# Intangible Capital, Relative Asset Shortages and Bubbles*

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**Abstract**

We analyze an OLG economy with financial frictions and accumulation of both physical and intangible capital. The key difference between these two types of capital is that intangible capital cannot be used as collateral for borrowing. As intangibles become more important relative to physical capital in production, interest rates decline, creating the conditions for the emergence of rational bubbles in equilibrium. The model predicts that the dynamics of capital accumulation in developed economies together with the low pledgeability of intangible capital induces an abundance of low-yield assets and a scarcity of high-yield ones. In our view, this is an important mechanism behind the long-lasting sequence of asset price bubbles observed in the last decades. Our paper stresses this relative imbalance and links it to purely technological factors. We also analyze the question of dynamic efficiency, demonstrating that, in the presence of financial frictions, neither the interest rate test nor the test proposed by Abel et al. (1989) are appropriate. Finally we show that, in general, rational bubbles are not Pareto improving in our framework.

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# 1 Introduction

There is enormous fascination among economists by the topic of asset price bubbles, both at the theoretical and practical level. This is attested by the large volume of studies on the subject, recently surveyed in Brunnermeier (2009). Yet, our knowledge about the causes and consequences of these important phenomena is still limited. The recent boom and bust of the housing sector in the United States and its widespread effects worldwide warn us about the importance of understanding why bubbles emerge, how they gain momentum and finally why they burst. An important open question in the literature is whether there are structural conditions in the global economy stimulating the creation of bubbles and, if that is the case, what governments should do.

The central contribution of our paper is to highlight that the dynamics of capital accumulation in *developed* countries might have induced a shortage of high-yield financial assets and a surplus of low-yield securities in capital markets, creating the conditions for the existence of asset price bubbles. Our theory builds on the fact that, in the recent decades, the importance of intangible capital – research and development (R&D), information technology, advertising, firm-brand, etc. – has increased relative to physical capital as an input in the production processes of advanced economies. By its very nature, intangible capital is not as good a source of collateral as tangibles. Other things equal, a higher dependence on intangibles distorts the economy's ability to issue financial assets in order to borrow funds and invest, affecting the mix of stores of value available to investors. The resulting scarcity of high-yield securities is the key mechanism to sustain bubbly equilibria.

More specifically, we analyze an OLG economy in which entrepreneurs raise funds to invest both in physical and intangible capital. Due to a simple moral hazard problem, borrowing requires collateral, which can only be provided by physical capital. As the importance of intangibles in production grows relative to tangibles, entrepreneurs face a harder time supplying assets in order to raise funds to invest in intangible capital. Since, in equilibrium, savings are to be invested, they end up financing an over-accumulation of physical structures. In turn, the high stock of physical capital drives down its marginal productivity, reducing the interest rate faced by lenders. Under those circumstances, rational bubbles can be sustained in equilibrium[1].

From a methodological perspective, the present paper is closely related to a recent strand

[1]Our focus on rational bubbles is not meant to fully capture the pattern of speculation in financial assets observed in reality. We restrict the analysis to fully rational environments because they impose more discipline on the researcher: intuitively, the conditions for rational bubbles are more stringent than those for behavioral ones. This, in a certain sense, reinforces the main findings in the paper. If technological evolution and the consequent relative asset shortage are capable of inducing bubbles even in fully rational environments, it is not hard to imagine that they can do even more once investors are allowed to make systematic mistakes.

of literature in economics suggesting that speculative bubbles and other important phenomena[2] result from a chronic problem of asset shortage in the world economy. According to Caballero (2006) *"Asset supply is having a hard time keeping up with the global demand for stores of value and collateral by households, corporations, governments, insurance companies, and financial intermediaries more broadly"*. A simple consequence of this mismatch between supply and demand is price appreciation, inducing speculation and the frequent occurrence of bubbles. If this is actually the case, one wonders why the world economy would be unable to increase the supply of assets at the same pace as demand for them rises.

Caballero et al. (2006) and Caballero et al. (2008a, 2008b) provide an answer. The authors argue that heterogeneity in the degree of financial development across countries coupled with the fast growth and high savings rates in developing economies are at the root of the matter. These fast growing and high savings areas have accumulated substantial wealth in the recent decades, increasing their demand for stores of value. Because financial markets and legal institutions in general are less developed in these regions, the issuance of new assets to accommodate the increased demand has not grown fast enough. In stylized theoretical contexts, this phenomenon results in the existence of equilibria featuring rational bubbles[3].

We shift the focus of the analysis in two important dimensions. First, we emphasize that the crucial aspect of the problem is not an *absolute* shortage of stores of value, but instead a *relative* scarcity of high-yield securities and an abundance of low-yield ones. Second, we claim that at least in part the source of this imbalance is found in the developed nations whose production processes rely extensively on intangible capital. This, in a certain sense, addresses an important criticism to the asset shortage literature, which falls short in explaining why the highly developed financial markets of advanced economies would be unable to properly accommodate the inflow of savings from emerging countries. In our model, we show that advanced economies can in fact absorb the excess savings from abroad. The central issue is that they can only do so by over-investing in relatively unproductive assets, since frictions prevent an efficient allocation of funds.

Another difference of our model relative to most previous papers is that we consider a production technology that displays declining marginal productivity of factors. This is not a minor technical extension but instead is an important part of our analysis. The existing models of asset shortage have relied extensively on linear technologies with marginal productivities that are forced to be higher than the equilibrium interest rate lenders face. Because of asymmetric information, any additional funds can only partly be used for productive investment, creating a gap between the supply and demand for stores of value, which provides the

[2]Caballero et al. (2008a) study the topic of global imbalances.

[3]Mendoza and Rios-Rull (2008) present a model with similar predictions.

conditions for the emergence of bubbles. In our model, on the contrary, lenders can always allocate their funds to real investments. However, after a certain point, moral hazard constraints become relevant and savings can only be channeled to the accumulation of physical capital, driving down its marginal productivity and the rate of interest creditors obtain.

The downward pressure on interest rates is the key mechanism linking technological advances to bubbles in our paper. As pointed out by Tirole (1985) and Santos and Woodford (1997), the low interest rate is a necessary condition for equilibria featuring rational bubbles when information is symmetric. In the OLG model of Tirole, for example, rational bubbles require the equilibrium interest rate to be below the growth rate of the economy in steady-state, which was traditionally interpreted as a sign of dynamic inefficiency resulting from the over-accumulation of capital.

This close connection between rational bubbles and inefficiency has been a source of criticism to models of rational bubbles. For instance, the classical paper of Abel, Mankiw, Summers and Zeckhauser (1989) – henceforth AMSZ – provides evidence that actual economies are not inefficient. More specifically, AMSZ compared the inflow versus the outflow of funds from the corporate sectors of OECD economies and found that they consistently generate more resources than what they absorb, a clear sign of efficiency. Moreover, in the context of their paper, AMSZ prove that their empirical test is equivalent to the interest rate criterion in Tirole's economy. However, in the presence of financial frictions, the link between interest rates and dynamic efficiency is severed, as shown by Woodford (1990), Farhi and Tirole (2008), and Martin and Ventura (2011). These papers demonstrate that economies which are efficient according to the AMSZ criterion can sustain rational bubbles in equilibrium.

Our paper builds extensively on this result, combining it with the role of intangible capital. In particular, we prove that, in our framework, the parameter space displays a separation into two mutually exclusive and complementary regions according to the importance of intangibles relative to physical capital. When intangible capital has little importance, rational bubbles cannot exist unless the bubbleless economy fails the AMSZ empirical benchmark. As intangibles become more relevant, on the other hand, rational bubbles can always be sustained in equilibrium. Hence, technological evolution moves the economy into a bubbly region and yet allows it to be productive, in accordance with the existing evidence. The reason why our model passes the AMSZ test despite the low interest rate is that the high stock of physical capital makes intangibles highly productive.

We also contribute to the literature on dynamic efficiency, complementing the previous papers and formally proving that, in the presence of financial frictions, not only the interest rate but also the AMSZ test of dynamic efficiency is not appropriate. Despite our stylized framework, we argue this result is much more general. Financial frictions create hetero-

geneity in the investment opportunities that different individuals face over their lifetime, and introduce an additional margin to improve allocations. Building on this intuition, we demonstrate that the AMSZ test is not equivalent to dynamic efficiency, even when potential reallocations are required to obey the existing financial constraints. In spite of that, we argue that rational asset price bubbles are not Pareto improving because they cannot replicate the type of reallocation that we propose.

Our paper is part of a broader literature on the interaction between bubbles and economic growth. Grossman and Yanagawa (1993), King and Ferguson (1993), and Olivier (2000), for example, analyze models of endogenous growth due to externalities in the accumulation of capital. In these papers, production externalities create a wedge between the private and social rates of return on investments, allowing for bubbly equilibria. Our work assumes no externalities but instead focuses on the role of financial frictions as the source of wedges that permit the co-existence of rational bubbles and efficiency in the AMSZ sense.

