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Abstract

We examine a multi-issue dynamic decision-making process that involves endogenous commitment. Our primary focus is on actions that impact delay, an extreme form of lack of commitment. Delay is strategically interesting when decision makers with asymmetric preferences face multiple issues and have limited resources for influencing outcomes. A delayed decision becomes part of the subsequent agenda, thereby altering the allocation of resources. The opportunity to delay decisions leads the players to act against their short-run interests when they have strongly asymmetric preferences. Two classes of strategic activity emerge: *focusing* (reductions in delay) and *pinning* (increases in delay). We characterize these equilibria, explore how strategic delay alters the benefits to agenda setting, and develop implications for settings where bargaining is feasible. Our analysis applies directly to group, hierarchical, and coalitional decision making settings and illuminates a range of multi-market competitive interactions.

1 Introduction

In 1974, amid pressure from the Watergate scandal which ultimately scuttled his presidency, an embattled Richard Nixon proposed a comprehensive national health insurance plan in his State of

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the Union address. Nixon's backing gave national health care insurance legislation, a traditionally Democratic issue, a real chance for passage, but Democratic support did not materialize and a plan was never passed. The timing of formal administration support coupled with the attractiveness of the bill to the Democrats have led some cynics to interpret Nixon's proposal as strategic.¹ If the Democrats wanted to make genuine progress on such legislation, they needed to focus legislative attention on health care and they needed the support of the Republican administration. As a result, action on Watergate would likely have been delayed and would certainly have been reduced in intensity. Delay could have saved Nixon's presidency.

While delayed action and deferred decisions rarely involve the toppling of a president, delay is ubiquitous in politics, business, and personal life. Delay as a justification for more informed decision making would be important, yet unremarkable, but this justification also provides cover for strategic uses of delay. This paper addresses questions regarding such strategic uses of delay in multi-issue settings. Why is delay valuable and who will initiate it? Does conflict lead to strategic delay and how does the order of issues considered affect delay and outcomes?

Delay itself is representative of a class of decision problems involving endogenous commitment. Deferring a decision until the next decision point is equivalent to no commitment, while a decision, if irreversible, constitutes full commitment.² In the former case, the delayed decision becomes part of the subsequent decision agenda and potentially changes each decision maker's allocation of influence activity. Thus, the analysis of delay in dynamic settings has wide-ranging implications.

¹Light (1991; p. 256) [9] states, for example, that "struggling to distract a Democratic Congress from the Watergate crisis, Nixon offered national health insurance as a last-second bargain to save his Presidency." National health care insurance legislation was more attractive to the Democratic party than the Republican party, though Nixon had been a supporter as well. The leading Democrat pushing for a national health care insurance plan was Ted Kennedy who had run for President in 1972. It has been reported that the 1974 failure to pass a national health care insurance plan was Kennedy's "biggest regret." (Washington Post, August 28, 2009)

²In corporate settings, a failed proposal that can be reintroduced is an example of a decision that lacks commitment. For example, rejected proposals of subordinates are sometimes quietly maintained in hope that changed circumstances will allow the proposal to be revisited. Burgelman (1991) [6], for example, argues that the RISC processor project at Intel was kept alive despite the company's explicit strategy of not pursuing such a processor. Our model can also be used to explore the dynamics of whether a proposal is placed on the agenda in the first place. Proposals that do not make it on the agenda have not been officially killed and can therefore be interpreted as "delayed." In the political system, rejection of legislation may be viewed as a decision that lacks commitment as such bills are frequently reintroduced in subsequent legislative sessions.

The structure of our complete information model isolates the effect of commitment while allowing for endogenous interaction. We abstract from the specifics of various decision structures and build a spartan strategic ark involving two players, two decisions, and two periods. Each player allocates a stock of non-storable influence resources (e.g., attention) over the available decisions and seeks to maximize their two-period payoff. In the first period, resource allocations affect the likelihood that a proposal is adopted (i.e., commitment). Opposing (reinforcing) use of resources increase (decrease) the probability of delay. Absent commitment, the proposal is deferred to the second period where it is considered along with an unrelated proposal. In the second period, players allocate resources and all proposals on the table are resolved permanently.

Actions that alter the subsequent agenda through delay are strategically valuable because they shape the actions of the other decision makers. The endogenous possibility of delay leads the agents to allocate their resources differently than they would in a single-period setting. Two main tactics emerge: *pinning* and *focusing*.

In equilibria characterized by pinning, one player expends first-period resources against static self-interest to decrease the probability that a long-term commitment is made (and hence decrease the probability that the decision will leave the agenda). The pinning player undertakes this strategy when the second-period issue involves conflict and the first-period issue is relatively unimportant to her but is relatively important to her rival. Pinning increases the probability that the rival's resources will continue to be allocated to the first issue, leaving less resources for the rival to contest the pinning player on the second issue.

If delaying commitment on the first issue is sometimes valuable, then securing commitment on that issue should be valuable in other circumstances. Suppose, for example, the players' interests align on the second issue. Then, it may be optimal for a player to decrease the likelihood of delay to free the rival's resources for (supportive) use on the second issue. We refer to this dual of pinning as *focusing*. The focusing incentive can be sufficiently strong that a player will support the rival's efforts to get the initial proposal accepted, even though that player prefers rejection of the proposal. Effectively, the focusing player sacrifices one decision outcome to focus the rival's attention and resources on the other decision. Focusing has the desirable feature of avoiding waste associated with offsetting use of resources and provides a time-consistent explanation for logrolling which does not rely on reputations or other outside-the-immediate-interaction considerations. A necessary condition for focusing is that preferences directionally align on the second-period issue, while a necessary condition for pinning

requires conflict on the second-period issue. When these necessary conditions are met, we show that focusing and pinning occur when the players differ significantly with respect to which issue is more important. Furthermore, pinning and focusing also emerge in a model in which both issues are considered in the first period but can be delayed to the second period.

We also assess the value to an agenda setter of reordering the sequence in which two decisions are considered across periods. The order question is most interesting when the players have partially conflicting and strong relative preferences so that one order would lead to focusing while the other would lead to pinning. We show that the player who would be focused always prefers the focusing agenda, whereas the other player generally prefers the pinning agenda.

When bargaining is feasible, our non-cooperative equilibrium results provide a foundation for understanding the threat points for bargaining. Decision outcomes based on static rather than dynamic self-interest do not provide appropriate threat points when there are strong relative preferences and some underlying conflict. For example, under Nash bargaining we show that pinning and focusing threats lead the players to experience a discontinuous shift in their shares of a smoothly changing overall surplus when the equilibrium shifts into a focusing or pinning regime from a regime where pursuit of static self-interest is optimal.

A general point of the analysis is that there is frequently value to resolution in a world with limited commitment and multiple decisions. Even if a current issue is of limited concern, delay in that issue's resolution means a claim on future resources of an ally or a rival. Therefore, a player may be quite willing to expend significant resources to secure resolution of an issue of no direct importance. With limited commitment, then, resolution of an issue will sometimes loom larger than the way in which the issue was resolved.

Examples of pinning and focusing can be found in numerous decision-making and market competition settings. The Nixon national health insurance proposal can be interpreted as a pinning move designed to divert Congressional attention away from actions relating to the Watergate scandal.³ Focusing provides an incentive compatible means of maintaining a political coalition or a business joint

³In terms of our model, Nixon's State of the Union address adds national health insurance (NHI) into the active legislative agenda alongside the possibility of Watergate impeachment hearings. Nixon's gambit can be interpreted as an attempt to use influence resources to increase the probability that NHI would become sufficiently attractive as a legislative option that the Democrats would divert their influence resources towards NHI and away from Watergate hearings (i.e., pinning). Ultimately, however, the fate of the Nixon NHI proposal was resolved early when it failed to gain Kennedy's support, so the attempt to pin did not succeed.

venture. It is common for a coalition to navigate proposals over which there is agreement and proposals over which there is disagreement. By sequencing the consideration of the proposals to create incentives for one party to focus the other, both parties can more easily support each other's key proposals without the need for other enforcement mechanisms.⁴

Firms that compete over multiple product or geographic markets frequently pin each other's resources by opening second fronts in markets that are important to their rivals (or focus their rivals by ceding them markets). Such market-based competitions lack the explicit timing of a final resolution point. However, when viewed through a commitment lens, market outcomes that involve long-term shifts in the nature of competition (e.g., if one firm withdraws from a market, a market with network externalities may tip to one firm) are similar to the resolved decisions in our multiple-round strategic interactions.⁵

Our analysis of delay can also be used to understand litigation between horizontal competitors for which the first-period decision involves expenditures on litigation and the second-period decision involves expenditures relevant to the ongoing competition. Litigation expenditures influence the duration and outcome of the legal dispute. Resolution or non-resolution, in turn, affects the marginal value of investments in competition, for example, via the impact of uncertainty about the outcome of litigation on the ability of the players to attract complementary investments from third parties. Protracted litigation can be used to pin a rival to a less favorable resource environment.⁶

⁴In presidential and parliamentary political systems with modest party discipline, influence activities involve keeping party members in line with party positions on the various items currently on the legislative agenda. Where party discipline is very strong, keeping party members in line is easy and a key use of party leadership resources involves allocating leadership attention to issues that require negotiation prior to their introduction.

⁵MacMillan, van Putten, and McGrath (2003) [11] call this class of tactics feints in which an attack in one market diverts resources from another market. They describe how Philip Morris attacked R.J. Reynold's U.S. position in premium cigarettes thereby diverting RJR's resources away from important Eastern European markets. The authors also discuss a competitive interaction between Gillette and Bic which could be interpreted as roughly consistent with focusing. Gillette, by withdrawing from the disposable lighter market, induced Bic to focus on that market and pull resources out of disposable razors. This example is in the spirit of our focusing model, with a resolved decision outcome being a change in market conditions that caused Bic to alter its relative interests—and hence resource use—from disposable razors to disposable lighters.

⁶The protracted litigation between AMD and Intel over AMD's access rights to Intel intellectual property as a former second source for the 286 microprocessor can be interpreted as Intel pinning AMD to more expensive R&D and customer

Little, if any, work has used analytical models to explore the effects of attention and endogenous commitment on organizational decision making. Our analysis connects research on influence activity to that of agenda setting. The influence activity models of Milgrom and Roberts (1988 [13], 1990 [14]) focus on the design of incentives to agents who, given the incentive structure, optimally split their time across current production and influence activities that impact all of the agents' payoffs. Our interest in endogenous commitment and their interest in organizational design lead to quite different models. We build a dynamic model to explore deferred decisions, but do not address various optimal organizational designs that structure the nature of the intra and inter-period decision-maker interactions.

Agenda setting models in the economics and the formal political science literatures (see, e.g. Plott and Levine 1978 [16], List 2004 [10]) explore the effects of decision order. While we explore select consequences of the ordering of decisions, our primary focus is not on how an agenda can be manipulated by an agenda setter. Instead we focus on how the agenda is altered as the result of strategic choices and the ensuing direct decision consequences. Agenda-setting models also typically focus on the influence of decision order when decision payoffs and outcomes are linked across decisions. We focus on the impact of deferred commitments on the allocation of influence which does not require any outcome or payoff link across decisions. Stated alternatively, we focus on across-meeting issues whereas most of the agenda setting literature is within-meeting focused.

The tensions between conflict and cooperation among decision making parties which we address in this paper are explored in a number of articles. In the evolution of cooperation literature, for example, Skaperdes (1992) [18] examines how different levels of marginal productivity in the use of resources can lead to varying degrees of equilibrium cooperation in a two-player static model where each player divides its resources between joint production of a product and increasing the probability that the value of the product will be "won" by that party. In the literature on moral hazard in teams, Bonatti and Hörner (2011) [2] build a dynamic model of moral hazard in which team (possible) production of a public good is delayed because the equilibrium time path of investment is slowed by free-riding incentives and pessimistic updating of the value of investment. Both of these papers focus on a single good (issue) and, therefore, cannot address the strategic exploitation of limited commitment through delaying (or accelerating) actions designed to manipulate a rival's use of resources across issues.

In the next two sections we develop and analyze our basic endogenous commitment model. Section

development under conditions of uncertainty. This litigation was finally settled after the access to the rights was no longer competitively significant.

4 discusses which combination of preferences leads to pinning and to focusing and then characterizes the conditions under which an order that induces pinning is preferred to an order that induces focusing. Section 5 examines some implications of the analysis for bargaining and Section 6 examines a symmetric model of decision making and establishes that strongly asymmetric and partially conflicting preferences necessarily result in an equilibrium with simultaneous pinning and focusing. In Section 7 we discuss the organizational context of our analysis as well as applications and limitations. Section 8 concludes.

2 Model

Our model consists of two players, A and B, who, over two periods, independently allocate their respective attention resources to influence the outcomes of two unrelated proposals, X and Y . Two players is usually thought of as the smallest number needed for decision conflict; two periods is the minimal time structure that can capture the effects of delay; while two decisions are needed to provide for allocation of attention.

We begin with a sequential model in which one proposal is on the agenda in the first period while the other is added in the second period. The allocation choices in the first period result in the proposal being accepted or delayed to the second period, while in the second period the allocation choices result in proposals being either accepted or rejected. We denote by $X \rightarrow Y$ and $Y \rightarrow X$ the sequential models in which proposals X and Y , respectively, are first on the agenda. These simple sequential models are sufficient for focusing and pinning to emerge in equilibrium and also allow us to examine the implications of such strategic actions for the preferred ordering of the proposals. We defer until Section 6 the examination of a more general symmetric model in which both proposals are on the agenda in period 1 and each proposal may be accepted, rejected or delayed.

