Absolute auctions and secret reserve prices: Why are they used?

Philippe Jehiel* and Laurent Lamy†

Abstract

Absolute auctions, i.e. auctions with a zero public reserve price, and auctions with secret reserve prices are frequently used by various auction sites. However, it is hard to find a rationale for these formats. By introducing the possibility that buyers differ in their understanding of how the participation rate varies with the auction format, and how reserve prices are distributed when secret, we show in a competitive environment that these auction formats may endogenously emerge. We also analyze how the objects of various quality sort into the different auction formats in equilibrium and the cognitive characteristics of the buyers who participate in the various auctions, thereby offering a range of testable predictions.

Keywords: Auctions, absolute auctions, secret reserve prices, endogenous entry, rational expectations, cognitive limitations.

JEL classification: D03, D44

1 Introduction

EBay provides a wonderful large-scale laboratory for the analysis of the kind of auction formats and instruments that are used by real sellers to sell their goods. One observation that can be made is that auctions with no or very low reserve prices are frequently used even in cases in which it would seem that the reservation value of the seller is above the chosen reserve price.[1] Another is that secret reserve prices are also often used, and especially for items of high quality (Bajari and Hortaçsu, 2003). Furthermore, field experiments have shown that those formats may be profitable for the seller depending on the characteristics of the good.[2]

*Paris School of Economics and University College London. e-mail: jehiel@pse.ens.fr.

†Paris School of Economics. e-mail: lamy@pse.ens.fr.

[1] See the survey in Hasker and Sickles (2010).

[2] For collectible trading cards, Reiley (2006) finds that absolute auctions raise 25% higher revenue than auctions with a reserve approximatively equal to the book value of the card in the case of low-quality cards;

Auctions with no reserve price are sometimes referred to as absolute auctions. An informal argument proposed in their favor is that not posting a reserve price is one way of attracting more participants in the auction, which is advantageous to sellers. That more participation is to be expected with lower reserve prices seems obviously right, and this is actually confirmed in the empirical literature.[3] However, greater participation is not the seller's objective per se, and if the good then happens to be sold at too low a price (below the seller's valuation), the seller would have been better off keeping the object. Theoretical models of auctions with endogenous participation (Levin and Smith 1994, McAfee 1993)[4] all conclude that in scenarios with rational buyers, endogenous participation should lead rational sellers to post reserve prices at their own valuation level. These theoretical models do not therefore explain why absolute auctions (or more generally, any reserve price below the seller's valuation) are used.

An informal argument sometimes offered in favor of secret reserve prices is that if one would like to post a high reserve price then it is better to keep this reserve price secret so as not to discourage participation. However, if bidders are fully rational, they should anticipate this, thereby canceling out the effect of disclosure choice. An unraveling argument should therefore be expected: the sellers who offer the most attractive (i.e. lowest) reserve prices amongst the secret prices would prefer to disclose this reserve price publicly, thereby rendering the use secret reserve prices unstable in equilibrium.[5] It should also be noted that, in private-value environments with endogenous participation, a rational seller choosing a secret reserve price should choose this price strictly above her valuation so as to exploit better her monopoly power, which in turn, if bidders' expectations are correct, would lead to suboptimal levels of participation, as shown by Levin and Smith (1994) and McAfee (1993). In these contexts, auction models with fully rational buyers do not then explain the attraction of secret reserve prices.

This paper provides a simple rationale for the use of both absolute auctions and secret

this difference is small and insignificant for cards of higher quality. The evidence on the profitability of the use of secret reserve prices is mixed (see the discussion in Bajari and Hortaçsu, 2004). It should be underlined that field experiments analyzing the effect of secret reserves (see e.g. Katkar and Reiley (2006)) suffer from a major concern in that they compare a public reserve to a secret reserve set at the same level, whereas the optimal secret and public reserves differ in models with endogenous entry (see the analysis below).

[3] We abstract from signalling issues that may invalidate this argument. These will be discussed later.

[4] These consider respectively the case where potential entrants learn their valuation after and before their participation decision. The respective terminologies are auctions with entry and auctions with participation costs. See also Peter's (2011) survey on competing mechanisms.

[5] Two previous attempts to solve the secret reserves 'puzzle' rely on the possibility of committing ex ante to stick to secret reserves, i.e. before the seller learns her valuation. Li and Tan's (2000) argument is based on risk aversion and works for first-price auctions but not for second-price or English auctions. The example developed by Vincent (1995) relies on interdependent valuations.

reserve prices while maintaining a simple pure private value framework. In our model, goods of heterogeneous quality are sold, and potential buyers must decide in which auction to participate. A key feature of our model is that not all bidders are assumed to be fully rational. Specifically, bidders are distinguished according to their statistical understanding of what the observable characteristics of the auction imply for participation, and what not disclosing the reserve price implies for the distribution of reserve prices. Bidders are also distinguished according to whether or not they observe (or pay attention to) all characteristics of the objects when deciding in which auction to participate.

Specifically, our various types of bidders are described as follows. Fully-rational bidders have a perfect understanding of how the observable characteristics of the auction relate to both participation and the distribution of the reserve price if this price is not disclosed. Fully-coarse bidders know the aggregate distribution of participation over all auctions, but not how it depends on the reserve price whether public or secret. Partially-coarse bidders know how participation varies across the various auction formats, but only know the aggregate distribution of reserve prices (not how reserve prices are distributed when secret).

To make our points more simply, we assume that fully-coarse bidders do not observe the quality of the object, and think that quality is independent of the proposed auction format.[6] Partially-coarse and fully-rational bidders do observe quality. We also assume that bidders learn the valuation of the object after having chosen in which auction to participate and that they know what these distributions are. Finally, sellers are assumed to be fully rational (i.e. they make the optimal choice of format given the behaviors of the various types of bidders, as described).

We establish the existence of a competitive equilibrium in such an environment. In any such competitive equilibrium, low-quality objects are sold through absolute auctions (even though sellers' valuations are strictly positive), secret reserve prices are used for very high-quality objects (with a reserve price set at Myerson's optimal level, i.e. above the valuation), and public reserve prices set at the seller's valuation are used for goods of intermediate quality. Fully-coarse bidders all choose absolute auctions. Fully-rational bidders all choose the objects which are sold with a strictly positive public reserve price, and partially-coarse bidders choose objects which are sold either with a secret reserve price or a strictly positive public reserve price.

[6] That is, these bidders are also coarse in that dimension. We briefly discuss that our main insights would remain the same were instead these bidders to make accurate inferences about the quality distribution (from the observable characteristics of the auction format).

The predictions of our model are consistent with many empirical and experimental observations. As pointed out by Bajari and Hortaçsu (2003) and Hossain (2008), secret reserve prices tend to be used for higher-quality objects, and Simonsohn and Ariely (2008) and Choi et al. (2010) note that the higher the public reserve price, the more experienced (and presumably the more knowledgeable) the bidders who choose it. In Section 5, we discuss in more detail the link between our results and the empirical literature.

The form of bounded rationality exhibited by our non-fully rational buyers is somewhat related to some well-known issues in econometrics. In the real world, variables such as the participation decision or the distribution of reserve prices when secret are likely to depend on a number of dimensions such as the observable characteristics of the objects for sale and the format of the auction. The econometrician faced with the task of estimating these links typically faces the well-known curse of dimensionality (it is hard/impossible to estimate statistical relations that depend on too many dimensions). To get around this problem, econometricians typically use dimension-reducing techniques: the standard in the structural econometrics of auction data (see the textbook of Paarsch and Hong (2006)) consists in creating a single-index and assuming implicitly that the distributions to be estimated depend solely on the auction characteristics via this index. An additional difficulty in forming correct expectations regarding the strategies used by other players comes from the fact that past data may be affected by unobserved heterogeneity insofar as a variable which played a key role and was observable by the agents at the time the auction took place may no longer be accessible to the current-day analyst. This is currently a hot topic for econometricians.[7] A related difficulty arises for secret reserve prices as the reserve price is usually not observed ex-post, at least when the reserve price is not reached during the course of the auction, making it difficult to estimate the distribution of such prices.

It seems plausible that real bidders face the same kinds of problems as do econometricians and that, in a first stage and based on both their initial (ad hoc) beliefs and the amount of data which they have available, they thus decide to drop some variables that do not seem (to them) to be of primary importance for their estimations. From this perspective, we interpret our various types of bidders as those who use different econometric specifications, where more experienced agents make better modeling choices and form their expectations conditional on more and more relevant variables. From a theoretical point of view, the competitive equilibrium we define in our environment with cognitive limitations can be seen as an adaptation of the analogy-based expectation equilibrium (Jehiel, 2005)

[7]See, e.g., Krasnokutskaya (2011) and An et al. (2010) for methods to deal with respectively an unobserved common component and an unobserved number of potential participants.

to the case of a continuum of players.

The remainder of the paper is organized as follows. Section 2 describes the auction framework, and Section 3 analyzes the case of fully-rational bidders, confirming that neither secret reserve prices nor absolute auctions prevail in this scenario. Section 4 then provides the main results of the paper, and Section 5 discusses our results in the light of existing evidence. Section 6 briefly sketches some extensions, including the analysis of a dynamic setting in which bidders make their participation decisions sequentially. Last, Section 7 concludes.

2 The model

We consider a model of competition between many sellers auctioning goods of various qualities to many buyers. We study a limit economy with a continuum of buyers and sellers, whereby a single deviation by one seller has no effect on buyers' overall participation decisions. Accordingly, we propose a notion of equilibrium for this limit economy that we refer to as a competitive equilibrium.

We first describe the underlying economic environment, and then analyze the competitive equilibrium first for rational buyers (Section 3) and then a mixed population of buyers who vary in their cognitive sophistication (Section 4).

Let b denote the total mass of buyers, while the mass of sellers is normalized to 1. Each seller owns one good whose quality q is distributed according to a continuously-differentiable cumulative distribution G with support $[\underline{q}, \overline{q}] \subseteq (0, \infty)$. The quality q of the various objects is assumed to be observed by everyone in the benchmark model. We will later on consider the case in which some buyers do not observe this quality q.

We assume that q fully determines both the distribution of bidders' valuations and how much the seller values the object (possibly because, for the seller, the value is fully determined by the expected selling price in future auctions). We let $v_s(q) = q + H(q)$ denote the valuation of the seller of a good of quality q. $H(.)$ is assumed to be positive and continuously differentiable, with $H' = h > -1$ so that the seller's valuation is above the quality q (which is also the minimum possible valuation of buyers, see below) and is strictly increasing in q. Buyers are assumed to participate in just one auction and learn how much they value the object for sale after they make their participation decision.[8]

[8]One motivation for these assumptions is that to assess how valuable a good is, bidders need to spend time inspecting the characteristics of the good as well as the characteristics of the seller (which may affect amongst other things shipping costs and the chance that the good will be delivered), thereby making it too costly to participate in several auctions. The private-value assumption is justified by the valuation being

The valuation of a buyer of a good of quality q is given by $q + \varepsilon$, where ε is drawn from a random variable that is independently distributed across buyers according to a continuously-differentiable cumulative distribution function F whose support is $[0, \infty)$ and such that the mean of F is well defined (which guarantees that the various integrals appearing below are well defined).[9] Note that the distribution of ε does not depend on q, which simplifies somewhat the analysis. Again, for simplicity, we assume that the function $x - \frac{1-F(x)}{f(x)}$ (where $f(x) = F'(x) > 0$ denotes the density) is strictly increasing in x. The latter assumption corresponds to Myerson's (1981) regularity assumption, which helps characterize the optimal reserve price when secret. We next use the notation $F^{(i:n)}$ for the CDF of the i^{th} order statistic of n independent draws from the distribution F, i.e. $F^{(1:n)}(x) = F^n(x)$, $F^{(2:n)}(x) = n \cdot F^{n-1}(x) - (n-1) \cdot F^n(x)$, etc.

The timing of events is as follows. Sellers simultaneously choose their auction format. The choice of the auction format bears on the reserve price r and whether to disclose it to buyers or keep it secret. The potential buyers simultaneously decide in which auction to participate, based on what they observe about the auction. As already mentioned, we assume that buyers can participate in just one auction. The remainder of the auction follows the rules of a second-price auction with the chosen reserve price. That is, bidders submit bids simultaneously. The bidder with highest bid wins if this bid exceeds the reserve price and pays the maximum of the second highest bid and the reserve price. This description fits with the practice on eBay and more generally on many Internet auction sites.[10] With this auction format, once a buyer participates, he bids his valuation.

A reserve price policy consists of a reserve price $r \in R_+$ and a disclosure policy $d \in \{public, secret\}$ for each level of quality q. Each seller's strategy space $R_+ \times \{public, secret\}$ is denoted by S. We also denote by $S^* = R_+ \cup \{secret\}$ the set which captures what buyers observe from the proposed auctions in addition to the quality q, i.e. either the level of the public reserve price $r \in R_+$ or that the reserve price has been set secretly, $d = secret$.[11] For any $s = (r, d) \in S$, let $\widehat{s} \in S^*$ denote the observable characteristics of the reserve price policy s. That is, $\widehat{s} = r$ if $d = public$ and $\widehat{s} = secret$ if $d = secret$.

In the following, we distinguish three classes of auction formats: absolute auctions for

fully determined after the inspection of the buyer.

[9] After integration by parts, note that the integrals which have the form $\int_s^\infty U(x)(1 - F(x))dx$, where $U(.)$ is a bounded function, are thus well defined.

[10] There is a mild difference between eBay and our format: we did not consider the possibility of using minimum bids when the reserve price is kept secret. However, positive minimum bids would never arise in our model. We discuss this in Section 5.

[11] In an auction with a secret reserve on eBay, the listing shows the message "Reserve not met" until a bid above the reserve has been submitted, thereby making it clear that the format is one with a secret reserve price.

which the reserve price is null and public, open reserve auctions for which the reserve price is strictly positive and public, and secret reserve price auctions for which the reserve price is not disclosed. The corresponding acronyms are AA, OR and SR.

The expected payoff of a seller with a good of quality q from holding a second-price auction with a reserve price of r (either public or secret) when she is matched with exactly n buyers is given by

$$\Phi_n(r,q) = q + H(q) \cdot F^{(1:n)}(r-q) + (r-q) \cdot [F^{(2:n)}(r-q) - F^{(1:n)}(r-q)] + \int_{r-q}^{\infty} xd[F^{(2:n)}(x)]. \tag{1}$$

She will obtain q in any event, an extra $H(q)$ when no bidder has a valuation greater than r (that is, $\varepsilon < r-q$ for all bidders, which occurs with probability $F^{(1:n)}(r-q)$), an extra $r-q$ when only one bidder has a valuation greater than r (this occurs with probability $[F^{(2:n)}(r-q) - F^{(1:n)}(r-q)]$) and an extra $\int_{r-q}^{\infty} xd[F^{(2:n)}(x)]$ when at least two bidders have a valuation above r.

Using standard results from auction theory, a buyer with valuation $u \geq r$ who participates in the seller's auction with n other buyers when the reserve price is r and the quality is q will receive the expected payoff of $\int_r^u F^n(x-q)dx$.[12] The ex ante payoff of a buyer entering such an auction, i.e. the expected payoff before knowing what his valuation will be, is thus given by

$$V_n(r,q) = \int_{r-q}^{\infty} F^n(x)(1-F(x))dx. \tag{2}$$

The sum $\Phi_n(r,q) + nV_{n-1}(r,q)$ corresponds to ex ante welfare, denoted by $W_n(r,q)$, which is also given by

$$W_n(r,q) = q + H(q) \cdot F^{(1:n)}(r-q) + \int_{r-q}^{\infty} xd[F^{(1:n)}(x)]. \tag{3}$$

It is well-known from the work of Myerson (1981) that, whatever the number n of bidders, maximizing the expected seller's payoff $\Phi_n(r,q)$ with respect to r leads to

$$r^M(q) = q + \varepsilon_q^M \tag{4}$$

[12]This is the integral of the interim probability that a bidder with valuation x wins the object as x varies from r to u.

where ε_q^M corresponds to the (unique) solution of the equation

$$\varepsilon_q^M - \frac{1 - F(\varepsilon_q^M)}{f(\varepsilon_q^M)} = H(q).$$

This result has a simple implication for the choice of the reserve price if secret. With a secret reserve price, participation is independent of r (since r is not observed), which implies that the seller should optimally pick a reserve price equal to $r^M(q)$.[13]

A key ingredient of the following analysis lies in the understanding of how many bidders participate in the various auction formats and how this is perceived by buyers at the time they choose their auction. Of course, on average over all public characteristics there are b bidders per auction, since buyers participate in just one auction and the ratio of buyers to sellers is b. But, for a specific auction, the effective number of participants is taken to be the realization of a random variable following a Poisson distribution with parameter $\mu \geq 0$. That is, the probability that there are n bidders in the auction is $e^{-\mu}\frac{\mu^n}{n!}$.[14] For a given auction with public characteristics in $S^* \times [\underline{q}, \overline{q}]$, the parameter μ is chosen to match the ratio of the density (or measure if appropriate) of auctions with such characteristics to the density (or measure if appropriate) of bidders choosing an auction with these characteristics. The Poisson distribution corresponds to the limit as the number of bidders goes to infinity of the distribution of the number of participants in a given auction, assuming that the bidders' participation decision is randomized uniformly over auctions with identical observable characteristics.[15]

3 Equilibrium with rational buyers

3.1 Definition

In this section we define a competitive equilibrium assuming that all buyers are fully rational. We first consider the case in which the quality q is observed by buyers, and then suggest how the analysis would be modified were a positive share of buyers not to observe q. This section serves to establish that when buyers are fully rational, neither the use of absolute auctions nor the use of secret reserve prices can be rationalized. As we shall see,

[13] This is actually true when there is a positive number of entrants with strictly positive probability. To simplify the presentation, we also assume that the seller selects $r^M(q)$ if there are no entrants. This can be viewed as a trembling-hand refinement.

[14] Note that the sum of two independent Poisson distributions with parameters μ_1 and μ_2 is a Poisson distribution with parameter $\mu_1 + \mu_2$.

[15] Wolinsky (1988) considers a related limit model in a search environment.

in equilibrium, a seller of quality q with reservation utility $v_s(q)$ opts for a public reserve price set at the socially-optimal level $r = v_s(q)$, and this remains an equilibrium even if we introduce a (sufficiently) small fraction of buyers who do not observe q. The results of this section can be viewed as a confirmation of the insights obtained in earlier models, in particular Levin and Smith (1994) and Peters and Severinov (1997),[16] even though it should be noted that the earlier literature did not consider the possibility of heterogeneous qualities nor (a fortiori) uninformed buyers (secret reserve prices were also not considered, with the exception of McAfee (1993) who allows for arbitrary mechanisms).[17]

As noted above, we abstract away from any subtleties due to the presence of finitely many buyers and sellers by directly considering the limit of an arbitrarily large number of buyers and sellers, i.e. we appeal to a large market hypothesis.[18] Accordingly, a deviation by a single seller cannot affect the expected utility of buyers, which is thus taken as given by every individual seller. Sellers pick the auction format so as to maximize their expected payoff given their expectations over participation. Moreover, the auction format affects participation so as to equate the buyers' expected utility from participating in the auction with their expected equilibrium utility, and this is correctly anticipated by sellers.

To define the competitive equilibrium formally, we introduce for each q, the strategy of sellers of the good of quality q that we denote by ρ_q. This is a measure over possible auction formats S (possibly concentrated on just one $s \in S$). We introduce for each q a Poisson parameter function $\mu_q : S^* \to R_+ \cup \{\infty\}$, where $\mu_q(\widehat{s})$ characterizes the distribution of participation for an auction with observable characteristic $\widehat{s}$ of a quality q object, where $\widehat{s}$ is derived from the auction format $s = (r, d)$ as above.

Definition 1 *A competitive rational-expectation equilibrium (CRE-equilibrium) is defined as* $(\rho_q, \mu_q)_{q \in [\underline{q}, \overline{q}]}$*, where* ρ_q *stands for the strategy of a quality* q *seller and* $\mu_q : S^* \to R_+ \cup \{\infty\}$ *describes the distributions of participation in the various auction formats (of goods of quality* q*) where*[19]

[16] In their section devoted to competing auctions with entry, Peters and Severinov (1997) provide a series of conditions for competitive equilibria. Although their formal analysis is correct, they wrongly conclude in their comments that the equilibrium reserve price lies strictly above the seller's reservation value.

[17] Peters (1997) generalizes McAfee (1993), in particular by allowing heterogeneity among sellers' reservation values.

[18] See Hernando and Veciana (2005) and Virag (2011) for formal results which lend support to this assumption. Under conditions that guarantee the existence of a symmetric pure strategy equilibrium between (homogenous) sellers, Virag (2011) shows that the equilibrium reserve price rises as the market gets smaller. In small markets, pure strategy equilibria may not exist, as shown by Burguet and Sakovics (1999), who also demonstrate that the reserve prices proposed in (possibly mixed) equilibria always lie strictly above the sellers' reservation value.

[19] Throughout the paper we follow the convention: $\sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} \Phi_n(r, q) := \infty$ and $\sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} V_n(r, q) := 0$ if $\mu = \infty$.

1. *(Profit maximization for sellers) for any* $q \in [\underline{q}, \overline{q}]$,

$$Supp(\rho_q) \subset Arg \max_{s=(r,d)\in S} \sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} \Phi_n(r,q). \tag{5}$$

2. *(Profit maximization for buyers) for any* $q \in [\underline{q}, \overline{q}]$ *and* $\widehat{s} \in S^*$

$$\mu_q(\widehat{s}) > 0 \Longrightarrow \sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} V_n^{FR}(\widehat{s}, q) = V^{FR}; \text{ and } \mu_q(\widehat{s}) = 0 \Longrightarrow V_0^{FR}(\widehat{s}, q) \leq V^{FR}, \tag{6}$$

where $V^{FR} := \int_{\underline{q}}^{\overline{q}} \left[\int_S \left(\sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} V_n^{FR}(\widehat{s}, q) \right) \cdot \frac{\mu_q(\widehat{s})}{b} \rho_q(s) ds \right] dG(q)$, $V_n^{FR}(\widehat{s} = r, q) := V_n(r, q)$ *and* $V_n^{FR}(\widehat{s} = secret, q) := \frac{\int V_n(r,q)\rho_q(r,secret)dr}{\int \rho_q(r,secret)dr}$ *if* $\int \rho_q(r, secret)dr \neq 0$.

3. *(Matching condition)* $\int_{\underline{q}}^{\overline{q}} \left[\int_S \mu_q(\widehat{s}) \rho_q(s) ds \right] dG(q) = b$.

