A Theory of Charitable Fund-raising with Costly Solicititations

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Abstract

We present a theory of charitable fund-raising in which it is costly to solicit donors. We fully characterize the optimal solicitation strategy that maximizes donations net of fundraising costs. The optimal strategy dictates that the fund-raiser target only those individuals whose equilibrium contributions exceed their solicitation costs. We show that as the income inequality increases, so does the level of the public good, despite a non-monotonic fund-raising effort. This implies that costly fund-raising can provide a novel explanation for the non-neutrality of income redistributions and government grants often found in empirical studies. We also show that in large economies, only the "most willing" donors are solicited; and the average donation converges to the solicitation cost of these donors, which is strictly positive.

Keywords: fund-raising, solicitation cost, charitable giving.
JEL Classification: H00, H30, H50

1 Introduction

Charitable fund-raising[1] is a costly endeavor. Andreoni and Payne (2003, 2011) indicate that an average charity spends 5 to 25 percent of its donations on fund-raising activities, including direct mailing, telemarketing, face-to-face solicitations, and staffing.[2] For instance, every

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[1] Charitable sector is a significant part of the U.S. economy. For instance, in 2008, total donations amounted to \$307 billion. \$229 billion of this total came from individuals, corresponding to 1.61% of GDP (*Giving USA*, 2009). See Andreoni (2006a) and List (2011) for an overview of this sector and the literature.

[2] Various watchdog groups such as *BBB Wise Giving Alliance* and *Charity Navigator* regularly post these cost-to-donation percentages for thousands of charities in the U.S. They often recommend a benchmark of 30-35 percent for a well-run charity.

year more than 115,000 nonprofit organizations hire fund-raising staff and consultants, paying them 2 billion dollars (Kelly, 1998).[3] It is thus strongly believed that both donors and charities dislike fund-raising, but view it to be a "necessary evil" for the greater good: fund-raising diverts resources away from charitable services while informing, or otherwise persuading, donors of the cause and fund-drive. Despite its significance, however, fund-raising costs have not been fully incorporated into the theory of charitable giving. This is the gap we aim to fill in this paper, and in doing so, we offer a new (and complementary) theory of charitable fund-raising.

Our formal setup adds an "active" fund-raiser to the "standard" model of giving in which donors care only about their private consumptions and the total supply of the public good.[4] In particular, unlike the standard model, we assume that each donor becomes aware of the charitable fund-drive only if solicited by the fund-raiser.[5] The solicitation is, however, costly. Our first observation is that the charity will contact an individual if he is expected to give more than his respective solicitation cost, or become a "net contributor" in equilibrium. We then show that identifying these net contributors in our model is equivalent to identifying the contributors in the standard model (without fund-raising cost) except that each donor's wealth is reduced by his solicitation cost. This important equivalence allows us to appeal to Andreoni and McGuire's (1993) elegant algorithm to solve for the latter. Our characterization of the optimal fund-raising strategy is simple because it does not require any equilibrium computation. More importantly, it pinpoints the exact set of donors to be targeted based on their preferences, incomes, and solicitation costs.[6]

Using our characterization, we next address two policy-related issues, one about income redistribution and the other about government grants. When individuals differ only in their incomes, we show that the fund-raising strategy reduces to assigning each individual a cutoff cost of solicitation that depends on the incomes of others richer than him. Intuitively, with only the income heterogeneity, the charity considers contacting first the richest donor; and

[3] The estimated number of paid workers employed by charities in 2004 was 9.4 million, which is more than 7% of the U.S. workforce (Sherlock and Gravelle, 2009).

[4] See, e.g., Warr (1983), Roberts (1984), Bergstrom, Blume, and Varian (1986), and Andreoni (1988).

[5] In addition, donors and fundraisers often report that one of the most effective fundraising techniques is directly asking people. See Andreoni (2006a) for a discussion; and Yoruk (2009), and Meer and Rosen (2011) for empirical evidence.

[6] This is consistent with the fact that fundraising professionals often recommend a careful study of donor base for an effective campaign (Kelly 1998). For instance, several software companies such as *DonorPerfect* (www.donorperfect.com), *DonorSearch* (http://donorsearch.net), and *Target Analytics* (www.blackbaud.com/targetanalytics) compile donor databases and sell them to charities along with programs to identify the prospect donors.

once this donor is in the "game", the charity becomes more conservative about contacting the second richest donor due to the free-riding incentive, which depends on their income difference. Iteratively applied, this logic implies that unlike the well-known neutrality result predicted by the standard theory (e.g., Warr 1983, and Bergstrom et al. 1986), an income redistribution is likely to affect the fund-raising strategy and thus the provision of the public good. In particular, as the income distribution becomes more unequal in the sense of Lorenz dominance (defined below), we find that the level of the public good strictly increases in the presence of costly fund-raising despite a non-monotonic fund-raising effort. Such non-neutrality of the public good provision also manifests itself in response to a government grant to the charity. We show that a more generous grant partially crowds out fund-raising effort, leaving some donations unrealized, as well as reducing the amount of the realized donations. The importance of this additional fund-raising channel for crowding-out has been recently evidenced by Andreoni and Payne (2003, 2011).

Given that many charities have a large donor base, we also investigate optimal fund-raising in replica economies. We show that in a sufficiently large economy, only the donors who like the public good "the most" are contacted, whose identity jointly depends on preference, income, and solicitation cost. Thus, even in a large economy, it is not necessarily the highest income and/or the lowest solicitation cost donors who will be contacted; rather it is a combination of all the three attributes that will define the fund-raiser's strategy. In particular, while the public good level (net of fund-raising costs) converges to a finite level, the average donation converges to the respective solicitation cost, which, unlike in the standard model, is strictly positive.

Aside from the papers mentioned above, our work relates to a relatively small theoretical literature on strategic fund-raising as a means of: providing prestige to donors (Glazer and Konrad 1996, Harbaugh 1998, and Romano and Yildirim 2001), signaling the project quality (Vesterlund 2003, and Andreoni 2006b), and organizing lotteries (Morgan 2000). Our work is more closely related to the models of strategic fund-raising to overcome zero-contribution equilibrium under non-convex production either by securing seed money (Andreoni 1998), or by collecting donations in piece-meals (Marx and Matthews 2000). None of these papers, however, consider endogenous, costly solicitations.

Our work is most closely related to Rose-Ackerman (1982) and Andreoni and Payne (2003). Rose-Ackerman is the first to build a model of costly fund-raising in which donors, as in ours, are unaware of a charity until they receive a solicitation letter. She, however, does not construct donors' responses from an equilibrium play. Andreoni and Payne (2003) endogenize both the fund-raiser and donors' responses as in our model, but they view solicitation letters to be randomly distributed. Their main theoretical result is that a government grant may discourage fund-raising, which is in line with one of our results. Unlike them, we fully characterize the optimal strategy that involves targeted solicitations, and provide a richer set of results, regarding the non-neutrality and large economies.

In addition to the theoretical literature, there is a more extensive empirical and experimental literature on charitable giving, to which we will refer below. For recent surveys of the literature, see the reviews by Andreoni (2006a) and List (2011).

The rest of the paper is organized as follows. In the next section, we set up the model. In Section 3, we characterize donor equilibrium for an arbitrary set of solicitations. In Section 4, we determine the optimal fund-raising strategy as a modified Andreoni and McGuire algorithm. In Sections 5 and 6, we consider the impacts of income redistribution and government grants, respectively. We examine large replica economies in Section 7, and conclude in Section 8. The proofs of all the formal results are relegated to the Appendix.

2 Model

Our formal setup extends the standard model of privately provided public goods (e.g., Warr 1983, Roberts 1984, Bergstrom et al. 1986, and Andreoni 1988). Thus, it is useful to briefly review this basic framework before introducing fund-raising costs.

Standard Model. There is a set of individuals, $N = \{1, ..., n\}$, who each allocate his wealth, $w_i > 0$, between a private good consumption, $x_i \geq 0$, and a gift to the public good or charity, $g_i \geq 0$. Units are normalized so that $x_i + g_i = w_i$. At the outset, every person is fully aware of the charitable fund-drive and thus in the "contribution game". Letting $G = \sum_{i \in N} g_i$ be the supply of the public good, individual i's preference is represented by the utility function $u_i(x_i, G)$, which is assumed to be strictly increasing, strictly quasi-concave, and twice differentiable. Both public and private goods are normal. Thus, individual i's (Marshallian) demand function for the public good, denoted by $f_i(w)$, satisfies the strict normality: $0 < f_i'(w) < 1$ for all $w > 0$. The equilibrium gifts, $\{g_1^*, ..., g_n^*\}$, are determined through a simultaneous play. Under strict normality, Bergstrom et al. (1986) showed that there is a unique Nash equilibrium.

To isolate any source of zero provision, we will assume that the standard model produces a positive level of the public good in equilibrium, $G^* > 0$. One sufficient condition for this is that $f_i(0) = 0$ for all $i \in N$, which we will maintain throughout. Together with strict normality, this condition implies that in a *one*-person economy, each individual allocates some positive amount of his wealth to the public good.

Costly Fund-raising. Note that there is no role for fund-raising in the standard model, as everyone is already aware of the fund-drive.[7] Thus, as with Rose-Ackerman (1982), and Andreoni and Payne (2003), we assume that each person i becomes informed of the fund-drive only if solicited by the fund-raiser.[8] It, however, costs $c_i > 0$ to do so,[9] which may reflect such expenses as telemarketing, direct mails, or door-to-door visits. While, for technical purposes (to break the indifference), we do not allow for $c_i = 0$, which could potentially represent repeat donors, one can always take the limits.[10]

Let $F \subseteq N$ be an arbitrary set of donors who are contacted by the fund-raiser, or simply the *fund-raiser set*. In our basic model, we assume that each contacted donor is also informed of the entire set F;[11] *but* we later show that even if the fund-raiser set is unobservable to donors, they can perfectly infer the "optimal" set in equilibrium. Let $g_i^*(F)$ be donor i's equilibrium gift in the simultaneous play in F. Then, the total fund-raising cost and the gross level of donations are defined respectively by,

$$C(F) = \sum\nolimits_{i \in F} c_i \text{ and } G^*(F) = \sum\nolimits_{i \in F} g_i^*(F),$$

where $C(\varnothing) = 0$ and $g_i^*(\varnothing) = 0$ by convention. Thus, the supply of the public good or charitable services is given by:

$$\overline{G}^*(F) = \max\left\{G^*(F) - C(F), 0\right\}. \tag{1}$$

The charity's objective is to choose the set F that maximizes $\overline{G}^*(F)$. Eq. (1) implies that if insufficient funds are received to cover the cost, then no public good is provided, which

[7] Alternatively, since, in the standard model, the equilibrium provision can never decrease by the inclusion of a new donor despite the free-rider problem (e.g., Andreoni and McGuire 1993), the fund-raiser would trivially ask everyone for donations.