Similarly to the present paper, other authors have studied the role of frictions in capital markets and their interaction with bubbles as well. Woodford (1990), Farhi and Tirole (2008) and Martin and Ventura (2011) demonstrate how bubbles can arise to relax borrowing constraints. Contrary to ours, these papers feature one type of capital with limited pledgeability. Kiyotaki and Moore (1997) and Lorenzoni (2008) study the macroeconomic implications of financial frictions, through the fluctuation in the value of collateral and fire-sale externalities. These papers focus on the additional macroeconomic volatility stemming from incomplete risk sharing, whereas we concentrate on the effect of frictions on capital accumulation and bubbles.

Several studies have analyzed rational bubbles in the context of asymmetric information, where the existence of the bubble is not common knowledge (Allen, Morris and Postlewaite (1993), Conlon (2004)) or where speculators are more informed than their investors (Allen and Gorton (1993), Allen and Gale (2000), Barlevy (2008)). Harrison and Kreps (1978) and Sheinkman and Xiong (2003), on the other hand, focus on the interaction between heterogeneous beliefs and short sale constraints, which increase the value of assets beyond fundamentals thanks to the option to sell them in the future to agents with different beliefs. Our model is distinct from those since both the information structure and beliefs are symmetric.

The rest of the paper is structured as follows. Section 2 discusses the evidence on the increased importance of intangible capital and the role of collateral and financing constraints. Section 3 presents the model, while Section 4 characterizes its multiple dynamic equilibria. Section 5 builds on the technical results derived before to expose our connection between technological progress and bubbles. Section 6 discusses the subject of dynamic efficiency

and presents some positive and normative considerations regarding bubbles, while Section 7 concludes.

# 2 Evidence on Intangible Capital and Financing Constraints

There are two central assumptions in our model. First, in a sense to be made clear below, we postulate that intangible capital has become more important relative to physical capital in the production processes of developed countries. Second, we assume that physical capital is a better source of collateral compared to intangibles, and therefore determines the entrepreneurs' ability to borrow funds and invest. In what follows, we discuss several papers that present evidence supporting these two hypotheses.[4]

## 2.1 The Increased Importance of Intangible Capital

A building block in the model we present later is the technology entrepreneurs use to produce the single final good

$$y = Ak^{\alpha}z^{\beta}l^{1-\alpha-\beta} \tag{1}$$

where A is a measure of productivity, $k$ represents physical capital, $z$ represents intangible capital and $l$ stands for labor. Several papers have employed similar versions of this specification to analyze the effects of intangible capital.

McGrattan and Prescott (2007) adopt an identical technology to evaluate the ability of the otherwise conventional neoclassical growth model to account for the business cycle in the U.S. economy during the 1990s. The authors show that including intangibles significantly improves the performance of the calibrated model. Hall (2000) introduces the concept of E-capital as an input for final production, which for many purposes is equivalent to our notion of intangibles. Relying on a non-parametric approach to calculate the stock of E-capital, Hall claims it might be an important common element behind events in the U.S. stock and labor markets in the 1990s.

In competitive equilibria the exponents of the Cobb-Douglas production function represent the share of total income that accrues to each factor. This is a measure of the importance of each factor for production, and it is the one we use in the paper. Hence, the notion that the

[4] As it will become clear in the next Section, we make several additional assumptions in order to obtain closed form expressions and to illustrate the main theoretical results in the paper. We emphasize such assumptions are made for the sake of simplicity and are by no means crucial to obtain the intuition behind our findings. What is central is the increased importance of intangibles relative to tangibles and the interaction between the low collateralizability of intangibles and the borrowing constraints.

importance of intangible capital has increased over time relative to physical capital translates, in our paper, in an increase of $\beta$ relative to $\alpha$. Note that this assumption is not equivalent to saying that the stock of $z$ has increased more than the stock of $k$, because such a movement could result from differences in initial conditions and general equilibrium effects, even without any change in the structure of the production function. The statement that $\beta$ has increased relative to $\alpha$ is essentially an assumption about a change in the nature of the production process in developed countries. Interestingly, we find strong evidence suggesting this is actually the case.

Corrado et al. (2006) carefully measure the share of output that is earned by the owners of intangibles in the United States. Their definition of investment in intangible capital includes expenses on firm brand, firm-specific resources, scientific and non-scientific R&D and computerized information. Using data on national accounts and estimates presented in Corrado et al. (2005), the authors show that investment on intangibles has increased substantially in the recent decades, much more so than investment in physical capital.

More importantly, their calculations indicate that the fraction of non-farm business output in the United States accruing to intangible capital has increased from 9.4% on average in the 1973-1995 period to approximately 14% in the 1995-2003 period. They also estimate that, if one focuses exclusively on the years 2000-2003, the share of income earned by the owners of intangible capital reaches 15%, while the owners of physical capital obtain 25% and the remaining 60% is absorbed by labor. Since there is no evidence that the share of income earned by physical capital has increased, when translated to our model, these results confirm the notion that the ratio $\frac{\beta}{\alpha}$ has increased in the United States in the last decades . At the same time, the data also point to a general shift of production from labor to capital (an increase in $\alpha + \beta$). Table 1 summarizes this information.

Belhocine (2008) reproduces Corrado et al. (2006) and finds qualitatively similar evidence for Canada, where estimated investment in intangibles has become as large as the investment in physical capital. Fukao et al. (2007) and Marrano and Haskel (2006) present similar findings for Japan and the United Kingdom respectively.

## 2.2 Intangible Capital, Collateral and Financing Constraints

Collateral is an important instrument for borrowers to raise funds in imperfect capital markets. Moral hazard and adverse selection reduce lenders willingness to allocate capital to borrowers whose quality or actions are not completely observable. Collateral plays an important role in overcoming this failure since it allows lenders to seize assets in the event of default. Moreover, it increases the incentives of borrowers to be diligent and perform well.

As emphasized by Shleifer and Vishny (1992), what turns an asset into good collateral

is its redeployability in the contingency of liquidation. In other words, good collateral is an asset which can be costlessly seized and sold in the market for a price similar to its value in the first best use. There is ample empirical evidence associating redeployability of collateral and access to financing. Benmelech et al. (2005) show that increased redeployability of commercial real state increases the duration of mortgages, reduces the cost of borrowing and increases the loan-to-value ratio. Benmelech (2009) shows that leverage was associated with redeployability of assets for U.S. railroads in the 19th century, while Benmelech and Bergman (2008) estimate the impact of collateral redeployability on airline companies' bargaining power during episodes of financial duress.

Intangible capital is, in general, a worse source of collateral compared to physical capital. First and foremost, intangible capital is, by its very nature, hard if not impossible to seize. Furthermore, it is fundamentally intrinsic to the firm or business that produced it. Even if in some cases lenders can obtain control over the intangibles of defaulting borrowers, the redeployability of these assets tend to be small since their value in alternative uses is reduced by the lack of specific knowledge or competences. Obviously, different sorts of physical capital suffer, to some extent, of the same problems.[5] However, plants, equipment and machines are generally accepted by financial intermediaries as collateral for lending.

The second fundamental assumption in our paper builds on this reasoning to argue that, other things equal, the more firms rely on intangible capital the more the financing constraints bind. Several studies have documented this fact. Aghion et al. (2007) show that R&D investment correlates with financing constraints in a large sample of French firms. Carpenter and Petersen (2002) find that small high-tech U.S. firms suffer more from financing constraints. Canepa and Stoneman (2008) present similar evidence for firms in the United Kingdom.

# 3 Model

## 3.1 The OLG Structure

The economy is represented by an overlapping-generation (OLG) model with one final good ($y$) and three inputs: labor ($l$), physical capital ($k$), and intangible capital ($z$). At every period $t$, a new unit-measure generation is born with an endowment of 1 unit of time and no endowment of goods. Each generation is composed by a fraction $\pi$ of entrepreneurs and a fraction $1-\pi$ of households. Individuals live for three periods: young, middle age, and old. They only consume when old. All individuals will try to use at best production and

[5] As exemplified by Shleifer and Vishny(1992), a satellite, which is formally considered physical capital, has low redeployability and does not constitute good collateral.

investment opportunities during their lifetime to convert the initial endowment of time into end-of-life consumption. For simplicity, we assume there is no uncertainty in the economy.

Figure 1 illustrates the model by describing the evolution of the generation born at time $t$. When young, both households and entrepreneurs supply labor inelastically to the existing firms run by the middle-aged entrepreneurs of generation $t-1$ in a competitive labor market. The labor services are paid only at the beginning of period $t+1$, at a wage rate $w(t+1)$. The labor supply of the young will represent the aggregate labor supply of the economy in each period.