Part 1 of the definition implies that a seller of quality q is required to pick a format which maximizes her expected payoff given the participation rate $\mu_q(\widehat{s})$ attached to any format $s = (r, d)$ with observable characteristic $\widehat{s}$. In Part 2, V^{FR} is the expected equilibrium payoff of a buyer. Condition (6) implies that whatever the format, either the participation rate is positive and delivers an expected equilibrium utility to buyers of V^{FR}, or the participation rate is zero and the corresponding expected payoff of a buyer (with no other entrant) $V_0^{FR}(\widehat{s}, q)$ is lower than V^{FR}. Part 3 reflects the constraint that buyers must participate in one and only one auction and the aggregate ratio of buyers to sellers is b.

It should be noted that the participation rates $\mu_q(\widehat{s})$ are defined irrespective of whether a format $s = (r, d)$ is offered in equilibrium (by a seller of quality q). It is determined to ensure that a buyer who participates in such an auction would obtain his equilibrium utility V^{FR}. This specification of the participation rates (covering also non-chosen formats) is a simple way to capture the trembling hand refinements that rule out non-meaningful equilibria.

3.2 Analysis

As already explained in Section 2, if a quality q seller chooses a secret reserve price, she must pick Myerson's reserve price $r^M(q)$. Thus, if there exists some CRE-equilibrium where secret reserve prices are used then by replacing the secret reserve strategy by its public reserve counterpart $r^M(q)$ we can build a CRE-equilibrium where the secret reserve strategy is not used. In a first step, we characterize the CRE-equilibria where only public reserve prices are selected. We will later on argue that secret reserve prices cannot be used in equilibrium.

From (5) and (6), any chosen public reserve price solves the maximization program:

$$\max_{r\geq 0} TW_q(\mu_q(r), r) \tag{7}$$

$$\text{where} \qquad TW_q(\mu, r) = \big[\sum_{n=0}^{\infty} e^{-\mu}\frac{\mu^n}{n!}W_n(r,q)\big] - \mu \cdot V^{FR} \tag{8}$$

is the total expected welfare (net of the expected opportunity cost of participation, which is equal to V^{FR} per buyer) associated with the auction with reserve price r proposed by a seller of quality q when the participation rate is μ .

As buyers obtain the incremental surplus they generate in the second-price auction with the reserve price r set at the seller's valuation $r = v_s(q)$, it is readily checked that the function $\mu \to TW_q(\mu, v_s(q))$ is maximized for the equilibrium participation rate $\mu = \mu_q(v_s(q))$. Thus,

Lemma 3.1 $Arg \max_{\mu\geq 0} TW_q(\mu, v_s(q)) = \{\mu_q(v_s(q))\}$

Furthermore, it can readily be checked from (3) that, for $n \geq 1$, maximizing expected welfare $W_n(r, q)$ with respect to r requires setting the reserve price at the seller's valuation: $r = v_s(q)$. If $\mu_q(v_s(q)) > 0$, then it follows that the maximization program $\max_{\mu\geq 0, r\geq 0} TW_q(\mu, r)$ has a unique solution given by $\mu = \mu_q(v_s(q))$ and $r = v_s(q)$ which coincides with the solution of program (7).

This result also implies that secret reserve prices are not used. Indeed, were a secret reserve price to be used for a quality q object, it would be set at $r^M(q)$ and a public reserve price of $r = v_s(q)$ would be strictly preferable, as we have just shown.

At this stage, we have shown the uniqueness of the sellers' strategy in a CRE-equilibrium: sellers propose open reserve price auctions in which the reserve price is set at their valuations.[20] From now on, we refer to such OR auctions as truthful-open auctions or TO. Furthermore, the first-best is implemented, i.e. for any $r \in Supp(\rho_q(., public))$ we have $(\mu_q(r), r) \in \text{Arg}\max_{\mu, r\geq 0} TW_q(\mu, r)$. To complete the description of the CRE-equilibrium, it remains to determine the equilibrium participation rate at the various qualities q and buyers' equilibrium utility V^{FR}. These are determined by requiring that, in equilibrium, buyers should be indifferent across all auctions, which defines a differential equation for the relationship between the participation rate and quality q. Moreover, the matching condition

[20]Uniqueness is actually slightly abusive: there is some slack w.r.t. to the sellers' strategy once $\mu_q(v_s(q)) = 0$. These sellers are obviously indifferent between all auctions leading to no participation. Regarding the expected payoff of the various agents, these indifferences are innocuous.

fixes the constant which is left unspecified in the differential equation. This is summarized in the following proposition.[21]

Proposition 3.2 *There is a unique CRE-equilibrium. Sellers propose truthful-open auctions while the participation rate in an auction for a quality q object $\mu^*(q) := \mu_q(v_s(q))$ is characterized as the unique solution of the differential equation*

$$y'(q) = -h(q) \cdot \frac{(1 - F(H(q))) \cdot e^{-y(q)(1-F(H(q)))}}{\int_{H(q)}^{\infty} (1 - F(x))^2 \cdot e^{-y(q)(1-F(x))} dx} \tag{9}$$

at any q such that $\mu^*(q) > 0$ together with the condition $\int_{\underline{q}}^{\overline{q}} \mu^*(q) \cdot dG(q) = b$. The equilibrium so obtained implements the first-best.

Remark. A special case of our analysis is obtained when $v_s(q)$ coincides with the minimum buyer's valuation, i.e. $H(\cdot) \equiv 0$. In this case $h(q) = 0$, and thus from (9) $\mu^*(q)$ is constant: this pertains as the reserve price of q makes all auctions (whatever q) equally attractive if the distribution of participants is the same.

Before turning to the main part of the analysis, introduces buyers with limited cognitive sophistication, we wish to consider a natural perturbation of the model while maintaining the rational expectation paradigm. More precisely, we consider the possibility that some share of buyers do not observe q when making their participation decision, but remain perfectly rational in every dimension. Specifically, compared to the model above, we assume that there is a share λ^{UN} of buyers who do not observe q while the remaining share $\lambda^{FR} := 1 - \lambda^{UN}$ of buyers do perfectly observe q, with all buyers being perfectly rational. We have the following result (see Appendix C for a formal definition of a competitive equilibrium in this case):

Proposition 3.3 *Consider an environment where $\mu^*(q)$ is bounded away from zero in the CRE-equilibrium derived in Proposition 3.2.*[22]

If either λ^{UN} is small enough, ceteris paribus, or if for a given $\lambda^{UN} < 1$, b is large enough, ceteris paribus, then there exists a competitive equilibrium implementing the first-best: sellers propose truthful-open auctions while the joint participation rates of informed and uninformed buyers correspond to the (efficient) rates derived in Proposition 3.2.

[21] Proposition 3.2 characterizes the participation rate $\mu^*(q)$ in any auction of a quality q object. We can derive V^{FR} from this using the expression for V^{FR} shown in Definition 1.

[22] These equilibria are the only relevant ones bearing in mind that organizing an auction involves some positive sunk costs such that a minimum participation rate is required to recover those costs.

When the share of uninformed buyers is small or the ratio of buyers to sellers is large, the equilibrium is identical to that obtained when all buyers observe q. This comes about as, were a seller of a quality q object to deviate by proposing the auction selected, in equilibrium, by a seller of a quality $q' \neq q$ object, then uninformed buyers will participate as if the quality were q', but informed rational buyers adapt their participation so that, in the end, the same joint participation rate as with only informed buyers will prevail in equilibrium. This makes such a deviation non-profitable, explaining Proposition 3.3.[23]

Proposition 3.3 does not address the case of a large share of uninformed buyers. When this share is sufficiently large, we would expect sellers to post reserve prices above their valuations due to a strategic desire to signal higher quality via a higher reserve price. However, a complete analysis of this signaling problem goes beyond the scope of this paper.[24]

Overall then, the possibility that (some) buyers be uninformed of the quality q rationalizes neither the use of absolute auctions nor the use of secret reserve prices when (all) buyers are assumed to be fully rational.[25]

4 Equilibrium with rational and coarse buyers

4.1 Definition

The main contribution of this paper is to illustrate how the introduction of boundedly-rational buyers may make absolute auctions and secret reserve prices desirable for (some) sellers. The analysis will also reveal for which qualities the various auction formats are used and the cognitive characteristics associated with the various auctions, thereby offering a set of predictions for the proposed model. We will later on review the existing empirical evidence and check that it is consistent with the predictions of our model. In the following analysis, we will make the following assumptions:

[23]Proposition 3.3 says nothing about the share of informed buyers needed to stabilize the market. This depends on the primitives of the model, such as the CDF F, the reservation values $v_s(q)$ and the mass of buyers b. We can illustrate how large λ^{UN} can be without destabilizing the equilibrium via a simple example. Consider the uniform distribution on $[0, 1]$ for F and $H(.) \equiv 0$. We consider the candidate equilibrium where both informed and uninformed buyers enter with, respectively, the Poisson parameter intensities $(1 - \lambda^{UN})b$ and $\lambda^{UN}b$. For any distribution G and for respectively, $b = 1, 1.5, 2, 3, 4, 5$, the above candidate equilibrium with a mixture of informed and uninformed buyers is an equilibrium if λ^{UN} is respectively smaller than 0.46, 0.60, 0.71, 0.85, 0.93 and 0.97. In this example, the equilibrium participation rate $\mu^*(.)$ derived in Proposition 3.2 is a constant function so that it is sufficient to check that the public reserve price $r^M(q)$ is not profitable.

[24]This qualitative insight comes out in Cai et al.'s (2007) model, in which entry is exogenous, valuations are (possibly) interdependent and no buyer observes the quality q; we suspect that it also emerges in our environment with endogenous entry and private values.

[25]In Section 5, we also briefly consider the case of risk averse (but fully rational) buyers and suggest that risk aversion does not convincingly explain the emergence of absolute and secret reserve price auctions.

Assumption A 1 $h(q) \geq 0$.

Assumption A 2 *The solution* y^* *of the differential equation (9) on the interval* $[\underline{q}, \overline{q}]$ *with the condition* $\int_{\underline{q}}^{\overline{q}} y^*(q) \cdot dG(q) = \lambda^{FR} \cdot b$ *satisfies* $y^*(q) > 0$ *for all* $q \in [\underline{q}, \overline{q}]$.

Note that in the case in which all buyers are fully rational, Assumption 1 ensures that the higher the quality the lower the participation rate (see Proposition 3.2), and Assumption 2 ensures in the rational case (and also in the case analyzed here) that all quality q sellers choose auctions that attract non-zero participation in equilibrium. Assumption 1, while not necessary, allows us to simplify the way in which we obtain the sorting of sellers into the various auction formats.

The main distinctive feature of the model we study now is that buyers differ in their understanding of how the auction format (r, d) affects participation, and how reserve prices are distributed when it is only known that the reserve price is set secretly. To simplify the analysis, we also assume that the buyers who are the least sophisticated in their understanding of the determinants of participation are also those who do not observe the quality q (we will later on relax this assumption in the case of homogeneous qualities, and find that the same qualitative insights result). All buyers are otherwise assumed to know the distribution from which valuations are drawn, and are thus able to compute $V_n(r, q)$, for any reserve price r and any quality q.[26] Sellers, on the other hand, are assumed to be perfectly rational.[27]

More precisely, we consider the following types of coarse understanding for buyers. In the first type, only the aggregate participation rate over all auctions is known, that is, it is not known how the reserve price affects participation. This corresponds to buyers who only look at how many bidders participated in past auctions without relating this number to the reserve price.[28] In the second type, when the reserve price is not observed (because it is secret), it is believed that the distribution of reserve prices matches the aggregate distribution of reserve prices whether public or secret. This corresponds to buyers who

[26]Once buyers make the correct assumption that valuations are drawn according to q plus a noise, then their expectations over the distribution of valuations (which rely, e.g., on estimations from the observation of historical data) are unbiased independently of how they aggregate their expectations over auction formats and qualities. This comes from our assumption that the CDF F does not depend on q. From this perspective, we are consistent when assuming that both rational and coarse buyers make correct expectations over the distribution of their opponents' valuations.

[27]Bounded rationality on the side of sellers is not required to explain the emergence of the AA and SR formats. Introducing such bounded rationality would only obscure the main message of the paper.

[28]For those buyers who do not observe the quality of the object, we also consider that when looking at past auctions, they do not relate the quality to the format through which the good was sold. This assumption can be relaxed as argued later on.

when looking at past auctions remember the reserve price but not whether it was public or secret.[29]

More precisely, we consider three types of buyers:

- Fully-coarse buyers (FC). These buyers do not observe the quality q and only know the aggregate participation rate over all auction formats. As we shall see, it is immaterial what these agents assume about the distribution of reserve price when the reserve price is set secretly. We also assume, for simplicity, that FC buyers do not relate the auction format to the quality of the good, even though our main qualitative insights would not be much affected by alternative assumptions.

- Partially-coarse buyers (PC). These buyers observe the quality q, are perfectly aware of how the participation rate varies with the auction format, but expect the reserve price to be distributed according to the aggregate distribution of reserve prices (whether public or secret) when the reserve price is set secretly.

- Fully-rational buyers)FR) who are fully rational and are also assumed to observe the quality q.

We let $\lambda^i > 0$ ($i = FC, PC, FR$) denote the share of buyers who are respectively fully coarse, partially coarse and fully rational ($\sum_{i=FC,PC,FR} \lambda^i = 1$). A competitive equilibrium in this environment is defined in a similar way as in Section 3, taking into account the (possibly coarse) expectations of the various types of buyers.

Definition 2 *A competitive analogy-based equilibrium (CAB-equilibrium) is defined as* $(\rho_q, \mu_q^i)_{q\in[\underline{q},\overline{q}]}$, *where* ρ_q *stands for the strategy of a quality q seller and* $\mu_q^i : S^* \to R_+ \cup \{\infty\}$ *describes the distributions of participation of buyers of type* $i \in \{FC, PC, FR\}$ *in the various auction formats (of goods of quality q) where*

1. *(Profit maximization for sellers) for any* $q \in [\underline{q}, \overline{q}]$,

$$Supp(\rho_q) \subset Arg \max_{s=(r,d)\in S} \sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} \Phi_n(r, q), \;\; where \;\; \mu_q(\widehat{s}) = \mu_q^{FC}(\widehat{s}) + \mu_q^{PC}(\widehat{s}) + \mu_q^{FR}(\widehat{s}) \tag{10}$$

[29] A major issue in practice which makes it difficult for buyers to estimate the distribution of reserves if secret, is that the reserve price is never disclosed when the good remains unsold. This creates a selection bias which could lead buyers to use auctions with public reserves to estimate this distribution. On eBay, there is public feedback once the reserve price has been reached so that the reserve price can be recovered from the bidding history once a good has been sold. On the contrary, it is typically not observed in traditional auction houses. The difficulties in understanding the distribution of secret reserve prices is also suggested by how the experimental literature deals with the issue of comparing the performance of secret versus public reserve prices (see footnote 2).

2. *(Profit maximization for PC and FR buyers) for any $i = PC, FR$, $q \in [\underline{q}, \overline{q}]$ and $\widehat{s} \in S^*$,*

$$\mu_q^i(\widehat{s}) \underset{(resp.\ =)}{>} 0 \Longrightarrow \sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} V_n^i(\widehat{s}, q) \underset{(resp.\ \leq)}{=} V^i, \tag{11}$$

where for $i = PC, FR$, $V^i := \int_{\underline{q}}^{\overline{q}} \left[\int_S \left(\sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} V_n^i(\widehat{s}, q)\right) \cdot \frac{\mu_q^i(\widehat{s})}{\lambda^i \cdot b} \rho_q(s) ds\right] dG(q)$, $V_n^i(\widehat{s} = r, q) = V_n(r, q)$, $V_n^{FR}(\widehat{s} = secret, q) := \frac{\int V_n(r,q)\rho_q(r,secret)dr}{\int \rho_q(r,secret)dr}$ if $\int \rho_q(r, secret)dr \neq 0$, and

$$V_n^{PC}(\widehat{s} = secret, q) := \int_0^{\infty} V_n(r, q)\overline{\rho}(r)dr \quad where \quad \overline{\rho}(r) := \int_{\underline{q}}^{\overline{q}} (\rho_q(r, public) + \rho_q(r, secret)) \cdot dG(q). \tag{12}$$

3. *(Profit maximization for FC buyers) for any $\widehat{s} \in S^*$, $\mu_q^{FC}(\widehat{s})$ is independent of q; yhid is denoted $\mu^{FC}(\widehat{s})$ and satisfies:*

$$\mu^{FC}(\widehat{s}) \underset{(resp.\ =)}{>} 0 \Longrightarrow E_q\left(\sum_{n=0}^{\infty} e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(\widehat{s}, q)\right) \underset{(resp.\ \leq)}{=} V^{FC},[30] \tag{13}$$

where $V^{FC} := \int_{\underline{q}}^{\overline{q}} \left[\int_S \left(\sum_{n=0}^{\infty} e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(\widehat{s}, q)\right) \cdot \frac{\mu^{FC}(\widehat{s})}{\lambda^{FC} \cdot b} \rho_q(s) ds\right] dG(q)$, $V_n^{FC}(\widehat{s}, q) := V_n^{PC}(\widehat{s}, q)$ and

$$\overline{\mu} := \int_{\underline{q}}^{\overline{q}} \left[\int_S \mu_q(s)\rho_q(s)ds\right] \cdot dG(q) \tag{14}$$

4. *(Matching conditions) for $i = FC, PC, FR$, $\int_{\underline{q}}^{\overline{q}} \left[\int_S \mu_q^i(\widehat{s})\rho_q(s)ds\right] dG(q) = \lambda^i \cdot b$.*

It is instructive to compare the definitions of the CRE- and CAB- equilibria. In both cases, sellers and rational buyers reason in the same way: they seek to maximize their expected payoffs given their correct understanding of how participation varies with the format and how reserve prices are distributed when secret (for buyers). The difference between the two definitions lies in how the partially- and fully-coarse buyers reason, thereby leading to new expressions for what these buyers expect their payoff to be when they participate in the different auction formats.

Specifically, partially coarse buyers differ from rational buyers in how they assess the expected payoff from participating in an auction with a secret reserve price. Instead of taking the correct distribution of reserve prices conditional on being secret, those buyers

[30] Note that the expectation w.r.t. q is not conditional on $\widehat{s}$, reflecting the assumption that FC buyers are coarse on the mapping from $\widehat{s}$ to q (as well as to μ). In addition, for FC buyers, it might a priori occur that, for a format that is not proposed in equilibrium, their perceived expected utility from chosing this format is strictly above V^{FC} for any participation rate $\mu^{FC}(\widehat{s})$. When this is the case, we should have $\mu^{FC}(\widehat{s}) = \infty$. However, this never occurs in equilibrium (because otherwise sellers would propose such a format).

reason as if the distribution of reserve prices there coincided with the aggregate distribution of reserve prices over all auctions (of all qualities and all formats). This can be seen in (12) in which $\overline{\rho}(.)$ denotes the aggregate distribution of reserve prices over all auctions.

Fully-coarse buyers do not observe the quality q and as such their participation rate is independent of q. They assess their payoff from participating in an auction $s \in S$ by expecting a constant participation rate irrespective of the auction format, where this rate matches the aggregate rate, as reflected in (14), and also by expecting the same distribution of quality (assumed to match the aggregate distribution of qualities) irrespective of the auction format. In our model, the total mass of buyers and sellers is exogenously given and the ratio of the two is b. This immediately implies that $\overline{\mu} = b$ via the matching condition.[31]

The CAB-equilibrium concept is in the spirit of the analogy-based expectation equilibrium (Jehiel, 2005) developed for games whereby players bundle various decision nodes or states in order to form their expectations about others' behaviors. From this perspective, our fully-coarse buyers bundle all participation decisions of buyers at all auction formats and for all qualities into the same analogy class, and partially-coarse buyers put all reserve price decisions of all quality q objects into the same analogy class.[32]

Comments: 1) Depending on the feedback regarding previous auctions that prevails in a particular environment where auctions with secret reserves can be used (see footnote 29), we could also consider variants where aggregation is over all public reserves or over all auctions but with a weight reflecting the frequency with which the reserve is publicly disclosed. Our insights do not qualitatively change in these variants. 2) In our baseline model, we assume that FC buyers do not observe q and that PC buyers aggregate the distribution of reserve prices over all qualities to obtain their expectation over the reserve price when secret. These assumptions have been made in order to obtain easily-testable relations between the auction format proposed and the quality of the good. In particular, in Section 6, we will see that our main insights are robust to all expectations being made conditional on quality. More precisely, we consider the case with fully homogenous goods. The equilibrium then involves mixed strategies for sellers where absolute and secret reserve are still used. We also consider a variant of the model where sellers differ in their (privately-known) outside options when they keep the good rather than in the quality of the good, which also has an impact on buyers' valuations. In equilibrium, it is then by the seller's

[31] More generally, if these total masses were endogenously determined (through, say, how the different agent types assess the attractiveness of the auction market), the expression for $\overline{\mu}$ in (14) would depend on these masses.

[32] To make this fit the framework of the analogy-based expectation equilibrium, we have to decompose into two decision nodes the choice of disclosure policy d and reserve price r, and in addition decompose the participation decisions in the various auction formats.

valuation rather than quality that sellers sort into the various auction formats.

4.2 Analysis

We make a number of observations that will help us to understand the structure of CAB-equilibria. First, as before, if a secret reserve price is chosen by a quality q seller, it must be set at Myerson's level $r = r^M(q)$. Second, we observe that in equilibrium FR buyers participate only in TO auctions; and by a similar argument we observe that if a partially-coarse buyer selects an auction with a public reserve price, it must be a TO auction. That is,

Lemma 4.1 *Consider $s = (r, d) \in Supp(\rho_q)$. If $\mu_q^{FR}(\widehat{s}) > 0$, then $r = v_s(q)$ and $d = public$. If $d = public$ and $\mu_q^{PC}(\widehat{s}) > 0$, then $r = v_s(q)$.*

The intuition behind Lemma 4.1 is similar to that developed in the rational case. If FR buyers were to participate in an auction with a different reserve price, sellers could raise their profits by offering a public reserve price set at the seller's valuation, with a similar conclusion holding for PC buyers as their assessments of auctions with public reserve prices is the same as that of FR buyers.

We next note that in equilibrium FC buyers opt for absolute auctions. This follows as these buyers do not perceive that participation is affected by the auction format, so the auction with a zero public reserve price is obviously the format that looks most attractive to FC buyers.[33] Of course, as there is a positive share of FC buyers, this implies (via sellers' maximization) that absolute auctions are indeed offered in equilibrium (as otherwise a deviation to an absolute auction would attract infinitely many FC buyers). To sum up,

Lemma 4.2 *Fully-coarse buyers select only absolute auctions: if $\mu^{FC}(\widehat{s}) > 0$, then $s = (0, public)$.*

Lemmas 4.1 and 4.2 imply that in equilibrium sellers propose either an AA auction expecting only FC buyers to participate, or a TO auction expecting FR and possibly some PC buyers to participate, or a SR auction with reserve price $r^M(q)$ expecting only PC buyers to participate (in equilibrium, SR auctions are not attractive to FR buyers nor to FC buyers, following lemmas 4.1 and 4.2). This is summarized by:

[33] Here we use the fact that FC buyers do not make inferences (from the auction format) about the quality of the object (as otherwise FC buyers might possibly prefer different auction formats that would be associated with better qualities). We will later on argue that the equilibrium we derive remains fully unchanged in the extension where FC buyers make inferences about q, as long as the quality of goods is not too heterogeneous.

