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COST INNOVATION: SCHUMPETER AND EQUILIBRIUM.
PART 1. ROBINSON CRUSOE

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# Cost Innovation: Schumpeter and Equilibrium. Part 1. Robinson Crusoe

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### Abstract

Modifying a parallel dynamic programming approach to a simple deterministic economy, we consider the effect of an innovation in the means of production. The success of the innovation is assumed to depend on the availability of financing, locus of financial control, the amount of resources invested, and on a random event. The relationship between money and physical assets is critical. In this first part stress is laid on the innovation behavior of Robinson Crusoe in a premonetary economy, then on his actions in a monetary economy in partial equilibrium. Part 2 considers the closed monetary economy with several differentiated agents.

*JEL Classification.* C73, D24, G32

*Keywords:* Cost innovation, Schumpeter, Circular flow, Strategic market games

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# 1 Context and circular flow

The title Schumpeter and equilibrium almost appears to be an oxymoron. In two linked, but independent, papers we construct simple models that achieve a mathematization of a fundamental insight that Schumpeter had almost a hundred years ago on the need to break the circular flow of finance required in a closed economy in equilibrium when there is the possibility of innovation. Our key concern is to be able to illustrate the relationship between real assets and money and debt. This requires investigating the nature of the cash flows and how the amount of money, credit and prices change in even in greatly simplified models of innovation. Stress is laid not merely on the control of the money and credit supply, but their relation to present and future physical assets (and implicitly) the evaluation aspects of the financing of innovation.

This first essay concentrates on the physical resource aspects of innovation. The second addresses the financial aspects.

The remainder of Section 1 is devoted to a general discussion of some of the issues that arise when innovation is introduced in economic models. Section 2 is a brief discussion of the ways in which innovation can be financed. Section 3 explains the simple technique that we use to model innovation in the improvement of a production process. In Section 4 we study innovation models for Robinson Crusoe acting as a single agent producing for his own consumption without the apparatus of finance. Section 5 then treats models in which Crusoe is viewed as a small firm or the owner-manager of a small firm in a large market economy. The final section is a brief discussion of some additional issues to be addressed in the second of these essays.

## 1.1 Equilibrium or disequilibrium

These essays are devoted to the specific task of providing some formal theory on the financing of innovation. But the emphasis is on the formalization and to solving some basic models to illustrate different control structures involved in innovation.

We intend to utilize some of these models to construct experimental games where process innovation is strategically feasible. Our goal is limited to being able to provide a simple theory and to analyze simple models of the financing of innovation that formalize the breaking of the circular flow of capital. A related experimental game can be constructed and utilized to obtain experimental data to contrast with the theory.

At even the simplest level there are many details to be covered and

distinctions to be illustrated before we are able to show the nature of the control system in a monetary economy with innovation. For this reason we first lay out the problems in innovation prior to the introduction of money and the possible separation of ownership, evaluation and control.

The success or failure of an innovation in production is here modeled as a random event with the probability of success being a function of the amount of real resources invested. There is a single random event in our model. If, as Schumpeter [10] implicitly suggested, there is a sequence of random events, the modelling would be far more complex. Even a satisfactory definition of what constitutes a reasonable solution is not immediately obvious Fortunately for the purpose of considering many of the basic problems in the financing of innovation the single random variable model is adequate.

## 1.2   The evolution of control

Although our prime immediate goal is as noted above, a general comment on the emergence of control and the increasing complexity of an innovating or evolving economy is called for.

We begin with a study of Robinson Crusoe, who as a solitary individual cannot use finance.[1] His optimization problem has constraints imposed by real resources and his production technology. A mass economy faces problems in coordination far beyond those of Crusoe. The introduction of a commodity money and markets provides a means for coordination that leaves the control of the quantity of money to the private sector of gold and/or silver producers. The introduction of a fiat money provides a means of exchange where much of the control of issue is centralized into the hands of the government and a private banking system, if it is permitted to vary the money supply.

In a mass market, Crusoe's optimization is replaced with a similar type of optimization but with more financial constraints imposed by money and weaker constraints relaxed by the presence of more commodities available in the markets. Fiat money is more relevant and realistic in the study of a modern economy; however the use of a commodity as money makes it easier to study concepts such as the meaning of reserves and the quantity of money in the system. The utilization of a commodity money without other forms of credit imposes a well defined physical resource constraint on the economy. This constraint can be removed by replacing the gold with generally accepted paper. This imposes somewhat different constraints on

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[1]Although he may find accounting useful as an aide memoire, and with a stretch of the imagination could set up a virtual market to calculate vitrual prices for himself.

the economy and provides a government with considerable economic control, but saves the deadweight loss of using an otherwise useful commodity for transactions or reserves. In going from a real commodity money to a fiat money an extra degree of freedom is introduced into the economy and aspects of that freedom can be controlled by the banking system.

By fixing default rules and monetary issue rules a government can bound the price system from below and above in an economy utilizing fiat money. In general the price levels in a system with any uncertainty cannot be uniquely specified.[2] Furthermore both size of issue and default penalties must change in a growing or shrinking economy if prices are to adjust appropriately..

The system with gold is not as flexible as the system with fiat. But the flexibility of the fiat system poses problems in the political economy of control and evaluation. With gold both the mathematics and the physics of the constraints on economic dynamics are well described. With fiat the selection of constraints on the optimization becomes an exercise in political economy. With gold government control is lessened and in particular the use of gold considerably limits the opportunities for inflation.[3]

## 1.3   On simple well-defined models and the playable game test.

Our approach is to construct simple but detailed models specifying every feasible move and all information conditions. Even so, with the microeconomic detail of economic reality, they represent a gross oversimplification. We attempt to construct the simplest mechanisms for which the phenomena of relevance appear. As they are well-defined models they should either manifest the basic properties ascribed to more realistic models or otherwise they should serve to indicate why some phenomena do not appear until a higher level of complexity is attained. The stress is on process analysis. The economy is viewed as a fully defined game of strategy.

The stress on the construction of an experimental game is made for two basic reasons. The first is to see if we can predict behavior in these simple economic environments. The second is that when dealing with economic dynamics it is easy to overlook apparently minor details that may have considerable influence on behavior. These include being specific about time lags, default conditions, constraints on borrowing, terminal conditions and

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[2] It will depend on details of initial conditions and asset structure as well as default and issue conditions.

[3] Whether inflation is good or bad for the economy is a question that is not addressed in this essay.

other micro-economic details that can be easily overlooked without the full specification of a process model or a playable game.