The advantages and disadvantages of delaying a decision depend on the nature of conflict between the two players. We focus on the strategically interesting case of *partial conflict* in which the players' preferences conflict on issue X , but align on issue Y . Within this general context, of course, a substantial variation may exist regarding the relative intensities of conflict or alignment. We assume that player A has a utility $u_X < 0$ when proposal X is accepted and $u_Y > 0$ when proposal Y is accepted. Player B 's utilities upon acceptance are similarly represented by $v_X > 0$ and $v_Y > 0$.

Condition 1 $v_X > 0 > u_X$ (*Conflict on X*), $u_Y > 0$ and $v_Y > 0$ (*Alignment on Y*).

Utility for the rejection of a proposal is normalized to 0. Consequently, the acceptance utilities are more precisely viewed as the incremental utility or disutility of accepting versus rejecting the proposal. Additionally, we make three simplifying assumptions: no discounting occurs; the utility associated with each proposal is independent of the outcome of the other proposal; and the preferences of each player are known to the other player. Each player maximizes the two-period sum of expected utility.

In each period a player allocates resources (e.g., attention or effort) to influence the outcome of proposals on the agenda. We model each player as choosing probability influence increments.⁷ A player supports (opposes) a proposal when she chooses a positive (negative) probability increment. We assume that each player's allocations have a direct effect which is linear and additive and that influence is neither cumulative nor storable across periods. This simple structure has the advantage of isolating the across-period strategic effects since additivity eliminates strategic interaction in a static single-period setting. Thus, when proposal i is on the agenda and player A chooses a_i and B chooses b_i , proposal i is accepted with probability

$$p_i = z + a_i + b_i \tag{1}$$

where z is a shift parameter reflecting exogenous factors that affect the probability of acceptance.⁸ We differentiate first-period actions and second-period actions by using lower case and upper case proposal-identifying subscripts, respectively.

When the agenda only contains proposal i , A chooses a_i while B chooses b_i . When the agenda contains both proposals, player A chooses probability increments a_X and a_Y and B chooses b_X and b_Y . To reflect the assumption that total influence is constrained in a multi-issue setting, we assume that there is a probability influence frontier g .

Condition 2 *Influence choices for player A (similarly for B) satisfy $|a_Y| \leq g(|a_X|)$ for $a_X \in [-\bar{p}, \bar{p}]$, where the probability frontier g satisfies (i) g is decreasing and concave over the interval $[0, \bar{p}]$, with*

⁷This structure with probability increments and the assumed properties for the resource frontier can be derived as endogenous properties for an underlying model of effort choice in which additional effort yields a diminishing marginal effect on probability and the agent is equally effective at influencing adoption of one issue or the other.

⁸For example, in a hierarchical setting involving two subordinates who try to influence a superior, one can think of z as the superior's initial bias regarding the decisions at issue.

$g(0) = \bar{p}$ and $g(\bar{p}) = 0$, (ii) g is symmetric around the 45° line: $a_Y = g(a_X) \Leftrightarrow a_X = g(a_Y)$, and (iii) $g'(0) = 0$ and $g'(\bar{p}) = -\infty$.

Under this resource constraint, the maximum probability influence on a single issue is equal to \bar{p} . The advantage of g 's concave frontier structure is that influence allocation choices in a multiple issue setting will be interior to the interval $(-\bar{p}, \bar{p})$. Symmetry of g with respect to the 45° line comparably situates each issue. In the first period, Condition 2 reduces to the requirement that influence choices on the single issue lie in $[-\bar{p}, \bar{p}]$.

Finally, we assume that uncertainty cannot be eliminated.

Condition 3 $2\bar{p} < z < 1 - 2\bar{p}$ (feasible influence choices never lead to deterministic outcomes).

Thus, for any choices a_i and b_i on proposal i , the probability p_i is always in $(0, 1)$. In the sequential agenda model, the first period has no reject possibility, so the probability of delaying proposal i is $1 - p_i$. In the second period, there is no delay state, so the probability of rejection is also $1 - p_i$.

2.1 The Static Equilibrium Benchmark

The optimal second-period actions provide building blocks for the dynamic analysis and a benchmark case in which strategic interaction is absent. These actions depend on whether there was delay in the first period. We begin with the simplest case of no delay. If only the aligned issue Y remains in period 2 (i.e., given the $X \rightarrow Y$ agenda, X was accepted), then the players have directionally common interests as u_Y and v_Y are both positive. Clearly, each player will choose \bar{p} such that the likelihood of accepting Y is maximized. The resulting payoffs associated with Y are then $U_Y = u_Y(z + 2\bar{p})$ and $V_Y = v_Y(z + 2\bar{p})$. If, instead, only the conflict issue remains (i.e., Y was accepted under the $Y \rightarrow X$ agenda), then the players have opposing interests since $u_X < 0 < v_X$ and they will take offsetting actions ($a_X = -\bar{p}$, $b_X = \bar{p}$). This results in payoffs of $U_X = u_X z$ and $V_X = v_X z$.

Now consider the two-proposal case which arises whenever the first-period proposal has been delayed. We solve for a Nash equilibrium in which each player allocates their own probability influence over each of the two issues. Given a choice by player A , say a_X and a_Y , player B 's problem is to choose influence levels to $\max_{(b_X, b_Y)} v_X [z + a_X + b_X] + v_Y [z + a_Y + b_Y]$ over feasible influence levels relative to the probability frontier. Since the actions of player A only have an additive effect on this payoff, the optimal choice by player B is given by

$$-\frac{v_X}{v_Y} = g'(b_X^*) \tag{2}$$

on issue X and $b_Y^* = g(b_X^*)$ on issue Y . Player A faces a similar problem except that A will seek to oppose proposal X . Thus, $-a_X \in [0, \bar{p}]$ and the solution is

$$\frac{u_X}{u_Y} = g'(-a_X^*) \quad (3)$$

on issue X and $a_Y^* = g(-a_X^*)$ on issue Y . The magnitude of a choice depends only on the preference intensity, defined by $u \equiv \left| \frac{u_X}{u_Y} \right|$ and $v \equiv \left| \frac{v_X}{v_Y} \right|$ and optimal choices equalize the probability trade-off and utility trade-off between X and Y . As the preference intensity for X rises, an increase in u or v , the magnitude of the action on X rises, while that for Y falls. The sign of a choice always follows the sign of the utility effect.

These choices constitute the Nash equilibrium for the static game. Precisely because the other player's action does not impact the marginal benefit of one's own action, the two players optimize independently of each other and there is no strategic interaction. Critically, however, a player's payoff does depend on the other player's actions. This is the channel for dynamic strategic effects in our model: anticipating that another player will support or oppose an issue that remains unresolved, there is an incentive to take action today to influence the other player's future move. To analyze this channel, we need the payoff outcomes for the simple static Nash equilibrium and we define $U_{XY} = u_X(z + a_X^* + b_X^*) + u_Y(z + a_Y^* + b_Y^*)$ and $V_{XY} = v_X(z + a_X^* + b_X^*) + v_Y(z + a_Y^* + b_Y^*)$.

2.2 Dynamic Equilibrium Choice

The static equilibrium strategies described above are also the optimal second-period equilibrium strategies. We now turn to the first-period actions. Consider the $X \rightarrow Y$ sequence ($Y \rightarrow X$ is analogous). From our analysis of the static case, we have the continuation payoffs for the players across the two possible states according to whether proposal X was delayed to the second period. The probabilities of each state are given by: $\{Y\}$ with p_x and $\{X, Y\}$ with $(1 - p_x)$. The payoffs for the players at a candidate set of period 1 choices are then given by the sum of the expected period 1 and 2 payoffs:

$$U^a \equiv [u_X + U_Y](z + a_x + b_x) + U_{XY}(1 - (z + a_x + b_x)) \quad (4)$$

$$V^b \equiv [v_X + V_Y](z + a_x + b_x) + V_{XY}(1 - (z + a_x + b_x)) \quad (5)$$

The incentives for players A and B to allocate influence in period one are, respectively:

$$\frac{\partial U^a}{\partial a_x} = u_X + U_Y - U_{XY} \quad (6)$$

$$\frac{\partial V^b}{\partial b_x} = v_X + V_Y - V_{XY}. \quad (7)$$

We are now ready to examine equilibrium choices and strategic delay in period 1.

3 Player Preferences and Strategic Delay

We first analyze focusing in a $X \rightarrow Y$ agenda and then turn to pinning in a $Y \rightarrow X$ agenda.

3.1 Partial Conflict and Focusing ($X \rightarrow Y$ Agenda)

The first step is to determine the optimal second-period allocations of attention. This was done in the static benchmark analysis above. The next step is to analyze the optimal first-period allocations. The linear structure of influence implies that the objective functions for players A and B are maximized by allocating all influence in support of issue X if and only if $u_X + U_Y - U_{XY} > 0$ and $v_X + V_Y - V_{XY} > 0$, respectively. See (6) and (7).

Lemma 1 *Under the $X \rightarrow Y$ agenda, player B 's optimal first-period allocation b_x is \bar{p} . (See Appendix for all proofs).*

Lemma 1 shows that B 's strategic and myopic interests coincide. If B were to use negative influence, she would increase the probability that issue X will be delayed in the first period, which is costly. The effect in the second period would also be negative because there would now be a higher probability that both X and Y are on the agenda in which case B will be opposed by A on issue X .

Player A has a direct incentive to oppose X in the first period. But if X is off the agenda in the second period, player B will allocate all of its attention to supporting issue Y which benefits A . This incentive to *focus* player B 's attention on issue Y becomes relatively stronger as A 's intensity of preference is greater for issue Y than issue X .

Definition 1 (*Focusing*) *A player focuses his rival on proposal j when the player's first-period allocation on i is greater than his static optimal allocation. A player follows static self-interest when the player chooses a first-period allocation equal to his static optimal allocation.*

The incentives for focusing can be fruitfully characterized as a function of the ratios of the preference intensities for X to Y for each of the two players.

Proposition 1 *Under the $X \rightarrow Y$ agenda: (a) For any preference ratio v for player B , there exists a focusing cut-off preference ratio \bar{u}_F below which in equilibrium it is optimal for player A to focus B by selecting $a_x = \bar{p}$ and above which A follows static self-interest by selecting $a_x = -\bar{p}$; B always follows static self-interest by selecting $b_x = \bar{p}$; (b) The focusing cut-off for Player A , $\bar{u}_F(v)$, is increasing in v , satisfies $\bar{u}_F(0) = 0$ and $\bar{u}_F(v) < v$ for $v > 0$, and is bounded above by $2\bar{p}/(1 - z)$; (c) Focusing only occurs when both players' preferences are aligned over the issue that is first introduced in the second period; (d) For $u < \bar{u}_F(v)$, the unique equilibrium involves A focusing B where we have $a_x = \bar{p}$ and $b_x = \bar{p}$; For $u > \bar{u}_F(v)$, the unique equilibrium has both players following static self-interest.*

Proposition 1 establishes that there is always a region of preferences for A and B in which A will uniquely focus B . Let \mathcal{E}_F denote this region, the set of (u, v) with $u \leq \bar{u}_F(v)$.

The incentive for A to focus B depends on both the intensity of preference A has for its key issue (Y) and the relative gain A gets from focusing—the difference in the payoff to the $\{Y\}$ state and the $\{X, Y\}$ state. Because the $\{X, Y\}$ state payoff depends on B 's intensity of preference, as v increases B allocates less resources to issue Y in the $\{X, Y\}$ state and hence the benefits to A of focusing increase (part b). If, instead, B cared much more about issue Y than issue X , the gain to focusing B on issue Y would not be great, as B would have been relatively focused on Y regardless. Part (c) establishes that preference alignment on issue Y is necessary for a focusing equilibrium.⁹ Focusing increases the probability that the second-period agenda will consist of issue Y alone. Y alone is attractive under alignment since both players work together, whereas under conflict their efforts will offset.

Focusing is inherently cooperative, but the strategic outcome does not achieve social efficiency or Pareto-optimality because there is always a positive probability that in the second period both issues will be on the agenda with the resulting wasteful offset of resources on the conflict issue X . Focusing is, however, an efficiency improvement over the static benchmark. Clearly, the focusing player is better off since focusing is an optimal strategy. The focused player benefits because of the increase in the probability that the favored issue of the focused player is accepted in the first period. One interpretation of focusing is that it is endogenous incentive-compatible log-rolling. Focusing also facilitates an agenda design that can support stable coalitions.

⁹It is not, however, necessary that there be conflict over the first issue. This is because the benefit of focusing derives from taking issue X off the agenda and is greatest to a player when it has a preference intensity ratio strongly favoring Y while the other player does not.

When we move from a single issue to a multiple-issue first-period setting (Section 6), first-period choices interact strategically and this leads to a more subtle set of incentives than in our base model. The incentive for focusing remains, but it is less dramatic than that forced by our base structure in which optimal first-period choices are cornered.

We now develop an example with a preference structure that we will also use to illustrate pinning and to discuss agenda preferences.