At the beginning of period $t+1$, households have no option but to lend their accumulated labor income at a gross rate $R(t+2)$ – to be paid at date $t+2$ – to the contemporaneous entrepreneurs in order to transfer funds to the last period of life. Hence, at date $t+2$, a household of generation $t$ consumes her total wealth, represented by $w(t+1)R(t+2)$, and then dies.

Entrepreneurs, on the other hand, have different investment opportunities. They use their own funds and borrow additional resources from the contemporaneous households to accumulate physical and intangible capital. They combine accumulated capital with labor services from the newborn of generation $t+1$ to produce final goods. Production by generation $t$ takes place between periods $t+1$ and $t+2$ and generates output of the final good at the beginning of time $t+2$, denoted $y_t(t+2)$. Note that $y_t(t+2)$ refers to the output produced by a single entrepreneur.

Finally, at date $t+2$, the total output produced by generation $t$ is distributed among the owners of the factors. Part of it goes to the old households of generation $t$ who lent funds to the entrepreneurs. Another part remunerates the labor services provided by generation $t+1$. The remaining output is retained by the old entrepreneurs of generation $t$, who consume and die.

## 3.2 Production and Financial Frictions

Each middle-aged entrepreneur of generation $t$ has access to a Cobb-Douglas production technology that combines physical capital $k$, intangible capital $z$ and labor $l$ as follows

$$y_t(t+2) = A(t+2)\,k_t(t+1)^{\alpha}\,z_t(t+1)^{\beta}\,l_t(t+1)^{1-\alpha-\beta} \tag{2}$$

where $A$ is a productivity parameter that grows at the constant rate $n$ and $l_t(t+1)$ represents the individual demand for labor in period $t+1$. At the beginning of $t+1$, a typical middle-aged entrepreneur uses her own funds $w(t+1)$ and borrows additional resources $d_t(t+1)$ to accumulate $z_t(t+1)$ and $k_t(t+1)$. This is the first stage of production. In a second

and final stage[6], she hires labor from the newborn of generation $t+1$. After production is completed, both $k$ and $z$ fully depreciate. The marginal rate of transformation of final goods into $k$ and $z$ equals 1.

We introduce a friction in the lending process by allowing entrepreneurs to default on their debt. In particular, we make the following assumptions. First, in the initial stage of production, an entrepreneur can default on her debt, close the firm, and bring with her all intangible capital $z$ she has accumulated.[7] In this case, the lender will seize the physical capital that has been pledged for the loan. Once she defaults on the debt, the entrepreneur can open a new firm, borrow funds from another lender, and start production (of course, she might choose to default again). Second, after production is completed, the lender can seize output $y$, provided the original firm has not been closed. This ensures repayment of the loan once production is completed.[8] Third, we assume that during the first stage of production, the lender has the ability to determine how the funds will be utilized by the borrower. This captures the fact that lenders frequently monitor the use borrowers make of loaned funds, and covenants specify the allocation of the resources they provide.[9]

Given these assumptions, it is clear that when loans are made, the lender will always require them to be completely invested in and backed by tangible capital. If any part of the loan was used to invest in intangibles, the entrepreneur would have the incentive to close the firm, keeping the intangible capital accumulated, and open a new firm with another lender who would lend to her against physical capital only. When the entrepreneur receives a loan backed by tangible capital, she has no incentive to default before production, because the tangible capital would be seized by the lender. Nor has she incentive to default after production occurs, because the lender can seize the output of production. Finally, note that competition among lenders implies that the return on these loans equals the marginal productivity of tangible capital.

Under those assumptions, we have:

**Lemma 1.** *Borrowing requires collateral in the form of physical capital. In other words, before investing, each middle-aged entrepreneur of generation t faces the following borrowing constraint:*

$$d_t(t+1) \leq k_t(t+1) \tag{3}$$

[6]Note that both stages happen between periods $t+1$ and $t+2$.

[7]This assumption is invoked to capture the fact that, in reality, intangible capital is intrinsic to the entrepreneur who accumulates it and has low redeployability value for third parties.

[8]Note that capital fully depreciates so it cannot be used to ensure repayment after production.

[9]Often, loans are explicitly tied to particular uses, as in the case of commercial or residential mortgages.

Hence, the problem of a middle-aged entrepreneur of generation $t$ can be written as

$$max_{(k,z,l,d)} \left\{ y_t \left( t+2 \right) - R \left( t+2 \right) d_t \left( t+1 \right) - w \left( t+2 \right) l_t \left( t+1 \right) \right\}$$

$$s.t.$$

$$k_t \left( t+1 \right) + z_t \left( t+1 \right) \leq w \left( t+1 \right) + d_t \left( t+1 \right)$$

$$d_t \left( t+1 \right) \leq k_t \left( t+1 \right)$$

Since the first constraint always binds in equilibrium, the borrowing restriction can be rewritten as

$$z_t \left( t+1 \right) \leq w \left( t+1 \right) \tag{4}$$

Simply put, the possibility of default together with the lack of pledgeability implies that intangible capital is financed with internal funds only.

In our model, we assume that entrepreneurs cannot issue equity in order to invest, but are constrained to issue collateralized debt only.[10] The crucial aspect here is not the distinction between debt and equity financing, but rather the asymmetry between inside and outside investors. In this respect, physical capital works as a mechanism through which managers commit not to expropriate investors, a property that cannot be mimicked by intangible capital.

It is useful to compare the assumptions above with the environment of other papers. Martin and Ventura (2011), for example, prevent borrowing by the agents with better investment opportunities relative to their peers due to an "*unspecified market imperfection*". Lorenzoni (2008) introduces limited risk-sharing by assuming that entrepreneurs can default *after* they produce, losing part of the output and remaining capital. Because we assume limited pledgeability of only one type of capital, in our model the default decision must be allowed to happen when K and Z are still differentiated. This is in fact similar to Kiyotaki and Moore (1997), where the borrower can close the firm and take away his human capital at any time, leaving behind the land (the tangible part). In their model, if the borrower defaults before production is completed, output is not produced and the unutilized land is seized and sold by the lenders on the market; therefore, creditors will make sure that the total amount to be repayed is at most as high as the future value of the land. In our model, lenders can seize physical capital and transfer it to another producer if default occurs, so the outside value of the collateral is $k_t \left( t+1 \right)$, while the present value of promised repayment is

[10] Implicitly, this assumption captures the idea that equity issuance is more exposed to asymmetric information and adverse selection problems compared to debt issuance, a fact that has been widely discussed in the corporate finance literature (Mayers and Majluf (1984) and Shleifer and Vishny (1997)).

$d_t\,(t+1)$, resulting in Lemma 1.[11]

# 4 Existence and Characterization of Equilibria

This Section proves the existence and characterizes the dynamic competitive equilibria of the model. In what follows, aggregate variables are denoted by upper case letters. For instance, the aggregate output of generation $t$ is given by $Y_t(t+2) = \pi y_t(t+2)$. Moreover, because labor is supplied inelastically every period, we omit the individual labor supply – which is normalized to 1.

## 4.1 The Bubbleless Economy

A bubbleless competitive equilibrium is defined as follows:

**Definition.** *The sequence of allocations*

$$\{l_t\,(t+1)\,, k_t\,(t+1)\,, z_t\,(t+1)\,, d_t\,(t+1)\}_{t=0}^{\infty}$$

*constitutes a* ***dynamic bubbleless competitive equilibrium*** *of the economy if there exists a sequence of price vectors* $\{w\,(t+1)\,, R\,(t+1)\}_{t=0}^{\infty}$ *such that, at each t:*

- *Each agent is maximizing her utility subject to her budget constraint.*
- *The labor market clears:* $1 = L_t\,(t+1) = \pi l_t\,(t+1)$.
- *The savings market clears:* $w\,(t+1) = K_t\,(t+1) + Z_t\,(t+1) = \pi(k_t\,(t+1) + z_t\,(t+1))$.

Solving in closed-form for the equilibrium is possible because of the simplifying assumptions we have made. When young, agents supply labor inelastically. When middle-aged, households supply their savings to the entrepreneurs inelastically as well.[12] Only entrepreneurs take active decisions, and they coincide with profit maximization for the firm. Therefore, the equilibrium wage and interest rate are determined by the F.O.Cs of the firm:

$$w\,(t+2) = (1-\alpha-\beta)\frac{y_t\,(t+2)}{l_t\,(t+1)} = (1-\alpha-\beta)Y_t\,(t+2) \tag{5}$$

[11] In other words, since a unit of capital seized by the lender will be lent to another entrepreneur in a loan backed by tangible capital, the $t+2$ outside value of capital – the revenue it will generate at $t+2$ under the alternative use – is $k_t(t+1)R(t+2)$, while the promised repayment is $d_t(t+1)R(t+2)$. The comparison between the two again reduces to equation (3).