## 1.4  The circular flow and equilibrium

In a modern economy much of economic activity calls for the use of money and credit, both for decentralization and control. Money, credit and financial institutions provide the link between statics and equilibrium and dynamics and disequilibrium.

General equilibrium deals precisely with equilibrium states. In spite of its elegance and abstraction, as was noted by Koopmans [5], general equilibrium theory is pre-institutional. Because the economic world is highly complex and multivariate, radical simplification is called for in the mathematization of the models studied. When process models of general equilibrium are mathematically formulated even the convergence to equilibrium from positions out of equilibrium in simple dynamic models may be difficult to establish. In contrast the literature on innovation is always process oriented. There are some simulations of these processes, but the predominant approach to understanding innovation is via the essay, often bolstered with empirical studies analyzed statistically.

Although originally written nearly a hundred years ago, Schumpeter's work on *The Theory of Economic Development* [10] provided an insightful description (in essay form) of a plausible dynamic process involving the interaction of the financial and physical processes of the economy intermixed with the socio-psychological factors of optimism and pessimism. No formal mathematical model was developed.

In the last twenty to thirty years there has been a surge in the writing on innovation as is evinced in the works of Nelson and Winter [9], Dosi et al. [4], Nelson [8], Lamoreaux and Sokoloff [7], Baumol [2] and many others. Beyond these works an understanding of the analogy between economic innovation and biological mutation is growing.

## 1.5  Types of innovation

The study of innovation cannot be approached monolithically. There are at least four distinct types of
innovation, namely:

- radically new product innovation;

- engineering variation of current product;

- distribution, network, information and communication innovation;

- organization, cost reduction or other process innovation influencing efficiency.

In terms of uncertainty they are highly different. The most difficult to handle by conventional economic analysis are radical product and network innovations. Both the production procedures and the demand acceptance are unknown. There often is little, if any, precedence. The subjective probabilities for success, if any, may be cooked up by stretched analogy with other products and networks that have succeeded or failed; and only can be quantified for the purpose of the construction of imaginary or pro forma financial statements used to persuade potential investors. They are also often subject to "winner take all increasing returns", as suggested by the insightful work of Arthur [1] who develops a plausible probabilistic increasing returns to scale model where chance determines who inherits the market, and the best technology does not necessarily emerge.

More or less standard product variation fits reasonably well into the current theory of oligopolistic competition. The large firms selling, say, refrigerators have products that are close to being identical. It is the job of marketing and the production engineers to have a spice shelf full of technically known modifications or additions that can help to differentiate the product. Costs and demand can be reasonably estimated for such innovations. Innovation can also fit into a modified model of a competitive market, as has been shown by Boldrin and Levine [3]. The cost innovation discussed here can be considered in competitive markets, especially when one takes into account that the appropriation by others of new ideas, industrial secrets and expertise is by no means instantaneous.

By far the most prevalent form of innovation in most modern economies is process innovation involving organization and frequently reducing costs of production by orders of magnitude. New inventions call for expensive prototypes. Even if the market for the new product is clearly present, over the first few years, especially with mass market possibilities, there is a considerable focus on unit cost reduction. The prototype is highly expensive and the first batch for sale, though cheaper than the prototype, is usually produced at nowhere near the intended cost. The possibility to quantify a reasonable gaming experiment with cost innovation and to provide a reasonable scenario appears to be far easier than trying to construct an experimental game to illustrate radical product innovation. Here we restrict our concern to cost reduction innovation in a competitive environment.

## 1.6  Some behavioral considerations

Much of the work in mathematical economics and in game theory has been based explicitly or implicitly on an abstract *homo economicus* or von Neumann man. This individual has perfect recall and an ability to compute everything. In actuality there are many different behavioral types that are worth considering. (See [11] for a discussion.)

Here for simplicity we will restrict attention to the von Neumann player. In a projected companion paper on an experimental game with innovation we wish to compare the human (non-experts) with the "rational" noncooperative players of our theory.

## 1.7  Property rights, information and appropriation

> Drive for show, but putt for dough.
> Old golf saying

The modeling and analysis of innovation is replete with difficulties. In much of the mythology of purely competitive markets adjustments usually take place immediately. In fact, in a dynamic system profits are made by innovators having the lead ahead of the myriads of time lags in the diffusion of information and expertise. The time it takes for an industrial secret to leak, and the delays and barriers caused by legal, accounting and tax considerations, are considerable.

Virtually everything is permeable at some point. Thus patent protection, must be looked at as a time delay device and other barriers to entry as delay devices. Law cases are often brought merely as time delay instruments.

In Crusoe's world none of these details exist. At the level of abstraction in the next essay, these items, often critical to any serious deal, are abstracted away. We also will avoid the introduction of taxes and subsidies that are a part of everyday life. In finance many of the profits lie in the taking care of these details that arise out of equilibrium.

# 2  How to finance innovation.

In a modern economy there are many different ways in which innovation is financed. They depend on many empirical details concerning the nature of the money and credit, transactions costs, knowledge, liquidity, evaluation ability, attitude towards risk, laws, taxation and other factors. In a complex economy such as that of the United States many different specialists may be

involved. They include inventors, their families and friends, entrepreneurs, venture capitalists, large and small firms, bankers and the government.

Among the many ways to finance we note five forms of financing and analyze several. They are financing by:

(1) the owners with their own resources;

(2) the owners utilizing a capitalist or an investment banker;

(3) the firm utilizing retained earnings;

(4) the firm using a capital market;

(5) the firm borrowing from government.

In current United States practice much financing for cost innovation is either self-finance by the firm's management and/or owners or an arrangement between a firm and its financiers. Government may encourage innovation and may subsidize the firms rather than be a direct investor.

In the models of Section 4 below, Crusoe is not bothered with these institutional details. For him innovation involves physical goods and his ideas and ability, not finance, or complex ownership and expertise conditions.

## 3   Models with cost innovation

Assume that the probability of the success of an improvement in the efficiency of production (which in a monetary economy can be interpreted as a cost reduction) and its size can be estimated reasonably well. To be specific, we suppose that from the initial production function $f$ for Crusoe, a new improved production function of the form $(1 + \theta)f$ is obtained with probability $\xi(k)$ after a successful innovation. Here the probability $\xi(k)$ of the improvement is a function of the resources $k$ invested in innovation. With probability $1 - \xi(k)$, the innovation fails and the production function is unchanged. For a given investment the improvement may be two-dimensional, there being a trade-off between the size of the improvement and its probability of success for a given investment. We consider the one dimensional cross section where a percentage cost improvement goal $\theta$ is given and the function $\xi(k)$ is the probability of success which increases with expenditure. We assume that $\xi(0) = 0$ so that an investment of zero corresponds to no attempt at innovation.

