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HOW FINANCIAL INNOVATION CAN CAUSE BUBBLES AND CRASHES

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July 23, 2011

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# Tranching, CDS and Asset Prices: How Financial Innovation can Cause Bubbles and Crashes.

Ana Fostel \*      John Geanakoplos<sup>†</sup>

July 23, 2011

## Abstract

We show how the timing of financial innovation might have contributed to the mortgage boom and then to the bust of 2007-2009. We study the effect of leverage, tranching, securitization and CDS on asset prices in a general equilibrium model with collateral. We show why tranching and leverage tend to raise asset prices and why CDS tend to lower them. This may seem puzzling, since it implies that creating a derivative tranche in the securitization whose payoffs are identical to the CDS will raise the underlying asset price while the CDS outside the securitization lowers it. The resolution of the puzzle is that the CDS lowers the value of the underlying asset since it is equivalent to tranching cash.

**Keywords:** Financial Innovation, Endogenous Leverage, Collateral Equilibrium, CDS, Tranching and Asset Prices.

**JEL Codes:** D52, D53, E44, G01, G10, G12

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# 1 Introduction

In this paper we propose the possibility that the mortgage boom and bust crisis of 2007-2009 might have been caused by financial innovation. We suggest that the astounding rise in subprime and Alt A leverage from 2000 to 2006, together with the remarkable growth in securitization and tranching throughout the 1990s and early 2000s, raised the prices of the underlying assets like houses and mortgage bonds. We further raise the possibility that the introduction of Credit Default Swaps, CDS, in 2005 and 2006 brought those prices crashing down with just the tiniest spark.

Securitization and tranching did not happen over night. The securitization of mortgages by the government agencies Fannie Mae and Freddie Mac began in earnest in the 1970s, when the first pools of mortgages were assembled and shares were sold to investors. In 1986 Salomon and First Boston created the first tranches, buying Fannie and Freddie Pools and cutting them into four pieces. This was no simple task, because it involved not only special tax treatment by the government, but also the creation of special legal entities and trusts which would collect the homeowner payments and then divide them up among the bondholders. By the middle 1990s the greatest mortgage powerhouse was the investment bank Kidder Peabody, cutting hundreds of billions of dollars worth of mortgage pools into over 90 types of tranches called CMOs (collateralized mortgage obligations). These tranches bore esoteric names like floater, inverse floater, IO, PO, inverse IO, Pac, Tac, etc. The young traders, often in their mid 20s, who collectively engineered this multi-trillion dollar operation were motivated by the profits they could make buying pools of mortgages and cutting them up into more valuable tranches. They would find out the needs of various buyers and tailor make the tranches to deliver money in just those states of nature that the buyers wanted them. In short, they exploited the heterogeneous needs of their buyers by creating heterogeneous pieces out of a homogeneous pie.

The impetus driving the tranching machine was not a demand for riskless assets; on the contrary, it was a demand for contingent assets. The Fannie and Freddie principal mortgage payments were guaranteed against homeowner default by Fannie and Freddie, enabling the tranches to be rated AAA. But that hardly meant they were riskless. Changes in interest rates or changes in prepayments by the underlying homeowners could radically alter the cash flows of the tranches. Gradually Wall Street came to see that default risk was just one among many risks, and pools and tranching began to be undertaken without government guarantees, for example for

jumbo mortgages that were not eligible for purchase by Fannie and Freddie and for credit cards and other assets.

Spurred on by these private securitizations, Wall Street dreamt up the idea in the mid 1990s of pooling and tranching subprime mortgages, with no government guarantees at all. Through a cleverly constructed architecture of pooling, senior pieces were still able to get AAA bond ratings because they were protected by junior tranches that absorbed the losses in case of homeowner defaults. The subprime mortgage market grew from a few million dollars to a trillion by 2006.

In the 1990s credit default swaps were invented for corporate bonds and sovereign bonds. It was not until 2005, however, that credit default swaps were standardized for mortgages. A CDS is a kind of insurance on an asset or bond. It is the promise to take back the underlying asset at par once there is a default, that is, to make up the losses of the underlying asset.

Our approach, like many papers in economics that take technological innovation as exogenous, is to take the financial innovations in the mortgage market between 1986 and 2010 as exogenous and investigate their consequences for asset pricing. Under this view, the tranching of subprime mortgages couldn't have begun earlier, because it had to wait for the innovation of CMO tranching. In later work we hope to explain why the innovations came when they did and why for example CDS seem to appear in various markets only after the risk of default is generally recognized to be significant.

The size of these financial innovations is certainly staggering, and leaves one wondering what their effects might have been. Consider first the history of subprime and Alt A leverage and housing prices from 2000-2008 shown in Figure 1, taken from Geanakoplos (2010b). Leverage went from about 7 in 2000 (14% average downpayment for the top half of households) to about 35 in the second quarter of 2006 (2.7% downpayment on average for the top half of households). Next consider the growth of securitization and tranching presented in Figure 2, especially in the late 1990s and early 2000s. These amounted to trillions of dollars a year. Finally, consider the growth of the CDS market presented in Figure 3, especially after 2005. The available numbers are not specific to mortgages, since most of these were over the counter, but the fact that subprime CDS were not standardized until late 2005 suggests that the growth of mortgage CDS in 2006 is likely even sharper than Figure

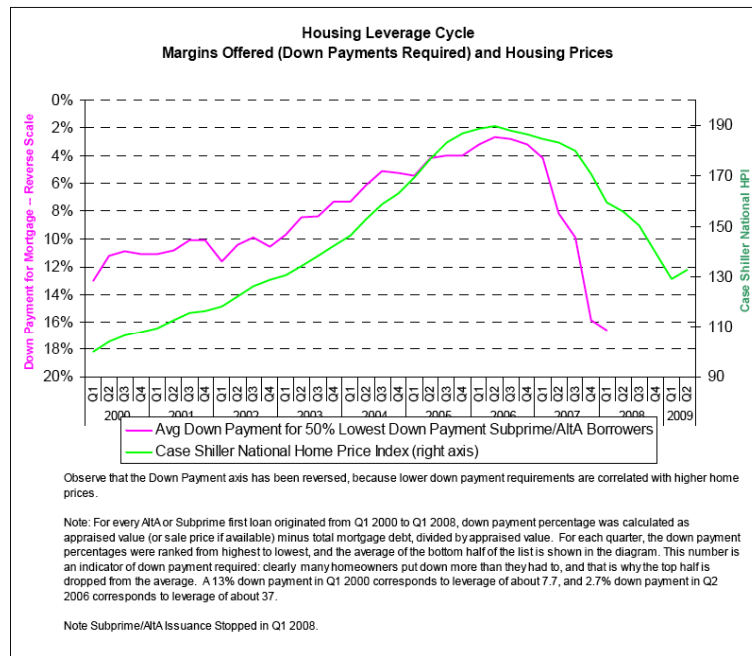


Figure 1: Housing Leverage.

3 suggests. What is clear is that the explosive growth of the CDS market came after the explosive growth of securitization.

Many people who are aware of these numbers have linked securitization and CDS to the crisis of 2007-2009. While we agree with much of their view, our analysis is based on entirely different considerations. And we wish to explain the boom as well as the bust.

Some problems that many critics have noted with tranching are i) the standing opportunity for the original lender to sell his loans into a securitization destroys his incentive to choose good loans and ii) once a pool is tranching it becomes very difficult for the bond holders to negotiate with each other (for example to write down principal).

Many observers have pointed to the creation of CDS as the source of many problems, to mention a few: i) important financial institutions wrote trillions of dollars of CDS insurance; the economy could not run smoothly after they lost so much money on their bad bets, ii) writers of CDS insurance did not even post enough collateral to

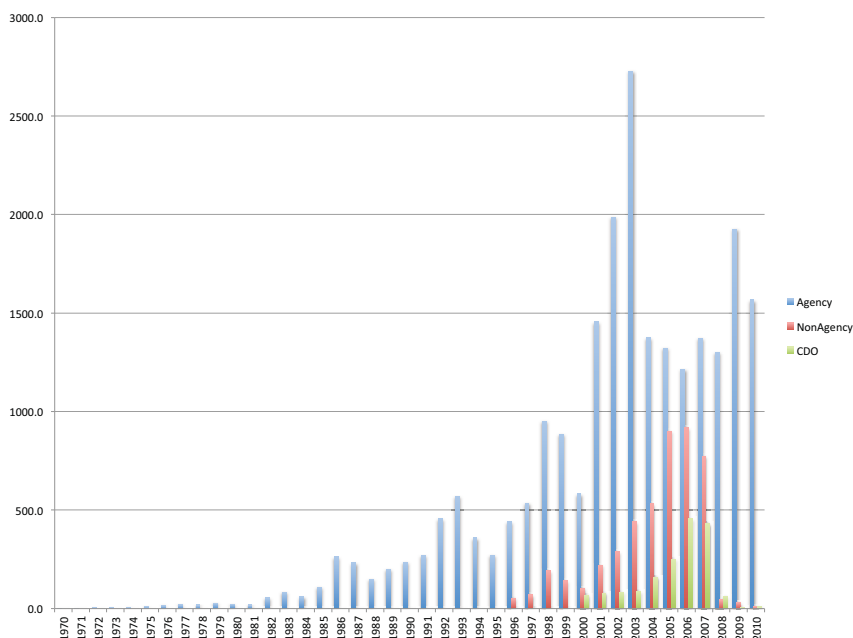


Figure 2: Securitization/Tranching.

cover their bets, forcing the government to bail out the beneficiaries, iii) CDS were traded on OTC markets, with a lack of transparency that enabled price gouging, iv) CDS give investors (at least those who wrote much more insurance than the underlying assets were worth) the incentive to manipulate markets, for example to avoid paying off a big insurance amount by directly paying off the bonds. George Soros went as far as calling CDS “instruments of destruction that should be outlawed” and claimed that “...some derivatives ought not to be allowed to be traded at all. I have in mind credit default swaps. The more I’ve heard about them, the more I’ve realized they’re truly toxic,”<sup>1</sup>

The first main contribution of this paper is to show that tranching and leverage raise the price of the underlying collateral even if they have no effect on the total cash flows coming from the collateral. All the difficulties with tranching and CDS pointed out by others in the last paragraphs rely on the pernicious effect of securitization and tranching on the basic cash flows. So our thesis is quite different. But it is really common sense. Indeed the historically enthusiastic government support for

<sup>1</sup>Wall Street Journal, March 24, 2009.

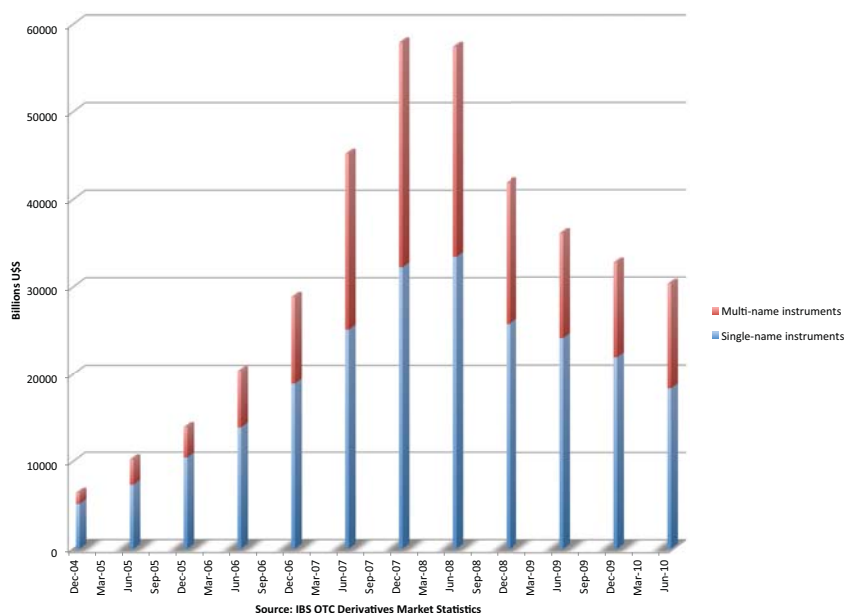


Figure 3: CDS Markets. Outstanding notional amount.

the tranching of mortgages was doubtless due to an understanding that it raises the price of the underlying mortgage assets and therefore reduces the borrowing costs to the homeowner.