[12] The model can also be solved in closed form if households have log utility of consumption in different periods. All the results derived below are valid under this more general context, apart of a renormalization of parameters. The proof of this claim is available upon request.

$$R\left(t+2\right)=\alpha\frac{y_t\left(t+2\right)}{k_t\left(t+1\right)}=\alpha\frac{Y_t\left(t+2\right)}{K_t\left(t+1\right)} \tag{6}$$

where $K_t(t+1)=\pi k_t(t+1)$.

Two markets are open in period $t+1$. One is the market for labor, in which the young of generation $t+1$ supply labor to generation $t$'s firms. We have already imposed its equilibrium condition above. The other market is the one for savings which achieves equilibrium when:

$$K_t(t+1)+Z_t(t+1)=w(t+1)=\left(1-\alpha-\beta\right)Y_{t-1}(t+1) \tag{7}$$

The borrowing constraint (whose Lagrangian multiplier we label $\lambda_t$ referring to generation $t$)

$$z_t(t+1)\leq w(t+1) \tag{8}$$

coupled with the optimal choice of intangible capital by entrepreneurs implies:

$$IRR(t+2)\equiv R(t+2)+\lambda_t=\frac{\beta y_t(t+2)}{z_t(t+1)} \tag{9}$$

$$\lambda_t\left[z_t(t+1)-w(t+1)\right]=0 \tag{10}$$

$$\lambda_t\geq 0 \tag{11}$$

These conditions show that, whenever the financing constraint binds, the marginal productivity of intangible capital is higher than the interest rate prevailing in the market for funds, which in equilibrium equals the marginal productivity of physical capital. We call that shadow rate of return IRR, to stress that it is the return an additional dollar could generate if freely transferred to an entrepreneur. Note that

$$IRR(t+2)-R(t+2)=\lambda_t \tag{12}$$

so $\lambda_t$ captures the spread between the return on a unit of internal funds and the return on a unit of funds raised through collateralized debt.

While analyzing equilibrium allocations, we focus exclusively on the aggregate variables $(Y,K,Z)$ and on the prices $(w,R)$. There is no loss of generality in doing so. Knowing the path of these aggregates is enough to fully describe the economy, since all other variables are immediately derived from them.

In order to characterize the behavior of the economy along an equilibrium path, it is necessary to determine under what conditions the borrowing constraint binds. Fortunately, the model features an interesting separation of the parameter space into two mutually exclusive and complementary regions, one where the constraint always binds and another where it never does. The next definition sets the stage for the formal proof of this result.

**Definition.** *Let* $\Theta = \{(\alpha, \beta, \pi, n) : (\alpha, \beta, \pi) \in (0,1)^3, 0 < \alpha + \beta < 1, 0 < n\}$ *be the parameter space in the model. Define two subsets:*

$B = \left\{(\alpha, \beta, \pi, n) \subset \Theta : \frac{\beta}{\alpha} \geqslant \frac{\pi}{1-\pi}\right\}$
$NB = \left\{(\alpha, \beta, \pi, n) \subset \Theta : \frac{\beta}{\alpha} < \frac{\pi}{1-\pi}\right\}$

Note that $\Theta = B \cup NB$ and $B \cap NB = \oslash$ .

Proposition 1 builds on this definition to analyze the existence of a dynamic bubbleless equilibrium. It is straightforward to show that, if a steady-state exists, all real variables must grow at the rate

$$(1+g) = (1+n)^{\frac{1}{1-\alpha-\beta}} \tag{13}$$

**Proposition 1.** *There always exists a trivial equilibrium in which* $K, Z,$ *and* $Y$ *are equal to zero. Moreover:*

*- If* $(\alpha, \beta, \pi, n) \in B$ *the financing constraint is binding along all equilibrium paths. The economy has a unique non-trivial bubbleless globally stable steady-state (where the borrowing constraint binds).*

*- If* $(\alpha, \beta, \pi, n) \in NB$ *the financing constraint is slack along all equilibrium paths. The economy has a unique non-trivial bubbleless globally stable steady-state (where the borrowing constraint is slack).*

*Proof.* See Appendix. □

According to Proposition 1, for a given value $\pi$, as the ratio $\frac{\beta}{\alpha}$ increases, the economy moves from equilibria in which financing constraints are irrelevant to ones in which they are binding. This result is intuitive. Other things equal, the demand for intangible capital is increasing in $\beta$. The higher the ratio $\frac{\beta}{\alpha}$, the more entrepreneurs want to accumulate intangible capital instead of physical capital. After a certain point, all internal funds are invested in $Z$, and financing constraints become binding.

As will become clear in the following Sections, Proposition 1 contains the central result behind our connection between intangible capital accumulation, relative asset shortages, and bubbles. The increased importance of intangible capital due to technological change, represented by increases in the ratio $\frac{\beta}{\alpha}$[13], moves the economy towards a constrained region of the

[13]The evidence for which was presented in Section 2.

parameter space. At this point, the relative supply of stores of value is distorted because households' savings can only be allocated to physical capital despite the higher productivity of intangibles. The over-accumulation of $K$ reduces its marginal productivity and the rate of interest in the market for funds, which is key for rational bubbles to exist.

## 4.2 The Bubbly Economy

Let us now analyze the existence of bubbly equilibria.

**Definition.** *A **rational bubble** is a security in unit supply which lasts forever, pays no dividends, and has price B(t)>0 at time t. Moreover, any individual can trade this security in the market.*

The presence of a bubble changes some of the equilibrium conditions outlined above. In particular, the savings market equilibrium requires that, at each period, there are enough savings to finance physical and intangible capital accumulation as well as the acquisition of the bubbly security:

$$K_t(t+1) + Z_t(t+1) + B\,(t+1) = (1-\alpha-\beta)\,Y_{t-1}\,(t+1) \tag{14}$$

Because any individual can trade the bubbly security in the market, its rate of return - the rate at which its price evolves over time - is equalized to the return on the worst investment opportunities available to investors. This is always given by the return $R$ that households obtain on their savings. Therefore, in any dynamic equilibrium the value of the rational bubble evolves according to:

$$B\,(t+1) = R\,(t+1)\,B\,(t) \tag{15}$$

In case financial frictions are binding, entrepreneurs will never be interested in the bubble, since their funds are invested at a rate

$$IRR = R + \lambda > R$$

Hence, in any constrained equilibrium, the bubble is held in its entirety by the households of each generation.

At this stage, we establish some notation to facilitate the derivation of the equilibrium.

**Definition.** *Let*

$$\hat{Y}_t(t+2) = \frac{Y_t(t+2)}{A(t+2)^{\frac{1}{1-\alpha-\beta}}}$$

$$\hat{K}_t(t+1) = \frac{K_t(t+1)}{A(t+2)^{\frac{1}{1-\alpha-\beta}}}$$

$$\hat{Z}_t(t+1) = \frac{Z_t(t+1)}{A(t+2)^{\frac{1}{1-\alpha-\beta}}}$$

$$B^*(t+1) = \frac{B(t+1)}{Y_{t-1}(t+1)}$$

The first three definitions formalize the usual concept of a variable per efficiency unit, while the last one defines the bubble per unit of output. Note that we are comparing the price of the bubble that is exchanged at time $t+1$ with the output generated at time $t+1$ (by generation $t-1$).

The next Lemma provides some helpful technical results.

**Lemma 2.** *i) The vector* $[Y_{t-1}(t+1), B(t+1)]$ *is a state vector for the economy. The vector* $[\hat{Y}_{t-1}(t+1), B^*(t+1)]$ *is a state vector as well, given an initial value of technology* $A(0)$.

*ii) If, at time t,* $[\hat{Y}_{t-1}(t+1), B^*(t+1)]$ *belongs to a dynamic equilibrium where* $(\alpha, \beta, \pi, n) \in NB$*, the financing constraint is slack in the next period.*

*iii) If, at time t,* $[\hat{Y}_{t-1}(t+1), B^*(t+1)]$ *belongs to a dynamic equilibrium where* $(\alpha, \beta, \pi, n) \in B$*, there exists a threshold* $B^*_{max,b}$ such that:

- For $B^*(t+1) \leq B^*_{max,b}$, the financing constraint is binding in period t+1.
- For $B^*(t+1) > B^*_{max,b}$, the financing constraint is slack in period t+1.

*Proof.* See Appendix. □

If the economy is in the NB region, the financing constraints are slack along any equilibrium path. The case where the economy belongs to the B region is more involved. Intuitively, for small enough bubbles, the financing constraint is binding, since the bubble does not drain too many funds, which means that the stock of capital will be high, inducing entrepreneurs to fully invest in intangibles. On the other hand, if the bubble is large enough, households will finance a small amount of physical capital. In this case, it is in the best interest of entrepreneurs to invest in $K$ as well, which means that the borrowing constraint becomes slack.