In our models we assume that at the start of the game there is the opportunity for innovation. In essence the first move is a strategic decision to take or reject a gamble to try to improve efficiency. The innovation is modeled as a random event whose value depends on the size of investment in an attempt to innovate.

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## 3.1  Control of innovation

Before we can stress the role of finance in the coordination and control of economic activity in Part 2, the models here show Crusoe in a nonfinancial environment and then Crusoe reinterpreted as a small owner-managed firm in a competitive economy.

Dealt with below are Robinson Crusoe

1. in a nonmonetary economy without and with risk aversion;

2. in a partial equilibrium or open monetary economy as a small owner-manager with a money and borrowing or depositing.

   In Part 2 we will consider an individual

3. in a closed economy with representative agents with a fiat money and borrowing or depositing;

4. in a closed economy with a many small independent owner-managers in control of their firms.

## 3.2  Understanding money, prices and cash flows

Before dealing with the models noted above, some motivation is offered as to why they are worth distinguishing and contrasting. Economics is primarily about production and the distribution of resources. Finance enters in as part of the enabling mechanism, but while the trading of paper for paper is important in the distribution of risk, at some point the connection of the paper with the physical economy must be made. History tells us that the economies of the world passed through a nonmonetary stage and then through a stage with commodity money and from there to fiat money and an array of other financial instruments some of which are close money substitutes. A reasonable question to consider is how much does the nature of the monetary arrangements influence prices and profits. The models presented here serve to illustrate the distinctions.

The models of Section 4 have no money, markets or prices. They pose pure operations research problems including that of the adjustment process of an isolated individual. The models of Section 5 involve mass markets and progressively more complex monetary and banking arrangements. The unintended consequence of a more sophisticated monetary structure was to provide government with a control mechanism over the society. The models here are purposely as simple as we can make them in the belief that the

problems and contrasts among the different monetary mechanisms rise at a basic level. In particular the first models call for pure economic physics; there is production and consumption by an individual but prices, markets and redistribution among individuals are not called for.

The models of Section 5 provide a first step toward a mass monetary economy with a given market price and a passive government bank that is required to set an interest rate at $1 + \rho = 1/\beta$ and to lend or accept deposits at that rate. This is essentially an open or "partial equilibrium" model without full feedback.

In Part 2 we lead off with a model that has a representative agent in a closed economy. Hence price is no longer constant. Another model considers independent agents in a closed economy with varying prices.

## 4  Robinson Crusoe in a nonmonetary economy

As a preliminary to a market economy and to financial control of such an economy, innovation by a single individual risking his resources, is considered first.

Consider a model in which a single agent, Robinson Crusoe, produces a good for his personal consumption. Suppose he begins with $q \geq 0$ units of the good, puts $i$ units into production, and consumes the remaining $x = q - i$ thereby receiving $u(q - i)$ in utility. The agent begins the next period with $f(i)$ units of the good and the game continues. (Both the utility function $u$ and the production function $f$ are assumed to be concave, nondecreasing on $[0, \infty)$ with $f(0) = 0$.) The value of the game $V(q)$ to Robinson Crusoe is the supremum over all strategies of the payoff function

$$\sum_{n=1}^{\infty} \beta^{n-1} u(x_n),$$

where $x_n$ is the amount of the good consumed in period $n$ and $\beta \in (0, 1)$ is a discount factor. For this model without the possibility of innovation, the value function $V$ satisfies the Bellman equation

$$V(q) = \sup_{0 \leq i \leq q} [u(q - i) + \beta V(f(i))].$$

Assume that there is an input $i_1$ such that $f'(i_1) = 1/\beta$. (This is certainly the case if $f'(0) = \infty$ and $\lim_{i \to \infty} f'(i) = 0$, as is often assumed.) Let $q_1 = f(i_1)$.

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**Theorem 1.** *(Karatzas et al., 2006) If the initial value of the good is $q_1$, then an optimal strategy is to input $i_1$ in every period. Consequently,*

$$V(q_1) = \frac{1}{1 - \beta} \cdot u(q_1 - i_1).$$

## 4.1 Innovation by Robinson Crusoe

Assume now that our single agent with goods $q$ is allowed to input $i$ for production and invest $j$ in innovation, where $0 \leq i \leq q$, $0 \leq j \leq q - i$. The agent consumes the remainder $q - i - j$. The innovation is successful with probability $\xi(j)$ resulting in an improved production function $g = (1 + \theta)f$, where $\theta > 0$. The innovation fails and the production function is unchanged with probability $1 - \xi(j)$. Let $V_1$ be the value function for the game with production function $f$ without innovation as in the previous section and let $V_2$ be the value function for the game with the improved production function $g$. Then the value function $V$ of the game with innovation satisfies

$$V(q) = \sup_{\substack{0 \leq i \leq q \\ 0 \leq j \leq q - i}} [u(q - i - j) + \beta\{\xi(j)V_2(f(i)) + (1 - \xi(j))V_1(f(i))\}].$$

Let $\psi(i, j)$ be the function of $i$ and $j$ occurring inside the supremum. For an interior optimum we must have the Euler equations:

$$\frac{\partial \psi}{\partial i} = \frac{\partial \psi}{\partial j} = 0.$$

To find a solution to Crosoe's innovation problem, we must calculate the values of $V_1$ and $V_2$ where the quantity of goods is the amount $f(i)$ yet to be determined. Theorem 1 only gives an expression for the value at one equilibrium point, which is different for the two production functions $f$ and $g$. This situation is a mathematical reflection of Schumpeter's insight that the circular flow must be broken.

The next two sections treat special cases where the value function can be found for all values of $q$ and the innovation problem can then be solved explicitly.

## 4.2 A risk-neutral Crusoe

If the agent is risk-neutral, then there is a simple description of the optimal strategy at every value of $q$.