Tranching makes the underlying collateral more valuable because it can be broken into pieces that are tailor made for different parts of the population, just as the traders in the 1990s realized. Splitting plain vanilla into strawberry for one group and chocolate for another should raise the value of the scarce ice cream. Leverage is an imperfect form of tranching and one would guess that therefore leverage would not raise the asset price as much as tranching.

We first build a static two-period model with heterogeneous agents in which we can investigate the circumstances under which these common sense conclusions hold true. We compute equilibrium prices without leverage, with leverage, then with tranching, then with tranching and CDS. We find that once the tranching technology is invented and freely available, it will inevitably proceed in equilibrium all the way to Arrow securities, or at least all the way to commonly verifiable events. Leverage and asset tranching always raise asset prices above their CDS and no-leverage levels.

Somewhat surprisingly, however, we find that this fine tranching does not always raise the asset price above the leverage price when all the general equilibrium effects are taken into account. We prove that if there is more heterogeneity among the pessimists than among the optimists, then tranching always yields a higher price than leverage which in turn is higher than the no-leverage price.

Furthermore, we show that with tranching, the price of the underlying collateral can rise above what any agent thinks it is worth, while with leverage the price of the asset typically rises to what a more optimistic agent thinks it is worth. This tranching hyper price fits the definition of a bubble given in Harrison and Kreps (1978), where a bubble is defined as an asset price that is higher than any agent thinks the asset is worth. In Harrison and Kreps the explanation for bubbles turned on the ability of agents to resell the asset to others who would think it was worth more, whereas in our two-period model there is no resale of the asset.

Third, we show that the introduction of CDS dramatically lowers the asset price, even below the non-leverage level, provided that the median investor thinks that the asset is more than 50% likely to have a good payoff. This seems counterintuitive at first glance. Tranching creates derivatives of the underlying asset, and presumably one of those tranches could have exactly the same payouts as the CDS. Indeed that happens in our model. The tranche and the CDS are perfect substitutes in every agent's mind. Yet when the CDS is created exclusively inside the securitization as a tranche of the asset, it raises the asset price. When the CDS is created outside the securitization it lowers the asset price. How could this be?

We show that on second thought this is not surprising at all. When agents sell CDS and put up cash as collateral, they are effectively tranching cash! That raises the value of cash relative to the reference asset. When every asset (more precisely, when all future cash flows) can be perfectly tranching, we get the Arrow-Debreu equilibrium, and all asset bubbles disappear. The depressing effect of CDS on asset prices is most dramatic when the asset is not tranching, but is held outright or levered, because in that case the buyers of the asset will divert their wealth into writing CDS, which is a perfect substitute for holding the asset.

In Section 2 we present our two period model of collateral equilibrium. In Section 3 we show how specifying the collateral technology in different ways allows the same model to encompass leverage, tranching, and CDS in one simple framework. In Section 4 we explain why leverage and securitization raise asset prices, and why



CDS lowers them. Finally in Section 5 we describe a dynamic model in which a non-levered initial situation is followed by the unexpected introduction of leverage, then securitization and tranching and finally CDSs. All the way through very small bad shocks are occurring. Nevertheless, prices rise dramatically during the initial three phases, then come crashing down with the introduction of CDSs.

The timing of the financial innovation was crucial. Tranching and securitization came first, raising asset prices, then CDS followed much later, crushing their prices. Figures 1, 2 and 3 show empirical evidence suggesting that this was indeed the timing in securitization and CDS markets.<sup>2</sup> The timing of innovation was disastrous because it caused a crash, forcing many people into bankruptcy or underwater. Had the CDS come at the same time as the securitization, asset prices would never have gotten so high, and the crash would have been milder, as we show in Section 5.

Our financial innovation theory of booms and busts complements the Leverage Cycle theory proposed in Geanakoplos (2003, 2010a and 2010b) and developed further in Fostel-Geanakoplos (2008, 2010 and 2011). The financial innovation theory is similar in many ways to the leverage cycle, but unlike that theory, it does not rely on any kind of shock, much less on a shock that also increases the volatility of future shocks. In the financial innovation story of the boom and bust told here, innovation could have generated the entire cycle on its own, with no external triggers.

In the leverage cycle story of the recent boom and bust told in Geanakoplos (2010a and 2010b), a prolonged period of low volatility led to high leverage and therefore high asset prices and the concentration of assets in the hands of the most optimistic investors. When bad news came in 2007 in the form of a spike in delinquencies of subprime homeowners, it not only directly reduced prices but it also reduced leverage, because it created more uncertainty about what would happen next, which had an indirect effect on asset prices. The combination of bad news, losses by the hyper-leveraged optimists, and plummeting leverage led to a huge fall in asset prices, much bigger than could be explained by the bad news alone.

Geanakoplos (2010b) attributed the rise in leverage to many factors besides low volatility. One of these was the almost explicit government guarantees to Government Agencies like Fannie Mae and Freddie Mac, and another was the implicit guarantees to the big banks who were too big to fail. Yet another was low interest rates and

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<sup>2</sup>Academic papers describing the financial crisis all agree on this fact, as in Brunnermeier (2009), Gorton (2010), Gorton and Metrick (2010), Geanakoplos (2010) and Stultz (2009).

the resulting pursuit of yield. But most important he said was securitization. Yet he provided no model for the connection between securitization and leverage. Geanakoplos 2010a, 2010b, also suggested that the introduction of standardized credit default swaps in 2005 had a sharp negative impact on the prices of assets.

Here we make rigorous the connection between leverage, tranching, and asset prices by extending the model in Geanakoplos (2003). In the language of Fostel-Geanakoplos (2008), tranching increases the collateral value of the underlying asset. Leverage is an imperfect form of tranching and so raises the underlying asset value less than ideal tranching. CDS is a form of tranching cash, and so raises the relative value of cash, thus lowering the value of the reference asset.

Our paper is more generally related to a literature on leverage as in Araujo, Kubler and Schommer (2011), Acharya and Viswanathan (2011), Adrian and Shin (2010), Brunnermeier and Pedersen (2009), Cao (2010), Fostel and Geanakoplos (2008, 2010 and 2011), Geanakoplos (1997, 2003 and 2010), Gromb and Vayanos (2002) and Simsek (2010). It is also related to work that studies the asset price implications of leverage as Hindy (1994), Hindy and Huang (1995) and Garleanu and Pedersen (2009).

Our paper is also part of a growing theoretical literature on CDS. Bolton and Oehmke (2011) study the effect of CDS on the debtor-creditor relationship. The proposition that CDS tends to lower asset prices was demonstrated in Geanakoplos (2010a), and confirmed in exactly the same model by Che and Sethi (2010).

## 2 General Equilibrium Model with Collateral

The model is a two-period general equilibrium model, with time  $t = 0, 1$ . Uncertainty is represented by a tree  $S = \{0, U, D\}$  with a root  $s = 0$  at time 0 and two states of nature  $s = U, D$  at time 1.

There are two assets in the economy which produce dividends of the consumption good at time 1. The riskless asset  $X$  produces  $X_U = X_D = 1$  unit of the consumption good in each state, and the risky asset  $Y$  produces  $Y_U = 1$  unit in state  $U$  and  $0 < Y_D = R < 1$  unit of the consumption good in state  $D$ . Figure 4 shows asset payoffs.

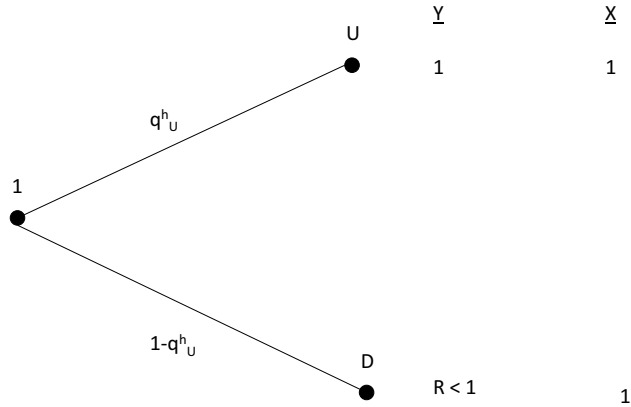


Figure 4: Asset Payoffs.

Each investor in the continuum  $h \in H = (0, 1)$  is risk neutral and characterized by a linear utility for consumption of the single consumption good  $x$  at time 1, and subjective probabilities,  $(q_U^h, q_D^h = 1 - q_U^h)$ . The von-Neumann-Morgenstern expected utility to agent  $h$  is

$$U^h(x_U, x_D) = q_U^h x_U + q_D^h x_D \tag{1}$$

We shall suppose that  $q_U^h$  is strictly monotonically increasing and continuous in  $h$ . Examples are  $q_U^h = 1 - (1 - h)^2$ ,  $q_D^h = (1 - h)^2$  and  $q_U^h = h$ ,  $q_D^h = 1 - h$ .

Each investor  $h \in (0, 1)$  has an endowment of one unit of each asset at time 0 and nothing else. Since only the output of  $Y$  depends on the state and  $1 > R$ , higher  $h$  denotes more optimism. Heterogeneity among the agents stems entirely from the dependence of  $q_U^h$  on  $h$ .

The reader may be aghast by the simplicity of the model, and in particular by heterogeneous priors, risk neutrality and the lack of endowment of the consumption good in states 1 and 2. We hasten to assure such a reader that we are using the

simplest model we can to illustrate our point. None of the results depend on risk neutrality or heterogeneous priors. By assuming common probabilities and strictly concave utilities, and adding large endowments in state  $D$  vs state  $U$  for agents with high  $h$  and low endowments in state  $D$  vs state  $U$  for agents with low  $h$ , we could reproduce the distribution of marginal utilities we get from differences in prior probabilities. We have chosen to replace the usual marginal analysis of consumers who have interior consumption with a continuum of agents and a marginal buyer. Our view is that the slightly unconventional modeling is a small price to pay for the simple tractability of the analysis.

## 2.1 Arrow Debreu Equilibrium

Arrow-Debreu equilibrium is easy to describe for our simple economy. It is given by present value consumption prices  $(p_U, p_D)$ , which without loss of generality we can normalize to add up to 1, and by consumption  $(x_U^h, x_D^h)_{h \in H}$  satisfying

1.  $\int_0^1 x_U^h dh = 1 + 1$
2.  $\int_0^1 x_D^h dh = 1 + R$
3.  $(x_U^h, x_D^h) \in B_W^h(p_U, p_D) = \{(x_U, x_D) \in \mathbb{R}_+^2 : p_U x_U + p_D x_D \leq p_U(1 + 1) + p_D(1 + R)\}$
4.  $(x_U, x_D) \in B_W^h(p_U, p_D) \Rightarrow U^h(x) \leq U^h(x^h), \forall h$

The interpretation of Arrow-Debreu equilibrium is that at time 0 agents trade contingent commodities forward. An agent with high  $h$  for example might sell a future claim for  $D$  consumption in exchange for  $U$  consumption. It is taken for granted that  $h$  will deliver the goods in  $D$  if that state occurs.

We can easily compute Arrow-Debreu equilibrium. Because of linear utilities and the continuity of utility in  $h$  and the connectedness of the set of agents  $H = (0, 1)$ , at state  $s = 0$  there will be a *marginal buyer*,  $h_1$ , who will be indifferent between buying the Arrow  $U$  and the Arrow  $D$  security. All agents  $h > h_1$  will sell everything and buy only the Arrow  $U$  security. Agents  $h < h_1$  will sell everything and buy only the Arrow  $D$  security. This regime is showed in Figure 5.