Based on that, we can characterize in detail the bubbly dynamic equilibria. Let us start with Proposition 2, which restates Tirole's (1985) result.

**Proposition 2.** *(Tirole 1985) If the bubbleless steady-state features*

$$\pi IRR + (1-\pi) R < (1+g) \tag{16}$$

*there exist dynamic bubbly equilibria.*

Whenever the average rate of return of the economy is below its growth rate in the bubbleless steady-state, rational bubbles can exist. This well known condition has been traditionally interpreted as a sign of dynamic inefficiency, since a low rate of return results from excessive accumulation of capital. It is important to note that the above Proposition defines the necessary and sufficient conditions for the existence of bubbly equilibria in the NB region - where $R$ is equalized to $IRR$. In the B region, on the other hand, it is possible for bubbly equilibria to survive even if the bubbleless steady-state features $\pi IRR + (1-\pi) R \geqslant (1+g)$ as shown by Farhi and Tirole (2008).

It turns out that in the bubbleless steady-state, the comparison between the average rate of return and the growth rate of the economy is equivalent to the AMSZ test about the productivity of the corporate sector. AMSZ compare, at every period, the amount of funds absorbed by the corporate sector in the form of capital expenditures versus the excess of its output over labor costs. The authors present strong evidence that the corporate sectors of OECD economies are highly productive since they generate more funds than what they absorb. We require our model to satisfy this empirical benchmark.[14]

**Lemma 3.** *In any bubbleless equilibrium, the economy satisfies the AMSZ test if and only if*

$$\alpha + \beta \geqslant \frac{1}{2} \tag{17}$$

*which is equivalent to a bubbleless steady-state in which:*

$$\pi IRR + (1-\pi) R \geqslant (1+g) \tag{18}$$

*Proof.* See Appendix. □

In what follows, we characterize the bubbly equilibria in the B region subject to the AMSZ empirical benchmark, which, for our purposes, defines the relevant region of the parameter space. We acknowledge that this restriction implies a counterfactually small share of labor income relative to output. However, this results from some of the stark assumptions we have imposed in order to make the model tractable[15] and are not directly linked to the most important aspects of our analysis. We omit the presentation of the dynamic equilibria in the other regions of the parameter space, since they can be derived in similar ways.

**Proposition 3.** *In the B region with $\alpha + \beta \geqslant \frac{1}{2}$, there exist dynamic bubbly equilibria that*

[14] From Proposition 2 it is easy to conclude that this restriction makes it harder for our model to generate rational bubbles.

[15] For instance, our assumption of full depreciation of capital is partly responsible for this difficulty.

*converge to a steady-state in which*

$$B^* = (1 - \alpha - \beta)(1 - \pi) - \alpha \tag{19}$$

*and the borrowing constraint is binding at all times, as long as* $B^* > 0$. *There also exist dynamic equilibria such that the bubble-to-output ratio is always non-negative but converges to zero over time.*

*Proof.* See Appendix. □

Figure 2 represents graphically the relevant regions of the parameter space in terms of $\alpha$ and $\beta$, for given $\pi$ and $n$. The B/NB line has slope $\frac{\pi}{1-\pi}$, and divides the space $\Theta$ into the B and NB regions.

Proposition 3 nicely extends and qualifies the result of Proposition 1 - about bubbleless equilibria - to bubbly equilibrium paths, by identifying an additional division of the parameter space, related to the AMSZ line. This line separates economies in which the AMSZ benchmark is satisfied ($\alpha + \beta \geq \frac{1}{2}$) from those in which it is not.

The dynamics of the bubbly equilibrium paths can be described as follows. In the region where the economy's average rate of return is low relative to its growth rate, i.e. below the AMSZ line, all of Tirole's (1985) standard results apply, whether the constraint binds or not. There can be bubbles that disappear asymptotically and bubbles that grow forever together with the economy. We do not focus on this area in our study, since it contradicts the empirical evidence.

In the area above the AMSZ line, the separation between NB and B defines the dynamics of the bubbly equilibria, just like in the case of the bubbleless equilibria. In the blue area of the B region, the bubbleless steady-state features $R < (1+g)$. This condition guarantees that there exist equilibrium paths in which the bubble-to-output ratio is positive but disappears asymptotically, as well as equilibrium paths that converge to a steady-state with a positive bubble-to-output ratio. In all bubbly paths the financing constraints are binding. In the rest of the B region and in the whole NB region, instead, no rational bubbles can be sustained in equilibrium.

Figures 3 represents the dynamics of the economy in the B region, above the AMSZ line, for $B^* > 0$. The red line represents the bubbleless equilibrium path, that converges to the bubbleless steady-state (red dot). All the other equilibrium paths have a positive bubble. They either converge to the bubbleless steady-state (the bubble disappears asymptotically), or they reach the bubbly steady-state through a saddle path, where the bubble is constant as a fraction of output.

## 4.3 Stochastic Bubbles

The previous results can be extended to the case of stochastic bubbles. In order to prove that, we consider an environment in which, at every period, the rational bubble survives with probability $p$ or bursts with probability $1-p$ as in the classic model of Blanchard and Watson (1982). Naturally, as long as the bubble exists, it has to grow at a rate $\frac{R}{p}$, so its expected return is equal to $R$.

Let us start with the definition of a conditional steady-state.

**Definition.** A conditional steady-state is an allocation such that a stochastic bubble exists and then bursts at some point. While the bubble exists, all variables in the economy $\left\{\hat{Y}_t(t+2), \hat{K}_t(t+1), \hat{Z}_t(t+1), B^*(t+1)\right\}$ are constant. After the bubble bursts, the economy converges to a bubbleless steady-state as shown in subsection 4.1.

Naturally, unless the economy starts at the conditional steady-state, it will almost surely not reach it, since the probability of that event happening converges to zero. The conditional steady-state is, notwithstanding, a useful benchmark because it allows us to determine the trajectory of a bubbly economy while the bubble lasts. In what follows, we characterize, through a series of propositions, the dynamics of the economy when bubbles are stochastic. We will focus on the case where the stochastic bubble has not burst until the most recent period, since the dynamics of the economy after the bubble bursts have been already explored in Section 4.1.

**Lemma 4.** *Along any bubbly path with initial condition* $[Y_{t-1}(t+1), B(t+1)]$*, all the results in Lemma 2 hold.*

*Proof.* See Appendix. □

Lemma 4 highlights that the dynamics of the economy are not materially affected by the stochastic nature of bubbles. Whether the continuation is binding or not depends only on the bubble-to-output ratio and the region of the parameter space where the economy lies.

**Proposition 4.** *There are no stochastic bubbly equilibria in the NB region if AMSZ is satisfied.*

*Proof.* See Appendix. □

As before, economies that are productive in the AMSZ sense cannot sustain stochastic bubbly equilibria if they are in the NB region of the parameter space.

**Proposition 5.** *In the B region with* $\alpha + \beta \geqslant \frac{1}{2}$*, there exist dynamic bubbly equilibria that converge to a conditional steady-state in which*

$$B^* = (1 - \alpha - \beta)(1 - \pi) - \frac{\alpha}{p} \tag{20}$$

*and the borrowing constraint is binding at all times, as long as* $B^* > 0$*. There also exist dynamic equilibria such that the bubble-to-output ratio is always non-negative but converges to zero over time.*

*Proof.* See Appendix. □

These results show that economies that satisfy the AMSZ criterion can sustain stochastic bubbles. Obviously, the conditions for the existence of stochastic bubbles are more stringent relative to the deterministic case. As mentioned before, conditional on not bursting, stochastic bubbles grow at the rate $\frac{R(t+1)}{p} > R(t+1)$. Hence, for these bubbles to not outgrow the economy in finite time, equilibrium interest rates in the bubbleless equilibrium have to be even lower compared to the economy's growth rate in the long-run (the more so the higher the chance the bubble will burst). This can achieved by having a ratio $\frac{\beta}{\alpha}$ high enough and binding financing contraints.

# 5 Intangible Capital, Technological Change, and Bubbles

Having developed the technical results on the dynamics of our model economy, we now provide intuition about the link between bubbles and the importance of intangible capital.[16] The analysis also considers the case of stochastic technological change, as well as the relationship between technological progress, the global savings glut, and bubbles.

