Cowles Foundation
for Research in Economics
at Yale University

Cowles Foundation Discussion Paper No. 1900

SEQUENTIAL INFORMATION DISCLOSURE IN AUCTIONS

Dirk Bergemann and Achim Wambach

July 2013

Sequential Information Disclosure in Auctions*

Dirk Bergemann† Achim Wambach‡

This Version: July 2013
First Version: April 2012

Abstract

We consider the design of an optimal auction in which the seller can determine the allocation *and* the disclosure rule of the mechanism. Thus, in contrast to the standard analysis of a optimal auctions, the seller can explicitly design the disclosure of the information received by each bidder as his private information.

We show that the optimal disclosure rule is a sequential disclosure rule, implemented in an ascending price auction. In the optimal disclosure mechanism, each losing bidder learns his true valuation, but the winning bidder only learns that his valuation is sufficiently high to win the auction. We show that in the optimal auction, the posterior incentive and participation constraints of all the bidders are satisfied. In the special case in which the bidders have no private information initially, the seller can extract the entire surplus.

JEL Classification: C72, D44, D82, D83

Keywords: Independent Private Value Auction, Sequential Disclosure, Ascending Auctions, Information Structure, Interim Equilibrium, Posterior Equilibrium.

*We thank Aron Tobias for excellent research assistance.

†Department of Economics, Yale University, New Haven, CT 06511, dirk.bergemann@yale.edu

‡Department of Economics, University of Cologne, wambach@wiso.uni-koeln.de

1 Introduction

We consider the design of an optimal auction for a single object and a finite number of bidders with independent and private values. Importantly, we extend the design of the optimal mechanism to include the determination of the allocation *and* the information. The present analysis is motivated by the observation that in many instances the seller of an object has considerable control over the information that the buyers have when bidding for the object under consideration. In fact, in some auctions, the seller intentionally limits the amount of information regarding the object sold to such an extent that they are commonly referred to as "blind auctions", see for example Kenney and Klein (1983) and Blumenthal (1988) for the licensing of motion pictures and Kavajecz and Keim (2005) for trading of large asset portfolios.

Interestingly, the relevant information is frequently disclosed sequentially *and* systematically linked to the bidding mechanism. In an auction practice referred to as indicative bidding, the seller (or an agent of the seller) initially invites "indicative" bids on the basis of a prospectus with a limited description of the asset and subsequently grants access to additional and more precise information only on the basis of sufficiently strong interest as expressed in the early rounds of bidding, see Ye (2007).

Here, we shall investigate the nature of the revenue maximizing mechanism when the seller can jointly determine the allocation and the disclosure rule which form the optimal mechanism. Importantly, we shall explicitly allow for sequential disclosure rules, i.e. disclosure rules which depend on the current (and past) bids, and hence in a direct mechanism on the current (and past) disclosed information.

In earlier work, Bergemann and Pesendorfer (2007) analyzed the present auction setting but restricted attention to static disclosure rules, i.e. disclosure rules in which each bidder only received a single signal. In contrast, in the present contribution we explicitly allow the seller to release additional signals in the course of the bidding process. In Bergemann and Pesendorfer (2007), the agents' initial private information was restricted to be the common prior distribution of the true valuations of the bidders. Thus, initially the agents did not possess any private information at all and any private information had to be generated by the disclosure rule of the mechanism. Subsequent work, notably by Eső and Szentes (2007) and Gershkov (2009), generalized the analysis to encompass private information which is not subject to the control of the disclosure rule and in due course we shall relate these two scenarios to each other. However, in the introduction it might be useful to briefly explain the role of sequential disclosure in the absence of any private information.

Consider for a moment an ascending auction, say in the form of the Japanese button auction, in which the asking price is raised continuously over time, see Cassady (1967). At each point in time and associated current price, each bidder has to make a decision as to whether he is staying in the auction or exiting the auction, i.e. whether he continues to press the button or whether he releases the button. We may now ask how much information would a bidder minimally need to participate in an efficient bidding mechanism, i.e. a mechanism which would support the efficient allocation of the object across the bidders. Now, given a current price, all he would need to know is whether his value is above or below the current price. If indeed he were in the possession of this information at all past and hence lower price points, then the sequential disclosure policy that supports this information structure is simply that at price p the true value p is revealed. Thus as the current price increases, and a bidder learns his value, he will rationally drop out (at the next price point) and the only remaining bidders are those who know that their true value are above the current price. It is now clear that this ascending auction reaches its natural stopping point when all but one of the bidders have dropped out, and the remaining bidder is the natural winner of the auction. The associated assignment of the object is efficient as his value is larger than that of everybody else. Now, given the information that he has, his expected valuation is the conditional expectation of his value, given that it is larger than or equal to the current price p. In the canonical ascending auction he indeed would pay p, but given his current information, his willingness to pay is his conditional expectation, which is strictly larger than p. In fact, the seller can charge him his exact conditional expectation and thus extract the entire surplus of the bidder, while satisfying the incentive and participation constraints, given the current information.

From the point of view of the seller, she would like the bidders to have and hence to provide just enough private information to identify which bidder has the largest valuation. At the same time, she does not want to give the bidder with the largest valuation too much information on his valuation so as to minimize the informational rent of the winning bidder. In the above procedure, this is achieved by giving the bidder at each point in time a binary information partition. Thus at any point in time, each bidder learns whether his valuation is above or below some threshold. The subsequent game is such that if the valuation of the bidder lies below the threshold, it is optimal for him to exit the contest. Increasing the threshold for all bidders until only one bidder remains, and then charging the winning bidder his expected valuation conditional on the valuation being larger than the final threshold, is the final outcome of the disclosure mechanism. Thus, each bidder learns either his true valuation, namely the losing valuations, or that he is the winning bidder and has the largest valuation, yet without learning its exact value.[1]

If bidders have private information, their respective type, from the very beginning of the mechanism, then the procedure needs to be generalized. First, the bidders have to report their types. Then, based on the reports, the thresholds in the sequential procedure are determined. These thresholds typically vary with the reports and hence differ across the bidders. Otherwise, the procedure works as above. Bidders obtain more and more information, and those who learn their true valuations exit the process. The final winner only learns that his valuation exceeds the final threshold. The winner will then be charged a price which is larger than this threshold but smaller than his expected value, conceding the informational rent he obtains with regard to his interim information. Determining the thresholds and the price is the critical step in the analysis to ensure that the bidder with the highest "shock-adjusted virtual valuation" wins, and to ensure that truthtelling both with regard to the initial, interim information and to the information obtained in the sequential procedure prevails.

Bergemann and Pesendorfer (2007) consider the standard independent private value auction for a single object with I risk-neutral bidders. Their objective is to derive the revenue maximizing mechanism. In contrast to the received analysis of the optimal mechanism, see the seminal contribution of Myerson (1981), they allow the seller to determine the allocation rule *and* the disclosure rule of the mechanism simultaneously. The disclosure rule of the mechanism determines the nature of the *private signal* that each agent receives about his true value, or willingness-to-pay for the object. Bergemann and Pesendorfer (2007) refer to the disclosure rule as the information structure of the mechanism. We shall refer to a pair of allocation and disclosure rules as a

[1] The information revelation mechanism analyzed here bears some similarity to the "bisection auction" recently proposed by Grigorieva, Herings, Müller, and Vermeulen (2007). In the bisection auction, each bidder is asked whether his valuation is above a threshold. If more than one says yes, the same question is asked with a higher threshold. If only one bidder says yes, he will obtain the good.

disclosure mechanism.

The disclosure rule controls the informativeness of the private signal about the valuation. Importantly, while the seller determines the disclosure rule, the seller does not observe the realization of the private signal of each bidder. Formally, the disclosure rule is a mapping, one for each agent, from the value of the object to a distribution over a set of possible signals. The set of feasible disclosure rules includes the *full disclosure rule*, in which each agent learns his value perfectly, and the *zero disclosure rule*, in which each agent learns nothing above the common prior over the valuation. Between these two extreme disclosure rules are many other feasible disclosure rules, including deterministic and stochastic disclosure rules. In Bergemann and Pesendorfer (2007), the seller chooses among all feasible disclosure *and* allocation rules to maximize her expected revenue.[2]

The canonical revenue maximizing problem, as pioneered by Myerson (1981), can then be viewed as the special case where the seller happens to adopt the full disclosure rule. The disclosure mechanism is subject to the standard incentive and participation constraints of the agents. In other words, given the disclosed private information, each bidder has an incentive to report his private information truthfully, and given the private information, each bidder is willing to participate, i.e. his expected net utility is at least as large as his utility from not participating. We shall refer to these constraints as the *posterior incentive* and *posterior participation* constraints, as each agent is conditioning his report and his participation on the private information revealed in the disclosure mechanism. These notions of posterior constraints were first introduced by Green and Laffont (1987) to reflect the possibility that the mechanism may reveal some, but not necessarily all, payoff-relevant information to the agents.[3]

Bergemann and Pesendorfer (2007) analyze the optimal disclosure mechanism subject to

[2]Kamenica and Gentzkow (2011) consider a related class of problems referred to as "Bayesian Persuasion". They consider the interaction between a principal and a *single* agent, where the principal can determine the disclosure rule, but the allocation is determined by the agent. Thus the game is "given" rather than "designed" as in the current analysis, but of course the action taken by the agent can be influenced through the disclosure rule adopted by the principal.