At  $s = 0$  aggregate revenue from sales of the Arrow  $U$  security is given by  $p_U \times 2$ . On the other hand, aggregate expenditure on it by the buyers  $h \in [h_1, 1)$  is given by  $(1 - h_1)(2p_U + (1 + R)(1 - p_U))$ . Equating we have

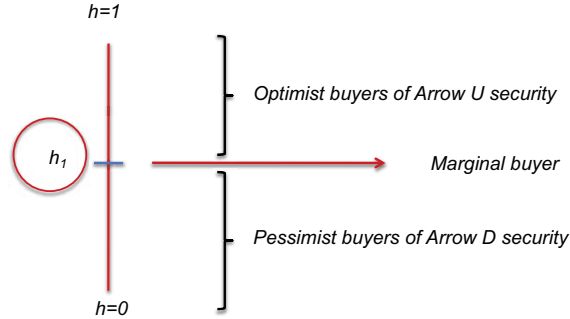


Figure 5: Arrow-Debreu Equilibrium Regime.

$$(2p_U + (1 + R)(1 - p_U))(1 - h_1) = 2p_U \quad (2)$$

The next equation states that the marginal buyer is indifferent between buying the Arrow  $U$  and the Arrow  $D$  security.

$$q_U^{h_1}/p_U = (1 - q_U^{h_1})/(1 - p_U) \quad (3)$$

Hence we have a system of two equations and two unknowns: the price of the Arrow  $U$  security,  $p_U$ , and the marginal buyer,  $h_1$ . For the probabilities  $q_U^h = 1 - (1 - h)^2$  and  $R = .2$ , we get  $h_1 = .33$  and  $p_U = .55$ . The implicit prices of  $X$  and  $Y$  are  $p_X = p_U1 + p_D1 = 1$  and  $p_Y = p_U1 + p_DR = .64$ .

## 2.2 Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral. In Arrow Debreu equilibrium the question of why agents repay their loans is ignored. We suppose from

now on that the only enforcement mechanism is collateral.

At time 0 agents can trade financial contracts. A financial contract  $(A, C)$  consists of both a promise,  $A = (A_U, A_D)$ , and an asset acting as collateral backing it,  $C \in \{X, Y\}$ . The lender has the right to seize as much of the collateral as will make him whole once the promise comes due, but no more: the contract therefore delivers  $(\min(A_U, C_U), \min(A_D, C_D))$  in the two states. The significance of the collateral is that the borrower must own the collateral  $C$  at time 0 in order to make the promise  $A$ .

We shall suppose every contract is collateralized either by one unit of  $X$  or by one unit of  $Y$ . The set of promises  $j$  backed by one unit of  $X$  is denoted by  $J^X$  and the set of contracts backed by one unit of  $Y$  is denoted by  $J^Y$ . In the next section we will analyze different economies obtained by varying the set  $J = J^X \cup J^Y$ .

We shall denote the sale of promise  $j$  by  $\varphi_j > 0$  and the purchase of the same contract by  $\varphi_j < 0$ . The sale of a contract corresponds to borrowing the sale price, and the purchase of a promise is tantamount to lending the price in return for the promise. The sale of  $\varphi_j > 0$  units of contract type  $j \in J^X$  requires the ownership of  $\varphi_j$  units of  $X$ , whereas the purchase of the same number of contracts does not require any ownership of  $X$ .

## 2.3 Budget Set

Each contract  $j \in J^C$  will trade for a price  $\pi^j$ . An investor can borrow  $\pi^j$  today by selling contract  $j$  in exchange for a promise of  $A^j$  tomorrow, provided he owns  $C$ .

We can always normalize one price in each state  $s \in S = \{0, U, D\}$ , so we take the price of  $X$  in state 0 and the price of consumption in each state  $U, D$  to be one. Thus  $X$  is both riskless and the numeraire, hence it is in some ways analogous to a durable consumption good like gold, or to money, in our one commodity model. Given asset and contract prices at time 0,  $(p, (\pi^j)_{j \in J})$ , each agent  $h \in H$  decides his asset holdings  $x$  of  $X$  and  $y$  of  $Y$  and contract trades  $\varphi_j$  in state 0 in order to maximize utility (1) subject to the budget set defined by

$$\begin{aligned}
 B^h(p, \pi) = \{ & (x, y, \varphi, x_U, x_D) \in R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+ : \\
 & (x - 1) + p(y - 1) \leq \sum_{j \in J} \varphi_j \pi^j \\
 & \sum_{j \in J^X} \max(0, \varphi_j) \leq x, \sum_{j \in J^Y} \max(0, \varphi_j) \leq y
 \end{aligned}$$

$$\begin{aligned}
x_U &= x + y - \sum_{j \in J^X} \varphi_j \min(A_U^j, 1) - \sum_{j \in J^Y} \varphi_j \min(A_U^j, 1) \\
x_D &= x + yR - \sum_{j \in J^X} \varphi_j \min(A_D^j, 1) - \sum_{j \in J^Y} \varphi_j \min(A_D^j, R)
\end{aligned}$$

At time 0 expenditures on the assets purchased (or sold) can be at most equal to the money borrowed selling contracts using the assets as collateral. The assets put up as collateral must indeed be owned. In the final states, consumption must equal dividends of the assets held minus debt repayment.

Notice that there is no sign constraint on  $\varphi_j$ ; a positive (negative)  $\varphi_j$  indicates the agent is selling (buying) contracts or borrowing (lending)  $\pi^j$ . Notice also that we are assuming that short selling of assets is not possible,  $x, y \geq 0$ .

## 2.4 Collateral Equilibrium

We suppose that agents are uniformly distributed in  $(0, 1)$ , that is they are described by Lebesgue measure. A *Collateral Equilibrium* in this economy is a price of asset  $Y$ , contract prices, asset purchases, contract trade and consumption decisions by all the agents  $((p, \pi), (x^h, y^h, \varphi^h, x_U^h, x_D^h)_{h \in H}) \in (R_+ \times R_+^J) \times (R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+)^H$  such that

1.  $\int_0^1 x^h dh = 1$
2.  $\int_0^1 y^h dh = 1$
3.  $\int_0^1 \varphi_j^h dh = 0 \quad \forall j \in J$
4.  $(x^h, y^h, \varphi^h, x_U^h, x_D^h) \in B^h(p, \pi), \forall h$
5.  $(x, y, \varphi, x_U, x_D) \in B^h(p, \pi) \Rightarrow U^h(x) \leq U^h(x^h), \forall h$

Markets for the consumption good in all states clear, assets and promises clear in equilibrium at time 0, and agents optimize their utility in their budget sets. As shown by Geanakoplos and Zame (1997), equilibrium in this model always exists under the assumptions we made so far.

## 2.5 Tranching

One of the most important financial innovations has been the “tranching” of assets or collateral. In tranced securitizations the collateral dividend payments are divided among a number of bonds which are sold off to separate buyers. So far in our analysis we have assumed that each collateral can back just one promise, so tranching seems out of the picture. But in fact the collateral holder gets the residual payments after the promise is paid, so effectively we have been tranching into two bonds all along. And with two states of nature, we shall show that there is no reason to have more pieces. So as long as there is no restriction on the nature of the promise, our collateral equilibrium includes tranching.

In practice houses have been tranced into first and second mortgages, and sometimes third mortgages. These tranches have the property that they all move in the same direction: good news for the house value is good news for all the tranches. But when mortgages are tranced, the tranche values often move in opposite directions. The more a floater pays, the less an inverse floater pays and so on. Even when the tranches of subprime mortgages appear to have the form of debt for the higher tranches, and equity for the lower tranches, the presence of various triggers which move cash flows from one tranche to another can make the payoffs go in opposite directions.

Thus in what follows we shall assume in our analysis that tranching has reached a degree of perfection that permits Arrow security tranches to be created.<sup>3</sup> The tranching of mortgages in the CMO revolution of the 1990s moved far along in that direction. And to the extent that the mortgage principal amount is nearly as high as the house price, as often occurred in the 2000s, the mortgage already includes the entire future value of the house. Thus we shall not distinguish between tranching the asset or tranching a mortgage written on the asset. In short we shall assume that the tranching is directly backed by the asset.

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<sup>3</sup>Of course, in reality Arrow securities cannot be created. But the reason has to do with the lack of verifiability and the cost of writing complex contingencies into a contract, both of which are ignored in our analysis.



### 3 Leverage, Securitization, and CDS

In this section we study the effect of leverage and derivatives on equilibrium by considering four different versions of the collateral economy introduced in the last section, each defined by a different set of feasible contracts  $J$ . We describe each variation and the system of equations that characterizes the equilibrium. In the next section we compare equilibrium asset prices across all the economies.

#### 3.1 No-Leverage Economy

We consider first the simplest possible scenario where no promises at all can be made,  $J = \emptyset$ . Agents can only trade assets  $Y$  and  $X$ . They cannot borrow using the assets as collateral.

Let us describe the system of equations that characterizes the equilibrium. Because of the strict monotonicity and continuity of  $q_U^h$  in  $h$ , and the linear utilities and the connectedness of the set of agents  $H = (0, 1)$ , at state  $s = 0$  there will be a unique *marginal buyer*,  $h_1$ , who will be indifferent between buying or selling  $Y$ . In equilibrium it turns out that all agents  $h > h_1$  will buy all they can afford of  $Y$  while selling all their endowment of  $X$ . Agents  $h < h_1$  will sell all their endowment of  $Y$ . This regime is shown in Figure 6.

At  $s = 0$  aggregate revenue from sales of the asset is given by  $p \times 1$ .<sup>4</sup> On the other hand, aggregate expenditure on the asset is given by  $(1 - h_1)(1 + p)$ , which is total income from the endowment of one unit of  $X$ , plus revenue from the sale of one unit of asset  $Y$  by buyers  $h \in [h_1, 1)$ . Equating supply and demand we have

$$p = (1 - h_1)(1 + p) \tag{4}$$

The next equation states that the price at  $s = 0$  is equal to the marginal buyer's valuation of the asset's future payoff.

$$p = q_U^{h_1} 1 + (1 - q_U^{h_1})R \tag{5}$$

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<sup>4</sup>All asset endowments add to 1 and without loss of generality are put up for sale even by those who buy it.

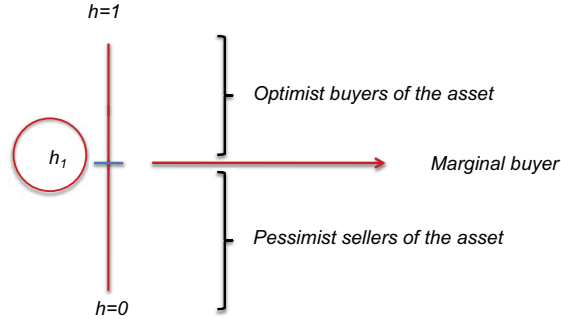


Figure 6: Non-Leverage Economy Equilibrium Regime.

Hence we have a system of two equations and two unknowns: the price of the asset,  $p$ , and the marginal buyer,  $h_1$ . For the probabilities  $q_U^h = 1 - (1 - h)^2$  and  $R = .2$ , we get  $h_1 = .54$  and  $p = .83$ .

### 3.2 Leverage Economy

Agents now are allowed to borrow money to buy more of the risky asset  $Y$ . We let them issue non-contingent promises using the asset as collateral. In this case  $J = J^Y$ , and each  $A_j = (j, j)$  for all  $j \in J = J^Y$ . The following result regarding leverage holds.

**Proposition 1:** *Suppose that in equilibrium the max min contract  $j^* = \min_{s=U,D}\{Y_s\} = R$  is available to be traded, that is  $j^* \in J = J^Y$ . Then  $j^*$  is the only contract traded, and the risk-less interest rate is equal to zero, this is,  $\pi^{j^*} = j^* = R$ .*

**Proof:** See Geanakoplos (2003) and Fostel-Geanakoplos (2010 and 2011).

Leverage is endogenously determined in equilibrium. In particular, the proposition *derives* the conclusion that although all contracts will be priced in equilibrium, the only contract actively traded is the max min contract, which corresponds to the *Value at Risk equal zero rule*,  $VaR = 0$ , assumed by many other papers in the literature. Hence there is no default in equilibrium.

Taking the proposition as given, let us describe the system of equations that characterizes the equilibrium. As before, there will be a *marginal buyer*,  $h_1$ , who will be indifferent between buying or selling  $Y$ . In equilibrium all agents  $h > h_1$  will buy all they can afford of  $Y$ , i.e., they will sell all their endowment of the  $X$  and borrow to the max min using  $Y$  as collateral. Agents  $h < h_1$  will sell all their endowment of  $Y$  and lend to the more optimistic investors. The regime is showed in Figure 7.

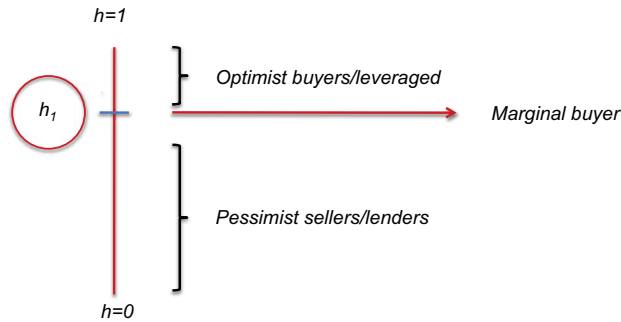


Figure 7: Leverage Economy Equilibrium Regime.

