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THE PRESENT AND FUTURE OF GAME THEORY

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Abstract

A broad nontechnical coverage of many of the developments in game theory since the 1950s is given together with some comments on important open problems and where some of the developments may take place. The nearly 90 references given serve only as a minimal guide to the many thousands of books and articles that have been written. The purpose here is to present a broad brush picture of the many areas of study and application that have come into being. The use of deep techniques flourishes best when it stays in touch with application. There is a vital symbiotic relationship between good theory and practice. The breakneck speed of development of game theory calls for an appreciation of both the many realities of conflict, coordination and cooperation and the abstract investigation of all of them.

Keywords: Game theory, Application and theory, Social sciences, Law, Experimental gaming, conflict, Coordination and cooperation

JEL Classification: C7, C9

1. INTRODUCTION

The purpose of this article is to present an overview of the theory of games.

Game theory has been a victim of its own successes. It is now firmly entrenched as a method of analysis as an important tool in economics, political science, law, social psychology and other disciplines. This overview attempts to indicate the probable directions of development and to suggest where some of the challenges lie ahead.

A broad sketch of many of the current areas of specialization is given with no attempt at an in-depth discourse on the proliferation of subspecializations.

The stress is on what we may want and expect for the future, keeping in mind the distinction between desire and feasibility

2. SUBDISCIPLINES

There are many subdisciplines in game theory. The boundaries among them are not firm, but fuzzy and there is a considerable mix involving, substantive areas such as law or economics. **Conversational**, as well as **formal game theory**, still abounds and this is not merely indicative of preformal investigation prior to formal modeling; but even more importantly it shows that in everyday business and politics the language of game theory is influencing practitioners and the public in general about the concepts of strategic analysis.

The first listing below touches on areas where formal models already exist, and special results have been obtained. The second listing below indicates substantive topics where game theoretic applications have been utilized. In each instance one or a few references are noted as early papers or exemplars of work in that specialty.

2.1. High Abstraction, with Some Fairly Clearly Formalized Contexts

1. Cooperative solution theory [1, 2, 3, 4]
2. Bargaining theory [5, 6, 7, 8]
3. Noncooperative theory [9, 10]
4. Assignment problems [11, 12, 13]
5. Preference theory and its relationship with game theory [14, 15]
6. Game theory as pure mathematics [16, 17]
7. Voting theory [3, 18, 19, 20]
8. Incomplete knowledge of rules of the game [21, 22]
9. Dynamic games [23, 24]
10. Asymmetric information and Agency theory [25, 26, 27]
11. Mechanism design [28, 29, 30]
12. Theory of incomplete contracts [31,32, 77]
13. Evolutionary game theory [34, 35]
14. Network games [38]
15. Agents with limited abilities [35 71]

16. Game theory and computer science [36, 38, 55, 56]
17. Finance [39]

Although the distinction is not tight all of the topics noted above, have and can be, for the most part studied with out a heavy investment in empirical material. Every one of them can claim at least a loose descriptive connection to some set of empirical problems, however the development of the mathematical structures and the stress on theorem proving are not necessarily in close coordination with applied and institutionally rich problems.

The topics noted below, in contrast to most of the above have a higher institutional content. A safe rule of thumb in the behavioral sciences is that anything involving dynamics requires a description of the carrier of the process and that carrier include the environment of an ecology, the institutions of a society, or the organisms of a biological structure.

2.2. Context-rich Studies

A broad sweep of the context rich studies include:

1. Oligopoly theory [40, 41, 42]
2. New Industrial Organization [42]
3. Experimental gaming [44, 45, 46, 47]
4. Operational gaming and mechanism testing [48]
5. Political science [49, 50, 51, 52, 53]
6. Evolutionary game theory and biology [54]
7. Macroeconomics [57]
8. Theory of money and financial institutions [58, 59, 60, 61]
9. Operations research: duels, military O.R. [62, 63, 64]
10. Law [65, 66, 67, 68]
11. Psychology [44]
12. Anthropology and Sociology [69]
13. Theology [70]

The references above, given the range of topics are clearly eclectic and sparse. For example, there are many thousands of articles on the Prisoners' Dilemma alone. Furthermore, there are many experiments than can equally well be classified as publications in psychology, as well as social psychology. The classification of the literature on sociology and anthropology is difficult. It is easy to find lip service paid to the ideas of game theory, but difficult to find articles where one could say that game theoretic analysis was central to the sociological or anthropological analysis.

