Evolutionary Dynamical Pattern of ‘Coyness and Philandering’: Evidence from Experimental Economics

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‘Coyness and Philandering’, a two populations game, is one of the most typical game in evolutionary game theory. The evolution dynamic provides the velocity field and then the evolution trajectories of the game with beautiful patterns. However, this game has never been empirically detected in laboratory experimental economics and the patterns have never been obtained. We design and conduct this two populations game with 192 human subjects. With our instantaneous velocity at position (VAP) metric, in which the velocity is of time reversal anti-symmetry, we find that, the empirical velocity vectors fall into a global cyclic pattern in the ‘Coyness and Philandering’ game. The significant of the global velocity pattern together with the VAP metric might be helpful for understanding evolutionary dynamic better.

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1 Evolutionary Game Theory, Experimental Economics and Velocity

The 'Coyness and Philandering' game original suggested by Dawkins in 1976 [10] is a typical model for evolutionary game theory for decades (e.g., [21, 16, 24, 15, 27]). In the paper of 'Coyness, Philandering and stable strategies', published in *Animal Behaviour* in 1981 [21], Schuste & Sigmund analysis the game to describe the evolution of strategies in the conflict of the sexes over parental investment with replicator dynamic [26], and the ordinary differential equations (ODE) is

$$\dot{x}_i = x_i \left(\sum_j a_{ij} x_j - \sum_{kj} x_k a_{kj} x_j \right), \tag{1}$$

where, the $\sum_{kj} x_k a_{kj} x_j$ is the mean payoff of the given population, and $\sum_j a_{ij} x_j$ is the mean payoff of the agent who use strategy i. The left hand side of Eq.(1), $\dot{x}_i$, is the velocity, which indicates the changing of the frequencies of strategies [4, 23, 28]. Given the payoff matrix in Table 1 and the replicator dynamic, phase diagram of the game can be presented [21]. Figure 1 is a reproduction of the phase diagram. This indicates that when the dynamic equation given, for each point in state space, the velocity is determined [1]. The strategies should be oscillating and the velocity field should be in cyclic pattern, it is predicted in different theoretical models [20].

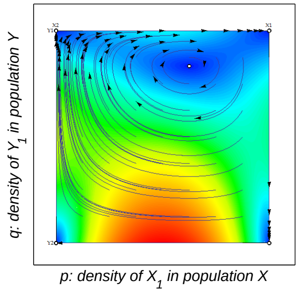

Figure 1: Phase diagram for the replicator dynamics of the 'Coyness and Philandering' game. The background colors represent speed of motion, the magnitude of velocity: blue is slowest, red is fastest. Instantaneous velocity should always be tangential to the trajectory. Black lines indicate the trajectories of the evolution. The white dots are unstable equilibrium. Figure is made by the game dynamics program Dynamo [20].

Laboratory experimental economics removes evolutionary game theory from its abstract setting and links the theory to observed behavior [19]. The observed behavior in the experiment was systematic, replicable, and roughly consistent with the dynamical systems approach [9, 29, 30, 3, 2, 6]. Empirically, however, the velocity pattern has not been seen yet.

We conducted the 'Coyness and Philandering' game in laboratory experimental economics, and then, in our *velocity at position* (VAP) metric, we measured *velocity* vectors in the data. In this paper we will report the graphical results and some additional results as literatures in experimental economics. The main aim of this paper is to report, firstly, the existence of global velocity pattern in the 'Coyness and Philandering' game. In discussion, the advantage of the VAP metric method is comparing with the existent metrics is shown; and conclusion last.

2 Design and procedure

2.1 Experiment Design and Theoretical Predict

Experiment Design

As the traditional experimental setting in experimental economics [9, 12, 13, 30, 2], with Dawkins'e matrix in Table 1, we employed a two populations game in which strategic interaction among two population of agents by introducing random matching [20]. Each session consists of two populations, each population includes 8 subjects, one is the male population and the other is female population. Males have the two strategies X_1 (faithful) and X_2 (philanderer); females have the another two strategies Y_1 (coy) and Y_2 (fast). We set 300 rounds repeated, as long as possible practically, for each of the sessions.