Example 1 (*X* → *Y* Agenda): Suppose that resources are traded off according to $g(a, \bar{p}) = \sqrt{\bar{p}^2 - a^2}$ with $\bar{p} = 0.1$, $z = 0.2$, and let the preferences be $u_X = -0.075$, $u_Y = 1$, $v_X = 1$, and $v_Y = 0.075$. Then the equilibrium allocations are

<i>Eq Alloc</i>	<i>1st period</i>	<i>2nd {Y}</i>	<i>2nd {XY}</i>
<i>Issue X</i>	$a_x = 0.1$	NA	$a_X = -.007$
	$b_x = 0.1$	NA	$b_X = 0.099$
<i>Issue Y</i>	NA	$a_Y = 0.1$	$a_Y = 0.099$
	NA	$b_Y = 0.1$	$b_Y = 0.007$

In this example both players are strongly concerned about the outcome of one proposal but not the other, and the primary concern of one player is the secondary concern of the other. The intensity ratio v is large which means that, when faced with both proposals in the second period, B would allocate most of her resources to proposal X . Player A , therefore, receives an incremental benefit from focusing B on proposal Y . Of course, player A 's low intensity of preference on proposal X makes it less costly to support proposal X against his static interest. In the example, player A 's payoff is about five percent greater under focusing than under a myopic (suboptimal static self-interest) strategy; Focusing boosts player B 's payoff by about thirty percent.

3.2 Partial Conflict and Pinning (*Y* → *X* Agenda)

Now consider the $Y \rightarrow X$ agenda. The analysis is analogous to that for $X \rightarrow Y$, except that the issue order has been reversed. Conflict is now over the *second* issue X and the alignment issue Y is handled first. Note that by Proposition 1c focusing cannot occur with this configuration of preferences.

If player B followed her preferences, she would support issue Y in the first period. However, dynamic considerations will sometimes cause B to oppose Y in the first period to improve the strategic situation in the second period. Essentially, B works to keep issue Y on the agenda because if both

X and Y are on the agenda in period two, then A will allocate more influence to Y and less influence to opposing B on issue X . B pins A to an issue that is important to A .

Definition 2 (*Pinning*) *A player pins her rival to proposal i when the player's first-period action is less than the player's static optimal action.*

Player B 's decision to resist her static preference depends on the relative strengths of the incentive to accept issue Y and the dynamic benefits of pinning player A to issue Y by delaying it to the second period. B chooses b_y to maximize $(v_Y + V_X)(z + a_y + b_y) + V_{XY}(1 - (z + a_y + b_y))$ where $v_Y + V_X$ captures the value of accepting Y in the first period and V_{XY} the value of delaying Y to the second period. It is clear from the objective function that $V_{XY} > v_Y + V_X$ is a necessary and sufficient condition for $b_y = -\bar{p}$ (which goes against B 's preference).

Because there is conflict over X , $V_X = v_X z$. V_{XY} , of course, depends on the optimal static allocations (see (2) and (3)). Following a similar solution approach as above, we establish existence of a unique pinning equilibrium, a comparative static result, and a necessary condition for pinning.

Proposition 2 *Under the $Y \rightarrow X$ agenda: (a) For any preference ratio u for Player A , there exists a pinning cut-off preference ratio $\bar{v}_P(u)$ above which in equilibrium it is optimal for B to pin A by selecting $b_y = -\bar{p}$ and below which B follows static self-interest $b_y = \bar{p}$; (b) The pinning cut-off for Player B , $\bar{v}_P(u)$, is increasing in u and satisfies $\bar{v}_P(u) > u$ for $u \geq 0$; (c) Pinning can only occur when both players' preferences conflict over an issue that is first introduced in the second period; (d) For $v > \bar{v}_P(u)$, the unique equilibrium involves B pinning A where we have $b_y = -\bar{p}$ and $a_y = \bar{p}$; For $\bar{v}_P(u) > v > u$, the unique equilibrium has both players following static self-interest.*

The key element here is the relative size of the preference intensities. Existence requires that v is large relative to u : we must have preference intensities in the set $\mathcal{E}_P \equiv \{(u, v) \mid v \geq \bar{v}_P(u)\}$. That is, compared to player A , player B has a stronger relative preference for X . In turn, this relative preference implies that in the static $\{XY\}$ game, the net impact of influence on issue X will be positive and $a_X^* + b_X^* > 0$ holds. In contrast, with X alone the player allocations cancel each other. Thus, B pins A by going negative on Y in period 1, acting against (static) interest, to increase the likelihood that issue Y is alive for the second period. Even as $v \rightarrow \infty$, so that player B does not care at all about issue Y , B will still have an incentive to affect the outcome associated with Y because of that outcome's indirect resource implications for the outcome of issue X . Thus, one should not be surprised to see influence activity by a player on an issue of little relative importance.

The gain to B from delaying issue Y depends on how A splits his resources in period 2 when both of X and Y are on the agenda. This gain is largest when $u = 0$ since A will then support Y fully and devote no resources to opposing X . As u rises, A shifts resources from supporting Y to opposing X and, consequently, the gain to B from pinning A will decline. This makes pinning unattractive for a wider range of B preferences and the cutoff, \bar{v}_P , must rise with u (part b). Finally, a preference conflict on the second issue is necessary for a pinning equilibrium (part c).¹⁰

In contrast to the focusing equilibrium, there is no direct analog of Lemma 1 to guarantee that the other player will always act in accord with preference and support issue Y . Instead, it is easy to show that Players A and B are symmetric with respect to pinning incentives.¹¹ For any pair of u and v utility preferences intensities, however, at most one of the players will have an incentive to pin since pinning requires that one intensity be sufficiently greater than the other. When the intensities are comparable in magnitude both players will follow interest (part d).

A pinning strategy is inherently defensive. Therefore, given opposition in the first period on the alignment issue Y and wasteful offsetting use of resources in the second period involving issue X , pinning equilibria are not socially efficient. Finally, unlike the focusing equilibria, pinning equilibria are not Pareto-improving versus static allocations. While the pinning player's expected utility is improved, the pinned player's utility declines.

The same preference structure and parameters in Example 1 can be used to illustrate pinning in the $Y \rightarrow X$ agenda. In this $Y \rightarrow X$ agenda example, player B has a weak preference over Y and, with these preferences, will pin player A to proposal Y in the second period. The optimal first period allocation of Player B is $b_y = -0.1$ which is the opposite of B 's static self-interest choice of 0.1. Player A 's first-period allocation of $a_y = 0.1$ is consistent with A 's static self-interest.¹²

¹⁰Pinning can also occur in $Y \rightarrow X$ settings involving pure conflict. Suppose player B prefers to accept both issues whereas player A prefers to reject both issues. When, for example, B 's preference intensity ratio heavily favors issue X while player A 's intensity ratio heavily favors issue Y , it is optimal for player B to delay issue Y and, by so doing, increase the probability that player A will be pinned to issue Y .

¹¹We employ notation that identifies which player is pinning, but there is a common cut-off function, $\bar{u}_P(w) = \bar{v}_P(w)$, for any utility intensity $w \geq 0$. In part (d) of Proposition 2 we need only interchange u and v to identify when A pins B .

¹²The second-period $\{XY\}$ allocations are $a_X = -0.007$, $a_Y = 0.099$, $b_X = 0.099$, and $b_Y = 0.007$. The second-period $\{X\}$ allocations are $a_X = -0.1$ and $b_X = 0.1$.

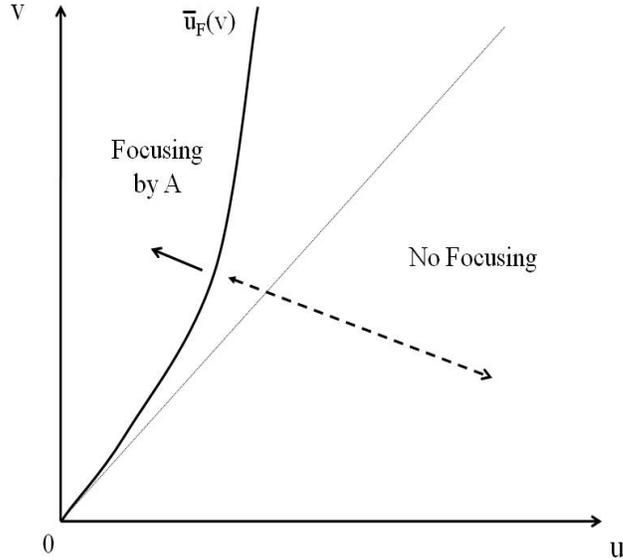


Figure 1: Equilibrium Taxonomy Under $X \rightarrow Y$ Agenda

4 Decision Order, Payoffs, and Agenda Selection

The analysis of partial conflict in the previous section highlighted the dynamic incentives associated with decision delay in a multi-issue setting. In this section we explore which preference configurations lead to focusing and to pinning and then examine the effect of issue order on player payoffs.

In our model, there is a potential structural advantage to a player to have his favored issue considered first because overall more resources are potentially available to influence acceptance of that issue relative to the other issue. However, strategic actions may offset such benefits. In period 2 the single-issue state is a liability when the players conflict over the issue, but a possible benefit when there is alignment. The incentive of A to expend initial resources on X in the $X \rightarrow Y$ agenda thus depends on the sign of $u_X + U_Y - U_{XY}$ in (6). When positive, player A will focus player B to increase the probability of $\{Y\}$. Focusing is possible because the shadow of possible cooperation in the second period may induce “cooperation” in the first period over an otherwise contentious issue X . Now consider the $Y \rightarrow X$ agenda. If $v_Y + V_X - V_{XY}$ in (7) is negative, then a player B with a strong intensity of preference regarding issue X has an incentive to delay issue Y to increase the probability of the $\{X, Y\}$ state through pinning.

Figure 1 divides the preference-intensity space into focusing and no focusing regions under an

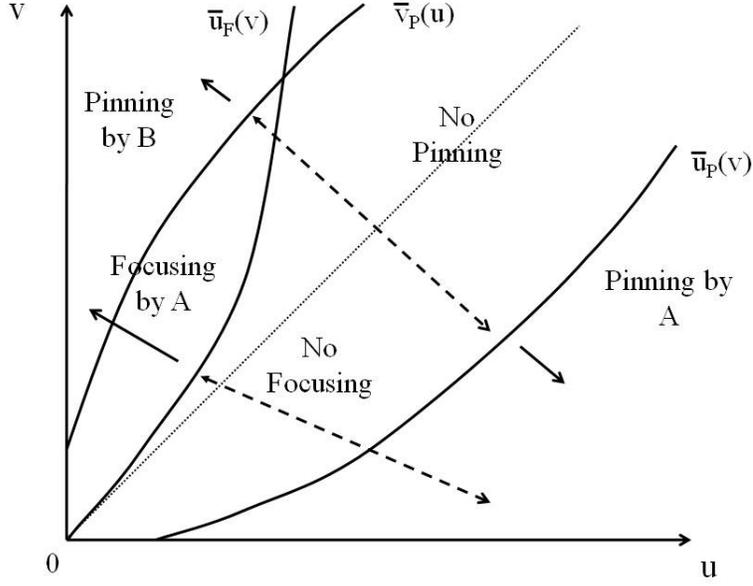


Figure 2: Equilibrium Taxonomy Under Both $X \rightarrow Y$ and $Y \rightarrow X$ Agendas

$X \rightarrow Y$ agenda. These regions are separated by the cutoff function $\bar{u}_F(v)$. In Figure 2 the $Y \rightarrow X$ agenda is overlaid on Figure 1 to locate the regions in preference-intensity space for which different types of equilibria exist under both the $X \rightarrow Y$ and the $Y \rightarrow X$ agendas. For the $Y \rightarrow X$ agenda addition, there are three regions: pinning by B (demarcated by $\bar{v}_P(u)$), pinning by A (demarcated by $\bar{u}_P(v)$), and no pinning by either player. Proposition 1c (2c) rules out focusing (pinning) for the $Y \rightarrow X$ ($X \rightarrow Y$) agenda. Figure 2 is helpful for thinking about agenda choice, as for a given set of preference intensities, one can get a feel for whether focusing or pinning will occur.

In the upper left (northwest) region of Figure 2 we see that a $X \rightarrow Y$ agenda results in focusing by A while a $Y \rightarrow X$ agenda results in pinning by B . This set is given by $\mathcal{E}_{FP} \equiv \{(u, v) \mid v \geq \max\{[\bar{u}_F]^{-1}(u), \bar{v}_P(u)\}\}$. We now explore this interesting region to determine how anticipated strategic action affects the ordering preferred by each player. Analysis of the regions where static self-interest prevails is straightforward.

The question of order preference hinges on whether it is better to put the conflict or the alignment issue first. Player A , for example, must consider whether it is better to be pinned by B under $Y \rightarrow X$ or to focus B under $X \rightarrow Y$, and this choice has several subtle aspects since A must take a costly action to focus B , while pinning has B taking action to neutralize A 's efforts. For B , the comparison is

intuitively much simpler since the focusing equilibrium has A going against myopic interest to support X and this should work to B 's benefit.

Proposition 3 *Suppose $(u, v) \in \mathcal{E}_{FP}$ so that equilibrium under the $X \rightarrow Y$ agenda has focusing by player A and equilibrium under the $Y \rightarrow X$ agenda has pinning by B . Then B always prefers the $X \rightarrow Y$ (focusing). A always prefers the $Y \rightarrow X$ (pinning) if either (i) $z \leq 2/5$ or (ii) $z > 2/5$ and $z + 2\bar{p} < 1 - 2\bar{p}^2/z$.*

In \mathcal{E}_{FP} , B strongly favors X , while A strongly favors Y . Under the $X \rightarrow Y$ agenda the conflict issue comes first and A will focus B by helping to accept X . Under the $Y \rightarrow X$, B will pin A . By choosing to pin, clearly, B prefers the payoff associated with the $\{X, Y\}$ second-period state to that associated with $v_Y + V_X$. Then, since the worst outcome for B in the $X \rightarrow Y$ agenda is bounded below by the payoff associated with the $\{X, Y\}$ state, a preference configuration that leads B to pin implies that B always prefers the $X \rightarrow Y$ agenda.