The next two sections are devoted to a few comments on a selection of the topics noted.

3. PRIMITIVE CONCEPTS

Before formal theory can be developed for any of the topics noted a few comments are made on the nature of some of the basic assumptions that must be implicitly or explicitly taken as given. They are, for most part

Assumptions concerning:

AGENTS

a. What constitutes a player: individual, formal organization, etc.?
b. Individual rationality
c. Preference structure
d. Ability to perceive
e. Ability to compute
f. Any aspects of personality?

Assumptions concerning:

RULES OF THE GAME AND CONTEXT

a. Are there formal or informal rules?
b. Are they fixed or do they evolve?
c. Is there un-modeled context outside of the game that matters?
d. Does language matter?

Assumptions concerning:

INDIVIDUAL OR GROUP BEHAVIOR

a. Are the players hardwired automata?

b. Are the players anonymous individuals playing an individual game against a Leviathan or mass societal phenomenon?

c. Do the players form coalitional groups with direct interpersonal bargaining and communication?

Clearing away this modeling underbrush, the modelers utilizing the many facets of game theory usually break their investigations into three broad classes.[1]

The three classes are game theory models and solutions where:

1. **The extensive form** or some variant is taken as given.
2. **The strategic form** or some variant is taken as given.
3. **The coalitional form** or some variant is taken as given.

Commenting broadly:

The extensive form is used for problems involving explicit concern with dynamics and process. At the most precise and constrained it is used to formulate mathematically highly challenging problems such as two person constant-sum multi-stage games with non-symmetric information. At the other extreme, in one form or another, it is used for "soft game theory" dominated with general intuitive discussion about multistage games where the order of moves may not be known as in "soft gaming" such as politico-military exercises (PME) or operational war games.

Soft extensive form is the domain of verbal applications to diplomacy and even theology. These "soft games" provide a rich location where application, operational planning and theory meet. This is where the practical men and women of the world, such as top bureaucrats, industrialists, generals, diplomats, and politicians can challenge each other and the theorists in exercises aimed at challenging both practice and theory.

[1] These variants often split into many sub variants as notational considerations emphasizing one or another feature of the specific problem at hand call for modification of the usual extensive, strategic or coalitional forms.

The strategic form is most heavily used in elementary exposition of game theory and in many economic applications. “A zero sum-game” is now a term in common speech. In spite of the challenge by Harsanyi and Selten to explain all cooperative games within a structure of noncooperative theory this program has not succeeded and it is my belief it will not succeed.

The coalitional form is especially dedicated towards the study of bargaining, coalitional formation and cooperative behavior. As noncooperative theory swept the field in many applications, cooperative theory virtually disappeared from many textbooks. But in spite of many premature reports of its impending death it still appears to be flourishing especially with the **core** and various **value solutions.**

4. OPEN QUESTIONS AND FUTURE DIRECTIONS

In this section, a few questions are considered looking towards the future.

There are many solutions suggested for dynamic games, repeated matrix games with and without uncertainty, or games in some variant of the extensive form with various terminal conditions. **There is no single universally accepted solution.** There may be many solutions that appear to be reasonable if judged from a specific context point of view. As von Neumann intimated any good applied mathematics has its assumption or axioms rooted in observation. The relationship between context and the concept of solution is critical

4.1. Solutions to Games in Extensive Form

In the development of multistage games; one must ask what are the questions to be answered? One question might be the proof of the existence of a unique equilibrium point for all games played by agents fitting into the often used model of the rational economic agent. This dispenses with the utilization of much psychology or social-psychology. For many applications of dynamic game theory it appears that at least one, if not many models of **a behavioral agent** with limits on perception, intelligence, ability to remember, ability to compute, and ability to act is called for. This has posed one of the greatest difficulties in the development of a viable applied dynamics in many topics. Needed is a mixture of gaming, game theory, cognitive psychology and social psychology. This is emerging, but at this time is far from fully developed. It depends on a joint understanding among professionals in different disciplines that has hardly been developed.

A key specific example is provided concerning the role of threats and the relevance of perfect equilibria. It is easy to run an experiment illustrating that in some contexts individuals in an experimental game do not select the obvious unique perfect equilibrium.