Table 1: Payoff matrix of the 'Coyness and Philandering' game

	Coy(Y_1)		Fast(Y_2)	
Faithful(X_1)	2		5	
		2		5
Philanderer(X_2)	0		15	
		0		– 5

Strategy Space and Strategy Position

Because there are 8 human subjects in each of the two populations, there would be 9 potential density states for each population, and 81 combinative strategies state points of two populations in a square 9×9 two-dimensional Euclidian grid. This Euclidian grid is the two populations strategy set in our experiment situation. Each of the 81 combinative strategies state points denotes one observable position which can be described as $x_{pq} = p\vec{e}_{X_1} + q\vec{e}_{Y_1}$, or (p, q), or x_{pq} in which $(p, q) \in \{0, 1/8, 2/8, \ldots, 1\} \otimes \{0, 1/8, 2/8, \ldots, 1\}$.

Theoretical Velocity Pattern

An evolution dynamic provides an instantaneous velocity of each position of the grid. When the positions are limited to the 81 points, as in the situation of our experiment, the instantaneous velocity can also be captured by the evolution dynamic equations.

Practically, with the ODE as Eq.(1) of replicator dynamic and the payoff matrix as Table 1 of the 'Coyness and Philandering' game, the Eq.(2) can be archived,

$$\begin{cases} \dot{p} = & p(1-p)(-10+12q) \\ \dot{q} = & q(1-q)(5-8p) \end{cases} \tag{2}$$

where the p indicates the density (or proportion, or frequency) of the subjects who using strategy X_1 in male population, and the q indicates the density of the subjects who using strategy Y_1 in female population. Point (p, q) is one state of the combinative strategies state of the two populations. Now, the instantaneous velocity at each of the 81 positions can be calculated from Eq.(2), and the result of the theoretical instantaneous velocity for each of the 81 positions is exhibited in Fig. 2.

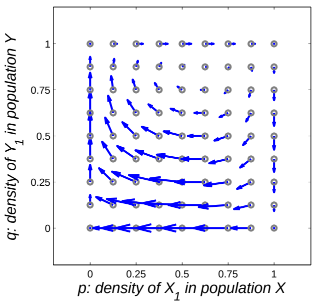

Figure 2: Theoretical instantaneous velocity field at the 81 state positions in strategy space from the replicator dynamics equation of the 'coyness and philandering' game as Eq.(2). Plotting ratio is 1:10.

2.2 Practical Velocity Measurement

In general physics, the unit of time is hour, minute, or second in generally. In laboratory experiment, the unit of time is the interval between two rounds in laboratory experimental economics game, and usually $\triangle t = 1$.

In general physics, again, velocity is the measurement of the rate and direction of change in the position of an object. Instantaneous velocity is always tangential to trajectory. Velocity is odd in time reversal symmetry transition, or, velocity is time reversal asymmetry. In experimental economics, in the discrete strategy space (in finite subject pool in population games), in the stochastic processes, there is no well accepted metric for velocity till now. We develop a metric for the variable, velocity, as following.

Velocity in a Microscope Process

We denote the observed populations state vector x_{pq} at t as x^o_{pq}, then denote the observed state at its last round $t+1$ state as x_{t+1} and observed state at $t-1$ as x_{t-1} respectively. For the fixed x^o_{pq}, we define the forward change, as the change from x^o_{pq} to x_{t+1}, as

$$x^+_{pq} = (x_{t+1} - x^o_{pq}); \tag{3}$$

and the backward change, as the change from x^- to x^o_{pq}, as

$$x^-_{pq} = (x^o_{pq} - x_{t-1}). \tag{4}$$

We define the *one observation of the instantaneous velocity at* x_{pq}, v^o_{pq}, explicitly as,

$$v^o_{pq} := (x^+_{pq} + x^-_{pq})/(2 \triangle t). \tag{5}$$

where $\triangle t = 1$ which means time interval in two closest rounds in a experimental session is set to 1. Practically, $\triangle t = 1$ is the smallest interval of time which could be archived.

No loss of generality, Fig. 3 illustrates one observation of an instantaneous velocity v^o_{pq} in a microscope process, meanwhile Fig. 3 demonstrates also one observation of forward change x^+ in Eq.(3) and one observation of backward change x^- in Eq.(4), respectively.