Corollary 4.3 *Consider $s = (r, d) \in Supp(\rho_q)$ such that $\mu_q(\widehat{s}) > 0$. If $d = public$, then either $r = 0$ and $\mu_q(r) = \mu_q^{FC}(r)$ or $r = v_s(q) > 0$ and $\mu_q(r) = \mu_q^{PC}(r) + \mu_q^{FR}(r)$. If $d = secret$, then $r = r^M(q)$ and $\mu_q(r) = \mu_q^{PC}(r)$.*

Let $\mu_q^{AA} := \mu_q(0)$, $\mu_q^{TO} := \mu_q(v_s(q))$ and $\mu_q^{SR} := \mu_q(secret)$ be the entry Poisson parameters associated with the choices of AA, TO and SR auctions in equilibrium. In the following analysis, it is convenient for $k = AA, TO, SR$, to define $\Pi_k^S(\mu, q)$ as the expected payoff of a seller with quality q when she chooses the AA, TO and SR auctions, respectively, and when the entry Poisson parameter is equal to μ:

$$\Pi_k^S(\mu, q) = \sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} \Phi_n(r_k(q), q), \tag{15}$$

where $r_k(q)$ denotes the reserve price chosen for a good of quality q in format k (i.e., $r_{AA}(q) = 0$, $r_{TO}(q) = v_s(q)$ and $r_{SR}(q) = r^M(q)$). We also let $\Pi_k^S(q)$ denote the corresponding equilibrium seller's payoff when the participation rate is at the equilibrium level: $\Pi_k^S(q) := \Pi_k^S(\mu_q^k, q)$.

It remains to determine how the sellers with various qualities q sort into the various auction formats AA, TO and SR. Making use of Assumptions 1 and 2 (which so far have played no role), we will show that the interval $[\underline{q}, \overline{q}]$ can be divided into three subintervals such that in the lower range sellers of quality q objects choose AA, in the upper range sellers of quality q objects choose SR, while TO are preferred for intermediate quality good sellers.[34]

First, we observe that from Assumption 2, we have $\mu_q^{TO} > 0$ for any quality q so that sellers offering TO auctions would attract FR buyers. This further implies that $\Pi_{TO}^S(q) > v_s(q)$ (given that a TO auction with strictly positive participation rate generates strictly more than $v_s(q)$ in expectation). Finally, SR auctions must be proposed in equilibrium as otherwise a top quality good seller offering a SR auction would attract more buyers than with a TO auction and would thus make more profits.[35] The following lemma formally proven in the Appendix summarizes these observations:

Lemma 4.4 *$\mu_q^{TO} > 0$ for any $q \in [\underline{q}, \overline{q}]$. As a corollary, $\Pi_{TO}^S(q) > v_s(q)$. Furthermore, SR*

[34]Without Assumption 2, it might be the case that for some qualities the good would never sell in equilibrium. This would thus open the door to a fourth subinterval where sellers are indifferent between TO and SR auctions. It would also open the door to equilibria where SR auctions are somehow vacuous insofar as they are used (and influence PC buyers' expectations) but do not attract any participants.

[35]The perception of PC buyers regarding the expected reserve price is lower than $v_s(q)$ for top-quality good sellers, thereby inducing a participation rate which is higher than that in the corresponding TO auction.

auctions are proposed in equilibrium with strictly positive probability and a positive measure of PC buyers selects those auctions.

Another key lemma in establishing how the various quality goods sort into the various auction formats is:

Lemma 4.5 *The functions $\Pi^S_{TO}(q) - \Pi^S_{AA}(q)$ and $\Pi^S_{SR}(q) - \Pi^S_{TO}(q)$ are quasimonotone increasing where a function $\psi : [\underline{q}, \overline{q}] \to R$ is quasimonotone increasing if for any pair (x, x') with $x > x'$ we have that $\psi(x) \leq 0$ implies that $\psi(x') < 0$.*

Lemma 4.5 can be viewed as establishing a form of single-crossing condition between the functions $\Pi^S_{TO}(q)$, $\Pi^S_{AA}(q)$ and $\Pi^S_{SR}(q)$. Lemmas 4.4 and 4.5 together imply that for any candidate equilibrium, we can define in a unique manner three parameters, two thresholds q^* and q^{**} with $\underline{q} < q^* < q^{**} < \overline{q}$ and a share $\tau^* \in [0, 1)$,[36] such that: 1) FC buyers choose AA; 2) FR buyers and a share τ^* of PC buyers choose TO; 3) a share $1 - \tau^*$ of PC buyers choose SR; 4) sellers with $q < q^*$ propose AA; 5) sellers with $q \in (q^*, q^{**})$ propose TO; and 6) sellers with $q > q^{**}$ propose SR. The first threshold q^* is defined such that a seller of quality q^* is indifferent between AA and TO, and the second threshold is defined such that a seller of quality q^{**} is indifferent between TO and SR. That is,[37]

$$\Pi^S_{TO}(q^*) = \Pi^S_{AA}(q^*) \;\text{ and }\; \Pi^S_{TO}(q^{**}) = \Pi^S_{SR}(q^{**}). \tag{16}$$

It should also be the case that PC buyers find their choice of auction best given their perception. That is, $\tau^* \underset{(resp.\ >)}{=} 0$ implies that

$$\sum_{n=0}^{\infty} e^{-\mu^{SR}_{q^{**}}} \frac{[\mu^{SR}_{q^{**}}]^n}{n!} V^{PC}_n(secret, q^{**}) \underset{(resp.\ =)}{\geq} \sum_{n=0}^{\infty} e^{-\mu^{TO}_{q^{**}}} \frac{[\mu^{TO}_{q^{**}}]^n}{n!} V^{PC}_n(v_s(q^{**}), q^{**}) \tag{17}$$

where the left-hand (resp. right-hand) side of (17) represents the expected utility as perceived by PC buyers of choosing a SR (resp. TO) auction for a quality q^{**} object. Note that the aggregate distribution of reserve prices $\overline{\rho}(.)$ which is used to compute $V^{PC}_n(secret, q)$

[36]Since FC (resp. FR) buyers only select AA (resp. TO) auctions, then AA (resp. TO) auctions are proposed in equilibrium with positive probability, i.e. $\underline{q} < q^*$ (resp. $q^* < q^{**}$). From Lemma 4.4, we have $q^{**} < \overline{q}$ and $\tau^* < 1$.

[37]Since there is a strictly positive measure of buyers participating in either AA, TO or SR auctions, each of those formats should be proposed with positive probability. Then for any $k \in \{AA, TO, SR\}$, $k \in \operatorname{Arg\,max}_{k'=AA,TO,SR} \Pi^S_{k'}(q)$ on a positive measure of qualities. From Lemma 4.5, and noting that the $\Pi^S_k(.)$ functions are continuous, we obtain that q^* and q^{**} are well-defined for any given equilibrium.

itself depends on q^* and q^{**}.[38]

Conversely, we show in Appendix H that any triple $(q^*, q^{**}, \tau^*) \in T$ where $T := \{(q_1, q_2, \tau) \in [\underline{q}, \overline{q}]^2 \times [0,1] | q_2 \geq q_1\}$ which satisfies (16) and (17) induces a CAB-equilibrium. The Appendix also establishes the existence of such a triple. Our discussion is summarized in the following proposition.

Proposition 4.6 *There exists a CAB-equilibrium. Any CAB-equilibrium is characterized by a triple $(q^*, q^{**}, \tau^*) \in T$ with $\underline{q} < q^* < q^{**} < \overline{q}$, $\tau^* < 1$ such that: 1) (16) and (17) jointly hold; 2) sellers with qualities in $[\underline{q}, q^*)$ propose absolute auctions; 3) sellers with qualities in (q^*, q^{**}) propose open reserve price auctions with the reserve price set at $v_s(q)$; 4) sellers with qualities in $(q^{**}, \overline{q})$ propose secrete reserve price auctions with the reserve price set at $r^M(q)$; 5) fully-coarse buyers select absolute auctions; 6) fully-rational buyers select open reserve auctions; and 7) partially-coarse buyers may mix between the open reserve auctions (with probability τ^*) and the secrete reserve auctions (with probability $1 - \tau^*$).*

Remarks. 1) Suppose in contrast to our main model that FC buyers make correct inferences about the quality (through Bayesian updating from the observable characteristics of the auction) while they are still coarse regarding the participation dimension. In the equilibrium shown in Proposition 4.6, the perceived expected utility of such FC buyers for AA is then given by $V^{FC} = \sum_{n=0}^{\infty} e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} \cdot \int_{\underline{q}}^{q^*} V_n^{FC}(0,q) \frac{dG(q)}{G(q^*)}$. Assuming also that such FC buyers expect the lowest quality for formats that are not proposed by sellers on the equilibrium path, it is readily checked that they would not be attracted by any given OR auction with a reserve price of $r > 0$: if the auction is proposed on the equilibrium path, then we have $r = v_s(\widehat{q})$ with $\widehat{q} \geq q^*$ and their perceived expected utility is lower than V^{FC} since $V_n^{FC}(0,q) \geq V_n^{FC}(v_s(q), q) \geq V_n^{FC}(v_s(\widehat{q}), \widehat{q})$ for any $q \leq q^*$, where the last inequality results from $h(q) \geq 0$; if the auction is not proposed on the equilibrium path, then the conclusion follows directly from $V_n^{FC}(0,q) \geq V_n^{FC}(r, \underline{q})$. If the qualities are not too heterogeneous, we also have $V_n^{FC}(0,q) \geq V_n^{FC}(secret, \widehat{q})$ for any $q \leq q^*$ and $\widehat{q} \geq q^{**}$ and we thus obtain that $\int_{\underline{q}}^{q^*} V_n^{FC}(0,q) \frac{dG(q)}{G(q^*)} \geq \int_{q^{**}}^{\overline{q}} V_n^{FC}(secret, q) \frac{dG(q)}{1-G(q^{**})}$ for any n, which ensures that these FC buyers are not attracted by SR auctions either, and thus that the equilibrium in Proposition 4.6 continues to hold with these (slightly more sophisticated) FC buyers. 2) If we consider that a small share of FR buyers is uninformed about q, then the equilibria derived in Proposition 4.6 remains the same: the reserve price in TO auctions

[38] The distribution $\overline{\rho}(r)$ in (12) is defined as: $\overline{\rho}(0) = G(q^*) \cdot \delta(0)$ (where $\delta(.)$ denotes the Dirac distribution); $\overline{\rho}(r) = g(q)$ for $r = v_s(q)$ with $q \in (q^*, q^{**})$; $\overline{\rho}(r) = g(q)$ for $r = r^M(q)$ with $q > q^{**}$and $\overline{\rho}(r) = 0$ almost everywhere.

fully reveals the quality so that uninformed FR buyers behave exactly like informed FR buyers. As in Proposition 3.3, it can be shown that sellers have no profitable deviations. These equilibria are also robust to the introduction of a small share of PC buyers who are uniformed about q: e.g., uninformed PC buyers can be spread (uniformly) over all SR auctions while informed PC buyers enter the various SR auctions in such a way that the total participation rate μ_q^{SR} satisfies $\sum_{n=0}^{\infty} e^{-\mu_q^{SR}} \frac{[\mu_q^{SR}]^n}{n!} V_n^{PC}(secret, q) = V^{PC}$, where the expected utility V^{PC} is the same as that in the original equilibrium with only informed PC buyers. 3) Our analysis with only rational buyers remains entirely unchanged were we to relax the constraint on the set of available mechanisms: the optimal mechanism for a given seller continues to coincide with a welfare-maximizing mechanism that induces efficient entry, which is implemented by the pivotal mechanism that coincides here with the TO auction. By way of contrast, the auction design perspective we adopt by restricting the sellers' strategy to a reserve price policy plays a role in our analysis with coarse buyers. In an unrestricted mechanism-design perspective, entry subsidies are one way to make the auction more attractive. For example, in the present analysis, a seller who proposes a fixed entry subsidy that is shared among entrants will attract all FC buyers and thus benefit from this deviation. These subsidies do raise implementability issues from a practical perspective insofar that buyers (and also possibly the seller) could choose to enter the mechanism via numerous identities (shill bidders) in order to collect the fees (see also Jehiel (2011) for further considerations on manipulative auction design in a monopolistic environment).

4.3 Main properties

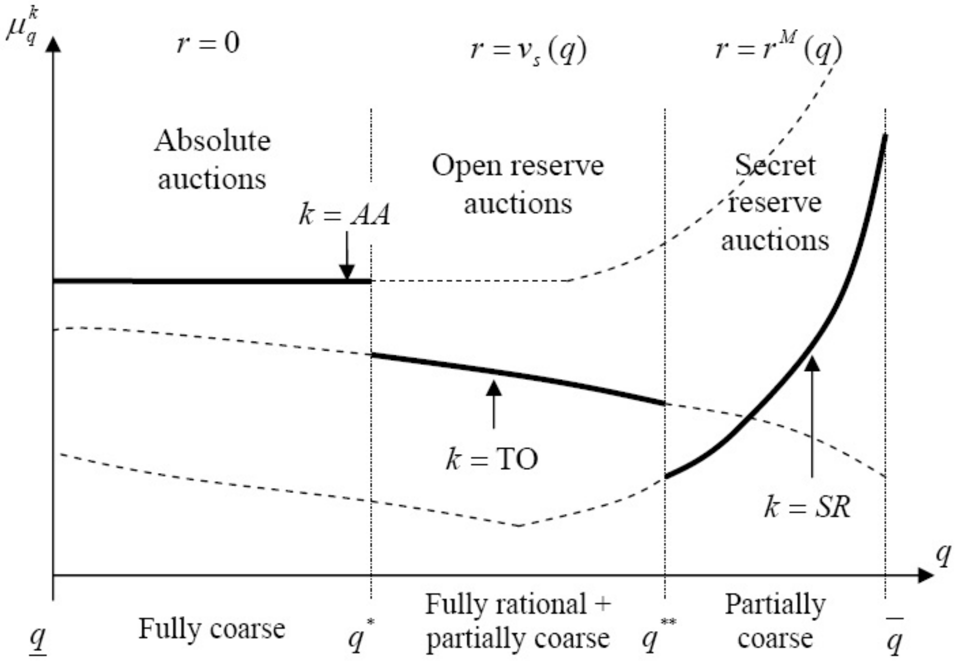
We consider three main questions in this Subsection: 1) How do the participation rates in the various auctions formats vary with quality? 2) How disappointed buyers are depending on their cognitive type, i.e. how do real and perceived expected payoffs differ? 3) How will changing the share of the various types of buyers affect sellers' and buyers' payoffs?

We start by considering how participation rates vary with quality, and how they compare to each other in AA, TO and SR auctions. The proposition is illustrated in Figure 1.

Proposition 4.7 *Equilibrium participation rates satisfy the following:*

- μ_q^{AA} *is constant and equal to* $\mu^{FC}(0) = \frac{\lambda^{FC} \cdot b}{G(q^*)}$ *on* $[\underline{q}, q^*]$ *and is then nondecreasing on* $[q^*, \overline{q}]$*;*
- μ_q^{TO} *is nonincreasing on* $[\underline{q}, \overline{q}]$, $\mu_q^{TO} < \mu_q^{AA}$ *for all* q *and* $\mu_q^{TO} > \mu_q^{SR}$ *for all* $q \in [\underline{q}, q^{**}]$*;*

Figure 1: Equilibrium form



- μ_q^{SR} *is nondecreasing and equal to* $\mu_q^{PC}(secret)$ *on* $[q^{**},\overline{q}]$*;*
- $\mu_q^{AA} > \mu_q(r)$ *for all* $r \in (0,\infty)$ *and* $q \in [\underline{q},\overline{q}]$, $\mu_q^{AA} > \mu_q(secret)$ *for all* $q \in [\underline{q}, q^{**}]$.

We now consider how costly it is to participate in an AA or SR auction. To this end, for $k = AA, TO, SR$, we let $\Pi_k^B(q)$ denote the true expected payoff of a buyer entering an auction of type k when the good is of quality q. For auctions in which FR buyers participate, this corresponds to V^{FR} since perceived and true expected payoffs coincide for FR buyers: in particular, we have $\Pi_{TO}^B(q) = V^{FR}$ for $q \in [q^*, q^{**}]$. It is straightforward that for any proposed format the expected payoff of a given entrant cannot be strictly larger than V^{FR}. This would otherwise represent a contradiction since it would imply that FR buyers would strictly benefit from participating in such formats. The next proposition states that FR buyers would be strictly worse off by participating in AA and SR auctions (this follows as the only reason why some sellers propose AA and SR auctions (as opposed to TO auctions) is that this allows them to induce more participation than the corresponding reserve price policies would produce were buyers to be fully rational, but then this implies that FR buyers are not tempted by such auctions).

Proposition 4.8 $\Pi_{AA}^B(q) < V^{FR}$ *for* $q \in [\underline{q}, q^*]$ *and* $\Pi_{SR}^B(q) < V^{FR}$ *for* $q \in [q^{**},\overline{q}]$.

A related but different issue regarding coarse buyers is how their perceived expected payoff compares to the true expected payoff they derive in the auctions in which they participate. We say that a buyer experiences disappointment if his perceived expected payoff is strictly smaller than his true expected payoff. From the equilibrium condition (17), we have that PC buyers weakly prefer the SR auctions proposed in equilibrium to TO auctions, or equivalently that $V^{PC} \geq V^{FR}$ and from Proposition 4.8 we have $V^{FR} > \Pi^B_{SR}(q)$. Together, this implies that:

Proposition 4.9 *Partially-coarse buyers experience disappointment in any secret reserve price auction in which they participate in equilibrium. That is,* $\Pi^B_{SR}(q) < V^{PC}$ *for any* $q \in [q^{**}, \overline{q}]$.

Turning to FC buyers, there are two forces driving their potential disappointment: on the one hand they misperceive the quality of the good, thereby overestimating the quality of the goods auctioned through AA in expectation;[39] on the other hand, they possibly underestimate the participation rate whenever $\overline{\mu} = b < \mu^{FC}(0)$, where this typically holds when λ^{PC} is not too large.[40]

Proposition 4.10 *If* $\mu^{FC}(0) > b$ *in equilibrium, then fully-coarse buyers experience disappointment in expectation:* $E_q[\Pi^B_{AA}(q)|q \leq q^*] < V^{FC}$.

Finally, we obtain comparative statics with respect to the vector $\Lambda = (\lambda^i)_{i=FC,PC,FR}$. We are particularly interested in whether a larger share of less sophisticated buyers makes sellers and/or buyers worse or better off.

In order to derive sharper comparative statics, we consider limiting cases where there are either no PC or no FC buyers. This is to make the comparative statics tractable, as otherwise the equilibrium conditions on (q^*, q^{**}, τ^*) would involve a fixed point in a three dimensional space.[41] By contrast, we have respectively $q^{**} = \overline{q}$ and $q^* = \underline{q}$ when $\lambda^{PC} = 0$ and $\lambda^{FC} = 0$, thereby allowing for sharper characterizations.

We first consider how the shares of auction formats AA, TO and SR vary with the shares of buyer types. In short, we find that the share of TO auctions increases with the share of FR buyers.

[39] This channel for disappointment vanishes in the variant of our model where FC buyers make correct inferences about quality.

[40] In the limit where $\lambda^{PC} = 0$, we have $\mu_q(\widehat{s}) < \mu^{FC}(0)$ for any $\widehat{s} \neq 0$ with $s = (r, d) \in \text{Supp}(\rho_q)$ (Proposition 4.7), which, in turn, implies that $\overline{\mu} < \mu^{FC}(0)$. However, for general $\lambda^{PC} > 0$ we may have $\mu^{SR}_q > \mu^{FC}(0)$ if q is large enough.

[41] In particular, for a given fixed λ^{PC}, moving the relative share of FC and FR buyers has an impact on the payoffs of PC buyers, which then induces an indirect impact on FC and FR buyers.

Proposition 4.11 • *Assume $\lambda^{PC} = 0$. For any $\lambda^{FR} \in [0,1]$, there is a unique CAB-equilibrium and q^* is strictly decreasing in λ^{FR}.*

- *Assume $\lambda^{FC} = 0$. For any $\lambda^{FR} \in [0,1]$, there is at most one CAB-equilibrium with $\tau^* = 0$. If such a CAB-equilibrium exists (which is guaranteed if λ^{PC} is small enough), then q^{**} is strictly increasing in λ^{FR}.*

We next turn to the impact of Λ on the expected payoffs of the various agents. To make this tractable, we consider the special case in which $\lambda^{PC} = 0$ and $h(.) \equiv 0$. In this case, the CAB-equilibria have a very simple form: the participation rates μ_q^{AA} (for $q \in [\underline{q}, q^*]$) and μ_q^{TO} (for $q \in [\underline{q}, \overline{q}]$) do not depend on q and are then denoted by $\mu^{AA} := \frac{\lambda^{FC} \cdot b}{G(q^*)}$ and $\mu^{TO} := \frac{\lambda^{FR} \cdot b}{1-G(q^*)}$. We have that $\mu^{AA} \geq b \geq \mu^{TO}$ with at least one of the inequalities being strict (Lemma G.1 and the matching condition). The threshold q^* is then characterized by the equilibrium condition

$$\Pi_{AA}^{S}(\frac{\lambda^{FC} \cdot b}{G(q^*)}, q^*) = \Pi_{TO}^{S}(\frac{\lambda^{FR} \cdot b}{1 - G(q^*)}, q^*). \tag{18}$$

It is instructive to compare the equilibria with almost only FR buyers to those with almost only FC buyers. With almost only FR buyers, we have $\mu^{AA} > \mu^{TO} = b$ and $q^* = \underline{q}$; with almost only FC buyers, we have $\mu^{AA} = b > \mu^{TO}$ and $q^* = \overline{q}$. Sellers thus face the same equilibrium participation rate in both cases: however with FC buyers only they set no reserve price, and with FR buyers only they set a reserve price at their valuation level. Clearly, sellers prefer the case with only FR buyers (and this case is also best from a total welfare perspective). By way of contrast, it is clear that all buyer types are better off in the case with (almost) only FC buyers, as compared to the case with (almost) only FR buyers. Based on these limiting cases, we might conjecture that both μ^{AA} and μ^{TO} are increasing in λ^{FR}, which implies that buyer and seller payoffs fall and rise, respectively, with λ^{FR}. This turns out to be true for μ^{TO} but not necessarily for μ^{AA}.[42]

Proposition 4.12 *Assume that $\lambda^{PC} = 0$ and that $h(.) \equiv 0$. μ_q^{TO} is strictly increasing in λ^{FR}. As a corollary, V^{FR} is decreasing in λ^{FR} and a seller proposing a TO auction is*

[42] The sign of $\frac{d\mu^{AA}}{d\lambda^{FR}}$ is ambiguous. To see this note that the matching condition yields $\frac{d\mu^{AA}}{dq^*} = -\frac{g(q^*)}{G(q^*)} \underbrace{[\mu^{AA} - \mu^{TO}]}_{>0} - \frac{1-G(q^*)}{G(q^*)} \underbrace{\frac{d\mu^{TO}}{dq^*}}_{<0}$. From (39), we can check that $\frac{d\mu^{TO}}{dq^*}$ is bounded away from zero when λ^{FC} is bounded away from zero. At a point where $g(q^*)$ is small enough, this implies that $\frac{d\mu^{AA}}{dq^*} > 0$ or equivalently $\frac{d\mu^{AA}}{d\lambda^{FR}} < 0$ (given that $\frac{dq^*}{d\lambda^{FR}} < 0$).

better off if λ^{FR} increases.