[8] Unlike Andreoni and Payne (2003), we assume for simplicity that each solicitation reaches the donor with certainty; or the fund-raiser ensures that this is the case.

[9] Aside from its informational value, fund-raising may also prevent willing donors from procrastinating (O'Donoghue and Rabin 1999). As Andreoni (2006a, p. 1257) states, an iron law of fund-raising is that people tend not to give unless thay are asked. See Yoruk (2009), and Meer and Rosen (2011) for some empirical evidence in favor of this so-called "the ask" principle.

[10] We could also include a fixed setup cost of fund-raising; but its analysis would be similar to that of a (negative) government grant considered in Section 6.

[11] For instance, the fund-raiser such as a private university may organize an alumni reunion at which contacted donors meet each other; or the fund-raiser may strictly instruct its volunteers to solicit donors from certain neighborhoods.

simply refers to a failed fund-raising in our model.[12] We assume that the charity dislikes fund-raising in that when indifferent between two fund-raiser sets, it strictly prefers the one with the lower cost.[13]

Our fund-raising game proceeds as follows. First, the charity decides whether or not to launch a fund-drive. If one is launched, then the charity reaches out to a (optimal) set F^o of potential donors, which then becomes common knowledge among them. Finally, given the fund-raising cost of $C(F^o)$, the contacted donors simultaneously contribute to the public good, leading to equilibrium gifts $\{g_i^*(F^o)\}_{i\in F^o}$ and the public good $\overline{G}^*(F^o)$. Our solution concept is subgame perfect Nash equilibrium.

Next, we briefly explore the donors' behavior for an arbitrary subset of the population, and then consider the optimal fund-raiser set.

3 Donor Equilibrium

Suppose that an arbitrary subset F of population has been contacted for donations. Note that if fund-raising were costless, the contribution game in F would coincide with the standard model discussed above, and have a unique and positive level of public good in equilibrium, $\overline{G}^*(F) > 0$. The fund-raising cost, however, introduces a threshold to the provision of the public good (see eq.(1)) and the potential for a zero-contribution equilibrium.

To characterize the equilibrium contributions, consider first person i's solo decision to cover the entire fund-raising cost, C. Note that person i would receive utility $u_i(w_i, 0)$, if he contributed nothing. Otherwise, he would have to choose $g_i \geq C$ to maximize $u_i(w_i - g_i, g_i - C)$. Let $V_i(w_i - C)$ be i's indirect utility in the latter case, which is increasing in the (net) income. For $C = 0$, clearly $V_i(w_i) > u_i(w_i, 0)$ because $f_i(w_i) > 0$, whereas for $C = w_i$, we have $V_i(0) \leq u_i(w_i, 0)$. Hence, there is a unique cutoff cost, $\widehat{C}_i \in (0, w_i]$ such that when alone, person i would consume some public good if and only if $C < \widehat{C}_i$.[14] To highlight the collective nature of the fund-raising cost among contributors, we also make

Assumption S. $c_i < \widehat{C}_i$ *for all* $i \in N$.

[12] For simplicity, it is assumed that donations are not refunded in the case of a failed fundraising, or they are used for other causes that donors do not care about.

[13] One justification for this could be that the charity has some concern about its cost/donation rating by the watchdog groups. Formally, if $F' \neq F$ are two fundraiser sets such that $G' - C' = G - C$ and $C' > C$, then it follows that $C'/G' > C/G$.

[14] For the CES utility: $u_i = (x_i^{\rho_i} + (\overline{G})^{\rho_i})^{1/\rho_i}$, with $\rho_i < 1$, it is easily verified that $\widehat{C}_i = [1 - (1/2)^{\frac{1-\rho_i}{\rho_i}}]w_i$ for $\rho_i \in (0, 1)$, and $\widehat{C}_i = w_i$ for $\rho_i \leq 0$ (including the Cobb-Douglas specification at $\rho_i = 0$).

This assumption is stronger than we need for our results, but it is easier to interpret.[15] It indicates that in a one-person economy, each donor would always prefer to cover his solicitation cost and consume some public good than consume only the private good. Nevertheless, the following result shows that donors together may contribute nothing.

Proposition 1. *Fix an arbitrary fund-raiser set, $F \neq \varnothing$, whose fund-raising cost is $C(F)$. If $C(F) < \max_{i \in F} \widehat{C}_i$, then $\overline{G}^*(F) > 0$. If, on the other hand, $\max_{i \in F} \widehat{C}_i \leq C(F)$, then there is always zero-contribution equilibrium, generating $\overline{G}^*(F) = 0$.*

Proposition 1 simply says that if at least one person can bear the entire fund-raising cost alone, then the equilibrium provision of the public good is positive. However, if no person can bear the cost alone, then the zero-contribution profile becomes an equilibrium despite our Assumption S. This is because absent an explicit cost-sharing agreement, each donor cares about the total fund-raising cost, $\sum_{i \in F} c_i$, at the time of giving – not just his own cost, c_i – which may well be greater than his threshold $\widehat{C}_i$.

Proposition 1 demonstrates that fund-raising expenses may indeed deter donors from giving and render a fund-drive unsuccessful. Hence, when fund-raising entails significant costs, a carefully planned strategy of who to ask for donations seems to be of utmost importance both to control the expenses and to encourage giving.[16] We illustrate this point with a numerical example, which also motivates our subsequent analysis.

Example 1. *Let $N = \{1, 2, 3\}$ and $u_i = x_i^{1-\alpha} (\overline{G})^\alpha$, with $\alpha = 0.3$. Individuals' wealth and solicitation costs are such that $(w_1, w_2, w_3) = (18, 18, 20)$ and $(c_1, c_2, c_3) = (0.01, 4, 6.9)$. The following table reports donor equilibrium, and highlights the optimal fund-raiser set.*

F	$g_1^*(F)$	$g_2^*(F)$	$g_3^*(F)$	$G^*(F)$	$C(F)$	$\overline{G}^*(F)$
$\{1\}$	5.41	–	–	5.41	0.01	5.40
$\{2\}$	–	8.2	–	8.2	4	4.2
$\{3\}$	–	–	10.83	10.83	6.9	3.93
$\{\mathbf{1}, \mathbf{2}\}$	4.83	4.83	–	9.66	4.01	**5.65**
$\{1, 3\}$	5.20	–	7.20	12.40	6.91	5.49
$\{2, 3\}$	–	6.84	8.84	15.68	10.9	4.78
$\{1, 2, 3\}$	4.85	4.85	6.85	16.55	10.91	5.64

Table 1: Donor Equilibrium

[15] In the Appendix, Lemma A3 presents a weaker condition that allows for $c_i > \widehat{C}_i$.

[16] Since the fundraising cost introduces a threshold to the public good provision, Proposition 1 is a reminiscent of the equilibrium characterization in Andreoni (1998). Unlike his model, however, the provision point in ours will be endogenous to fundraising strategy as opposed to being a capital requirement.

Tables 1 reveals that it is optimal to contact only the donors 1 and 2. Donor 3 is not included in the set because of his high solicitation cost even though he would give the most. It also reveals that with the optimal solicitations, the C/G ratio is 41%, which is not the lowest. Finally, it is clear that even with three donors, a direct approach to identifying the optimal fund-raiser set promises to be computationally demanding. In the next section, we make some key observations about the optimal fund-raising strategy and derive a simple algorithm to find it.

4 Optimal Fund-raising

Our first observation is that although donors may end up contributing nothing for an arbitrary fund-raiser set, the same cannot happen if the set is optimally chosen.

Lemma 1. *In a fund-raising equilibrium, $F^o \neq \varnothing$ if and only if $\overline{G}^*(F^o) > 0$.*

Lemma 1 says that in a world of complete information, an optimizing charity would never start fund-raising if it did not expect that donations would exceed the cost. Together with Proposition 1, this means that in our model, the charity can fail to provide the public good despite fund-raising only because it suboptimally sets the fund-raising strategy.[17]

While enlightening, Lemma 1 does not inform us about the composition of individual contributions. As hinted by Example 1, the optimal fund-raiser set is likely to depend on this composition. The following result offers some significant insights in this direction.

Proposition 2. *If $F^o \neq \varnothing$, then it is unique and exactly identified by these two conditions:*

(C1) *every individual i in F^o is a "net contributor" in the sense that $g_i^*(F^o) - c_i > 0$; and*

(C2) *any individual i outside F^o would be a "net free-rider" if added to F^o, in the sense that $g_i^*(F^o \cup \{i\}) - c_i \leq 0$.*

Proposition 2 indicates that the charity will contact person i if he is expected to give more than his own cost in equilibrium. As such, the charity classifies donors as *net* contributors and *net* free-riders with respect to their solicitation costs even though there is no explicit cost-sharing agreement among them. The equilibrium gifts exhibit some (implicit)

[17] As noted in the Introduction, charities spend billions of dollars on professional fundraisers to presumably have a well-planned fund-drive. For instance, the Association of Fundraising Professionals (AFP) represent 30,000 such fundraisers.

cost-sharing due simply to the charity's decision of who to solicit. Note, however, that net contributors and net free-riders are defined in equilibrium. Thus, if the fund-drive is expected to be too costly, it is possible that donors give nothing, and in turn no fund-drive begins, $F^o = \varnothing$.[18]

Proposition 2 essentially offers an algorithm to determine the optimal set. Consider Table 1 above. We see that $F^o \neq \{1\}$ because if included in this set, person 2 would also be a net contributor $(4.83 - 4 > 0)$. $F^o \neq \{1, 2, 3\}$ either; because person 3 would be a net free-rider $(6.85 - 6.9 \leq 0)$. As a result, $F^o = \{1, 2\}$.