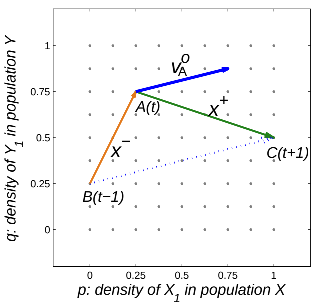

Figure 3: Illustration of an observation of an instantaneous velocity at a microscope process. No loss of generality, suppose that, during a process P the obtained state at t round is $\vec{A}$, and at $(t-1)$ is $\vec{B}$ and at $(t+1)$ is $\vec{C}$, Eq.(5) says, an instantaneous velocity vector, $v^o_A := (\vec{C} - \vec{B})/2$, is observed at site A. v^o_A (in blue), including both processes of a backward change (x^-, jump-in to A, in orange) and a forward change (x^+, jump-out from A, in green), is a time reversal asymmetric vector. In the time reversed process P^T of above process P, at A, the velocity should be $v^{oT}_A = (\vec{B} - \vec{C})/2$ and equals to $-v^o_A$ in P, and it is time reversal asymmetric. The Eq.(6) is an aggregation form for a given site.

VAP Metric

We define VAP (Velocity At a given Position) metric for the mean velocity at the given position x_{pq} as

$$\upsilon_{pq} = \frac{\sum_o \upsilon_{pq}^o}{\Omega_{pq}}, \tag{6}$$

where υ_{pq}^o is an *one observation of velocity vector at position* (p, q) and the summarization of each velocity vector is adding up once whenever (p, q) is obtained; Ω_{pq} is the obtained times (adding up 1 whenever the (p, q) state is obtained in all of the experimental sessions, or called as occupation times). For each of the υ_{pq}^o is odd under T-symmetry, υ_{pq} is anti-symmetry (also asymmetry) under time reversal transition. Comparing with existing metrics for evolution pattern in data of experimental economics is in the last section. Till now, we have finished to build the metric, VAP, for each position x_{pq}, to measure the velocity vector practically from experimental data.

For comparison, similarly, we define the mean vector of forward change (called also as mean forward change, jump out) per round at position (p, q) is

$$\upsilon_{pq}^{out} = \frac{1}{\Omega_{pq}} \sum_o \frac{x_{pq}^+}{\triangle t}. \tag{7}$$

For comparison again, similarly, the mean vector of backward change (called also as mean backward change, jump in) per round at position (p, q) is

$$\upsilon_{pq}^{in} = \frac{1}{\Omega_{pq}} \sum_o \frac{x_{pq}^-}{\triangle t}; \tag{8}$$

For Eq.(7) and Eq.(8) the $\triangle t = 1$ and the summarization of each velocity vector is carried over whenever a (p, q) state is obtained.

2.3 Experiment Procedure

The experiment includes 12 independent sessions. Each session involves 16 human subjects split randomly into two populations no matter what gender of a subject is. Each populations includes 8 subjects. The role of subjects in one population is called as player X and in the other, player Y. Once a subject's role is generated it is persisted on during a session, meanwhile, we only tell the subjects the payoff matrix. Each session consists of 300 rounds of the game repeatedly with a random matching for each round. The experiments were conducted with a computerizing controlled environment. Each subject sat in an isolated seat with a computer. No communication among was allowed during the experiment. The software for the experiments was developed as a Web base system by the authors. In each session, the player X has two options, X_1 and X_2, while the player Y has Y_1 and Y_2. The player X selected to use 'X_1' or 'X_2'

button and player Y selected to use 'Y_1' or 'Y_2' button simultaneously. After each round one received the feedback including his or her own strategy used in the previous round, his or her opponent's strategy in the previous round, and his or her own payoff from the previous round. The payoff is calculated according the payoff matric in Table1. The subjects were asked to document the information on his/her experimental records. At the end of the experiments, one could obtain the accumulated score which would be changed into RMB currency, as the subject's payments after a session. The exchange rate is 25 experimental points for 1 RMB, in addition, every participant was paid 5 Yuan RMB as the showup fee.

We conducted the laboratory experimental sessions in the Center of Social Science Laboratory of Zhejiang Gongshang University in Hangzhou China from January to March 2011. Each session lasts about 1.5 to 2 hours including the introduction stage in both the written and oral forms to inform the participating subjects the game protocol as well as a test drive for the subjects to be familiarized with the game and the experiments. There were in total 192 undergraduate students of Zhejiang Gongshang University majoring in different areas recruited into these experiments with each subject participating in only one session. The average payoff for each participant was 33.92 Yuan RMB.