5 Discussion of the empirical/experimental literature

This Section summarizes the main empirical findings on competitive auctions and relates them to our theoretical results. In the following discussion, the references to our results that are consistent with the corresponding empirical insights appear in square brackets.

As highlighted in the introduction, one important puzzle is the use of secret reserve prices. Bajari and Hortaçsu (2003) and Hossain (2008) underline that goods of higher quality are more often associated with secrete reserves [Proposition 4.6]. Furthermore, according to their counterfactual estimates, Bajari and Hortaçsu (2003) find that the expected revenue difference between SR and OR auctions is increasing in the book value of the good [Lemma 4.5]. They also find that secret reserves yield higher expected revenue to the seller, while Katkar and Reiley (2006) finds the opposite in some field experiments: this is consistent with our theory which does not predict any performance premium for SR relative to OR, but rather that secret reserves are profitable for high-quality goods [Proposition 4.6]. Bajari and Hortaçsu (2003) suggest that the fact that the sale rate is much lower in SR auctions compared to OR auctions reflects that the reserve price is much higher in the former than the latter [Corollary 4.3]. Furthermore, buyers bidding in SR auctions experience disappointment [Proposition 4.9]: Bajari and Hortaçsu (2003) report that eBay claimed to receive too many complaints for those formats, which led the company to impose extra fees for SR auctions. Consistent with this, they find that sellers using secret reserves receive more negative feedback.[43]

Another important puzzle is the use of absolute auctions as emphasized by Hasker and Sickles (2010).[44] Low reserves have often been perceived as supporting the theory of endogenous as opposed to exogenous entry à la Myerson, where sellers should set reserves that are strictly above their reservation values. However, in many cases we observe reserve prices that are much lower than any reasonable reservation value of the seller, which are difficult to justify based on classic explanations, but come out naturally in our setup with cognitive limitations. We mentioned in the introduction the field experimental results in Reiley (2006) which are consistent with our work insofar as the expected revenue difference

[43] It is interesting to note that secret reserves were not available for sellers in more than half of the hundred or so sites surveyed by Lucking-Reiley (2000) in the early days of Internet auctions.

[44] In a separate study with computer monitors on eBay, they find that 12% of the sellers use absolute auctions for goods that sell on average for 134$. In the sample of 167 auctions for a board game analyzed by Malmendier and Lee (2011), 26% (resp. 44%) of auctions had a reserve price below 1$ (resp. 10$), while the average final price was 132$.

between AA and OR auctions falls with quality [Lemma 4.5]: this drops from 2.70$ to 3.40$, and from 10.05$ to 9.93$, for respectively low- and medium-value cards when we move from an OR to an AA auction. A number of field experiments suggest that AA auctions may maximize the seller's revenue (Walley and Fortin 2005, Barrymore and Raviv 2009). For homogenous goods, Ariely and Simonsohn (2008) find nearly identical revenues for AA and OR auctions [see also Section 6.1]. With heterogenous goods, taking advantage of a natural experiment with a discontinuity in the reserve price policy for second-hand car auctions in U.K., Choi et al. (2010) are able to compare AA and OR auctions. As in Ariely and Simonsohn (2008), they find that more experienced buyers participate more in auctions with higher reserves. This is consistent with our results [Proposition 4.6] if we have in mind that the degree of rationality should be correlated with experience. Without any structural model, Ariely and Simonsohn (2008) find that the buyer's expected payoff is higher in OR than in AA [Proposition 4.8], since they observe that buyers in AA are less likely to win and also pay more on average when they do win, which is inconsistent with a model with fully rational buyers (as in the model of Levin and Smith (1994)) but is also compatible with our model. As expected, these authors also find that participation is greater in AA as compared to the OR auctions proposed by the sellers.[45]

Bajari and Hortaçsu (2003) confirm the theory of Levin and Smith (1994) by finding that the OR auctions that maximizes the expected payoff of the seller is the TO auction, and this holds independently of the seller's reservation value. This is also consistent with our results: in any CAB-equilibrium, the optimal OR auction is the TO, as the bidders entering OR auctions behave like rational buyers so that some parts of our analysis then match closely the standard model with only rational buyers.

The structural model in Bajari and Hortaçsu (2003) is based on a regression of the participation rate on various variables, including a proxy for the quality (the book value) and the reserve price policy. This specification does not fit our theory (in which there is no linear relation between the participation rate and the quality, say). Nevertheless, their results are somewhat consistent with Proposition 4.7: they find that there is a significantly positive effect of quality on participation for SR auctions, while the effect is much smaller for the full sample (see also Figure 1). Nevertheless, we would like to emphasize that the comparative statics of μ_q^{TO} w.r.t. q is not a critical feature of our theory. In particular,

[45] Ariely and Simonsohn (2008) propose an informal herding explanation with a dynamic perspective, in particular because they additionally find that, conditional on the current price, auctions that start at a lower minimum bid surprisingly receive more new bids. However, those 'new' bids include additional bids from bidders who had already entered the auction, thereby making the herding phenomenon less clear.

we should not view evidence against it as invalidating our theory.[46] This result relies critically on Assumption 1, an assumption which has been made for technical reasons but whose absence may not invalidate the way in which various qualities sort into the different formats as in Proposition 4.6 (see e.g. the proof of Lemma 4.5 where other channels play in favor of the quasimonotonicity properties).

From an empirical perspective, we would like to mention that our model should not be taken to the letter. In particular, by absolute auctions, we should have in mind more generally the set of reserve prices that clearly lie below the seller's reservation value. In the same vein, sellers use sometimes minimum bids in SR auctions. However, they are typically set at very low levels: Bajari and Hortaçsu (2003) report that, on average, the ratio of minimum bids to the book value is 150% higher under a public reserve than a secret reserve, while the book value is shown to be a reliable predictor of the winning price.

Risk aversion is often viewed as the best model to explain the discrepancy between the equilibrium with risk-neutral bidders and experimental data for first-price auctions (see, e.g., Bajari and Hortaçsu, 2005). In our model of second-price auctions with entry, it is clear that risk-aversion cannot play in favor of secret reserves (risk-averse buyers would not like the less transparent formats, and the unraveling argument mentioned in the introduction would certainly destabilize the presence of SR auctions in a world with only fully-rational buyers). The effect of risk aversion on absolute auctions is less straightforward, but the attractiveness of an AA relative to a TO auction in a world with only rational buyers seems to result mainly from the small probability of being the sole entrant and thus winning the good for free. Intuition suggests that payoffs are 'more uncertain' in an AA than a TO auction, thereby making it unlikely that risk aversion alone can explain the emergence of AA.

The quantal response equilibrium (QRE) concept is another popular model used to explain anomalous behaviors, especially in experimental settings, and may be combined with risk aversion in the context of auctions (see Goeree et al. 2002, for first-price auctions). We briefly discuss what might be expected if we consider that entry decisions are taken according to QRE.[47] Our intuition is that for a given quality, the equilibrium participation

[46] Reiley (2006) actually found that collectible trading cards of higher values receive considerably more bids than their low value counterpart when the reserve price is set at 90% of the market price value. In our model, quality has an additive effect on buyers' valuations. In environments where quality has a multiplicative effect, we should rather expect participation to increase with quality. With a multiplicative model (and mild additional assumptions), we still obtain the analog of Proposition 4.6, but with μ_q^{TO} being increasing in q.

[47] On the contrary, we maintain that sellers maximize their expected payoffs while buyers bid their valuations at the bidding stage. Note that in the second-price auction with private values, QRE predicts that buyers should bid both above and below their valuations.

rate as a function of the public reserve should be flatter under QRE than Nash equilibrium,[48] which suggests that the optimal reserve price should be larger in QRE than in Nash equilibrium (where it is set at the seller's valuation). Note that in the limit where the QRE error parameter goes to infinity, participation becomes irresponsive to the announced format, in which case the reserve price should be set at Myerson's optimal level.[49]

6 Variants and extensions

In our main model, buyers differed in both their cognitive sophistication in analyzing data from previous auctions and whether they observed the quality q of the auctioned good. In Subsection 6.1 below, we consider the case in which all goods are of the same quality so that agents differ only in their cognitive sophistication. As we will see, this does not affect our main insights (in particular regarding the emergence of absolute auctions and secret reserve prices). In subsection 6.2, we then consider a variant of the model in which quality is homogeneous among objects but sellers differ in their valuations, and characterize the consequent sorting of sellers into the various auction formats. In subsection 6.3, we depart from our static model and develop a dynamic model with sequential entry decisions in which we argue that most of our insights remain robust. Finally, subsection 6.4 briefly touches on the issue of shill bidding.

6.1 Homogeneous qualities

We here assume that there is no heterogeneity in the quality of the good, i.e. $\overline{q} = \underline{q} = q$. The analysis with only rational buyers can immediately be extended without modification: sellers propose TO auctions. The analysis with coarse buyers is also adapted easily: the difference from the analysis of Section 4 is that sellers will now use mixed strategies. Specifically, any CAB-equilibrium is characterized by a triple $(\nu^*, \nu^{**}, \tau^*) \in [0,1]^3$ with $\nu^* < \nu^{**}$ such that sellers propose AA with probability $\nu^* > 0$, TO with probability $\nu^{**} - \nu^* > 0$ and SR with probability $1 - \nu^{**} > 0$, and such that FC buyers select AA, FR buyers select TO while PC buyers mix between the TO (with probability

[48] This is analogous to flatter responses to asymmetric payoffs within QRE in matching pennies games (Goeree et al., 2003).

[49] The level-k framework which has been used to explain overbidding at the auction stage in various models (Crawford and Iriberri, 2007) does not seem particularly useful in explaining participation decisions in contexts with a continuum of auctions (such models usually insist that there are just a few levels of cognitive sophistication, and as such it would be hard to reproduce the feature that buyers spread their participation over a continuum of auctions). Combining level-k models with QRE is not easy to effect.

$\tau^* < 1$) and SR (with probability $1-\tau^*$). In a CAB-equilibrium $(\nu^*, \nu^{**}, \tau^*)$ has to satisfy:

$$\sum_{n=0}^{\infty} e^{-\mu^{AA}} \frac{[\mu^{AA}]^n}{n!} \Phi_n(0,q) = \sum_{n=0}^{\infty} e^{-\mu^{TO}} \frac{[\mu^{TO}]^n}{n!} \Phi_n(v_s(q),q) = \sum_{n=0}^{\infty} e^{-\mu^{SR}} \frac{[\mu^{SR}]^n}{n!} \Phi_n(r^M(q),q), \tag{19}$$

and $\tau^* \underset{(resp.\ >)}{=} 0$ implies

$$\sum_{n=0}^{\infty} e^{-\mu^{SR}} \frac{[\mu^{SR}]^n}{n!} [\nu^* \cdot V_n(0,q) + (\nu^{**} - \nu^*) \cdot V_n(v_s(q),q) + (1-\nu^{**}) \cdot V_n(r^M(q),q)] \underset{(resp.\ =)}{\geq} \sum_{n=0}^{\infty} e^{-\mu^{TO}} \frac{[\mu^{TO}]^n}{n!} V_n(v_s(q),q), \tag{20}$$

where the participation rates μ^{AA}, μ^{TO} and μ^{SR} are uniquely characterized from the matching conditions $\mu^{AA} = \frac{\lambda^{FC} \cdot b}{\nu^*}$, $\mu^{TO} = \frac{(\lambda^{FR} + \tau^* \lambda^{PC}) \cdot b}{\nu^{**} - \nu^*}$ and $\mu^{SR} = \frac{(1-\tau^*)\lambda^{PC} \cdot b}{1-\nu^{**}}$. Conditions (19) and (20) are the analogs of (16) and (17), respectively, in the main model. Thus, the emergence of AA and SR as well as the sorting of buyers in the various auction formats remain qualitatively the same as those in the main model.

6.2 Heterogeneous reservation values

Consider a variant of the model in which the quality of the good is homogenous, and buyers and sellers are heterogeneous in their valuations. Buyers' valuations are drawn from a fixed CDF F for all goods, and sellers' valuations, denoted by X_S, are distributed according to a continuously-differentiable cumulative distribution G with support $[\underline{X}_S, \overline{X}_S]$. The bulk of our previous analysis remains completely unchanged. What remains to be determined here is how the sellers with various reservation values X_S sort into the different auction formats AA, TO and SR. As in our main model, it can be shown that the interval $[\underline{X}_S, \overline{X}_S]$ can be divided into three subintervals such that in the lower range $[\underline{X}_S, X_S^*)$ sellers choose AA, in the upper range $(X_S^{**}, \overline{X}_S]$ sellers choose SR, while TO are preferred by sellers with intermediate reservation values in (X_S^*, X_S^{**}). This relies in particular on the analog to Lemma 4.5. The formal argument is sketched in the Supp. Mat.

6.3 Dynamic bidding

Another extension of our basic model is to allow for sequential participation decisions. To simplify the analysis, we assume that sellers are homogeneous with a common reservation value X_S, and that bidders are also homogenous with a common valuation $X_B > X_S$. Entry opportunities arise exogenously and buyers' strategies may then depend on the bidding history of the complete set of auctions at the time they receive the opportunity to enter the auction market. As in the main model, we continue to assume that buyers are not able to switch from one auction to another, so that each buyer bids his valuation in the auction in which he participates.[50] The current price of an auction at a given time t corresponds to the minimum amount that a buyer should bid to enter the auction at t, and is equal to the maximum of the reserve price and the second-highest bid submitted up to t.[51]

As in the main model, we also consider three types of buyers, still labeled as fully coarse (FC), fully rational (FR) and partially coarse (PC) depending on their ability to make inferences. As before, the share of the three types of buyers is denoted by $\lambda^i > 0$, $i = FC, FR, PC$. Fully-coarse buyers are those who consider only the current price of the auction without taking into account whether (and how many) other bids were submitted in order to make their participation decision. This induces confusion for such buyers between an auction with an open reserve price r in which there are no other participants and one in which there is only one other bidder who has submitted a bid above r (in both cases, the current price is r). Given their perception, fully-coarse buyers participate in the available auction with minimum current price, as long as this price is below their valuation X_B. Fully-rational and partially-coarse buyers are modeled as before: FR buyers choose the format that is best given what is observed; PC buyers are rational in all dimensions except that when the reserve price is secret, they think it is distributed according to the aggregate distribution of reserve prices. Finally, we assume that there are more sellers than buyers ($b < 1$)[52] which guarantees that sellers' expected payoffs lie strictly below X_B. And to simplify the analysis, we assume the following timing. First, sellers simultaneously select their auction format (as in the main model); buyers then enter in sequence, first FC buyers, followed by FR buyers and finally PC buyers. We also assume that $\frac{\frac{\lambda^{FC}}{2}}{\lambda^{FR}+\lambda^{PC}} < \frac{X_S}{X_B}$ which guarantees that all buyers whatever their type entering AA auctions is not an equilibrium.

Analysis. For $(\nu^*, \nu^{**}, \tau^*, r^*) \in [0,1]^3 \times [X_S, X_B]$ with $0 < \nu^* \leq \nu^{**} < 1$, $\tau^* < 1$, consider the following constraints:

$$\nu^{**} - \nu^* = \max\{0, (\lambda^{FR} + \tau^*\lambda^{PC}) \cdot b - \psi_0 \cdot \nu^*\}, \tag{21}$$

[50] The inability to switch from one auction to another can be rationalized by the costs inherent to an interaction with a new seller (see footnote 8).

[51] Regarding online auctions, we abstract from bidding increments. See Hickman (2010) for an analysis of the strategic implications of bidding increments.

[52] As will be clear from the proof of Proposition 6.1, if $(1 - \lambda^{FC})b \geq \overline{\psi}_0$, where $\overline{\psi}_0$ is characterized by $\overline{\psi}_0 \cdot (2 - ln[\overline{\psi}_0]) = 2 - \lambda^{FC}b$ if $2 - \lambda^{FC}b \geq$ and 0 otherwise, then every seller posting a reserve price equal to X_B would be an equilibrium, which is precisely the kind of equilibrium we want to avoid. The previous condition is equivalent to b being larger than a given threshold $\widehat{b}$, where $\widehat{b} \in (1, \frac{2}{2-\lambda^{FC}})$.

$$r^* = \psi_2 \cdot X_B = X_S + \frac{(1-\tau^*)\lambda^{PC} b}{1-\nu^{**}}(X_B - X_S), \tag{22}$$

where ψ_0 and ψ_2 are defined by $\psi_0 \cdot (2 - ln[\psi_0]) = 2 - \frac{\lambda^{FC} b}{\nu^*}$, and $\psi_2 = \frac{\lambda^{FC} b}{\nu^*} + \psi_0 - 1$, and $\nu^{**} > \nu^*$ and $\tau^* \underset{(resp.\ >)}{=} 0$ imply that

$$\nu^* \cdot X_B + (\nu^{**} - \nu^*) \cdot (X_B - r^*) \underset{(resp.\ =)}{\geq} (X_B - r^*). \tag{23}$$

The equilibria are characterized in the following proposition:

Proposition 6.1 *Any equilibrium is characterized by a 4-uple* $(\nu^*, \nu^{**}, \tau^*, r^*) \in [0,1]^3 \times (X_S, X_B)$ *with* $0 < \nu^* \leq \nu^{**} < 1$, $\tau^* < 1$ *satisfying (21), (22) and (23). It is such that: 1) a share* ν^* *[resp.* $\nu^{**} - \nu^*$ *and* $1 - \nu^{**}$*] of the sellers propose AA auctions [resp. OR auctions with the reserve* r^* *and SR auctions with the reserve* X_B*]; 2) Sellers' expected payoff is* $r^* \in (X_S, X_B)$*, which corresponds also to the expected payoff of the OR auctions with the reserve* r^* *in the case where* $\nu^{**} > \nu^*$*; 3) Fully-coarse buyers select AA auctions; 4) Fully-rational buyers select auctions with no participants – first AA auctions and then OR auctions if any; and 5) Partially-coarse buyers select auctions with no participants – first AA auctions if any and then mix between OR auctions (with probability* τ^**) and SR auctions (with probability* $1 - \tau^*$*).*

Several comments are required here. First, if a SR auction is selected, it is clear that the reserve price is set at X_B. Second, were AA not to be proposed in equilibrium, then by offering AA a seller would immediately attract two FC buyers (as AA looks like the most attractive format to FC buyers when there are 0 or 1 other participants), thereby yielding the maximum possible revenue X_B to such a seller. AA auctions are thus proposed in equilibrium. FC buyers then enter AA auctions which have 0 or 1 bidders (with no distinction). Once an AA auction receives two bids the price shifts to X_B and there are no further entrants. If the entry dynamic of FC buyers into AA is such that all AA auctions receive two bids, then these sellers would raise a strictly larger profit than other sellers which would imply a contradiction. At the end of the entry by FC buyers, a positive share of AA auctions should then end with a price of zero while having no bidders. This latter share corresponds precisely to ψ_0 (whose calculation is relegated to the Supp. Mat.). FR and PC buyers would first select AA auctions with no other bidders (this is because there are no FC buyers left, and thus these buyers do not expect further entry (after their own) in such auctions, and these auctions produce the maximum possible payoff for them). However, FR buyers enter first. So they fill the remaining AA auctions with zero participants and then start filling in the OR auctions once the AA auctions are exhausted. Finally, PC buyers exhaust the remaining AA auctions with zero participants, if any, and then choose SR auctions and may also sometimes mix choosing OR auctions (with no other participants). It cannot occur that PC buyers strictly prefer the OR auctions proposed in equilibrium to the SR auctions, as otherwise sellers would (strictly) benefit (compared to their assumed equilibrium payoff) by proposing OR auctions with a slightly higher reserve price. When they mix, the PC buyers' perceived payoff from choosing an SR auction should be equal to the payoff they receive in an OR auction (that is, $X_B - r^*$). This is formally described in (23) which is the analog of (17). Condition (22), which is the analog of (16), reflects sellers' indifference between the various formats proposed in equilibrium. In particular, the expected payoff of sellers choosing OR is r^* (because OR auctions must be filled in with probability one, as otherwise a slight deviation to a lower reserve price would attract for sure a bidder, thereby ensuring a strictly larger payoff), the expected payoff of a seller choosing AA is given by $\psi_2 \cdot X_B$, where ψ_2 corresponds to the share of AA auctions that receive two bids, and the expected payoff of a seller choosing SR is $\frac{(1-\tau^*)\lambda^{PC}b}{1-\nu^{**}}X_B$, where $\frac{(1-\tau^*)\lambda^{PC}b}{1-\nu^{**}}$ represents the ratio of the measure of PC buyers choosing an SR auction to the measure of sellers choosing SR (which is also the probability that a single seller choosing SR receives one bidder). Finally, condition (21) is a matching condition reflecting that OR auctions receive exactly one entrant (when they arise in equilibrium). In other words, the measure of OR auctions proposed in equilibrium, $\nu^{**} - \nu^*$, should be equal to the measure of buyers entering OR auctions, which corresponds to the right-hand side in (21).

It may come as a surprise that the reserve price set in OR differs from X_S (in contrast to what happens in the main model). This is due to the sequential nature of the entry decisions. In equilibrium, all OR auctions must be filled in with at least one bidder (as otherwise a slight deviation to a lower public reserve price would be a profitable deviation, as it would attract for sure an FR buyer), so that OR sellers enjoy monopoly power vis-a-vis FR and PC buyers. The reserve price r^* and the other parameters ν^*, ν^{**}, τ^* of the equilibrium are determined from the condition that sellers be indifferent between all three formats AA, SR and OR.

