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# BIOLOGY AND THE ARGUMENTS OF UTILITY\*

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PRELIMINARY

## Abstract

Why did evolution not give us a utility function that is offspring alone? Why do we care intrinsically about other outcomes, such as food, and what determines the intensity of such preferences? A common view is that such other outcomes enhance fitness and the intensity of our preference for a given outcome is proportional to its contribution to fitness. We argue that this view is incomplete. Specifically, we show that in the presence of informational asymmetries, the evolutionarily most desirable preference for a given outcome is determined not only by the significance of the outcome, but by the Agent's degree of ignorance regarding its significance. Our model also sheds light on the phenomena of peer effects and prepared learning, whereby some peer attitudes are more influential than others.

KEYWORDS: Utility, Biological Evolution.

JEL Codes: D01, D80

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# 1 Introduction

Despite a steady trickle of research on the issue over the last 20 years, it remains unconventional to consider the biological basis of utility or preferences.<sup>1</sup> This approach holds out the promise of generating a utility function with the key economic properties of being exogenous and invariant. At first blush, such a biologically derived utility would value commodities in accordance with their impact on fitness—we should value food, for example, in terms of its marginal contribution to biological success. However, on reflection, a serious conceptual problem arises—why have we been made to care about *anything other* than offspring?<sup>2</sup> Why, that is, if we are rational and intelligent, are we not programmed to like *only* offspring and then to treat food, for example, as purely instrumental, as a means to an end? There seems little doubt that there are many biologically intermediate goods that are nevertheless valued intrinsically. In modern times, indeed, we readily sacrifice expected offspring to increase consumption of other commodities. The recent “demographic transition,” during which incomes rose but fertility fell is dramatic *prima facie* evidence on this score.

We consider a solution to this conundrum in terms of information. On the one hand, there are relevant aspects of the environment that are “recurrent signals” in the sense of having a long evolutionary history. For example, sunlight has long had an effect in aiding Vitamin D production and thereby enhancing health and fitness.<sup>3</sup> Nature then has had the opportunity to incorporate a liking for the sun in the utility function. On the other hand, there are relevant aspects of the environment that are “transient signals” in the sense of being local in time and space, and having arisen only rarely before, or perhaps never. These signals might concern likely locations in which to sunbathe, for example. Natural selection could not have been brought to bear directly on these aspects of the environment.

We assume that utility is shaped by Nature in the light of the recurrent signals only. The individual maximizes the expectation of his utility, conditioning on both the recurrent and transient signals, but using arbitrary beliefs. We derive two general results. First, despite Nature’s inability to anticipate the transient signal, and despite the Individual’s holding arbitrary beliefs, the equilibrium action chosen by the Individual is the best possible in the sense of maximizing expected fitness conditional on both signals. Second, the resulting marginal utility of an action is determined not by the marginal contribution of that action

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<sup>1</sup>Robson (2001) argues that a utility function serves as a method for Nature to partially decentralize control, thus achieving a flexible response to novelty.

<sup>2</sup>The number of offspring is assumed to be appropriately adjusted for quality throughout the paper.

<sup>3</sup>The full importance of Vitamin D was only recently recognized.

to fitness, but by the Individual’s degree of ignorance regarding such contribution.

Our theoretical approach is an instance of the principal-agent problem. In this interpretation, the principal (Nature), maximizes the productivity of the agent (Individual) by choosing the agent’s utility function. While the principal-agent perspective is illuminating, there are no formal techniques or results that can be directly imported from the existing literature.<sup>4</sup> In fact, the specific principal-agent problem we consider is not meaningful in conventional economic applications. Most significantly, we assume that (i) the principal has the power to fully shape the agent’s preferences, (ii) all actions of the agent are contractible, and (iii) the principal has information that cannot be directly communicated to the agent, despite the parties having parallel interests in the information.

A paper that can be described in analogous terms is Samuelson & Swinkels (2006), who also consider an environment in which both Nature and the Individual possess relevant information. In an ideal case, the Individual would maximize expected fitness. In their work, unlike ours, there is an emphasis on second-best solutions that provide a rationale for behavioral biases.

Our focus throughout is on “primary” rather than “secondary” arguments of utility. That is, we consider arguments that are desired as ends in themselves rather than as means to an end. There are many primary arguments, of course. For example, Linden’s list includes food, prestige, temperature, a view, and sex (see Linden, 2011). Bentham lists 26 categories of “simple” pleasures and pains (Bentham, 1789). Perhaps the most salient example of a secondary argument is money, which is fundamentally only a means to an end from the perspective of the individual.<sup>5</sup>

The remainder of the paper is organized as follows:

Section 2 outlines the basic model and considers a different direct mechanism by which evolution could implement the optimal choice. We dismiss as unrealistic the possibility that the Individual’s beliefs match the true distribution. This leaves us with a mechanism in which Nature shapes utility in the light of her recurrent information and the individual maximizes expected utility conditional on arbitrary beliefs.

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<sup>4</sup>In spirit, ours is a model of delegation. See, for example, Holmstrom (1984), Aghion & Tirole (1997), Dessein (2002), Alonso & Matouschek (2008), and Armstrong & Vickers (2010).

<sup>5</sup>Not all of Bentham’s categories seem clearly primary. For example, he nominates wealth as a simple pleasure, but then defends this choice in terms of what the money can buy.

Since since money is a very familiar means, it induces a rather automatic response. It was once thought that the fMRI signature of money could not be distinguished from that of sex. However, Sescousse *et al.* (2010) show that money and sex have subtly distinct fMRI signatures, reflecting the instrumental role of money.

Section 3 considers the problem of existence of a utility function that guarantees first-best actions. Although existence is delicate in a continuous formulation, it is a straightforward question in a discrete approximation.

Section 4 relies on a continuous formulation while placing a monotone structure on the problem. We show that optimal choice can be generated by a utility function that has a simple additive form.

Section 5 considers various simple examples with quadratic fitness functions and normal distributions for the signals. These examples illustrate that the marginal utility of a given action depends on the precision of the recurrent signals relative to the precision of the transient ones. In a labor-leisure example, the perceived cost of effort may vary with the task, although its actual fitness cost does not.

Section 6 shows how the present framework can be readily adapted to consider the evolution of interdependent preferences. We derive a “desire for conformity.” In a simple example, we show how greater precision for the recurrent signal relative to the transient signal may lead to a greater weight being placed on another individual’s choice. This is of interest in light of the psychological literature on “prepared learning.”

Section 7 discusses a type of preference misalignment that is likely to arise in modern times owing to modern humans being more knowledgeable, relative to our hunter-gather ancestors, of the link between actions and fitness. We argue that when our ancestors underestimated (overestimated) the marginal fitness value of a given action, modern humans will select an excessively large (small) action relative to its fitness-maximizing level. Section 8 concludes.