[3]By contrast, the ex-post incentive and participation constraints are evaluated under complete information about the realized ex post valuation of each agent. Further, and by convention, we refer to *ex-ante* as the moment in which each bidder only knows the common prior, and to *interim* as the moment in which each bidder only knows his own private type.

posterior incentive and posterior participation constraints. They find that the optimal disclosure mechanism uses a deterministic, but coarse, disclosure rule. In other words, each agent receives only limited information about his true value, and the resulting revenue strictly exceeds the revenue of the full disclosure rule. In addition, the deterministic disclosure rule can be represented as a finite partition over the set of values, where each element of the partition is an interval, and hence a connected set, of the real line. The optimality of the coarse information is shown to arise from the nature of the information rent. In the full disclosure rule, each agent is informed of his true value, and while this can guarantee an efficient allocation, it also allows the agent to receive a substantial information rent. By limiting the private information, it is shown that the seller can reduce the information rent without substantially lowering the efficiency of the allocation. In fact, Bergemann and Pesendorfer (2007) show that the optimal disclosure rule always induces an asymmetric partition of the values across the bidders, even in an otherwise symmetric environment. The asymmetry of the partition allows the seller to rank the bidders, and hence approximately maintain efficiency, while fitting each signal of a given bidder between competing signals (from below and above) of the other bidders, which enhances the competition and hence depresses the information rent of each agent.

Gershkov (2002) reconsiders the optimal disclosure mechanism of Bergemann and Pesendorfer (2007) under a weaker participation constraint, namely the ex-ante participation constraint, while maintaining the posterior incentive constraints. With the ex-ante participation constraint, the seller can charge each bidder a participation fee before the release of any private information. The participation fee essentially allows the seller to extract the entire expected surplus from the agents. Gershkov (2002) establishes that in the presence of the ex-ante participation constraint, the optimal disclosure rule is the full disclosure rule, and the optimal allocation rule is the efficient assignment of the object under the standard second price auction. The participation fee charges each bidder his expected net utility of the subsequent second price auction, and hence extracts the entire surplus from the bidders. To wit, the resulting transfer rule necessarily violates the posterior participation constraint, as all but one of the bidders, namely the winning bidder, make a payment, the participation fee, but do not receive the object, and hence realize a negative net utility.[4]

[4]The nature of the solution in Gershkov (2002) is reminiscent to the analysis of the efficient regulation of a

In an important contribution, Eső and Szentes (2007) pursue the analysis of the optimal information disclosure in the context of an informational environment which encompasses Bergemann and Pesendorfer (2007) and Gershkov (2002). In their model, each agent has two possible sources of private information, an initial private signal of the true value of the object, the type, and subsequently the realization of the true value. Importantly, the disclosure of the initial signal, the type, cannot be affected by the disclosure mechanism, it is only the disclosure of the subsequent signal, possibly the true value of the object, that is controlled by the disclosure mechanism. Eső and Szentes (2007) show that the additional, or incremental, information that is contained in the true value of the object, relative to the initial signal, can be represented as a signal that is orthogonal to, i.e. independent of, the initial signal. Based on this representation of the private information of each agent, namely the initial signal and the incremental and independent signal, they suggest a sequential screening contract, in which each agent first reveals his initial information, and then in a second step the additionally disclosed information. The design of the optimal disclosure mechanism is subject to the posterior incentive constraints and the interim participation constraints. Thus, each bidder is willing to participate given his initial private information only, and is reporting truthfully his initial information and the additional disclosed information. Surprisingly, they show that the optimal disclosure mechanism is the full disclosure mechanism. Yet, even though each agent is receiving two distinct and independent private signals, they also show that the net utility of each agent is due to the information rent of the initial signal only. In consequence, the main result in Eső and Szentes (2007) is that the optimal disclosure mechanism generates as much revenue as an optimal mechanism could in which the incremental information of each agent was observable by the seller.[5][6]

The strong equivalence result, based on the orthogonalization of the initial and the incremental signals, again relies on the interim participation constraint, similar to the role of the ex-ante

natural monopoly offered by Demsetz (1968) and Loeb and Magat (1979), which suggests the *ex ante* sale of all future rents.

[5] Gershkov (2009) obtains a similar result in a setting where the incremental signal of each agents pertains to common value component in the valuation of each bidder.

[6] In a very recent contribution, Li and Shi (2013) extend the analysis of the optimal disclosure process by permitting the disclosure process to depend not only on the reported type, but also on the true, but unknown value of the object. In this case, they show that the optimal policy can involve partial *and* discriminatory rather than complete *and* uniform information disclosure.

participation constraint in Gershkov (2002). In particular, the mechanism requires each bidder to pay a participation fee, or an option fee, which modifies the probability of winning, and the transfer conditional on winning. Importantly, the mechanism necessitates a payment from the losing bidders, and hence violates the posterior participation constraint. Thus, this result leaves open the question what can be achieved under stricter participation constraints. Krähmer and Strausz (2011) pursue this question in the sequential screening environment of Courty and Li (2000), which is a single agent setting. In contrast to the previously discussed literature, they maintain the full disclosure rule, and thus do not investigate the nature of the optimal disclosure policy. Rather, they investigate the nature of the optimal screening mechanism, when the seller is required to satisfy the posterior participation and posterior incentive constraints. Now, given the full disclosure rule and the single agent setting, the posterior constraints actually coincide with the ex post participation and incentive constraints. Krähmer and Strausz (2011) conclude that under the stronger participation constraint, the benefits of sequential screening completely disappear, and the optimal sequential contract is equivalent to the optimal static contract in which the agent reports the initial and the incremental signals simultaneously. The decomposition between the initial and the incremental signal proved, by itself, to be an important tool in the analysis of sequential screening contracts, see Pavan, Segal, and Toikka (2011) for a recent contribution on revenue maximizing mechanism design in a general environment with an infinite time horizon.

We proceed as follows. In the next section we present the model, the payoff and the information environment, which closely follow Eső and Szentes (2007). We also describe the framework of sequential information disclosure. In Section 3 we analyze the case without interim private information by the bidders; and here the first best allocation can be implemented. The general case is analyzed in Section 4, where we also provide an example, which compares the sequential information disclosure procedure with the handicap auction proposed by Eső and Szentes (2007). Section 5 concludes.

2 The Model

2.1 Payoffs, Types and Signals

There is one seller with a single object for sale and there are n potential bidders, indexed by $i \in \{1, 2, ..., n\}$, which are risk-neutral and with quasi-linear utility. The seller can commit to a mechanism to sell the object to one of the competing bidders.

The true valuation of bidder i is given by $V_i \in \mathcal{V}_i$, where $\mathcal{V}_i$ is a compact and convex subset of $\mathbb{R}_+$, which we assume without loss of generality to be equal to the unit interval $\mathcal{V}_i = [0, 1]$ for all i. The prior distribution of V_i is denoted by H_i and corresponding density h_i. The valuations are independently distributed across the agents.

Importantly, each agent only receives a noisy signal v_i of his true valuation V_i before he enters the mechanism. We assume that the type v_i is distributed, again without loss of generality on the unit interval $[0, 1]$ with distribution F_i and corresponding density f_i. We denote by $H_{iv_i} \triangleq H_i \left(V_i \left| v_i \right. \right)$, the distribution of V_i conditional on v_i, with the corresponding conditional density $h_{iv_i} \triangleq h_i \left(V_i \left| v_i \right. \right)$. We refer to v_i as the *type*, or interim information, of agent i.

In addition, each agent i may receive additional information which resolves the residual uncertainty about the value V_i during the bidding process. Eső and Szentes (2007) suggested that the additional information can be described by a random variable s_i which is statistically independent of the initial information, v_i. Formally, s_i can be written as:

$$s_i(v_i, V_i) = H_{iv_i}(V_i \left| v_i \right.) \triangleq s_i. \tag{1}$$

Thus s_i is the percentile of the true valuation conditional on the type v_i. We refer to the random variable as the *signal* $s_i \in S_i = (0, 1]$. By providing the signal $s_i(v_i) = H_{iv_i}(V_i)$ the bidder learns his valuation, while the seller, assuming that she could observe the signal, would still not know the exact valuation of the bidder. Denote the function which computes the valuation given the signal and initial type by

$$u_i(v_i, s_i) \triangleq H_{iv_i}^{-1}(s_i).$$

Thus by construction, it has the property that for all v_i and s_i, the resulting conditional expectation satisfies $\mathbb{E}\left[u_i(v_i, s_i) \left| v_i, s_i \right. \right] = V_i$, i.e. after observing type v_i and signal s_i, bidder i knows his true valuation V_i. We observe that by construction of (1), the distribution of s_i is simply the uniform distribution on $[0,1]$.

Importantly, we assume that the seller can control the time and the precision of the additional disclosed information. But, as in Bergemann and Pesendorfer (2007) and Eső and Szentes (2007), while the seller can control the precision (and now the timing) of the information, she does not observe the realization of the additional signal, which remains private information to each bidder i. In the next subsection we describe a specific procedure of sequential information disclosure of the signal s_i. The disclosure of the random variable s_i is going to be sequential in that the disclosure mechanism determines for *every* realization of the signal s_i the time at which the realization is disclosed. In particular, higher realizations of s_i are going to be disclosed later in time.

2.2 Sequential Mechanism: Disclosure and Allocation

We consider the following sequential disclosure *and* allocation mechanism which ends with the allocation of the object. The disclosure component determines the time by which the signal s_i is revealed. The allocation component determines the final allocation of and payments for the object. As in the ascending auction, the object is awarded to the final participating bidder.

Disclosure The sequential mechanism asks each bidder to initially report his type v_i and then to report his signal realization s_i as soon as it is disclosed by the mechanism. The disclosure part of the mechanism determines the time $t \in [0,1]$ at which the signal realization s_i is disclosed. We first define the sequential disclosure component which determines the time at which the signal realization s_i is disclosed. For every agent i, we define a disclosure function $\xi_i \triangleq \xi_i(t, \widehat{v}_i, s_i)$:

$$\xi_i : [0,1] \times [0,1] \times (0,1] \to [0,1], \tag{2}$$

which determines the disclosure of the signal realization as a function of time $t \in [0,1]$, reported type $\widehat{v}_i \in [0,1]$ and signal realization $s_i \in (0,1]$. The function $\xi_i \triangleq \xi_i(t, \widehat{v}_i, s_i)$ is assumed to be a step function in time t, with a single jump, from 0 (which represent the event of no signal disclosure yet) to $s_i > 0$ at a particular disclosure time $t_i(\widehat{v}_i, s_i)$:

$$t_i(\widehat{v}_i, s_i) \triangleq \min\{t \in [0,1] \,|\xi_i(t, \widehat{v}_i, s_i) > 0\},$$

and constant everywhere else in t. Thus the disclosure time $t_i(\widehat{v}_i, s_i)$ is the time at which the signal realization s_i is disclosed to bidder i given a reported type $\widehat{v}_i$.