Comments: 1) Models with dynamic bidding may seem closer to how online auctions work. This view should be treated cautiously however. In online auctions, it is known that (most) bidders submit their bids at the very last minute (this is referred to as sniping). To the extent that all bidders behave this way, our previous formulation in static terms may be more appropriate. 2) In the case with only fully-rational buyers, there is a unique equilibrium which involves only OR auctions: we have $r^* = X_S$ (this relies crucially on $b < 1$). Since $r^* > X_S$ in Proposition 6.1, we obtain that sellers benefit from the presence of coarse buyers.[53] 3) While the entry dynamics would be much more complex in the model extension where buyers have uncertain valuations, we conjecture that similar insights would also obtain in that case.

6.4 Robustness to shill bidding

Shill bidding is a pervasive phenomenon in second-price auctions (Lamy, 2009, 2010). Even though it is illegal, some sellers are ready to employ shill bidding to raise their expected payoffs. In a pure private values environment, this is equivalent to the possibility of raising the reserve price after bidders have made their entry decisions. Anecdotal evidence (Lamy, 2010) suggests that the usual strategy of such fraudulent sellers consists in proposing absolute auctions so as to attract more buyers, and then putting the reserve price at its optimal level once bidders have become captive, while being prepared to buy their own good and pay the transaction fees (if no other larger bid is submitted). This informal argument seems to rely implicitly on the bidders' failure to form rational expectations: otherwise they would anticipate that shill bidding will occur more in absolute auctions and reduce their participation levels in those formats leading fraudulent sellers to prefer alternative formats.

A full analysis of competitive equilibria with possible shill bidding is beyond the scope of this section. Even so, we below make a series of comments suggesting why in the presence of shill bidding, fully-coarse bidders may have a stabilizing and welfare-enhancing role.

Consider first the case of fully-rational buyers and suppose that all sellers use shill bidding. It is then readily checked that a quality q seller will eventually set a reserve price at Myerson's level $r^M(q)$ (either directly or through the shill bid). This is because the shill bid is essentially similar to a secret reserve price in this setting. Suppose next that because shill bidding is illegal, not every seller resorts to it: only a share α of sellers consider shill bidding. Assuming that buyers are fully rational, there are a priori many equilibria if deviations from the equilibrium path are interpreted as meaning that there is a greater chance that the seller be a shill bidder. However, reasonable restrictions on these off-path interpretations yield the prediction that no matter how small is α, the only equilibrium is that which is as if all sellers had selected Myerson's reserve price $r^M(q)$.[54]

[53] For b in the left neighborhood of 1, the picture would be different. The unique equilibrium with FR buyers still involves solely OR auctions, but with $r^* = X_B$. On the contrary, with coarse buyers, there will be an equilibrium similar to those derived in Proposition 6.1, which would thus be less profitable for sellers.

[54] The required selection idea is that if a seller proposes an off-the-path reserve price, then she is perceived

Consider next the case with FC buyers. A natural specification for the FC buyers' beliefs is that they ignore shill bidding (where shill bids are perceived as regular bids). Note that with shill bidding, the perceived participation rate $\overline{\mu}$ for FC buyers is not equal to b due to shill bidding, which latter inflates the average number of bidders per auction. It should be clear that CAB-equilibria are robust to shill bidding provided that α is small enough: only FC buyers enter the AA auctions proposed by the shill sellers and also a share of non-shill sellers. From the shill-bidding perspective, FC buyers may thus raise welfare by stabilizing the market (as this allows shill sellers to target FC buyers through AA auctions while leaving other formats which are immune to shill bidding).

7 Conclusion

We have here illustrated how the presence of non-fully rational buyers may explain the emergence of absolute auctions and secret reserve price auctions in competitive environments with experienced sellers. We have also reviewed the empirical literature on competitive auctions and checked that the most salient findings there can all be explained within our framework. We believe that approaches similar to that presented here can be used to shed new light on seemingly odd phenomena in other applications.

It is a stylized fact in certification/grade-disclosure environments that a non-negligible proportion of subjects with bad signals prefer to hold them back. The standard explanation for the absence of complete unraveling (which standard theory predicts) is the fact that certification may be costly, so that those who receive the worst grades do not pay for it. However, this type of argument is less compelling in environments in which the grades are already available to the subjects for free. The estimates in Conlin and Dickert-Conlin (2010) reveal that colleges underestimate the relationship between an applicant's action in submitting (or not) his SATI score and the actual score. In auctions for baseball cards, Jin and Kato (2006) provide empirical evidence of buyers' naïveté: some buyers overestimate the quality of the card when sellers do not pay to be graded by a professional certifier, especially if the seller also claims that the quality is high. For example, sellers claiming top qualities instead of nothing raise their revenue by 50%. Jin and Kato (2006) also note that the average winner of a graded-card auction is more experienced than winners of ungraded-card auctions. These findings can be related to our analysis of SR auctions and how PC buyers form their beliefs over the distribution of reserve prices when secret.

to use shill bidding with a probability of at most α (which can be rationalized on the grounds that fraudulent sellers, who are likely to be more active/experienced players, are less likely to "tremble").

It has long been observed that people do not react rationally to the characteristics of lotteries/contests. In particular, participants seem to under-react to an increase in the number of other contestants (see Lim et al. (2009) and the references therein). This feature remains in the lab once we remove charity motives and the small probabilities/large prize effects that are often inherent to lotteries. This puzzle can be rationalized if we view subjects as putting in the same analogy class lotteries with a varying number of participants. This is to an extent similar to our modeling of FC buyers.

The analysis in greater detail of these applications is a clear subject for further research.

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Appendix 1

A Proof of Lemma 3.1

$\frac{\partial TW_q(\mu, v_s(q))}{\partial \mu} = \sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} [W_{n+1}(v_s(q), q) - W_n(v_s(q), q)] - V^{FR}$, which is also equal to $\sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} V_n(v_s(q), q) - V^{FR}$ as the second-price auction with the reserve price of $v_s(q)$ corresponds to the pivotal mechanism (also called *the* Vickrey auction). Since $V_n(v_s(q), q)$ is decreasing in n (see Eq. (2)), we obtain that $\mu \to TW_q(\mu, v_s(q))$ is strictly concave. Finally, from Eq. (6) we have that its optimum is reached at $\mu = \mu_q(v_s(q))$.

B Proof of Proposition 3.2

Take $\widehat{q} \in \operatorname{Arg\,min} H(q)_{q \in [\underline{q}, \overline{q}]}$ and $\widetilde{q} \in \operatorname{Arg\,max} H(q)_{q \in [\underline{q}, \overline{q}]}$. For any n, we thus have $\widehat{q} \in \operatorname{Arg\,max} V_n^{FR}(v_s(q), q)$. This implies that for any function $m(.) \geq 0$ on $[\underline{q}, \overline{q}]$ such that there exists a constant C with $\sum_{n=0}^{\infty} e^{-m(q)} \frac{[m(q)]^n}{n!} V_n^{FR}(v_s(q), q) = C$ whenever $m(q) > 0$ while $V_0^{FR}(v_s(q), q) \leq C$ otherwise, we have: if there exists $q \in [\underline{q}, \overline{q}]$ such that $m(q) > 0$ then $m(\widehat{q}) > 0$.

We are left with the characterization of $\mu^*(.)$ which then guarantees the uniqueness of the equilibrium entry strategy of the buyers. First the differentiation of Eq. (6) leads to Eq. (9) on any point with $\mu^*(q) > 0$. From the Cauchy-Lipschitz Theorem the existence and uniqueness of the solution $y(.)$ of the differential equation Eq. (9) on $[\underline{q}, \overline{q}]$ with an initial condition of the form $y(\widehat{q}) = \overline{\mu}$ is guaranteed. Let $m[\overline{\mu}](q)$ denote the pointwise maximum of this solution and 0. From the previous paragraph, we have that $\overline{\mu} \leq 0$ implies that $m[\overline{\mu}](.)$ is uniformly equal to 0 on $[\underline{q}, \overline{q}]$. It remains to show that there is a one-to-one (strictly increasing) mapping between $\overline{\mu} \geq 0$ and $\int_{\underline{q}}^{\overline{q}} m[\overline{\mu}](q) \cdot dG(q)$, and that $\int_{\underline{q}}^{\overline{q}} m[\overline{\mu}](q) \cdot dG(q)$ goes from 0 to infinity as $\overline{\mu}$ goes from 0 to infinity. Consider two initial points $\overline{\mu}_1$ and $\overline{\mu}_2$ with $\overline{\mu}_1 > \overline{\mu}_2$. At any point where $m[\overline{\mu}_i](q) > 0$ $(i = 1, 2)$, we have $m[\overline{\mu}_1](q) > m[\overline{\mu}_2](q)$ (if not, by continuity, we should have a point q' where $m[\overline{\mu}_1](q') = m[\overline{\mu}_2](q')$ which would imply a contradiction since $m[\overline{\mu}_1](.)$ and $m[\overline{\mu}_2](.)$ would then be equal everywhere by the Cauchy-Lipschitz Theorem). As a corollary, we obtain that $\int_{\underline{q}}^{\overline{q}} m[\overline{\mu}_1](q) \cdot dG(q) > \int_{\underline{q}}^{\overline{q}} m[\overline{\mu}_2](q) \cdot dG(q)$. First note that $\int_{\underline{q}}^{\overline{q}} m[0](q) \cdot dG(q) = 0$. Second, since $|\frac{dm[\overline{\mu}](q)}{dq}|$ is bounded by $\max_{q \in [\underline{q}, \overline{q}]} |h(q)| \cdot \frac{[1 - F(H(\widehat{q}))]}{\int_{H(\widehat{q})}^{\infty} (1 - F(s))^2 ds}$, we obtain that $\min_{q \in [\underline{q}, \overline{q}]} m[\overline{\mu}](q)$ goes to infinity as $\overline{\mu}$ goes to infinity, and so finally that $\int_{\underline{q}}^{\overline{q}} m[\overline{\mu}](q) \cdot dG(q)$ goes to infinity as $\overline{\mu}$ goes to infinity. Overall there is a unique $\overline{\mu}$ such that $\int_{\underline{q}}^{\overline{q}} m[\overline{\mu}](q) \cdot dG(q) = b$ and thus a unique solution for $\mu^*(.)$.

C Extension with uninformed buyers

We do not provide an exhaustive list of the properties that the beliefs of an uninformed rational buyer should satisfy. The idea is that such a buyer would base his beliefs over the quality, participation and the secret reserve, if any, according to Bayesian updating from the mixture of the underlying distributions selecting a given format. For the present analysis and since $v_s(.)$ is (strictly) increasing, we are only interested in equilibria in separating strategies where sellers of different qualities chose different formats. The formal definition is given below. In these equilibria, uninformed rational buyers have the same beliefs as informed rational buyers on the equilibrium path. On the contrary, if a seller of quality q deviates by behaving as if she were of quality q' then an uninformed rational buyer would form his beliefs as if the quality were q'. If the seller behaves in a way that is incompatible with any equilibrium behavior for uninformed buyers, then we do not impose any restriction on their beliefs.

Definition 3 *A competitive (rational expectation) equilibrium with λ^{UN} uninformed buyers in separating public strategies is defined as $(r_q, \mu_q^i)_{q\in[\underline{q},\overline{q}],}$, where $r_q \in R_+$ stands for the public reserve price chosen by a quality q seller with $r_q \neq r_{q'}$ if $q \neq q'$ and $\mu_q^i : S^* \to R_+ \cup \{\infty\}$ describes the distributions of participation of buyers of type $i \in \{FR, UN\}$ in the various formats (of goods of quality q) where*

1. *(Profit maximization for sellers) for any $q \in [\underline{q}, \overline{q}]$,*

$$(r_q, public) \in Arg \max_{s=(r,d)\in S} \sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} \Phi_n(r, q), \text{ where } \mu_q(\widehat{s}) = \mu_q^{FR}(\widehat{s}) + \mu_q^{UN}(\widehat{s}). \tag{24}$$

2. *(Profit maximization for informed buyers) for any $q \in [\underline{q}, \overline{q}]$ and $r \in R_+$,*

$$\mu_q^{FR}(r) \underset{(resp.\ =)}{>} 0 \Longrightarrow \sum_{n=0}^{\infty} e^{-\mu_q(r)} \frac{[\mu_q(r)]^n}{n!} V_n(r, q) \underset{(resp.\ \leq)}{=} \int_{\underline{q}}^{\overline{q}} \left[\sum_{n=0}^{\infty} e^{-\mu_q(r_q)} \frac{[\mu_q(r_q)]^n}{n!} V_n(r_q, q) \right] \cdot \frac{\mu_q^{FR}(r_q)}{\lambda^{FR} \cdot b} dG(q). \tag{25}$$

3. *(Profit maximization for uninformed buyers) for any $r \in R_+$, $\mu_q^{UN}(r)$ is independent of q; it is denoted $\mu^{UN}(r)$ and for any $q \in [\underline{q}, \overline{q}]$ it satisfies*

$$\mu^{UN}(r_q) \underset{(resp.\ =)}{>} 0 \Longrightarrow \sum_{n=0}^{\infty} e^{-\mu_q(r_q)} \frac{[\mu_q(r_q)]^n}{n!} V_n(r_q, q) \underset{(resp.\ \leq)}{=} \int_{\underline{q}}^{\overline{q}} \left[\sum_{n=0}^{\infty} e^{-\mu_q(r_q)} \frac{[\mu_q(r_q)]^n}{n!} V_n(r_q, q) \right] \cdot \frac{\mu^{UN}(r_q)}{\lambda^{UN} \cdot b} dG(q). \tag{26}$$

4. (Matching conditions) for $i = FR, UN$, $\int_{\underline{q}}^{\overline{q}} \mu_q^i(r_q)dG(q) = \lambda^i \cdot b$.

Proof of Proposition 3.3 As an equilibrium candidate, consider the following strategy profile: for any $q, q' \in [\underline{q}, \overline{q}]$, let $r_q := v_s(q)$, $\mu_q^{FR}(v_s(q')) := \max\{\widetilde{\mu}_q(v_s(q')) - \lambda^{UN} \cdot \widetilde{\mu}^*(q), 0\}$, $\mu^{UN}(v_s(q)) := \lambda^{UN} \cdot \widetilde{\mu}^*(q)$, and for any $\widehat{s} \in S^*$ such that $\widehat{s} \notin \{v_s(q) | q \in [\underline{q}, \overline{q}]\}$, let $\mu_q^{FR}(\widehat{s}) := \widetilde{\mu}_q(\widehat{s})$ and $\mu^{UN}(\widehat{s}) := 0$, where $\{\widetilde{\mu}_q(.)\}_{q \in [\underline{q}, \overline{q}]}$ corresponds to the distributions of participation in the CRE-equilibrium with only informed buyers and $\widetilde{\mu}^*(q) := \widetilde{\mu}_q(v_s(q))$. Note in particular that $\mu_q^{FR}(\widehat{s}) > 0$ implies $\mu_q(\widehat{s}) = \widetilde{\mu}_q(\widehat{s})$ and that $\mu^{UN}(\widehat{s}) > 0$ implies the existence of q such that $\widehat{s} = v_s(q)$ and $\mu_q(v_s(q)) = \widetilde{\mu}^*(q)$. We can then readily check that the parts 2, 3 and 4 in definition 3 hold, and we are thus left with (24).

From the way in which we define our equilibrium candidate, it is straightforward that any deviation $\widehat{s}$ with $\mu^{UN}(\widehat{s}) = 0$ (which implies that $\mu_q(\widehat{s}) = \widetilde{\mu}_q(\widehat{s})$) is not profitable. Consider then that a given seller with quality q proposes the reserve $r = v_s(q')$ with $q' \neq q$. If some informed buyers participate after the deviation then $\mu_q(r) = \widetilde{\mu}_q(r)$, i.e. participation coincides with that which would prevail in an environment with only informed buyers and for which we know that deviations can not be profitable. We are thus left with deviations such that only uninformed buyers participate.

Since we have assumed that on the equilibrium path participation was bounded away from zero and thus uniformly on $[\underline{q}, \overline{q}]$, there then exists $\Pi > 0$ such that the profit of a seller with quality q is above $v_s(q) + \Pi$ if she sets the public reserve $v_s(q)$. If λ^{UN} is small enough, then the participation rate $\lambda^{UN} \cdot \widetilde{\mu}^*(q')$ can be made as small as possible such that the expected profit is smaller than $v_s(q) + \Pi$ for any q and q' so that informed buyers do not enter. On the whole we obtain that any deviation where informed buyers do not enter is not profitable. As argued above, deviations where informed buyers participate with a strictly positive probability can never be strictly profitable. Condition (24) thus holds when λ^{UN} is small enough.

For the remaining part of the proposition, it is sufficient to show that if b is large enough then $\mu_q^{FR}(v_s(q')) > 0$ for any $q, q' \in [\underline{q}, \overline{q}]$. The above inequalities are equivalent to

$$\widetilde{\mu}_{q'}(v_s(q')) - \widetilde{\mu}_q(v_s(q')) < (1 - \lambda^{UN}) \cdot \widetilde{\mu}^*(q') \quad \text{for any} \quad q, q'. \tag{27}$$

We first have:

$$\frac{d\mu_q(v_s(q'))}{dq} = \frac{(1 - F(v_s(q'))) \cdot e^{-\mu_q(v_s(q') - q)(1 - F(v_s(q') - q))}}{\int_{v_s(q') - q}^{\infty} (1 - F(s))^2 \cdot e^{-\mu_q(v_s(q'))(1 - F(s))} ds}.$$

Take then $\alpha > \max_{q,q'}\{v_s(q') - q\}$. We have $\frac{d\mu_q(v_s(q'))}{dq} \leq \frac{e^{-\mu_q(v_s(q'))(1 - F(v_s(q') - q))}}{\int_{v_s(q') - q}^{\infty} (1 - F(s))^2 \cdot e^{-\mu_q(v_s(q'))(1 - F(s))} ds} \leq$

$\frac{e^{-\mu_q(v_s(q'))(1-F(v_s(q')-q))}}{\int_\alpha^\infty (1-F(s))^2 \cdot e^{-\mu_q(v_s(q'))(1-F(\alpha))}ds} \leq \frac{e^{-\mu_q(v_s(q'))(F(\alpha)-F(v_s(q')-q))}}{\int_\alpha^\infty (1-F(s))^2 ds} \leq \frac{1}{\int_\alpha^\infty (1-F(s))^2 ds}$. Second, if b is large enough, $\min_q\{\widetilde{\mu}^*(q)\}$ can be made as large as possible. Finally we obtain that $|\frac{\widetilde{\mu}_q(v_s(q'))}{q}| < (1-\lambda^{UN}) \cdot \frac{\min_{q' \in [\underline{q},\overline{q}]}\{\widetilde{\mu}^*(q')\}}{\overline{q}-\underline{q}}$ for any q, q' once b is large enough, which guarantees that (27) holds. Condition (24) thus holds when b is large enough which concludes the proof. **Q.E.D.**

D Proof of Lemma 4.1

Take $s = (r, d) \in \text{Supp}(\rho_q)$ with $\mu_q^{FR}(\widehat{s}) > 0$ and suppose that $(r, d) \neq (v_s(q), public)$. We first note that $\mu_q^{FR}(\widehat{s}) > 0$ implies $\sum_{n=0}^\infty e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} V_n^{FR}(\widehat{s}, q) = V^{FR}$. In the same way as in Section 3, we obtain that the seller's expected payoff equals $TW_q(\mu_q(\widehat{s}), r)$ if $d = public$. If $d = secret$, then $r = r^M(q)$ and $(r^M(q), secret) \in \text{Supp}(\rho_q)$ implies that $V_n^{FR}(secret, q) = V_n(r^M(q), q)$ and we finally obtain that the seller's expected payoff equals $TW_q(\mu_q(secret), r^M(q))$. We show that the seller would strictly benefit from choosing the reserve policy $(v_s(q), public)$. If the seller chooses $(v_s(q), public)$, we then have $\sum_{n=0}^\infty e^{-\mu_q(v_s(q))} \frac{[\mu_q(v_s(q))]^n}{n!} V_n(v_s(q), q) \leq V^{FR}$. Let $\overline{m} := \max\{m, 0\}$ where m is characterized as the solution of the equation $\sum_{n=0}^\infty e^{-m} \frac{m^n}{n!} V_n(v_s(q), q) = V^{FR}$. Note that $\overline{m} \leq \mu_q(v_s(q))$ and thus the expected equilibrium payoff of the seller is larger than the payoff she would obtain were the participation rate to be equal to $\overline{m}$. In this latter case, and in the same way as in Section 3, the seller's expected payoff would equal $TW_q(\overline{m}, v_s(q)) = \max_{\mu \geq 0, r \geq 0} TW_q(\mu, r)$. Furthermore, and still as in Section 3, if $\overline{m} > 0$ [resp. $\overline{m} = 0$], then $\text{Arg}\max_{\mu \geq 0, r \geq 0} TW_q(\mu, r) = \{(\overline{m}, v_s(q))\}$ [resp. $= \{(0, r) | r \geq 0\}$]. In any case we obtain that $TW_q(\overline{m}, v_s(q)) > TW_q(\mu_q(r, d), r)$, which raises a contradiction since this means that the seller would strictly benefit from choosing the public reserve $v_s(q)$.

The second part of the lemma follows from the first after noting that PC buyers have the same beliefs as FR buyers for public reserve prices.

E Proof of Lemma 4.2

We first show that $\text{Arg}\max_{\widehat{s} \in S^*} E_q(\sum_{n=0}^\infty e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(\widehat{s}, q)) = \{0\}$. The inclusion $\{0\} \subseteq \text{Arg}\max_{\widehat{s} \in S^*} E_q(\sum_{n=0}^\infty e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(\widehat{s}, q))$ is straightforward since $V_n(r, q)$ is decreasing in r for any n and q so that

$$E_q(\sum_{n=0}^\infty e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(\widehat{s}, q)) \leq E_q(\sum_{n=0}^\infty e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(0, q)) \tag{28}$$

for any $\widehat{s} \in S^*$. Since $V_0(r,q)$ is *strictly* decreasing in r for any q and the event where a buyer is the unique participant in the auction is expected to occur with positive probability by FC buyers ($\overline{\mu} = b < \infty$), we obtain that the inequality (28) is strict for any $\widehat{s} \in R_+$. From the mass equilibrium condition for FR buyers and Lemma 4.1, we obtain $\int_{\underline{q}}^{\overline{q}} \mu_q^{FR}(v_s(q)) \cdot \rho_q(v_s(q), public)dG(q) = \lambda^{FR} \cdot b > 0$, so that positive reserve prices are used with positive probability and thus $V_0^{FC}(0,q) > \frac{\int_{\underline{q}}^{\overline{q}} \int_0^\infty V_0(r,q)\rho_{q'}(r,public)drdG(q')}{\int_{\underline{q}}^{\overline{q}} \int_0^\infty \rho_{q'}(r,public)drdG(q')}$. Combined with $V_0^{FC}(0,q) \geq \frac{\int_{\underline{q}}^{\overline{q}} \int_0^\infty V_0(r,q)\rho_{q'}(r,secret)drdG(q')}{\int_{\underline{q}}^{\overline{q}} \int_0^\infty \rho_{q'}(r,secret)drdG(q')}$, we obtain the strict inequality $V_0^{FC}(0,q) > V_0^{FC}(secret,q)$ for any q, which finally implies that the inequality (28) is strict for $\widehat{s} =$ secret. We have thus shown that

$$\operatorname{Arg}\max_{\widehat{s}\in S^*} E_q(\sum_{n=0}^{\infty} e^{-\overline{\mu}} \frac{\overline{\mu}^n}{n!} V_n^{FC}(\widehat{s},q)) = \{0\}. \tag{29}$$

Suppose now that AA auctions are never proposed by sellers. From (13) and (29), we obtain then that $\mu^{FC}(0) = \infty$ which raises a contradiction since any seller would then find profitable to deviate and propose an AA auction. We obtain finally that some AA auctions are proposed and then from (29) that FC buyers participate only in those auctions.