## 2 Model

There are two players: a *principal* (Nature) and an *agent* (the Individual). The agent faces a one-shot opportunity to produce fitness  $y \in \mathbb{R}$  (representing quality-adjusted offspring). Fitness is a function of the Individual’s action  $x \in \mathbb{R}^N$  and an underlying state  $\sigma \in \mathbb{R}$ :

$$y = \varphi(x, \sigma).$$

The players have only partial knowledge of the state. That knowledge has two components:  $s \in \mathbb{R}$  and  $t \in \mathbb{R}$ . These components represent, respectively, recurrent and transient aspects of the environment. The essential distinction is that the utility function given to the

Individual can depend on the recurrent component,  $s$ , but not on the transient component,  $t$ . From Nature’s point of view, the (“true”) distribution of  $\sigma$  conditional on  $(s, t)$  is given by the pdf  $f(\sigma | s, t)$ ; from the Individual’s point of view, the (“subjective”) distribution of  $\sigma$  conditional on  $t$  is given by the pdf  $g(\sigma | s, t)$ .

A more detailed interpretation of  $s$  and  $t$  is as follows. On the one hand,  $s$  represents observed aspects of the environment whose implications have remained constant over evolutionary time.<sup>6</sup> For example,  $s$  might represent exposure to sunlight. Exposure to UV light, in particular, aids in the production of Vitamin D, which is essential for health. The pdf  $f$  incorporates the true effect of sunlight on fitness. The individual, however, can be largely or completely ignorant of this causal chain, having arbitrary beliefs as captured by the pdf  $g$ . The recurrent nature of this causal chain means, however, that Nature can shape the utility function in the light of  $s$ .

On the other hand,  $t$  represents observed aspects of the environment that may be important but have a one-off quality. For instance, a hunter observes the exact position and strength of his prey, as well as the current abundance of alternative sources of food. By observing these precise local conditions, in the language of Binmore (1994), the individual serves as Nature’s “eyes.” The rarity, or even complete novelty, of such an aspect means that Nature cannot shape utility in the light of  $t$ .

If  $t$  is transient, how can evolution generate a choice that is optimal given the true distribution conditional on  $t$  as well as  $s$ ? If  $t$  represents completely unrestricted novelty, there is no hope of attaining optimality. For this to be possible, there must be a suitable restriction on the novelty. Consider the following example. For simplicity, suppose there is no  $s$  signal, but only  $t$ . Further, suppose  $t$  is a perfectly accurate signal of the state,  $\sigma$ , which is the exact direction in which game is to be found. If the Individual hunts for game in that direction, choosing then an action  $x = t$ , fitness is 1; otherwise, it is zero. Since the exact direction is plausibly rare or entirely novel, the utility function under the present approach cannot be made conditional on  $t$ . It seems, nevertheless, highly plausible that an animal would evolve, via an appropriate reward for fitness, to hunt in the direction of the signal,  $t$ .

This scenario is then consistent with the present approach, in which Nature’s criterion

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<sup>6</sup>It need not be that  $s$  itself is unchanging over time. For example, Nature knows that, if you are warm, cold drinks would be helpful; if you are cold, hot. Nature makes the appropriate drink seem pleasurable in each circumstance. Kandel *et al.* (2000) describe the neurological basis of this phenomenon. This corresponds to conditioning upon an  $s$  that can have a number of possible realizations that vary over time.

employs the true distribution of the state given each signal, namely, that  $\sigma = t$ , and the optimal action is taken for each value of  $t$ .

The model proceeds in three stages:

1. Nature selects a utility function  $U(x, y, s)$  for the Individual which can depend on the as yet undetermined realization of  $s$  but not on that of  $t$ . The goal of the principal is to maximize the agent's true expected fitness, as expressed via the pdf  $f$ .
2. The signals  $s, t$  are realized.
3. The agent learns his utility function  $U$ , observes  $s, t$  and selects an action  $x$ . The goal of the agent is to maximize his expected utility conditional on the information available to him, as expressed in the pdf  $g$ .
4. The state  $\sigma$  is drawn and the payoffs of both players – fitness for the principal and utility for the agent – are realized.

We interpret this setting as a metaphor for the long-run outcome of an evolutionary process in which the utility functions of humans are heritable and are the object of natural selection. Over time, through a trial-and-error process, those individuals endowed with utility functions that best promote their own fitness dominate the population. Rather than explicitly modelling such trial-and-error process, we suppose Nature can directly “choose” a fitness-maximizing utility function for each human being. In this metaphor, Nature has had time to learn the recurrent features of the evolutionary environment (represented by  $s$ ). That is, evolution has had time to shape the utility function appropriately.

From the principal's perspective, the ideal choice of  $x$  solves

$$\max_x \mathbb{E}_f [y \mid s, t], \tag{1}$$

where  $\mathbb{E}_f$  means that the expectation is taken with respect to the true pdf  $f$ . For simplicity, we assume that, for each pair  $(s, t)$ , this problem has a unique solution, denoted  $x^*(s, t)$ . If a function  $U$  implements  $x^*(s, t)$  for all  $(s, t)$ , we say it is optimal.

*The Individual is fully informed*

If the Individual is fully informed, so that  $g = f$ , his objective is

$$\max_x \mathbb{E}_f [U(x, y, s) \mid s, t]. \tag{2}$$

A trivially optimal utility function is then

$$U(y) \equiv y,$$

which perfectly aligns the agent’s objective (2) with the principal’s objective (1).

Such resolution, however, is not a realistic description of humans. Most obviously, perhaps, we do not value only offspring intrinsically, viewing sex, for example, purely as an instrumental means to the end of producing more offspring. Less obviously, but perhaps more convincingly, consider how the experimental results of Wedekind *et al.* (1995) imply that our utility functions have arguments other than fitness. These results are that males with compatible immune systems appear to smell good to women. In the language of Barash (1979), a pleasant smell produces a “whispering within” that motivates them to select such mates. This amounts to this smell being an argument of utility. We are not born knowing that compatibility between parental immune systems is relevant to the fitness of offspring, or with any knowledge about how to check such compatibility.

*The Individual is not fully informed*

When the Individual holds arbitrary beliefs  $g$  his problem becomes

$$\max_x \mathbb{E}_g [U(x, y, s) | s, t] = \max_x \int U(x, y, s) g(\sigma | s, t) d\sigma,$$

where  $\mathbb{E}_g$  means that the expectation is taken with respect to the subjective pdf  $g$ , and where  $y = \varphi(x, \sigma)$ . Note that  $t$  affects the Individual’s decision exclusively through the conditional distribution of  $\sigma$ , whereas  $s$  serves also as a parameter of the utility function.

In the remainder of the paper, we restrict attention to this scenario. We will show that this mechanism is constrained optimal, for hunter-gatherers, so there would have been no selection pressure for modification.

### 3 First-Best Implementation – Finite Case

In general, a basic theoretical question is whether there exists a function  $\bar{U}$  that depends only on  $x, \sigma$ , and  $s$  and that satisfies the integral equation

$$\int \varphi(x, \sigma) f(\sigma | s, t) d\sigma = \int \bar{U}(x, \sigma, s) g(\sigma | s, t) d\sigma,$$



where the functions  $\varphi(x, \sigma)$ ,  $f(\sigma|s, t)$  and  $g(\sigma|s, t)$  have been specified exogenously. If  $y = \varphi(x, \sigma)$  were strictly monotonic in  $\sigma$ , for each  $x$ , then the existence of a function  $U(x, y, s)$  that implements the fitness-maximizing action  $x^*(s, t)$  for all  $s, t$  would be a consequence. However, the existence of a solution for  $\bar{U}(x, \sigma, s)$  to such a “Fredholm equation of the first kind” (Hochstadt (1973)) is a delicate issue.