The comparison for A is more subtle. There is a natural bias in favor of $Y \rightarrow X$ and pinning because $u_Y + U_X > u_X + U_Y$. But the cooperative element in focusing means that X is accepted and, hence, period 2 has only the Y issue, with probability $z + 2\bar{p}$; under pinning the corresponding probability for Y and then X only is z . Proposition 3 provides a strong result on the relative importance of these effects: pinning is preferred to focusing by A for any preferences in \mathcal{E}_{FP} provided only that $z + 2\bar{p}$ is not too close to 1. Even when (i) and (ii) do not hold, A still prefers pinning to focusing whenever we are near the focusing boundary (where $u_X + U_Y - U_{XY} = 0$). Intuitively, focusing only dominates pinning for A when we have near certainty in the outcome ($z + 2\bar{p} \rightarrow 1$) and only Y matters ($u \rightarrow 0$). In this limiting case, the probability effect overwhelms the natural bias for pinning under $Y \rightarrow X$ since focusing under $X \rightarrow Y$ leads to Y being accepted with probability one. Except for this limiting case, A will prefer pinning.

To see the effect of various agenda orders, consider again Example 1 and its $Y \rightarrow X$ analog from the previous section. The individual payoffs for those examples are in the Table below. For comparison, we also provide the payoffs to a disequilibrium set of strategies in which each player merely allocates according to myopic self-interest.

Table 1: Payoffs under Example 1 Preferences with $X \rightarrow Y$ and $Y \rightarrow X$ Agendas

Agenda	Strategy	A payoff	B payoff
$X \rightarrow Y$ (<i>Ex. 1</i>)	A focuses B	0.301	0.601
	myopic	0.293	0.458
$Y \rightarrow X$	B pins A	0.425	0.307
	myopic	0.565	0.299

In both of these agendas, the incentives for focusing and pinning for the “strategic” player are modest, but the effects of those actions on the other player are substantial.

The above comparison of various agenda orders provides insight into the value of being able to change the agenda or being able to impose costs on those that do.¹³ First, Proposition 3 asserts that conflict between the agents over agenda choices will be common, especially with asymmetric issue preferences. Even when there are some proposals for which the parties have aligned preferences, strategic action implies that conflict rather than consensus will prevail with regard to agenda choice. In addition, and as the example illustrates, the value of an agenda to a player has a subtle component as the preference across agendas hinges not on creating an opportunity to engage in strategic behavior but, rather, to put one’s rival in a position where the rival will be led to focus or to pin.

5 Bargaining and Asymmetric Preferences

When the ability to strike cooperative agreements is limited, equilibrium outcomes such as focusing or pinning are predicted by our model. In circumstances where bargaining is possible, focusing or pinning outcomes serve as threat points or disagreement outcomes. In such cases, predictions employing threat points based on static self interest will be misleading. One critical element here is the payoff impact on the focused or pinned player that arises as one transitions from preference cases where all players follow static self-interest to the more asymmetric preference cases that lead to focusing and pinning in equilibrium (recall Figure 2). We find that even with movements in preference space that increase social surplus, focusing and pinning lead to discontinuous decreases in negotiational payoffs for one player (and corresponding increases for the other).

The Nash bargaining solution (NBS) provides a convenient and familiar framework for addressing

¹³Of course, decision situations are rarely fully malleable to changes in the agenda. Timing frequently reflects some underlying flow of relevant information that provides a natural ordering to decisions and oftentimes one proposal is clearer in its likely parameters at an earlier time than other proposals.

the split-of-surplus issue. Even under other bargaining protocols, we expect similar insights because the NBS result stems from changes in the threat points of the bargaining parties. To begin, we specify the threat points and set of feasible agreements. Assume that the threat points are given by the equilibrium payoffs of the $X \rightarrow Y$ agenda, denoted by t_A for A and t_B for B ; these threat points involve focusing whenever $u < \bar{u}_F(v)$ and static self-interest, otherwise. A feasible agreement can specify (i) the agenda, either $X \rightarrow Y$ or $Y \rightarrow X$, (ii) the actions for each player in period 1 and also in period 2, contingent on period 1 outcomes, and (iii) a transfer payment between A and B .¹⁴ Social surplus, S , is defined as the sum of the expected utilities for the players and under (i-iii) we solve directly for the maximized social surplus, S^* . The generalized NBS is the division of S^* with shares for each player such that $s_A = t_A + \alpha(S^* - t_A - t_B)$ for A and $s_B = t_B + (1 - \alpha)(S^* - t_A - t_B)$ for B where α parameterizes the bargaining power of A .

To find S^* , we begin with the period-2 contingent actions. Since Y is the alignment issue, we have $w_Y \equiv u_Y + v_Y > 0$ and positive actions of $a_Y = b_Y = \bar{p}$ are optimal when only issue Y is on the agenda. For the conflict issue X , suppose that $w_X \equiv u_X + v_X > 0$; then actions of $a_X = b_X = \bar{p}$ are optimal when only issue X is on the agenda (we omit the case where $w_X < 0$ as it has a similar analysis). Finally, when both issues are on the agenda, we need only apply the logic of the static benchmark choice with two issues to the social surplus values of w_X and w_Y . This logic yields the optimal actions of $a_i = b_i > 0$ for $i = X, Y$ via $g'(a_X) = -w_X/w_Y \equiv -w$ and $a_Y = g(a_X)$; denote these solutions by $a_i(w)$. The joint payoffs for the period-2 states are then

$$\begin{aligned} W_i &= (z + 2\bar{p})w_i & \text{for } i = X, Y \\ W_{XY} &= (z + 2a_X(w))w_X + (z + 2g(a_X(w)))w_Y. \end{aligned}$$

Maximized social surplus is then found by solving

$$S^* = \max_{(X \rightarrow Y, Y \rightarrow X)} \{S_{X \rightarrow Y}^*, S_{Y \rightarrow X}^*\}.$$

where

$$\begin{aligned} S_{X \rightarrow Y}^* &\equiv \max_{(a_x, b_x)} \{(w_X + W_Y)p_x + W_{XY}(1 - p_x)\}, \\ S_{Y \rightarrow X}^* &\equiv \max_{(a_y, b_y)} \{(w_Y + W_X)p_y + W_{XY}(1 - p_y)\}. \end{aligned}$$

¹⁴For simplicity, we are assuming transferable utility and this implies a linear Pareto frontier. Deriving the set of feasible payoffs with non-transferable utility can be done but the resulting structure is much more complicated.

The resulting optimal agenda is then $X \rightarrow Y$ when $w_X > w_Y$ and $Y \rightarrow X$ when $w_X < w_Y$ and in each case the optimal actions on the first issue are \bar{p} . We focus on the case of $w_X > w_Y$ (the other has a similar logic).

We now carry out the comparative statics exercise on bargaining outcomes with respect to preference asymmetry. First, consider a point on the 45° line where $u = v$. (See Figure 2.) From Proposition 1, we know the threat outcome will be the static self-interest (SI) equilibrium for $X \rightarrow Y$, with payoffs $t_A^{SI} = (u_X + U_Y)z + U_{XY}(1 - z)$ and $t_B^{SI} = (v_X + V_Y)z + V_{XY}(1 - z)$. Now suppose u_X rises (becomes less negative) and, hence, that issue Y becomes more important relative to X for A . We then move to the left from the 45° line but, initially, remain to the right of the focusing cut-off level of $\bar{u}_F(v)$. In this range, the threat payoffs vary smoothly with u_X ; at the same time, maximal social surplus rises smoothly, reflecting the added value from the rise in u_X . Once u_X rises sufficiently that we hit the focusing boundary, the threat outcome shifts to focusing (F) with payoffs $t_A^F = (u_X + U_Y)(z + 2\bar{p}) + U_{XY}(1 - z - 2\bar{p})$ and $t_B^F = (v_X + V_Y)(z + 2\bar{p}) + V_{XY}(1 - z - 2\bar{p})$. At the boundary, when player A is just willing to shift from static self-interest on X in period 1 to focusing, we necessarily have $u_X + U_Y - U_{XY} = 0$ and t_A varies smoothly with u_X (i.e., $t_A^F = t_A^{SI}$ on the focusing boundary). The maximal social surplus also continues to rise smoothly. In contrast, t_B jumps up in value by $(v_X + V_Y - V_{XY})2\bar{p} > 0$, by Lemma 1, once focusing commences. As a result, at the boundary the NBS for player B jumps up while that for A jumps down.

Thus, focusing leads to a strong shift in the bargaining outcome. When B faces a rival who has a strong preference for the alignment issue Y relative to B 's preference, the shift in equilibrium behavior to focusing corresponds to a shift in the threat points that favors B . Interestingly, even though A 's increased preference for Y implies an increase in the social surplus that can be shared, the focusing effect on bargaining threats leads to a lower payoff for A .

The formal version of this comparative statics result identifies the role of the variation in preference intensity, (u, v) , relative to the boundary for self-interest and focusing, $\bar{u}_F(v)$, and that for pinning, $\bar{v}_P(u)$. To keep track of the optimal agenda choice it is helpful to define the set

$$\mathcal{F} \equiv \{(u, v) \mid v \geq 1 + (1 + u)(u_Y/v_Y)\};$$

given any u_Y and v_Y for the alignment issue, $X \rightarrow Y$ is the efficient agenda for (u, v) in \mathcal{F} as the defining condition reduces to $w_X > w_Y$. Finally, $\omega \equiv (u_Y + v_Y)/(v_Y - u_Y)$ is where the lower boundary of \mathcal{F} intersects the 45° line. The role of ω is to provide a reference point in \mathcal{F} so that as we perform the comparative static, by varying u or v , we know that $X \rightarrow Y$ continues to be the optimal agenda

and that the threat point shifts from self-interest to focusing or to pinning. We then have

Proposition 4 *Consider any given Y preferences where $v_Y > u_Y$. Then, (a) \mathcal{F} contains all points on the 45° line where $u = v \geq \omega$; (b) For any $v \geq \omega$, \mathcal{F} contains all (u, v) such that $u \leq v$ and, hence, \mathcal{F} contains the focusing boundary $\bar{u}_F(v)$; (c) For any $u \geq \omega$, \mathcal{F} contains all (u, v) such that $v \geq u$ and, hence, \mathcal{F} contains the pinning boundary $\bar{v}_P(u)$; (d) For any $(u, v) \in \mathcal{F}$ on the focusing boundary, $u = \bar{u}_F(v)$, the NBS shares are discontinuous with*

$$s_B^F - s_B^{SI} = -(s_A^F - s_A^{SI}) = \alpha 2\bar{p}(v_X + V_Y - V_{XY}) > 0;$$

where F and SI index the shares for focusing and static self-interest threat points, respectively; (e) For any $(u, v) \in \mathcal{F}$ on the pinning boundary, $v = \bar{v}_P(u)$, the NBS shares are discontinuous with

$$s_B^P - s_B^{SI} = -(s_A^P - s_A^{SI}) = (1 - \alpha)2\bar{p}(u_X + U_Y - U_{XY}) > 0$$

where P indexes the share for the pinning threat point.

Away from the focusing and pinning boundaries, we have continuity in (u, v) for the threat payoffs. The shift in strategic behavior when focusing or pinning commences, however, always has a discrete impact on the focused or the pinned player. Thus, player B benefits by being focused, through the shift in t_B ; A suffers by being pinned, through the shift in t_A . The proposition also identifies the critical aspect of preference intensity, as measured by the position of (u, v) relative to the focusing and pinning boundaries, rather than preferences with respect to one issue for one player.¹⁵

6 A Symmetric Model of Acceptance, Rejection, and Delay

In this section we show that the intuition from the asymmetric model carries over to symmetric decision settings. To establish the robustness of our earlier results, we modify the asymmetric model to include both proposals in the first period and to allow each proposal to be accepted, rejected or delayed in the first period, effectively eliminating the role of issue order. We find that focusing and pinning continue

¹⁵A number of the special assumptions we made can easily be modified without affecting the main insight regarding the jump in bargaining outcomes. If $Y \rightarrow X$ is the surplus maximizing agenda, then we need only observe that this also varies smoothly with (u, v) . We can then do a comparative static with player B in which the (u, v) variation is due to changes in u_Y and v_Y ; for instance, when v_Y falls and, hence, so does social surplus, B 's share will again jump up as we cross the pinning boundary.

to emerge in such settings and that for sufficiently asymmetric preferences the equilibrium necessarily involves focusing by one player and pinning by the other.

Including both issues in the first period introduces an initial tradeoff between X and Y in terms of the optimal use of resources (recall Condition 2 and the g function). To include rejection, we model delay, acceptance and rejection as follows: first, with probabilities d_i and $1 - d_i$, respectively, decision i is delayed to the second period or it is resolved. If resolved, then the proposal is accepted with probability p_i and rejected with probability $1 - p_i$.¹⁶ Delay is modeled as $d_i = z_D - \gamma a_i b_i$ where z_D is the exogenous decision delay probability and γ is a scaling factor for the endogenous delay effect caused by conflict or agreement over issue i . The multiplicative functional form used here implies that agreement reduces delay while disagreement increases delay.¹⁷ We rule out deterministic outcomes and assume that $\gamma \bar{p}^2 < z_D < 1 - \gamma \bar{p}^2$.