Importantly, the disclosure time $t_i(\widehat{v}_i, s_i)$ will be constructed to be component-wise strictly increasing, that is $t_i(\widehat{v}_i, s_i)$ is strictly increasing in both the reported type $\widehat{v}_i$ and the signal realization s_i. Thus, a higher reported type slows down the disclosure of information, and a higher realizations of s_i is going to be disclosed later than a low realization of s_i. In this sense, the initial report $\widehat{v}_i$ influences the speed of disclosure, and as time goes by, the bidder continues to update his estimate, even in the absence of a disclosed signal. The disclosure function ξ_i and disclosure time t_i for different realization of the type v_i and signal s_i are illustrated in Figure 1.

Insert Figure 1: Disclosure function ξ_i and disclosure time t_i here.

The state of the disclosure process at time t, given by $\xi_i(t, \widehat{v}_i, s_i)$, is privately observable to bidder i, and it is either 0 (which means disclosure has not yet occurred) or s_i (which means disclosure has occurred).

A reporting (or message) strategy $m_i = (r_i, d_i)$ of bidder i consists of an initial report r_i and a (continued) participation decision d_i for bidder i. The strategy of each bidder i depends on the private state (or history) of bidder i. The private history of bidder i at $t = 0$ is simply his type v_i, or $h_i^0 = (v_i)$ and at all subsequent times $t > 0$, his type v_i, his reported type $\widehat{v}_i$ and the state of the disclosure process $\xi_i(t, \widehat{v}_i, s_i)$, or

$$h_i^t = (v_i, \widehat{v}_i, \xi_i(t, \widehat{v}_i, s_i)).^7 \tag{3}$$

Formally, then the initial report r_i is defined as a mapping:

$$r_i : [0, 1] \to [0, 1] \tag{4}$$

and the continued participation decision d_i is defined as:

$$d_i : [0, 1] \times [0, 1] \times [0, 1] \times [0, 1] \to \{0, 1\}. \tag{5}$$

[7] We use the term private state or private history here interchangeably. Formally, the definition of the private state represents a sufficient statistic of the entire private history, as it summarizes, without loss of generality, the evolution of the disclosure process in terms of its present value.

The decision of the bidder is either to stay in the bidding process: $d_i(\cdot) = 1$ or to exit the bidding process: $d_i(\cdot) = 0$. The participation decision depends on the time $t \in [0,1]$, the true type v_i, the reported type $\widehat{v}_i \in [0,1]$, and the state of the disclosure process $\xi_i(t, \widehat{v}_i, s_i) \in [0,1]$. The exit decision is irrevocable, and hence d_i, as a function of time, is restricted to be weakly decreasing in t.

Allocation The object is assigned as soon as all but one of the bidders have exited the bidding process. As time t progresses, we can track the exit decision of the agents. At time $t < 1$, agent i has exited the bidding process if the exit time $\tau_i(t)$ of bidder i:

$$\tau_i(t) \triangleq \min\{\{t' \leq t \,|d_i(t', \cdot) = 0\} \wedge 1\}, \tag{6}$$

satisfies $\tau_i(t) \leq t$. To wit, if the agent has not yet exited, then at time t, we assign him the exit time 1, which simply represent the fact that at t he is still participating in the bidding. For the individual bidder i, the disclosure process $\xi_i(\cdot)$ stops as soon as bidder i decides to exit the auction, or $\xi_i(t, \widehat{v}_i, s_i) = \xi_i(\tau_i, \widehat{v}_i, s_i)$ for all $t \geq \tau_i$.

The mechanism determines the allocation at the first time, τ, at which all but one of the agents have exited the auction:

$$\tau \triangleq \min\{t > 0 \,|\exists k, \text{ s. th. } \tau_j(t) \leq t, \forall j \neq k, \tau_k(t) > t\}.$$

This definition of the stopping time (and the subsequent definition of the allocation rule) excludes events in which *all of* the remaining bidders stop at the same time. These are zero probability events and hence can be omitted without loss of generality. At the expense of additional notation, we could complete the description by introducing a uniform random allocation in case of such a zero probability event, essentially a tied bid.

The assignment of the object is described by a probability vector $x = (x_1, ..., x_n)$, and the assignment probabilities x_i :

$$x_i : [0,1]^n \to \{0,1\} \tag{7}$$

are required to sum to less than or equal to one,

$$\sum_{i=1}^{n} x_i(\cdot) \leq 1.$$

The allocation itself depends only on the stopping time τ, i.e.

$$x_i(\tau_1, ..., \tau_n) \triangleq 0 \Leftrightarrow \tau_i \leq \tau, \quad x_i(\tau_1, ..., \tau_n) \triangleq 1 \Leftrightarrow \tau_i > \tau.$$

Similarly, the transfers are described by a vector $p = (p_1, ..., p_n)$, where each p_i is formally defined by

$$p_i : [0,1] \times [0,1] \times [0,1] \to \mathbb{R}_+. \tag{8}$$

The transfer payments will have the property that only the winning bidder is making a positive payment, i.e. $p_i(\widehat{v}_i, \tau_i, \tau) = 0$ if $\tau_i \leq \tau$, and that the payment of the winning bidder will only depend on his initial report $\widehat{v}_i \in [0,1]$, and the stopping time $\tau \in [0,1]$, of course conditionally on $\tau_i > \tau$.

Incentive and Participation Constraints We now define "truthtelling" behavior as follows:

$$r_i^*(v_i) \triangleq v_i,$$

and

$$d_i^*(t, v_i, v_i, \xi_i(t, v_i, s_i)) \triangleq \begin{cases} 1, & \text{if} \quad \xi_i = 0; \\ 0, & \text{if} \quad \xi_i > 0. \end{cases}$$

In other words, each agent reports truthfully his own type, and then stays in the bidding process as long as he has not yet received the additional signal s_i, and exits as soon as a signal has been received. We refer to this as "truthtelling" behavior as the individual exit time reveals the value of the signal. We can now define the incentive and participation constraints. We require that truthtelling be a best response along every private history h_i^t:

$$\mathbb{E}\left[x_i\left(\left(m_i^*, m_{-i}^*\right)\right)V_i - p_i\left(\left(m_i^*, m_{-i}^*\right)\right)\left|h_i^t\right.\right] \geq \mathbb{E}\left[x_i\left(\left(m_i, m_{-i}^*\right)\right)V_i - p_i\left(\left(m_i, m_{-i}^*\right)\right)\left|h_i^t\right.\right], \forall m_i, \forall h_i^t, \tag{9}$$

and that truthtelling satisfy the participation constraint along every private history h_i^t :

$$\mathbb{E}\left[V_i x_i\left(\left(m_i^*, m_{-i}^*\right)\right) - p_i\left(\left(m_i^*, m_{-i}^*\right)\right)\left|h_i^t\right.\right] \geq 0, \forall h_i^t. \tag{10}$$

In minor abuse of notation, we describe the assignments x_i and the transfers p_i in (9) and (10) as dependent on the entire reporting strategy profile, but of course the strategy profile m^* generates the reports $r^*(v)$ and the exit times $(..., \tau_i, ...)$, which determine the assignment and transfer prices. We observe that the above incentive and participation constraints imply that the *interim* participation and incentive constraints are satisfied, i.e. at the outset of the game when each agent only observes his type v_i: $h_i^0 = v_i$, as well as the *posterior* participation and incentive constraints when bidder i either exited, $\tau_i \leq \tau$, or won the bidding process, $\tau_i > \tau$.

We may summarize the sequential mechanism as follows. For each bidder i, nature initially draws (v_i, s_i). Bidder i initially observes v_i but not s_i. Bidder i reports $\widehat{v}_i \triangleq r_i(v_i)$ according to the reporting strategy $r_i(\cdot)$ (whether or not $\widehat{v}_i = v_i$). Then, the disclosure policy $\xi_i(\cdot)$ uses the *reported* type $\widehat{v}_i$ (and *not* the true type v_i) and the signal s_i to generate the disclosure time $t(\widehat{v}_i, s_i)$. The mapping specified by the disclosure policy (that is, the *time* at which a signal realization will be disclosed as a function of the reported type) is common knowledge. At any point of time t, the bidder *either* knows that $s_i > s_i'$ for the critical signal s_i' such that $t = t(\widehat{v}_i, s_i')$ *or* that the value is s_i, namely if $t(\widehat{v}_i, s_i) \leq t$.

The allocation mechanism is thus a version of an ascending auction, in the format of the "Japanese" or "button" auction in which the price uniformly increases over time. In the button auction, if a bidder releases the button, he reveals his type, and the auction ends for him. The ascending disclosure mechanism modifies the button auction in two important aspects: (i) it associates a disclosure process with the price process, (ii) the final price paid is personalized, and related to, but not necessarily equal to the valuation of the final remaining competitor.

A special, but important, case with which we begin the analysis in Section 3 is the case of uninformed bidders. Here, the initial information, the type v_i, is simply a singleton, and thus merely represents the prior information contained in the common prior H_i, and does not contain any additional information.

3 Bidding without Interim Information

We begin our analysis with bidders who do not possess interim private information. In other words, the initial information of each agent is the common prior $H = (H_1, ..., H_n)$ over the valuations. This informational environment with ex-ante uninformed bidders was analyzed by Bergemann and Pesendorfer (2007), but they restricted attention to *static* disclosure mechanisms. In this section we revisit their setting but allow for the possibility of *sequential* information disclosure.