**Theorem 2.** *Assume that $u(x) = x$ for all $x$. Then an optimal strategy is to input $q$ if $q \le i_1$ and to input $i_1$ if $q > i_1$. For $q \ge i_1$, the value of the game is*

$$V(q) = q - i_1 + \frac{\beta}{1 - \beta} \cdot (q_1 - i_1).$$

*Proof.* A player with goods $q > q' \ge 0$ can always consume $q - q'$ and then play from $q'$. Hence,

$$V(q) \ge q - q' + V(q').$$

Consider now $q \le i_1$, and a strategy that inputs $i < q$. The best possible return from such a strategy is

$$q - i + \beta V(f(i)).$$

But an input of $q$ gives a best return of

$$
\begin{aligned}
\beta V(f(q)) &\ge \beta \cdot [f(q) - f(i) + V(f(i))] \\
&\ge \beta \cdot [f'(q)(q - i) + V(f(i))] \\
&\ge q - i + \beta \cdot V(f(i))
\end{aligned}
$$

since $f'(q) \ge f'(i_1) = 1/\beta$. So it is optimal to input $q$ when $q \le i_1$.

Now suppose that $q > i_1$. Since $u' = 1$, the Euler equation reduces to $f'(i) = 1/\beta$ or $i = i_1$. The appropriate transversality condition is trivially satisfied since $q_n = q_1$ for all $n \ge 1$. It is easy to check that the strategy is interior and therefore optimal $\qquad\square$

Consider next the innovation problem of the previous section for our risk-neutral agent with $u(x) = x$.

Assume that $f'(i_1) = 1/\beta$ and $g'(i_2) = 1/\beta$ Then by Theorem 2, $V_1'(q) = V_2'(q) = 1$ for $q \ge \max\{i_1, i_2\}$. Thus if $f(i) \ge \max\{i_1, i_2\}$, we have

$$
\begin{aligned}
\frac{\partial \psi}{\partial i}(i, j) &= -1 + \beta\{\xi(j)f'(i) + (1 - \xi(j))f'(i)\} \\
&= -1 + \beta f'(i), \\
\frac{\partial \psi}{\partial j}(i, j) &= -1 + \beta \xi'(j)\{V_2(f(i)) - V_1(f(i))\}.
\end{aligned}
$$

Hence, in this case, the solutions to the Euler equations are

$$i^* = (f')^{-1}(1/\beta) = i_1 \text{ and } j^* = (\xi')^{-1}(1/\beta[V_2(f(i^*)) - V_1(f(i^*))]).$$

To illustrate the solution, we calculate it below for a very simple example. We will revisit essentially the same example for several other models.

### 4.2.1 A numerical example

Assume that the initial production function is $f(i) = 2\sqrt{i}$ and $\theta = .1$ so that, after a successful innovation, the production function is $g = 2.2\sqrt{i}$. Set $\beta = .95$. Solve

$$f'(i_1) = 1/\beta \qquad \text{and} \qquad g'(i_2) = 1/\beta$$

to get

$$i_1 = .9025, \qquad i_2 = 1.092$$

and

$$q_1 = f(i_1) = 1.9, \qquad q_2 = g(i_2) = 2.299.$$

For $q \geq i_2 > i_1$, it follows from Theorem 2 that

$$V_2(q) - V_1(q) = \frac{i_1 - i_2}{1 - \beta} + \frac{\beta}{1 - \beta}(q_2 - q_1) = 3.791.$$

Assume now that the probability of successful innovation from investing $j$ is $\xi(j) = j/(1+j)$. As noted above, the first Euler equation has the solution $i^* = i_1 = .9025$ so that $f(i^*) = f(i_1) = q_1 = 1.9$. Since $1.9 > i_2 > i_1$,

$$V_2(f(i^*)) - V_1(f(i^*)) = 3.791$$

and the solution to the second Euler equation is $j^* = (\xi')^{-1}(1/(.95)(3.791)) = .8977$. Thus $\xi(j^*) = .8977/1.8977 = .473$ is the probability that the innovation is successful.

We can use the formula from Theorem 2 to calculate

$$V_2(f(i^*)) = V_2(1.9) = 23.741,$$

and

$$V_1(f(i^*)) = V_1(1.9) = 19.95.$$

These values together with the values for $i^*$ and $j^*$ can be substituted in the formula for the value of the game with innovation to get $V(q) = q + 18.86$ for $q \geq i^* + j^*$. The value of the game without innovation can also be calculated as $V_1(q) = q + 18.05$, which shows the value of the possibility of innovation in this instance.

## 4.3 A risk-averse Robinson Crusoe with proportional production

Many of the interesting features of investment call for the consideration of risk-averse individuals. In general, it is not possible to achieve the sort of instant adjustment to a stationary state that can be obtained with a risk-neutral Robinson Crusoe. However, analytic solutions are available when the utility function has constant elasticity and production is directly proportional to the input.

In this section we take $u(x) = \log x$ and $f(i) = \alpha i$, where $\alpha$ is a positive constant. (The full class of constant elasticity utilities is considered in a nice article of Levhari and Srinivasan [6].) Thus the Bellman equation is

$$V(q) = \sup_{0 \leq i \leq q} [\log(q - i) + \beta V(\alpha i)].$$

The Euler equation for an interior solution $i = i(q)$ takes the form

$$\frac{1}{q - i(q)} = \frac{\beta \alpha}{\alpha i(q) - i(\alpha i(q))}.$$

The solution is $i(q) = \beta q$ and does not depend on $\alpha$. Thus the optimal plan is for Crusoe to input $\beta q$ for production whenever he holds $q$ units of the good. Under this plan Crusoe's successive positions are

$$q_1 = q, \; q_2 = (\alpha\beta)q, \; \ldots, \; q_n = (\alpha\beta)^{n-1}q, \; \ldots,$$

and the optimal return is

$$V(q) = \sum_{n=1}^{\infty} \beta^{n-1} \log(q_n - \beta q_n) = \sum_{n=1}^{\infty} \beta^{n-1} \log((\alpha\beta)^{n-1}(1 - \beta)q).$$

Using properties of the log function and geometric series, we can rewrite the return as

$$V(q) = \frac{\log q}{1 - \beta} + \frac{\log(1 - \beta)}{1 - \beta} + \frac{\beta}{(1 - \beta)^2}[\log \alpha + \log \beta].$$

### 4.3.1 Innovation by a risk-averse Robinson Crusoe

Consider now the situation of an agent who begins with the utility $u(x) = \log x$ and production function $f(i) = \alpha i$ as in the previous section, and contemplates the possibility of an innovation leading to an improved production function $g(i) = (1 + \theta)\alpha i$. As in section 1.1, let $V_1$ and $V_2$ be the original