The optimal static equilibrium strategies are, as before, also the optimal second-period equilibrium strategies for this symmetric model. With both X and Y in the first period, we now have four possible agenda states in the second period. The probabilities of each state are given by: \emptyset with $(1 - d_X)(1 - d_Y)$, only $\{X\}$ with $d_X(1 - d_Y)$, only $\{Y\}$ with $(1 - d_X)d_Y$, and $\{X, Y\}$ with $d_X d_Y$. The payoff for player A (similarly for player B) at a candidate set of period 1 actions is then given by

$$\begin{aligned}
 U^a \equiv & (1 - d_X)d_Y [p_X u_X + U_Y] + d_X (1 - d_Y) [p_Y u_Y + U_X] \\
 & + d_X d_Y U_{XY} + (1 - d_X)(1 - d_Y) [p_X u_X + p_Y u_Y].
 \end{aligned} \tag{8}$$

The incentives for player A for allocating influence across the two proposals are:

$$\frac{\partial U^a}{\partial a_x} = u_X - u_X d_X + \gamma b_x [-U_X + (U_X - U_Y - U_{XY})d_Y + p_x u_X]$$

¹⁶As the sum of the probabilities of the possible decision consequences must sum to one, this particular acceptance-rejection-delay structure distributes the changes in delay probabilities proportionately across accept and reject outcomes.

¹⁷Most observers have found a positive correlation between the desire to attain decision consensus and delay. Conflict which makes consensus more difficult would then seem also positively correlated with delay. Our delay assumption seems particularly appropriate for environments in which decision makers favor some degree of consensus over pure formal authority or adherence to strict voting rules. For example, in a study of a medical school, Bucher (1970 p. 45 [5]) observed that “most of the opposition to an idea is worked through...or else the proposal dies” The positive relationship between conflict and delay is not, however, uncontroversial. Eisenhardt’s (1989) [8] study of decision making speed in microcomputer firms found both examples where conflict slowed decisions – where the firms valued consensus – and where it did not.

$$\frac{\partial U^a}{\partial a_y} = u_Y - u_Y d_Y + \gamma b_y [-U_Y + (U_X - U_Y - U_{XY})d_X + p_y u_Y]$$

with analogous incentives for player B .

Consider first the benchmark case of no endogenous delay ($\gamma = 0$). Here, players maximize an objective function strictly analogous to that faced in the second period (static) setting. Hence we have

Lemma 2 *Consider the symmetric model. If delay is exogenous, $\gamma = 0$, then the optimal first-period actions are the same as the corresponding optimal actions in the static equilibrium when issues X and Y are both on the agenda.*

This result means that the effect of exogenous delay on optimal actions is isolated in the model from the effects of strategic delay. Hence, we can attribute changes in first-period actions relative to the optimal static equilibrium actions as resulting from strategic choices.

We now show that focusing and pinning occur in the symmetric model with sufficiently extreme relative preferences.

Proposition 5 *For u sufficiently small ($u \rightarrow 0$) and v sufficiently large ($v \rightarrow \infty$), every equilibrium involves (i) A focusing B on issue Y , by acting against static self-interest on X in period 1, $a_x^* > 0$ and $b_x^* > 0$ and (ii) B pinning A on Y by acting against static self-interest on Y in period 1, $a_y^* > 0$ and $b_y^* < 0$.*

Proposition 5 highlights preference-intensity settings in which pinning and focusing necessarily occur in equilibrium (for sufficiency, an existence result is provided in the appendix). In Figure 2, the case of a small u for A and a large v for B corresponds to the regions for focusing and pinning equilibria under the $X \rightarrow Y$ and $Y \rightarrow X$ agendas, respectively. Weaker forms of focusing and pinning occur when a player does not allocate resources at the static self-interest levels but does maintain the direction of support (or opposition) that is consistent with static self-interest.

7 Discussion

In this section we consider how the model provides insight for a broad range of settings and, in so doing, address some limitations of the analysis. Throughout this paper we have emphasized strategic opportunities posed by decisions which may be delayed rather than resolved. The strategic use of focusing and pinning to influence delay can be interpreted more broadly as action that increases or

decreases the probability of a commitment. For example, the decision at issue may be a choice between a course of action that is difficult to reverse (a public commitment to launch a product combined with purchase of specialized assets) and one that is easy to reverse (a private decision to launch a product with no immediate supporting actions), rather than merely a decision to adopt a proposal or not. In the asymmetric model p_i (probability of acceptance) would then be a measure of the likelihood of first-period commitment regarding matter i , while in the symmetric model this measure would be $1 - d_i$ (one minus the probability of delay). Modeling more nuanced levels of commitment is potentially an interesting extension.

Our model applies to settings in which players who choose to influence decisions (or that make decisions) have limited resources such as a limitation in time and attention. Such limitations have been emphasized by the organizational decision-making literature as central to decision making. Simon (1947, p.294) [19], for example, views “[a]ttention...[as] the chief bottleneck in organizational activity” and argues that “the bottleneck becomes narrower and narrower as we move to the tops of organizations...”¹⁸ The importance of attention for organizational decision making has also been highlighted in more political conceptualizations of organizations such as Pfeffer’s (1978) [15] micropolitics model or the organized anarchy (garbage-can) model of Cohen, March, and Olsen (1972) [7]. March and Olsen (1979) [12], for example, regard participation in various choice decisions as dependent on organizational obligations, various symbolic aspects of decision making, and rational action regarding the allocation of attention across various alternatives.

“There are almost no decisions that are so important that attention is assured...The result is that even a relatively rational model of attention makes decision outcomes highly contextual...Substantial variation in attention stems from other demands on the participants’ time (rather than from features of the decision under study). If decision outcomes depend on who is involved..., if the attention structures are relatively permissive and unsegmented, and if individuals allocate time relatively rationally, then the outcomes of choices

¹⁸Divided attention is a common theme in the decision making literature. Wood and Peake (1998) [20] find, for example, that presidential attention to important unresolved foreign policy issues declines when other foreign policy issues become more prominent. Redman (1973) [17] (pp. 55-57) also delineates numerous examples illustrating the effects of divided attention in the legislative setting. He describes, for example, how an “amendment in committee” strategy for grafting a National Health Service Corps onto another health bill in 1970 was derailed by the U.S. invasion of Cambodia.

will depend on the availability and attractiveness of alternative arenas for activity. The individuals who end up making the decision are disproportionately those who have nothing better to do..." (pp. 46-47).¹⁹

While March and Olsen's comment regarding the influence of the idle reflects an element of whimsy, there is a serious undercurrent in it regarding the use of resources that are non-storable. Our model adopts the starkest version of attention resources: there is no marginal cost of use up to a fixed maximum. One can think of the model as being directly applicable when the benefit or cost of the less important decision outcome exceeds the marginal cost of effort. Because our results depend on relative rather than absolute issue preferences, this zero marginal cost-of-use assumption is not particularly limiting. Alternatively, one could think of the marginal cost of effort as a filter that limits the number of decisions that are important enough to attract attention of the focal decision influencers.

In terms of decision-making settings, we see our model as applying to decision making both by committee and within a hierarchy. Consider a committee structure. While committee decision making typically involves more than two players, the impact of different preferences on strategic delay actions is arguably captured in our two-player model. In our model a player with unbalanced preferences has an incentive to take a strategic action against myopic interest when the other player also has unbalanced preferences. Other players who have more balanced preferences have an incentive to take actions consistent with their static self-interest. The actions of these "other" committee members can then be interpreted as being captured by z , the exogenous probability parameter.

Next consider an extension of the two-player model to accommodate N decision makers each of whom may have unbalanced preferences. We conjecture that equilibria exist in such models which involve multiple players taking focusing or pinning actions while others act with static self-interest. When expanding from a two-player setting to multiple-player settings, one has to account for a more complex preference set. Recall that there are two factors that determine whether a player will focus (or pin): the relative intensity of own preferences by the possibly focusing (pinning) player and the incremental value of such a strategic action relative to the baseline of acting with static self-interest. Incremental value depends on the anticipated actions which will be taken by the other players in the single-issue-only state and in the multiple-issue state. One can propose a multi-player equilibrium and

¹⁹See Bendor, Moe, and Shotts (2001) [1] for a critical review of the research program addressing the garbage-can model of organizational decision making.

then check deviations by examining the incentives of each player based on their respective preferences and the “net” actions implied by the equilibrium for the other players. The additive separability inherent in the model’s structure helps enormously with this task.

Hierarchical decision making represents the other extreme in which, political models of organizational decision making aside, a single person is the decision maker. Within this context, each of the two players in our model can be interpreted as taking actions to influence the ultimate decision maker. Subordinates within such settings have considerable latitude regarding the influence and attention they devote to any given decision. Bower (1970) [3], for example, describes strategy choice as a resource-allocation process in which a firm’s strategy emerges from a decision making system in which upper management primarily controls organizational level decisions (e.g., a firm’s overall direction or its culture) but implicitly relies on the judgment of middle managers who compete over project-level decisions. Decision making from this perspective is seen as “decidedly multilevel and multiperson,” (Bower, Doz, and Gilbert 2005, p.13) [4]. In this interpretation z would constitute the bias of the decision maker.

8 Conclusion

“In a minute there is time [f]or decisions and revisions which a minute will reverse”
(*The Love Song of J. Alfred Prufrock*, TS Eliot 1917)

When an important decision is made that does not involve real commitment, the decision remains either explicitly on the agenda because the decision was deferred or implicitly on the agenda because the decision is reversible (e.g., the Obama health care plan). Important but reversible decisions continue to attract decision making attention thereby affecting future influence allocations and, hence, future outcomes. Consequently, anticipating such future effects, decision makers may alter the allocation of current resources. Such decision dynamics lead to two closely related strategies: taking actions against myopic interest to pin a rival’s future attention to a proposal carried over from the current round or taking actions against myopic interest to remove a distracting issue and focus a rival’s future attention on a particular issue. These strategic actions emerge in equilibrium when decision participants have strong relative preferences for one issue over another. Strategies of pinning and focusing also alter the value of having one issue precede another issue. The analysis, therefore, has implications for across-meeting agenda setting, rather than for the more commonly analyzed problem

of within-meeting agenda setting.

There is much room to extend the theoretical analysis to multiple participants with varying resources as well as to consider additional issues. In addition to exploring the effect of deferring decisions empirically, other arguably interesting avenues would be to examine the effect of related decisions in which adoption of one proposal changes the utilities associated with other proposals and to explore the incentives for introducing proposals which are selected or designed to take dynamic advantage of decision participant preferences. Finally, allowing for incomplete information regarding preferences on issues would lead naturally to a role for signaling and reputations.

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Appendix

The following simple result characterizes the optimal static actions for any configuration of player preferences. We make frequent use of this result in subsequent proofs, including those for Propositions 1 and 2.

Lemma A1 Let $u \equiv \left| \frac{u_X}{u_Y} \right|$ and $v \equiv \left| \frac{v_X}{v_Y} \right|$ denote the preference intensities. Then, the strategies in the static Nash equilibrium when X and Y are on the agenda are given by

- i) $g'(|a_X|) = -u$ and $g'(|b_X|) = -v$
- ii) $g(|a_X|) = |a_Y|$ and $g(|b_X|) = |b_Y|$
- iii) $sgn(a_i) = sgn(u_i)$ and $sgn(b_i) = sgn(v_i)$ for $i = X, Y$.