## 5.1 The Importance of Intangible Capital and Bubbles

The results in Section 4 show that economies which satisfy the AMSZ condition can sustain bubbles only if intangible capital is sufficiently important in the production process. With a

[16] Our connection between bubbles and technological changes is also different from the work of Pastor and Veronesi (2008), which is not based on financial frictions. The authors develop a model where the interaction between technological evolution and learning in financial markets induces asset prices to display bubble-like behavior. However, as the authors argue, this is misleading since, in their model, the market value of traded securities reflects fundamentals conditional on the information available to investors. In our model, on the other hand, there is no learning. The technological shift allows the bubbly security to have a positive market price despite its fundamental value being zero at every period. Moreover, the asset shortage approach highlighted here is absent in their work.

high $\beta$ relative to $\alpha$, financing constraints are active, and the equilibrium interest rate is low – which allows rational bubbles to be sustained in equilibrium.

To better illustrate the point, we calibrate the model[17] and calculate the steady-state rates of return and growth rate of the bubbleless economy for different values of $\alpha$ and $\beta$. In this exercise, the fraction of income that goes to intangible capital $\beta$ is increased by 15 percentage points, while $\alpha$ is decreased by 12 percentage points, the remaining 3 percentage points being taken from labor in order to preserve constant returns to scale. This calibration mimics the change in the share of output accruing to the different factors documented for the US in the last 30 years by Corrado et al. (2006). Figure 4 shows how the bubbleless steady-state rates $R$ and $IRR$ shift as $\beta$ gradually increases and at the same time $\alpha$ decreases – the horizontal axis representing the change in $\beta$. It also compares the growth rate of the economy $1+g$ with the equilibrium average rate of return, $(1-\pi)R + \pi IRR$.[18]

The green vertical threshold separates the NB region (left) from the B region (right). For small values of $\beta$, the economy is in the NB region. The returns on tangible and intangible capital are equalized, so that $R = IRR$. Since the AMSZ condition is satisfied, $R > 1+g$ in the steady-state, and no bubbles can exist. For larger values of $\frac{\beta}{\alpha}$, on the other hand, the economy is in the B region. The wedge between $R$ and $IRR$ increases with $\frac{\beta}{\alpha}$ and might drive $R$ below $1+g$ in steady-state if $\beta$ is sufficiently high. After this point, bubbly equilibria are possible.

The change in $\frac{\beta}{\alpha}$ we calibrate here represents a simple comparative statics exercise, but its implications are identical to an unexpected and exogenous technological shift in the economy. In particular, suppose that the economy is originally in the NB region of the parameter space, and satisfies the AMSZ condition. No bubble can exist at this time, and the economy is on a bubbleless path. Suppose all agents originally expect technology to be constant forever, but, at a certain time $T$, $(\alpha, \beta)$ suddenly shifts to $(\alpha', \beta')$ such that $(\alpha' + \beta') > 0.5$ – ensuring the new economy satisfies the AMSZ condition as well. Finally, suppose no more changes are due. The arrow in Figure 5 represents this shift in the space of $(\alpha, \beta)$. Under these assumptions, we can prove the following:

**Proposition 6.** For any value of $(\alpha' + \beta') \in (0.5, 1)$, there exist parameters $(\alpha', \beta')$ with $\frac{\beta'}{\alpha'}$ large enough which allow for the creation of a bubble at the beginning of $T$. Before time $T$, there can be no bubbles.

*Proof.* See Appendix. □

[17] Further details of the calibration are reported in the Appendix.

[18] Note that in our calibration the economy always satisfies the AMSZ test, since the average rate – the dotted line – is always above $1+g$.

This Proposition shows that bubbly equilibria can arise upon technological innovation. In the proof, we consider the case in which the entrepreneurs of generation $T-1$ are the ones who issue the bubbly security, selling it to the households of the same generation at the beginning of date $T$. The dynamics of the economy change, and (depending on the size of the bubble that is created) the economy will jump to one of the bubbly paths depicted in Figure 3.

The issuance of a bubble by the entrepreneurs has three effects. First, it immediately alleviates the financing constraints of the existing entrepreneurs, reducing distortions in capital allocation and directly benefiting all individuals from generation $T-1$. Second, there is a positive spillover impact on future generations since the increased production raises labor income, relaxing future financing constraints as well. Because of decreasing returns to capital, though, its importance dies out over time. Finally, there is a competition effect: as the bubble grows, it competes for funds with the entrepreneurs, and therefore has a negative impact on the accumulation of capital and production of future generations.[19]

## 5.2 Stochastic Technological Progress and Bubbles

In this subsection, we extend the analysis above and investigate the case in which agents anticipate that technological change can occur with a certain probability. In particular, we demonstrate that even the *possibility* of experiencing a technological change which increases $\beta$ relative to $\alpha$ is already enough to allow for bubbly equilibria. Interestingly, bubbles can arise even if the economy is originally in the $NB$ region and if it satisfies the AMSZ at all times.

Consider an economy that, as long as technological change has not occurred yet, belongs to the $NB$ region of the parameter space and also satisfies the AMSZ condition. At every period, there is a probability $q > 0$ that technological progress will take place, changing the original parameters of the production function $(\alpha, \beta)$ to new values $(\alpha', \beta')$. Once this happens, no further changes occur and the economy becomes completely deterministic. The new specification $(\alpha', \beta')$ belongs to the $B$ region and also satisfies the AMSZ criterion – $(\alpha' + \beta' > 0.5)$. Additionally, we assume that entrepreneurs know exactly their technology at the time they make investment decisions. Hence, if technological change has not taken place until the beginning of period $t+1$, for example, the middle-aged entrepreneurs of generation $t$ will produce with the old technology and uncertainty might only affect the entrepreneurs of future generations. Under these assumptions, we can prove the following:

[19] This is easy to verify by looking at the equation of equilibrium in the savings market, from which it is clear that the bubble subtracts resources to the investment in the two types of capital.

**Proposition 7.** For any value of $(\alpha' + \beta') \in (0.5, 1)$, there exist parameters $(\alpha', \beta')$ with $\frac{\beta'}{\alpha'}$ large enough to allow for bubbly equilibria.

*Proof.* See Appendix. □

In the situation described above, the probability of the economy experiencing technological progress is positive and constant at all times until the change takes place. As a consequence, technological progress occurs with probability 1 in the long-run. If $\frac{\beta'}{\alpha'}$ is large enough, thus, the steady-state will feature low interest rates almost surely. We can show, however, that bubbles may arise even if there is uncertainty about interest rates in the long-run. To see that, let us assume a similar model as before, but suppose that the permanent technological progress occurs with probability $q > 0$ at the very beginning of time $T$ only, where $T$ is a future period. For any other date, $q = 0$ and there is no uncertainty. Under those assumptions, we can prove the following:

**Proposition 8.** For any value of $(\alpha' + \beta') \in (0.5, 1)$, there exist parameters $(\alpha', \beta')$ with $\frac{\beta'}{\alpha'}$ large enough to allow for bubbly equilibria such that:

i) if technological change does not occur at date $T$, the bubble immediately bursts and the economy converges to the bubbleless steady-state thereafter.

ii) if technological change occurs at date $T$, the bubble is positive at all times.

*Proof.* See Appendix. □

Proposition 8 illustrates how the stochastic nature of bubbles might be purely linked to the randomness of technological progress. If there is a technological "disappointment" and the production structure is unchanged at time $T$, the bubble immediately bursts.

The intuition for the results in Propositions 6-8 is straightforward. If technological progress shifts the production function to be heavily biased towards intangibles relative to physical capital, it will move the economy to a region of the parameter space where the bubbleless steady-state features binding financing constraints and very low interest rates, despite high $IRR$ – as illustrated by the red arrow in Figure 5. This change allows for the co-existence of bubbles and efficiency. Interestingly, the very possibility of such a change is already sufficient to generate paths with a positive bubble even before the technological shift takes place.

## 5.3 Relation to the Savings Glut Explanation

One recent alternative explanation for asset price bubbles is based on the so called "savings glut" phenomenon – see Bernanke (2005). It points to the fast development of emerging

markets like China and oil-exporting countries, that have large amounts of savings but are unable to supply sound stores of value due to underdeveloped domestic financial markets. The excess funds from these countries have flooded the capital markets of developed nations, putting downward pressure on interest rates and stimulating the appearance of speculative bubbles. Besides identifying a separate underlying story for the emergence of bubbles in developed economies, our model differs from the savings glut explanation along two main dimensions.

First, our model hinges on a *relative* imbalance of high-yield versus low-yield assets in developed economies – a surplus of $K$ and a shortage of $Z$ – rather than on an absolute lack of sound stores of value. While the latter is certainly reasonable for developing countries, it seems less relevant for developed nations. If anything, one would expect that, as economies develop, their capacity to create stores of value increases. In this case, more securities can be issued and more accumulation of capital would be possible.