Before we consider any sequential disclosure mechanism it is useful to describe the benchmark allocation, which the seller could achieve, if the valuation V_i of each bidder were observable by the seller. In the case of observable valuations, the seller could directly identify the bidder i with the highest valuation V_i, and offer him the object at a price equal to his valuation V_i. The resulting allocation would satisfy the posterior, in fact the ex post participation and incentive constraints of all the bidders, and the seller would be able to extract the entire social surplus. The resulting optimal revenue, the social surplus, is given by:

$$S^* \triangleq \int_{V_1} \cdots \int_{V_n} \max\{V_1, ..., V_n\}\, dH_1(V_1) \cdots dH_n(V_n). \tag{11}$$

The resulting allocation is socially efficient, that is bidder i with valuation V_i obtains the good if and only if all other bidders have valuations less than V_i.

We now adapt (and simplify) the *sequential mechanism,* defined earlier by (2), (7) and (8) to the present environment. In particular, without interim information v_i, the disclosure function can depend on time t and signal s_i alone, and without loss of generality, we can take the signal s_i to be equal to the valuation V_i. With this, the disclosure function can now be written as:

$$\xi_i : [0,1] \times [0,1] \to [0,1], \tag{12}$$

which determines the disclosure of the valuation as a function of time $t \in [0,1]$ and of the valuation $V_i \in [0,1]$. The disclosure function $\xi_i(t, V_i)$ is constructed as a step function in time t, with a single jump, from 0 to V_i at the disclosure time t_i of valuation V_i, where

$$t_i(V_i) \triangleq \min_t \{t \in [0,1] \,|\xi_i(t, V_i) \geq V_i\}. \tag{13}$$

Thus the disclosure time $t_i(V_i)$ is the first time at which the valuation V_i is privately disclosed to bidder i.

In the absence of ex-ante private information, we can choose the disclosure functions $\{\xi_i\}_{i=1}^n$ to be identical for all of the agents and define

$$\xi_i(t, V_i) \triangleq \begin{cases} 0, & \text{if} \quad t < V_i; \\ V_i, & \text{if} \quad t \geq V_i. \end{cases} \tag{14}$$

Thus, bidder i with valuation V_i receives a perfectly informative signal about his valuation at $t = V_i$, whereas at all times t with $t < V_i$, he will infer that his expected valuation is given by the conditional expectation, $\mathbb{E}[V_i | V_i \geq t]$.

The assignment of the object to agent i depends only on his exit time τ_i and the stopping time τ :

$$x_i(\tau_i, \tau) \triangleq \begin{cases} 0, & \text{if } \tau_i \leq \tau; \\ 1, & \text{if } \tau_i > \tau. \end{cases} \tag{15}$$

The transfer payments request a single positive payment p_i at the stopping time τ from the winning bidder only:

$$p_i(\tau_i, \tau) \triangleq \begin{cases} 0, & \text{if } \tau_i \leq \tau; \\ \mathbb{E}[V_i | V_i \geq \tau], & \text{if } \tau_i > \tau. \end{cases} \tag{16}$$

A sequential mechanism is then defined by (14)-(16), and we shall refer to it as the *ascending disclosure mechanism*.

Without interim information, the participation decision d_i depends only on the time t and the state of the disclosure process at time t, represented by $\xi_i(t, \cdot)$.

$$d_i : [0,1] \times [0,1] \to \{0,1\},$$

The decision of the bidder is either to stay in the bidding process: $d_i(\cdot) = 1$ or to exit the bidding process: $d_i(\cdot) = 0$. We can now explicitly describe the incentive and participation constraints in this environment. We begin with the incentive constraints and require that "truthtelling" be a best response for every private history $h_i^t = (t, \xi_i(t, \cdot))$. Thus if $\xi_i(t, \cdot) = 0$, then;

$$\mathbb{E}[x_i(1,t)(V_i - p_i((1,t))) | t, \xi_i(t, \cdot) = 0] \geq \mathbb{E}[x_i((t,t))(V_i - p_i((t,t))) | t, \xi_i(t, \cdot) = 0], \tag{17}$$

and if $\xi_i(t, \cdot) > 0$, then:

$$\mathbb{E}[x_i((t,t))(V_i - p_i((t,t))) | t, \xi_i(t, \cdot) = V_i] \geq \mathbb{E}[x_i((1,t))(V_i - p_i((1,t))) | t, \xi_i(t, \cdot) = V_i]. \tag{18}$$

In other words, it is optimal to stay in the bidding process if no information has been revealed: $\xi_i(t, \cdot) = 0$; and it is optimal to exit rather than to continue if information has been disclosed: $\xi_i(t, \cdot) = V_i$. Now given that $x_i(1,t) = 1$, $x_i(t,t) = 0$, $p_i(1,t) = \mathbb{E}[V_i | V_i \geq t]$ and $p_i(t,t) = 0$, we can simplify (17) and (18) to read:

$$\mathbb{E}[(V_i - \mathbb{E}[V_i | V_i \geq t]) \mid V_i \geq t] \geq 0,$$

and if $\xi_i(t,\cdot) > 0$, then:

$$0 \geq V_i - \mathbb{E}\left[\widehat{V}_i \middle| \widehat{V}_i \geq t\right].$$

We also require that in either case, the expected net utility for the bidder is always nonnegative, or

$$\mathbb{E}\left[\left(V_i - \mathbb{E}\left[V_i \left| V_i \geq t\right.\right]\right) \mid V_i \geq t\right] \geq 0, \tag{19}$$

and

$$\mathbb{E}\left[x_i\left((t,t)\right)\left(V_i - p_i\left((t,t)\right)\right) \left| t, \xi_i(t,\cdot) = V_i\right.\right] \geq 0. \tag{20}$$

We refer to the above constraints as the *posterior* incentive and participation constraints, as each agent is willing to report truthfully, given the information the agent has, and has been provided by the sequential mechanism at every time t. We refer to the constraints as the posterior constraints rather than as the ex post constraints, as the agent may not know his true valuation at the time of the assignment, but given the information at the time of the assignment, his constraints are met.

The revenue of the ascending disclosure mechanism, provided that all the bidders report truthfully is denoted by R^*. We can now state our first result in the setting with bidders without interim information.

Proposition 1 *The ascending disclosure mechanism satisfies the* posterior *incentive and participation constraints for all agents and the seller extracts the entire social surplus:*

$$S^* = R^*.$$

Proof. We first observe that if all the bidders follow the truthtelling strategy, then the posterior participation constraint is satisfied for the losing and the winning bidders. A losing bidder does not receive the object, see allocation rule (15), and by the payment rule (16) faces a zero payment, and hence his net utility is equal to zero. The winning bidder receives the object with probability one, see allocation rule (15), but given the payment rule (16) has to pay his expected conditional valuation at the stopping time τ. Thus, again, given the information disclosed by the mechanism at time τ, the net utility of the winning bidder is zero, and hence the posterior participation constraint is satisfied.

We then consider the posterior incentive constraints in the ascending disclosure mechanism. Every losing bidder learns his value and immediately exits to receive a net utility of zero. Clearly, exiting before learning the valuation V_i does not improve the net utility of bidder i, as bidder i would merely exit earlier, and still receive zero net utility. But if he were to stay longer, and not stop his own disclosure process, then the auction could reach the stopping point $\tau > \tau_i = V_i$, and ask bidder i to pay more than his true valuation. Clearly, this does not improve his net utility either. Finally, consider the winning bidder. He cannot change the price conditional on winning, he can only lower his probability of winning by exiting the auction before his valuation is revealed. But if he were to exit the auction, he would receive zero net utility as well, thus exiting early does not constitute a profitable deviation either. Thus staying in the mechanism is an optimal strategy.

Finally, let us consider the revenue of the ascending disclosure mechanism. The seller receives revenue from bidder i when all the other bidders have a valuation below him. Thus, the allocation is efficient, and as every bidder, winning or losing receive zero expected utility, it follows that the seller receives the entire social surplus. ■

We observe that in the ascending disclosure mechanism, the participation and incentive constraints of the losing bidders are not merely satisfied as posterior constraints, but even hold as ex post constraints. In other words, given the truthful reports of all the agents, a losing bidder would not want to change his reporting behavior, even after he learned his true valuation V_i. In contrast, for the winning bidder, the surplus extraction result crucially relies on the fact that the winning bidder does not learn his true valuation V_i, but rather is limited to knowing that his true valuation is in the interval $[\tau, 1]$ and hence forms his conditional expectation on the basis of the disclosed information.

Having shown that with ex-ante uninformed bidders, the ascending information disclosure leads to the revenue maximizing allocation, we now generalize the procedure to the case where the bidders have some private, or interim, information before they enter the mechanism.

4 Bidding with Interim Information

We now return to the general model in which each bidder i receives a noisy signal v_i of his valuation V_i, his interim information. This is the informational environment analyzed in Eső and Szentes (2007) and we maintain their distributional assumptions, namely that the density $f_i(v_i)$ associated with the distribution $F_i(v_i)$ of the buyer's type v_i is positive everywhere and that the distribution satisfies the monotone hazard condition, that is $f_i(v_i)/(1-F_i(v_i))$ is weakly increasing in v_i. We also maintain their assumptions about the relationship between the initial type and final valuation, namely that $(\partial H_{iv_i}(V_i)/\partial v_i)/h_{iv_i}(V_i)$ is increasing in v_i and V_i. They establish that in the revenue maximizing mechanism, the seller makes all additional information s_i available to the bidders. Yet, surprisingly, the seller can achieve the same expected revenue as if the private signal s_i were directly observable by the seller. The objective of this section is to provide a sequential implementation of the revenue maximizing mechanism. The ascending disclosure mechanism differs from the static disclosure mechanism in Eső and Szentes (2007) in two essential aspects: (i) the signal s_i is not completely disclosed, and (ii) the participation constraint of each bidder is satisfied at the posterior level rather than the interim level.