At  $s = 0$  aggregate revenue from sales of the asset is given by  $p \times 1$ . On the other hand, aggregate expenditure on the asset is given by  $(1 - h_1)(1 + p) + R$ . The first term is total income (endowment of  $X$  plus revenues from asset sales) of buyers  $h \in [h_1, 1)$ . The second term is borrowing, which from proposition 1 is  $R$  (recall that the interest rate is zero). Equating we have

$$p = (1 - h_1)(1 + p) + R \tag{6}$$

The next equation states that the price at  $s = 0$  is equal to the marginal buyer's valuation of the asset's future payoff.

$$p = q_U^{h_1} 1 + (1 - q_U^{h_1})R \tag{7}$$

We have a system of two equations and two unknowns: the price of the asset,  $p$ , and the marginal buyer,  $h_1$ . Notice how equation (6) differs from equation (4). Optimists now can borrow  $R$ . This will imply that in equilibrium a fewer number of optimists can afford to buy all the asset in the economy. Hence, the marginal buyer in the Leverage economy will be someone more optimistic than the marginal buyer in the No-Leverage economy. We will discuss this in detail in Section 4. For the probabilities  $q_U^h = 1 - (1 - h)^2$  and  $R = .2$ , we get  $h_1 = .63$  and  $p = .89$ .

Finally, notice that buying the asset while leveraging to the max min is equivalent to buying the Arrow  $U$  security. Since the owner needs to pay back  $R$  in period 1, his net payoffs are  $1 - R$  at  $U$  and 0 at  $D$ . Hence, optimistic investors who are desperate to transfer their wealth to the  $U$  state can very effectively do that by leveraging the asset to the max min. In equilibrium, the implicit price of the Arrow  $U$  security is given by  $p_U = \frac{p-R}{1-R} = .86$ .

### 3.3 Asset-Tranching Economy

In this economy we suppose that the risky asset  $Y$  can be *trached* into arbitrary contingent promises, including the riskless promises from the last section and all Arrow promises. The holder of the asset can sell off any of the tranches he does not like and retain the rest. This is a step forward from the leverage economy, in which investors holding a leveraged position on the asset could synthetically create the Arrow  $U$  security. Now they can also synthetically create the Arrow  $D$  security.

To simplify the analysis we suppose at first that  $J = J^Y$  consists of the single promise  $A = (0, R)$ , tantamount to a multiple of the Arrow  $D$  security. Notice that by buying the asset  $Y$  and selling off the tranche  $(0, R)$ , any agent can obtain the Arrow  $U$  security. Our parsimonious description of  $J = J^Y$  therefore already includes

the possibility of tranching  $Y$  into Arrow securities. We shall see shortly that once that is possible, there is no reason to consider further tranches.

Let us describe the system of equations that characterizes the tranching equilibrium with the single tranche  $A = (0, R)$ . In this case it is easy to see that there will be two *marginal buyers*  $h_1$  and  $h_2$ . In equilibrium all agents  $h > h_1$  will buy all of  $Y$ , and sell the down tranche  $A = (0, R)$ , hence effectively holding only the Arrow  $U$  security. Agents  $h_2 < h < h_1$  will sell all their endowment of  $Y$  and purchase all of the durable consumption good  $X$ . Finally, agents  $h < h_2$  will sell their assets  $Y$  and  $X$  and buy the down tranche from the most optimistic investors. The regime is showed in Figure 8.

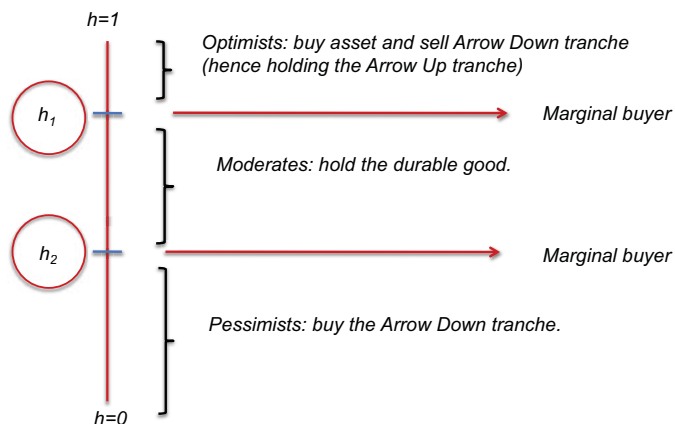


Figure 8: Asset Tranching Economy Equilibrium Regime.

The system of equations that characterizes equilibrium is the following. Let  $\pi_D$  denote the price of the down tranche. Equation (8) states that money spent on the asset should equal the aggregate revenue from its sale. The top  $1 - h_1$  agents are buying the asset and selling off the down tranche. They each have wealth  $1 + p$  plus the revenue from the tranche sale  $\pi_D$ . Finally, there is 1 unit of total supply of the asset. Hence we have

$$(1 - h_1)(1 + p) + \pi_D = p \tag{8}$$

Notice that the implicit price of the Arrow  $U$  security, which the top  $1 - h_1$  agents are effectively buying, equals  $p_U = p - \pi_D$ , the price of the asset minus the price of the down tranche  $A = (0, R)$ .

Equation (9) states that total money spent on the down tranche should equal aggregate revenues from their sale. The bottom  $h_2$  agents spend all their endowments to buy all the down tranches available in the economy (which is one since there is one asset), at the price of  $\pi_D$ .

$$h_2(1 + p) = \pi_D \tag{9}$$

Equation (10) states that  $h_1$  is indifferent between buying the Arrow  $U$  security and holding the durable consumption good. So his expected marginal utility from buying the Arrow  $U$  security, the probability  $q_U^{h_1}$  multiplied by the delivery of 1, divided by its price,  $p - \pi_D$ , equals the expected marginal utility of holding the durable consumption good divided by its price, 1.

$$\frac{q_U^{h_1}}{p - \pi_D} = 1 \tag{10}$$

Finally, equation (11) states that  $h_2$  is indifferent between holding the down tranche and the durable consumption good  $X$ . So his expected marginal utility from buying the down tranche, which is the probability  $1 - q_U^{h_2}$  multiplied by the payoff  $R$ , divided by its price,  $\pi_D$ , equals the expected marginal utility of holding the durable consumption good divided by its price, 1.

$$\frac{(1 - q_U^{h_2})R}{\pi_D} = 1 \tag{11}$$

We have a system of four equations and four unknowns: the price of the asset,  $p$ , the price of the down tranche  $\pi_D$ , and the two marginal buyers,  $h_1$  and  $h_2$ .

Finally, notice that despite the fact that both Arrow securities are present, markets are not complete. Arrow securities are *created through the asset*. Hence, agents cannot sell all the Arrow securities they desire and the Arrow-Debreu allocation cannot be implemented. Tranching the asset is not enough to complete markets.

For the probabilities  $q_U^h = 1 - (1 - h)^2$  and  $R = .2$ , we get  $h_1 = .58, h_2 = .08, p = 1$  and  $\pi_D = .17$ . The asset price is much higher even than it was with leverage. The simple reason is that leverage is an imperfect form of tranching. When the owner of the asset  $Y$  can create pieces even better suited to heterogenous buyers it makes the asset still more attractive.

One important conclusion to be drawn from combining equations (10) and (11) is that the tranching asset price is

$$p = q_U^{h_1} 1 + q_D^{h_2} R \quad (12)$$

Interestingly, the asset price can be higher than any agent in the economy thinks it is worth!. Defining the implicit Arrow security prices  $p_U = p - \pi_D = .83$  and  $p_D = \pi_D/R = .83$ , we see that  $p = p_U + p_D R > 1$ . We discuss how this could happen in Section 4.

A moment's reflection should convince the reader that in our two state economy, completely tranching  $Y$  is tantamount to allowing the asset to back a promise of  $R$  in the down state. The asset holder on net then retains the  $U$  Arrow security. By buying  $y$  units of  $Y$  and selling off  $y$  units of the tranche  $A = (0, R)$ , and also buying  $z/R$  units of the down tranche (perhaps created by somebody else), any agent who has enough wealth can effectively purchase the arbitrary consumption  $x_U = y, x_D = z$ . If it were possible to create different tranches beyond the two Arrow securities, no agent would have anything to gain by doing so. In the end his new tranches would not offer a potential buyer anything the buyer could not obtain for himself via the Arrow tranches, as we just saw. With unlimited and costless tranching, tranching into Arrow securities always drives out all alternative tranching schemes, a point made in Geanakoplos-Zame (2011). Since  $Y = \{(0, R)\}$  already embodies Arrow tranching, there is no reason to consider any more complicated tranching schemes.

### 3.4 CDS Economy

A CDS on the asset  $Y$  is a contract that promises to pay 0 at  $s = U$  when  $Y$  pays 1 and promises  $1 - R$  at  $s = D$  when  $Y$  pays only  $R$ . Figure 9 describes a comparison between the underlying asset payoffs and the CDS payoffs.

A CDS is thus an insurance policy for  $Y$ . A seller of a CDS must post collateral, typically in the form of money. In a two period model buyers of the insurance would

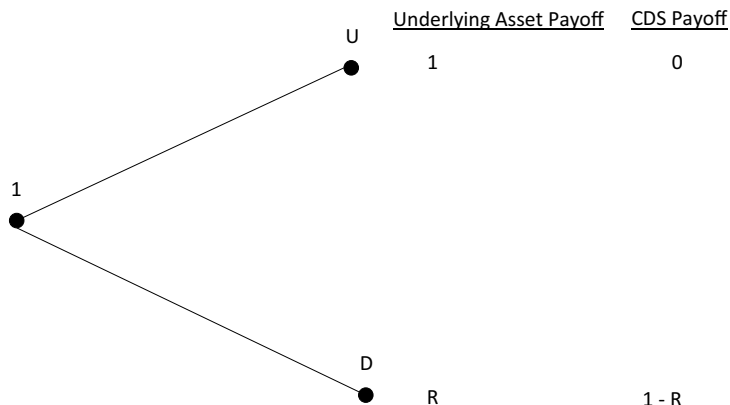


Figure 9: CDS Payoffs.

insist on  $1 - R$  of  $X$  as collateral. Thus for every one unit of payment, one unit of  $X$  must be posted as collateral. We can therefore incorporate CDS into our economy by taking  $J^X$  to consist of one contract called  $c$  promising  $(0, 1)$ .

We shall maintain our hypothesis that the asset  $Y$  itself can be tranching, so we continue to suppose that  $J^Y$  consists of the single promise  $(0, R)$  called the down tranche or  $D$ . Of course that is equivalent to supposing that  $(1 - R)/R$  units of the asset  $Y$  can be put up as collateral for 1 CDS promising  $1 - R$  in state D. In other words, the down tranche in the securitization of  $Y$  is identical to the CDS. Yet we shall show that the two have very different effects on the price of  $Y$ .

Equilibrium requires that buyers recognize that  $D$  and  $c$ , the CDS, are essentially proportional and hence in equilibrium their prices must be in the same proportion. Equation (13) states that

$$\pi_D = R\pi_c \tag{13}$$

Once equation (13) holds, it must also be the case that buyers recognize that



there are two equivalent ways of effectively buying the Arrow  $U$  security: *tranching the asset  $Y$*  and *tranching cash  $X$* . Hence we must have equation (14)

$$\frac{1}{p - \pi_D} = \frac{1}{1 - \pi_c} \quad (14)$$

Given these identities, it is evident that in equilibrium there will be a *marginal buyer  $h_1$*  such that all agents  $h > h_1$  will buy all of  $Y$  and  $X$  and sell the down tranche  $D$  backed by the  $Y$  and the CDS contract  $c$  collateralized with  $X$ . Agents  $h < h_1$  will sell all their endowment of  $Y$  and  $X$  and buy  $D$  and  $c$ . The regime is showed in Figure 10.