Second, most of the models that attribute the emergence of rational bubbles to an increase in the supply of funds feature a single-factor economy (physical capital), with linear technology ($AK$) and limited pledgeability of output. The linearity assumption, however, is not innocuous. With limited pledgeability of output, as new funds enter the economy capital keeps accumulating, albeit at a lower level than it would be possible without frictions. In an economy where physical capital has decreasing marginal returns, this accumulation would in fact *lower* the internal rate of return of funds. This means that, at some point, the economy would have a low enough productivity of capital to fail the $AMSZ$ test, thereby rendering it inconsistent with the available evidence. The assumption of linear technology shuts down this channel and keeps the internal return on funds artificially high.

In our framework, however, the savings glut explanation reinforces the technology-based approach. In the latter, the separation between the two types of capital allows the accumulation of physical capital to actually *increase* the wedge between $R$ and $IRR$: the marginal productivity of physical capital decreases, while the marginal productivity of intangible capital increases. To the extent that foreign investors cannot overcome the moral hazard frictions in the domestic economy, an inflow of funds from abroad increases the stock of $K$ and pushes the $IRR$ up. This allows the $AMSZ$ condition to be satisfied even for very low interest rates.[20] We formalize this intuition through a stylized example of an open economy presented in the Appendix.

[20]Interestingly, for our effects to take place, there is no need for global savings to increase. As physical capital becomes less attractive due to technological change, even a stable amount of total savings is capable of pushing interest rates down. Hence, our results do not contradict Laibson and Mollerstrom (2009) who argue that there is no global savings glut since global investment has not increased as a fraction of global GDP since mid 1990s.

# 6 Dynamic Efficiency

## 6.1 Traditional Tests of Dynamic Efficiency

As discussed in Section 4, bubbly equilibria can exist even if the economy satisfies the AMSZ empirical benchmark. In AMSZ's original paper as well as in Tirole (1985), this restriction corresponds to the condition that the equilibrium interest rate is not below the growth rate of the economy in the long-run, which has been been traditionally interpreted as a sign of dynamic efficiency. In particular, in that setting the following three statements are equivalent:

1. The economy satisfies the AMSZ condition.
2. The interest rate is higher than the growth rate of the economy in the long-run.
3. The economy is dynamically efficient.

In the presence of frictions, however, the equivalence of these statements is broken. For instance, in a similar environment, Farhi and Tirole (2008) have shown that statements 1 and 2 are delinked. We extend their analysis by proving that, in our model, statements 1 and 3 are not equivalent either. Moreover, we qualify the relation between 1 and 2. While our results are derived under a stylized setting, they allow us to make the general point that, by creating heterogeneity in investment opportunities, financial constraints can decisively influence the validity of traditional tests of dynamic efficiency.[21]

Lemma 3 formalizes the connection between the AMSZ restriction and rates of return. Note that as long as $\alpha + \beta \geq \frac{1}{2}$, the AMSZ test will be satisfied. This does not, however, guarantee that the interest rate condition will be met at all times, since Lemma 3 only holds in the steady-state. Nonetheless, analyzing steady-states is central since they contain important information about the efficiency properties of allocations.

We now formally prove that the AMSZ test is not sufficient to guarantee the efficiency of allocations once financial frictions are taken into account. For this purpose, it is important to discuss the dimensions a planner could act on in order to look for potential improvements over market allocations.

A benevolent planner can operate on at least two margins. First, whenever the borrowing constraints bind, the planner can make intra-generational transfers from unconstrained households to constrained entrepreneurs. This, however, has no major implications for the topic of dynamic efficiency, since it does not address the issue of over or under-accumulation of capital. Much more interesting, though, is to consider improvements in the intertemporal margin while respecting existing borrowing constraints.

[21] We do not attempt to derive an exact test of efficiency since our set up is too simplified, and any rules would probably not apply in more general contexts.

**Definition.** *An allocation is* constrained efficient *if it cannot be Pareto improved by a planner who is not allowed to transfer resources from the unconstrained households to the constrained entrepreneurs of a given generation.*

Based on the original literature on dynamic efficiency, one could expect that the constrained efficiency requirement would coincide with the AMSZ test. The following proposition proves that this is not the case.

**Proposition 9.** *If financial frictions are binding, the AMSZ criterion is not a sufficient condition to guarantee* constrained efficiency*.*

*Proof.* See Appendix. □

The reason why the AMSZ criterion does not guarantee efficiency in our model is related to the heterogeneity of investment opportunities created by financial frictions. In the binding region of the parameter space, there is a wedge between the interest rate obtained by the creditors and the rate of return earned by entrepreneurs. Looking at the average rate of return in the economy – as in the AMSZ test – is not sufficient to judge the efficiency property of market allocations, because there might be over-accumulation among those who cannot park funds on productive assets. The social planner can improve an allocation by transferring funds across households from different generations, compensating all entrepreneurs along the way. This can be done even if the growth rate of the economy is lower than the average rate of return, as long as it is greater than R.

One could argue, though, that our suggested transfer scheme does not do justice to the AMSZ test. On the one hand, we do not allow the planner to overcome the inefficiency resulting from moral hazard, but on the other hand, she is allowed to identify the different groups according to their investment opportunities and target them for reallocation purposes. To address this criticism, we analyze the efficiency properties of allocations that satisfy the AMSZ test in a setting where the planner cannot target specific groups.

**Definition.** *An allocation is* blind constrained efficient *if it cannot be Pareto improved by a planner who can only transfer resources across generations without identifying households or entrepreneurs.*

Interestingly, even when we consider only blind reallocations, it is still the case that the AMSZ benchmark does not coincide with efficiency, this time for the opposite reason: there exist allocations that do not satisfy the AMSZ benchmark, but cannot be improved by a blind social planner.

**Proposition 10.** *If financial frictions are binding, the failure to satisfy the AMSZ criterion is not a sufficient condition to guarantee that an allocation is* not blind constrained efficient*.*

*Proof.* The proof mimics the proof of Proposition 1 in AMSZ, taking into account that because $IRR > R$, in any intertemporal transfer scheme entrepreneurs should be compensated more than households, which forces the planner to transfer IRR to both groups (since she cannot distinguish between them). □

The intuition behind this result is straightforward. The AMSZ test compares the average rate of return on the economy with its growth rate in the long-run. However, a planner trying to improve over market allocations who cannot target specific groups in society is required to transfer resources over time at the highest rate of return every period. This is the only way to ensure that those who face the best investment opportunities are not harmed by reallocations. Hence, the fact that the economy fails the AMSZ test does not imply there are feasible improvements for blind planners, unless $IRR < (1 + g)$ in steady-state.

## 6.2 Normative and Positive Implications in a Bubbly Economy

The previous subsections have shown that it is possible for the economy to satisfy the AMSZ benchmark while being inefficient. Moreover, Sections 4 and 5 demonstrated that the economy can sustain bubbly equilibria even if we impose the AMSZ condition. A natural question, therefore, is whether rational bubbles are capable of restoring efficiency. As in Farhi and Tirole (2008), we argue this is not the case in general.

To see that, consider an economy where borrowing constraints bind. In this context, the introduction of a rational bubble has redistributive effects that affect negatively some individuals. On the one hand, bubbles improve the investment opportunities of savers. On the other hand, as the bubble grows, it competes for funds with entrepreneurs who borrow resources to make real investments in future periods. Therefore, typical rational bubbles are not Pareto improving in our model.

From a practical perspective, the present paper suggests one natural approach to mitigate the relative shortage of assets and prevent the emergence of bubbles: to improve the pledgeability of future cash flows, particularly those stemming from activities that are intensive in intangible capital. The increased pledgeability of future output allows funds to be allocated to their most productive ends, increasing the productivity in the overall economy and raising the rate of return faced by outside investors. This is crucial to preempt the emergence of new bubbles. Policies targeted to strengthen property rights and the enforcement of contracts are obvious candidates to achieve this goal. So are measures that facilitate investments in education and human capital which tend to be productive, but in general are subject to important financing constraints.

# 7 Conclusion

Following the emerging literature on asset shortages, we believe that one of the structural causes of endless speculation in financial markets around the globe is the inability of the world economy to properly supply the stores of value necessary to accommodate the increased wealth it has generated. Caballero et al. (2008) point to the developing world as a source of imbalances in the supply and demand for assets. The present paper suggests that developed countries might have played a significant role in this process as well. Our analysis stresses that one important element favoring asset price bubbles in developed countries is not an absolute lack of stores of value, but rather a difficulty in generating high-yield assets relative to low-yield ones.

Acknowledging this tendency of the global economy to induce asset price bubbles is of utmost importance for policymakers and academics. In an environment in which bubbles bridge the gap between high and low-yield securities and alleviate financial distortions, it might be useless or even harmful to try to chase and prick them. Hence, the best strategy for central banks might be to smoothly manage high valuation episodes, curbing excessive speculation without triggering huge drops in asset prices. We believe that checking the accuracy of these statements in formal models of optimal monetary and fiscal policy might be a very interesting research topic.