We proceed in three steps. In Subsection 4.1, we recall the relevant aspects of the revenue maximizing allocation in which the signal profile $\mathbf{s}$ is directly observable by the seller, as derived by Eső and Szentes (2007).[8] In Subsection 4.2, we present the ascending disclosure mechanism with interim information. In Subsection 4.3, we show that the ascending disclosure mechanism implements the revenue maximizing allocation.

4.1 Observable Signal

The benchmark case is the situation where the seller can observe the signal s_i of each bidder. Eső and Szentes (2007) show that in the second best, where the seller can observe the so-called 'shocks' s_i, the optimal mechanism has the following property: the object is rewarded to the bidder with the largest non-negative "shock-adjusted virtual valuation" $W_i(v_i, s_i)$:

$$W_i(v_i, s_i) = u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}(v_i, s_i), \tag{21}$$

[8] Eső and Szentes (2007) proceed to show that this second best allocation can also be implemented when the signal profile $\mathbf{s}$ is unobservable to the seller.

where $u_{i1}(v_i, s_i)$ is the partial derivative of $u_i(v_i, s_i)$ with respect to the first argument. We next describe some properties of the virtual valuation.

Lemma 1 (Virtual Valuation)

1. *The virtual valuation $W_i(v_i, s_i)$ is strictly increasing in v_i and s_i;*

2. *If $u_i(v_i, s_i) = u_i(v_i', s_i')$ and $v_i \geq v_i'$, then $W_i(v_i, s_i) \geq W_i(v_i', s_i')$;*

3. *If $W_i(v_i, s_i) = W_i(v_i', s_i')$ and $v_i \geq v_i'$, then $u_i(v_i, s_i) \leq u_i(v_i', s_i')$.*

Proof. (1.) - (3.) follow directly from Lemma 1 and Corollary 1 of Eső and Szentes (2007). ■

The above monotonicity of the virtual utility $W_i(v_i, s_i)$ implies that for a given vector of types $\mathbf{v} = (v_1, ..., v_n)$ and vector of signals $\mathbf{s}_{-i} = (s_1, .., s_{i-1}, s_{i+1}, ..., s_n)$, bidder i obtains the good whenever his signal s_i is larger than a threshold value $\underline{s_i}(\mathbf{v}, \mathbf{s}_{-i})$ of the signal s_i. This threshold is defined by:

$$\underline{s_i}(\mathbf{v}, \mathbf{s}_{-i}) \triangleq \min\left\{\min\left\{s_i \in [0,1] | W_i(v_i, s_i) \geq 0 \text{ and } \forall j \neq i,\ W_i(v_i, s_i) \geq W_j(v_j, s_j)\right\}, 1\right\}. \quad (22)$$

We note that in the above we take the minimum over s_i and 1, as v_i might be small, and hence there might be no signal $s_i \in [0,1]$ that would turn bidder i into a winner. Given that the virtual valuation does only depend on $\mathbf{v}$ and $\mathbf{s}$ and in particular is not a function of the distributional property of $\mathbf{s}$, we can construct the optimal (static) mechanism for every realization of $\mathbf{s}$. The optimal allocation is then determined by the virtual valuations and the bidder obtains the good whenever his type is larger than a threshold $\underline{v_i}(\mathbf{v}_{-i}, \mathbf{s})$:

$$\underline{v_i}(\mathbf{v}_{-i}, \mathbf{s}) \triangleq \min\left\{\min\left\{v_i \in [0,1] | W_i(v_i, s_i) \geq 0 \text{ and } \forall j \neq i,\ W_i(v_i, s_i) \geq W_j(v_j, s_j)\right\}, 1\right\}. \quad (23)$$

We construct incentive compatible transfers, which are only paid in case of winning, by asking the winner to pay the valuation of the lowest type $\underline{v_i}(\mathbf{v}_{-i}, \mathbf{s})$, given the signals $\mathbf{s}$, which would have won the contest:

$$p_i(\mathbf{v}_{-i}, \mathbf{s}) \triangleq u_i\left(\underline{v_i}(\mathbf{v}_{-i}, \mathbf{s}), s_i\right). \quad (24)$$

The payment $p_i(\mathbf{v}_{-i}, \mathbf{s})$ therefore has the Vickrey property that the payment of the winner i is independent of his true type v_i, conditional on the event $v_i \geq \underline{v}_i(\mathbf{v}_{-i}, \mathbf{s})$. The payment rule described by (24) therefore has the property that it implements truthtelling with respect to v_i if the signals $(s_1, ..., s_n)$ are publicly revealed.

4.2 Ascending Disclosure Mechanism

We next construct the sequential information disclosure with the important property that the virtual valuations of all participating bidders are equalized at all times t until bidding ends at τ. Given the initial reports of all bidders, truthful or not, we reveal to each bidder i whether his signal s_i is above a current threshold at a speed such that at all times the virtual utility of all participating bidders evaluated at the current threshold are identical. In this context, the initial report $\widehat{v}_i$ of bidder i simply determines the speed at which the disclosure process is running through the signals. Formally, we explicitly define the disclosure function $\xi_i(t, \widehat{v}_i, s_i)$ through the virtual valuation $W_i(\widehat{v}_i, s_i)$ and the associated disclosure time $t_i(\widehat{v}_i, s_i)$ for all $i, \widehat{v}_i, s_i$:

$$t_i(\widehat{v}_i, s_i) \triangleq \begin{cases} 0, & \text{if } W_i(\widehat{v}_i, s_i) < 0; \\ W_i(\widehat{v}_i, s_i), & \text{if } W_i(\widehat{v}_i, s_i) \geq 0; \end{cases} \tag{25}$$

and thus

$$\xi_i(t, \widehat{v}_i, s_i) = \begin{cases} 0, & \text{if } t < t_i(\widehat{v}_i, s_i); \\ s_i, & \text{if } t \geq t_i(\widehat{v}_i, s_i). \end{cases} \tag{26}$$

We use the static payments (24) in the ascending mechanism, but only via the (conditioning) information available at the stopping time τ. The individual exit times of the losing bidders, $\tau_j \leq \tau$, implicitly define the reported signal realizations $\widehat{s}_j$ via (25), namely:

$$W_j(\widehat{v}_j, \widehat{s}_j) = \tau_j.$$

Thus, the winning bidder i pays for all realizations of s_i above the threshold $\underline{s_i}(\widehat{\mathbf{v}}, \widehat{\mathbf{s}}_{-i})$, and we define the transfer function $P_i(\widehat{\mathbf{v}}, \widehat{\mathbf{s}}_{-i})$ by:

$$P_i(\widehat{\mathbf{v}}, \widehat{\mathbf{s}}_{-i}) \triangleq \mathbb{E}\left[p_i(\widehat{\mathbf{v}}_{-i}, \widehat{\mathbf{s}}) \,\middle|\, s_i \geq \underline{s_i}(\widehat{\mathbf{v}}, \widehat{\mathbf{s}}_{-i})\right]. \tag{27}$$

In particular, this implies that the winning bidder pays in expectations as much as he does in the static mechanism with observable signals.

If we consider the allocation and payment rules, as encoded by (22) and (24), then it is apparent that all the decisions with respect to bidder i, whether they concern the disclosure of information or the allocation, only depend on the competing bidders in a very limited way; namely via the largest virtual utility among the competing bidders. Thus, to the extent that the other bidders are truthtelling, a sufficient statistic of the profile $(\mathbf{v}_{-i}, \mathbf{s}_{-i})$ is the resulting maximal virtual utility

$$w(\mathbf{v}_{-i}, \mathbf{s}_{-i}) \triangleq \max_{j \neq i} \{W_j (v_j, s_j), 0\}.$$

It follows that to verify the posterior incentive and participation constraints of bidder i, it is entirely sufficient to represent the competitors via a distribution of competing (maximal) virtual utilities w, which we denote by $G(w)$. For the remainder of this section, it will therefore be sufficient to consider a single agent competing against a virtual valuation w. In consequence we can drop the subscripts everywhere and rewrite the relevant notation, in particular (24) and (23), as:

$$\underline{s}(\widehat{v}, w) \triangleq \min \{s | W(\widehat{v}, s) \geq \max\{w, 0\}\}, \tag{28}$$

and

$$\underline{v}(s, w) \triangleq \min \{v | W(v, s) \geq \max\{w, 0\}\}. \tag{29}$$

Consequently, the transfer payment given by (24) can be written as:

$$p(s, w) \triangleq u(\underline{v}(s, w), s), \tag{30}$$

where the transfer has a Vickrey property with respect to v but not with respect to s.

Now, as s is not observable in the ascending disclosure mechanism, if the bidder with a reported type $\widehat{v}$ wins against the virtual valuation of w, then his true signal s has to be sufficiently high, namely $s \geq \underline{s}(\widehat{v}, w)$, and the transfer payment is formed by the conditional expectation:

$$P(\widehat{v}, w) \triangleq \mathbb{E}[p(s, w) | s \geq \underline{s}(\widehat{v}, w)] = \frac{1}{1 - \underline{s}(\widehat{v}, w)} \int_{\underline{s}(\widehat{v}, w)}^{1} u(\underline{v}(s, w), s) \, ds, \tag{31}$$

where here and in all future integral expressions, we use the property that s is uniformly distributed on the unit interval, see (1). By the construction of the payment $P(\widehat{v}, w)$ in (31), it follows that

$$p(\underline{s}(\widehat{v}, w), w) \leq P(\widehat{v}, w), \tag{32}$$

as well as

$$u(\widehat{v}, \underline{s}(\widehat{v}, w)) - P(\widehat{v}, w) \leq 0, \tag{33}$$

where we note that by construction $\widehat{v} = \underline{v}(\underline{s}(\widehat{v}, w), w)$. For later use, we collect some properties of the threshold signal and the payment.