Proof: We prove the results for Agent A; the proof for B involves a simple change of labels. Property (iii), $sgn(a_i) = sgn(u_i)$, is trivial. If $u_i > 0$ but $a_i < 0$, then $u_i a_i < 0 < -u_i a_i$ and $-a_i > 0$ is a better choice for A. Similarly, if $u_i < 0$ but $a_i > 0$, then $-a_i < 0$ is again a better choice. For (ii), $g(|a_X|) = |a_Y|$, suppose not. Then, by feasibility, we have $g(|a_X|) > |a_Y|$. If $u_Y > 0$, then a choice of a_X and $\hat{a}_Y = g(|a_X|)$ yields a higher payoff. Similarly, if $u_Y < 0$, then using the slack in resources to set $\hat{a}_Y = -g(|a_X|)$ increases the payoff. Because the objective, $u_X a_X + u_Y a_Y$, is linear and the constraint set, $g(|a_X|) \geq |a_Y|$ for $0 \leq |a_X| \leq \bar{p}$, is symmetric, Properties (ii) and (iii) of Lemma A1 imply that we can solve A's choice problem for any (u_X, u_Y) by first solving the problem for the case of $u_X > 0$ and $u_Y > 0$ and then making an adjustment of sign on the optimal influence choices. Thus, for $u_X > 0$ and $u_Y > 0$ the choice problem of A reduces to

$$\max [u_X a_X + u_Y g(a_X)] \quad s.t. \quad 0 \leq a_X \leq \bar{p}$$

This is a continuous objective on a compact set and therefore has a solution. Since g is strictly concave, the solution is uniquely determined by the first-order condition $u_X + u_Y g'(a_X) = 0$. By part (iii) of Condition 2 for g , the solution is interior. For reference, we use $a_X(u)$ and $a_Y(u)$ to denote the solution for any ratio $u \equiv |u_X/u_Y| > 0$. Comparative statics are straightforward. Defining $G(u) \equiv [g']^{-1}(-u)$, these are given by $a'_X(u) = -1/g''(G(u)) > 0$ and $a'_Y(u) = u/g''(G(u)) < 0$. Finally, note that $u a'_X(u) + a'_Y(u) = 0$ (Envelope Theorem).■

Proof of Lemma 1: By Condition 1, we have $V_X = z v_X$ since $a_X = -\bar{p}$ and $b_X = \bar{p}$ are the optimal choices. Similarly, $V_Y = (z + 2\bar{p})v_Y$, since $a_Y = b_Y = \bar{p}$. Finally, $V_{XY} = v_X(z_X + a_X^* + b_X^*) + v_Y(z_Y + a_Y^* + b_Y^*)$, by the optimal static choices (denoted by *) from Lemma A1 when X and Y are both on the agenda. Player B chooses b_x to maximize $(z + a_x + b_x)(v_X + V_Y) + [1 - (z + a_x + b_x)]V_{XY}$. Clearly, since the objective is linear, $b_x = \bar{p}$ iff $v_X + V_Y > V_{XY}$. Simplifying, this inequality reduces

to

$$v_X + (z + 2\bar{p})v_Y > (z + a_X^* + b_X^*)v_X + (z + a_Y^* + b_Y^*)v_Y$$

This is valid because (1) $1 > z + a_X^* + b_X^*$, by Condition 3; (2) $z + 2\bar{p} \geq z + a_Y^* + b_Y^*$ by $\bar{p} \geq a_Y$ and $\bar{p} \geq b_Y$, and (3) each of $v_X > 0$ and $v_Y > 0$ holds by Condition 1. ■

Proof of Proposition 1, Part A: Player A chooses a_x to maximize $\{(z + a_x + b_x)(u_X + U_Y) + [1 - (z + a_x + b_x)]U_{XY}\}$. The solution is $a_x = \bar{p}$ iff $u_X + U_Y > U_{XY}$ (it is $a_x = -\bar{p}$ when $u_X + U_Y < U_{XY}$). Substituting for U_Y and U_{XY} with the optimal static actions, rearranging terms, and dividing by $u_Y > 0$, we have

$$u_X + U_Y > U_{XY} \Leftrightarrow (2\bar{p} - a_Y^* - b_Y^*) + \frac{u_X}{u_Y}(1 - z - a_X^* - b_X^*) > 0.$$

Now, using the definitions of $u \equiv -\frac{u_X}{u_Y} > 0$ and $v \equiv \frac{u_X}{v_Y} > 0$, and writing the the optimal choices in the $\{XY\}$ state in terms of the solutions to the first-order conditions from Lemma A1, that is $(a_X^*, a_Y^*) = (-a_X(u), a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), b_Y(v))$, our condition for $a_x = \bar{p}$ becomes

$$h(u, v) \equiv [2\bar{p} - a_Y(u) - b_Y(v)] - u[1 - z + a_X(u) - b_X(v)] > 0.$$

We claim that, for any $v > 0$, the function $h(u, v)$ is (1) decreasing in u , (2) positive at $u = 0$, (3) negative as $u \rightarrow \infty$, and, hence, (4) there $\exists!$ $u \ni h$ crosses 0. To show (1), differentiate h w.r.t. u and apply the envelope theorem, $a_Y'(u) + ua_X'(u) = 0$, to find $h_u = -[1 - z + a_X(u) - b_X(v)] < 0$, as follows from Condition 2 for interior probabilities. For (2), let $u \rightarrow 0$ and note that $a_X(u) \rightarrow 0$ and $a_Y(u) \rightarrow \bar{p}$, so that $h(0, v) = [\bar{p} - b_Y(v)] > 0$. For (3), letting $u \rightarrow \infty$ in $h(u, v)$ and noting $a_X(u) \rightarrow \bar{p}$ and $a_Y(u) \rightarrow 0$, we see $h(u, v) \rightarrow -\infty$. Then, (4) follows by continuity and $h(u, v)$ crosses zero one time at a unique $u = \bar{u}_F(v) \in (0, \infty)$. Thus, $h(u, v) > 0$ holds for $0 < u < \bar{u}_F(v)$ and then $a_x = \bar{p}$, while $h(u, v) < 0$ holds for $u > \bar{u}_F(v)$ and then $a_x = -\bar{p}$.

Part (b): To verify that $\bar{u}_F(v)$ is increasing, simply note that $\bar{u}'_F(v) = -h_v/h_u$, the ratio of partials for h . From above, we know $h_u < 0$. Calculating, we find $h_v = ub'_X(v) - b'_Y(v) > 0$. Hence, $\bar{u}'_F(v) > 0$ holds. To verify that $\bar{u}_F(0) = 0$, observe that $b_x(0) = 0$ and $b_y(0) = \bar{p}$ so that $h(u, 0) = \bar{p} - a_Y(u) - u[1 - z + a_X(u)]$. At $u = 0$, we have $a_X(0) = 0$ and $a_Y(0) = \bar{p}$. This implies $h(0, 0) = 0$ and, hence, $\bar{u}_F(0) = 0$. To show that $\bar{u}_F(v) < v$ for any $v > 0$, it is sufficient to show that $h(v, v) < 0$ since this implies h crosses zero to the left of v . Simplifying $h(u, v)$ at $u = v$, we have

$$h(v, v) = 2[\bar{p} - b_Y(v)] - v[1 - z].$$

Since g is concave, we have $\bar{p} - b_Y(v) < -g'(b_X(v))b_X(v) = vb_X(v)$, where the last step is by the first-order condition for B . This implies $2\bar{p} < 2vb_X(v) + 2b_Y(v)$. Since $1 - z > 2\bar{p}$ by Condition 3 and $\bar{p} \geq b_X(v)$, we have $v[1 - z] > 2vb_X(v)$ and, thus, the inequality for $h(v, v) < 0$ is valid. Finally, for the upper bound on $\bar{u}_F(v)$, write $h(\bar{u}_F(v), v) = 0$ as (suppressing arguments) $\bar{u}_F = [2\bar{p} - a_Y - b_Y]/[1 - z + a_X - b_X]$. Since a_Y and b_Y are non-negative and $a_X(\bar{u}_F) < b_X(v)$ for $\bar{u}_F < v$, the upper bound of $\bar{u}_F < 2\bar{p}/(1 - z)$ follows directly.

Part (c): Suppose Y is a conflict issue. In a focusing equilibrium, player A chooses $a_x = \bar{p}$ against own interest based on $u_X < 0$. The choice $a_x = \bar{p}$ is optimal iff $u_X + U_Y - U_{XY} > 0$. Substituting for U_Y and U_{XY} , noting that $\hat{a}_Y + \hat{b}_Y = 0$ [where \hat{a}_Y and \hat{b}_Y are the optimal actions when only (the conflict) issue Y is on the second-period agenda] as players A and B choose oppositely in Y , and rearranging terms yield $u_X + U_Y - U_{XY} > 0 \Leftrightarrow$

$$u_X[1 - (z + a_X^* + b_X^*)] - (a_Y^* + b_Y^*)u_Y > 0. \quad (9)$$

To show that alignment in Y is necessary, we show that the first order condition for focusing (9) cannot hold when players A and B conflict on issue Y . There are two cases for conflict (A) $u_Y > 0 > v_Y$ and (B) $u_Y < 0 < v_Y$.

Case A ($u_X < 0$ and $u_Y > 0 > v_Y$): Consider (9). Substitute with $u = -u_X/u_Y > 0$ and simplify with the solutions to the first-order conditions, $(a_X^*, a_Y^*) = (-a_X(u), a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), -b_Y(v))$, to see that (9) holds iff

$$[b_Y(v) - a_Y(u)] - u[1 - (z - a_X(u) + b_X(v))] > 0$$

This expression is strictly decreasing in u since the partial (applying the Envelope Theorem) is $-[1 - (z - a_X(u))] < 0$. At $u = 0$, the expression reduces to $b_Y(v) - \bar{p} < 0$. Hence, the expression is never positive, which is a contradiction.

Case B ($u_X < 0$ and $u_Y < 0 < v_Y$): Consider (9). Substitute with $u = u_X/u_Y > 0$ and simplify with the solutions to the first-order conditions, $(a_X^*, a_Y^*) = (-a_X(u), -a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), b_Y(v))$, to see that (9) holds iff

$$[b_Y(v) - a_Y(u)] - u[1 - (z - a_X(u) + b_X(v))] > 0.$$

Since this is the same expression as in Case A, we have a contradiction.

Part (d): By Lemma 1, B always chooses $b_x = \bar{p}$. Since A chooses a_x according to the cut-off function, $\bar{u}_F(v)$, the equilibrium result follows directly. ■

Proof of Proposition 2, Part (a): The partial conflict pinning assumptions are: $v_X > 0 > u_X$, $u_Y > 0$ and $v_Y > 0$. Player B chooses b_y to maximize $(z + a_y + b_y)(v_Y + V_X) + [1 - (z_Y + a_y + b_y)]V_{XY}$. Thus, $b_y = -\bar{p}$ iff $v_Y + V_X < V_{XY}$. Substituting for V_X and V_{XY} with the optimal static actions, rearranging terms and dividing through by $v_Y > 0$, we have

$$v_Y + V_X < V_{XY} \Leftrightarrow 1 - [z + a_Y^* + b_Y^*] - \frac{v_X}{v_Y}(a_X^* + b_X^*) < 0.$$

Using $u \equiv -\frac{u_X}{u_Y} > 0$ and $v \equiv \frac{v_X}{v_Y} > 0$, and writing the the optimal choices in the $\{XY\}$ state in terms of the solutions to the first-order conditions, that is $(a_X^*, a_Y^*) = (-a_X(u), a_Y(u))$ and $(b_X^*, b_Y^*) = (b_X(v), b_Y(v))$, our condition for $b_x = -\bar{p}$ becomes

$$k(v, u) = 1 - [z + a_Y(u) + b_Y(v)] - v[b_X(v) - a_X(u)] < 0.$$

Next, we claim that, for any $u > 0$, the function $k(v, u)$ is (1) increasing in v for $v < u$ and decreasing in v for $v > u$, (2) positive at $v = 0$, (3) negative as $v \rightarrow \infty$, and, hence, (4) there $\exists!$ $v \ni k$ crosses 0. To show (1), differentiate k w.r.t. v and apply the envelope theorem, $b'_Y(v) + vb'_X(v) = 0$, to find $k_v = a_X(u) - b_X(v)$. From the proof of Lemma A1, we know that $a_X(u) \geq b_X(v)$ as $u \geq v$ since both are increasing in the utility intensity. Then, (1) follows directly. For (2), let $v \rightarrow 0$ and note that $b_X(v) \rightarrow 0$ and $b_Y(v) \rightarrow \bar{p}$, so that $k(0, u) = 1 - [z + a_Y(u) + \bar{p}] > 0$, by Condition 2. For (3), letting $v \rightarrow \infty$ in $k(v, u)$ and noting $b_X(v) \rightarrow \bar{p}$ and $b_Y(v) \rightarrow 0$, we see $k(v, u) \rightarrow -\infty$. Then, (4) follows by continuity and $k(v, u)$ crosses zero one time at a unique $v = \bar{v}_P(u) \in (0, \infty)$. Thus, $k(v, u) > 0$ holds for $0 < v < \bar{v}_P(u)$ and then $b_y = \bar{p}$, while $k(v, u) < 0$ holds for $v > \bar{v}_P(u)$ and then $b_y = -\bar{p}$. Note that, by property (1), for a given u , the maximum of k over all $v \geq 0$ occurs at $v = u$. Since $k(u, u) > 0$, it follows that k crosses zero in v to the right of $v = u$ and we therefore have $\bar{v}_P(u) > u$. Finally, note that $k(0, 0) > 0$, so that we have $\bar{v}_P(0) > 0$.

Part (b): Implicit differentiation of $k(\bar{v}_P, u) = 0$ yields $\bar{v}'_P(u) = -k_u/k_v$, the ratio of partials. We know $k_v < 0$ holds when $k(\bar{v}_P, u) = 0$. Also, we easily find that $k_u = -a'_Y(u) + va'_X(u) > 0$. Hence, $\bar{v}'_P(u) > 0$. Finally, $\bar{v}_P(u) > u$ was shown just above.

Part (c): Suppose X is an alignment issue. In a pinning equilibrium player B chooses $b_y = -\bar{p}$ against own interest based on $v_Y > 0$. We know $b_y = -\bar{p}$ is optimal when the condition $v_Y + V_X - V_{XY} < 0$ holds. Substituting for V_X and V_{XY} and rearranging terms $v_Y + V_X - V_{XY} < 0 \Leftrightarrow$

$$v_Y[1 - (z + a_Y^* + b_Y^*)] + v_X(\hat{a}_X + \hat{b}_X) - v_X(a_X^* + b_X^*) < 0 \quad (10)$$

where \hat{a}_X and \hat{b}_X are the optimal actions where only issue X is on the second-period agenda. There are two cases of alignment for X :

Case 1 ($u_X > 0, v_X > 0$): This implies $\hat{a}_X = \hat{b}_X = \bar{p}$. But then $v_Y[1 - (z + a_Y^* + b_Y^*)] + v_X(2\bar{p} - a_X^* - b_X^*) > 0$, which contradicts (10).