Nevertheless, Proposition 2 is not a full characterization of the optimal strategy because it involves equilibrium choices. It does, however, point out that the optimal strategy should exactly identify the set of net contributors, or equivalently the set of net free-riders. A similar identification problem would arise in the standard model if one were to detect the (pure) free-riders. For that case, Andreoni and McGuire (1993) offer an elegant algorithm that does not require equilibrium calculation for each subset of donors. Although our point of investigation here is very different from theirs, we draw a connection owing to Proposition 2.

When finding the optimal set, it is clear from Proposition 2 that the fund-raiser can imagine each individual i tentatively paying for c_i. Then, the optimal set problem reduces to "identifying the net free-riders" with residual incomes, $w_i - c_i$, by using Andreoni and McGuire algorithm.[19] Let $G_i^0 > 0$ be the "drop-out" level of the public good for person i, which, given that $0 < f_i' < 1$ and $f_i(0) = 0$, uniquely solves[20]

$$f_i(w_i - c_i + G_i^0) = G_i^0. \tag{2}$$

One interpretation of G_i^0 in our context is that person i becomes a net contributor if and only if the sum of others' net contributions stays below G_i^0. Without loss of generality, index individuals in a descending order of their dropout levels: $G_1^0 \geq G_2^0 \geq ... \geq G_n^0$. Next, define

[18] This is simply the subgame perfection argument. Some fundraising may never start because, given the cost, the fundraiser believes the donors would play the zero equilibrium. In fact, given our convention that $g_i^*(\varnothing) = 0$, Proposition 2 is also consistent with $F^o = \varnothing$: C1 would trivially hold while C2 would imply that everyone would be a net free-rider, resulting in $F^o = \varnothing$.

[19] Note that $w_i - c_i > 0$ because $c_i < \widehat{C}_i$ by Assumption S. In general, any donor with $w_i - c_i \leq 0$ would automatically be excluded from the fundraiser set.

[20] Technically, we need $0 < f_i' \leq \theta < 1$ (Andreoni 1988) to guarantee a solution here.

$$\Phi_i(G) = \sum_{j=1}^{i} (\phi_j(G) - G) + G,$$

where $\phi_j = f_j^{-1}$ (inverse demand), $\phi_j' > 1$, and thus $\Phi_i'(G) > 0$. The following result fully characterizes the optimal fund-raising strategy.

Proposition 3. *Define $\Delta_i \equiv \Phi_i(G_i^0) - \sum_{j=1}^{i}(w_j - c_j)$. Then, we have $\Delta_1 \geq \Delta_2 \geq ... \geq \Delta_n$, with $\Delta_1 > 0$. Moreover, letting $k \in N$ be the largest number such that $\Delta_k > 0$, the optimal fund-raiser set is $F^o = \{1, ..., k\}$. This set generates the public good, $\overline{G}^* = \Phi_k^{-1}(\sum_{j=1}^{k}(w_j - c_j))$.*

To understand how the optimal strategy works, note that Δ_i can be interpreted as a measure of person i's incentive to pay for his solicitation cost. In particular, as in Bergstrom et al. (1986), $\Phi_i(G)$ is the minimum level of total wealth needed to sustain public good G as an equilibrium among agents, $1, ..., i$. This means that if the actual total wealth available to these agents is strictly less than $\Phi_i(G_i^0)$, namely $\Delta_i > 0$, then the dropout value of person i, G_i^0, cannot be reached, making him a net contributor and thus a *candidate* for the fund-raiser set. Given $\Phi_i'(G) > 0$ by the strict normality, these incentives are monotonic in that $\Delta_i \geq \Delta_{i+1}$, and therefore, the fund-raiser considers the largest set of individuals with a positive incentive. This set will be optimal if, given the total fund-raising cost, $\sum_{j=1}^{k} c_j$, incurred, each individual decides to contribute rather than consume only the private good; i.e., if, in equilibrium, his net cost, $\sum_{j=1}^{k} c_j - G_{-i}^*$, is strictly less than his cutoff, $\widehat{C}_i$. Since everyone else in the set is expected to give more than his solicitation cost, this net cost cannot exceed his own cost, which, by Assumption S, is less than his cutoff, $\widehat{C}_i$.[21] As a point of reference, it is worth observing that if the charity could *force* each contacted donor to at least pay for his solicitation cost, then there would be no incentive constraint, $\Delta_k > 0$, because no donor would be able to give less than his cost. This means that providing donors with the incentives to be net contributors is the reason why some in the population may not be solicited in our model.

The optimal fund-raising strategy in Proposition 3 is easy to apply given that it does not require any equilibrium computation. Moreover, it ends in at most n steps, which is often much smaller than the number of all donor subsets, $2^n - 1$. Re-consider Example 1

[21] Lemma A3 in the Appendix offers a weaker cost condition than Assumption S. It relies on a lower bound on voluntary provision in a group.

above. From eq.(2), it is easily verified that $G_1^0 = 7.71$, $G_2^0 = 6.0$, and $G_3^0 = 5.61$. Using these, we find that $\Delta_1 = 7.71$, $\Delta_2 = 2.01$, and $\Delta_3 = -.17$, which implies that $F^o = \{1, 2\}$, as previously observed.

The optimal fund-raising strategy also has some intuitive comparative statics. Since eq.(2) implies that all else equal, G_i^0 is higher (1) the richer the person; (2) the greater his demand for the public good;[22] and/or (3) the lower his solicitation cost, the fund-raiser is more likely to contact such a person. This is consistent with the anecdotal evidence that schools often exclusively solicit alumnus and parents; religious organizations first target their members; and health charities primarily ask former patients and their families for donations.

Proposition 3 also raises an interesting fund-raising question when donors' preferences and incomes are negatively correlated. Consider, for instance, two individuals, a and b, with Cobb-Douglas utilities, and let the solicitation cost be c for each. Then, $G_i^0 = \beta_i(w_i - c)$ where $\beta_i = \frac{\alpha_i}{1-\alpha_i}$. Suppose that $\beta_a > \beta_b$ but $w_a < w_b < \frac{\beta_a}{\beta_b} w_a$. It can be verified that there is some $c^* > 0$ such that $G_a^0 > G_b^0$ for $c < c^*$, and $G_a^0 < G_b^0$ for $c > c^*$. That is, while for small costs, the higher preference individual is more likely to be solicited than the richer one, the order switches for large costs. The reason is that the fund-raising cost has a direct income effect, which is larger for the higher preference individual.

4.1 Unobservability of the Fund-raiser Set

While our assumption that the fund-raiser set is observable to donors is reasonable in some settings, in others, it may be less so. In particular, it may be difficult or infeasible for donors to monitor the charity's solicitations, in which case they can only hold beliefs about them. Given the unique optimal set, F^o, one natural belief system is as follows: if a donor in F^o is contacted, he learns about the fund-drive and believes that the rest of F^o will also be contacted, whereas, if a donor outside F^o is contacted, he attributes this to a mistake and believes that he is the only one contacted besides F^o.[23] Formally, letting $\mathcal{F}_i$ be donor i's belief about the fund-raiser set when he is contacted, we have $\mathcal{F}_i = F^o$ if $i \in F^o$, and $\mathcal{F}_i = F^o \cup \{i\}$ if $i \notin F^o$. Under these beliefs, the following result shows that the

[22] Formally, person i has a greater demand for the public good than j if $f_i(w) \geq f_j(w)$ for all $w > 0$.

[23] These beliefs are similar to "passive" beliefs often used in bilateral contracting in which one party privately contracts with several others (e.g., Cremer and Riordan 1987; McAfee and Schwartz 1994). One justification for such beliefs in our context is that the fundraiser assigns a different staff member to contact different donors so that mistakes are perceived to be uncorrelated.

unobservability of the fund-raiser set is of no consequence in equilibrium.

Proposition 4. *Suppose that the fund-raiser set is unobservable to donors. Then, under the beliefs, $\mathcal{F}_i$, described above, F^o is sustained as a perfect Bayesian equilibrium.*

The intuition behind Proposition 4 mainly comes from Proposition 2. When solicitations are unobservable, the fund-raiser would contact a donor outside the optimal set if he were expected to be a net contributor. But, this would contradict the fact that he was not included in the optimal set in the first place. Proposition 4 says that the fund-raiser does not necessarily have an incentive to "fool" donors about the solicitations, and run into a commitment problem about its targeting strategy.

Armed with the optimal fund-raiser behavior, we next address two policy-related issues, the first one being the role of an income redistribution.

5 Income Redistribution and Non-neutrality

Suppose that individuals may differ only in their incomes. In particular, they possess identical preferences and identical costs of solicitation, i.e., $u_i = u$ and $c_i = c$ for all i. This means $f_i = f$ and $\phi_i = \phi$. Without loss of generality, rank incomes as $w_1 \geq w_2 \geq .. \geq w_n$. Since $f' > 0$, from (2), this implies that $G_1^0 \geq G_2^0 \geq ... \geq G_n^0$. Applying Proposition 3, the fund-raising strategy then simplifies to a cutoff solicitation cost for each donor.

Lemma 2. *Define $\widehat{\phi}(G) = \phi(G) - G$, where $\widehat{\phi}(0) = 0$ and $\widehat{\phi}'(G) > 0$. Let the following be the cutoff solicitation cost for donor i:*

$$\bar{c}_i = \max\{w_i - \widehat{\phi}(\sum_{j=1}^{i}(w_j - w_i)), 0\}. \tag{3}$$

Then, we have $\bar{c}_1 \geq \bar{c}_2 \geq ... \geq \bar{c}_n$. Moreover, $F^o = \{i \in N | c < \bar{c}_i\}$.

That is, based on incomes, the charity determines a cutoff strategy for each individual, whereby individual i is solicited whenever the actual cost, c, falls below his cutoff. In general, the cutoff is strictly less than one's income except for the richest agent, and the gap increases for lower income individuals. This is because the fund-raising strategy follows a pecking order: for a given c, the charity first asks the richest person(s) for donations, and once this person is in the "game", it becomes more conservative in asking the second richest person because of the free-rider problem, which is a function of the wealth difference.