Lemma 2 (Payment and Signal Threshold)

1. *If $v > v'$, then $\underline{s}(v, w) < \underline{s}(v', w)$ for all w.*

2. *$p(s, w)$ is increasing in s and w.*

3. *$P(v, w)$ is increasing in w and decreasing in v.*

Proof. (1.) By Lemma 1.1, the virtual valuation is strictly increasing in v and s, and hence it follows that the signal thresholds $\underline{s}(\cdot, w)$ have to have the reverse ranking of v.

(2.) The transfer function $p(s, w)$ is given by $u(\underline{v}(s, w), s)$, see (30). By Lemma 1.3, it follows that if s is increasing, then $u(\underline{v}(s, w), s)$ is increasing as well. By Lemma 1.1, $W(v, s)$ is strictly increasing in v and s, and hence $\underline{v}(s, w)$ is increasing in w, and since $u(v, s)$ is increasing in v, the result follows.

(3.) For a given v, the transfer function $P(v, w)$, see (31), is defined as a conditional expectation over all signal realization s above a threshold $\underline{s}(v, w)$. This threshold is increasing in w by the monotonicity of $W(v, s)$, see Lemma 1.1. But by the previous argument, (2.), $p(s, w)$ is increasing in both s and w, and hence the conditional expectation over $p(s, w)$ is increasing in w. After all, an increase in w raises the expectation, given that the function $p(s, w)$ is increasing in s for a given w, but also the function $p(\cdot, w)$ is shifted upwards by a shift in w.

For a given w, the transfer function $P(v, w)$, is defined as a conditional expectation over all signal realization s above a threshold $\underline{s}(v, w)$. This threshold is decreasing in v by the monotonicity of $W(v, s)$, see Lemma 1.1. But by the previous argument, (2.), $p(s, w)$ is increasing in s, and hence the conditional expectation over $p(s, w)$ is decreasing in v. ■

4.3 Posterior Implementation

We now establish that the ascending disclosure mechanism leads to truthtelling with respect to v and s. This will establish our main result:

Proposition 2 (Posterior Implementation)
The ascending disclosure mechanism satisfies the posterior incentive and participation constraints for all agents. The seller extracts as much revenue as in the revenue maximizing auction with observable signals.

The proof proceeds in several steps. We show in Lemma 3 that if the bidder reports both his type and his signal truthfully, then he obtains the same allocation and expected utility as in the revenue maximizing mechanism of Eső and Szentes (2007). In Lemma 4 we show that if the bidder reports his type v truthfully, then he will also report his signal s truthfully, that is he will exit the process as soon as he learns his true signal s. Then, Lemma 5 establishes that if the bidder reports his signal s truthfully, he will also report his type v truthfully. The final step of the argument, presented in Lemma 6, shows that lying both with respect to the type and the signal is not profitable either.

Lemma 3 (Revenue Equivalence)
Given truthtelling of (v, s)*, the allocation and the expected net utility is identical to the revenue maximizing mechanism with observable signals.*

Proof. The equivalence follows directly from the stipulated behavior at (29) and the expected payment stipulated by (30). In the static mechanism a bidder with type v obtains

$$\int_0^1 \left[\int_0^{\max\{0, W(v,s)\}} \left[u(v, s) - u(\underline{v}(s, w), s) \right] dG(w) \right] ds. \tag{34}$$

In the present sequential procedure, the bidder with type v obtains:

$$\int_0^1 \left[\int_{\underline{s}(v,w)}^1 \left[u(v, s) - u(\underline{v}(s, w), s) \right] \right] ds dG(w). \tag{35}$$

The equivalence of (34) and (35) now follows after exchanging the order of integration. ■

We can now verify that every agent reports his information truthfully in equilibrium.

Lemma 4 (Truthful Signal Report)
Given truthtelling of type v*, the bidder is truthtelling about signal* s*.*

Proof. Suppose the sequential procedure reaches w and $s > \underline{s}(v,w)$, then we assign the object to the bidder and ask him to pay:

$$P(v,w) = \frac{1}{1-\underline{s}(v,w))}\int_{\underline{s}(v,w)}^{1} u\left(\underline{v}\left(s,w\right),s\right) ds,$$

and since he does not know the signal realization s either, the expected net utility is

$$\frac{1}{1-\underline{s}(v,w))}\int_{\underline{s}(v,w)}^{1} \left[u\left(v,s\right)-u\left(\underline{v}\left(s,w\right),s\right)\right] ds. \tag{36}$$

But since the virtual utility is increasing in s, see Lemma 1, it follows that

$$\frac{\partial \underline{v}\left(s,w\right)}{\partial s} < 0,$$

and hence for all $s > \underline{s}(v,w)$

$$u\left(v,s\right)-u\left(\underline{v}\left(s,w\right),s\right) > 0,$$

since $v > \underline{v}(s,w)$, and thus the bidder expects a positive utility, and is staying in the auction.

On the other hand, suppose he were to learn that his true signal is $s = \underline{s}(v,w)$, then he would quit the auction immediately, because his expected utility if he were to win at some later point $w' \geq w$ is given by

$$\begin{aligned} & u\left(v,\underline{s}(v,w)\right)-P(v,w') \leq u\left(v,\underline{s}(v,w)\right)-P(v,w) \\ = \; & u\left(v,\underline{s}(v,w)\right)-\int_{\underline{s}(v,w)}^{1} u\left(\underline{v}\left(s,w\right),s\right)\frac{ds}{1-\underline{s}(v,w)} \leq 0. \end{aligned}$$

Now,

$$u\left(v,\underline{s}(v,w)\right)-u\left(\underline{v}\left(s',w\right),s'\right) < 0,$$

since with $s' > \underline{s}(v,w)$ and $v' < v$ such that $u\left(v,\underline{s}(v,w)\right) = u\left(v',s'\right)$, $W\left(v,\underline{s}(v,w)\right) > W\left(v',s'\right)$, by Lemma 1.2. But this means that $\underline{v}\left(s',w\right) > v'$, and hence

$$u\left(v,\underline{s}(v,w)\right)-u\left(\underline{v}\left(s',w\right),s'\right) < u\left(v,\underline{s}(v,w)\right)-u\left(v',s'\right) = 0,$$

which completes the proof. ■

We are now in a position to verify that, conditional on reporting truthfully in the ascending auction, each bidder is also willing to report truthfully about his type v.

Lemma 5 (Truthful Type Report)

Given truthtelling of the signal s, the bidder is truthtelling about his type v.

Proof. Suppose for now that the bidder knows the value of w. Suppose also that the bidder misreports $\widehat{v} \neq v$ but continues to report his signal truthfully, that is he exits whenever his signal s has been disclosed to him, i.e. $d\left(t, v, \widehat{v}, \xi\left(t, \widehat{v}, s\right)\right) = 0$ if and only if $\xi\left(t, \widehat{v}, s\right) > 0$. Then, the agent will fail to win the object if $s < \underline{s}(\widehat{v}, w)$, which happens with probability $\underline{s}(\widehat{v}, w)$. Now, if $\underline{s}(\widehat{v}, w) = 1$, then the proof is complete, since in this case this deviation yields a zero net payoff, and thus not profitable. Now suppose that $\underline{s}(\widehat{v}, w) < 1$. The agent wins the auction if $s \geq \underline{s}(\widehat{v}, w)$ which happens with probability $1 - \underline{s}(\widehat{v}, w)$, in which case he pays

$$\frac{1}{1 - \underline{s}(\widehat{v}, w)} \int_{\underline{s}(\widehat{v}, w)}^{1} u(\underline{v}(s, w), s) \mathrm{d}s.$$

Therefore, his *ex-ante* expected payment is

$$\int_{\underline{s}(\widehat{v}, w)}^{1} u(\underline{v}(s, w), s) \mathrm{d}s.$$

His *ex-ante* gross utility derived from the object is

$$\int_{0}^{\underline{s}(\widehat{v}, w)} 0 \mathrm{ds} + \int_{\underline{s}(\widehat{v}, w)}^{1} u(v, s) \mathrm{d}s,$$

so that his *ex-ante* net expected utility is

$$\int_{\underline{s}(\widehat{v}, w)}^{1} \left[u(v, s) - u(\underline{v}(s, w), s)\right] \mathrm{d}s. \tag{37}$$

Note that $u(v, s) - u(\underline{v}(s, w), s) \geq 0$ if and only if $v \geq \underline{v}(s, w)$, and in turn if and only if

$$W(v, s) \geq W(\underline{v}(s, w), s) = W(v, \underline{s}(v, w))$$

if and only if $\underline{s}(v, w) \leq s$. Therefore, the integral (37) is maximized if it is performed only on the interval on which the integrand is non-negative, which is by construction $[\underline{s}(v, w), 1]$. In other words, setting $\widehat{v} = v$ maximizes this integral. Since this holds for any w, it must also hold in expectation over all w. ■

For further analysis it is worth noting that the above proof establishes that reporting the true type is not just optimal in expectation over all possible competing virtual valuations w, but in fact for each realization of the virtual valuation w. The initial report $\hat{v}$ determines the speed by which the bidder runs through his signals. Now, for every w, an overreport is associated with a lower threshold for the critical signal $\underline{s}(v,w)$ by Lemma 2.1: $\hat{v} > v \Leftrightarrow \underline{s}(\hat{v},w) < \underline{s}(v,w)$. Similarly, for every w, an underreport is associated with a higher threshold for the critical signal $\underline{s}(v,w)$ by Lemma 2.1: $\hat{v} < v \Leftrightarrow \underline{s}(\hat{v},w) > \underline{s}(v,w)$. Thus, if the bidder overreports his type, $\hat{v} > v$, the disclosure process ends earlier for the bidder, as the threshold for the disclosed signals s is lower, $\underline{s}(\hat{v},w) < \underline{s}(v,w)$. Thus, the bidder receives less private information, than if he were to report truthfully. By contrast, if the bidder underreports his type, $\hat{v} < v$, then the disclosure process ends later for the bidder, as the threshold for the disclosed signals is higher, $\underline{s}(\hat{v},w) > \underline{s}(v,w)$. The initial reporting strategy of the bidder therefore influences the amount of private information that he will receive in the disclosure process. But the next result establishes that the advantage of increasing or decreasing the information is offset by less favorable transfer payments associated with underreports and overreports, respectively.