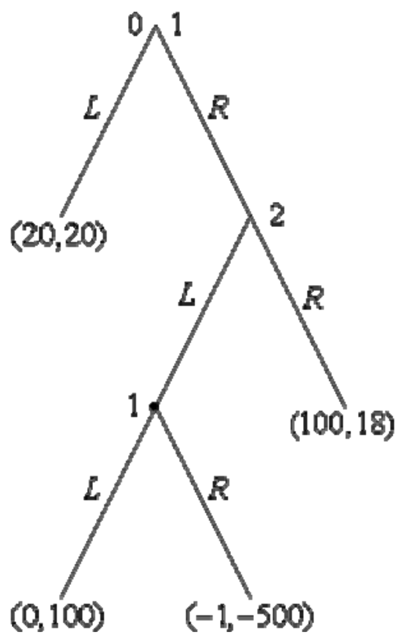

Figure 1: Threat Equilibria

4.2. Backward and Forward Planning: Recall and Strategy

Explicit forward planning is heavily short term. It rarely exceeds five years into the future. Longer range plans are almost always revised before five years have passed. Much of memory is episodic, depending on "events that stand out." These observations indicate that more investigation is called for, of multistage games played with updating strategies where individual look back no further than T1 periods and project forward no more than T2 periods where at most 4 or 5 years are considered. In business School and Corporate Planning literature there has been considerable work on planning processes.

Almost all of it is evolutionary. The projections are revised as the environment changes. This work is currently far from formal game theory.

4.3. Modeling Expertise

In the game of Go (and to a lesser extent in chess) there is a substantial literature on expertise, complete with a handicapping system that enables individuals with different expertise to play each other on a more or less evened up basis. This alone is sufficient to indicate the importance of expertise.

As long ago as in 1947 von Neumann and Morgenstern [76, p. 614] considered bargaining with different refinements of perception.

Within the current formal structure of the extensive form it is easy to model one rudimentary form of expertise by giving one individual a strict refinement of the information possessed by her competitor. An example of a simple classroom game shows that finer perception can be modeled as a refinement of information and its influence on payoffs and play can be easily illustrated as shown in the example below

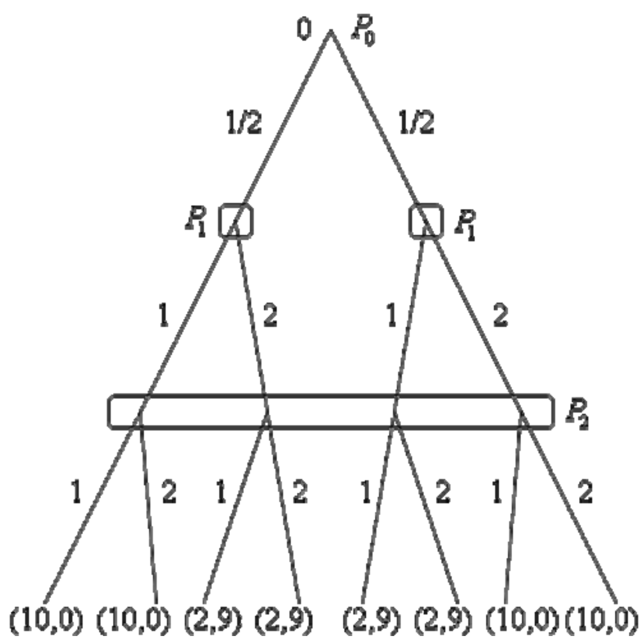

Figure 2

It is my belief that the role of expertise calls for considerable collaborative work in both of game theory and psychology. Even a casual glance at society and the economy illustrates that for many basic questions expertise, experience and superior perception cannot be ignored.

4.4. The Role of Language

A critical, but as yet lightly studied aspect of game theory has been the treatment of language and gesture. Some consideration has been given to “cheap talk” and to costly information but chiefly because it is extremely difficult and probably requires interdisciplinary collaboration the role of language is virtually ignored in the current development of game theory [but see 78, 79]. Although verbal communication and language as a coordinating device, have been recognized for many years in actual behavior. Expressions such as “a man of his word” or “his word is his bond” raise deep questions in the analysis of threats and bargaining.

I conjecture that even permitting a highly restricted formal set of messages associated with even mild penalties for “failing to live up to your word,” can start to show some of the features of language in game theory.