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value function and that after a successful innovation. Then the value function $V_1(q)$ is given by the formula of the previous section and $V_2(q)$ is given by the same formula with the constant $\alpha$ multiplied by $1 + \theta$. Thus

$$V_2(q) = V_1(q) + \frac{\beta}{(1-\beta)^2} \log(1+\theta),$$

and the final term above represents the value to Crusoe of having the improved production function. The value function $V$ for the game with innovation can now be written as

$$V(q) = \sup_{\substack{0 \le i \le q \\ 0 \le j \le q-i}} [\log(q-i-j) + \beta\{\xi(j)V_2(\alpha i) + (1-\xi(j))V_1(\alpha i)\}]$$

$$= \sup_{\substack{0 \le i \le q \\ 0 \le j \le q-i}} \left[\log(q-i-j) + \beta\{V_1(\alpha i) + \xi(j)\frac{\beta}{(1-\beta)^2}\log(1+\theta)\}\right].$$

The Euler equations for an interior solution $i = i(q), j = j(q)$ can be obtained by letting $\psi(i,j)$ be the function inside the supremum and setting its two partial derivatives equal to zero. Here is the result.

$$\frac{1}{q-i-j} = \frac{\beta}{(1-\beta)}\frac{1}{i} = \frac{\beta^2}{(1-\beta)^2}\log(1+\theta)\xi'(j).$$

The first equation can be solved for $i$ to get

$$i = \beta(q-j).$$

This expression for $i$ can then be substituted back in to obtain

$$\xi'(j) = \frac{1-\beta}{\beta^2\log(1+\theta)} \cdot \frac{1}{q-j}.$$

This equation can be solved explicitly if, as in Section 4.2.1, $\xi(j) = j/(j+1)$. In this case, the equation above for $j$ becomes a quadratic. Using $\beta = .95, \theta = .1$ as in Section 4.2.1. and setting $q = 2$, the positive root of this quadratic equation is $j^* = .57$ and, for this value, the chance of a successful innovation is $.57/1.57 = .36$.

## 4.4   A comment on saving and assets

We have so far modeled Crusoe without durable assets. Prior to introducing money, this is done to illustrate the simple point that without durables

Crusoe's wealth is limited by immediate production. Hence his ability to innovate calls for his cutting back on immediate production and/or consumption. If he is able to store durables, there is no bound to his wealth. A perfect durable may be regarded as having a linear storage production function such that $x_t \rightarrow x_{t+1}$. The concept of utility or end use consumption involves flows rather than stocks. Durable assets may provide a flow of consumption or production services over time. If Crusoe's island contains a deserted town he might derive little if any direct consumption value from its presence, but it could supply assets for innovation.

In the model of the next section, Crusoe has both a durable and a nondurable asset. The consumption value of the durable asset is represented by a parameter $\gamma$, which may be extremely small. However, the asset can be used to increase the probability of success of the innovation.

In a modern economy the predominant form of real asset is a production asset such as a steel plant or bank or computer center that yields no direct consumption value. Furthermore consumer assets such as houses, automobiles and appliances yield a stream of daily services that are relatively small in comparison with their asset value in a multistage dynamic economy.

## 4.5 Crusoe innovates using a long term asset

Suppose that in addition to his holdings of $q$ units of a nondurable good, Crusoe also has $r$ units of a durable good that yield a utility of $\gamma r$ in each period, where $\gamma$ is a positive constant.

If only the nondurable good is used for production then his optimal reward $V_1(q, r)$ will satisfy

$$
\begin{aligned}
V_1(q, r) &= \sup_{0 \leq i \leq q} [u(q - i) + \gamma r + \beta V_1(f(i), r)] \\
&= V_1(q) + \frac{\gamma r}{1 - \beta}.
\end{aligned}
$$

Here $V_1(q)$ is the value of the previous sections in which Crusoe held only one good and had production function $f$.

Likewise if Crusoe has production function $g(i) = (1 + \theta)f(i)$, then his optimal reward is
$$
V_2(q, r) = V_2(q) + \frac{\gamma r}{1 - \beta},
$$

where $V_2(q)$ is the corresponding value when he holds only $q$.

Now assume that Crusoe can invest any quantity $j \in [0, r]$ of the durable good in an attempt at innovation. As in section 4.1 the investment of $j$ is

successful with the probability $\xi(j)$ resulting in the improved production function $g(i) = (1 + \theta)f(i)$. With probability $1 - \xi(j)$ the innovation fails and the production function is unchanged. The optimal reward will take the form

$$V(q, r) = \sup_{\substack{0 \leq i \leq q \\ 0 \leq j \leq r}} [u(q-i)+\gamma(r-j)+\beta\{\xi(j)V_2(f(i),r-j)+(1-\xi(j))V_1(f(i),r-j)\}].$$

Let $\psi(i,j)$ denote the expression occurring inside the supremum. It can be rewritten as

$$\psi(i,j) = u(q - i) + \frac{\gamma(r - j)}{1 - \beta} + \beta\{\xi(j)[V_2(f(i)) - V_1(f(i))] + V_1(f(i))\}.$$

### 4.5.1  The numerical example with a long-term asset

As in the example of Section 4.2.1, let $u(x) = x$, $f(i) = 2\sqrt{i}$, $\theta = .1$, $\beta = .95$ and $\xi(j) = j/(j + 1)$. Also set $\gamma = .1$. It follows from the calculation in Section 4.2.1 that, for $q$, $f(i)$ and $r$ sufficiently large, that

$$\psi(i,j) = q - i + 2(r - j) + .95\{\xi(j)(3.791) + V_1(2\sqrt{i})\}.$$

Setting

$$\frac{\partial\psi}{\partial i} = \frac{\partial\psi}{\partial j} = 0,$$

we find that the optimal values are

$$i^* = i_1 = .9025, \qquad j^* = (\xi')^{-1}(2/(.95)(3.791)) = .34$$

with the success probability $\xi(.34) = .34/1.34 = .25$.

## 5  Crusoe as a single small firm in a large economy

We model Crusoe's entry into the market in two ways. In the first model, he is a utility maximizing owner-manager of a small firm. In the second, he acts as a profit maximizing small firm.

### 5.1  Crusoe as a utility maximizing owner of a small firm

Suppose Crusoe owns a small firm operating in a large market for a non-durable consumption good. At the start of each period Crusoe holds a

nonnegative amount of cash $m$ and goods $q$. The goods are sold in the market at a fixed price $p > 0$. Crusoe also bids an amount $b$ in the market to purchase an amount $b/p$ of goods that can be either consumed or used as input for production. Some portion of the goods purchased can also be invested in innovation. In the monetary economy Crusoe can buy the desired goods rather than have them in inventory.