Lemma 6 (Joint Deviations)
The bidder cannot increase his utility by overreporting $\hat{v} > v$ or by underreporting $\hat{v} < v$.

Proof. We fix w and consider the utility the bidder obtains as a function of his own signal s, if observed. We claim that for any misreport, the bidder obtains a lower utility for every w than he would have obtained reporting his true type. We begin with overreporting $\hat{v} > v \Leftrightarrow \underline{s}(\hat{v},w) < \underline{s}(v,w)$. It is useful to consider two separate cases, and thus let

$$V^{+} \triangleq \{v \in [0,1] | \widehat{v} \geq v \text{ and the bidder wants the object upon learning that } s > \underline{s}(\widehat{v},w)\}, \quad (38)$$

and conversely let

$$V^{-} \triangleq \{v \in [0,1] | \widehat{v} \geq v \text{ and the bidder rejects the object upon learning that } s > \underline{s}(\widehat{v},w)\}. \quad (39)$$

Note that the agent prefers to receive the object upon learning that $s > \underline{s}(\widehat{v},w)$ if and only if

$$\mathbb{E}\left[u(v,s) - P(\widehat{v},w) | s > \underline{s}(\widehat{v},w)\right] = \frac{1}{1-\underline{s}(\widehat{v},w)} \int_{\underline{s}(\widehat{v},w)}^{1} \left[u(v,s) - u(\underline{v}(s,w),s)\right] \mathrm{d}s \geq 0.$$

Now suppose that $\widehat{v} \in V^{-}$ and the agent learns that $s \leq \underline{s}(\widehat{v},w)$, and hence $s \leq \underline{s}(\hat{v},w) < \underline{s}(v,w)$, then it is optimal for the bidder to exit after s has been revealed. After all, by overreporting $\hat{v} > v$, it follows that $u(\hat{v},s) > u(v,s)$, for all s. But if $s \leq \underline{s}(\hat{v},w)$, then by (33),

$u(\hat{v}, s) - P(\hat{v}, w) < 0$, and hence it follows that $u(v, s) - P(\hat{v}, w) < 0$ as well, and thus exit is an optimal response, with the resulting zero net expected utility. If $\widehat{v} \in V^-$ and $s > \underline{s}(\widehat{v}, w)$, then the agent will refrain from claiming the object by construction of (39), as well. Therefore, any deviation $\widehat{v} \in V^-$ is unprofitable.

Now suppose that $\widehat{v} \in V^+$. Again, if $s \leq \underline{s}(\widehat{v}, w)$, then the agent will *truthfully* refrain from claiming the object. If $s > \underline{s}(\widehat{v}, w)$, then he will *truthfully* claim the object by construction. Therefore, if $\widehat{v} \in V^+$, then the bidder will optimally report his signal truthfully in the second stage for any realization of the signal. Now if $\widehat{v} \in V^+ \setminus \{v\}$ would constitute a strictly profitable deviation, then we would have established a contradiction to Lemma 5, which established the optimality of truthtelling of the type, given thruthtelling of the signal.

Next consider the case of underreporting: $\hat{v} < v \Leftrightarrow \underline{s}(\hat{v}, w) > \underline{s}(v, w)$. This implies that the bidder will learn more as compared to the case where he reported truthfully. If the signal s is sufficiently small, then $s \leq \underline{s}(v, w) < \underline{s}(\hat{v}, w)$. Now, we observe that if the true signal had been $s = \underline{s}(v, w)$, then the bidder would not want to receive the object if offered at $P(v, w)$, since

$$u(v, \underline{s}(v, w)) - P(v, w) < 0,$$

and by Lemma 2.3, $P(\hat{v}, w) > P(v, w)$, and a fortiori would want to drop out of the auction. Suppose then that the true signal s is sufficiently large, or $s > \underline{s}(v, w)$. Now, there must exist a signal $\tilde{s}$ with $\underline{s}(v, w) < \tilde{s} \leq \underline{s}(\hat{v}, w)$ such that the bidder buys the good (for the given w) if and only if his true signal is above $\tilde{s}$. Now, consider a type $\tilde{v}$ with $W(\tilde{v}, \tilde{s}) = w$. By construction, the bidder who underreported v to $\hat{v}$ obtains the object for the same set of signals as the truthful type $\tilde{v}$ would have. Note, however, that the payment of the type v who underreports to $\hat{v}$ is larger than the payment of the $\tilde{v}$ type, again by Lemma 2.3. So, the utility of the bidder with type v, who underreports with regard to his type, and then behaves optimally with regard to his reported signal is smaller than if the bidder still underreported to $\tilde{v}$ and then reported his signal truthfully. But given Lemma 5, even the resulting net utility is smaller than the bidder would obtain if he were to report his type truthfully. Thus underreporting is not profitable either. ■

4.4 Single Agent Example

We conclude this section with an example of a single buyer with a utility function that is additive in the type v and the signal s. The example is meant to illustrate the impact of the sequential information disclosure on the reporting incentives and the structure of payments. We also illustrate how the transfers in the static disclosure environment of Eső and Szentes (2007) compare with the transfers in the sequential disclosure environment.

Thus, we assume that the valuation of the single bidder/buyer is determined by:

$$u(v,s) = v + s,$$

with v uniformly distributed on $[0,1]$ and s uniformly distributed on $[-1,1]$. (The support assumption on the signal s, and consequently the valuation $u(v,s)$ here does not agree with the normalization to the unit interval in the earlier sections, but this is without consequence and merely assists in computing the present example.) The signal s therefore has an expected value of 0 and can readily be interpreted as the incremental information over and above the initial type v.

Selling with an Observable Signal We saw earlier in (21) that the buyer should receive the good whenever his virtual utility is nonnegative, or:

$$W(v,s) = u(v,s) - \frac{1-F(v)}{f(v)} u_1(v,s) = 2v + s - 1 \geq 0. \tag{40}$$

Thus, in particular, if the signal s were observable by the seller, then the bidder should receive the object if and only if

$$2v + s - 1 \geq 0 \Leftrightarrow v \geq \frac{1-s}{2}. \tag{41}$$

We observe that even though type v and signal s receive the same weight in the valuation of the buyer, the virtual valuation is biased towards the initial type v as the information rent of the buyer only arises due the type v. In the presence of an observable signal s, the incentive compatible payment, essentially a posted price, is:

$$p(s) = \frac{1-s}{2} + s = \frac{1+s}{2}.$$

In consequence, the net utility of the buyer, conditional on receiving the object at a given realization of v and s is:

$$u(v,s) - p(s) = v + s - \frac{1+s}{2} = v - \frac{1-s}{2}. \tag{42}$$

The latter is always nonnegative, given (41), and hence satisfies the interim participation constraint of the buyer. Moreover, the interim expected utility of a buyer with type v is given by

$$\begin{aligned} \mathbb{E}\left[u(v,s) - p(s)\,|v\right] &= \mathbb{E}\left[\left[u(v,s) - p(s)\right] \times \mathbb{I}\left\{W(v,s) \geq 0\right\}|v\right] \\ &= \int_{1-2v}^{1} \frac{1}{2}\left(v - \frac{1-s}{2}\right) ds = \frac{1}{2}v^2. \end{aligned} \tag{43}$$

Static Disclosure: The Handicap Sale In the presence of private information of the signal s, Eső and Szentes (2007) suggest a handicap auction which implements the revenue maximizing outcome. With a single bidder, the case we study here, the handicap auction works like a menu of options. More precisely, the seller offers the buyer a menu of option contracts, where each option contract is specified by a pair (f,p), namely an initial fee, denoted by f, and an associated strike price, denoted by p, to acquire the object. In the direct revelation mechanism, the fee f and the strike price are determined as function of the reported type v only, and the signal s simply controls whether the buyer exerts the option at the given strike price p or not.[9]

The handicap auction implements the revenue maximizing allocation by choosing the strike price p so that the agent buys the object if and only if the virtual utility is nonnegative, $W(v,s) \geq 0$. This determines the strike price for a given v as the bidder should receive the object if and only if his virtual valuation is larger than zero, or $v + s \geq 1 - v$, and thus

$$p(v) \triangleq 1 - v. \tag{44}$$

The associated fee $f(v)$ for the option to strike at price $p(v) = 1 - v$, the price of the option, is then given by:

$$f(v) \triangleq \frac{1}{2}v^2. \tag{45}$$

[9] In Eső and Szentes (2007), the additive environment appears in Example 1 to illustrate payoff environments that satisfy the relevant monotonicity conditions. The additive environment reappears in the discussion of the handicap auction, which they solve explicitly for the case of v being distributed uniformly and s being distributed logistically. Here, we solve the symmetric environment where v and s are both uniformly distributed.

The option price $f(v)$ is determined by the revenue maximization problem of the seller, subject to the incentive constraint that the buyer is purchasing the "right" option:

$$\int_{-1}^{1} \max\{v+s-p(v),0\}\frac{1}{2}ds - f(v) \geq \int_{-1}^{1} \max\{v+s-p(\widehat{v}),0\}\frac{1}{2}ds - f(\widehat{v}), \text{ for all } v, \widehat{v}.$$

We find from (44) and (45), that a buyer with a high type is paying a high up front fee $f(v)$ in exchange for a lower strike price $p(v)$.

By the taxation principle, we can achieve the outcome of the direct mechanism, the handicap auction, with an indirect mechanism, namely a menu of option contracts, $\{f, p(f)\}$, in which a higher initial fee purchases a lower strike price, with $p(f) = 1 - \sqrt{2f}$, or conversely, where a lower strike price requires a higher option fee, $\{f(p), p\}$, with $f(p) = (1-p)^2/2$.