3 Results

In all, there are 12 sessions, each session is 300 rounds repeated, and we have total 3600 records from the two populations game. From the the requirement of Eq. 5, observation number reduces to 3576 for the velocity field measurement (excluding the first and the last rounds in each of the 300 rounds' 12 session). We now report the mean velocity vectors and its distributions at all of the observable positions from the data.

3.1 Velocity at Positions

According to Eq.(6), we measure the velocity, υ_{pq}, at each given position (p, q). Fig. 4 exhibits the results from our data according to Eq.(6). The empirical pattern, called as velocity vector field following, is exactly a cyclic and grossly can be captured by evolutionary population dynamics models (e.g., Fig.2 in [21] and Fig. 3 in [34]).

For comparison, we also compute the mean forward change according to Eq.(7) and the mean backward change according to Eq.(8) for all 81 positions in our data. Fig. 5 provided the patterns. Both of them are quit faraway from the the theoretical pattern, cycle-pattern, likes the pattern in Fig. 2 and Fig. 1.

Adding up the forward vector field and backward vector field together at each (p, q) position, the cyclic pattern υ_{pq} is rebuilded. It is the mathematical logic within the Equations set, Eq.(6), Eq.(7) and Eq.(8).

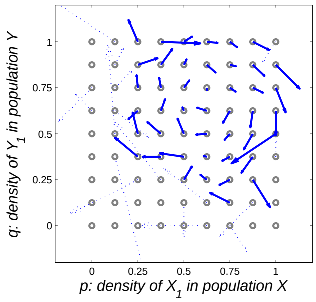

Figure 4: Experimental observation of the velocity field of the 'Coyness and Philandering' game. Each velocity vector (υ_{pq}) denotes the speed and direction at (p, q). At sites with $\Omega_{pq} > 5$ the mean velocity vectors are plotted in heavy arrows, and others in thin arrows. The plotting ratio (measured:plotting) is 1:3.

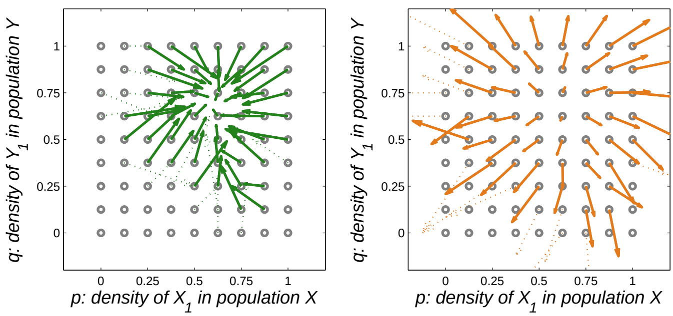

Figure 5: Change in given sites (a) mean forward change (in green) vector, and (b) mean backward change (in orange) vector at each strategy site ($\vec{x}_{pq}$) in strategy space. At sites with $\Omega_{pq} > 5$ the vectors are plotted in heavy arrows and others in thin arrows. The plotting ratio is 1:1.

3.2 Occupation at Positions

The mean density is, in mean±Std.Dev., $(0.604 \pm 0.061, 0.681 \pm 0.11)$, and the theoretical prediction is $(5/8, 5/6)$ or $(0.625, 0.833)$ [21]. Table 2 reports the mean observation of the density for each sessions. We measure also the observations of occupation, Ω_{pq}, at each population strategy point (p, q), from the total 3576 observations. Fig. 6 is the results. These data could be useful for understanding the evolutionary velocity reported above and for the learning models' competing, e.g., [11, 22, 5].

Table 2: Mean Value of (p, q) by Sessions

SessionID	1	2	3	4	5	6
p	0.487	0.614	0.649	0.517	0.635	0.573
q	0.682	0.784	0.762	0.733	0.479	0.710
SessionID	7	8	9	10	11	12
p	0.654	0.610	0.654	0.559	0.697	0.600
q	0.482	0.680	0.855	0.639	0.660	0.704

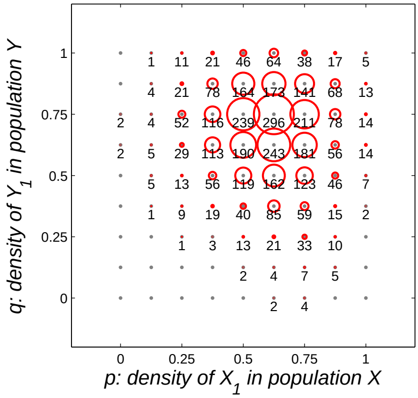

Figure 6: Distribution of observation in the lattice of the strategy space. At each (p, q) point, the size of the circle and the number means the obtained times, Ω_{pq}, during the 12 experimental sessions. For example, $\Omega_{(1,0.5)} = 7$ means, at $(1, 0.5)$ point (all in the male X population choice faith X_1 and half in the female Y population choice coyness Y_1), in the total 3576 observations (excluding the first and the last rounds in each of the 12 session), this state is obtained (occupied) 7 times.