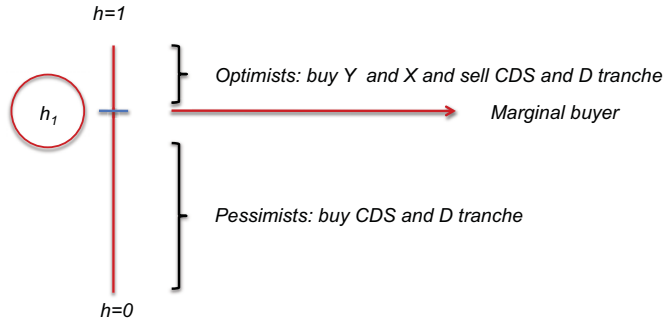


Figure 10: CDS Economy Equilibrium Regime.

Equation (15) states that the total money spent on  $Y$  and  $X$  has to equal the revenues from their sale. Agents  $1 - h_1$  buy both. They use their endowments as before but now they also receive income from the sales of  $D$  and  $c$ , using  $Y$  and  $X$  as collateral respectively. The total wealth represented by the endowments of  $X$  and  $Y$  is  $1 + p$  per person, and the total revenue from the sale of  $D$  and  $C$  is  $\pi_D + \pi_c$ .

This must equal the purchase cost of all the  $X$  and  $Y$  in the economy, which is also  $1 + p$ . Hence

$$(1 - h_1)(1 + p) + \pi_D + \pi_c = 1 + p \quad (15)$$

Equation (16) states that the marginal buyer should be indifferent between buying the Arrow  $U$  security (either way he can) and buying the down tranche or the CDS.

$$\frac{q_U^{h_1}}{p - \pi_D} = \frac{(1 - q_U^{h_1})}{\pi_c} \quad (16)$$

A succinct way of describing the difference between this economy and the previous one is that CDS allow for the tranching of cash in addition to the previous tranching of assets. As a consequence, with only two states of the world, CDS and tranching allow the economy to implement the Arrow-Debreu equilibrium. Hence, for the probabilities  $q_U^h = (1 - h)^2$  and  $R = .2$ , we get the same equilibrium as the one described in the Arrow-Debreu section. The price of the asset is given by  $p = p_U + Rp_D = .64$ .

Finally, a CDS can be “covered” or “naked” depending on whether the buyer of the CDS needs to hold the underlying asset. Our previous discussion corresponds to the case of “naked” CDS. When CDS must be “covered”, agents willing to buy CDS need to hold the asset. But notice that holding the asset and buying a CDS is equivalent to holding the risk-less bond, which was already available without CDS. “Covered” CDS have no effect on equilibrium. For the rest of the paper we will focus only on the “naked” CDS case.

## 4 Financial Innovation and Asset Pricing

We solve for equilibrium with probabilities  $q_U^h = 1 - (1 - h)^2$  in all the economies just described as  $R$  varies. Figure 11 displays the  $Y$  asset prices  $p$  for different values of  $R$ .<sup>5</sup>

For all economies the asset price increases as  $R$  increases and disagreement disappears. This is not surprising, since the asset clearly makes more payments the

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<sup>5</sup>Complete results are presented in Table 1 in Appendix C.

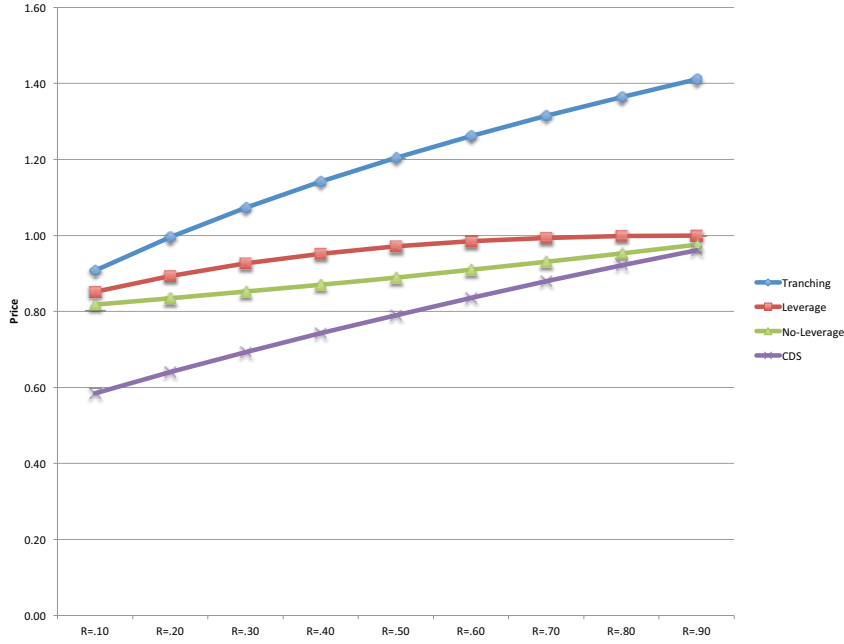


Figure 11: Asset Prices in all economies for different values of  $R$ .

higher is  $R$  and so naturally its price should increase. We come back to how far it should increase shortly.

By far the most important implication of our numerical simulations is that leverage and tranching make the asset price higher than it would be without leverage, and still higher than it would be in the CDS or Arrow-Debreu economy. Leverage and derivatives thus have a profound effect on asset prices. And therefore so does financial innovation. We now investigate how general these results are.

**Proposition 2:** *The asset price in the Leverage economy is higher than in the No-Leverage economy for all strictly monotonic and continuous  $q_U^h$ , and all  $0 < R < 1$ .*

**Proof:** From equation (4) we see that in the No-Leverage economy

$$\frac{1}{h_1^{NL}} = 1 + p^{NL}$$

while in the Leverage economy, from equation (6),

$$\frac{1}{h_1^L} + \frac{R}{h_1^L} = 1 + p^L$$

Assuming  $R > 0$ ,  $p^{NL} \geq p^L$  only if  $h_1^L > h_1^{NL}$ . But from equations (5) and (7) (which say that the asset price is equal to the marginal buyer's valuation) these last two inequalities are not compatible. **QED.**

As discussed before, the possibility of borrowing against the asset makes it possible for fewer investors to hold all the asset in the economy. Hence, the marginal buyer is someone more optimistic than in the No-Leverage economy, raising the price of the asset. This effect was first identified in Geanakoplos (1997, 2003). This connection between leverage and asset prices is precisely the Leverage Cycle theory discussed in Geanakoplos (2003, 2010a, 2010b) and Fostel-Geanakoplos (2008, 2010 and 2011). This theory can rationalize the housing market behavior during the crisis. Leverage on housing increased dramatically from 2000 to 2006 and housing prices increased dramatically during the same time. Leverage on housing collapsed in 2007 and the same happened to housing prices.

The boost in the price of  $Y$  from leverage is greatest for intermediate values of  $R$ , a region that can be characterized as “normal times” and when disagreement is not negligible. For too low values of  $R$ , agents can borrow very little against the asset and hence the marginal buyer will be someone very close to the marginal buyer when borrowing is not possible. On the other extreme, when  $R$  is very high, though borrowing is very important, agents almost agree on the outcome of the asset, pushing even the no-leverage price up near to 1. Next we turn to tranching.

**Proposition 3:** *The asset price in the Tranching economy is higher than in the No-Leverage economy for all strictly monotonic and continuous  $q_U^h$ , and for all  $0 < R < 1$ .*

**Proof:** From equation (8) in the Tranching economy, we see that

$$\frac{1}{h_1^T} + \frac{\pi_D^T}{h_1^T} = 1 + p^T$$

From equation (4) we see that in the No-Leverage economy

$$\frac{1}{h_1^{NL}} = 1 + p^{NL}$$

Then  $p^{NL} \geq p^T$  only if  $h_1^L > h_1^{NL}$ . But from equations (5) and (12) these last two inequalities are not compatible. **QED.**

In the numerical simulations, tranching raised the asset price even above the Leverage economy price. But this need not always be the case. Consider for example the beliefs given by

$$q_U^h = \max\{1 - (1 - h)^2, 1 - (1 - .60)^2\}$$

The Leverage economy equilibrium calculated earlier is still the same, since the marginal buyer was  $h_1^L = .63$ . But in the Tranching economy, the marginal buyer is  $h_1^T < .60$  and the price will therefore be  $.84(1) + .16(.2) = .872 < .89$ . In general,  $h_1^T < h_1^L$ . The tranching price becomes higher than the leverage price when  $q_U^{h_1^T} + q_D^{h_2^T} \gg 1$ , because then the tranching price  $p^T = q_U^{h_1^T} 1 + q_D^{h_2^T} R$  can be very large. In the example just given the probabilities are cooked up so that  $q_U^{h_1^T} = q_U^{h_2^T}$ .

All this suggests that the tranching price is higher than the leverage price when there is more heterogeneity at the bottom, among the pessimists, than there is at the top among the optimists. Indeed the following is true.

**Proposition 4:** *If the probabilities  $q_U^h$  are concave in  $h$ , as well as strictly monotonic and continuous, then the asset price in the Tranching economy is higher than in the Leverage economy for all  $0 < R < 1$ .*

**Proof:** From equations (7) and (12) and the fact that  $h_1^T > h_2^T$ , we have that

$$p^L = q_U^{h_1^L} 1 + q_D^{h_1^L} R = R + q_U^{h_1^L} (1 - R)$$

$$p^T = q_U^{h_1^T} 1 + q_D^{h_2^T} R > R + q_U^{h_1^T} (1 - R)$$

Assume temporarily that  $p^L \geq p^T$ . Then we must have  $q_U^{h_1^L} > q_U^{h_1^T}$ , so  $h_1^L > h_1^T$ , and hence  $(q_D^{h_1^T} - q_D^{h_1^L}) > 0$ . Putting the above two equations together,

$$p^T - p^L = -(q_U^{h_1^L} - q_U^{h_1^T}) 1 + [(q_D^{h_2^T} - q_D^{h_1^T}) + (q_D^{h_1^T} - q_D^{h_1^L})] R$$

To get our desired contradiction, it suffices to show that  $(q_U^{h_1^L} - q_U^{h_1^T}) < (q_D^{h_2^T} - q_D^{h_1^T}) R$ . From the hypothesized concavity of  $q^h$  in  $h$  and from  $q_D^{h_1^L} > q_D^{h_1^T} > q_D^{h_2^T}$ , it suffices to show that  $h_1^L - h_1^T < (h_1^T - h_2^T) R$ . From equations (6) and (8) we know that

$$(1 - h_1^L)(1 + p^L) + R = p^L$$

$$\frac{1+R}{1+p^L} = h_1^L$$

$$(1-h_1^T)(1+p^T) + \pi_D = p^T$$

$$\frac{1+\pi_D}{1+p^T} = h_1^T$$

If  $p^L \geq p^T$ , then

$$h_1^L - h_1^T \leq \frac{R - \pi_D}{1 + p^T} < \frac{R}{1 + p^T}$$

On the other hand, from equations (10) and (11) and Walras Law, we know that in the tranching equilibrium, the agents between  $h_1^T$  and  $h_2^T$  must hold all the X, hence

$$h_1^T - h_2^T = \frac{1}{1 + p^T}$$

From the last two equations and the concavity of  $q_U^h$ ,

$$q_U^{h_1^L} - q_U^{h_1^T} < R(q_U^{h_1^T} - q_U^{h_2^T}) = R(q_D^{h_2^T} - q_D^{h_1^T})$$

From the equation above and our earlier calculations, we conclude  $p^T - p^L > 0$ . **QED.**

The idea of the proof is that the tranching price tends to be lower than the leverage price because  $q_U^{h_1^L} > q_U^{h_1^T}$ , since  $h_1^L > h_1^T$ . But the tranching asset price rises because  $q_D^{h_2^T} > q_D^{h_1^T}$  since  $h_1^T > h_2^T$ . By concavity the gap between  $h_1^T$  and  $h_2^T$  has a bigger effect than the gap between  $h_1^L$  and  $h_1^T$ .