Crusoe can also obtain loans or make deposits in a bank that charges and pays an interest rate $\rho > 0$. For simplicity we assume deposit and loan rates are the same. Crusoe's limit for a one period loan is given by his expected discounted cash earnings which are $pq/(1+\rho)$. It is a commercial loan that finances circulating capital. Crusoe can spend in the market any of the cash he holds together with what he can borrow. Thus his bid $b$ must belong to the interval $[0, m + pq/(1+\rho)]$

After choosing $b$, Crusoe selects an amount of goods $i$ to put into production and consumes the remaining $b/p - i$. He begins the next period with cash $\tilde{m} = (1+\rho)(m-b) + pq$ and goods for sale $\tilde{q} = f(i)$.

Given a utility function $u$ for consumption, the value $V_1(m, q)$ is given by

$$V_1(m,q) = \sup_{\substack{0 \leq b \leq m + pq/(1+\rho) \\ 0 \leq i \leq b/p}} \left[ u\left(\frac{b}{p} - i\right) + V_1((1+\rho)(m-b) + pq, f(i)) \right].$$

At an interior optimum, the actions $b$ and $i$ will satisfy the Euler equations:

$$u'\left(\frac{b}{p} - i\right) = \beta(1+\rho)u'\left(\frac{\tilde{b}}{p} - \tilde{i}\right)$$

$$u'\left(\frac{b}{p} - i\right) = \beta f'(i)\frac{1}{1+\rho}u'\left(\frac{\tilde{b}}{p} - \tilde{i}\right).$$

Here $\tilde{b}$ and $\tilde{i}$ are the optimal actions at the next stage. It follows that the optimal input $i$ must satisfy $f'(i) = (1+\rho)^2$ for any utility function $u$ with a strictly positive derivative.

Consider next the possibility that Crusoe devotes some quantity $j$ of the $b/p$ units of the good to innovation that with probability $\xi(j)$ results in an improved production function $g(i) = (1+\theta)f(i)$. Let $V_2(m, q)$ be the value function corresponding to the production function $g$ Then the value function

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$V(m, q)$ for the game with the possibility of innovation is given by

$$\sup_{\substack{0 \leq b \leq m + pq/(1+\rho) \\ 0 \leq i \leq b/p \\ 0 \leq j \leq b/p - i}} [u(b/p - i - j) + \beta\{\xi(j)V_2(\tilde{m}, f(i)) + (1 - \xi(j))V_1(\tilde{m}, f(i))\}],$$

where $\tilde{m} = (1+\rho)(m-b) + pq$ is Crusoe's cash at the beginning of the next period.

### 5.1.1   A simple example for the utility maximizing owner

Assume $u(x) = x$, $f(i) = 2\sqrt{i}$, $\theta = .1$ and $\beta = .95$ as in previous examples. Furthermore set $p = 1$ and $1 + \rho = 1/\beta$.

The Euler equations are satisfied for $i = i_1 = (f')^{-1}((1+\rho)^2) = .81$ and any allowed value for $b$. One optimal strategy is to always bid the maximum so that the initial bid is $m + q/(1 + \rho)$ and the bid is $q_1/(1 + \rho)$ at every later stage, where $q_1 = f(i_1) = 1.805$. Thus, for $m + q/(1 + \rho) \geq i_1$,

$$V_1(m, q) = m + \frac{q}{1 + \rho} - i_1 + \frac{\beta}{1 - \beta} \cdot \left( \frac{q_1}{1 + \rho} - i_1 \right)$$

$$= m + .95q + 16.38.$$

Similarly, for $i_2 = (g')^{-1}((1+\rho)^2) = .9855$, $q_2 = g(i_2) = 2.184$, and $m + q/(1 + \rho) \geq i_2$,

$$V_2(m, q) = m + \frac{q}{1 + \rho} - i_2 + \frac{\beta}{1 - \beta} \cdot \left( \frac{q_2}{1 + \rho} - i_2 \right)$$

$$= m + .95q + 19.71.$$

Now let $V(m, q)$ be the value of the innovation game as defined at the end of the previous section, and let $\psi(b, i, j)$ be the function inside the supremum in the formula for $V(m, q)$. For our simple example, we have

$$\psi(b, i, j) = b - i - j + \beta \cdot \{\xi(j)V_2(\tilde{m}, f(i)) + (1 - \xi(j))V_1(\tilde{m}, f(i))\}$$

$$= b - i - j + (.95)\{\xi(j) \cdot (19.71 - 16.38) + V_1(\tilde{m}, f(i))\},$$

for $m + q/(1 + \rho)$ sufficiently large. Since $\psi$ is increasing in $b$, the optimal value is its maximum, namely $b^* = m + (.95)q$. Also,

$$\frac{\partial \psi}{\partial i} = -1 + (.95)f'(i) \cdot \frac{\partial V_1}{\partial q} = -1 + (.95)^2 f'(i).$$

So the optimal value is $i* = i_1 = (f')^{-1}(1/(.95)^2) = .81$. Finally,

$$\frac{\partial \psi}{\partial j} = -1 + (.95)(3.37)\xi'(j).$$

If $\xi(j) = j/(j+1)$, then this partial derivative is zero for $j = .79$. The chance of a successful innovation for this optimal value of $j$ is $\xi(.79) = .79/1.79 = .44$.

## 5.2   Crusoe as a profit maximizing firm

Consider now Crusoe as a small firm in a large economy. In each period the firm holds goods $q \geq 0$ which are to be sold in a market for cash at a fixed price $p > 0$. The firm also bids cash in the market to buy goods as input for production. The firm holds no cash, but can borrow from a bank at interest rate $\rho \geq 0$. The bound on the firm's loan is the amount $pq/(1+\rho)$, which is the most that the firm can pay back with interest at the end of the period. Thus the firm borrows and bids an amount $b \in [0, pq/(1+\rho)]$ to purchase goods $i = b/p$ as input for production. The profit $\pi$ of the firm in the period is its income less what it must pay back to the bank, namely

$$\pi = pq - (1+\rho)b.$$

The profit is paid in each period to the firm's owners, and the firm begins the next period with goods $\tilde{q} = f(i)$.

The object of the firm is to maximize its total discounted profit given by

$$\sum_{n=1}^{\infty} \frac{1}{(1+\rho)^{n-1}} \cdot \pi_n,$$

where $\pi_n$ is the profit in the nth period. The firm has the Bellman equation:

$$V(q) = \sup_{0 \leq b \leq pq/(1+\rho)} [pq - (1+\rho)b + \frac{1}{(1+\rho)} V(f(b/p))].$$

Assume that there is an input $i_3$ such that $f'(i_3) = (1+\rho)^2$, and let $q_3 = f(i_3)$. The following theorem is analogous to Theorem 2 as is its proof.