4 Summary

Till now, the main results are reported. In summary, we first compare our VAP metric in Eq.(6) with three most relevant metrics in experimental economics literatures; then, discuss on velocity and the pattern; and last, conclusion.

4.1 Comparison of Metrics for Evolution

Understanding how a system change, evolution, in time and in the strategy space is long expected [23]. Experimental economics also develops metrics to detect the change, e.g., [17]. Mainly, there are three class metrics:

- Metric **M1** is for (a) each round's frequency of strategies on time (round) and/or (b) N-round average frequency on time (round);
- Metric **M2** is for (c) the time path of frequency in strategy grid and/or (d) N-round average of frequency pathes in strategy space;

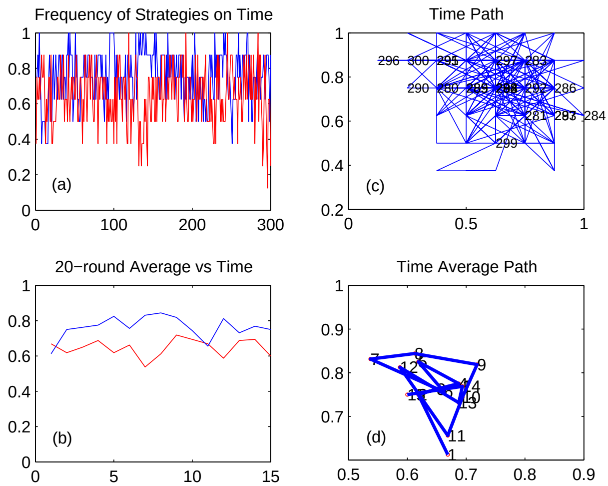

Figure 7: Samples of traditional metric **M1** and **M2**. Metric **M1** presentations are (a) frequency of strategies on time for each round, (b) N-round average on time, red and blue lines are frequencies of X_1 and Y_1 respectively. Metric **M2** presentations are (c) time path in strategy grid and (d) N-round average path in strategy space. Data from the 3rd session in our experiment and $N = 20$. Numeric labels in (c) are the last 20 rounds. In above subplots, x, y-axis label the observed values of strategy frequency, except in the two subplots in left, the x-axis are labeled along experimental rounds.

- Metric **M3** is a for the average of change of x, $\triangle x$, at given x.

The metrics **M1** and **M2** are used to present social evolution [30, 8, 29] with which the sample results in our data are show in subplot (a)-(d) in Fig. 7. Both of **M1** and **M2** are useful for distinguish convergence, however, in complex game environment, as in the four sub-figures, the patterns of evolution are fuzzy. The **M3** [2] is the most relative to our VAP metric and the comparison of **M3** and VAP will be provided following.

The **M3** method, 'the change of x, $\triangle x$, at given x' has been employed for evaluating the evolution dynamics for $\dot{x}$ [2]. In their one population games, both of the best responde and the logit ($\lambda = 1$) dynamics predicts there exists a separatrix $\triangle x$ (at 0.8 that is state x_6 near the payoff dominant equilibrium in Fig. 4 in ref. [2]). The Fig. 4 presents how $\dot{x}$ dependent on x. Numerical result of the average of the change in x, denoted by $\triangle x$, for each state for each treatment (see the Table III in [2]) is reported, and as a result, however, the authors do not find the separatrix (see footnote 9 in [2]).

Now we compare the two metrics mathematically, then report the results in both experimental data with both metrics, comparing roughly with both of the theoretical pattern form evolution dynamics, respectively.

With **M3**, the metric 'change in given site' metric can be presented as Eq.(7), we have:

- in their data, one can replicate their results (the Table III and the Figure 5 in [2]). In Table 3 in this paper includes the Table III in [2] in (FV)-block, from which we can not find the separatrix in the (FV)-block either, see the numerics in bold. At the same time, at each of the end position of the populations strategy space (x_0 and x_8) in all of the three treatments, the velocity biases from zero, at $x_0 > 0$ and $x_8 < 0$; but all the theoretical prediction are zero.