The choice of a continuous formulation over a discrete one here is mainly a matter of convenience. Indeed, from a conceptual point of view, a discrete formulation seems unobjectionable. In such a formulation, existence can be readily addressed. Suppose, then, that  $\sigma$  and  $t$  are restricted to  $\{1, \dots, S\}$ . Given  $s$ , the problem is to find  $\bar{U}(x, \sigma, s)$  such that

$$\sum_{\sigma} \overbrace{\varphi(x, \sigma)}^{1 \times S} \overbrace{f(\sigma|s, t)}^{S \times S} = \sum_{\sigma} \overbrace{\bar{U}(x, \sigma, s)}^{1 \times S} \overbrace{g(\sigma|s, t)}^{S \times S}, \text{ for all } x.$$

This equation has a unique solution for the row vector  $\overbrace{\bar{U}(x, \sigma, s)}^{1 \times S}$  if and only if the matrix  $\overbrace{g(\sigma|s, t)}^{S \times S}$  is non-singular, which is a condition that holds generically.<sup>7</sup>

More generally,  $\sigma$  and  $t$  might be restricted to finite sets of different sizes,  $\{1, \dots, S\}$  and  $\{1, \dots, T\}$ . Perhaps the plausible alternative case is where the number of signals is less than the number of states, and so  $S > T$ . That is, there are more unknowns, as in  $\overbrace{\bar{U}(x, \sigma, s)}^{1 \times S}$ , than there are equations, where there is one for each signal,  $t$ . If the matrix  $\overbrace{g(\sigma|s, t)}^{S \times T}$  has full rank,  $T$ , then there is again no problem of existence; rather there is an issue of multiplicity—there are many solutions for  $\overbrace{\bar{U}(x, \sigma, s)}^{1 \times S}$ .

We have then proved a simple but illuminating result—

**Proposition 1** *Suppose  $S \geq T$ . If matrix  $\overbrace{g(\sigma|s, t)}^{S \times T}$  has full rank,  $T$ , there exists a solution for the row vector  $\overbrace{\bar{U}(x, \sigma, s)}^{1 \times S}$ .*

The approach we take below renders this discussion moot. We maintain the continuous

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<sup>7</sup>Generically in  $\mathbb{R}^{S^2}$ , the matrix  $\overbrace{g(\sigma|t)}^{S \times S}$  has a nonzero determinant. Now normalize each column by dividing by the sum of the entries in that column to obtain  $\overbrace{g(\sigma|t)}^{S \times S}$ . This normalization does not affect the determinant.

formulation, but we sidestep the most general issue of existence by imposing substantive additional structure. This structure is mainly intended to generate a tractable model, and to make available further results, but, as a side effect, it ensures existence.

## 4 A Monotone Environment

We revert to seeking an optimal utility function of the form  $U(x, y, s)$ . We first relax the requirement of matching the entire function  $\int \varphi(x, \sigma) f(\sigma|s, t) d\sigma$ , asking only that expected utility be maximized by  $x^*(s, t)$ . That is, we require only that, given  $s$ ,

$$x^*(s, t) = \arg \max_x \int \varphi(x, \sigma) f(\sigma|s, t) d\sigma = \arg \max_x \int U(x, y, s) g(\sigma|s, t) d\sigma, \text{ for all } t$$

This relaxation of the restrictions on utility is helpful. It is reasonable as well, since there would have been no biological selection that did more than this.

**Assumption 1** *i) The pdf  $f(\sigma|s, t)$  is continuously differentiable in  $(s, t)$  and strictly increasing, in the sense of first-order stochastic dominance, in  $s$  and in  $t$ .<sup>8</sup>*

*ii) The pdf  $g(\sigma|s, t)$  is continuously differentiable in  $(s, t)$  and strictly increasing, in the sense of first-order stochastic dominance, in  $s$  and in  $t$ .*

*iii) Fitness  $\varphi(x, \sigma)$  is twice continuously differentiable and strictly concave in  $x$ .<sup>9</sup>*

*iv) Increasing the state  $\sigma$  increases the marginal product of each action:  $\varphi_{x_i\sigma}(x, \sigma) > 0$ , for  $i = 1, \dots, N$ .*

*v) Actions are complements in that  $\varphi_{x_i x_j}(x, \sigma) \geq 0$ , for all  $i, j = 1, \dots, N, i \neq j$ .*

**Lemma 1** *Under Assumption 1,  $x_i^*(s, t)$  is differentiable and  $\frac{\partial x_i^*(s, t)}{\partial s} > 0$ ,  $\frac{\partial x_i^*(s, t)}{\partial t} > 0$ , for all  $i$ . Furthermore, if  $\hat{x}(s, t) = \arg \max_x \int \varphi(x, \sigma) g(\sigma|s, t) d\sigma$ , it similarly follows that  $\hat{x}_i(s, t)$  is differentiable and  $\frac{\partial \hat{x}_i(s, t)}{\partial s} > 0$ ,  $\frac{\partial \hat{x}_i(s, t)}{\partial t} > 0$ , for all  $i$ .*

**Proof.** See the Appendix. ■

### *The Main Result in the Monotone Environment*

In order to set the stage for this result, consider an arbitrary  $x \in \mathbb{R}^N$ . We associate to the component  $x_i$  the value of  $t \in \mathbb{R}$  such that the  $i$ th component of  $x^*(s, t)$  is  $x_i$ .

<sup>8</sup>Specifically, we require that  $\int v(\sigma) \frac{\partial f(\sigma|s, t)}{\partial s} d\sigma > 0$  for all continuous and strictly increasing functions  $v$ ; similarly for  $t$ .

<sup>9</sup>Specifically, we require that the matrix of second derivatives of  $\varphi(\cdot, \sigma)$  is everywhere negative definite.

**Definition 1** Let  $t^i : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  be such that  $x_i^*(s, t^i(x_i, s)) \equiv x_i$ , for all  $i$ .

We now demonstrate the existence of a simple utility function which is uniquely maximized at the optimal  $x$ .

**Theorem 1** Under Assumption 1, for all  $s, t$  the following utility function implements  $x^*(s, t)$ —

$$U(x, y, s) = y + \alpha(x, s) = \varphi(x, \sigma) + \alpha(x, s).$$

The “adjustment term” is

$$\alpha(x, s) = \sum_i \alpha^i(x_i, s)$$

where

$$\alpha^i(x_i, s) = - \int_0^{x_i} \int \varphi_{x_i}(x^*(s, t^i(z, s)), \sigma) g(\sigma | s, t^i(z, s)) d\sigma dz.^{10}$$

**Proof.** See Appendix. ■

This is the simplest utility function that delivers  $x^*(s, t)$ , in that  $\alpha(x, s)$  is deterministic (independent of  $\sigma$ ), additively separable from  $y$  and across the components of  $x$ . Note also that such an additive term must depend on both  $x$  and  $s$  and thus cannot be further simplified.