F Proof of Lemma 4.4

From (11), the matching condition with respect to FR buyers and Lemma 4.1, we obtain that μ_q^{TO} is characterized as the solution of the differential equation (9) with the matching condition $\int_{\underline{q}}^{\overline{q}} \mu_q^{TO} \cdot k(q)dG(q) = \alpha$, where $\alpha \geq \lambda^{FR} \cdot b$ denotes the mass of buyers participating in TO auctions and $k(q) \leq 1$ denotes the percentage of sellers with quality q who select the corresponding TO auction. If $\mu_q^{TO} < y^*(q)$ for some q, then the strict inequality will hold for any q, then $\int_{\underline{q}}^{\overline{q}} \mu_q^{TO} \cdot k(q)dG(q) < \int_{\underline{q}}^{\overline{q}} y^*(q)dG(q) = \lambda^{FR} \cdot b \leq \alpha$ and we have thus raised a contradiction. Finally we obtain from Assumption 2 that $\mu_q^{TO} > 0$ for any $q \in [\underline{q}, \overline{q}]$. From Eq. (15) and after noting that $\Phi_n(v_s(q), q) \geq v_s(q)$ for any n where the inequality is strict if $n \geq 2$, we obtain that $\Pi_{TO}^S(q) > v_s(q)$.

Suppose that the probability of proposing SR auctions is zero such that $\overline{\rho}(r) := \int_{\underline{q}}^{\overline{q}} \rho_q(r, public) \cdot dG(q)$. From Lemma 4.1, TO auctions are proposed with positive probability. Let thus $\widetilde{q} :=$ sup $\{q \in [\underline{q}, \overline{q}] | (v_s(q), public) \in \text{Supp}(\rho_q)\}$. If $(v_s(\widetilde{q}), public) \in \text{Supp}(\rho_{\widetilde{q}})$, then the proof is complete: we can raise a contradiction by showing that a seller with quality $\widetilde{q}$ would strictly benefit from proposing a SR auction, as she will thereby attract more entrants $(\mu_{\widetilde{q}}(secret) > \mu_{\widetilde{q}}(v_s(\widetilde{q}))$ since $V_n^{PC}(secret, \widetilde{q}) > V_n^{PC}(v_s(\widetilde{q}), \widetilde{q})$ for any n, where the latter inequalities result from $\overline{\rho}(r) = 0$ for $r > v_s(\widetilde{q}))$ and she sets a better reserve price for any given participation rate ($r^M(\widetilde{q})$ instead of $v_s(\widetilde{q})$). If $(v_s(\widetilde{q}), public) \notin \mathrm{Supp}(\rho_{\widetilde{q}})$, we can conclude similarly by selecting q in the neighborhood of $\widetilde{q}$ such that $(v_s(q), public) \in \mathrm{Supp}(\rho_q)$. By continuity, we still have $V_n^{PC}(secret, q) > V_n^{PC}(v_s(q), q)$ if q is close enough to $\widetilde{q}$ which would raise a contradiction. Overall we have shown that SR auctions are proposed with strictly positive probability. If the mass of PC buyers entering those auctions is zero, then the participation rate is zero in some SR auctions that are proposed in equilibrium so that the seller's expected payoff would be $v_s(q)$. Since $\Pi_{TO}^S(q) > v_s(q)$, this raises a contradiction with the seller's optimization program.

G Proof of Lemma 4.5

Straightforward calculation leads to:

$$\Pi_{AA}^S(q) = q \cdot (1 - \mu_q^{AA} e^{-\mu_q^{AA}}) + H(q) \cdot e^{-\mu_q^{AA}} + \sum_{n=1}^{\infty} e^{-\mu_q^{AA}} \frac{[\mu_q^{AA}]^n}{n!} \int_0^{\infty} ud[F^{(2:n)}(u)]. \quad (30)$$

At any point q where $\mu_q^{AA} = \mu^{FC}(0)$ (e.g. if $(0, public) \in \mathrm{Supp}(\rho_q)$), we have:

$$\frac{d\Pi_{AA}^S(q)}{dq} = 1 + h(q) \cdot e^{-\mu_q^{AA}} - \mu_q^{AA} \cdot e^{-\mu_q^{AA}}. \quad (31)$$

In the same way as Eq. (7) was established and from Lemma 3.1, we obtain:

$$\Pi_{TO}^S(q) = \max_{\mu} \left(q + [\sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} (H(q) F^n(H(q)) + \int_{H(q)}^{\infty} ud[F^{(1:n)}(u)])] - \mu \cdot V^{FR} \right). \quad (32)$$

From the envelope Theorem, the differentiation w.r.t. q leads to

$$\frac{d\Pi_{TO}^S(q)}{dq} = 1 + h(q) \cdot e^{-\mu_q^{TO}(1-F(H(q)))} \quad (33)$$

Recalling that $\frac{\partial \Phi_n}{\partial r}(r^M(q), q) = 0$, we obtain from (15)

$$\frac{d\Pi^S_{SR}(q)}{dq} = 1+h(q)\cdot e^{-\mu_q^{SR}(1-F(\varepsilon_q^M))}+\frac{d\mu_q^{SR}}{dq}\underbrace{[\sum_{n=0}^{\infty} e^{-\mu_q^{SR}}\frac{[\mu_q^{SR}]^n}{n!}(\Phi_{n+1}(r^M(q),q)-\Phi_n(r^M(q),q))]}_{\equiv J(\mu_q^{SR},q)} \quad (34)$$

Lemma G.1 formalizes the tradeoff between a larger reserve price and enhancing participation.

Lemma G.1 $\Pi^S_{AA}(q) \geq \Pi^S_{TO}(q) \Rightarrow \mu_q^{AA} = \mu^{FC}(0) > \mu_q^{TO}$ *and* $\Pi^S_{TO}(q) \geq \Pi^S_{SR}(q) \Rightarrow \mu_q^{TO} > \mu_q^{SR}$.

Proof Suppose that $\Pi^S_{AA}(q) \geq \Pi^S_{TO}(q)$ and $\mu_q^{AA} > \mu^{FC}(0)$. This implies that either $\mu^{FR}(0) > 0$ or $\mu^{PC}(0) > 0$. Following the way in which we proved Lemma 4.1, this would mean that the seller would strictly benefit from proposing the auction $(v_s(q), public)$ instead of $(0, public)$, which raises a contradiction. We have thus shown that $\Pi^S_{AA}(q) \geq \Pi^S_{TO}(q) \Rightarrow \mu_q^{AA} = \mu^{FC}(0)$.

We have $\Phi_n(0,q) \leq \Phi_n(v_s(q),q) \leq \Phi_n(r^M(q),q)$ and the inequalities are strict for $n \geq 1$. From (15), we obtain that $\Pi^S_{AA}(\mu,q) < \Pi^S_{TO}(\mu,q) < \Pi^S_{SR}(\mu,q)$ for any $\mu > 0$ and any quality q. When we apply the first inequality to $\mu = \mu_q^{AA} = \mu^{FC}(0) > 0$, we obtain that $\Pi^S_{AA}(q) \geq \Pi^S_{TO}(q)$ implies $\Pi^S_{TO}(\mu_q^{AA},q) > \Pi^S_{TO}(\mu_q^{TO},q)$ and finally $\mu_q^{AA} > \mu_q^{TO}$ since $\frac{\partial \Pi^S_{TO}(\mu,q)}{\partial \mu} > 0$. With an analogous argument but with the second inequality, we obtain that $\mu_q^{TO} > 0$ and $\Pi^S_{TO}(q) \geq \Pi^S_{SR}(q)$ imply $\mu_q^{TO} > \mu_q^{SR}$. We conclude the proof by noting that $\mu_q^{TO} > 0$ has been established in Lemma 4.4. **Q.E.D.**

In order to show that a given differentiable function is quasimonotone increasing, it is sufficient to show that its derivative is strictly positive at any point where the function is null. Consider q such that $\Pi^S_{TO}(q) = \Pi^S_{AA}(q)$. From Lemma G.1, we obtain that $\mu_q^{AA} = \mu^{FC}(0)$. From (31) and (33), we have $\frac{d[\Pi^S_{TO}(q)-\Pi^S_{AA}(q)]}{dq} = h(q)\cdot[e^{-\mu_q^{TO}(1-F(H(q)))} - e^{-\mu_q^{AA}}] + \mu_q^{AA}\cdot e^{-\mu_q^{AA}}$. The first term is positive since $h(q) \geq 0$ and $\mu_q^{AA} > \mu_q^{TO}$. The second term is strictly positive. Overall, we have $\frac{d[\Pi^S_{TO}(q)-\Pi^S_{AA}(q)]}{dq} > 0$. Consider now q such that $\Pi^S_{SR}(q) = \Pi^S_{TO}(q)$. From (33) and (34), we have $\frac{d[\Pi^S_{SR}(q)-\Pi^S_{TO}(q)]}{dq} = h(q)\cdot[e^{-\mu_q^{SR}(1-F(\varepsilon_q^M))} - e^{-\mu_q^{TO}(1-F(H(q)))}] + \frac{d\mu_q^{SR}}{dq}\cdot J(\mu_q^{SR},q)$. From lemma G.1 and since $\varepsilon_q^M > H(q)$, $\Pi^S_{SR}(q) = \Pi^S_{TO}(q)$ guarantees that $e^{-\mu_q^{SR}(1-F(\varepsilon_q^M))} > e^{-\mu_q^{TO}(1-F(H(q)))}$ which implies that the first term is positive. From Lemma 4.4, we have $\Pi^S_{TO}(q) > v_s(q)$ which implies that $\mu_q^{SR} > 0$ and thus that $J(\mu_q^{SR},q) > 0$. Furthermore, from (11) for $\widehat{s}$ = secret and PC buyers, we obtain $\frac{d\mu_q^{SR}}{dq} = \frac{\sum_{n=0}^{\infty} e^{-\mu_q^{SR}}\frac{[\mu_q^{SR}]^n}{n!}\int_0^{\infty} F^n(r-q)(1-F(r-q))\overline{\rho}(r)dr}{\sum_{n=0}^{\infty} e^{-\mu_q^{SR}}\frac{[\mu_q^{SR}]^n}{n!}(V_n^{PC}(secret,q)-V_{n+1}^{PC}(secret,q))} > 0$ and the second term is thus strictly positive. Finally, we have $\frac{d[\Pi^S_{SR}(q)-\Pi^S_{TO}(q)]}{dq} > 0$.

H Proof of Proposition 4.6

The proof contains four steps. We first derive a set of necessary conditions for any CAB-equilibrium. In a second step ('construction'), we build a full strategy profile $(\widehat{\rho}_q[q_1, q_2, \tau], \widehat{\mu}^i_q[q_1, q_2, \tau])_{q\in[\underline{q},\overline{q}], i\in\{FC,PC,FR\}}$ for any triple $(q_1, q_2, \tau) \in T$. In a third step ('verification'), we show that if a triple $(q^*, q^{**}, \tau^*) \in T$ satisfies (16) and (17), then the above strategy profile is a CAB-equilibrium. In a fourth step ('existence'), we show that there exists a triple $(q^*, q^{**}, \tau^*) \in T$ satisfying (16) and (17) by applying Kakutani fixed point theorem. This last part is relegated to the Supplementary Material since it is rather technical and does not add new insights. To lighten the notation we let $t := (q_1, q_2, \tau)$.

1/ A set of necessary conditions

We have already shown that, for any CAB-equilibrium, we have a triple $(q_1, q_2, \tau) \in T$ so that: 1) $\text{Supp}(\widehat{\rho}_q[t]) = (0, public)$ if $q < q_1$; 2) $\text{Supp}(\widehat{\rho}_q[t]) = (v_s(q), public)$ if $q \in (q_1, q_2)$; 3) $\text{Supp}(\widehat{\rho}_q[t]) = (r^M(q), secret)$ if $q > q_2$; 4) a share τ [resp. $(1-\tau)$] of the PC buyers participate in TO [resp. SR] auctions; 5) FC buyers participate only in AA; and 6) FR buyers participate only in TO. From profit maximization for buyers and the matching equilibrium conditions, the equilibrium participation rates $\widehat{\mu}^{AA}_q[t]$, $\widehat{\mu}^{TO}_q[t]$ and $\widehat{\mu}^{SR}_q[t]$ in the various formats AA, TO and SR for any quality q respectively in $[\underline{q}, q_1]$, $[q_1, q_2]$ and $[q_2, \overline{q}]$ necessarily have the following form:[55]

- For any $q \in [\underline{q}, q_1]$, we have

$$\widehat{\mu}^{AA}[t] := \widehat{\mu}^{AA}_q[t] = \frac{\lambda^{FC} \cdot b}{G(q_1)}. \tag{35}$$

- For any $q \in [q_1, q_2]$, we have $\widehat{\mu}^{TO}_q[t] = \widetilde{\mu}^{TO}_q[t]$, where $q \to \widetilde{\mu}^{TO}_q[t]$ is uniquely characterized as the solution of the differential equation (9) on the interval $[\underline{q}, \overline{q}]$ (this guarantees the indifference of FR and PC buyers regarding the various quality q objects that are auctioned off through TO auctions) with the matching condition:

$$\int_{q_1}^{q_2} \widetilde{\mu}^{TO}_q[t] \cdot dG(q) = (\lambda^{FR} + \tau \cdot \lambda^{PC}) \cdot b. \tag{36}$$

Let $\widehat{V}^{FR}[t] := \sum_{n=0}^{\infty} e^{-\widehat{\mu}^{TO}_q[t]} \frac{[\widehat{\mu}^{TO}_q[t]]^n}{n!} V_n(v_s(q), q)$, with $q \in [q_1, q_2]$, denote the corresponding expected utility of FR and PC buyers in TO auctions (which does not depend on q by construction).

[55]Uniqueness for $\widehat{\mu}^{TO}_q[t]$ and $\widehat{\mu}^{SR}_q[t]$ results from a similar argument to that presented in Appendix B.

- For any $q \in [q_2, \overline{q}]$, we have $\widehat{\mu}_q^{SR}[t] = \widetilde{\mu}_q^{SR}[t]$, where $q \to \widetilde{\mu}_q^{SR}[t]$ is uniquely characterized as the solution of the differential equation

$$y'(q) = -\frac{\sum_{n=0}^{\infty} e^{-y(q)} \frac{[y(q)]^n}{n!} \cdot \frac{\partial V_n^{PC}(secret,q;q_1,q_2)}{\partial q}}{\sum_{n=0}^{\infty} e^{-y(q)} \frac{[y(q)]^n}{n!} \cdot (V_n^{PC}(secret, q; q_1, q_2) - V_{n+1}^{PC}(secret, q; q_1, q_2))} \quad (37)$$

on the interval $[q_2, \overline{q}]$ where $V_n^{PC}(secret, q; q_1, q_2) := G(q_1) \int_{-q}^{\infty} F^n(x)(1 - F(x))dx + \int_{q_1}^{q_2} \left[\int_{v_s(u)-q}^{\infty} F^n(x)(1 - F(x))dx\right] dG(u) + \int_{q_2}^{\overline{q}} \left[\int_{r^M(u)-q}^{\infty} F^n(x)(1 - F(x))dx\right] dG(u)$ (this guarantees the indifference of PC buyers regarding the various quality q objects that are auctioned off through SR auctions) with the matching condition

$$\int_{q_2}^{\overline{q}} \widehat{\mu}_q^{SR}[t] \cdot dG(q) = (1 - \tau)\lambda^{PC} \cdot b. \quad (38)$$

Let $\widehat{V}^{PC}[t] := \sum_{n=0}^{\infty} e^{-\widehat{\mu}_q^{SR}[t]} \frac{[\widehat{\mu}_q^{SR}[t]]^n}{n!} V_n^{PC}(secret, q; q_1, q_2)$, with $q \in [q_2, \overline{q}]$, denote the corresponding expected utility of PC buyers in SR auctions (which does not depend on q by construction).

For $r \in R_+$, let $\widehat{\mu}_q^*[t](r)$ be defined as the solution (w.r.t. μ) of the equation $\sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} V_n(r, q) = \widehat{V}^{FR}[t]$ if any and zero otherwise. Let $\widehat{\mu}_q^*[t](secret)$ be defined as the solution of the equation $\sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} V_n(r^M(q), q) = \widehat{V}^{FR}[t]$ if any and zero otherwise. In any CAB-equilibrium, if some FR buyers participate to auctions with the observable characteristics $\widehat{s} \in S^*$, then $\widehat{\mu}_q^*[t](\widehat{s})$ corresponds to the equilibrium participation rate. This is also true for PC buyers if $\widehat{s} \neq secret$ since PC buyers correctly assess auctions with public reserves. Furthermore, equilibrium rates can never be strictly lower than these benchmarks since otherwise FR buyers would strictly prefer to participate in these auctions.

The equilibrium rates in any CAB-equilibrium thus satisfy for any $q \in [\underline{q}, \overline{q}]$: 1) $\widehat{\mu}_q^{AA}[t] = \max\{\widehat{\mu}^{AA}[t], \widehat{\mu}_q^*[t](0)\}$, 2) $\widehat{\mu}_q^{TO}[t] = \widehat{\mu}_q^*[t](v_s(q))$, 3) $\widehat{\mu}_q^{SR}[t] = \max\{\widetilde{\mu}_q^{SR}[t], \widehat{\mu}_q^*[t](secret)\}$.

We have also already shown that $\underline{q} < q_1 < q_2 < \overline{q}$ and $\tau < 1$.

2/ Construction We entirely specify $(\widehat{\rho}_q[q_1, q_2, \tau], \widehat{\mu}_q^i[q_1, q_2, \tau])_{q \in [\underline{q}, \overline{q}], i \in \{FC, PC, FR\}}$ in the following way:

- $\text{Supp}(\widehat{\rho}_q[t]) := \left\{ \begin{array}{lll} (0, public) & & q < q_1 \\ (v_s(q), public) & \text{if} & q \in [q_1, q_2] \\ (r^M(q), secret) & & q > q_2 \end{array} \right.$

- Let $\widehat{\mu}_q^{FC}[t](0) := \widehat{\mu}^{AA}[t]$ and $\widehat{\mu}_q^{FC}[t](\widehat{s}) := 0$ for any $\widehat{s} \in S^* \setminus \{0\}$.

- For $r \in (0,\infty)$, let $\widehat{\mu}_q^{FR}[t](r) := \frac{\lambda^{FR}}{\lambda^{FR}+\tau\lambda^{PC}} \cdot \widehat{\mu}_q^*[t](r)$. Let $\widehat{\mu}_q^{FR}[t](0) := \frac{\lambda^{FR}}{\lambda^{FR}+\tau\lambda^{PC}} \cdot \max\{\widehat{\mu}_q^*[t](0) - \widehat{\mu}^{AA}[t], 0\}$. Let $\widehat{\mu}_q^{FR}[t](secret) := 0$ if $\widehat{\mu}_q^*[t](secret) \leq \widetilde{\mu}_q^{SR}[t]$ and $\widehat{\mu}_q^{FR}[t](secret) := \widehat{\mu}_q^*[t](secret)$ otherwise.

- For $r \in (0,\infty)$, let $\widehat{\mu}_q^{PC}[t](r) := \frac{\tau\lambda^{PC}}{\lambda^{FR}+\tau\lambda^{PC}} \cdot \widehat{\mu}_q^*[t](r)$. Let $\widehat{\mu}_q^{PC}[t](0) := \frac{\tau\lambda^{PC}}{\lambda^{FR}+\tau\lambda^{PC}} \cdot \max\{\widehat{\mu}_q^*[t](0) - \widehat{\mu}^{AA}[t], 0\}$. Let $\widehat{\mu}_q^{PC}[t](secret) := \widehat{\mu}_q^{SR}[t]$ if $\widehat{\mu}_q^*[t](secret) \leq \widetilde{\mu}_q^{SR}[t]$ and $\widehat{\mu}_q^{PC}[t](secret) := 0$ otherwise.

We also let $\widehat{\mu}_q[t](.) = \sum_{i=FC,PC,FR} \widehat{\mu}_q^i[t](.)$. We can check that our equilibrium candidate satisfies the aforementioned necessary conditions. In particular, we have: $\widehat{\mu}_q[t](0) = \max\{\widehat{\mu}^{AA}[t], \widehat{\mu}_q^*[t](0)\}$, $\widehat{\mu}_q[t](r) = \widehat{\mu}_q^*[t](r)$ for any $r \in (0,\infty)$ and $\widehat{\mu}_q[t](secret) = \max\{\widehat{\mu}_q^*[t](secret), \widetilde{\mu}_q^{SR}[t]\}$.