Case 2 ($u_X < 0, v_X < 0$): This implies $\hat{a}_X = \hat{b}_X = -\bar{p}$. Then $v_Y[1 - (z + a_Y^* + b_Y^*)] - v_X(2\bar{p} + a_X^* + b_X^*) > 0$, which again contradicts (10).

Hence, pinning cannot occur with alignment over the second issue.

Part (d): A completely symmetric argument shows that Player A also has a cut-off value, denoted by $\bar{u}_P(v)$ and it is defined by the condition $k(\bar{u}_P, v) = 0$. The claim regarding a pinning equilibrium now follows directly. For $v > \bar{v}_P(u)$, we know Player B optimally chooses $b_y = -\bar{p}$. Because $\bar{v}_P(u) > u$, we see that $v > u$ holds. We then have $\bar{u}_P(v) > v > u$ and Player A optimally chooses $a_y = \bar{p}$. ■

Proof of Proposition 3: The payoff comparison for B across the two agendas, $X \rightarrow Y$ and $Y \rightarrow X$, is given by

$$\begin{aligned} V_F^b &\equiv (z + 2\bar{p})(v_X + V_Y) + [1 - (z + 2\bar{p})]V_{XY} > z(v_Y + V_X) + (1 - z)V_{XY} \equiv V_P^b \\ &\Leftrightarrow (z + 2\bar{p})(v_X + V_Y - V_{XY}) > z(v_Y + V_X - V_{XY}). \end{aligned}$$

By Lemma 1, we have $v_X + V_Y - V_{XY} > 0$. By existence of the pinning equilibrium, we have $v_Y + V_X - V_{XY} < 0$. Thus, the inequality holds.

The payoff comparison for A across the two agendas is more subtle. To begin, we have

$$\begin{aligned} U_F^a &\equiv (z + 2\bar{p})(u_X + U_Y) + [1 - (z + 2\bar{p})]U_{XY} < z(u_Y + U_X) + (1 - z)U_{XY} \equiv U_P^a \\ &\Leftrightarrow (z + 2\bar{p})(u_X + U_Y - U_{XY}) < z(u_Y + U_X - U_{XY}). \end{aligned}$$

Note that this inequality always holds at $u = \bar{u}_F(v)$ since this implies $u_X + U_Y - U_{XY} = 0$ while the right-hand side is positive since

$$u_Y + U_X - U_{XY} > 0 \Leftrightarrow u_Y[1 - (z + a_Y(u) + b_Y(v))] - u_X(b_X(v) - a_X(u)) > 0,$$

as follows from $u_Y > 0 > u_X$, $1 > z + a_Y(u) + b_Y(v)$ by feasible probabilities, and $b_X(v) > a_X(u)$ by $v > u$. To extend this to all (u, v) for which focusing and pinning equilibria exist, first simplify the inequality for $U_F^a < U_P^a$ by substituting for the U_X , U_Y and U_{XY} terms and note that $U_F^a < U_P^a$ iff

$$D(u, v) \equiv (z + 2\bar{p})^2 - z - 2\bar{p}(z + a_Y(u) + b_Y(v)) - u[z - z^2 + 2\bar{p} - 2\bar{p}(z - a_X(u) + b_X(v))] < 0.$$

Note that $D(u, v)$ is strictly decreasing in u and strictly increasing in v since the partial derivatives satisfy:

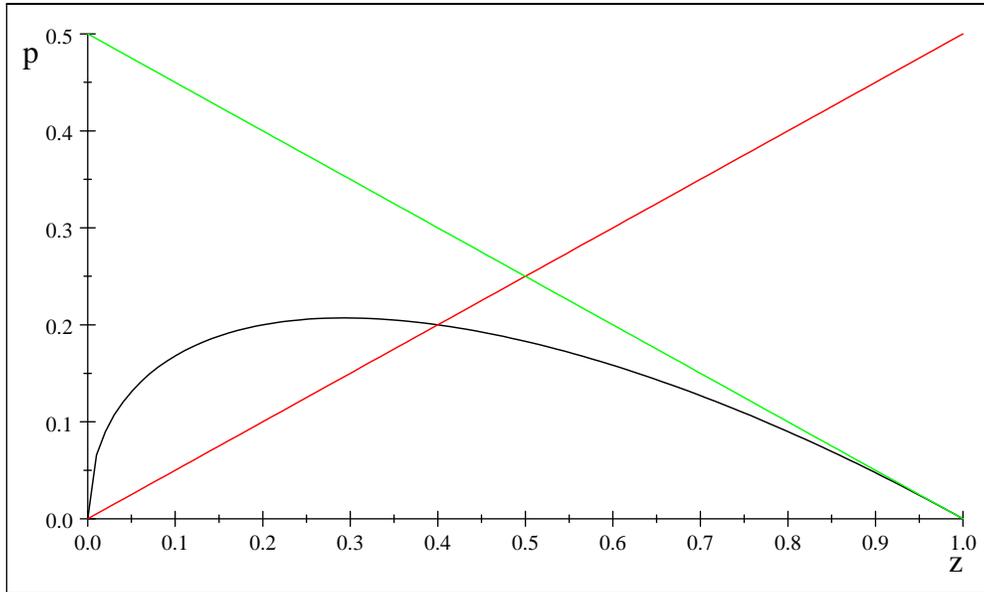
$$\begin{aligned} D_u &= -z(1 - z) - 2\bar{p}[1 - (z - a_X(u) + b_X(v))] < 0 \\ D_v &= 2\bar{p}[ub'_X(v) - b'_Y(v)] > 0, \end{aligned}$$

as follows from feasible probabilities and $b'_X(v) > 0 > b'_Y(v)$. If we can show $\lim_{v \rightarrow \infty} D(0, v) < 0$, then we will have $D(u, v) < 0$ for any $u \leq \bar{u}_F(v)$ since monotonicity in u and v implies $\lim_{v \rightarrow \infty} D(0, v) > D(0, v) > D(u, v)$. From the definition, we find

$$\lim_{v \rightarrow \infty} D(0, v) = 2\bar{p}(z + \bar{p}) - z(1 - z)$$

since $a_X(u) \rightarrow 0$ and $a_Y(u) \rightarrow \bar{p}$ as $u \rightarrow 0$, and $b_X(v) \rightarrow \bar{p}$ and $b_Y(v) \rightarrow 0$ as $v \rightarrow \infty$.

To characterize the limiting value of D in terms of the z and \bar{p} parameters, recall that our feasible set is given by z and \bar{p} in $(0, 1)$ for which $2\bar{p} < z < 1 - 2\bar{p}$ or, equivalently, $0 < \bar{p} < z/2$ for $z \leq 1/2$ and $0 < \bar{p} < (1 - z)/2$ for $z \geq 1/2$. Solving for \bar{p} in the implied quadratic $2\bar{p}(z + \bar{p}) - z(1 - z) = 0$ from the $\lim_{v \rightarrow \infty} D(0, v)$ expression, we see that $\lim_{v \rightarrow \infty} D(0, v) < 0$ holds when $\bar{p} \leq [\sqrt{2z - z^2} - z]/2$. As is easily verified, this necessarily holds when $z < 2/5$. Over full feasible parameter set for z and \bar{p} , we calculate that $\lim_{v \rightarrow \infty} D(0, v) < 0$ holds for approximately 89% of the region. Refer to the figure below. ■



Proof of Proposition 4: For part (a), we need only verify that any (u, v) with $v \geq u \geq \omega$ satisfies the condition for inclusion in \mathcal{F} . Since $v \geq u$, this reduces to $u \geq 1 + (1 + u)(u_Y/v_Y)$. This simplifies to $u \geq (u_Y + v_Y)/(u_Y - v_Y) = \omega$ and we are done. Note that an immediate implication is that \mathcal{F} contains all (u, v) where $0 \leq u \leq v$ for any $v \geq \omega$ as well as all (u, v) where $v \geq u$ for any $u \geq \omega$, since the lower boundary of \mathcal{F} is linear with slope less than 1 and crosses the 45° line at $u = v = \omega$. For part (b), we know from Proposition 1 that utility intensities in \mathcal{E}_F consist of (u, v)

such that $u \leq \bar{u}_F(v) \leq v$. Since we have $v \geq \omega$, we know from part (a) that \mathcal{F} contains all $u \leq v$ and, therefore, that any (u, v) in \mathcal{E}_F is also an element of \mathcal{F} when $v \geq \omega$. For part (c), we know from Proposition 2 that \mathcal{E}_P consists of (u, v) such that $v \geq \bar{v}_P(u) \geq u$. Since we have $u \geq \omega$, we know from part (a) that \mathcal{F} contains all $v \geq u$ and, therefore, that any (u, v) in \mathcal{E}_P is also an element of \mathcal{F} when $u \geq \omega$.

For part (d), the value of the discontinuity is a simple calculation from the NBS share formula using the threat point and social surplus, as described in the text. In order to be valid, however, we need to verify consistency with the underlying optimal agenda choice of $X \rightarrow Y$ as we cross the focusing boundary. This holds by construction: with $(u, v) \in \mathcal{F}$ we necessarily have $w_X > w_Y$ and, since Y is the alignment issue where $w_Y > 0$, we then have $w_X > 0$. Hence, $X \rightarrow Y$ is optimal in \mathcal{F} . The proof for part (e) is analogous as we need only employ the pinning threat payoffs in the calculation. ■

Proof of Lemma 2: $\gamma = 0$ implies that $d_X = d_Y = z_D$. $U^a = z_D^2 U_{XY} + z_D(1 - z_D)[U_X + p_Y u_Y] + (1 - z_D)z_D[p_X u_X + U_Y] + (1 - z_D)^2[p_X u_X + p_Y u_Y]$ which, after rearranging terms and simplifying gives $U^a = z_D[U_X + U_Y] - z_D^2[U_X + U_Y - U_{XY}] + (1 - z_D)[p_X u_X + p_Y u_Y]$. Similarly, $V^b = z_D[V_X + V_Y] - z_D^2[V_X + V_Y - V_{XY}] + (1 - z_D)[p_X v_X + p_Y v_Y]$. Maximizing U^a and V^b involves solving $\max_{a_X, a_Y} \{p_X u_X + p_Y u_Y\}$ and $\max_{b_X, b_Y} \{p_X v_X + p_Y v_Y\}$ with solutions that are the same as those for the static actions when both issues X and Y are on the agenda. ■

Proof of Proposition 5: To begin, we simplify U^a from the text and the analogous expression for V^b by collecting terms to obtain

$$\begin{aligned} U^a &= d_X d_Y U_{XY} + (1 - d_X) d_Y U_Y + d_X (1 - d_Y) U_X + (1 - d_X) p_X u_X + (1 - d_Y) p_Y u_Y \\ V^b &= d_X d_Y V_{XY} + (1 - d_X) d_Y V_Y + d_X (1 - d_Y) V_X + (1 - d_X) p_X v_X + (1 - d_Y) p_Y v_Y. \end{aligned}$$

The values when only X or only Y are on the agenda in period 2 are unchanged from before; also, values for U_{XY} and V_{XY} are determined by the preference ratios. We prove the proposition by taking limits as $u \rightarrow 0$ and $v \rightarrow \infty$. Since the relevant terms involve strict inequalities, our result holds in a neighborhood of these limiting values. For convenience, adopt the normalization of $u_Y = v_X = 1$ and let $u_X \uparrow 0$ while $v_Y \downarrow 0$. Then the limiting values for the $\{X, Y\}$ state in period 2 are $U_{XY} = (z + \bar{p})$ and $V_{XY} = (z + \bar{p})$, since the actions of A follow $a_X(u) \rightarrow 0$ and $a_Y(u) \rightarrow \bar{p}$ while those of B follow

$b_X(v) \rightarrow \bar{p}$ and $b_Y(v) \rightarrow 0$. Substituting in the payoffs above and simplifying yield

$$\begin{aligned} U^a &= d_Y [z + 2\bar{p} - d_X \bar{p}] + (1 - d_Y)p_Y \\ &= z + a_Y + b_Y + [z_D - \gamma a_Y b_Y] [2\bar{p} - a_Y - b_Y - \bar{p}(z_D - \gamma a_X b_X)] \end{aligned}$$

and

$$\begin{aligned} V^b &= d_X [z + d_Y \bar{p}] + (1 - d_X)p_X \\ &= z + a_X + b_X + [z_D - \gamma a_X b_X] [\bar{p}(z_D - \gamma a_Y b_Y) - a_X - b_X]. \end{aligned}$$

We can now employ a revealed preference argument to show that at any best response we have $a_Y \geq 0$ for A and $b_X \geq 0$ for B . For A , fix any given (b_X, b_Y) by B and compare the payoff U^a at (a_X, a_Y) where $a_Y > 0$ to that at $(a_X, -a_Y)$. Note that when (a_X, a_Y) is feasible then so is $(a_X, -a_Y)$. The payoff is larger with a_Y if and only if

$$2a_Y \{1 - z_D - \gamma b_Y [2\bar{p} - b_Y - \bar{p}d_X]\} > 0.$$

Since $a_Y > 0$ we need only show the bracketed term is positive. By feasibility, we have $1 - z_D > \gamma \bar{p}^2$ so it is sufficient to show $\bar{p}^2 > b_Y [2\bar{p} - b_Y - \bar{p}d_X]$. Over all $w \in [-\bar{p}, \bar{p}]$, the function $w [2\bar{p} - w - \bar{p}d_X]$ is strictly concave with an interior maximum at $w = \bar{p}(2 - d_X)/2$ where the function assumes its maximum value of $(\bar{p}(2 - d_X)/2)^2$. Our sufficient condition then reduces to $4 > (2 - d_X)^2$, which is clearly valid since we always have $d_X \in (0, 1)$. Thus, A will never choose $a_Y < 0$ in any best response.