The mechanism described in this paper identifies one potential structural cause of bubbles, but as economies develop and change other forces may become more or less important at the same time. For example, the pledgeability of tangible capital itself may be much lower in developing economies than in developed ones – so that for developing countries that might be the main source of frictions. At the same time, even among developed economies, the degree of pledgeability of intangible capital will vary. As nations develop better instruments to transfer ownership of intangible capital (for example, a more efficient patent system), the effective pledgeability of intangibles will improve, mitigating the frictions that in our model are at the source of the existence of rational bubbles. This interaction between technological and institutional progress is an interesting topic for future studies.

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# Tables

| | Conventional, without intangibles | Including intangibles |
|---|---|---|
| Shares of total income | | |
| Labor | 70.4 | 60.0 |
| Tangible capital | 29.6 | 25.0 |
| Intangible capital | | 15.0 |

Table 1: Shares of income to factor, non-farm business sector, 2000-2003, including and excluding intangible capital (from Corrado et al. 2006, Table 4)

# Figures

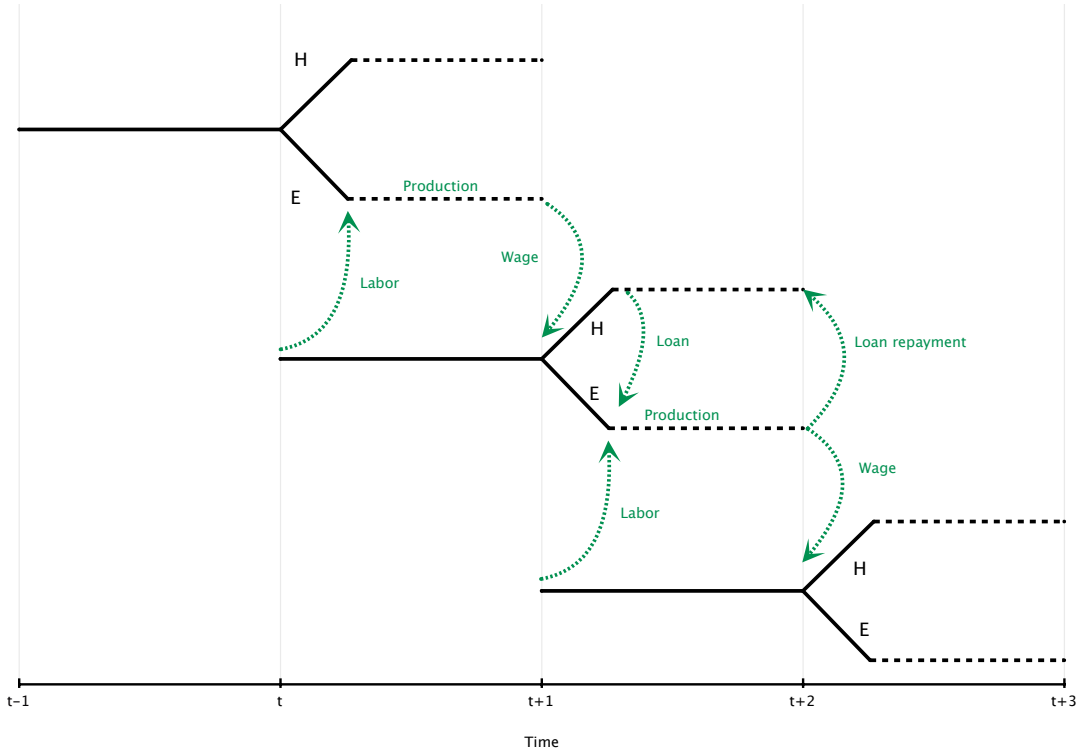


Figure 1: OLG model

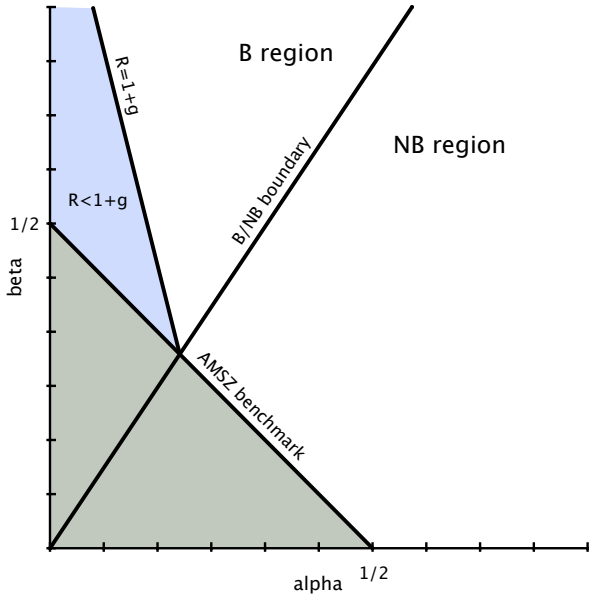


Figure 2: Characterization of the parameter space

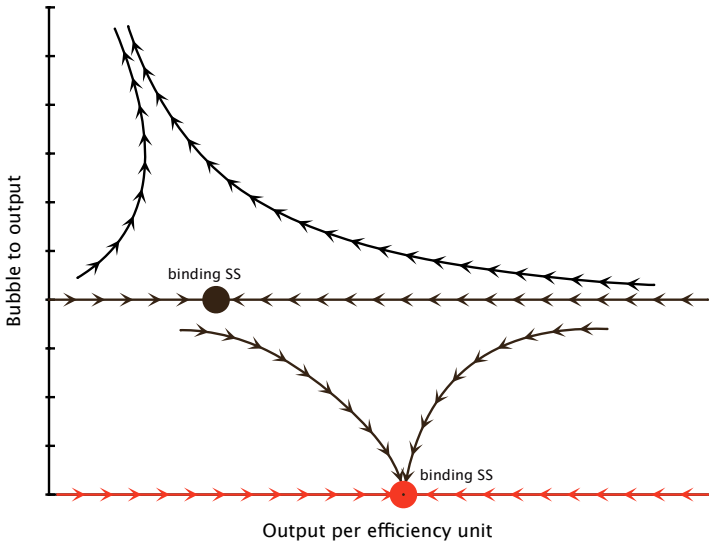


Figure 3: B region

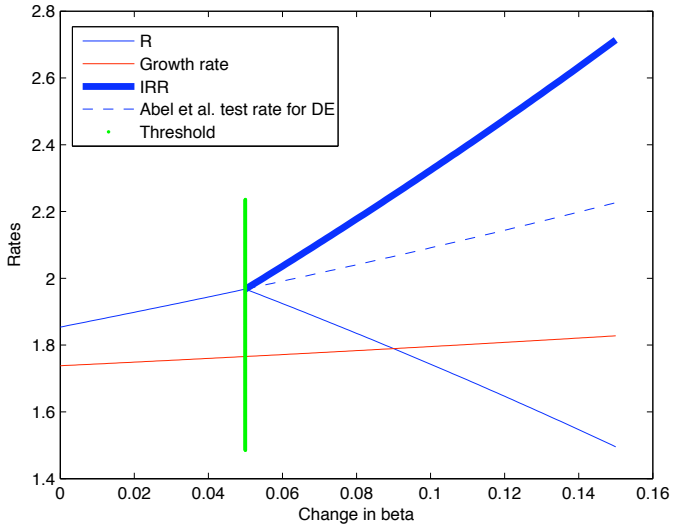


Figure 4: Steady states and technological transition

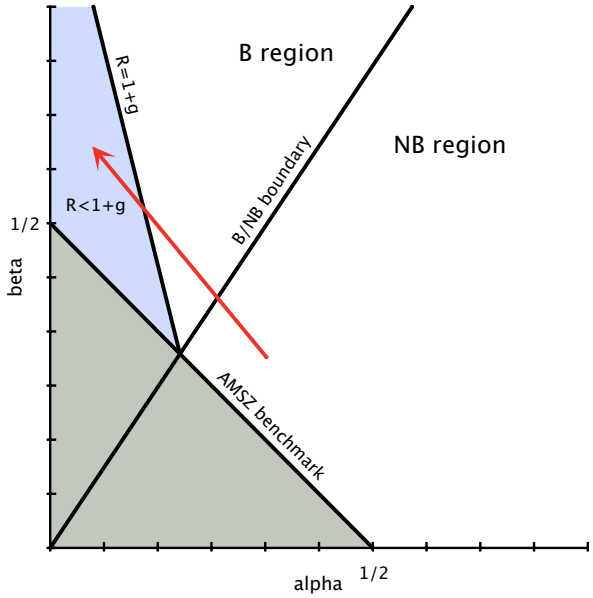


Figure 5: Technological change