Applied iteratively, this logic explains why person i's cutoff in (3) is decreasing in the sum of wealth differences between him and the others who are richer than him. One important implication of this observation is that a redistribution of income is likely to affect the fund-raising strategy and thus the equilibrium provision of the public good.

As first observed by Warr (1983), if the set of contributors and their total wealth do not change by an income redistribution, then neither does the level of the public good in the standard model of giving.[24] This striking theoretical prediction has, however, been at odds with empirical evidence on private charity.[25] As such, several researchers have modified the standard model to reconcile this discrepancy; but these modifications have mostly been confined to the donor side – the most prominent one being "warm-glow" giving in which people also receive a direct benefit from contributing.[26] Here, we show that strategic and costly fund-raising can provide a complementary explanation as to the endemic breakdown of neutrality.

To develop some initial intuition, suppose again that individuals have identical Cobb-Douglas preferences: $u_i = x_i^{1-\alpha}\overline{G}^{\alpha}$, and consider these two income distributions: $\mathbf{w}' = (w, w, ..., w)$ and $\mathbf{w}'' = (\varepsilon + n(w-\varepsilon), \varepsilon, ..., \varepsilon)$, with $\frac{1}{1+\alpha/[n(1-\alpha)]} < \frac{\varepsilon}{w} < 1$. It is readily verified that in the standard model, all individuals contribute under both income distributions and thus in equilibrium, $G^{*\prime} = G^{*\prime\prime} > 0$. This neutrality result should extend to costly fund-raising as long as c is small so that everyone is still contacted under both income distributions. For a sufficiently large c, however, the fund-raising strategy, and thus the public good provision, is likely to be affected by the income distribution. For instance, when $\varepsilon < c < w$, it is clear that whereas everyone is contacted under the egalitarian income distribution, $\mathbf{w}'$, only the richest individual is contacted under the unequal income distribution, $\mathbf{w}''$. This means that although there are more contributors under $\mathbf{w}'$, there are also more fund-raising expenses. In fact, given Cobb-Douglas utilities, trivial algebra shows that equilibrium public good levels are given respectively by $\overline{G}^{*\prime} = \frac{n\alpha}{n(1-\alpha)+\alpha}(w-c)$ and $\overline{G}^{*\prime\prime} = \alpha(\varepsilon + n(w-\varepsilon) - c)$, and comparing them reveals $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$; in particular $\overline{G}^{*\prime} \neq \overline{G}^{*\prime\prime}$. Note also that if the fund-raising were even costlier, $w \leq c < \varepsilon + n(w-\varepsilon)$, then the fund-raising effort would be reversed: no individual would be contacted under $\mathbf{w}'$,

[24] Subsequent work showed the robustness of this result with varying generality. See, e.g., Bergstrom et al. (1986), Bernheim (1986), Roberts (1987), Andreoni (1988), and Sandler and Posnett (1991), among others.
[25] See, e.g., Clotfelter (1985), Kingma (1989), Steinberg (1991), Brunner (1997), and Ribar and Wilhelm (2002).
[26] See, e.g., Cornes and Sandler (1984), Steinberg (1987), and Andreoni (1989).

whereas the richest person under $\mathbf{w}''$ would still be solicited. Nevertheless, the public good provision would again imply that $0 = \overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$. Of course, if $c \geq \varepsilon + n(w - \varepsilon)$, then no fund-raising takes place in either case.

Overall, it seems that when fund-raising cost is significant, the neutrality result is *unlikely* to hold. In particular, even if, following an income redistribution, the poorest individuals remain willing to give, the fund-raiser may no longer solicit donations from them in order to control the costs. It also seems that while the equilibrium number of solicitations responds non-monotonically to a more unequal distribution of income, the public good provision will always increase. To prove these observations generally, we first need an appropriate definition of income inequality. To do so, we employ the well-known concept of Lorenz dominance (see, e.g., Atkinson 1970, and Lambert 2001).

Definition. (Lorenz Dominance) *Let $\mathbf{w} = (w_1, w_2, ..., w_n)$ be a vector of incomes whose elements are indexed in a descending order, and define $L_i(\mathbf{w}) = \sum_{j=1}^{i} w_j$. Consider two income vectors $\mathbf{w}' \neq \mathbf{w}''$ such that $L_n(\mathbf{w}') = L_n(\mathbf{w}'')$. It is said that $\mathbf{w}''$ is more unequal than $\mathbf{w}'$ if $\mathbf{w}'$ Lorenz dominates $\mathbf{w}''$, i.e., $L_i(\mathbf{w}'') > L_i(\mathbf{w}')$ for all $i < n$.*

Intuitively, an income distribution $\mathbf{w}''$ is more unequal than $\mathbf{w}'$ if the total income is more concentrated in the hands of the few. In particular, the egalitarian income distribution Lorenz dominates all the others, whereas a perfectly unequal income distribution in which one person possesses all the wealth is dominated by all the others. Based on this inequality concept, we reach,

Proposition 5. *Let $\mathbf{w}' \neq \mathbf{w}''$ be two income vectors such that $\mathbf{w}''$ is more unequal than $\mathbf{w}'$ in the sense of Lorenz. Moreover, suppose that with the standard model, every person is a contributor under both $\mathbf{w}'$ and $\mathbf{w}''$ so that $G^{*\prime} = G^{*\prime\prime} > 0$. Then, $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime} > 0$ for $c \in [0, \overline{c}_n'')$, and $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$ for $c \in [\overline{c}_n'', \overline{c}_1'')$. For $c \geq \overline{c}_1''$, no fund-raising takes place, yielding $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime} = 0$.*

Proposition 5 generalizes our intuition from the above discussion. For a sufficiently small cost of fund-raising, every donor is solicited regardless of the income redistribution, resulting in the same level of the public good. When the cost is significant, however, the fund-raising strategy, and the level of public good, is influenced by the income redistribution. In particular, a more unequal income distribution produces a higher level of the public good.

Note from (3) that $\overline{c}''_n$ is likely to be much smaller than w''_n, whereas $\overline{c}''_1$ is equal to w''_1, which means that the cost interval $[\overline{c}''_n, \overline{c}''_1)$ can indeed be significant.

To illustrate Proposition 5 and the non-monotonicity of the fund-raising strategy, we present the following example.

Example 2. *Let $N = \{1, 2, ..., 10\}$ and $u_i = x_i^{1-\alpha}(\overline{G})^\alpha$, with $\alpha = 0.3$. Table 2 below records three income distributions, $\mathbf{w}'$, $\mathbf{w}''$, and $\mathbf{w}'''$. It is easy to verify that average income in each case is 19.27, and $L_i(\mathbf{w}') > L_i(\mathbf{w}'') > L_i(\mathbf{w}''')$ for all i. Thus, $\mathbf{w}'$ and $\mathbf{w}'''$ exhibit the least and the most inequality, respectively. Using (3), Table 2 reports the cutoff solicitation cost for each donor under these income distributions.*

Donor i	w'_i	w''_i	w'''_i	$\overline{c}'_i$	$\overline{c}''_i$	$\overline{c}'''_i$
1	20.8	21.2	23.5	20.8	21.2	23.5
2	20	19.8	20.97	18.13	16.53	15.07
3	19.8	19.7	18.7	17	15.97	2.2
4	19.7	19.7	18.55	16.2	15.97	1
5	19.5	19.5	18.5	14.13	13.9	0.49
6	18.7	18.65	18.5	4	3.13	0.49
7	18.55	18.58	18.5	1.75	2.08	0.49
8	18.55	18.57	18.5	1.75	1.91	0.49
9	18.55	18.5	18.49	1.75	0.53	0.29
10	18.55	18.5	18.49	1.75	0.53	0.29

Table 2: Solicitation Cost and Non-Monotone Fund-raising

From Table 2, note first that for a solicitation cost, $c \leq .29$, all donors are called upon regardless of the income distribution, which implies the neutrality: $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime} = \overline{G}^{*\prime\prime\prime}$, as it should. Second, for $1.75 < c < 1.91$, the fund-raising set is non-monotonic in income inequality because clearly, $F^{*\prime} = \{1, 2, ..., 6\}$, $F^{*\prime\prime} = \{1, 2, ..., 8\}$, and $F^{*\prime\prime\prime} = \{1, 2, 3\}$, revealing that $F^{*\prime\prime\prime} \subset F^{*\prime} \subset F^{*\prime\prime}$. Nevertheless, the monotonicity in public good provision holds: $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime} < \overline{G}^{*\prime\prime\prime}$, as predicted by Proposition 5. For instance, for $c = 1.8$, we have $\overline{G}^{*\prime} = 7.180$, $\overline{G}^{*\prime\prime} = 7.185$, and $\overline{G}^{*\prime\prime\prime} = 7.220$.

We should point out that strategic costly fund-raising offers a complementary explanation for the non-neutrality to those identified in the literature. In particular, as with Bergstrom et al. (1986), we draw attention to the endogenous nature of the contributor set to the income distribution; but unlike in their study of the standard model, the contributor set in ours is optimally chosen by the fund-raiser. This means, for instance, that the non-contributors in our model are not necessarily pure free-riders; rather they are not asked for donations due to solicitation costs. We should also point out that in their Theorem 1d, Bergstrom et al. also observe that "Equalizing income redistributions that involve any transfers from contributors to non-contributors will decrease the equilibrium supply of the public good."[27] However, as is clear from Proposition 5, under strategic costly fund-raising, the non-neutrality exists even when everyone remains contributors under both income distributions in the standard model.

6 Government Grants

A long-standing policy question in public economics is that if the government gives a grant to a charity, to what degree will it displace private giving? While, in light of the neutrality result, the standard model of giving predicts a complete (dollar-for-dollar) crowding out, there is overwhelming evidence that this is not the case (see Footnote 25). The empirical studies have, for the most part, attributed any crowding-out to the donors' responses. Recently however, Andreoni and Payne (2003, 2011) have shifted attention and empirically showed that a significant part of the crowding-out can be explained by the fund-raiser's response in the form of reduced fund-raising efforts. By a simple modification to our base model, we can theoretically address the same issue here and support their findings.[28]

Let $R > 0$ be the amount of the government grant, and F_R^o and F_0^o denote the optimal fund-raiser sets with and without the grant, respectively. The following result summarizes our findings in this section.