- in our data, the empirical pattern is in Fig. 5, which significant difference from the cycle pattern expected by evolution dynamics, like Fig. 2.

In words, with their metric **M3**, one should also fail to link the empirical patterns with theoretical evolutionary velocity field patterns either in their data or in our date.

Table 3: Results of the two metrics: the average changes in given x

	Treat-ment	x: The number of subjects choosing X								
		0[§]	1	2	3	4	5	6[†]	7	8[§]
FV[α]	0.6R	**0.55**	0.38	0.10	-0.08	-0.27	-0.03	**-0.34**	-0.20	**-0.16**
	R	**0.23**	-0.18	-0.22	0.14	-0.4	-0.37	**-0.15**	0.19	**-0.32**
	2R	**0.07**	-0.26	-0.46	0.16	-0.5	-0.57	**-0.38**	-1.75	
VAP[β]	0.6R	**-0.051**	0.071	-0.100	-0.038	-0.082	-0.008	**0.036**	0.014	**0.013**
	R	**-0.005**	-0.074	-0.156	-0.035	-0.195	-0.191	**-0.037**	0.104	**-0.011**
	2R	**-0.009**	-0.115	-0.346	-0.138	-0.196	-0.119	**0.143**	-0.125	

[α] Forward change, by Eq.(7), calculated from raw data from [2], values are exactly equal to Table III in ref. [2]; The strategy space is not be normalized to $[0, 1]$ but as $[0, 8]$ as the ref. [2];
[β] By Eq.(6), the mean velocity at a given position with VAP;
[†] The separatrix is theoretically expected at this x_6 in dynamics models in Fig. 4 in [2]
[§] Theoretical values in all of the three treatments at $x = 0$ and $x = 8$ should be zero (the Fig. 4 in [2]).

Alternatively, with the VAP metric, explicitly, as Eq.(6), we have:

- in their data, in Table 3 VAP-block, the missed separatrix near x_6, see the numerics in bold at the x_6 column, appears in each of the three treatments; at the same time, at each of the end position of the populations strategy space (x_0 and x_8) in all of the three treatments, the velocity closes to zero, see also the numerics in bold at the x_0 and x_8 column, as the theoretical prediction of the dynamics models.

- in our data, the figural pattern Fig. 4 is systematic and roughly consistent with the dynamical velocity pattern likes Fig. 2.

In words, with our metric VAP, the patterns of the velocity direction are roughly comparable with the theoretical evolutionary velocity field patterns both in their data and in our data.

Important difference between the **M3** and our VAP metric comes from T-symmetry of velocity. Comparing with VAP as Eq. (6), the backward-jumping vector v_{pq}^{in} is not included in Eq.(7) in **M3**, and so the **M3** is not time reversal anti-symmetry, which significantly violates the requirement of the T-symmetry of the velocity [18].

4.2 Discuss on Velocity and the Pattern

In the time scale, the default setting of $\triangle t = 1$ is for the interval of the rounds is one. Two aspects need to be noticed. First, reports from N-round average pathes, usually, the evolution pathes is still visible in the stochastic processes in experimental economics, the reason is not clear. Second, with data from recently developed continuous time game data [14], the pattern of the velocity field is not clear.

In our pilot research, with the VAP metric, we have found the widely existence of global patterns of velocity fields, e.g., in matching pennies game [32], in standard Rock-Paper-Scissor game [31] in Selten and Chmura 12 different 2×2 games data [22, 33] and also in market game data [6, 33]. We notice also that the dynamics models of evolutionary game theory could be evaluated with the empirical velocity pattern [31].

4.3 Conclusion

Our exploring indicates that evolutionary velocity field, by the 'Coyness and Philandering' game as an example, can not only can be illustrated in abstract models in mathematics [21, 16, 24, 15, 27] or in physics [7, 25], but also be obtained empirically in laboratory experimental economics.

In the practical VAP metric method, firstly, we report the significant dynamic pattern from laboratory experimental economics. Von Neumann and Morgenstern stress: 'A dynamic theory would unquestionably be more complete and therefore preferable'. For the dynamic theory, the significant velocity field pattern might be helpful.

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