The proof that $b_X \geq 0$ in any best response of B is similar and therefore omitted.

The following properties are straightforward to verify:

$$\begin{aligned} \text{A1:} \quad & \frac{\partial U^a}{\partial a_X} = \gamma \bar{p} b_X d_Y \geq 0 \quad \text{if } b_X \geq 0 \text{ (strict if } b_X > 0); \\ \text{A2:} \quad & \frac{\partial U^a}{\partial a_Y} = 1 - z_D - \gamma b_Y [\bar{p}(2 - d_X) - 2a_Y - b_Y] > 0 \quad \text{if } b_Y \leq 0; \\ \text{B1:} \quad & \frac{\partial V^b}{\partial b_X} = 1 - z_D - \gamma a_X [\bar{p}d_Y - a_X - 2b_X] > 0 \quad \text{if } a_X \geq 0 \text{ and } b_X \geq 0; \\ \text{B2:} \quad & \frac{\partial V^b}{\partial b_Y} = -\gamma \bar{p} a_Y d_X \leq 0 \quad \text{if } a_Y \geq 0 \text{ (strict if } a_Y > 0). \end{aligned}$$

Building on these properties, we can now show that i) in any best response to (b_X, b_Y) where $b_X \geq 0$, A always chooses such that $a_X \geq 0$, and ii) in any best response to (a_X, a_Y) where $a_Y \geq 0$, B always chooses such that $b_Y \leq 0$. We prove i) and omit the proof of ii), which is similar. There are two cases: $b_X > 0$ and $b_X = 0$. For $b_X > 0$, compare U^a at $a_X > 0$ and at $-a_X < 0$ for given

(b_X, b_Y) and a_Y . Then the payoff at $a_X > 0$ is larger if and only if $2\gamma a_X b_X \bar{p} d_Y > 0$. As $b_X > 0$ is given, $a_X > 0$ is by construction, and $d_Y \in (0, 1)$, we are done. For the case of $b_X = 0$, the payoff is independent of a_X since U^a simplifies to $U^a = d_Y [z_Y + 2\bar{p} - z_D \bar{p}] + (1 - d_Y)p_Y$. Now, if $b_Y \leq 0$, then we see from property A2 above that U^a is strictly increasing in a_Y . Hence, $a_Y = \bar{p}$ is optimal and, therefore, $a_X = 0$. If $b_Y > 0$, then observe that $\partial^2 U^a / \partial a_Y^2 = 2\gamma b_Y > 0$ and U^a is convex. The optimal choice of a_Y by A must then be one of the endpoints, either \bar{p} or $-\bar{p}$. Comparing U^a at these two choices, we find that \bar{p} is optimal if $\bar{p}^2 > b_Y[\bar{p}(2 - z_D) - b_Y]$. The maximum value for the right-hand side is $[\bar{p}(2 - z_D)/2]^2$, which occurs at $b_Y = \bar{p}(2 - z_D)/2$, and this is clearly less than \bar{p}^2 . Thus, A will never choose $a_X \leq 0$ in a best response and i) is established.

Summarizing, we have shown that in any equilibrium we necessarily have: $a_X \geq 0$, $a_Y \geq 0$, $b_X \geq 0$, and $b_Y \leq 0$. From this pattern, we now show that all of these inequalities are strict in equilibrium and, furthermore, that each agent does at least e on their dominant issue, where e is defined by $g(e) = e$, where g crosses the 45° line. Note that $g'(e) = -1$ and $g(e) > \bar{p}/2$.

We begin with B . Since we have $a_X \geq 0$ and $b_X \geq 0$ in equilibrium, property B1 implies that V^b is strictly increasing in b_X . This implies that $b_Y = -g(b_X)$ in any equilibrium. To see why, recall that $b_Y \leq 0$ holds in equilibrium. If we ever had $b_Y < -g(b_X)$ then the slack could be used to increase b_X and this would lead to a strict increase in V^b . Next, substituting with $b_Y = -g(b_X)$ for b_Y in V^b , the resulting variation with b_X is given by

$$\frac{\partial V^b}{\partial b_X} - g'(b_X) \frac{\partial V^b}{\partial b_Y}.$$

At $b_X = 0$, $g'(b_X) = 0$ holds (note that $\partial V^b / \partial b_Y$ is bounded by 1 in magnitude). Since $\partial V^b / \partial b_X > 0$ from B1, we see that B always chooses $b_X > 0$ in equilibrium. Incorporating $b_X > 0$, a similar argument allows us to conclude that $a_Y = g(a_X)$ and that $a_X > 0$ also holds. In turn, we can then show $a_Y > 0$ and $b_Y < 0$.

To show that each of a_Y and b_X exceed e , it is straightforward to substitute with g and reduce each of U^a and V^b to a function of only b_X and a_Y . We can then show that each of these functions is strictly increasing (in the action on the player's dominant issue) over the interval $[0, e]$. For A we

calculate

$$\begin{aligned}
U^a(a_Y, b_X) &= z + a_Y - g(b_X) + [z_D + \gamma a_Y g(b_X)] [\bar{p}(2 - z_D) - a_Y + g(b_X) + \gamma \bar{p} g(a_Y) b_X] \\
\frac{\partial U^a}{\partial a_Y} &= [1 - z_D + \gamma \bar{p}(2 - z_D)g(b_X) + \gamma g(b_X)^2] - 2\gamma g(b_X)a_Y \\
&\quad + \gamma \bar{p} z_D b_X g'(a_Y) + \gamma^2 \bar{p} b_X g(b_X)[g(a_Y) + a_Y g'(a_Y)] \\
\frac{\partial^2 U^a}{\partial a_Y^2} &= -2\gamma g(b_X) + \gamma \bar{p} z_D b_X g''(a_Y) + \gamma^2 \bar{p} b_X g(b_X)[2g'(a_Y) + a_Y g''(a_Y)] < 0
\end{aligned}$$

Since U^a is concave in a_Y , we need only show that $\partial U^a / \partial a_Y$ is positive at $a_Y = e$ for all $b_X \in [0, \bar{p}]$ to conclude that A chooses $a_Y > e$ in any best response. Evaluating and simplifying, we have

$$\left(\frac{\partial U^a}{\partial a_Y} \right)_{a_Y=e} = 1 - z_D + \gamma [\bar{p}(2 - z_D)g(b_X) + g(b_X)^2 - 2eg(b_X) - \bar{p}z_D b_X]$$

where we have used the properties $g(e) = e$ and $g'(e) = -1$.

Differentiating the above expression with respect to z_D yields $-1 - \gamma \bar{p}[g(b_X) + b_X] < 0$ and, therefore, the expression is bounded below by the value at $z_D = 1 - \gamma \bar{p}^2$, which is the maximum feasible value for z_D . Substituting with $z_D = 1 - \gamma \bar{p}^2$ in the original expression and simplifying, it is then sufficient to show

$$\bar{p}^2 + \bar{p}(1 + \gamma \bar{p}^2)g(b_X) + g(b_X)^2 - 2g(b_X)e - \bar{p}(1 - \gamma \bar{p}^2)b_X > 0$$

This expression is increasing in γ since $\bar{p}^3[g(b_X) + b_X] > 0$ and, therefore, it is bounded below by the value at $\gamma = 0$. It is then sufficient to show

$$\bar{p}^2 + \bar{p}g(b_X) + g(b_X)^2 - 2g(b_X)e - \bar{p}b_X > 0$$

This expression is increasing in \bar{p} since $2\bar{p} + g(b_X) - b_X > 0$ and is therefore bounded below by the value at $\bar{p} = 0$. As a result, it is sufficient to show $g(b_X)[1 - 2e] > 0$. Since feasibility implies $e < \bar{p}$ and $\bar{p} < 1/2$, we are done. This establishes that $\partial U^a / \partial a_Y > 0$ at $a_Y = e$ for all $b_X \in [0, \bar{p}]$.

To show that B always chooses a b_X that exceeds e , we calculate

$$\begin{aligned}
V^b(b_X, a_Y) &= z_X + g(a_Y) + b_X + [z_D - \gamma g(a_Y)b_X] [\bar{p}z_D - g(a_Y) - b_X + \gamma \bar{p}a_Y g(b_X)] \\
\frac{\partial V^b}{\partial b_X} &= [1 - z_D - \gamma \bar{p}z_D g(a_Y) + \gamma g(a_Y)^2] + 2\gamma g(a_Y)b_X \\
&\quad + \gamma \bar{p}z_D a_Y g'(b_X) - \gamma^2 \bar{p}a_Y g(a_Y)[g(b_X) + b_X g'(b_X)] \\
\frac{\partial^2 V^b}{\partial b_X^2} &= 2\gamma g(a_Y) + \gamma \bar{p}z_D a_Y g''(b_X) - \gamma^2 \bar{p}a_Y g(a_Y)[2g'(b_X) + b_X g''(b_X)]
\end{aligned}$$

V^b is not necessarily concave in b_X and the proof that $\partial V^b/\partial b_X > 0$ for $b_X \in [0, e]$ and $a_Y \in [0, \bar{p}]$ is more complicated than that for agent A . To begin, differentiating $\partial V^b/\partial b_X$ with respect to z_D yields $-1 - \gamma\bar{p}g(a_Y) + \gamma\bar{p}a_Yg'(b_X) < 0$. Hence, $\partial V^b/\partial b_X$ is decreasing in z_D and, therefore, bounded below by the value at $z_D = 1 - \gamma\bar{p}^2$, the maximum feasible value for z_D . Substituting and simplifying in $\partial V^b/\partial b_X$, it is sufficient to show

$$\bar{p}^2 - \bar{p}(1 - \gamma\bar{p}^2)g(a_Y) + g(a_Y)^2 + 2g(a_Y)b_X + \bar{p}(1 - \gamma\bar{p}^2)a_Yg'(b_X) - \gamma\bar{p}a_Yg(a_Y)[g(b_X) + b_Xg'(b_X)] > 0.$$

We claim this expression is increasing in γ . Differentiating with respect to γ , we need to show

$$\bar{p}^2g(a_Y) - \bar{p}^2a_Yg'(b_X) - a_Yg(a_Y)[g(b_X) + b_Xg'(b_X)] > 0.$$

This last expression is positive at $b_X = 0$ since $\bar{p}(\bar{p} - a_Y)g(a_Y) > 0$ and it is increasing in b_X since, differentiating with respect to b_X , we have

$$-\bar{p}^2a_Yg''(b_X) - a_Yg(a_Y)[2g'(b_X) + b_Xg''(b_X)] > 0.$$

Thus, we have shown the sufficient condition is increasing in γ .

As a result, the sufficient condition is bounded below by the value at $\gamma = 0$ and, in turn, it is now sufficient to show

$$\bar{p}^2 - \bar{p}g(a_Y) + g(a_Y)^2 + 2g(a_Y)b_X + \bar{p}a_Yg'(b_X) > 0$$

for $b_X \in [0, e]$ and $a_Y \in [0, \bar{p}]$. Observe that this last condition is increasing in \bar{p} since, by differentiation in \bar{p} , we have $2\bar{p} - g(a_Y) + a_Yg'(b_X) > 0$, as follows from $g'(b_X) \geq -1$ for $b_X \leq e$. Hence, $\bar{p} = 0$ provides a lower bound for the sufficient condition and we need only show $g(a_Y)^2 + 2g(a_Y)b_X > 0$, which clearly holds. We have thus established that V^b is increasing in b_X for $b_X \in [0, e]$ and that a best response by B will necessarily involve an action above e .

Existence of Equilibrium: we provide a simple pure-strategy existence result. To begin, note that the players have symmetric best-responses to extreme choices. It is straightforward to verify that the best-response of A to $b_X = 0$ is $a_Y = \bar{p}$ and, similarly, that the best-response of B to $a_Y = 0$ is $b_X = \bar{p}$. At the other extreme, the best response of A to $b_X = \bar{p}$ is interior and solves the first-order condition

$$0 = \left(\frac{\partial U^a}{\partial a_Y} \right) \Big|_{b_X=\bar{p}} = 1 - z_D + \gamma\bar{p}^2z_Dg'(a_Y).$$

Similarly, in response to $a_Y = \bar{p}$, the best-response problem for B is identical to that of A once we substitute $a_Y = \bar{p}$ in V^b .

As noted above, U^a is concave and A has a continuous best-response function that always exceeds e and is characterized by the unique solution to the first-order condition at any $b_X \in (0, \bar{p}]$, with $a_Y = \bar{p}$ in response to $b_X = 0$. It can be shown that A 's best response is decreasing in b_X for $b_X \in [0, e]$ but this need not hold at larger b_X values.

The complication with B is that V^b is not necessarily concave. If we make the stronger assumption on g that

$$\bar{p}(z_D - \gamma e \bar{p})g''(t) < -2 + \gamma e \bar{p}g'(t)$$

holds for $0 \leq t \leq \bar{p}$, then V^b is concave, as is easily verified from the above expression for $\partial^2 V^b / \partial b_X^2$. As a result, B now has a continuous best-response function, characterized by the solution to the first-order condition. We know from above that every best response of B is above e .

It follows directly from continuity and the common values of A and B in response to 0 and \bar{p} that the best-response functions cross each other and an equilibrium exists. ■