Proposition 6. *Suppose that, without a grant, some public good is provided, i.e., $\overline{G}_0^* > 0$. Then, with the grant, donor i is solicited if and only if $\Delta_i > R$*

(a) *there is less fund-raising with the grant, $F_R^o \subseteq F_0^o$;*

(b) *each donor gives strictly less with the grant, i.e., $g_i^*(F_R^o) < g_i^*(F_0^o)$ for every $i \in F_R^o$; and*

(c) *private giving is partially crowded out, i.e., $\overline{G}^*(F_R^o) < R + \overline{G}^*(F_0^o)$, but $\overline{G}^*(F_R^o) > \overline{G}^*(F_0^o)$.*

[27] Bergstrom et al. use direct transfers among donors, but it is well-known that such Daltonian transfers are equivalent to Lorenz dominance (Atkinson 1970).

[28] In their 2003 paper, Andreoni and Payne also investigate the issue theoretically and obtain similar results for government grants. But, as mentioned before, they assume random solicitations.

Since a government grant directly enters into public good production, part (a) implies that the charity optimally responds by soliciting fewer donors. This reduced fund-raising is not because the charity has diminishing returns to funds under a linear production, but because it anticipates that donors will be less willing to give, as reflected by the optimal strategy. While, all else equal, cutting back fund-raising increases the public good provision by cutting costs, it also leaves some donations unrealized. Moreover, despite a smaller fund-raiser set, and thus less severe free-riding, with the grant, part (b) indicates that each contacted donor gives strictly less than he would without the grant. This is due to diminishing marginal utility from the grant that simply overwhelms the small group effect. Part (c) shows that the two negative effects of a government grant, namely lower fund-raising and fewer donations, never neutralize its direct production effect on the public good. That is, the crowding-out is partial because of both the fund-raiser's and the donors' behavioral responses.

Two observations are in order. First, unlike income redistribution, a government grant monotonically reduces the optimal fund-raiser set. The reason is that through the public good, R uniformly affects all the donors. Second, Proposition 6 appears consistent with anecdotal evidence that in economic downturns, fund-raising efforts often increase. For instance, in the wake of the recent economic crisis, state legislatures across the U.S. cut back support for higher education, and in response, public colleges and universities are reported to have stepped up their fund-raising by hiring consultants, hunting down graduates, and mobilizing student phone banks (*New York Times* 2011, Jan. 15).

7 Large Replica Economies

Many charities have access to a large donor base. In particular, the advent of information technology has helped fund-raisers to better search and locate prospect donors. To understand fund-raising behavior in large economies, we consider a simple replica-economy in which there are r donors of each type represented by the triple (u_i, w_i, c_i), resulting in the drop-out value G_i^0 from (2). The following result is the main finding in this section.

Proposition 7. *Suppose* $G_1^0 > G_2^0 > ... > G_n^0 > 0$. *Then*

(a) *there are some replicas* $\underline{r}_n \leq ... \leq \underline{r}_2 < \infty$ *such that type-i donors are not solicited in any* $r \geq \underline{r}_i$ *replica economy.*

(b) *As $r \to \infty$, only type-1 donors are solicited, in which case each donation converges to the solicitation cost, c_1, but the public good level approaches G_1^0.*

Proposition 7 says that except for the most willing type, there is a large enough replication of the economy in which no other type is solicited. The reason is that as the economy is replicated, the higher types replace the lower ones in net contributions. Proposition 7 also says that in the limit, each donation from a type-1 person converges to his respective solicitation cost. While this means that the net contribution is approximately zero, the level of the public good approaches to a finite level G_1^0. Note that even in the limit, it is not the lowest cost and/or highest income donors who are solicited; rather it is a combination of all the three attributes that determine the highest type.

Within the standard model, Andreoni (1988) finds that, all else equal, in large economies, only the richest agents contribute and others free ride. Andreoni also finds that the average contribution decreases to zero.[29] With costly fund-raising, our result suggests that only the richest agents contribute because they will be the only ones to be solicited in large economies. Moreover, the average donation converges to the solicitation cost, which is strictly positive.

8 Conclusion

It is an unfortunate fact that charities need to spend money to raise money. Thus, a careful planning of who to ask for donations should be paramount for a charity aiming to control its fund-raising costs while maximizing donations. Perhaps, this is why the charitable sector has grown to be highly professional and innovative. Yet, the theory of charitable fund-raising has almost exclusively focused on its revenue side. In this paper, we take a first stab at filling this void by introducing an active fund-raiser to the standard model of voluntary giving as studied by Bergstrom et al. (1986). In particular, we assume that each donor becomes aware of the fund-drive only if solicited by the charity, which is costly to do. In this extended model, we fully characterize the optimal fund-raising strategy that can be easily computed from the donors' preferences and incomes, and the fund-raiser's solicitation costs. Using this characterization, we show that costly fund-raising can provide a novel explanation for the non-neutrality of income redistributions and crowding-out hypothesis often encountered in

[29] See also Fries, Golding, Romano (1991) for a characterization of large economies under the standard model.

empirical studies. We also show that in large replica economies, it is only the most willing types who are solicited for donations – not because others would free-ride per se but because they would not be cost effective to ask for donations. In addition, the average donation converges to the solicitation cost, which is strictly positive.

Our analysis is based on the standard model in which donors have purely altruistic motives of giving. This model allows us to clearly highlight the effects of fund-raising costs, and it is the framework most theoretical insights for public good provision are built on. Before making firm policy recommendations, however, other motives of giving such as "warm-glow" (Andreoni 1989) should also be taken into account. Nonetheless, we believe that the basic trade-offs identified in our investigation would continue to emerge in these enriched settings. For future research, it may also be worthwhile to consider the possibility of sequential solicitations where donations are collected upon each visit. Another promising, and perhaps more challenging, direction would be to investigate the competition between charities where donors' responses are fully accounted for.

A Appendix

Proof of Proposition 1. Fix an arbitrary fund-raiser set, $F \neq \varnothing$, whose total fund-raising cost is $C(F) > 0$. Suppose that $C(F) < \max_{i \in F} \widehat{C}_i$, but, on the contrary, that $\overline{G}^*(F) = 0$. Then, it must be that $g_i^*(F) = 0$ for all $i \in F$ (otherwise, any $g_i^*(F) > 0$ would be used by person i toward the private good). Since $C(F) < \widehat{C}_j$ for some $j \in F$, note that given $G_{-j}^*(F) = 0$, person j would strictly prefer to contribute, yielding a contradiction. Thus, $\overline{G}^*(F) > 0$. A similar argument shows that when $C(F) \geq \max_{i \in F} \widehat{C}_i$, the zero-contribution profile is an equilibrium, resulting in $\overline{G}^*(F) = 0$. ■

Proof of Lemma 1. Clearly, $\overline{G}^*(F^o) > 0$ implies that some agents have been contacted, and thus $F^o \neq \varnothing$. Conversely, suppose that in equilibrium, $F^o \neq \varnothing$, but $\overline{G}^*(F^o) = 0$. Then, since $C(F^o) > 0$, the charity has a strict incentive to choose $F = \varnothing$ and incur no cost. Hence, $\overline{G}^*(F^o) > 0$. ■

In what follows, we define the contributor set, $F_C = \{i \in F | g_i^*(F) > 0\}$.

Lemma A1. *If* $\overline{G}^*(F) > 0$*, then* $\Phi_{F_C}(\overline{G}^*(F)) = \sum_{i \in F_C} w_i - C(F)$.

Proof. Suppose $\overline{G}^*(F) > 0$. If $i \in F_C$, then $\overline{G}^*(F) = f_i(w_i + G_{-i}^*(F) - C(F))$, which, given that $\phi_i \equiv f_i^{-1}$, implies that $\phi_i(\overline{G}^*(F)) = w_i + G_{-i}^*(F) - C(F)$. Summing over all $i \in F_C$, we have

$$\sum_{i \in F_C} \phi_i(\overline{G}^*(F)) = \sum_{i \in F_C} w_i + (|F_C| - 1)G^*(F) - |F_C|C(F).$$

Arranging terms yields $\sum_{i \in F_C}(\phi_i(\overline{G}^*(F)) - \overline{G}^*(F)) + \overline{G}^*(F) = \sum_{i \in F_C} w_i - C(F)$, or equivalently yields $\Phi_{F_C}(\overline{G}^*(F)) = \sum_{i \in F_C} w_i - C(F)$, as stated. ■

Lemma A2. *Suppose that* $\overline{G}^*(F) > 0$*. Then, there is a unique value* $\underline{G}(F) > 0$ *that solves* $\Phi_F(\underline{G}(F)) = \sum_{i \in F} w_i - C(F)$*. Moreover,* $\underline{G}(F) \leq \overline{G}^*(F)$.

Proof. Since $\overline{G}^*(F) > 0$, we know that $G^*(F) - C(F) > 0$, and thus $\sum_{i \in F} w_i - C(F) > 0$. In addition, since $\Phi_F(0) = 0$ and $\Phi'_F(G) > 0$, there is a unique $\underline{G}(F) > 0$ that solves: $\Phi_F(\underline{G}(F)) = \sum_{i \in F} w_i - C(F)$.