**Theorem 3.** *An optimal strategy for the firm is to make the maximum bid $pq/(1+\rho)$ if $q/(1+\rho) < i_3$ and to bid $b^* = pi_3$ if $i_3 \leq q/(1+\rho)$. In the latter case, the value of the game is*

$$V(q) = pq - b^*(1+\rho) + \frac{1}{\rho}(pq_3 - b^*(1+\rho)).$$

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Notice that when $1 + \rho = 1/\beta$, the factor $1/\rho$ in the formula of Theorem 3 corresponds to the factor $\beta/(1 - \beta)$ in Theorem 2. However,

$$i_3 = (f')^{-1}((1 + \rho)^2) = (f')^{-1}(1/\beta^2),$$

whereas in Theorem 2, $i_1 = (f')^{-1}(1/\beta)$. The reason for this difference in inputs is that Robinson Crusoe is able to consume the production from his input in the next period, but the firm must wait an additional period to realize the profit from its input.

### 5.2.1 Innovation by Crusoe's firm

Suppose now that, in the first period, the firm bids $b + c \in [0, pq/(1 + \rho)]$ to purchase goods $(b + c)/p$, inputs $b/p$ for production and invests $c/p$ in innovation. If the innovation succeeds, the firm will then have the improved production function $g = (1 + \theta)f$ to use in future periods. Let $V_1$ be the firm's value function for the original production function $f$ and let $V_2$ be the value function corresponding to $g$. If $\xi(c/p)$ is the probability of a successful innovation, then the value function $V(q)$ of the game with innovation satisfies the Bellman equation:

$$V(q) = \sup_{\substack{0 \le b + c \le pq/(1+\rho) \\ b \ge 0, c \ge 0}} \psi(b, c),$$

where

$$\psi(b, c) = pq - (b+c)(1+\rho) + \frac{1}{1 + \rho}\{\xi(c/p)V_2(f(b/p)) + (1 - \xi(c/p))V_1(f(b/p))\}$$

At an interior maximum,
$$\frac{\partial \psi}{\partial b} = \frac{\partial \psi}{\partial c} = 0.$$

Suppose that $f'(i_3) = (1 + \rho)^2 = g'(i_4)$. If $f(b/p)/(1 + \rho) \ge \max\{i_3, i_4\}$, then, by Theorem 3,

$$\frac{\partial \psi}{\partial b} = -(1 + \rho) + \frac{1}{1 + \rho}f'(b/p)$$

and

$$\frac{\partial \psi}{\partial c} = -(1 + \rho) + \frac{1}{1 + \rho} \cdot \frac{1}{p} \cdot \xi'(c/p)[V_2(f(b/p)) - V_1(f(b/p))].$$

It follows that, for an interior maximum, we must have

$$b^*/p = i^* = i_3 = (f')^{-1}((1+\rho)^2)$$

and

$$c^*/p = j^* = (\xi')^{-1}\left(\frac{p(1+\rho)^2}{V_2(f(i^*)) - V_1(f(i^*))}\right).$$

### 5.2.2   A simple example for the firm

Assume as in Section 5.1.1 that $f(i) = 2\sqrt{i}, \theta = .1, p = 1, 1 + \rho = 1/(.95)$ and $\xi(j) = j/(j+1)$. Then

$$b^* = i^* = i_3 = (f')^{-1}(1/(.95)^2) = .81$$

and

$$q_3 = f(i_3) = 1.8.$$

Also, set

$$q_4 = (g')^{-1}(1/(.95)^2) = .99, \qquad q_4 = g(i_4) = 2.19.$$

Now use Theorem 3 to calculate

$$V_1(f(i^*)) = V_1(1.8) = 1.8 - (.81)/(.95) + 19(1.8 - (.81)/(.95)) = 18.95,$$

$$V_2(f(i^*)) = V_2(1.8) = 1.8 - (.99)/(.95) + 19(2.19 - (.99)/(.95)) = 22.57.$$

Hence,

$$j^* = (\xi')^{-1}\left(\frac{1/(95)^2}{22.57 - 18.95}\right) = .81,$$

and $\xi(j^*) = .81/(1.81) = .45$. Thus the firm's chance of a successful innovation is about the same as that of the risk-neutral utility maximizer in Section 5.1.1. This is not surprising since the firm's utility is also linear.

## 5.3   The firm's innovation financed by long-term loans

As in the previous section, Crusoe here plays the role of a profit maximizing small firm. As before the firm holds goods $q$ and can obtain short-term loans from a central bank at interest rate $\rho$ to finance production in each period. However, the firm's investment in innovation is now financed by a long-term loan from an investment bank that sets an upper bound $D \geq 0$ on such loans. If the firm borrows an amount $d \in [0, D]$ from the investment bank,

then it is required to pay back $\rho d$ in every period. Such a loan is sometimes called a perpetuity.

In any period after the first, Crusoe will begin with an amount of goods $q \geq 0$ and long-term debt $d \geq 0$. Since he is obligated to pay the investment bank $\rho d$ is each period, the central bank now sets the limit on his short-term loans at $pq/(1+\rho) - \rho d$ (or zero if this bound is negative). Crusoe's value function now becomes

$$V_1(q, d) = \sup_{0 \leq b \leq pq/(1+\rho) - \rho d} [pq - (1+\rho)b - \rho d + \frac{1}{1+\rho} V_1(f(b/p), d)],$$

when the production function is $f$.

Let $i_3$ be an input such that $f'(i_3) = (1+\rho)^2$ as in Section 5.2. If $pi_3 \leq pq/(1+\rho) - \rho d$, then the optimal action for the firm is $b = pi_3$ as in Theorem 3. Indeed, the only difference from the problem faced by the firm in Section 5.2 is the payment of $\rho d$ in every period for a total discounted cost of

$$\rho d + \frac{\rho d}{1+\rho} + \frac{\rho d}{(1+\rho)^2} + \cdots = d(1+\rho).$$

Hence,

$$V_1(q, d) = V_1(q) - d(1+\rho),$$

where $V_1(q)$ is the value of the game in 5.2.

