times 1$ vector of zeros, with the sole exception that its i-th element is equal to 1. The *matrix determinant lemma (Lemma 2)* implies that

$$\det\left(\mathbf{A} - \mathbf{1}_N \cdot \mathbf{0}_{\mathbf{1}_i}^T\right) = \left(1 - \mathbf{0}_{\mathbf{1}_i}^T \cdot \mathbf{A}^{-1} \cdot \mathbf{1}_N\right) \cdot \det(\mathbf{A}) \ . \tag{37}$$

Setting,

$$\mathbf{A}_i \equiv \mathbf{A} - \mathbf{1}_N \cdot \mathbf{0}_{\mathbf{1}_i}^T \ ,$$

and combining (36) with (37), we obtain

$$c^i(\Xi_0) = 1 - \frac{\det(\mathbf{A}_i)}{\det(\mathbf{A})} \ . \tag{38}$$

Equation (38) implies that, in order to characterize $c^i(\Xi_0)$, we must first characterize $\det(\mathbf{A}_i)$ and $\det(\mathbf{A})$. We start from characterizing $\det(\mathbf{A})$. Let

$$\tilde{\mathbf{A}} \equiv \mathbf{A} - \mathbf{1}_N \cdot \mathbf{1}_N^T \ ,$$

which implies that $\tilde{\mathbf{A}}$ is a diagonal matrix. Denoting the i-th diagonal element of $\tilde{\mathbf{A}}$ by $\texttt{diag}\left(\tilde{\mathbf{A}}\right)_i$, it is,

$$\texttt{diag}\left(\tilde{\mathbf{A}}\right)_i = \int_\Theta \frac{\delta\mu(\theta)}{1 - \delta\mu(\theta)} \xi_0^i(\theta)\,\mathrm{d}\theta \ . \tag{39}$$

Applying again the *matrix determinant lemma (Lemma 2)*,

$$\det(\mathbf{A}) = \det\left(\tilde{\mathbf{A}} + \mathbf{1}_N \cdot \mathbf{1}_N^T\right) = \left(1 + \mathbf{1}_N^T \cdot \tilde{\mathbf{A}}^{-1} \cdot \mathbf{1}_N\right) \cdot \det\left(\tilde{\mathbf{A}}\right) \ ,$$

which implies

$$\det(\mathbf{A}) = \left(1 \ + \ \sum_{i=1}^N \frac{1}{\texttt{diag}\left(\tilde{\mathbf{A}}\right)_i}\right) \cdot \prod_{i=1}^N \texttt{diag}\left(\tilde{\mathbf{A}}\right)_i \ . \tag{40}$$

In order to characterize $\det(\mathbf{A}_i)$, we use the definitions of $\mathbf{A}_i$ and to $\tilde{\mathbf{A}}$, noticing that $\mathbf{A}_i - \tilde{\mathbf{A}} = \mathbf{1}_N \cdot \left(\mathbf{1}_N^T - \mathbf{0}_{\mathbf{1}_i}^T\right)$, which implies,

$$\mathbf{A}_i = \tilde{\mathbf{A}} + \mathbf{1}_N \cdot \mathbf{1}_{\mathbf{0}_i}^T \ , \tag{41}$$

where $\mathbf{1}_{\mathbf{0}_i}$ is an $N \times 1$ vector of ones, with the sole exception that its i-th element is equal to 0. Combining (41) with the *matrix determinant lemma (Lemma 2)*,

$$\det\left(\mathbf{A}_i\right) = \det\left(\tilde{\mathbf{A}} + \mathbf{1}_N \cdot \mathbf{1}_{\mathbf{0}_i}^T\right) = \left(1 + \mathbf{1}_{\mathbf{0}_i}^T \cdot \tilde{\mathbf{A}}^{-1} \cdot \mathbf{1}_N\right) \cdot \det\left(\tilde{\mathbf{A}}\right) \ ,$$

which gives,

$$\det\left(\mathbf{A}_i\right) = \left(1 \ + \ \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{1}{\mathtt{diag}\left(\tilde{\mathbf{A}}\right)_j}\right) \cdot \prod_{i=1}^{N} \mathtt{diag}\left(\tilde{\mathbf{A}}\right)_i \ . \tag{42}$$

Uniqueness of $c^i\left(\Xi_0\right)$ for all $i \in \{1, ..., N\}$ is guaranteed due to the nonsingularity of matrix $\mathbf{A}$ in system (33). Equation (5) is derived after combining (38) with (42), (40), and (39). Equation (6) follows directly from (5), proving the proposition. $\square$

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