To prove that $\overline{G}^*(F) \geq \underline{G}(F)$, first note that if $F = F_C$, then $\Phi_F(\underline{G}(F)) = \sum_{i \in F} w_i - C(F) = \Phi_F(\overline{G}^*(F))$, implying that $\underline{G}(F) = \overline{G}^*(F)$. If, on the other hand, $F \neq F_C$, then we have $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = w_i - g_i^*(F)$ for $i \in F_C$, and $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) \geq w_i$ for $i \in F \backslash F_C$. Then, summing over all $i \in F$, we obtain,

$$\sum_{i \in F} w_i - C(F) \leq \sum_{i \in F} [\phi_i(\overline{G}^*(F)) - \overline{G}^*(F)] + \overline{G}^*(F) = \Phi_F(\overline{G}^*(F)).$$

Since $\Phi_F(\underline{G}(F)) = \sum_{i\in F} w_i - C(F)$, it follows that $\underline{G}(F) \le \overline{G}^*(F)$. ■

Proof of Proposition 2. Let F^o be the unique optimal fund-raiser set. Suppose that $i \in F^o$ but, contrary to C1, $g_i^*(F^o) \le c_i$. Note that the equilibrium public good must satisfy $\overline{G}^*(F^o) > 0$; otherwise we would have $F^o = \varnothing$, contradicting $i \in F^o$. Moreover, since $\overline{G}^*(F^o) > 0$ and $g_i^*(F^o) \le c_i$, there must exist $i' \ne i$ such that $g_{i'}^*(F^o) > c_{i'}$ for some $i' \in F^o$. Clearly, $F_C^o \ne \varnothing$ because $i' \in F_C^o$. Moreover, Lemma A1 implies that $\Phi_{F_C^o}(\overline{G}^*(F^o)) = \sum_{j\in F_C^o} w_j - C(F^o)$.

We first prove that $F^o = F_C^o$, i.e., everyone in F^o is a contributor. Since $F_C^o \subseteq F^o$ by definition, we only show $F^o \subseteq F_C^o$. Suppose not. Then, $j \in F^o$ but $j \notin F_C^o$ for some j. That is, person j is contacted even though $g_j^*(F^o) = 0$. Evidently, $\Phi_{F_C^o}(\overline{G}^*(F^o)) < \sum_{i\in F_C^o} w_i - (C(F^o) - c_j) = \Phi_{F_C^o}(\underline{G}(F_{-j}^o))$, where the equality follows from Lemma A1. Then, $\overline{G}^*(F^o) < \underline{G}(F_{-j}^o)$. Moreover, $\underline{G}(F_{-j}^o) \le \overline{G}^*(F_{-j}^o)$ by Lemma A2. Thus, $\overline{G}^*(F^o) < \overline{G}^*(F_{-j}^o)$, contradicting the optimality of F^o. Hence, $F^o = F_C^o$.

Next, recall our initial supposition that $i \in F^o$ and $g_i^*(F^o) \le c_i$. Since we now know $g_i^*(F^o) > 0$, it must be that $\phi_i(\overline{G}^*(F^o)) - \overline{G}^*(F^o) = w_i - g_i^*(F^o)$. Inserting this into the equilibrium condition in Lemma A1: $\Phi_{F^o}(\overline{G}^*(F^o)) \equiv \sum_{j\in F^o}(\phi_j(\overline{G}^*(F^o)) - \overline{G}^*(F^o)) + \overline{G}^*(F^o) = \sum_{j\in F^o} w_j - C(F^o)$, we obtain

$$\begin{aligned}
\Phi_{F_{-i}^o}(\overline{G}^*(F^o)) &\equiv \sum_{j\in F_{-i}^o}(\phi_j(\overline{G}^*(F^o)) - \overline{G}^*(F^o)) + \overline{G}^*(F^o) = \sum_{j\in F_{-i}^*} w_j - C(F^o) + g_i^*(F^o) \\
&= \sum_{j\in F_{-i}^o}(w_j - c_j) - \underbrace{(c_i - g_i^*(F^o))}_{\ge 0} \\
&\le \sum_{j\in F_{-i}^o}(w_j - c_j) = \Phi_{F_{-i}^o}(\underline{G}(F_{-i}^o)),
\end{aligned}$$

where the last equality follows from Lemma A2. Since $\Phi'_{F_{-i}^o} > 0$, this implies $\overline{G}^*(F^o) \le \underline{G}(F_{-i}^o)$. Moreover, by Lemma A2, $\underline{G}(F_{-i}^o) \le \overline{G}^*(F_{-i}^o)$. Hence, $\overline{G}^*(F^o) \le \overline{G}^*(F_{-i}^o)$. But, this contradicts the optimality of F^o either because $\overline{G}^*(F^o) < \overline{G}^*(F_{-i}^o)$, or because $\overline{G}^*(F^o) = \overline{G}^*(F_{-i}^o)$ and $C(F^o) > C(F_{-i}^o)$. As a result, $g_i^*(F^o) > c_i$.

To prove that F^o must also satisfy C2, suppose, by way of contradiction, that individual i is not in F^o, but that if added to F^o, i's contribution would satisfy $g_i^*(F^o \cup \{i\}) - c_i > 0$. Let $F^o \cup \{i\} \equiv F^+$ and $F_{C,-i}^+ \equiv F_C^+ \setminus \{i\}$. By definition, $F_{C,-i}^+ \subseteq F^o$. Moreover, since $c_i > 0$, we have $g_i^*(F^+) > 0$, which implies that $\phi_i(\overline{G}^*(F^+)) - \overline{G}^*(F^+) = w_i - g_i^*(F^+)$.

Inserting this into the equilibrium condition, $\Phi_{F_C^+}(\overline{G}^*(F^+)) = \sum_{j\in F_C^+} w_j - C(F^+)$, we obtain $\Phi_{F_{C,-i}^+}(\overline{G}^*(F^+)) = \sum_{j\in F_{C,-i}^+} w_j - C(F^o) + (g_i^*(F^+) - c_i)$. If $F_{C,-i}^+ = F^o$, then, since $g_i^*(F^+) - c_i > 0$,

$$\Phi_{F^o}(\overline{G}^*(F^+)) = \sum_{j\in F^o} w_j - C(F^o) + (g_i^*(F^+) - c_i) > \sum_{j\in F^o} w_j - C(F^o) = \Phi_{F^o}(\overline{G}^*(F^o)),$$

where the last equality follows because, by the first part of the proof, $F^o = F_C^o$. But, given that $\Phi'_{F^o} > 0$, we then have $\overline{G}^*(F^+) > \overline{G}^*(F^o)$, which contradicts the optimality of F^o.

Next, suppose that $F_{C,-i}^+ \neq F^o$, or equivalently $F_{C,-i}^+ \subset F^o$. Then, by definition of Φ_{F^o}, we have $\Phi_{F^o}(\overline{G}^*(F^+)) = \sum_{j\in F^o}(\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+)) + \overline{G}^*(F^+)$, or decomposing terms,

$$\begin{aligned}\Phi_{F^o}(\overline{G}^*(F^+)) &= [\sum_{j\in F_{C,-i}^+}(\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+)) + \overline{G}^*(F^+)] + \sum_{j\in F^o\setminus F_{C,-i}^+}(\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+)) \\ &= \Phi_{F_{C,-i}^+}(\overline{G}^*(F^+)) + \sum_{j\in F^o\setminus F_{C,-i}^+}(\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+)).\end{aligned}$$

Since $\Phi_{F_{C,-i}^+}(\overline{G}^*(F^+)) = \sum_{j\in F_{C,-i}^+} w_j - C(F^o) + (g_i^*(F^+) - c_i)$ and $\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+) \geq w_j$ (because $j \in F^o\setminus F_{C,-i}^+$ and thus a free-rider in the set F^+), it follows that

$$\Phi_{F^o}(\overline{G}^*(F^+)) \geq \sum_{j\in F^o} w_j - C(F^o) + (g_i^*(F^+) - c_i) > \sum_{j\in F^o} w_j - C(F^o).$$

Note again that $\sum_{j\in F^o} w_j - C(F^o) = \Phi_{F^o}(\overline{G}^*(F^o))$ because $F^o = F_C^o$. This implies that $\Phi_{F^o}(\overline{G}^*(F^+)) > \Phi_{F^o}(\overline{G}^*(F^o))$, which, in turn, implies that $\overline{G}^*(F^+) > \overline{G}^*(F^o)$, contradicting the optimality of F^o. As a result, i is in F^o, which means F^o also satisfies C2.

($\Longleftarrow$): We prove the uniqueness of the equilibrium fund-raiser set. Suppose, on the contrary, that there are two distinct sets F and F' each satisfying C1 and C2. Note that $F \subset F'$ or $F' \subset F$ cannot be the case: otherwise, C1 or C2 would be violated for at least one set. Next, take any i such that $i \in F'$ but $i \notin F$. By C2, i would be a net free rider in $F \cup \{i\} = F^+$. Then,

$$g_i^*(F^+) - c_i = f_i(w_i - c_i + G_{-i}^*(F^+) - C(F)) - (G_{-i}^*(F^+) - C(F)) \leq 0,$$

which implies that $G_i^0 \leq G_{-i}^*(F^+) - C(F))$, where G_i^0 is given by eq.(2). Therefore,

$$G_i^0 = f_i(w_i - c_i + G_i^0) \leq f_i(w_i - c_i + G_{-i}^*(F^+) - C(F)) \leq \overline{G}^*(F^+).$$

where the first inequality follows from the strict normality: $0 < f_i' < 1$, and the second one by definition of equilibrium. Note also that $\overline{G}^*(F^+) \leq \overline{G}^*(F)$ by the same argument we made in the first part above: removing a net free rider weakly increases the equilibrium public good. Therefore, $G_i^0 \leq \overline{G}^*(F)$

In addition, since i is a net contributor in F' by C1, it follows that

$$g_i^*(F') - c_i = f_i(w_i - c_i + G_{-i}^*(F') - C(F'_{-i})) - (G_{-i}^*(F') - C(F'_{-i})) > 0$$

Hence $G_i^0 > G_{-i}^*(F') - C(F'_{-i})$. Therefore,

$$G_i^0 = f_i(w_i - c_i + G_i^0) > f_i(w_i - c_i + G_{-i}^*(F') - C(F'_{-i})) = \overline{G}^*(F').$$

Thus, $G_i^0 > \overline{G}^*(F')$. Together, the two inequalities reveal that $\overline{G}^*(F) \geq G_i^0 > \overline{G}^*(F')$, which, in turn, reveals $\overline{G}^*(F) > \overline{G}^*(F')$. But, a symmetric argument shows that $\overline{G}^*(F) < \overline{G}^*(F')$, yielding a contradiction. Hence, $F = F'$. ■

Proof of Proposition 3. We first claim that if $\overline{G}^*(F) > 0$ for some F, then $i \in F$ is a net contributor in equilibrium, i.e., $g_i^*(F) - c_i > 0$ if and only if $G_i^0 > \overline{G}^*(F)$. In a positive equilibrium, $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = w_i - g_i^*(F)$, or equivalently $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = (w_i - c_i) - (g_i^*(F) - c_i)$ if $g_i^*(F) > 0$; and $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) \geq w_i$ if $g_i^*(F) = 0$. Since $\phi_i(G_i^0) - G_i^0 = w_i - c_i$ by eq.(2), and $\phi_i' > 1$, the claim follows.