Thus, in the handicap auction, the buyer first purchases the option, and then possibly pays the strike price to receive the object. Now, clearly, since the fee is nonnegative, $f(v) \geq 0$, if eventually the buyer fails to acquire the object, then his net utility is negative, and the ex post participation constraint is violated. But, even if the buyer acquires the object, he may not break even, as the sum of the payments $f + p$ may exceed his valuation, that is

$$u(v,s) - p(v) - f(v) \leq 0,$$

as the buyer will pick up the option, but not break even as long as

$$1 - 2v \leq s \leq 1 + \frac{1}{2}v^2 - 2v.$$

Thus, in the event of a failure to purchase, or a failure to receive a sufficiently high signal s, the ex post participation constraint of the buyer will be violated.

Sequential Disclosure: The Ascending Sale We now contrast the static disclosure with the sequential disclosure. In the sequential disclosure mechanism, the buyer initially reports a type v. As a function of the initial report v, the sequential disclosure mechanism determines how much the buyer will learn about his signal s. We showed earlier that the sequential mechanism assigns the object to the buyer if and only if the virtual utility is positive, and hence by (40), $s \geq 1 - 2v$. It follows that the mechanism sequentially discloses all signals below the critical value $\underline{s}(v)$ with

$$\underline{s}(v) \triangleq 1 - 2v. \tag{46}$$

Thus if his true signal s is below $\underline{s}(v)$, then the buyer will learn the value of s exactly, but in contrast, if his true signal s is above $\underline{s}(v)$, then he only learns this conditional information. Thus, if the disclosure mechanism reveals that the signal s satisfies $s \geq \underline{s}(v)$, then it will stop and offer the object to the buyer at a price denoted by $P(v)$. The price $P(v)$ takes the expectation over all s, conditional on $s \geq \underline{s}(v)$.

We established in (42) that the incentive compatible price given an observable s is given by:

$$p(s) = \frac{1+s}{2},$$

and thus the price $P(\widehat{v})$, conditional on the reported type $\widehat{v}$ takes the average:

$$P(\widehat{v}) \triangleq \frac{1}{\Pr(s \geq \underline{s}(\widehat{v}))} \int_{\underline{s}(\widehat{v})}^{1} p(s) \frac{1}{2} ds = \frac{1}{\widehat{v}} \int_{1-2\widehat{v}}^{1} \frac{1+s}{2} \frac{1}{2} ds = 1 - \frac{1}{2}\widehat{v}. \tag{47}$$

We then find from (46) and (47) that the initial report v affects the amount of disclosed information *and* the purchase price of the object. The disclosed information decreases in the initial report v, and hence lower initial reports lead to more disclosed information. But, the additional information is only obtained in exchange for a higher purchase price as $P(v)$ is decreasing in v as well, $P'(v) = -1/2 < 0$. Thus, a lower report "buys" more information and hence increases the informativeness of the mechanism, but at the cost of a higher price conditional on the transaction. This is the option character of the initial report. With his initial report the buyer then determines how much more he will learn about his true valuation and ultimately pay for the additional information with a higher purchase price. The expected utility of the truthtelling bidder, conditional on receiving the object is given by:

$$\begin{aligned} \mathbb{E}\left[u(v,s) - P(v) \,|v, s \geq \underline{s}(v)\right] &= \mathbb{E}\left[u(v,s)\,|v, s \geq \underline{s}(v)\right] - P(v) \\ &= \frac{1}{\Pr(s \geq \underline{s}(v))} \int_{\underline{s}(v)}^{1} (s+v) \frac{1}{2} ds - \left(1 - \frac{1}{2}v\right) \\ &= 1 - \left(1 - \frac{1}{2}v\right) = \frac{1}{2}v, \end{aligned}$$

which is nonnegative for every $v \geq 0$, and hence satisfies the posterior participation constraint of the buyer everywhere. And of course, as the interim probability of winning with a type v is given by $\Pr(s \geq \underline{s}(v)) = v$, it follows that the interim expected utility is given by $v^2/2$, confirming the earlier result of (43).

Finally, with the guidance of Lemma 4 - 6, it is straightforward to establish that it is indeed optimal to tell the truth in the sequential disclosure mechanism. In particular, the parametric nature of the example allows us to explicitly calculate the value of the misreporting strategies and to establish that none of them improves on truthtelling.

5 Conclusions

We extend the canonical mechanism design to allow the seller to also control the information that the bidder can receive about the object during the bidding contest. We exhibited a sequential disclosure mechanism associated with a sequential bidding mechanism which allowed the seller to extract almost the entire surplus of the allocation. The information rent of each bidder is restricted to the private information that each bidder was endowed with before entering the auction. The sequential disclosure process allowed us to assign the object in such a way as to maintain the posterior incentive and participation constraints of all the bidders. The disclosure mechanism allowed each bidder to obtain a sufficient amount of private information to find out whether his virtual valuation is larger or smaller than those of their competitors. Importantly, the winning bidder only learns the lower bound of his virtual utility, but never his exact valuation nor others' virtual valuations. This was achieved by informing the bidders in each round whether their valuations are below or above a given threshold. The threshold was increased in every round and thus the losing bidders learn that their valuations are below the threshold at the moment at which they exit the process. The winning bidder has to pay a price which is larger than if the signal were known to equal exactly the threshold value but smaller than (or in case with no private information equal to) his expected valuation.

References

BERGEMANN, D., AND M. PESENDORFER (2007): "Information Structures in Optimal Auctions," *Journal of Economic Theory*, 137, 580–609.

BLUMENTHAL, M. (1988): "Auctions with Constrained Information: Blind Bidding for Motion Pictures," *Review of Economics and Statistics*, 70, 191–198.

CASSADY, R. (1967): *Auctions and Auctioneering*. University of California Press.

COURTY, P., AND H. LI (2000): "Sequential Screening," *Review of Economic Studies*, 67, 697–717.

DEMSETZ, H. (1968): "Why Regulate Utilities?," *Journal of Law and Economics*, 11, 55–65.

ESŐ, P., AND B. SZENTES (2007): "Optimal Information Disclosure in Auctions," *Review of Economic Studies*, 74, 705–731.

GERSHKOV, A. (2002): "Optimal Auctions and Endogenous Information Structure," Discussion paper, Hebrew University, Jerusalem.

——— (2009): "Optimal Auctions and Information Disclosure," *Review of Economic Design*, 13, 335–344.

GREEN, J., AND J. LAFFONT (1987): "Posterior Implementability in a Two Person Decision Problem," *Econometrica*, 55, 69–94.

GRIGORIEVA, E., P. HERINGS, R. MÜLLER, AND D. VERMEULEN (2007): "The Private Value Single Item Bisection Auction," *Economic Theory*, 30, 107–118.

KAMENICA, E., AND M. GENTZKOW (2011): "Bayesian Persuasion," *American Economic Review*, 101, 2590–2615.

KAVAJECZ, K., AND D. KEIM (2005): "Packaging Liquidity: Blind Auctions and Transaction Cost Efficiencies," *Journal of Financial and Quantitative Analysis*, 40, 465–492.

Kenney, R., and B. Klein (1983): "The Economics of Block Booking," *Journal of Law and Economics*, 26, 497–540.

Krähmer, D., and R. Strausz (2011): "The Benefits of Sequential Screening," University of Bonn and Humboldt University of Berlin.

Li, H., and X. Shi (2013): "Discriminatory Information Disclosure," Discussion paper, University of British Columbia and University of Toronto.

Loeb, M., and W. Magat (1979): "A Decentralized Method of Utility Regulation," *Journal of Law and Economics*, 22, 399–404.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58–73.

Pavan, A., I. Segal, and J. Toikka (2011): "Dynamic Mechanism Design: Revenue Equivalence, Profit Maximization, and Information Disclosure," Discussion paper, Northwestern University and Stanford University.

Ye, L. (2007): "Indicative Bidding and a Theory of Two-Stage Auctions," *Games and Economic Behavior*, 58, 181–207.

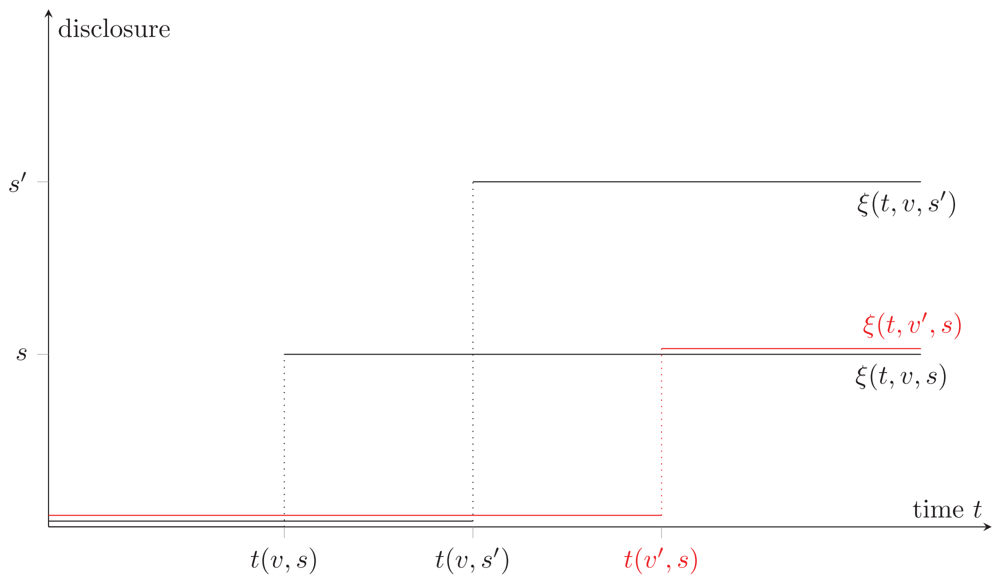

Figure 1: Disclosure function and disclosure time.