4.5. Mass Player Game Theory and Mass Particle Physics? How Good Is the Analogy?

What is meant by many? How many is many? How much is it context dependent?

It is easy to comment in lectures on analogies between physics or biology and economics. The mass particle methods of physics may appear to be highly attractive to those economists concerned with mass anonymous markets. But in asking “how many is many? Fundamental differences appear among particles in physics, people in economics and cells in biology. In human affairs 10e10 covers the world population of humans with some room to spare. In physics and biology 10e10 can easily be a small number a small number.

The nature of communication among individuals and among atoms is sufficiently different to require us to use the analogies with care. Is the “madness of mobs” and example analogous to polarization of particles [80]? How far does the analogy stretch and how useful is it? The burgeoning topic of network game theory may be of relevance.

We may expect that experimental gaming and simulation will flourish, in the investigation of mass behavior games, utilizing simulation and agent-based modeling to beat through otherwise intractable heterogeneous population models [37].

4.6. The Games within the Game

At one level, at least in political-economy the relationship between much of economic decision-making and political decision — making is, that to a considerable extent, the political decision-making takes place on a different time scale than the economic decision-making and the politics provides many of the rules and context for the economics. It is reasonable to consider that there is a game within a game. It is feasible to formalize a class of games where finite lived players (real persons) play in a game each period, but another set of players (legal persons or institutions) who can be considered with operationally infinite lives can modify some of the rules of the second level game every n periods. Structures with two or more levels of decision-making taking place on different time spans provide a setting where the influence of the polity on the economy and vice-versa can both be modeled and may eventually serve to provide a more satisfactory paradigm to link mathematical micro-economics to macro-economics than general equilibrium theory [81].

At the least conventional GE will be replaced by a game with a continuum of agents and one atomic player with financial instruments included. This treated fully game theoretically calls for political-economy as a game within a game. The economy being the basically competitive game and above it the polity being the rule maker for the game below. My guess is that 25% ± 5% is the “energy or money” consumption required by an elected government for a viable political economy. This is roughly like the energy consumption of the brain relative to the body [82].

It is worth noting that with this structure a mass of individual local optimizers could drive an evolving system that is continuously modifying the rules for the local optimizers with the system as a whole having no particular direction.

4.7. On Matrix Games, Cooperative and Noncooperative Theories

In Section 4.4 the games within the game were noted, basically with respect to economics and political science. Here it is suggested that it is possible that at a far higher level of abstraction the possibility of the relevance of a multistage organism may be implicit within the n-matrix representation of a game in strategic form. Although the generic inefficiency of the Noncooperative Equilibrium has been proved [72], to my knowledge few attempts [83, 84] have been made trying to measure how inefficient is the NCE in comparison with the cooperative game approach of von Neumann and Morgenstern that utilized measurable transferable utility. A way to approach this problem is to place the utility comparisons on the same level; thereby enabling one to sum the payoffs at any equilibrium point. In utilizing this approximation one can define the upper and lower bounds on the set of noncooperative equilibrium points associated with any matrix game and thus consider the upper and lower bounds to measure by what percentage the NCE falls short of the joint maximum (JM). This measure provides an indication of the resource gain available to all agents by switching to a cooperative game. But instead of switching to the cooperative game *per se* if the gap were large enough the agents could construct a mechanism in the form of a second stage to the game that provides coordination, signaling and possibly some other forms of control on the original matrix game in such a way that the players can pay for the administrative costs and still all be able to benefit from its existence. This might be regarded as a reconciliation between the benefits of unlimited individual behavior and a higher level of organization.

4.8. Voting or Evaluating Candidates

There is a considerable literature on voting and public choice replete with measures: impossibility results; and different voting structures suggested. In spit of Arrow's [53] elegant extension of Condorcet’s paradox, all that an impossibility theorem does is force us to ask if we have the right domain and the right description of the players and the right concept of solution. Recently a method has been suggested that places the stress on evaluating and grading the candidates rather than voting for them in a conventional manner [52]**.**

4.9. Agent Based Modeling and Simulation

Purely analytical methods in the behavioral sciences are usually of great help if one can get them. But their use is by no means an easy goal to achieve. This is especially true if one wishes to consider heterogeneous agents. The growth of simulation methods and the cheapness and speed of computation are beginning to provide the behavioral sciences with a new form of viewing device akin to the telescope and microscope in other sciences. The computer provides not only a means for computation, but a means for investigation in the behavioral sciences whether the simulation involves different types of optimizing agents or minimal intelligence automata, animal fights, market struggles or political voting contests, multi-agent models can be constructed and employed to provide insights and to sweeten the intuition in a way that even a decade ago was unthinkable.