Our examples where  $q_U^h = 1 - (1 - h)^2$  or  $q_U^h = h$ , both satisfy the concavity hypothesis. A striking consequence of the power of securitization to raise the asset price can be seen as  $R$  increases in our example with  $q_U^h = 1 - (1 - h)^2$ . As the graph shows, the price of the asset with tranching goes even above 1. This seems puzzling since the durable good  $X$  delivers at least as much as the asset in every state and its price in equilibrium equals 1. With leverage, the price rose because the marginal buyer became a more optimistic agent, and the price came to reflect his beliefs instead of the more pessimistic marginal buyer that obtained without leverage. However, with leverage the asset price can never rise above 1, since no agent values the asset at more than 1. But with tranching, the price can rise above what any agent thinks it is worth. How can this be? The answer is that with tranching there are two marginal buyers instead of one. The marginal buyer with leverage was indifferent between the asset and the two cash flows into which leverage split it, and his beliefs determined the price of the asset. In the tranching equilibrium the cash flows into

which the asset is split are held by different people, and nobody who wants one flow would touch the other.

The tranching of the asset  $Y$  that was begun with leverage is perfected by tranching into Arrow securities. Leverage is a precursor or primitive form of tranching. With leverage the asset can be used as collateral to issue non-contingent promises. In the tranching economy, the asset can be used as collateral to issue contingent promises raising even further its value as collateral.  $Y$  becomes so valuable because it can be broken into pieces that are tailor made for different parts of the population. Splitting plain vanilla into strawberry for one group and chocolate for another raises the value of the scarce ice cream.

This result parallels the result in Harrison and Kreps (78), who defined a bubble as a situation in which an asset is priced higher than any agent thinks its cash flows are worth. But we obtain the result in a static context without resale of the asset. They displayed bubbles in a dynamic context with heterogeneous agents: the most optimistic agent would buy the asset, but next period instead of suffering bad cash flows he could resell the asset to a different agent. In our context, tranching alone, without resale, creates collateral value and potentially bubbles. The gap in prices between leveraged (or securitized) assets and unleveraged assets is what Fostel-Geanakoplos (2008) called *Collateral Value*. When assets can be used as collateral to borrow and not just as investment, there are deviations from classical forms of Law of One Price. An asset with identical or inferior payoffs in the future can be priced higher if it has higher collateral capacities.

This result is in tune with developments in the financial market. For more than five years securitized mortgages traded at negative OAS.<sup>6</sup> Securitization and tranching dramatically increased from the 1990s to 2006, along with leverage, explaining, in our view, much of the rise in housing and related securities prices.

Finally, we turn our attention to the CDS Economy. The numerical simulations show that CDS can dramatically lower the asset price, even below the no-leverage

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<sup>6</sup>Option Adjusted Spread (OAS) is the fudge factor Wall Street firms add to their pricing model when all their risk factors are unable to explain market prices. Typically this number is positive, suggesting that the market is willing to pay less than the model values because of the fear of some unknown risk the modelers may have missed. But for mortgages around 2000 this OAS number turned negative. We are suggesting here the reason is that FNMA and Freddie Mac were willing to pay more for the mortgages than their cash flows warranted because they could use them to create pools which would then be cut by Wall Street into more valuable tranches. An alternative explanation is that because of their implicit government guarantees, FNMA and Freddie Mac could borrow at cheaper rates than anybody else and hence were willing to overpay for their assets.

level. In the simulations, the lower the  $R$ , the bigger is the price reduction from the introduction of the CDS. For very high  $R$ , the introduction of CDS has little effect on asset prices when compared to the no-leverage level. We have the following proposition.

**Proposition 5:** *The asset price in the CDS economy is lower than in the Leverage economy and lower than in the Tranching economy, for all strictly monotonic and continuous  $q_U^h$  and for all  $0 < R < 1$ .*

**Proof:** Recall from equations (6), (8) that

$$\frac{1 + R}{1 + p^L} = h_1^L$$

$$\frac{1 + q_D^{h_1^T} R}{1 + p^T} = h_1^T$$

Next, recall that the CDS equilibrium is the same as the Arrow-Debreu equilibrium. Hence  $p^{CDS} = p^{AD} = p_U 1 + p_D R$ . From equation (3),  $p_D = q_D^{h_1^{CDS}}$ . From equation (2) and Walras Law,

$$\frac{q_D^{h_1^{CDS}} (1 + R)}{1 + p^{CDS}} = h_1^{CDS}$$

Suppose that  $p^{CDS} \geq p^T$ . Then the marginal buyer  $h_1^{CDS} > h_1^T$ , hence  $q_D^{h_1^{CDS}} < q_D^{h_1^T}$ . But this contradicts the last two equations above. Similarly, if  $p^{CDS} \geq p^L$  then the marginal buyer  $h_1^{CDS} > h_1^L$ , contradicting the first and third equations above. **QED.**

The stunning fact that the introduction of CDS dramatically lowers the asset price below the leverage price and below the tranching price seems counterintuitive at first glance. Tranching creates exactly the same derivative payouts as the CDS. They are perfect substitutes in every agent's mind. Yet when the CDS is created exclusively inside the securitization as a tranche of the asset, it raises the asset price. When the CDS is created outside the securitization it lowers the asset price.

On second thought this is not surprising at all. CDS are a way of tranching  $X$ . When agents sell CDS and put up cash as collateral, they are effectively tranching cash! That raises the value of cash relative to other assets, lowering the others' prices.



However, the CDS price need not be lower than the no-leverage price. Indeed if agents think the bad state is very likely, then in the limit, by equation (4) and Walras Law, we would find that  $h_1^{NL} \rightarrow 1/(1+R)$ , while by equation (2) and Walras Law,  $h_1^{CDS} \rightarrow 1$ . But if the median agent regards the good state as more likely than the bad state, then the CDS price must be lower than the no-leverage price, as we show in the next proposition.

**Proposition 6:** *If the  $q_U^h$  are strictly monotonic and continuous in  $h$  and if  $q_U^{1/2} \geq 1/2$ , then the asset price in the CDS economy is lower than in the No-Leverage economy for all  $0 < R < 1$ .*

**Proof:** Recall that in the No-Leverage economy, the top  $1 - h_1^{NL}$  agents hold all the asset and the bottom  $h_1^{NL}$  agents hold all the durable good  $X$ , which is more valuable. Hence  $h_1^{NL} > 1/2$ . In the CDS economy,

$$h_1^{CDS} = \frac{(1+R)q_D^{h_1^{CDS}}}{2q_U^{h_1^{CDS}} + (1+R)q_D^{h_1^{CDS}}}$$

If  $h_1^{CDS} \geq 1/2$ , then by hypothesis  $q_U^{h_1^{CDS}} \geq 1/2$ . But that contradicts the above equation for  $R < 1$ . Hence  $h_1^{CDS} < 1/2 < h_1^{NL}$ , and therefore  $p^{NL} > p^{CDS}$ . **QED.**

Putting the last five propositions together, we get

**Theorem:** *If the  $q_U^h$  are strictly monotonic, concave, and continuous in  $h$  and if  $q_U^{1/2} \geq 1/2$ , then the Tranching asset price is greater than the Leverage asset price which is greater than the No-Leverage asset price which is greater than the CDS asset price, for all  $0 < R < 1$ .*

In Appendix A we give the equations for two more economies, which allow us to see the depressing effect of CDS on the asset price even more clearly. First we consider what happens if cash  $X$  can be tranching by CDS, but  $Y$  cannot be tranching or held as collateral for any kind of loan or derivative. Now the tranching boost to price goes the other way, and the price of  $Y$  relative to  $X$  plummets still further.

Finally, we consider the situation which obtains today for many assets like sovereign bonds. The underlying bond-asset is not tranching, but people can leverage their purchases of it. On the other hand they can use cash as collateral to write CDS on the asset-bond. As we see in Appendix A in Figure 16, the  $Y$  asset price is below the complete markets, tranching-CDS price, but above the price where we have CDS backed

by cash alone. The reason is that the asset  $Y$  can only be imperfectly tranced via leverage, whereas the cash is perfectly tranced.

## 5 Dynamic Asset Prices: Bubbles and Crashes.

As we said at the outset, securitization emerged over a period of 20 years, and then the CDS mortgage market suddenly exploded at the end of the securitization boom. To model the implications of that dynamic we need to examine a multi-period model. Needless to say, as long as nobody gets utility from holding collateral, leverage, tranching, and CDS are irrelevant without uncertainty. Hence the dynamic model must incorporate risk at each stage. To keep the model tractable (and thus to avoid exponential growth in the number of states) we suppose that in every period with high probability a tiny bit of bad news is received, which leaves the world a little worse but much like it was, or with low probability all the uncertainty is resolved and the good outcome obtains for sure from there on out. The most likely and most interesting history occurs along the path of consecutive pieces of bad news. This single history contains *all* the nodes at which the asset price is non-trivial. As we shall see, this model is tractable, it assumes volatility increases with bad news (and decreases with good news), and it makes leverage pro-cyclical.<sup>7</sup>

The risky asset payoffs are described in Figure 12, as are the probabilities of the agents. At each point in time, there is either good news (with low probability  $1 - (1 - h)^{2/N}$ ) or bad news (with high probability  $(1 - h)^{2/N}$ ). After good news, uncertainty is completely resolved and the risky asset  $Y$  pays 1. However, after bad news, the economy proceeds to the next period. After  $N$  consecutive periods of bad news output materializes at  $R < 1$ . Notice that each agent believes that the final output of  $Y$  will be  $R$  with probability  $(1 - h)^2$  and 1 with probability  $1 - (1 - h)^2$ , exactly as we had in the two period model of Section 2. Each agent in the continuum  $h \in (0, 1)$  begins at time 0 with one unit of  $Y$  and one unit of  $X$ , and has no further endowments. As before,  $X$  produces 1 unit of the consumption good in every terminal state. We suppose that agents care only about expected consumption in the terminal states.

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<sup>7</sup>Fostel-Geanakoplos (2010) show that this stochastic structure is not something crazy or ad-hoc. They show that investors, given the opportunity to choose between technologies that exhibit positive or negative correlations between first and second moments, they would mostly choose the latter.

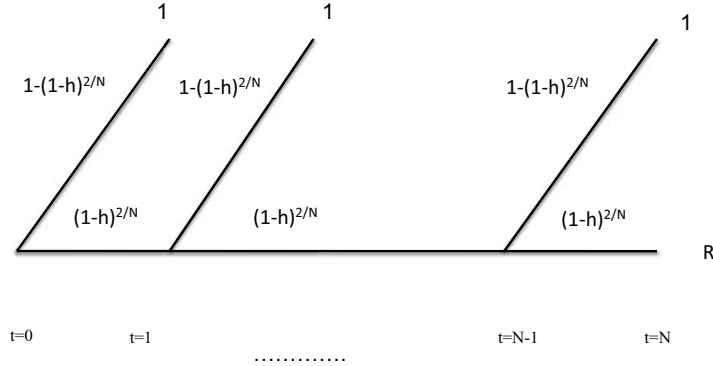


Figure 12: Dynamic Asset Payoffs.

Notice that as bad news comes, future output volatility increases. In other words, bad news is revealed very slowly and each piece of bad news comes attached with a spike in volatility. This type of technology, in which first moments and second moments of the asset distribution are negatively correlated, was introduced in Geanakoplos (2003) in a three-period tree, and studied extensively by Fostel-Geanakoplos (2010). This kind of economy induces pro-cyclical leverage, meaning that each piece of bad news also decreases leverage. One potential drawback (or advantage) to the model is that it presumes that if at the beginning one agent is more optimistic than another, then he remains more optimistic throughout.<sup>8</sup>

The No-Leverage, Leverage, Tranching, and CDS economies are defined by their contract structure, as in the static model. In each node  $s$  of the tree we are given a set of one period contracts  $J^X(s)$  using one unit of  $X$  as collateral and another set of one period contracts  $J^Y(s)$  using one unit of  $Y$  as collateral. For the No-Leverage economy, these sets are empty. For the Leverage economy,  $J^X(s)$  is empty and  $J^Y(s)$  consists of all promises  $(j, j)$  in the following two successor states. For

<sup>8</sup>This may be the reason why in our model agents would buy CDS contracts at the beginning if they were available, rather than waiting until default becomes more likely.

the Tranching economy  $J^X(s)$  is empty and  $J^Y(s)$  consists of all possible promises in the two successor states. In the CDS economy we would like to define the CDS as a contract promising  $1 - R$  in the terminal state following all bad news. But then the question is: how much  $X$  collateral will the market require at each node  $s$ ? The answer is just enough to cover the market value of the CDS at node  $sD$ , the successor node of  $s$  after bad news. With that insight we can reproduce the CDS equilibrium by assuming that  $J^X(s)$  consists of the one period promise  $(0, 1)$  for all  $s$ , and that  $J^Y(s)$  consists of all possible promises in the two successor states.