*Proof in One Dimension*

When  $x$  is one-dimensional, so  $N = 1$ , the first-order condition becomes

$$\int \varphi_x(x, \sigma) g(\sigma | s, t) d\sigma - \int \varphi_x(x^*(s, t(x, s)), \sigma) g(\sigma | s, t(x, s)) d\sigma = 0.$$

Since  $t(x, s)$  is the value of  $t$  that induces  $x$  as the solution to the constrained optimum, it follows that  $x^*(s, t(x, s)) = x$ . Hence if  $x = x^*(s, t)$  then  $t(x, s) = t$  and this first-order condition is satisfied. Further, if  $x < x^*(s, t)$ , then  $t(x, s) < t$ . Since  $g(\sigma | s, t)$  is increasing in  $t$  in the sense of first-order stochastic dominance, it follows that  $\int \varphi_x(x, \sigma) g(\sigma | s, t(x, s)) d\sigma > \int \varphi_x(x, \sigma) g(\sigma | s, t) d\sigma$ , so that marginal expected utility is then positive. Similarly, if  $x > x^*(s, t)$ , then marginal expected utility is negative. Hence  $x = x^*(s, t)$  is the global maximizer of expected utility.

*Sketch of Proof in Two Dimensions*

The case in which  $x$  is two-dimensional serves to demonstrate the intuitive idea of the general proof, although the general case is substantially more complex technically. With

<sup>10</sup>The lower limit in the outer integral is taken to be 0 to ensure convergence.

$N = 2$ , the first-order conditions for maximizing expected utility are

$$\int \varphi_{x_i}(x, \sigma)g(\sigma|s, t)d\sigma = \int \varphi_{x_i}(x^*(s, t^i(x_i, s)), \sigma)g(\sigma|s, t^i(x_i, s))d\sigma \text{ for } i = 1, 2.$$

As required, these first-order conditions are satisfied by  $x = x^*(s, t)$  since this implies  $t^i(x_i, s) = t$ , for  $i = 1, 2$ .

Indeed, it follows that  $x = x^*(s, t)$  is the unique global maximum. To see why, consider any  $x \neq x^*(s, t)$ . Figure 1 describes the directions in which expected utility unambiguously increases. These directions can be established by signing the corresponding derivatives. These directions lie in the NE quadrant and the SW quadrant relative to  $x^*(s, t)$ . There are two representative cases to consider. Case i)  $x \geq x^*(s, t)$ . In this case, Figure 2 sketches how it is possible to move from  $x$  to  $x^*(s, t)$  in a fashion that increases expected utility. That is, first reduce the coordinate that is too large relative to being on the  $x^*(s, \cdot)$  curve. Then move along this curve  $x^*(s, \cdot)$  to  $x^*(s, t)$ . (The case in the SW quadrant where  $x \leq x^*(s, t)$  is analogous.) Case ii)  $x_1 \geq x_1^*(s, t)$  and  $x_2 < x_2^*(s, t)$ . Refer to Figure 3. Consider a path from  $x$  to  $x^*(s, t)$  that first increases  $x_2$  to  $x_2^*(s, t)$ , as in Step 1 in Figure 3, and then reduces  $x_1$  to  $x_1^*(s, t)$ , as in Step 2. Step 2 is a limiting case from Figure 1 where expected utility must increase, but Step 1 is apparently ambiguous. Consider, however, Step 1', where  $x_2$  increases to  $x_2^*(s, t)$ , with  $x_1 = x_1^*(s, t)$ . Expected utility must increase in Step 1' since it is again a limiting case from Figure 1. The assumption that  $\varphi_{x_1x_2}(x, \sigma) \geq 0$  implies that expected utility must increase by at least as much in Step 1 as it does in Step 1', so it must increase in the two-step process—first Step 1 and then Step 2. The case in which  $x$  lies in the NW quadrant is analogous, so the sketch of the proof is complete.

#### *Observations*

Note that this particular decomposition of utility generates a particular trade-off between  $y$  and  $x$ , so the individual would sacrifice expected offspring for, say, more food. Furthermore, this decomposition into  $y$  and  $x$  is unique, within the additively separable class, even though  $y$  is itself a function of  $x$  as  $y = \varphi(x, \sigma)$ , under the additional assumption that utility cannot depend directly on  $\sigma$ .

**Remark 1** *A measure of the contribution of Nature to the Individual's decision is*

$$\left( \int \varphi_{x_i}(x^*(s, t), \sigma)g(\sigma|s, t)d\sigma \right)^2 .$$

This measure is expressed purely in terms of the fitness function and so is independent of the utility representation. It is a measure of how much the optimal choice of  $x^*(s, t)$  involves “adjusting” the Individual’s preferences away from expected fitness, generating then derivatives of expected offspring that differ from zero.

## 5 Examples

Consider first the case of  $N = 1$  and the simple quadratic fitness function  $\varphi(x, \sigma) = -(x - \sigma)^2$ . Suppose further that the state has a diffuse prior, so that  $\sigma \sim N(\mu_0, \infty)$  loosely speaking. For Nature,  $s = \sigma + \varepsilon_s$  where  $\varepsilon_s \sim N(0, v_s)$ , and  $t = \sigma + \varepsilon_t$ , where  $\varepsilon_t \sim N(0, v_t)$  and  $\varepsilon_s$  and  $\varepsilon_t$  are independent. For the Individual,  $s = \sigma + \varepsilon'_s$  where  $\varepsilon'_s \sim N(0, v'_s)$ , and  $t = \sigma + \varepsilon'_t$  where  $\varepsilon'_t \sim N(0, v'_t)$  and  $\varepsilon'_s$  and  $\varepsilon'_t$  are independent.

The true mean of  $\sigma$  after observing both  $s$  and  $t$  is<sup>11</sup>

$$\mu^* = \lambda s + (1 - \lambda)t \text{ where } \lambda = \frac{\frac{1}{v_s}}{\frac{1}{v_s} + \frac{1}{v_t}} \in (0, 1),$$

so that  $\lambda$  is the precision of Nature’s information relative to that of the Individual. With a quadratic fitness, the optimal  $x$  is, in general, the mean of the distribution of  $\sigma$ . Hence, it follows that  $x^*(s, t) = \lambda s + (1 - \lambda)t$ .

The mean of  $\sigma$  from the Individual’s point of view is

$$\mu' = \gamma s + (1 - \gamma)t \text{ where } \gamma = \frac{\frac{1}{v'_s}}{\frac{1}{v'_s} + \frac{1}{v'_t}} \in (0, 1).$$

Using the general formula  $\alpha(x, s) = -\int^x \int \varphi_x(z, \sigma)g(\sigma|s, t(z, s))d\sigma dz$  where  $\lambda s + (1 - \lambda)t(z, s) = z$ , so  $t(z, s) = \frac{z - \lambda s}{1 - \lambda}$ , we have

$$U(x, y, s) = y - \frac{\lambda - \gamma}{1 - \lambda}(x - s)^2.$$

Thus the (positive or negative) weight put on the term  $-(x - s)^2$ , which reflects how the individual’s choices are adjusted away from those that maximize expected fitness, depends on  $\lambda$  and  $\gamma$ .

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<sup>11</sup>Assuming that the variance of the prior distribution of  $\sigma$  is infinite implies that the mean of this prior is irrelevant.

If  $1 > \lambda > \gamma$ , then the individual is induced to make a choice that puts a greater weight on  $s$  than does  $\mu'$ . If, in the limit,  $1 = \lambda > \gamma$ , then  $x = s$ , since the transient signal  $t$  is then irrelevant.