These methods have been developing with increasing speed (see Epstein and Axtell [37]; the Santa Fe Institute double auction [73]. The Game of Life [74] to name a few). But they will often (but not always) be best utilized when linked with clear questions and analytical concern. Given the considerable difficulty in developing good analytical models of capacity constrained agents [85] the possibilities for game theory to benefit from automata simulations are considerable.

4.10. Computer Science, Game Theory, Combinatorics and Computation

The last topic noted is computer science and game theory. Anyone who has considered the proliferation of the number of alternative structures represented by matrix games knows that even the number of 2 x 2 games is imposingly large. Although in much of elementary teaching a handful of special games such as the Prisoners’ Dilemma, the

Battle of the Sexes, or the Stag Hunt play prominent roles in illustrating principles and paradoxes.

If we limit ourselves to strong ordinal preferences, just for the 2 x 2 matrix there are 4!*4! = 576 games that can be generated. Considerations of symmetry can reduce this large number to a somewhat more manageable 144 (or for some purposes [78]). But even if one only weakens assumptions on preferences to include ties the number of different games jumps to $4^4 * 4^4$ = 65,536 games. By symmetry considerations they can be reduced to 726 strategically different games. However the class of all 3 x 3 matrix games is already hyper-astronomical; but in order to understand the gap between individual non-coordinated behavior and correlated or cooperative behavior one needs to consider general domains such as the set of all k x k games. At this point methods of combinatorics and computer science are called for. Even simple questions such as finding if there exits a NCE that yields a payoff larger than some number for each player was shown to be NP-complete [75]. A useful survey has been provided by Halpern [76] covering bounded rationality, the computation of Nash equilibria and algorithmic mechanism design, adding to the usual economic discussions of mechanism design the estimates of computational requirements.

There is a similarity between the problems noted in Sections 4.6 and 4.7 above and the investigation of the price of anarchy considered in computer science.

It is my belief that work at this intersection of disciplines will proliferate in the next decade a part of the intermix between computation and conceptualization of new problems in game theory, combinatorics and computer science.

5. REPRISE

Only a few of the many topics noted in Sections 2.1 and 2.2 above have been discussed here. All of them are still expected to grow, many with increasing rates. An article attempting to provide anything more than superficial commentary on these many flourishing sub-disciplines would probably require more than 70 or 80 pages.

In the 1950s, when I was deeply impressed by the power of the formal models and the mathematics of game theory, I underestimated the importance of what I have called conversational game theory and its loose connections with operational gaming. The many subtle features of human behavior are not going to be mathematized easily, if at all. There is little doubt that a considerable amount of formal mathematics must still be developed; but not at the cost of cutting off the developers from understanding the reality they believe that the mathematics is meant to be portraying. In order to stay in touch with the various realities of application the game theorists must set up and maintain a dialogue with the practitioners. It is a two way street. Theory may influence practice, but practice must influence the development the appropriate models if the subject is to be more than an exercise in a minor branch of pure mathematics. A contact with the many realities of the politicians, entrepreneurs, financiers, generals, philosophers, religious leaders, hard and social scientists is an absolute must. The appropriate models cannot be built without an understanding of context and both the models and their formal analysis are called for to bring the influence of theory into the domain of public discourse and appreciation.

In this brief overview only a few less obvious items have been noted, but as the 70th anniversary of the publication of *The Theory of Games and Economic Behavior* approaches the speed of growth in both theory and applications is still accelerating. The applications are more varied and are forcing us towards more and more collaborations across all of the behavioral sciences in general and, in my opinion in biology and in computer science in particular.

Although the discussion here has been primarily concerned with the present and future, it is desirable to have at least, some understanding of the past. Recently a stimulating study of the precursors to the development of game theory has been provided by Robert Leonard [86] in his writing on Von Neumann, Morgenstern and the Creation of Game Theory. A rich context is provided to help us more fully appreciate the developments of today.

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