Next let $V_2(q, d)$ be the value when the firm has production function $g = (1+\theta)f$ and starts with goods $q$ and long-term debt $d$. If $i_4$ is an input such that $g(i_4) = (1+\rho)^2$ and $pi_4 \leq pq/(1+\rho) - \rho d$, then

$$V_2(q, d) = V_2(q) - d(1+\rho),$$

where again $V_2(q)$ is the value for the game of Section 5.2 with production function $g$.

Suppose now that Crusoe begins with goods $q > 0$ and considers the possibility of financing innovation by a long-term loan. His value function $V(q)$ is then the supremum over $b$ and $d$ such that $0 \leq d \leq D$ and $0 \leq b \leq pq/(1+\rho) - \rho d$ of the expression

$$pq - (1+\rho)b - \rho d + \frac{1}{1+\rho}\{\xi(d/p)V_2(f(b/p), d) + (1 - \xi(d/p))V_1(f(b/p), d)\}$$

or equivalently

$$pq - (1+\rho)(b+d) + \frac{1}{1+\rho}\{\xi(d/p)V_2(f(b/p)) + (1 - \xi(d/p))V_1(f(b/p))\}.$$

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This is the same algebraic expression, with $d$ in place of $c$, that we had for the value $V(q)$ in Section 5.2.1. So an interior solution to the short-term problem in Section 5.2.1 will also be a solution to the long-term financing problem. However, since Crusoe's daily payments of $\rho d$ for the long-term loan are smaller than the one payment of $(1 + \rho)d$ for the short-term loan, it may happen that the long-term loan is feasible when the short-term loan is not.

# 6 Ownership and control in a monetary economy

## 6.1 Three levels of modeling

There are several levels for the modeling of competitors. In this first essay we deal only with the first one

**The nonatomic agent**

The first has a minute or non-atomic individual agent manufacturer-consumer approach is strategically just a step up from Robinson Crusoe. This has an insignificantly sized owner run firm embedded in large markets. In essence there is no market feedback, it can be considered as a partial equilibrium or open model and one need not worry about conservation.

The next two are covered at the start of Part 2.

**The representative agent**

The second has a representative agent who is a price-taker like the nonatomic agents glued together, and the full macroeconomic feedback from the closed economy is considered.

**The full measure of independent agents**

The third model deals with a closed economy with full feed back from a multitude of independent agents.

The representative agent and full measure of independent agents appear to be the same when there is no exogenous uncertainty present, but the basic differences become clear as soon as uncertainty is introduced

## 6.2 Innovation over many periods

In all of the models analyzed above the basic theme has been that of a single individual, first in an isolated non-monetary world and then in a large monetary world . Our concern has been with his decision to innovate. We have however limited the analysis to a single decision. We close with one more model where the individual may have the opportunity to try several times until either she succeeds or her credit runs out

### 6.2.1  Repeated attempts at innovation by a firm

The model with repeated attempts at innovation until success is a direct extension of the model in 5.2.1. The only difference is that, after a failed attempt at innovation the firm is free to try again. The Bellman equation for the value function $V(q)$ will be the same as in 5.2.1 except that in the expression for the function $\psi(b, c)$, $V_1(f(b/p))$ must be replaced by $V(f(b/p))$. The reason is that after a failed attempt, the firm faces the same problem again but with the quantity of goods $q$ replaced by $f(b/p)$. For large enough values of $q$ the optimal initial bid $b^* = pi^*$ is the same as in 5.2.1, and the expression for the optimal $c^*$ is also the same except that $V_1(f(i^*))$ must be replaced by $V(f(i^*))$. However, we do not have an analytic expression for $V$ comparable to that of Theorem 3. So the calculation of $c^*$ seems to be more difficult. If $f(i^*)$ is sufficiently large so that the bids $b^*$ and $c^*$ are feasible (i.e. $b^* + c^* \leq pf(i^*)/(1 + \rho)$), then the optimal policy will be to continue to make these bids until the innovation attempt is successful. If he is rich enough, success will occur eventually with probability one if $\xi(c^*/p) > 0$.

A somewhat more complicated but economically reasonable model would include the possibility that a successful innovator would continue to try for a subsequent innovation. This opens up problems with increasing returns to scale [1] that we do not deal with in this essay although the model would be highly related to the one above.

### 6.3  Innovation and the emerging financial structure

From a view point of the economics the models have been quite simple; but in our layering on the complexities starting with the first non-monetary individualistic models where ownership and control are clearly unified we are able to trace the needs for the emergence of various financial instruments and their role and to contrast them with their counterparts, if any in an autarchic nonmonetary world. The specific observations are noted below:

1. The availability of durable assets are critical for Crusoe, but for the small individual in a large economy with many assets for sale; money is in general the surrogate for all marketable assets.

2. The problem of the separation between ownership and control does not appear in Crusoe's world. It comes into benign a multiperson economy that distinguishes real persons from corporate persons

3. Short term loans in our models are, in essence commercial loans for circulating capital, or "bills" with essentially no risk attached to them

that are repaid immediately after "goods-in-process" have been produced and sold. The constraints on how large they should be are made under the assumption that the bank can forecast accurately in the very short term

4. Long term loans are qualitatively different from short term loans and even assuming no exogenous uncertainty, are called for even when the dynamics promises a stream of profits easily sufficient to repay the loan with interest, but with cash flows that are insufficient to repay the loan in a single period.

5. The evaluation of the credit worthiness of an innovator who wishes to borrow beyond the liquidation value of his assets is virtually an art form for the venture capitalist This is why the parameter "D́" in our models has no formula attached to it. In our multistage model above in 6.2. an innovator who fails can only try again if his credit line now shrunk to D-d is large enough. In fact after the first failure there would be a readjustment of D based on the old banking adage of Character, Competence and Collateral.

6. As soon as there are two different types of loans, seniority questions must be specified. They are implicitly present in our models in the need for default conditions that we have not specified. We would get away without having to specify them when examining equilibrium conditions that are default free whereas the dynamics of these models could easily involve default. At the expense of several more constraints they can be made explicit in our models.

7. In general in the transformation from the self-sufficient Crusoe, the financial system of a monetary economy is the control and perception mechanism of a multi-person economy. The roles of finance both in perception and control must be accounted for. In the mathematics the constraints reflect the control; but for the most part the evaluation of the constraints lie out of the model depending on how risk assessment and due diligence is performed.

8. The last point concerns the financiers themselves. In this essay we have concentrated on Crusoe and the firm. The resource bounds an Crusoe's economy were only physical involving real assets. The resource bounds on the monetary economy involves the availability of real assets and money hence the creation and destruction of outside money, which is

a virtual real asset, by the government or banks plays a role that has not been covered in this essay.

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