If  $\gamma > \lambda > 0$ , then the individual is induced to make a choice that puts a lower weight on  $s$  than does  $\mu'$ . If, in the limit,  $\gamma > \lambda = 0$ , then  $x = t$ , since the recurrent signal  $s$  is then irrelevant.

In this example, the adjustment of the Individual's utility is measured by

$$\begin{aligned}\mathbb{E}_g [\varphi_{x_i}(x^*(s, t), \sigma)|s, t]^2 &= (\mathbb{E}_g [\sigma|s, t] - \mathbb{E}_f [\sigma|s, t])^2 \\ &= (\lambda - \gamma)^2 (t - s)^2\end{aligned}$$

Hence

$$\mathbb{E} \left[ \mathbb{E}_g [\varphi_{x_i}(x^*(s, t), \sigma)|s, t]^2 \right] = (\lambda - \gamma)^2 (v_s + v_t).$$

When  $\lambda \neq \gamma$ , the Individual sacrifices fitness, as judged using his beliefs, in order to obtain higher intrinsic utility from  $x$ .

The magnitude of this effect increases in  $|\lambda - \gamma|$ .

#### *An Anecdote*

To illustrate the implications of the relative precision of the information held by Nature and that held the Individual, consider the situation faced by an individual hiking across Baffin Island.<sup>12</sup> There are two main dangers faced on such a trek—polar bears and rivers. Of these, bears are more psychologically salient. A hiker inevitably worries more about encountering a bear than encountering a river. However, although rivers are less dangerous per encounter, there are many more encounters with them and the aggregate danger posed by rivers exceeds that posed by polar bears. One needs to take river crossings seriously.

In terms of the current model, it seems reasonable, on the one hand, that the information held by Nature concerning bears is rather precise relative to any information available to the Individual. Indeed, polar bears belong to the category of large carnivores with sharp yellow fangs and claws that one is instinctively afraid of. There are not many sufficiently good reasons to voluntarily approach a polar bear. That is, in terms of the example, it may well be that  $1 \approx \lambda > \gamma$ , so that the transient signal is essentially irrelevant. On the other hand, rivers can be useful. Thus the information held by the agent may well be decisive. There are

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<sup>12</sup>A more familiar example concerns rats' behavior in the face of either snakes or flowers, as in the literature on prepared learning. See the next section.

indeed many sufficiently good reasons to cross a river, and the overall evolutionary strategy is to rely to a much larger extent on the agent’s assessment of the local circumstances. Thus,  $\gamma > \lambda \approx 0$ , and so  $x$  is heavily dependent on  $t$ .

*An Example of Labor-Leisure Choice*

Assume that fitness  $y = \varphi(x, \sigma) = \sigma x - c(x)$ , where  $x$  is effort,  $\sigma x$  is a material output and  $c(x)$  is the cost of effort.

Suppose  $\mathbb{E}_f[\sigma|s, t] = \lambda s + (1 - \lambda)t$  and  $\mathbb{E}_g[\sigma|s, t] = \gamma s + (1 - \gamma)t$  as obtained in the previous example.

An optimal utility function is now

$$U(x, y, s) = y - \frac{\lambda - \gamma}{1 - \lambda} [c(x) - sx].$$

To interpret this function, suppose  $s$  represents the state of the immune system, and  $\lambda > \gamma$ . After illness, the immune system is depleted, so we assume  $s < t$ . The model predicts less effort in this circumstance.

Indeed, it is believed that, after illness, cytokines are released that inhibit effort, and the individual exhibits “anhedonia.” Evidence for the inhibitory role of cytokines comes from experiments that involve injecting cytokines into healthy rats. This makes them expend less effort for a given reward. (See Trivers, 2011.)

In this case, the Individual perceives a cost of effort that varies as  $\lambda$  varies with the task, even though the actual fitness cost of effort is invariant. It seems psychologically plausible, for example, that the perceived effort involved in walking 10km is less when there is a magnificent view than when walking to work.

## 6 Application to Conformity and Prepared Learning

Consider now how the present approach yields a framework for considering the evolution of interdependent preferences.<sup>13</sup> Consider two individuals who choose sequentially, with Individual 1 choosing in the light of the recurrent signal,  $s$ , and her transient signal,  $t_1$ , and Individual 2 choosing in the light of the recurrent signal, his transient signal,  $t_2$  and

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<sup>13</sup>Samuelson (2004) is a key antecedent in the literature on relative consumption. He also supposes that the observable but inherently irrelevant consumption of others conveys information about an unobservable but relevant state. In contrast to the model here, Samuelson adopts an informational structure in which there is no counterpart to the information of Nature and therefore no counterpart to the application to prepared learning outlined below.

Individual 1's choice,  $x_1$ . The transient signals are independent, so observing  $x_1$  is useful to Individual 2. Given the appropriate utility function, Individual 1 effectively maximizes  $\int \varphi(x_1, \sigma) f(\sigma|s, t_1) d\sigma$  by choosing  $x_1^*(s, t_1)$ . This appropriate utility is

$$U_1(x_1, y_1, s) = y_1 + \alpha(x_1, s) = \varphi(x_1, \sigma) + \alpha(x_1, s)$$

where  $\alpha(x_1, s) = - \int_0^{x_1} \int \varphi_x(z, \sigma) g_1(\sigma|s, t^1(z, s)) d\sigma dz$ , for all  $t$ , given  $s$ ,

exactly as in Theorem 1, where the pdf  $g_1(\sigma|s, t_1)$  represents Individual 1's beliefs. Following from Assumption 1, a key property is that  $\frac{\partial x_1^*(s, t_1)}{\partial t_1} > 0$ , so Individual 2 can infer  $t_1$  from any possible observed value of  $x_1$ . We can then formulate Individual 2's beliefs and choices in terms of  $x_1$ .

Consider now Individual 2's ideal choice,  $x_2^*(s, x_1, t_2)$ , given  $s$ ,  $x_1$ , and  $t_2$ . This is the choice that maximizes  $\int \varphi(x_2, \sigma) h(\sigma|s, x_1, t_2) d\sigma$ . In this expression,  $h$  represents the true pdf for  $\sigma$  conditional on  $s$ ,  $x_1$ , and  $t_2$ . Assume that increases in any of  $s$ ,  $x_1$  or  $t_2$  increase the distribution for  $\sigma$  in the sense of first-order stochastic dominance. The problem facing Individual 2 remains analogous to that described in detail in Section 4, with  $s$  and  $x_1$  together playing the role that was played by  $s$  alone and  $t_2$  playing the role of  $t$ .