Given the contract structure, equilibrium is defined by analogy to what we did in Section 2 for the two period model in which  $N = 1$ . However, for longer horizons agent optimization is much more complicated because agents must be forward looking, anticipating what the price of  $Y$  will be in the successor states and taking into account that they might not want to buy the risky asset  $Y$  today, even if its expected payoffs exceed its price, because they might do better by waiting to buy it next period when its price might be cheaper. All the equations that characterize the equilibrium for each economy are presented in Appendix B.

Figure 13 shows the asset price evolution for the different economies for  $R = .2$  and  $N = 10$ .<sup>9</sup> Note first that at time 0 the leverage price and the tranching price are much higher than they were at time 0 in the two-period economy. Multiple periods increase the power of leverage and tranching to raise asset prices. The no-leverage price at time 0 is exactly the same in the 10-period economy as in the two-period economy.

As before, the tranching price starts out higher than all the others, followed by the leverage price, then the non-leverage price, then the very low CDS price. As time goes and bad news keeps occurring, the prices all fall (there is no point in presenting the price after good news, since that is always 1). Since all the price lines end up at nearly the same price near .2, the fall is inversely related to the starting point. In the Leverage economy prices start higher and hence the the price crash is much bigger than in the No-Leverage and CDS economies. Since the no-leverage marginal buyer never changes (he has no reason to sell after bad news), the price always reflects his opinion of expected output. Thus, in the Leverage economy the bigger price decline must be from feedback effects through deleveraging and through wealth loss among the optimists. It is interesting that at the very beginning the leverage price is more

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<sup>9</sup>Complete results are presented in Table 2 in Appendix C.

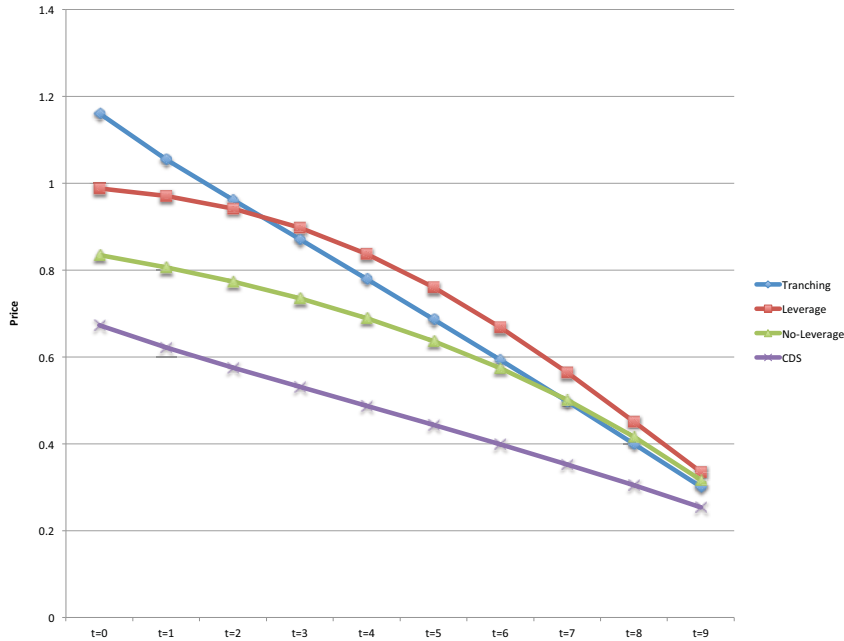


Figure 13: Asset Price Dynamics for  $N = 10$ .

stable than the non-leverage price, but picks up steam fast as more bad news comes in. However, the really dramatic fall in price on just a tiny bit of bad news happens in the Tranching economy.

By contrast the fall is much more modest in the CDS economy. Early securitization and tranching created a huge increase in asset prices. Had CDS been available from the very beginning this over-valuation would not have happened.

However, as discussed in the introduction, CDS were actually introduced later. Figure 14 shows the effect on asset prices if CDS were introduced to a Tranching economy in period  $t = 2$ . We distinguish two different cases: a sudden, un-expected introduction in which the economy is proceeding along as if CDS will never exist, and an expected introduction in which from the very beginning it is known that CDS will appear in period  $t = 2$ .

Without CDS, there would be a 17% drop in prices in the Tranching economy after two pieces of bad news, as we saw in Figure 13. This drop becomes much bigger if CDS appear in period  $t = 2$ , namely 28%, even if the appearance of CDS at time

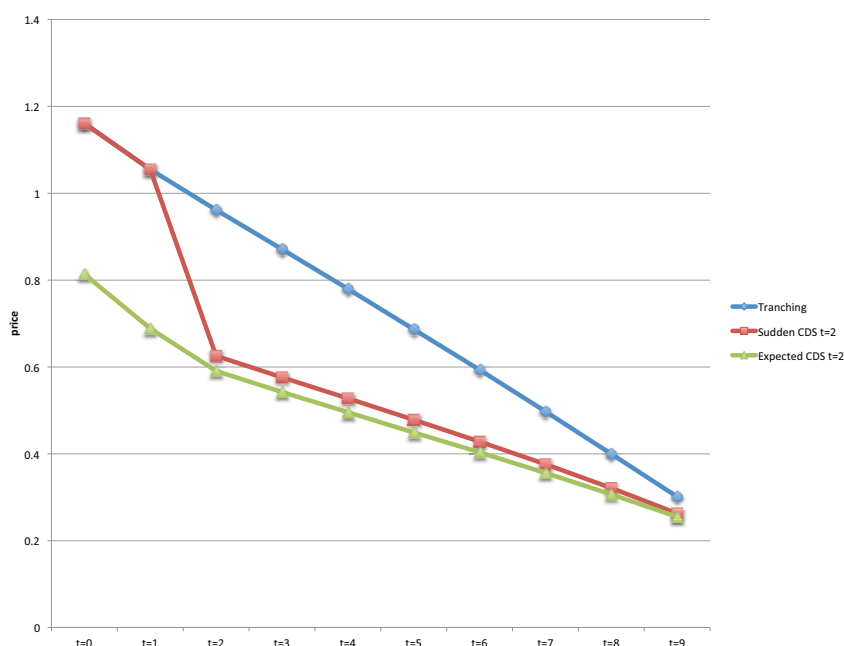


Figure 14: Late introduction of CDS at  $t = 2$ .

$t = 2$  is anticipated from the very beginning at time  $t = 0$ . The crash becomes a horrific 46% if the CDS appear in period  $t = 2$  as a surprise.

In our view the key to the size of the crisis is the order of the financial innovation that materialized. Securitization, with all the tranching of CDOs and leverage created a bubble and the introduction of CDS burst it, pushing pricing faster and further down than they would have gone had there never been tranching or leverage or CDS. Had CDS been there from the beginning, asset prices would never have gotten so high.

The most dramatic drop in price occurs with the historical timing. In Figure 15 we see the evolution of the risky asset price assuming that the economy that starts at  $t = 0$  with no leverage. At  $t = 1$  leverage is unexpectedly introduced and price goes up, even though some bad news arrives. In  $t = 2$  the risky asset tranching technology is unexpectedly introduced and the price climbs even further, despite more bad news. Finally, at  $t = 3$  CDS are unexpectedly introduced and with it the price crashes. Just one piece of bad news, on top of the introduction of CDS, reduces the price by nearly 50%. In our dynamic model we only keep track of price after bad

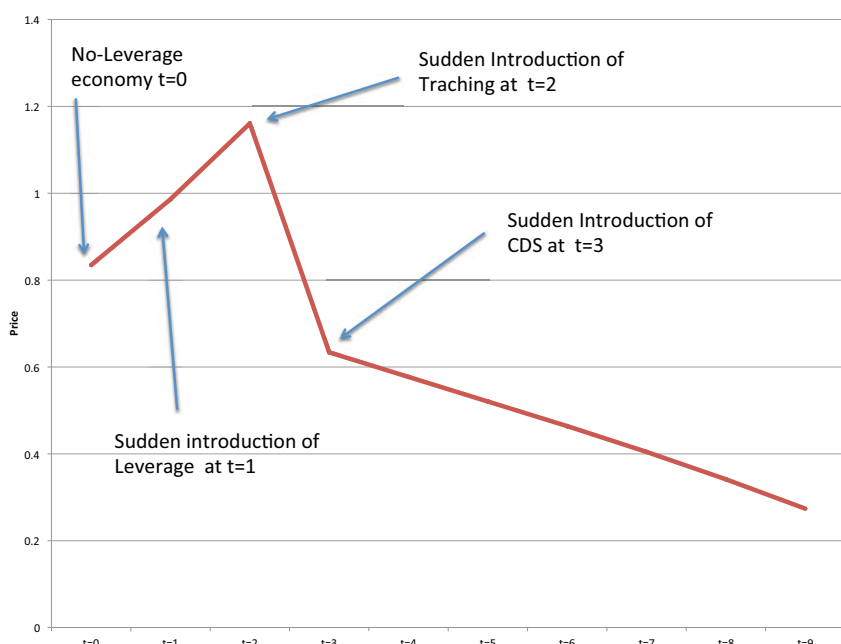


Figure 15: Anatomy of the Bubble and Crash.

news, yet the introduction of leverage and tranching is still strong enough to offset the bad news and raise prices. Obviously in reality, the leverage and tranching part of the cycle occurred mostly during pieces of “good news,” which made the bubble even more violent.

It would be very interesting to endogenize the order of financial innovation. We conjecture that with a transactions cost, CDS would not be traded at the beginning because there is a very small probability of paying off. As the likelihood increases and more disagreement is created, more people will want to trade them. This common sense thought does not really obtain in our model, as far as we can see, because in the model agents who are going to be the most pessimistic at the end are already the most pessimistic at the beginning. They would buy their CDS from the start if there were no transactions costs. Indeed CDS volume would decline, not increase over time. Clearly this merits more investigation.

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# A Two more CDS Economies

## A.1 CDS without Tranching

For many assets, like sovereign bonds nowadays, direct tranching is not available but CDS are written. As we said earlier, if the asset itself can be used as collateral by the writer of CDS insurance, then indeed the assets are indirectly tranced, and the analysis of the last section applies. There is good reason why the asset might be used as collateral for its own insurance (puzzling as that sounds), because the optimists most likely to own the bonds will be the same people writing the insurance. But in practice cash seems to be the collateral of choice, so for completeness we analyze also the case where only cash can be used as collateral for the CDS. We also do not permit leverage of the assets. In the next section we do.

To formalize our assumption that there is no tranching and that CDS must be collateralized with cash, we take  $J = J^X$  to consist of just the CDS  $c$  that promises  $(0, 1)$ . In equilibrium there will be two marginal buyers  $h_1 > h_2$ . Agents  $h > h_1$  will hold all of the “cash”  $X$  and use it all as collateral to write CDS  $c$  on  $Y$ , hence effectively holding only the Arrow  $U$  security. Agents  $h_2 < h < h_1$  will sell all their endowment of  $X$  and purchase all of the asset  $Y$ . Finally, agents  $h < h_2$  will sell their assets  $Y$  and  $X$  and buy the CDS from the most optimistic investors.

Equation (17) says that the top group of agents indeed holds all of  $X$ , and writes the maximal number of CDS.

$$(1 - h_1)(1 + p) + \pi_c = 1 \tag{17}$$

Note that these agents are effectively buying the Arrow Up security at an implicit price of  $p_U = 1 - \pi_c$ . Equation (18) says that the bottom group of agents holds all the CDS

$$h_2(1 + p) = \pi_c \tag{18}$$

Equation (19) says that  $h_1$  is indifferent between writing CDS backed by  $X$  (and thus synthetically holding the Arrow Up security) and holding the asset

$$\frac{q_U^{h_1}}{1 - \pi_c} = \frac{q_U^{h_1} + (1 - q_U^{h_1})R}{p} \tag{19}$$

Equation (20) says that  $h_2$  is indifferent between holding the asset and holding the CDS  $c$

$$\frac{q_U^{h_2} + (1 - q_U^{h_2})R}{p} = \frac{1 - q_U^{h_2}}{\pi_c} \quad (20)$$

For the probabilities  $q_U^h = (1 - h)^2$  and  $R = .2$ , we get  $h_1 = .65, h_2 = .29, p = .55$  and  $\pi_c = .36$ . The implicit arrow prices are given by  $p_U = .63$  and  $p_D = .45$ .