Now, note that since $\phi_{i+1}(G_{i+1}^0) - G_{i+1}^0 = w_{i+1} - c_{i+1}$ by eq.(2), we have

$$\Delta_i - \Delta_{i+1} = G_i^0 - G_{i+1}^0 + \sum_{j=1}^{i} [(\phi_j(G_i^0) - G_i^0) - (\phi_j(G_{i+1}^0) - G_{i+1}^0)].$$

Given that $G_i^0 \geq G_{i+1}^0$ and $\phi_j' > 1$, it follows that $\Delta_i \geq \Delta_{i+1}$. Moreover, $\Delta_1 = G_1^0 > 0$.

Next, let $k \in N$ be the largest number such that $\Delta_k > 0$. Since $\Phi_k(0) = 0$, $\Phi_k' > 0$, and $\sum_{j=1}^k (w_j - c_j) > 0$, there is a unique solution, $\overline{G}^* > 0$, to $\Phi_k(\overline{G}^*) = \sum_{j=1}^k (w_j - c_j)$. If this is an equilibrium, each $i = 1, ..., k$ is a net contributor because $G_i^0 > \overline{G}^*$. $\overline{G}^*$ is an equilibrium among these individuals if $\sum_{j=1}^k c_j - G_{-i}^* < \widehat{C}_i$ for $i = 1, ..., k$. Note that

$$\sum_{j=1}^{k} c_j - G_{-i}^* \leq \sum_{j=1}^{k} c_j - \sum_{j \neq i}^{k} c_j = c_i.$$

But, by Assumption S, $c_i < \widehat{C}_i$. Finally, Lemma A1 implies that $\overline{G}^* = \Phi_k^{-1}(\sum_{j=1}^k (w_j - c_j))$. ■

Lemma A3. *Let $k \in N$ be the largest number such that $\Delta_k > 0$ as in Proposition 3. Then, if the following condition holds:*

$$f_i(w_i - \widehat{C}_i) < \Phi_k^{-1}(\sum_{j=1}^{k}(w_j - c_j)), \tag{A-1}$$

then $\sum_{j=1}^{k} c_j - G_{-i}^* < \widehat{C}_i$ for $i = 1, ..., k$. Moreover, Assumption S implies (A-1).

Proof. By definition, $\sum_{j=1}^{k} c_j - G_{-i}^* = g_i^* - \overline{G}^*$. From Lemma A2, we know that $\overline{G}^* \geq \underline{G}$ where $\underline{G} = \Phi_k^{-1}(\sum_{j=1}^{k}(w_j - c_j))$. Moreover, in any equilibrium person i gives less than his stand-alone contribution at the cutoff cost: $g_i^* \leq \widehat{g}_i = \widehat{C}_i + f_i(w_i - \widehat{C}_i)$. This means that if $\widehat{g}_i - \underline{G} \leq \widehat{C}_i$ is satisfied, so is $\sum_{j=1}^{k} c_j - G_{-i}^* \leq \widehat{C}_i$. Inserting and simplifying terms, $\widehat{g}_i - \underline{G} \leq \widehat{C}_i$ turns into the condition (A-1).

Next, we show that $f_i(w_i - \widehat{C}_i) \leq \Phi_k^{-1}(\sum_{j=1}^{k}(w_j - \widehat{C}_j))$ is automatically satisfied for $i = 1, ..., k$. Let $\widehat{w}_i \equiv w_i - \widehat{C}_i$, which is nonnegative. If $\widehat{w}_i = 0$ for all i, then the inequality trivially follows because $f_i(0) = \Phi_k(0) = 0$. If $\widehat{w}_i > 0$ for some i, then, by Bergstrom et al. (1986), there is a unique equilibrium $G^* > 0$ and $\Phi_k(G^*) = \sum_{j=1}^{k} \widehat{w}_j$. But since $G^* \geq f_i(\widehat{w}_i + G_{-i}^*) \geq f_i(\widehat{w}_i)$ for $i = 1, ..., k$, the inequality again follows. Finally, if Assumption S is satisfied, i.e., $c_i < \widehat{C}_i$ for $i \in N$, we have $\Phi_k^{-1}(\sum_{j=1}^{k}(w_j - \widehat{C}_j)) < \Phi_k^{-1}(\sum_{j=1}^{k}(w_j - c_j))$, which implies (A-1). ■

Proof of Proposition 4. We will show that given the beliefs $\{\mathcal{F}_i\}_{i=1}^n$, contacting $j \notin F^o$ is not a profitable deviation for the fund-raiser. In particular, letting g_j^o be j's contribution in this case, we will show that $g_j^o \leq c_j$. To the contrary, suppose $g_j^o > c_j$. Then, upon being contacted, person j would expect others' gross contributions to be $G^*(F^o)$, resulting in

$$\phi_j(\overline{G}^*(F^o) + g_j^o - c_j) - (\overline{G}^*(F^o) + g_j^o - c_j) = w_j - g_j^o. \tag{A-2}$$

On the other hand, if the individuals in F^o knew about the presence of j before contributing, then, by Proposition 2, we would have: (1) $g_j^*(F_{+j}^o) \leq c_j$, and (2) $\overline{G}^*(F^o) \geq \overline{G}^*(F_{+j}^o)$, where we let $F_{+j}^o = F^o \cup \{j\}$. This is because F^o is optimal and $j \notin F^o$. Furthermore,

$$\phi_j(\overline{G}^*(F_{+j}^o)) - \overline{G}^*(F_{+j}^o) \geq w_j - g_j^*(F_{+j}^o). \tag{A-3}$$

Note that $g_j^*(F_{+j}^o) \leq c_j$ and $c_j < g_j^o$ imply that $g_j^*(F_{+j}^o) < g_j^o$, which in turn implies that $w_j - g_j^*(F_{+j}^o) > w_j - g_j^o$. Then, since $\phi_j' > 1$, eq.(A-2) and (A-3) reveal that $\overline{G}^*(F_{+j}^o) > \overline{G}^*(F^o) + g_j^o - c_j$. In addition, since $\overline{G}^*(F^o) \geq \overline{G}^*(F_{+j}^o)$ by optimality, we must have $g_j^o \leq c_j$, a contradiction. Hence, $g_j^o \leq c_j$. ■

Proof of Lemma 2. From Proposition 3, define $\Delta_i = \overline{\Delta}_i(c) = \Phi_i(G_i^0(c)) - \sum_{j=1}^{i} w_j + ic$ such that $i \in F^o$ if and only if $\overline{\Delta}_i(c) > 0$. Substituting for $\phi_i = \phi$, it follows that $\overline{\Delta}_i'(c) = -\frac{1}{\phi'(G_i^0)-1} < 0$ since $\phi' > 1$. Hence, $i \in F^o$ if and only if $c < \overline{c}_i$, where $\overline{\Delta}_i(\overline{c}_i) = 0$. Simplifying terms, $\overline{c}_i$ solves,

$$i[\phi(G_i^0) - G_i^0] + G_i^0 - \sum_{j=1}^{i} w_j + ic = 0.$$

Since $\phi(G_i^0) - G_i^0 = w_i - c$ from (2), we have $G_i^0(\overline{c}_i) = \sum_{j=1}^{i}(w_j - w_i)$. In addition, given that $\widehat{\phi}(G) \equiv \phi(G) - G$, we also have $\widehat{\phi}(G_i^0(\overline{c}_i)) = w_i - \overline{c}_i = \widehat{\phi}\left(\sum_{j=1}^{i}(w_j - w_i)\right)$, which simplifies to

$$\overline{c}_i = w_i - \widehat{\phi}(\sum_{j=1}^{i}(w_j - w_i)).$$

But, this solution is valid only if it is nonnegative; otherwise the cutoff cost is set to 0 to mean that person i never contacted for a positive cost, as in eq.(3).

To prove the last part, note from (2) that $\overline{c}_i - \overline{c}_{i+1} = w_i - w_{i+1} + \widehat{\phi}(\sum_{j=1}^{i+1}(w_j - w_{i+1})) - \widehat{\phi}(\sum_{j=1}^{i}(w_j - w_i))$. Since $w_i \geq w_{i+1}$ and $\widehat{\phi}' > 0$, it follows that $\overline{c}_i \geq \overline{c}_{i+1}$, as desired. ■

Lemma A4. *Let $u_i = u$ and $c_i = c$ for all $i \in N$. Moreover, let $\mathbf{w}' \neq \mathbf{w}''$ be two income distributions such that $\mathbf{w}'$ Lorenz dominates $\mathbf{w}''$. Then, $\overline{G}^{*\prime} \leq \overline{G}^{*\prime\prime}$. In addition, $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$, if one of the following conditions is satisfied: (1) $\varnothing \neq F_C' = F_C'' \neq N$, (2) $F_C'' \subset F_C' \neq N$, or (3) $F_C' \subset F_C''$.*

Proof. Let $|F_C'| = m'$ and $|F_C''| = m''$. First, consider $\varnothing \neq F_C' = F_C'' \neq N$. Since, $L_{m'}(\mathbf{w}') < L_{m''}(\mathbf{w}'')$, it follows that

$$\Phi_{m'}(\overline{G}^{*\prime}) = \sum_{i=1}^{m'}(w_i' - c) < \sum_{i=1}^{m''}(w_i'' - c) = \Phi_{m'}(\overline{G}^{*''}),$$

which implies that $\overline{G}^{*\prime} < \overline{G}^{*''}$. Next, suppose that $F_C'' \subset F_C' \neq N$, and by way of contradiction, that $\overline{G}^{*\prime} \geq \overline{G}^{*\prime\prime}$. Then,

$$\begin{aligned}
\sum_{i=1}^{m'}(w_i' - c) &= \Phi_{m'}(\overline{G}^{*\prime}) \geq \Phi_{m'}(\overline{G}^{*\prime\prime}) \\
&= \left[\sum_{i=1}^{m''}(\phi(\overline{G}^{*\prime\prime}) - \overline{G}^{*\prime\prime}) + \overline{G}^{*\prime\prime}\right] + \sum_{i=m''+1}^{m'}(\phi(\overline{G}^{*\prime\prime}) - \overline{G}^{*\prime\prime}) \\
&\geq \sum_{i=1}^{m''}(w_i'' - c) + \sum_{i=m''+1}^{m'}(w_i'' - c) \\
&= \sum_{i=1}^{m'}(w_i'' - c),
\end{aligned}$$

where the third line follows from the fact that individuals $\{m''+1, ..., m'\}$ are net free-riders under $\mathbf{w}''$. This implies that $L_{m'}(\mathbf{w}') \geq L_{m'}(\mathbf{w}'')$, which contradicts our hypothesis that $\mathbf{w}'$ Lorenz dominates $\mathbf{w}''$. Thus, $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$.