Extending Theorem 1 to this context, it follows that there exists a utility function whose expectation under Individual 2's beliefs, given by the pdf  $g_2(\sigma|s, x_1, t_2)$ , is uniquely maximized by  $x_2^*(s, x_1, t_2)$  of the form

$$U_2(x_2, y_2, s, x_1) = y_2 + \beta(x_2, s, x_1).$$

The following captures a ‘‘conformity’’ or ‘‘anticonformity’’ effect—

**Definition 2** *Define the (marginal) peer effect as  $\frac{\partial \beta(x_2, s, x_1)}{\partial x_2 \partial x_1}$ . If this peer effect is positive— $\frac{\partial^2 \beta(x_2, s, x_1)}{\partial x_2 \partial x_1} > 0$ , an increase in the action taken by Individual 1 spurs an increase in the marginal utility of the action taken by Individual 2, and there is a conformity effect. If this peer effect is negative— $\frac{\partial^2 \beta(x_2, s, x_1)}{\partial x_2 \partial x_1} < 0$ , there is an anticonformity effect.*

#### *An Example on Prepared Learning*

Suppose fitness for each individual is quadratic, of the form  $\varphi(x, \sigma) = -(x - \sigma)^2$ . Suppose that  $\sigma \sim N(\mu_0, \infty)$ ,  $s \sim N(\sigma, v_s)$ , and  $t_i \sim N(\sigma, v_t)$ ,  $i = 1, 2$ , where  $s, t_1$  and  $t_2$  are independent. The optimal choice for 1 is then  $x_1^* = \lambda_1 s + (1 - \lambda_1) t_1$ , for  $\lambda_1 = \frac{r}{1+r} \in (0, 1)$ , where  $r = \frac{1}{v_s} / \frac{1}{v_t}$  is the true precision of the recurrent signal relative to that of the transient



signal. Given beliefs by the Individual 1, this choice can be implemented by a suitable utility function, as described above, but the precise form of this function is not relevant to the present focus, which is on Individual 2.

Consider then Individual 2. The optimal choice for 2 in the light of  $s, t_1$  and  $t_2$  is  $x_2^* = \lambda_2 s + (1 - \lambda_2) \frac{t_1 + t_2}{2}$ , where  $\lambda_2 = \frac{r}{2+r} \in (0, 1)$ . There is an interesting implication of these optimal choices that is independent of Individual 2's beliefs. Note that  $x_2^*$  can be expressed as

$$x_2^* = \frac{1+r}{2+r} x_1 + \frac{r}{2+r} t_2,$$

where there is no explicit dependence of  $x_2^*$  on  $s$ . An increase in  $x_1$  spurs a positive but less-than-matching increase in  $x_2$ . The weight put on  $x_1$  in this expression for  $x_2^*$  is uniquely determined by the optimal weight that needs to be put on  $t_1$ . The absence of  $s$  in the above expression follows since the *relative* weight that  $x_2^*$  puts on  $s$  and  $t_1$  is the same as the *relative* weight that  $x_1^*$  puts on these two signals.

Consider some relevant results from the psychology literature on prepared learning. Monkeys do not exhibit an inborn fear of snakes or of flowers (less surprisingly). However, they readily learn to be afraid of snakes if they observe another monkey acting fearfully in the presence of a snake. It is much more difficult to teach them similarly to be afraid of flowers (Cook and Mineka, 1989, for example).

The example sheds light on these phenomena. Suppose that  $r$ , the precision of the recurrent signal relative to the transient signal, increases. It follows that Individual 2 is *more* responsive to Individual 1's choice.<sup>14</sup> That is, individuals are *more* influenced by peer choices concerning snakes, where the recurrent signal seems to be relatively precise, than they are by those concerning flowers, where it is less so.

Consider now the peer effect. This brings in the beliefs of Individual 2. Suppose that these beliefs are described by  $\sigma \sim N(\mu'_0, \infty)$ ,  $s \sim N(\sigma, v'_s)$ ,  $x_1 \sim N(\sigma, v'_x)$  and  $t_2 \sim N(\sigma, v'_t)$ , so that  $\mathbb{E}(\sigma | g_2(\sigma | s, x_1, t_2)) = \gamma s + \mu x_1 + (1 - \gamma - \mu) t_2$ , for some  $\gamma, \mu > 0$  where  $\gamma + \mu < 1$ .<sup>15</sup> It follows that the choice of  $x_2^*$  is implemented by the utility function

$$U_2 = y_2 - k(x_2 - \delta s - (1 - \delta)x_1)^2, \text{ where } k = (1 + r) - (2 + r)(\gamma + \mu) \text{ and } \delta = -\frac{\gamma}{k}.$$

The peer effect in this example is  $2k(1 - \delta) = 2k + 2\gamma$ , which can be either positive or

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<sup>14</sup>Namely,  $\frac{1+r}{2+r}$  is increasing in  $r$ .

<sup>15</sup>Indeed,  $\gamma = \frac{\frac{1}{v'_s}}{\frac{1}{v'_s} + \frac{1}{v'_x} + \frac{1}{v'_t}}$ , and  $\mu = \frac{\frac{1}{v'_x}}{\frac{1}{v'_s} + \frac{1}{v'_x} + \frac{1}{v'_t}}$ .

negative. In either case, since  $\frac{dk}{dr} = 1 - \gamma - \mu > 0$ , the peer effect is greater, the larger is the precision of Nature’s signal for the recurrent signal relative to the transient one. That is, the peer effect is larger if it is positive to start with, or less negative if it is negative to start with.

This result implies that the observations concerning prepared learning, as derived above for optimal choices, would apply in a modern environment despite the application of different beliefs that do not yield evolutionarily optimal choice. The topic of how the adjustment term in utility would manifest itself in modern choices that fail to be evolutionarily optimal is taken up next.

## 7 Application to Decisions in the Modern Environment

There are vast differences between the ancestral environment in which our basic preferences evolved and the present environment. A central, though informal, claim in the literature on evolutionary psychology is that such differences have led to “misalignments” in our preferences in the sense that we frequently make fitness-reducing choices. Notably, the preference misalignments studied thus far belong to a single category: those originating in technological advances that have increased both the general availability of resources and our control over reproduction. Common examples cited in the literature include the tendency of modern humans to consume excessive amounts of sugar and fat and their tendency to use contraceptives. In terms of our model, such misalignments would readily arise upon altering the Individual’s fitness function while holding his beliefs and utility function constant.

In this section, we investigate a different category of preference misalignments: those originating from our improved understanding, relative to our hunter-gather ancestors, of the link between actions and fitness. In our model, as we illustrate below, such misalignments arise when changing the Individual’s beliefs while holding her fitness and utility functions constant. Such misalignments generate a set of testable, revealed preference, predictions of the model.

For concreteness, we consider the problem of a perfectly-knowledgeable Individual who understands that the state  $\sigma$  is distributed according to  $f$ , not  $g$ , and is nevertheless endowed with an “ancestral” utility function. In this exercise, for simplicity, we assume that the fitness function  $\varphi$  is the same for the ancestral and modern environments.

Specifically, consider a monotone environment with  $N = 1$ , a quadratic fitness function  $\varphi(x, \sigma) = -(x - \sigma)^2$ , and  $g \neq f$ . Moreover, suppose the Individual is endowed with the utility function in Theorem 1:

$$U(x, y, s) = y + \alpha(x, s),$$

where  $\alpha(x, s) = \int_0^x \int 2(z - \sigma)g(\sigma | s, t(z, s))d\sigma dz$  for all  $x, s$ .