3/ Verification We first check that buyers' profit-maximization conditions are satisfied under the strategy profile $(\widehat{\rho}_q[t], \widehat{\mu}_q^i[t])_{q\in[\underline{q},\overline{q}], i\in\{FC,PC,FR\}}$. This is straightforward for FC buyers. From the definition of $\widehat{\mu}_q^*[t](.)$ we obtain that the equilibrium condition (11) for FR buyers is satisfied for any $\widehat{s} \in (0,\infty)$. If $\widehat{\mu}_q^{FR}[t](0) > 0$, then $\widehat{\mu}_q[t](0) = \widehat{\mu}_q^*[t](0)$ and (11) holds for $\widehat{s} = 0$. On the contrary, if $\widehat{\mu}_q^{FR}[t](0) = 0$, then $\widehat{\mu}_q[t](0) \geq \widehat{\mu}_q^*[t](0)$ such that (11) also holds for $\widehat{s} = 0$. If $\widehat{\mu}_q^{FR}[t](secret) > 0$, then $\widehat{\mu}_q[t](secret) = \widehat{\mu}_q^*[t](secret)$ and (11) holds for $\widehat{s} = secret$ and $i = FR$. On the contrary, if $\widehat{\mu}_q^{FR}[t](secret) = 0$ then $\widehat{\mu}_q[t](secret) \geq \widehat{\mu}_q^*[t](secret)$ such that (11) also holds for $\widehat{s} = secret$ and $i = FR$. We have thus finished with the analysis of FR buyers. If $\widehat{\mu}_q^{PC}[t](secret) > 0$, then $\widehat{\mu}_q[t](secret) = \widetilde{\mu}_q^{SR}[t](secret)$ and (11) holds for $\widehat{s} = secret$ and $i = PC$. On the contrary, if $\widehat{\mu}_q^{PC}[t](secret) = 0$, then $\widehat{\mu}_q[t](secret) \geq \widetilde{\mu}_q^{SR}[t](secret)$ such that (11) also holds for $\widehat{s} = secret$ and $i = PC$. From (17) and after noting that PC buyers have the same expectation as FR buyers for the auction with public reserves, we have finished the PC buyers' maximization program.

Second we check that sellers' profit maximization conditions are satisfied under the strategy profile $(\widehat{\rho}_q[t], \widehat{\mu}_q^i[t])_{q\in[\underline{q},\overline{q}], i\in\{FC,PC,FR\}}$. For $k = AA, OR, SR$, let $\widehat{\Pi}_k^S(q) := \Pi_k^S(\widehat{\mu}_q[t](r_k(q)), q)$. Combined with Eq. (16), the following lemma allows us to conclude.

Lemma H.1 *The functions $\widehat{\Pi}_{TO}^S[t](q) - \widehat{\Pi}_{AA}^S[t](q)$ and $\widehat{\Pi}_{SR}^S[t](q) - \widehat{\Pi}_{TO}^S[t](q)$ are quasi-monotone increasing.*

Proof The proof follows that in Lemma 4.5: all the properties of a CAB-equilibrium that are used to establish Lemma 4.5 are satisfied by our equilibrium candidate $(\widehat{\rho}_q[q_1, q_2, \tau], \widehat{\mu}_q^i[q_1, q_2, \tau])_{q\in[\underline{q},\overline{q}], i\in\{FC,PC,FR\}}$. **Q.E.D.**

4/ Existence See the supplementary material.

I Proof of Proposition 4.7

The monotonicity results w.r.t. the various participation rates μ_q^k, $k = AA, TO, SR$, come from the differential equations characterizing any CAB-equilibrium which appeared in the proof of Proposition 4.6. The two inequalities in the second bullet are a corollary of Lemma G.1. The last bullet pertains similarly: $\Pi_{AA}^S(q) \geq \sum_{n=0}^{\infty} e^{-\mu_q(\widehat{s})} \frac{[\mu_q(\widehat{s})]^n}{n!} \Phi_n(r, q)$ implies that $\mu_q^{AA} > \mu_q(\widehat{s})$ if $r \in (0, r^M(q)]$. We thus obtain that $\mu_q^{AA} > \mu_q(\widehat{s})$ for any $s = (r, d) \in S$ with $r \in (0, r^M(q)]$ and for $q \in [\underline{q}, q^*]$. Since $q \to \mu_q(r)$ (for $r \in (0, \infty)$) and $q \to \mu_q^{AA}$ are respectively nonincreasing and nondecreasing, the inequalities $\mu_q^{AA} > \mu_q(r)$ ($r \in (0, \infty)$) extend to $[\underline{q}, \overline{q}]$. Since $\mu_q^{TO} > \mu_q^{SR}$ on $[\underline{q}, q^{**}]$, $\mu_q^{AA} > \mu_q(secret)$ on $[\underline{q}, q^{**}]$ is a corollary.

J Proof of Proposition 4.8

If $\Pi_k^B(q) \geq V^{FR}$ ($k = AA, SR$), then the expected payoff of a seller with a quality q object is not larger than $TW_q(\mu_q^k, r_k(q))$. Since $\mu_q^{TO} > 0$ and $r_k(q) \neq v_s(q)$, we then have $TW_q(\mu_q^k, r_k(q)) < TW_q(\mu_q^{TO}, v_s(q)) = \Pi_{TO}^S(q)$. Overall we obtain that the expected payoff of the seller should be strictly smaller in the k format than in the TO auction. For $k = AA$ and $q \in [\underline{q}, q^*]$ or $k = SR$ and $q \in [q^{**}, \overline{q}]$ this would thus raise a contradiction.

K Proof of Proposition 4.10

If $b < \mu^{FC}(0)$, we obtain that $E_q[\Pi_{AA}^B(q)|q \leq q^*] = E_q[\sum_{n=0}^{\infty} e^{-\mu^{FC}(0)} \frac{[\mu^{FC}(0)]^n}{n!} V_n^{FC}(0, q)|q \leq q^*] < E_q[\sum_{n=0}^{\infty} e^{-b} \frac{b^n}{n!} V_n^{FC}(0, q)|q \leq q^*] \leq E_q[\sum_{n=0}^{\infty} e^{-b} \frac{b^n}{n!} V_n^{FC}(0, q)] = V^{FC}$, i.e. FC buyers experience disappointment in expectation.

L Proof of Proposition 4.12

From Proposition 4.11, it is sufficient to show that μ^{TO} is strictly decreasing in q^*. Eq. (18) can be equivalently expressed as a function of μ^{TO} and q^* only in the following way:

$$\Pi_{AA}^S(\mu^{TO} + \frac{b - \mu^{TO}}{G(q^*)}, q^*) = \Pi_{TO}^S(\mu^{TO}, q^*).$$

Differentiating w.r.t. q^* and μ^{TO} leads to

$$
[\underbrace{\frac{\partial \Pi^S_{TO}}{\partial \mu}}_{>0} + \underbrace{(\frac{1}{G(q^*)} - 1) \cdot \frac{\partial \Pi^S_{AA}}{\partial \mu}}_{\geq 0 \ \text{[Lemma } D.1]}] d\mu^{TO} = -[\underbrace{(\frac{\partial \Pi^S_{TO}}{\partial q} - \frac{\partial \Pi^S_{AA}}{\partial q})}_{>0 \ \text{[Lemma 4.5]}} + \underbrace{\frac{(b - \mu^{TO}) \cdot g(q^*)}{[G(q^*)]^2} \cdot \frac{\partial \Pi^S_{AA}}{\partial \mu}}_{\geq 0 \ \text{[Lemma } D.1]}] dq^*. \tag{39}
$$

We obtain finally that $\frac{d\mu^{TO}}{dq^*} < 0$.

Appendix 2 (Supplementary Material)

A End of the proof of Proposition 4.6: 4/ Existence

If $\underline{q} < q_1 < q_2 < \overline{q}$ and $\tau < 1$, then $\widehat{\mu}_q^{AA}[t]$, $\widehat{\mu}_q^{TO}[t]$ and $\widehat{\mu}_q^{SR}[t]$ are fully characterized as argued in Appendix H. In the remaining cases, the characterization extends straightforwardly: in particular, let $\widehat{\mu}_q^{AA}[t] = \infty$ if $q_1 = \underline{q}$, let $\widehat{\mu}_q^{TO}[t] = \infty$ if $q_1 = q_2$ and let $\widehat{\mu}_q^{SR}[t] = \infty$ if $q_2 = \overline{q}$ and $\tau < 1$.

We also let $\widehat{\widehat{\mu}}_q^{SR}[t; \widetilde{q}_2]$ be the unique solution of the differential equation

$$y'(q) = -\frac{\sum_{n=0}^{\infty} e^{-y(q)} \frac{[y(q)]^n}{n!} \cdot \frac{dV_n^{PC}(secret,q;q_1,\widetilde{q}_2)}{dq}}{\sum_{n=0}^{\infty} e^{-y(q)} \frac{[y(q)]^n}{n!} \cdot (V_n^{PC}(secret,q;q_1,\widetilde{q}_2) - V_{n+1}^{PC}(secret,q;q_1,\widetilde{q}_2))} \tag{40}$$

with the matching condition (38). The difference from the definition of $\widehat{\mu}_q^{SR}[t]$ is that q_2 has been replaced by $\widetilde{q}_2$ in the differential equation (40), but not in (38).

Lemma A.1 $\frac{\partial \widehat{\mu}_{q_1}^{AA}[q,q_2,\tau]}{\partial q}|_{q=q_1} \leq 0$, $\frac{\partial \widehat{\mu}_{q_1}^{TO}[q,q_2,\tau]}{\partial q}|_{q=q_1} \geq 0$, $\frac{\partial \widehat{\mu}_{q_2}^{TO}[q_1,q,\tau]}{\partial q}|_{q=q_2} \leq 0$ *and* $\frac{\partial \widehat{\widehat{\mu}}_{q_2}^{SR}[q_1,q,\tau;\widehat{q}^{**}]}{\partial q}|_{q=q_2} \geq 0$.

Proof From Proposition 4.8, we have $\widehat{\mu}_{q_1}^{AA}[t] > \widehat{\mu}_{q_1}^{*}[t](0)$ so that $\widehat{\mu}_q^{AA}[t] = \frac{\lambda^{FC} \cdot b}{G(q_1)}$ for q in the neighborhood of q_1. We have finally $\frac{\partial \widehat{\mu}_{q_1}^{AA}[q,q_2,\tau]}{\partial q}|_{q=q_1} = -\frac{\lambda^{FC} \cdot b \cdot g(q_1)}{[G(q_1)]^2} \leq 0$.

Consider $\underline{q} \leq q < q' \leq q_2$. Suppose that $\widehat{\mu}_{q'}^{TO}[q,q_2,\tau] > \widehat{\mu}_{q'}^{TO}[q',q_2,\tau]$, the differential equation (9) then implies that $\widehat{\mu}_{q''}^{TO}[q,q_2,\tau] > \widehat{\mu}_{q''}^{TO}[q',q_2,\tau] \geq 0$ for any $q'' \in [q',q_2]$. We thus obtain that $\int_{q'}^{q_2} \widehat{\mu}_u^{TO}[q,q_2,\tau] \cdot dG(u) > \int_{q'}^{q_2} \widehat{\mu}_u^{TO}[q',q_2,\tau] \cdot dG(u)$ and finally that $\int_{q}^{q_2} \widehat{\mu}_u^{TO}[q,q_2,\tau] \cdot dG(u) > (\lambda^{FR} + \tau \cdot \lambda^{PC}) \cdot b$, which raises a contradiction with the matching condition (36). We then obtain that $\frac{\partial \widehat{\mu}_{q_1}^{TO}[q,q_2,\tau]}{\partial q}|_{q=q_1} \geq 0$.

The proof is similar for the two remaining inequalities. **Q.E.D.**

Let $\overline{\Pi}_{AA}^{S}(q,q_2,\tau) := \Pi_{AA}^{S}(\widehat{\mu}_q^{AA}[q,q_2,\tau],q)$ and $\overline{\Pi}_{TO,1}^{S}(q,q_2,\tau) := \Pi_{TO}^{S}(\widehat{\mu}_q^{TO}[q,q_2,\tau],q)$.

Lemma A.2 *The function* $q \to \overline{\Pi}_{TO,1}^{S}(q,q_2,\tau) - \overline{\Pi}_{AA}^{S}(q,q_2,\tau)$ *is quasimonotone increasing on* $(\underline{q},q_2)$ *for any* $q_2 \in [\underline{q},\overline{q}]$ *and* $\tau \in [0,1]$.

Proof $\frac{d\left(\overline{\Pi}_{TO,1}^{S}(q,q_2,\tau) - \overline{\Pi}_{AA}^{S}(q,q_2,\tau)\right)}{dq} = \frac{d\left(\Pi_{TO}^{S}(\widehat{\mu}_q^{TO}(\widehat{q},q_2,\tau),q) - \Pi_{AA}^{S}(\widehat{\mu}_q^{AA}(\widehat{q},q_2,\tau),q)\right)}{dq}|_{\widehat{q}=q} + \frac{d\widehat{\mu}_q^{TO}(\widehat{q},q_2,\tau)}{d\widehat{q}}|_{\widehat{q}=q} \cdot \frac{\partial \Pi_{TO}^{S}(\widehat{\mu}_q^{TO}(q,q_2,\tau),q)}{\partial \mu} - \frac{d\widehat{\mu}_q^{AA}(\widehat{q},q_2,\tau)}{d\widehat{q}}|_{\widehat{q}=q} \cdot \frac{\partial \Pi_{AA}^{S}(\widehat{\mu}_q^{AA}(q,q_2,\tau),q)}{\partial \mu}$. In the same way as Lemma 4.5, we obtain that the first term is strictly positive at any point where $\overline{\Pi}_{TO,1}^{S}(q,q_2,\tau) =$

$\overline{\Pi}^S_{AA}(q, q_2, \tau)$. From Lemma A.1, the second term is always positive since we also have $\frac{\partial \Pi^S_{TO}(\mu,q)}{\partial \mu} \geq 0$ for any μ. From Lemma A.1, we are finished with the third term if we show that $\frac{\partial \Pi^S_{AA}(\widehat{\mu}^{AA}_q(q,q_2,\tau),q)}{\partial \mu} \geq 0$ once $\overline{\Pi}^S_{TO,1}(q, q_2, \tau) = \overline{\Pi}^S_{AA}(q, q_2, \tau)$ which comes from Lemma D.1. **Q.E.D.**

If $q_2 \neq \underline{q}$, $\overline{\Pi}^S_{TO,1}(q, q_2, \tau) - \overline{\Pi}^S_{AA}(q, q_2, \tau)$ goes to $-\infty$ as q goes to $\underline{q}$ and goes to $+\infty$ as q goes to q_2. As a corollary of lemma A.2, there is a unique solution in $(\underline{q}, q_2)$ to the equation

$$\overline{\Pi}^S_{TO,1}(q, q_2, \tau) = \overline{\Pi}^S_{AA}(q, q_2, \tau) \tag{41}$$

for any $q_2 \neq \underline{q}$ and any τ. Let $F_1(q_1, q_2, \tau)$ denote this solution and let $F_1(\underline{q}, \underline{q}, \tau) = \underline{q}$. Note that $F_1(.,.,.)$ is a continuous function of the variables q_1, q_2, τ on T since the expected profit function is continuous with respect to those variables.

Let $\overline{\Pi}^S_{TO,2}(q_1, q, \tau) = \Pi^S_{TO}(\widehat{\mu}^{TO}_q[q_1, q, \tau], q)$ and $\overline{\Pi}^S_{SR}(q_1, q, \tau; \widetilde{q}_2) = \Pi^S_{SR}(\widehat{\widehat{\mu}}^{SR}_q[q_1, q, \tau; \widetilde{q}_2], q)$.

Lemma A.3 *The function $q \to \overline{\Pi}^S_{SR}(q_1, q, \tau; \widetilde{q}_2) - \overline{\Pi}^S_{TO,2}(q_1, q, \tau)$ is quasimonotone increasing on $(q_1, \overline{q})$ for any $q_1, \widetilde{q}_2 \in [\underline{q}, \overline{q}]$ and $\tau \in [0, 1]$.*

Proof $\frac{d\left(\overline{\Pi}^S_{SR}(q_1,q,\tau;\widetilde{q}_2) - \overline{\Pi}^S_{TO,2}(q_1,q,\tau)\right)}{dq} = \frac{d\left(\Pi^S_{SR}(\widehat{\widehat{\mu}}^{SR}_q(q_1,\widehat{q},\tau;\widetilde{q}_2),q) - \Pi^S_{TO}(\widehat{\widehat{\mu}}^{TO}_q(q_1,\widehat{q},\tau),q)\right)}{dq}\Big|_{\widehat{q}=q} + \frac{d\widehat{\widehat{\mu}}^{SR}_q(q_1,\widehat{q},\tau;\widetilde{q}_2)}{d\widehat{q}}\Big|_{\widehat{q}=q} \cdot \frac{\partial \Pi^S_{SR}(\widehat{\mu}^{SR}_q(q_1,q,\tau;\widetilde{q}_2),q)}{\partial \mu} - \frac{d\widehat{\mu}^{TO}_q(q_1,\widehat{q},\tau)}{d\widehat{q}}\Big|_{\widehat{q}=q} \cdot \frac{\partial \Pi^S_{TO}(\widehat{\mu}^{TO}_q(q_1,q,\tau),q)}{\partial \mu}$. In the same way as Lemma 4.5, we obtain that the first term is strictly positive at any point where $\overline{\Pi}^S_{SR}(q_1, q, \tau; \widetilde{q}_2) = \overline{\Pi}^S_{TO,2}(q_1, q, \tau)$. From Lemma A.1, the second and the third terms are always positive since we also have $\frac{\partial \Pi^S_{SR}(\mu,q)}{\partial \mu} \geq 0$ and $\frac{\partial \Pi^S_{TO}(\mu,q)}{\partial \mu} \geq 0$ for any μ. **Q.E.D.**

If $\tau \neq 1$ and $q_1 \neq \overline{q}$, $\overline{\Pi}^S_{SR}(q_1, q, \tau; \widetilde{q}_2) - \overline{\Pi}^S_{TO,2}(q_1, q, \tau)$ goes to $-\infty$ as q goes to q_1 and goes to $+\infty$ once q goes to $\overline{q}$. As a corollary of lemma A.3, there is a unique solution in $(q_1, \overline{q})$ to the equation

$$\overline{\Pi}^S_{SR}(q_1, q, \tau; \widetilde{q}_2) = \overline{\Pi}^S_{TO,2}(q_1, q, \tau) \tag{42}$$

for any q_1, $\widetilde{q}_2$ and τ. Consider the solution of this equation when q_1 is fixed to $F_1(q_1, q_2, \tau)$, $\widetilde{q}_2$ to q_2 and τ remains our initial τ. This solution is then denoted by $F_2(q_1, q_2, \tau)$. To complete the definition, we let $F_2(\overline{q}, \overline{q}, \tau) := \overline{q}$ and $F_2(q_1, q_2, 1) := \overline{q}$ for any q_1, q_2. The function $F_2(.,.,.)$ is continuous on T.

Take $q_1 < \overline{q}$, $q_2 \in (q_1, \overline{q})$. We note first that $\frac{\partial \widehat{\widehat{\mu}}^{SR}_q(q_1,q_2,\tau)}{\partial \tau} < 0$ and $\frac{\partial \widehat{\mu}^{TO}_q(q_1,q_2,\tau)}{\partial \tau} > 0$. There is thus a unique solution $\widetilde{\tau}$ to the program

$$\widetilde{\tau} \underset{(resp.\ >)}{=} 0 \Rightarrow \sum_{n=0}^{\infty} e^{-\widehat{\widehat{\mu}}^{SR}_{q_2}(q_1,q_2,\widetilde{\tau})} \frac{[\widehat{\widehat{\mu}}^{SR}_{q_2}(q_1, q_2, \widetilde{\tau})]^n}{n!} V^{PC}_n(secret, q_2; q_1, q_2) \underset{(resp.\ =)}{\geq} \sum_{n=0}^{\infty} e^{-\widehat{\mu}^{TO}_{q_2}(q_1,q_2,\widetilde{\tau})} \frac{[\widehat{\mu}^{TO}_{q_2}(q_1, q_2, \widetilde{\tau})]^n}{n!} V^{PC}_n(v_s(q_2), q_2). \tag{43}$$

Let then $F_3(q_1, q_2, \tau) := \{\widetilde{\tau}\}$. If $q_2 = q_1$, let $F_3(q_1, q_2, \tau) = \{0\}$. If $q_1 < q_2 = \overline{q}$, let $F_3(q_1, q_2, \tau) = \{1\}$. Finally, let $F_3(\overline{q}, \overline{q}, \tau) = [0, 1]$. The correspondence $F_3(., ., .)$ is upper hemicontinuous on T.

We now have all of the elements required to apply a fixed point Theorem. Consider the correspondence F such that $F(q_1, q_2, \tau) = (F_1(q_1, q_2, \tau, F_2(q_1, q_2, \tau), F_3(q_1, q_2, \tau))$. The correspondence F is an upper hemicontinuous function from T, which is a convex compact subset of the Euclidian space, to itself. From the Kakutani fixed point Theorem the correspondence F has a fixed point. Note first that for any fixed point $t^* := (q^*, q^{**}, \tau^*)$, we have $\underline{q} < q^* < q^{**} < \overline{q}$ and $\tau^* < 1$. We conclude the proof by noting that the equations (41-43) guarantee that any fixed point $t^* := (q^*, q^{**}, \tau^*)$ of F satisfies (16) and (17).

B Proof of Proposition 4.11

Our analysis of CAB-equilibria in Subsection 4.2 applies to the cases where $\lambda^i > 0$ for any $i \in \{FC, PC, FR\}$. However, it extends straightforwardly to the cases $\lambda^{PC} = 0$ and $\lambda^{FC} = 0$.

Case $\lambda^{PC} = 0$. Any CAB-equilibrium is given by the threshold q^* which satisfies

$$\Pi^S_{AA}(\frac{(1-\lambda^{FR}) \cdot b}{G(q^*)}, q^*) = \Pi^S_{TO}(\widetilde{\mu}^{TO}_{q^*}[q^*, \overline{q}, 0], q^*). \quad (44)$$

Let $m_1(q, \lambda) := \widetilde{\mu}^{TO}_q[q, \overline{q}, 0]$, where $\lambda = \lambda^{FR}$. The dependence in λ^{FR} is restored for clarity. As a preliminary, we show that for any $\lambda > 0$, $m_1(q, \lambda)$ is strictly increasing in q. Suppose on the contrary that $q' > q$ and $m_1(q', \lambda) \leq m_1(q, \lambda)$. As both functions $x \to \widetilde{\mu}^{TO}_x[q, \overline{q}, 0]$ and $x \to \widetilde{\mu}^{TO}_x[q', \overline{q}, 0]$ satisfy the differential equation (9) this implies that $\widetilde{\mu}^{TO}_x[q', \overline{q}, 0] \leq \widetilde{\mu}^{TO}_x[q, \overline{q}, 0]$ for any x. Since $\widetilde{\mu}^{TO}_x[q, \overline{q}, 0] > 0$ for x in a neighborhood of q, we then obtain that $\int_q^{\overline{q}} \widetilde{\mu}^{TO}_x[q, \overline{q}, 0] \cdot dG(x) > \int_{q'}^{\overline{q}} \widetilde{\mu}^{TO}_x[q', \overline{q}, 0] \cdot dG(x)$ which raises a contradiction with $\int_q^{\overline{q}} \widetilde{\mu}^{TO}_x[q, \overline{q}, 0] \cdot dG(u) = \int_{q'}^{\overline{q}} \widetilde{\mu}^{TO}_x[q', \overline{q}, 0] \cdot dG(u) (= \lambda^{FR} \cdot b)$. Similarly, we have that for any $q < \overline{q}$, $m_1(q, \lambda)$ is strictly increasing in λ.