Finally, consider $F_C' \subset F_C''$. Let $\overline{G}_{m'}^{*\prime\prime}$ be the equilibrium level of the public good if they constituted the whole economy. Since individuals $m'+1, .., m''$ are also contributors under $\mathbf{w}''$, we obtain:

$$\begin{aligned}
\Phi_{m''}(\overline{G}_{m'}^{*\prime\prime}) &= \sum_{i=1}^{m'}(\phi(\overline{G}_{m'}^{*\prime\prime}) - \overline{G}_{m'}^{*\prime\prime}) + \overline{G}_{m'}^{*\prime\prime} + \sum_{i=m'+1}^{m''}(\phi(\overline{G}_{m'}^{*\prime\prime}) - \overline{G}_{m'}^{*\prime\prime}) \\
&< \sum_{i=1}^{m'}(w_i'' - c) + \sum_{i=m'+1}^{m''}(\phi(G_i^{0\prime\prime}) - G_i^{0\prime\prime}) \\
&= \sum_{i=1}^{m''}(w_i'' - c) = \Phi_{m''}(\overline{G}^{*\prime\prime}).
\end{aligned}$$

Then, $\overline{G}_{m'}^{*\prime\prime} < \overline{G}^{*\prime\prime}$. Now, assume, by way of contradiction, that $\overline{G}^{*\prime} \geq \overline{G}^{*\prime\prime}$. It follows that

$$\sum_{i=1}^{m'}(w_i'' - c) = \Phi_{m'}(\overline{G}_{m'}^{*\prime\prime}) < \Phi_{m'}(\overline{G}^{*\prime\prime}) \leq \Phi_{m'}(\overline{G}^{*\prime}) = \sum_{i=1}^{m'}(w_i' - c),$$

which implies that $L_{m'}(\mathbf{w}') > L_{m'}(\mathbf{w}'')$, contradicting Lorenz dominance hypothesis. Hence, $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$. ■

Proof of Proposition 5. From (3), $L_i(\mathbf{w}'') > L_i(\mathbf{w}')$ for every $i < n$ implies $c_n^{*\prime\prime} < c_n^{*\prime}$ and $c_1^{*\prime\prime} > c_1^{*\prime}$. Note also that $c_n^{*\prime\prime} > 0$ since she is a contributor for $c = 0$. Hence, for $c \in [0, c_n^{*\prime\prime})$ all individuals are contributors, $F_C^{0\prime} = F_C^{0\prime\prime}$ and $L_n(\mathbf{w}'') = L_n(\mathbf{w}')$. Therefore,

$\overline{G}^{*\prime} = \overline{G}^{*\prime\prime}$. For $c \in [c_n^{*\prime\prime}, c_1^{*\prime\prime})$, we clearly have that one of the conditions in Lemma A4 is satisfied, and the fact that $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$ follows directly from Lemma A4. ■

Proof of Proposition 6. To prove part (a), note that since $\overline{G}^*(F_R^o) = \max_F\{G(F) - C(F) + R, 0\}$, in equilibrium $G^*(F_R^o) - C(F_R^o) \geq 0$; otherwise $F_R^o = \varnothing$, which guarantees the public good level R. That is, even with an outside grant R, the fund-raiser tries to maximize $G(F) - C(F)$ in equilibrium. In particular, Proposition 2 still holds. Since $\overline{G}^*(F_R^o) > 0$, equilibrium condition for individual $i \in F_R^o$ implies that $\overline{G}^*(F_R^o) = f_i(w_i + G_{-i}^*(F_R^o) - (C(F_R^o) - R))$. This has two implications: First, as in Lemma A1, summing over all $i \in F_R^o$, we obtain $\Phi_{F_R^o}(\overline{G}^*(F_R^o)) = \sum_{i \in F_R^o}(w_i - c_i) + R$. Second, re-arranging terms, we have $g_i^*(F_R^o) - c_i = f_i(w_i - c_i + \sum_{j \neq i}(g_j^*(F_R^o) - c_j) + R) - [\sum_{j \neq i}(g_j^*(F_R^o) - c_j) + R]$. But, this means that $g_i^*(F_R^o) - c_i > 0$ if and only if $\sum_{j \neq i}(g_j^*(F_R^o) - c_j) + R > G_i^0$, where G_i^0 is exactly as defined in eq.(2) for $R = 0$. Thus, i is a net contributor in equilibrium if and only if $\Phi_{F_R^o}(G_i^0) > \sum_{i \in F_R^o}(w_i - c_i) + R$, or equivalently $\Delta_i > R$, where Δ_i is stated in Proposition 3.

To prove part (a), suppose $\overline{G}^*(F_0^o) > 0$. Then, $F_0^o \neq \varnothing$. If $F_R^o = \varnothing$, clearly $F_R^o \subset F_0^o$. Suppose $F_R^o \neq \varnothing$. If $i \in F_R^o$, then $\Delta_i > R$, which implies that $\Delta_i > 0$ and thus $i \in F_0^o$. Hence, $F_R^o \subseteq F_0^o$. Next, we prove part (b), i.e., $\overline{G}^*(F_0^o) < \overline{G}^*(F_R^o)$. Suppose not. That is, suppose $\overline{G}^*(F_0^o) \geq \overline{G}^*(F_R^o)$ even though $F_R^o \subseteq F_0^o$. Then,

$$\begin{aligned}
\sum_{i \in F_0^o}(w_i - c_i) &= \Phi_{F_0^o}(\overline{G}^*(F_0^o)) \\
&\geq \Phi_{F_0^o}(\overline{G}^*(F_R^o)) = \sum_{i \in F_0^o}(\phi_i(\overline{G}^*(F_R^o)) - \overline{G}^*(F_R^o)) + \overline{G}^*(F_R^o) \\
&\geq \sum_{i \in F_0^o}(w_i - c_i) + R,
\end{aligned}$$

implying that $R \leq 0$, a contradiction. Hence, $\overline{G}^*(F_0^o) < \overline{G}^*(F_R^o)$. Finally, note that in equilibrium $g_i^*(F_R^o) = w_i + \overline{G}^*(F_R^o) - \phi_i(\overline{G}^*(F_R^o))$. Since $\overline{G}^*(F_0^o) < \overline{G}^*(F_R^o)$ and $\phi_i' > 1$, it follows that $g_i^*(F_R^o) < g_i^*(F_0^o)$. As a result, by strict normality, $\overline{G}^*(F_R^o) - \overline{G}_0^* < R$. ■

Proof of Proposition 7. Suppose $G_1^0 > G_2^0 > ... > G_n^0 > 0$. Note first that the order of types is preserved under replicas since G_i^0 depends only on (u_i, w_i, c_i). In an $r-$replica economy, define $\Delta_i = \overline{\Delta}_i(r) \equiv \sum_{j=1}^{i} r(\phi_j(G_i^0) - G_i^0) + G_i^0 - \sum_{j=1}^{i} r(w_j - c_j)$, or re-arranging terms,

$$\overline{\Delta}_i(r) = G_i^0 - r\sum_{j=1}^{i}\left[(w_j - c_j) - (\phi_j(G_i^0) - G_i^0)\right]. \tag{A-4}$$

Since $G_j^0 > G_i^0$ by our indexing, and $\phi_j' > 1$ by strict normality, it follows that $w_j - c_j > \phi_j(G_i^0) - G_i^0$ for every $j < i$. Moreover, since $w_i - c_i = \phi_i(G_i^0) - G_i^0$ by definition of G_i^0, eq.(A-4) implies that for $i > 1$, $\overline{\Delta}_i(r)$ is *strictly* decreasing in r, whereas $\overline{\Delta}_1(r) = G_1^0$, which is independent of r.

To prove part (a), observe that since $\overline{\Delta}_1(r) = G_1^0 > 0$ for any r, type-1 donors are always solicited. Consider $i > 1$. Note that $\overline{\Delta}_i(1) \leq 0$ implies that $\overline{\Delta}_i(r) \leq 0$ for any $r \geq 1$. If, on the other hand, $\overline{\Delta}_i(1) > 0$, then there exists $\underline{r}_i < \infty$ such that $\overline{\Delta}_i(\underline{r}_i) \leq 0$. This means $\overline{\Delta}_i(r) < 0$ for $r \geq \underline{r}_i$. Moreover, given that $\overline{\Delta}_i(r) > \overline{\Delta}_{i+1}(r)$, it follows that $\underline{r}_{i+1} \leq \underline{r}_i$ for $i > 1$. Thus, type$-i$ donors are not solicited in a replica economy with $r \geq \underline{r}_i$ for $i > 1$.

To prove part (b), note that as $r \to \infty$, only type-1 will be asked for donations by part (a). Note also that given the symmetry within the limiting group, the equilibrium must be symmetric. Since $G_1^0 > 0$, it follows that $\overline{G}^* > 0$ and $\overline{G}^* < G_1^0$; otherwise $\overline{G}^* \geq G_1^0$ would imply no contribution, yielding a contradiction. The symmetric equilibrium means that each type-1 donor is a net contributor, which means that $\overline{G}^*$ is monotonically converging to G_1^0. Since G_1^0 is a finite level, we must have $g_1^* \to c_1$; otherwise, if, in the limit, $g_1^* - c_1 > 0$, then $\overline{G}^* \to \infty$, making everyone contribute nothing, a contradiction. ■

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