**Lemma 2** Consider a perfectly-knowledgeable Individual who solves, for all  $s, t$ ,

$$\max_x \mathbb{E}_f [\varphi(x, \sigma) | s, t] + \alpha(x, s).$$

The Individual's optimal choice, denoted  $x^M(s, t)$ , satisfies:

1. Given  $s, t$ ,  $x^M(s, t)$  is greater (smaller) than  $x^*(s, t)$  whenever  $\mathbb{E}_f [\sigma | s, t]$  is greater (smaller) than  $\mathbb{E}_g [\sigma | s, t]$ .
2. Suppose further that  $\mathbb{E}_f [\sigma | s, t] = \lambda s + (1 - \lambda)t$  and  $\mathbb{E}_g [\sigma | s, t] = \gamma s + (1 - \gamma)t$  for some  $\lambda, \gamma \in (0, 1)$ . Given  $s, t$ ,  $\frac{\partial}{\partial s}x^M(s, t)$  is greater (smaller) than  $\frac{\partial}{\partial s}x^*(s, t)$  whenever  $\lambda$  is greater (smaller) than  $\gamma$ .

**Proof.** Fix  $s, t$ . The Individual's objective is strictly concave and the corresponding first-order condition is

$$\mathbb{E}_f [\sigma | s, t] - \mathbb{E}_g [\sigma | s, t(x, s)] = 0.$$

Part 1 of the lemma follows from noting that  $\mathbb{E}_g [\sigma | s, t(x, s)]$  is increasing in  $x$  and  $\mathbb{E}_g [\sigma | s, t(x, s)] = \mathbb{E}_g [\sigma | s, t]$  when  $x = x^*(s, t)$ .

For part 2 of the lemma, we apply the Implicit Function Theorem to the Individual's first-order condition to obtain

$$\begin{aligned} \frac{\partial}{\partial s}x^M(s, t) &= -\frac{\frac{\partial}{\partial s}t(x, s)}{\frac{\partial}{\partial x}t(x, s)} + \frac{\frac{\partial}{\partial s}\mathbb{E}_f [\sigma | s, t] - \frac{\partial}{\partial s}\mathbb{E}_g [\sigma | s, t(x, s)]}{\frac{\partial}{\partial t}\mathbb{E}_g [\sigma | s, t(x, s)] \frac{\partial}{\partial x}t(x, s)} \\ &= \frac{\partial}{\partial s}x^*(s, t) + (\lambda - \gamma) \frac{1 - \lambda}{1 - \gamma}, \end{aligned}$$

which establishes the desired result. ■

Part 1 of the lemma indicates that when the Individual's ancestors underestimated (overestimated) the marginal fitness value of a given action, this Individual will select, once perfectly informed, an excessively large (small) action relative to its fitness-maximizing level.

An example is a modern human who sunbathes excessively upon learning that sunlight aids his production of Vitamin D. The Individual faces a double incentive: a primitive intrinsic preference for sunlight and a recently-acquired knowledge of its fitness benefits.

Part 2 of the lemma indicates that when the Individual’s ancestor had beliefs  $\mathbb{E}_g[\sigma | s, t]$  that under-reacted (over-reacted) to increases in  $s$ , this individual, once perfectly informed, will over-react (under-react) to increases in  $s$ . Consider the following example in which  $s$  is the observed action of a peer. Suppose a modern human develops an excessive aversion (a phobia) toward a spider upon learning that a peer fears that spider. We can understand this aversion in the light of our approach as arising from two independent reasons. First, there is an evolved utility adjustment that is triggered by his peer’s fear—that is, prepared learning as in the previous section. Second, there is an effect on his beliefs of his peer’s fear, since this fear signals that the spider is likely to be poisonous—as considered in this section.

## 8 Conclusions

The motivating question that we began with was: Supposing, for the sake of argument, that we are intelligent and rational, why is our evolved utility not simply offspring? We formulated a principal-agent model in which both Nature and the individual observed two signals—one recurrent and one transient—that bear on the fitness consequences of the agent’s choices. The agent, however, has arbitrary beliefs about the implications of the signals. One abstract option would be for Nature to explicitly and directly communicate her accurate beliefs to the agent, who could then choose optimally by maximizing expected fitness in the light of these beliefs. This option, however, is simply not realistic.

Alternatively, we consider the more realistic option that Nature shapes the utility function in the light of the recurrent signal. The individual then maximizes the expectation of this utility conditional her rather arbitrary beliefs. We show, remarkably enough, that this option could also generate optimal choice in the context of the model. That this is the method actually in evidence, despite the existence of a more direct way of achieving the same end, may then have been harmless phylogenetic happenstance.

We considered why utility is a “whispering within” urging individuals to take actions that reflect the evolutionary wisdom of a multitude of ancestors, in addition to accounting for local on-the-spot information. In our examples, the loudness of the whisper, or the force of push delivered by Nature, derives from the relative precision of the signals for Nature as

compared to the relative precision of the two signals from the point of view of the individual.

In vastly changed modern conditions—not the least of which involves contraception—the mechanism may no longer be evolutionarily optimal. Nevertheless, the legacy of this mechanism is plausible—namely utility functions that convey evolutionary information, rather than explicit transmission of the raw statistical information.

## 9 Appendix—Proofs

### 9.1 Proof of Lemma 1.

The function  $x^*(s, t)$  is characterized by the first-order conditions

$$\int \varphi_{x_i}(x^*(s, t), \sigma) f(\sigma|s, t) d\sigma = 0, \text{ for } i = 1, \dots, N.$$

Hence

$$\sum_j A_{ij} \frac{\partial x_j^*(s, t)}{\partial t} = b_i \text{ for } i = 1, \dots, N,$$

where

$$A_{ij} = \int \varphi_{x_i x_j}(x^*(s, t), \sigma) f(\sigma|s, t) d\sigma \text{ and } b_i = - \int \varphi_{x_i}(x^*(s, t), \sigma) \frac{\partial f(\sigma|s, t)}{\partial t} d\sigma < 0.$$

The  $n \times n$  matrix  $A$  is symmetric, negative definite, and has non-negative off-diagonal elements. Hence  $-A$  is a Stieltjes matrix, which must have a symmetric and non-negative inverse (see Varga (1962, p. 85)). Hence  $A^{-1}$  must be a symmetric and non-positive matrix. Since

$$\begin{bmatrix} \frac{\partial x_1^*(s, t)}{\partial t} \\ \dots \\ \frac{\partial x_N^*(s, t)}{\partial t} \end{bmatrix} = A^{-1} b,$$

it follows that  $\frac{\partial x_j^*(s, t)}{\partial t} \geq 0$ , for  $j = 1, \dots, N$ . Further, since  $A^{-1}$  is non-singular, it cannot have any row be entirely zero, and it must indeed be that  $\frac{\partial x_j^*(s, t)}{\partial t} > 0$ , for  $j = 1, \dots, N$ .

The proof that  $\frac{\partial x_j^*(s, t)}{\partial s} > 0$ , for  $j = 1, \dots, N$  is analogous, as are the corresponding properties of  $\hat{x}(s, t)$ .

## 9.2 Proof of Theorem 1

Select an arbitrary  $s$ . To simplify notation, we drop the dependence of  $x^*(\cdot)$  and  $\alpha(\cdot)$  on  $s$ . Define, for all  $x$  and  $t$ ,

$$V(x; t) = \mathbb{E}[\varphi(x, \sigma) \mid t] + \alpha(x),$$

where the expectation is taken over  $\sigma$  using the pdf  $g$ .