## A.2 CDS with Leverage

Finally we add the possibility of leverage for  $Y$ , maintaining the hypothesis that  $Y$  cannot be tranced and that only  $X$  can be used as collateral for CDS. To formalize this assumption, we take  $J^X$  to consist of just the CDS  $c$  that promises  $(0, 1)$ , and we take  $J^Y$  to be all promises of the form  $(j, j)$  for any  $j > 0$ . As we saw before only the promise  $(j^*, j^*) = (R, R)$  will be traded in equilibrium. We denote its price by  $\pi_{j^*}$ .

In equilibrium there will as usual be two marginal buyers  $h_1 > h_2$ . Agents  $h > h_1$  will hold all of  $Y$ , purchased entirely via leverage, and all of the cash  $X$ , using it all as collateral to write CDS  $c$  on  $Y$ , hence effectively holding only the Arrow  $U$  security. Agents  $h_2 < h < h_1$  will sell all their endowment of  $X$  and  $Y$  and lend it to the optimists, protected by the collateral of  $Y$ . Finally, agents  $h < h_2$  will sell their assets  $Y$  and  $X$  and buy the CDS from the most optimistic investors.

Equation (21) says that the top group of agents indeed holds all of  $Y$  via leverage and also all of  $X$ , and writes the maximal number of CDS.

$$(1 - h_1)(1 + p) + \pi_c + \pi_{j^*} = 1 + p \quad (21)$$

Note that these agents are effectively buying the Arrow Up security at an implicit price of  $p_U = 1 - \pi_c$ . But they are equally buying all the Arrow Up security via their leveraged purchase of  $Y$ . Hence Equation (22) requires that

$$\frac{1 - R}{p - \pi_{j^*}} = \frac{1}{1 - \pi_c} \quad (22)$$

Equation (23) says that the bottom group of agents holds all the CDS

$$h_2(1 + p) = \pi_c \quad (23)$$

Equation (24) says that  $h_1$  is indifferent between writing CDS backed by  $X$  (and thus synthetically holding the Arrow Up security) and lending

$$\frac{q_U^{h_1}}{1 - \pi_c} = \frac{R}{\pi_{j^*}} \quad (24)$$

Equation (25) says that  $h_2$  is indifferent between lending and holding the CDS  $c$

$$\frac{R}{\pi_{j^*}} = \frac{1 - q_U^{h_2}}{\pi_c} \quad (25)$$

For the probabilities  $q_U^h = (1 - h)^2$  and  $R = .2$ , we get  $h_1 = .38, h_2 = .28, p = .61, \pi_c = .45$  and  $\pi_{j^*} = .17$ . The implicit arrow prices are given by  $p_U = .54$  and  $p_D = .56$ .

Figure 16 presents asset prices values for different values of  $R$  as we did in the main text but for the six different cases.

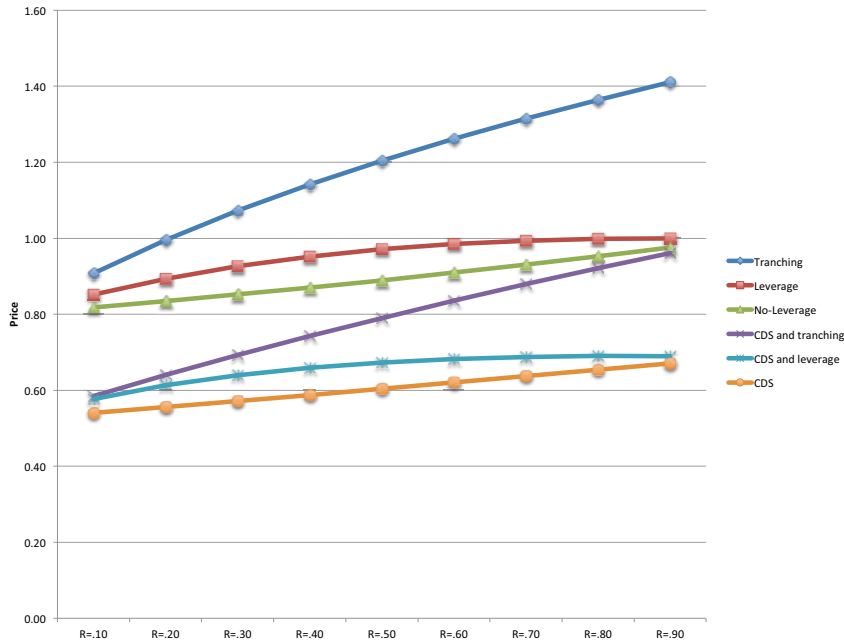


Figure 16: Asset Prices in all economies for different values of  $R$ .

## B Equilibrium Equations for different Economies in Section 5

### 1. CDS and Tranching Economy.

As noted before, this equilibrium corresponds to the Arrow-Debreu Equilibrium. The equations are the traditional equations in which we solve for arrow-prices for  $N + 1$  final states.

### 2. No-Leverage Economy.

We use the fact that the marginal buyer rolls over his debt at every node to build up the system and then verify that the guess is correct. Notice that the probability of good news in period  $k$  is given by  $(1 - (1 - h_k)^2)^{1/N}$ .

$$p_1 = ((1 - (1 - h_k)^2)^{1/N})^N + (1 - ((1 - (1 - h_k)^2)^{1/N})^N)R$$

$$p_1 = \frac{(1 - h_1) + R}{h_1}$$

⋮

$$p_k = ((1 - (1 - h_k)^2)^{1/N})^{N-k} + (1 - ((1 - (1 - h_k)^2)^{1/N})^{N-k})R$$

⋮

$$p_{N-1} = ((1 - (1 - h_{N-1})^2)^{1/N})^{N-(N-1)} + (1 - ((1 - (1 - h_{N-1})^2)^{1/N})^{N-(N-1)})R$$

### 3. Leverage Economy.

Notice that since the final probability of disaster is constant (regardless of  $N$ ), the probability of bad news in period  $k$  is given by  $(1 - h_k)^{2/k}$ . What is crucial is that the marginal buyers are decreasing in time. There is a perpetual wealth redistribution towards more pessimistic investors in each period after bad news since previous leveraged investors go bankrupt.

$$p_{N+1} = R$$

$$p_N = (1 - (1 - h_N)^{2/N}) + (1 - h_N)^{2/N} R$$

$$h_{N-1} = \frac{h_N(1 + p_N)}{1 + p_{N+1}}$$

$$p_{N-1} = \frac{(1 - (1 - h_{N-1})^{2/N}) + (1 - h_{N-1})^{2/N} \frac{(1 - (1 - h_{N-1})^{2/N})}{(1 - (1 - h_N)^{2/N})} p_N}{(1 - (1 - h_{N-1})^{2/N}) + (1 - h_{N-1})^{2/N} \frac{(1 - (1 - h_{N-1})^{2/N})}{(1 - (1 - h_N)^{2/N})}}$$

$$h_{N-2} = \frac{h_{N-1}(1 + p_{N-1})}{1 + p_N}$$

⋮

$$p_1 = \frac{(1 - (1 - h_1)^{2/N}) + (1 - h_1)^{2/N} \frac{(1 - (1 - h_1)^{2/N})}{(1 - (1 - h_2)^{2/N})} p_2}{(1 - (1 - h_1)^{2/N}) + (1 - h_1)^{2/N} \frac{(1 - (1 - h_1)^{2/N})}{(1 - (1 - h_2)^{2/N})}}$$

$$h_0 = \frac{h_1(1 + p_1)}{1 + p_2} = 1$$

#### 4. Tranching Economy

The probabilities are as before. The equilibrium in this economy is more challenging due to the multiplicity of marginal buyers in each period. It turns out that both marginal buyers are decreasing with time. Buyers of Arrow U security last only for one period. The second marginal buyer wealth evolution turns out to be more complicated. They buy gold in the future, but eventually, as time goes by, they buy Arrow U. In our example for  $N=10$ ,  $h_2^0$  buys gold from  $t = 1$  to  $t = 8$ , and buys Arrow U in the last trading period. Define  $q_U^h = (1 - (1 - h)^{2/N})$  and  $q_D^h = 1 - q_U^h$ .

At  $t = 0$

$$p_0 - p_1 p_{D1} = (1 - h_0^1)(1 + p_0)$$

$$h_0^2(1 + p_0) = p_1 p_{D1}$$

$$\frac{q_U^{h_0^1}}{p_0 - p_1 p_{D1}} = q_U^{h_0^1} + q_D^{h_0^1} \frac{q_U^{h_0^1}}{p_1 - p_2 p_{D2}}$$

$$\frac{q_D^{h_0^2}}{p_{D1}} C_0 = q_U^{h_0^2} + q_D^{h_0^2} C_0$$

where  $p_{D1}$  is the price of the Arrow down (which pays in period 1 after bad news) and  $C_0$  is the continuation value of  $h_0^2$  given his future investment

For  $t = k$  for  $k = 2, \dots, N - 2$ .

$$p_k - p_{k+1} p_{Dk+1} = \frac{h_{k-1}^1 - h_k^1}{h_{k-1}^1 - h_{k-1}^2}$$

$$\frac{h_k^2}{h_{k-1}^2} p_k = p_{k+1} p_{Dk+1}$$

$$\frac{q_U^{h_k^1}}{p_k - p_{k+1} p_{Dk+1}} = q_U^{h_k^1} + q_D^{h_k^1} \frac{q_U^{h_k^1}}{p_{k+1} - p_{k+2} p_{Dk+2}}$$

$$\frac{q_D^{h_k^2}}{p_{Dk+1}} C_k = q_U^{h_k^2} + q_D^{h_k^2} C_k$$

At  $t = N - 1$

$$p_{N-1} - p_N p_{DN} = \frac{h_{N-2}^1 - h_{N-1}^1}{h_{N-2}^1 - h_{N-2}^2}$$

$$\frac{h_{N-1}^2}{h_{N-2}^2} p_{N-1} = p_N p_{DN}$$

$$\frac{q_U^{h_{N-1}^1}}{p_{N-1} - p_N p_{DN}} = 1$$

$$\frac{q_D^{h_{N-1}^2}}{p_{DN}} = 1$$

## C Equilibrium Values in Section 4 and 5

Table 1 and 2 show the complete equilibrium information from Section 4 and 5 respectively.

**Table 1: Asset Prices in the Static Model**

	R=.10	R=.20	R=.30	R=.40	R=.50	R=.60	R=.70	R=.80	R=.90
<b>No-Lev</b>	0.82	0.83	0.85	0.87	0.89	0.91	0.93	0.95	0.98
<b>Lev</b>	0.85	0.89	0.93	0.95	0.97	0.99	0.99	1.00	1.00
<b>Tran</b>	0.91	1.00	1.07	1.14	1.20	1.26	1.31	1.36	1.41
<b>CDS</b>	0.58	0.64	0.69	0.74	0.79	0.84	0.88	0.92	0.96

**Table 2: Asset Prices in the Dynamic Model**

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
<b>No-Lev</b>	0.83	0.81	0.77	0.73	0.69	0.64	0.57	0.50	0.42	0.32
<b>Lev</b>	0.99	0.97	0.94	0.90	0.84	0.76	0.67	0.56	0.45	0.34
<b>Tran</b>	1.16	1.05	0.96	0.87	0.78	0.69	0.59	0.50	0.40	0.30
<b>CDS</b>	0.67	0.62	0.58	0.53	0.49	0.44	0.40	0.35	0.30	0.25
<b>CDS and Crashes</b>										
<b>Tran</b>	1.16	1.05	0.96	0.87	0.78	0.69	0.59	0.50	0.40	0.30
<b>Sud CDS t=2</b>	1.16	1.05	0.63	0.58	0.53	0.48	0.43	0.38	0.32	0.26
<b>Exp CDS t=2</b>	0.81	0.69	0.59	0.54	0.50	0.45	0.40	0.36	0.31	0.25
<b>Anatomy of a Bubble and Crash</b>										
	<b>No-Lev</b>	<b>Lev</b>	<b>Tran</b>	<b>CDS</b>						
<b>Price</b>	0.83	0.99	1.16	0.63	0.58	0.52	0.46	0.40	0.34	0.27