We first show that $\lambda^{FR}_1 \leq \lambda^{FR}_2$ implies that $q^*_1 \geq q^*_2$ where the pairs (λ^{FC}_i, q^*_i), $i = 1, 2$, stand for the solutions of (44). Suppose that $\lambda^{FR}_1 \leq \lambda^{FR}_2$ and $q^*_1 < q^*_2$. From Lemma 4.5 and (44) for the pair (λ^{FC}_2, q^*_2), we obtain that $\Pi^S_{AA}(\frac{(1-\lambda^{FR}_2)b}{G(q^*_2)}, q^*_1) > \Pi^S_{TO}(m_1(q^*_2, \lambda^{FC}_2), q^*_1)$. From Lemma D.1, we obtain that $\frac{\partial \Pi^S_{AA}(\mu, q^*_1)}{\partial \mu} \geq 0$ if $\mu \geq \frac{(1-\lambda^{FR}_2)b}{G(q^*_2)}$. Since $q^*_1 < q^*_2$, we have then $\Pi^S_{AA}(\frac{(1-\lambda^{FR}_2)b}{G(q^*_1)}, q^*_1) \geq \Pi^S_{AA}(\frac{(1-\lambda^{FR}_2)b}{G(q^*_2)}, q^*_1)$. Since $q^*_1 < q^*_2$ and $m_1(q, \lambda)$ is increasing in q, we have furthermore $\Pi^S_{TO}(m_1(q^*_2, \lambda^{FC}_2), q^*_1) \geq \Pi^S_{TO}(m_1(q^*_1, \lambda^{FC}_2), q^*_1)$ (). We then obtain

$$\Pi^S_{AA}(\frac{(1-\lambda_2^{FR})b}{G(q_1^*)}, q_1^*) > \Pi^S_{TO}(m_1(q_1^*, \lambda_2^{FR}), q_1^*). \tag{45}$$

Note that from Lemma D.1 Eq. (45) implies that $\frac{\partial \Pi^S_{AA}(\mu, q_1^*)}{\partial \mu} \geq 0$ if $\mu \geq \frac{(1-\lambda_2^{FR})b}{G(q_1^*)}$. From (44) for the pair (λ_1^{FC}, q_1^*) and since $\lambda_1^{FR} \leq \lambda_2^{FR}$ we obtain that

$$\Pi^S_{AA}(\frac{(1-\lambda_2^{FR})b}{G(q_1^*)}, q_1^*) \leq \Pi^S_{AA}(\frac{(1-\lambda_1^{FR})b}{G(q_1^*)}, q_1^*) = \Pi^S_{TO}(m_1(q_1^*, \lambda_1^{FR}), q_1^*) \leq \Pi^S_{TO}(m_1(q_1^*, \lambda_2^{FR}), q_1^*),$$

which raises a contradiction with (45). There is thus a unique CAB-equilibrium and q^* is nonincreasing in λ^{FR}.

Similarly, it can be shown that $\lambda_1^{FR} < \lambda_2^{FR}$ implies that $q_1^* > q_2^*$ which ends the proof of the first bullet.

Case $\lambda^{FC} = 0$. Any CAB-equilibrium with $\tau^* = 0$ is given by the threshold q^{**} which satisfies:

$$\Pi^S_{AA}(\widetilde{\mu}^{TO}_{q^{**}}[\underline{q}, q^{**}, 0], q^{**}) = \Pi^S_{TO}(\widetilde{\mu}^{SR}_{q^{**}}[\underline{q}, q^{**}, 0], q^{**}) \tag{46}$$

Let $m_2(q, \lambda) := \widetilde{\mu}^{TO}_q[\underline{q}, q, 0]$ and $m_3(q, \lambda) := \widetilde{\mu}^{SR}_q[\underline{q}, q, 0]$, where $\lambda = \lambda^{FR}$. As a preliminary, we show that for any $\lambda > 0$, $m_1(q, \lambda)$ is strictly increasing in q. In the same way as we above established the monotonicity properties for $m_1(q, \lambda)$, we have for any $\lambda \in (0, 1)$ and $q \in (\underline{q}, \overline{q})$: $m_2(q, \lambda)$ [resp. $m_3[q, \lambda)$] is strictly decreasing [resp. increasing] in q and strictly increasing [resp. decreasing] in λ.

We show that $\lambda_1^{FR} \geq \lambda_2^{FR}$ implies that $q_1^{**} \geq q_2^{**}$, where the pairs $(\lambda_i^{FR}, q_i^{**})$, $i = 1, 2$, stand for the solutions of (46). Suppose that $\lambda_1^{FR} \geq \lambda_2^{FR}$ and $q_1^{**} < q_2^{**}$. From Lemma 4.5 and (46) for the pair $(\lambda_2^{FR}, q_2^{**})$, we obtain that $\Pi^S_{TO}(m_2(q_2^{**}, \lambda_2^{FR}), q_1^{**}) > \Pi^S_{SR}(m_3(q_2^{**}, \lambda_2^{FR}), q_1^{**})$. Since $q_1^{**} < q_2^{**}$ and $m_2[\lambda, q](q)$ [resp. $m_3[\lambda, q](q)$] is decreasing [resp. increasing] in q, we have furthermore $\Pi^S_{TO}(m_2(q_1^{**}, \lambda_2^{FR}), q_1^{**}) \geq \Pi^S_{TO}(m_2(q_2^{**}, \lambda_2^{FR}), q_1^{**})$ and $\Pi^S_{SR}(m_3(q_2^{**}, \lambda_2^{FR}), q_1^{**}) \geq \Pi^S_{SR}(m_3(q_1^{**}, \lambda_2^{FR}), q_1^{**})$. We then obtain

$$\Pi^S_{TO}(m_2(q_1^{**}, \lambda_2^{FR}), q_1^{**}) > \Pi^S_{SR}(m_3(q_1^{**}, \lambda_2^{FR}), q_1^{**}). \tag{47}$$

From (46) for the pair $(\lambda_1^{FR}, q_1^{**})$, and since $\lambda_1^{FR} \geq \lambda_2^{FR}$, we obtain that

$$\Pi_{TO}^S(m_2(q_1^{**}, \lambda_2^{FR}), q_1^*) \leq \Pi_{TO}^S(m_2[q_1^{**}, \lambda_1^{FR}), q_1^*) = \Pi_{SR}^S(m_3(q_1^{**}, \lambda_1^{FR}), q_1^*) \leq \Pi_{SR}^S(m_3(q_1^{**}, \lambda_2^{FR}), q_1^*)$$

which raises a contradiction with (47). There is thus at most one CAB-equilibrium with $\tau^* = 0$ and q^{**} is nondecreasing in λ^{FR}.

Similarly, it can be shown that $\lambda_1^{FR} > \lambda_2^{FR}$ implies that $q_1^{**} > q_2^{**}$ which ends the proof of the second bullet.

C Heterogenous reservation values

The equations (48-51) below are the analogs of (1-4).

$$\Phi_n(r, X_S) = X_S \cdot F^{(1:n)}(r) + r \cdot (F^{(2:n)}(r) - F^{(1:n)}(r)) + \int_r^\infty x d[F^{(2:n)}(x)] \tag{48}$$

$$V_n(r, X_S) = \int_r^\infty F^n(x)(1 - F(x))dx \tag{49}$$

$$W_n(r, X_S) = X_S \cdot F^{(1:n)}(r) + \int_r^\infty x d[F^{(1:n)}(x)] \tag{50}$$

$$r^M(X_S) - \frac{1 - F(r^M(X_S))}{f(r^M(X_S))} = X_S \tag{51}$$

The participation rates in AA, TO and SR auctions, denoted respectively by $\mu_{X_S}^{AA}$, $\mu_{X_S}^{TO}$ and $\mu_{X_S}^{SR}$, are characterized similarly. For any X_S, we have $\mu^{AA} := \mu_{X_S}^{AA} = \frac{\lambda^{FC} b}{G(X_S^*)}$, $\mu^{SR} := \mu_{X_S}^{SR} = \frac{(1-\tau^*)\lambda^{PC} b}{1-G(X_S^{**})}$ (where τ^* denotes the equilibrium share of PC buyers who enter TO auctions) and finally $\mu_{X_S}^{TO}$ is characterized as the unique solution of the differential equation

$$y'(X_S) = -\frac{(1 - F(X_S) \cdot e^{-y(X_S)(1-F(X_S))}}{\int_{X_S}^\infty (1 - F(x))^2 \cdot e^{-y(X_S)(1-F(x))} dx} \tag{52}$$

on any point where $\mu_{X_S}^{TO}$ together with the condition $\int_{X_S^*}^{X_S^{**}} \mu_{X_S}^{TO} \cdot dG(X_S) = b$. From the analog of (15) and adopting similar notation, the equilibrium conditions $\Pi_{AA}^S(X_S^*) = \Pi_{TO}^S(X_S^*)$ and $\Pi_{TO}^S(X_S^{**}) = \Pi_{SR}^S(X_S^{**})$ imply that $\mu^{AA} > \mu_{X_S}^{TO} > \mu^{SR}$ for any $X_S \in [X_S^*, X_S^{**}]$. We have furthermore

$$\frac{d\Pi_{AA}^S(X_S)}{dX_S} = e^{-\mu^{AA}}, \tag{53}$$

$$\Pi^S_{TO}(X_S) = \max_{\mu}\left(\sum_{n=0}^{\infty} e^{-\mu}\frac{\mu^n}{n!}(X_S F^n(X_S) + \int_{X_S}^{\infty} x d[F^{(1:n)}(x)]) - \mu \cdot V^{FR}\right) \tag{54}$$

$$\frac{d\Pi^S_{TO}(X_S)}{dX_S} = e^{-\mu^{TO}_{X_S}(1-F(X_S))} \tag{55}$$

$$\frac{d\Pi^S_{SR}(X_S)}{dX_S} = e^{-\mu^{SR}(1-F(r^M(X_S)))} \tag{56}$$

We then obtain that:

- $\Pi^S_{AA}(X_S) = \Pi^S_{TO}(X_S) \Rightarrow \frac{d[\Pi^S_{TO}(X_S) - \Pi^S_{AA}(X_S)]}{dX_S} > 0$
- $\Pi^S_{TO}(X_S) = \Pi^S_{SR}(X_S) \Rightarrow \frac{d[\Pi^S_{SR}(X_S) - \Pi^S_{TO}(X_S)]}{dX_S} > 0$

We finally obtain that $\Pi^S_{TO}(X_S) - \Pi^S_{AA}(X_S)$ and $\Pi^S_{SR}(X_S) - \Pi^S_{TO}(X_S)$ are quasimonotone increasing, which is the key property for the derivation of the sorting property among AA, TO and SR auctions according to the seller's reservation value.

D Monotonicity of the seller's payoff w.r.t. the participation rate in AA

Lemma D.1 *If* $\Pi^S_{AA}(\mu, q) \geq v_S(q)$ *and* $\mu > 0$*, then* $\frac{d\Pi^S_{AA}(\mu,q)}{d\mu} \geq 0$.[56]

The lemma implies that once a seller proposes an AA auctions in equilibrium then she is better off as the participation rate increases.

Proof If $\mu > 0$, then $\Pi^S_{AA}(\mu, q) \geq v_S(q)$ is equivalent to $\sum_{n=1}^{\infty} \frac{e^{-\mu}}{1-e^{-\mu}} \frac{\mu^n}{n!} \cdot \Phi_n(0, q) \geq v_S(q) = \Phi_0(0, q)$. The left term can be viewed as a weighted sum of the terms $\Phi_n(0, q)$ with respect to the weights $w_n = \frac{e^{-\mu}}{1-e^{-\mu}} \frac{\mu^n}{n!}$ which sum to 1. The inequality $\frac{d\Pi^S_{AA}(\mu,q)}{d\mu} \geq 0$ can also be written equivalently as $\sum_{n=1}^{\infty} \frac{\mu^n}{n!}[\frac{n}{\mu} - 1] \cdot \Phi_n(0, q) \geq v_S(q)$. The left term can be viewed as a weighted sum of the terms $\Phi_n(0, q)$ with respect to the weights $w'_n = \frac{\mu^n}{n!}[\frac{n}{\mu} - 1]$ which sum to 1. Since $\Phi_n(0, q)$ is increasing in n for $n \geq 1$, in order to show that $\sum_{n=1}^{\infty} w'_n \cdot \Phi_n(0, q) \geq \sum_{n=1}^{\infty} w_n \cdot \Phi_n(0, q)$ it is sufficient to show that $\sum_{n=1}^{k} w_n \geq \sum_{n=1}^{k} w'_n$ for any $k \geq 1$. This can be rewritten equivalently as $D_k(\mu) := \sum_{n=1}^{k} \frac{\mu^n}{n!} - (1 - \frac{\mu^k}{k!}) \cdot (e^{\mu} - 1) \geq 0$. The proof is by induction on k. The inequality $D_1(\mu) \geq 0$ is equivalent to $e^{-\mu} \geq (1 - \mu)$, which is known to hold. Suppose now that $D_{k-1}(\mu) \geq 0$. We have $D_k(0) = 0$. Furthermore, $D'_k(\mu) = D_{k-1}(\mu) + \frac{\mu^k}{k!} e^{\mu} \geq D_{k-1}(\mu) \geq 0$. Finally we obtain that $D_k(\mu) \geq 0$ for any μ. **Q.E.D.**

[56] It is not true that $\frac{d\Pi^S_{AA}}{d\mu}(\mu, q) \geq 0$ for any $\mu \geq 0$. In particular, $\frac{d\Pi^S_{AA}(0,q)}{d\mu} = -v_S(q) < 0$

E Proof of Proposition 6.1

Since $b < 1$, we first note that a positive measure of sellers will fail to sell their good and thus that sellers' expected payoff should lie strictly below X_B in equilibrium. We obtain then that in equilibrium, FC buyers should participate solely in AA. If this were not the case, then a seller would sell the good for sure at the price X_B by proposing AA since she will receive two bids for sure. Since sellers can not be sure of selling the good at the price X_B, a positive measure of the goods proposed at AA should remain unsold. We have thus $0 < \nu^* < \frac{\lambda^{FC} b}{2}$. FC buyers then bid solely in auctions where the current price is zero, i.e. in AA auctions that did not receive strictly more than one bid. Note that once an auction with a public reserve has received two bids, then the price equals X_B and no additional bidder will enter the auction.

In a first step, we detail the dynamics of the number of bidders in AA auctions for any $0 < \nu^* < \frac{\lambda^{FC} b}{2}$. Let $\psi_0(x;s)$, $\psi_1(x;s)$ and $\psi_2(x;s)$ denote the share of AA auctions with respectively zero, one and two entrants when a share x of the FC buyers have already made their entry decision and where s denotes the ratio between the mass of FC buyers and the mass of AA auctions proposed (we have $s = \frac{\lambda^{FC} b}{\nu^*}$ in equilibrium). We have $\sum_{i=1}^{3} \psi_i(x;s) = 1$ such that we are left with the characterization of $\psi_0(x;s)$ and $\psi_1(x;s)$ which are uniquely characterized by the system of differential equations:[57]

$$\psi_0'(x;s) = \frac{-\psi_0(x;s)}{\psi_0(x;s) + \psi_1(x;s)} \cdot s$$

$$\psi_1'(x;s) = \frac{\psi_0(x;s) - \psi_1(x;s)}{\psi_0(x;s) + \psi_1(x;s)} \cdot s$$

with the initial conditions $\psi_0(0;s) = 1$ and $\psi_1(0;s) = 0$. At any point where $\psi_0(x;s) > 0$, we then have $\frac{\psi_1'(x;s)}{\psi_0'(x;s)} = \frac{\psi_1(x;s)}{\psi_0(x;s)} - 1$. With the initial conditions, this yields $\psi_0(x;s) \cdot ln[\psi_0(x;s)] = -\psi_1(x;s)$. Combined with the matching condition

$$2 \cdot \psi_2(x;s) + 1 \cdot \psi_1(x;s) = s \cdot x, \tag{57}$$

we obtain finally that $\psi_0(x;s)$ is characterized as the (unique) solution of the equation:

$$\psi_0(x;s) \cdot (2 - ln[\psi_0(x;s)]) = 2 - s \cdot x. \tag{58}$$

We can check that for any $s > 0$, $\psi_0(x;s)$ is a decreasing function on $[0,1]$. Note that if $s \geq 2$, then FC buyers will fill all AA auctions, i.e. $\psi_0(1;s) = \psi_1(1;s) = 0$ (this cannot occur

[57] In the case where $\psi_0(x;s) + \psi_1(x;s) = 0$, we let $\psi_0'(x;s) = \psi_1'(x;s) = 0$.

in equilibrium, as argued above). On the contrary, we have $s < 2$ and $\psi_0(1; s) \in (0, 1)$ and is decreasing in s. Furthermore, this goes to 1 [resp. 0] as s goes to 0 [resp. 2]. According to the notation introduced in Subsection 6.3, we have $\psi_0 = \psi_0(1; \frac{\lambda^{FC} b}{\nu^*})$ and $\psi_2 = \psi_2(1; \frac{\lambda^{FC} b}{\nu^*})$.

Once all FC buyers have entered, we then have a positive mass $\psi_0 \cdot \nu^*$ of AA auctions which received no bid at this stage. First FC buyers enter these remaining auctions until all AA auctions have received at least one bid. If there are some remaining AA auctions with no entrants, we then turn to the PC buyers. At this stage, our analysis does not exclude the existence of an equilibrium where buyers enter solely AA auctions. We proceed by contradiction. Suppose that $(\lambda^{FR} + \lambda^{PC})b \leq \psi_0 \nu^*$. The expected payoff of a seller proposing an AA auction is then given by $\Pi^S_{AA} = \psi_2 \cdot X_B + \psi_0(1 - \frac{(\lambda^{FR} + \lambda^{PC})b}{\psi_0 \nu^*}) \cdot X_S$. From the matching condition (57) with $x = 1$, we obtain that $\psi_2 \leq \frac{s}{2}$ and then that $\Pi^S_{AA} \leq \frac{\lambda^{FC} b}{2\nu^*} \cdot X_B + (\psi_0 - \frac{(\lambda^{FR} + \lambda^{PC})b}{\nu^*}) \cdot X_S \leq \frac{\lambda^{FC} b}{2\nu^*} \cdot X_B + (1 - \frac{(\lambda^{FR} + \lambda^{PC})b}{\nu^*}) \cdot X_S$. Combined with our assumption $\frac{\frac{\lambda^{FC}}{2}}{\lambda^{FR} + \lambda^{PC}} < \frac{X_S}{X_B}$, this leads to $\Pi^S_{AA} < X_S$ which raises a contradiction since sellers would strictly prefer proposing another format. We have thus proved that a positive measure of PC buyers should enter either some OR auctions or some SR auctions in equilibrium. We prove below that the OR auctions proposed in equilibrium (if any) all have a reserve equal or above X_S. There are two possibilities for the OR auctions with the lowest reserve: either they receive no bids, which means that these are the SR auctions which receive some bids (since AA can not receive all bids in equilibrium) but then the seller would strictly benefit from switching to a SR auction; or they receive sometimes one bid, which means that the seller would strictly benefit from raising his reserve to X_S to avoid a loss. If sellers propose OR auctions with different reserves then there will be a contradiction: if buyers enter both kinds of OR auctions with probability one, then the seller would strictly prefer the highest reserve, the only possibility is that buyers mix for the auction with the highest reserve between the two, however this would raise a contradiction since a seller would strictly benefit by proposing an auction with a reserve just below the highest of those two reserves. Let r^* denote the reserve of the OR auctions that are proposed in equilibrium, if any.

We show that there is a positive measure of SR auctions proposed in equilibrium. Note that once a SR auction has received one bid, then the price equals X_B and no additional bidder will enter the auction. If SR auctions were not proposed in equilibrium, then some OR auctions with reserve r^* will be proposed and some PC buyers should participate in those auctions. For PC buyers, SR auctions will appear as strictly more profitable (the expected distribution of secret reserve for FC buyers will lie strictly below r^* if there are no SR auctions). Sellers would thus strictly benefit from proposing a SR auction which raises a contradiction. Since there are some SR auctions in equilibrium and there is a positive measure of buyers entering them, we then obtain that $r^* > X_S$.

We show that the expected number of entrants in OR auction should be equal to one. It cannot be larger than one since OR auctions receive at most one bid; it cannot be below one since otherwise sellers proposing OR auctions would strictly benefit by undercutting the reserve price which would allow them to attract an entrant with probability one. Let τ^* denote the share of PC buyers who do not enter SR auctions. We hence obtain the matching condition:

$$\nu^{**} - \nu^* = \max\{0, (\lambda^{FR} + \tau^* \lambda^{PC}) \cdot b - \psi_0 \cdot \nu^*\}. \tag{59}$$

where $(\lambda^{FR} + \tau^* \lambda^{PC}) \cdot b - \psi_0 \cdot \nu^*$ corresponds to the measure of FR and PC buyers who enter neither AA nor SR auctions, i.e. the measure of buyers entering OR auctions.

We now compute the expected payoff of a seller for each kind of auctions proposed in equilibrium: $\Pi^S_{AA} = \psi_2 \cdot X_B$, $\Pi^S_{OR} = r^*$ if $\nu^{**} > \nu^*$, $\Pi^S_{OR} = 0$ if $\nu^{**} = \nu^*$ and $\Pi^S_{SR} = X_S + \frac{(1-\tau^*)\lambda^{PC} b}{1-\nu^{**}}(X_B - X_S)$. We thus obtain

$$r^* = \psi_2 \cdot X_B = X_S + \frac{(1-\tau^*)\lambda^{PC} b}{1-\nu^{**}}(X_B - X_S) \tag{60}$$

The final equilibrium equation is that reflecting the indifference of PC buyers between SR and OR auctions in the case where they participate in the two. Due to the dynamic nature of the game, this indifference does not come for free. In particular it may occur that conditional on no entrants PC buyers strictly prefer OR auctions to SR, but that once the former are all filled, it has no choice except to enter the SR auction. However, if this were the case, then sellers would strictly benefit from proposing an OR auction with a reserve slightly above r^*. The indifference of PC buyers between SR auctions and OR auctions is given by:

$$\nu^{**} > \nu^* \text{ and } \tau^* \underset{(resp. >)}{=} 0 \Rightarrow \nu^* \cdot X_B + (\nu^{**} - \nu^*) \cdot (X_B - r^*) \underset{(resp. =)}{\geq} (X_B - r^*). \tag{61}$$

Overall, we have then demonstrated all of the necessary conditions in Proposition 6.1.