We wish to show that  $V(x^*(t), t) > V(x, t)$  for all  $t$  and all  $x \neq x^*(t)$ .

**Remark 2** *Properties of  $V(x; t)$ . For all  $i$  and all  $t$ :*

1.  $\frac{\partial}{\partial x_i} V(x_i, x_{-i}; t)$  is weakly increasing in  $x_{-i}$  for all  $x_i$ .
2.  $\frac{\partial}{\partial x_i} V(x; t)$  is strictly increasing in  $t$  for all  $x$ .
3.  $\frac{\partial}{\partial x_i} V(x^*(t); t) = 0$ .

**Proof.** From the definitions of  $V(\cdot)$  and  $\alpha(\cdot)$  we obtain

$$\frac{\partial}{\partial x_i} V(x; t) = \mathbb{E}[\varphi_{x_i}(x_i, x_{-i}, \sigma) \mid t] - \mathbb{E}[\varphi_{x_i}(x^*(t^i(x_i)), \sigma) \mid t^i(x_i)]. \quad (3)$$

For property 1, note that the first term on the R.H.S. of (3) is weakly increasing in  $x_{-i}$  (since, by assumption,  $\frac{\partial^2}{\partial x_i \partial x_j} \varphi(x, \sigma) \geq 0$  for all  $x, \sigma$  and all  $i \neq j$ ), and the second term is independent of  $x_{-i}$ .

For property 2, note that the first term on the R.H.S. of (3) is increasing in  $t$  (since, by assumption,  $\frac{\partial^2}{\partial x_i \partial \sigma} \varphi(x, \sigma) > 0$  for all  $x, \sigma$  and all  $i$ , and the pdf  $g$  is increasing in  $t$  in first-order stochastic dominance), and the second term is independent of  $t$ .

For property 3, note that  $t^i(x_i^*(t)) = t$  (by definition) and therefore

$$\mathbb{E}[\varphi_{x_i}(x^*(t), \sigma) \mid t] = \mathbb{E}[\varphi_{x_i}(x^*(t^i(x_i)), \sigma) \mid t^i(x_i)].$$

■

Now select an arbitrary  $t$  and an arbitrary  $x \neq x^*(t)$ . Let  $\tau_i = t^i(x_i)$  for all  $i$ . Assume, WLOG, that  $\tau_1 \leq \tau_2 \leq \dots \leq \tau_N$ . Also, select two numbers  $\tau_0$  and  $\tau_{N+1}$  such that  $\tau_0 \leq \min\{\tau_1, t\}$  and  $\tau_{N+1} \geq \max\{\tau_N, t\}$ .

Define

$$\begin{aligned} M_+ &= \{i : x_i \geq x_i^*(t)\}, \\ M_- &= \{i : x_i < x_i^*(t)\}. \end{aligned}$$

Note that

$$V(x \vee x^*(t); t) = V(x; t) + \sum_{n \in M_-} \int_{\tau_n}^{\min\{\tau_{n+1}, t\}} \frac{d}{d\tau} V(x_{i \leq n}^*(\tau), x_{i > n}; t) d\tau,$$

and

$$V(x \vee x^*(t); t) = V(x^*(t); t) + \sum_{n \in M_+} \int_{\max\{\tau_{n-1}, t\}}^{\tau_n} \frac{d}{d\tau} V(x_{i \in M_-}^*(t), x_{i \in M_+}^*(\tau), x_{i \geq n}^*(\tau); t) d\tau,$$

where  $M_+^n$  is defined as the set  $\{i \in M_+ : i < n\}$ .

It follows that

$$\begin{aligned} V(x^*(t); t) - V(x; t) &= \sum_{n \in M_-} \int_{\tau_n}^{\min\{\tau_{n+1}, t\}} \frac{d}{d\tau} V(x_{i \leq n}^*(\tau), x_{i > n}; t) d\tau \\ &\quad - \sum_{n \in M_+} \int_{\max\{\tau_{n-1}, t\}}^{\tau_n} \frac{d}{d\tau} V(x_{i \in M_-}^*(t), x_{i \in M_+}^*(\tau), x_{i \geq n}^*(\tau); t) d\tau. \end{aligned} \quad (4)$$

We begin by showing that  $V(x^*(t); t) \geq V(x; t)$ , for which we proceed in two steps.

**Step 1.** We show that all terms in the first sum of (4) are nonnegative. Fix  $n \in M_-$ .

For all  $\tau \in (\tau_n, \min\{\tau_{n+1}, t\})$  (a possibly empty interval) we have

$$\begin{aligned} &\frac{d}{d\tau} V(x_{i \leq n}^*(\tau), x_{i > n}; t) = \\ &\quad \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}; t) \cdot \frac{d}{d\tau} x_j^*(\tau) \\ &\geq \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}(\min\{\tau_{n+1}, t\}); t) \cdot \frac{d}{d\tau} x_j^*(\tau) \\ &\geq \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \cdot \frac{d}{d\tau} x_j^*(\tau) > 0. \end{aligned} \quad (5)$$

(Recall that  $\frac{d}{d\tau} x_j^*(\tau) > 0$  for all  $j$ .)

The first weak inequality in (5) follows from property 1 of the remark:  $x_i \geq x_i(\min\{\tau_{n+1}, t\})$

for all  $i > n$  implies

$$\begin{aligned} & \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}; t) \geq \\ & \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}(\min \{\tau_{n+1}, t\}); t) \text{ for all } j \leq n. \end{aligned}$$

The second weak inequality in (5) follows from property 2 of the remark:  $t \geq \min \{\tau_{n+1}, t\}$  implies

$$\begin{aligned} & \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}(\min \{\tau_{n+1}, t\}); t) \geq \\ & \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}(\min \{\tau_{n+1}, t\}); \min \{\tau_{n+1}, t\}) \text{ for all } j \leq n. \end{aligned}$$

Finally, the strict inequality in (5) follows from combining all three properties of the remark:  $\tau < \min \{\tau_{n+1}, t\}$  implies  $x_i^*(\tau) < x_i^*(\min \{\tau_{n+1}, t\})$  for all  $i$  and therefore

$$\begin{aligned} & \frac{\partial}{\partial x_j} V(x_{i \leq n}^*(\tau), x_{i > n}(\min \{\tau_{n+1}, t\}); \min \{\tau_{n+1}, t\}) \geq \\ & \frac{\partial}{\partial x_j} V(x^*(\tau); \min \{\tau_{n+1}, t\}) > \frac{\partial}{\partial x_j} V(x^*(\tau); \tau) = 0 \text{ for all } j \leq n. \end{aligned}$$

**Step 2.** We show that all terms in the second sum of (4) are nonpositive. Fix  $n \in M_+$ . Note that for all  $\tau \in (\max \{\tau_{n-1}, t\}, \tau_n)$  (a possibly empty interval) we have

$$\begin{aligned} & \frac{d}{d\tau} V(x_{i \in M_-}^*(t), x_{i \in M_+^n}, x_{i \geq n}^*(\tau); t) = \tag{6} \\ & \sum_{j \geq n} \frac{\partial}{\partial x_j} V(x_{i \in M_-}^*(t), x_{i \in M_+^n}, x_{i \geq n}^*(\tau); t) \cdot \frac{d}{d\tau} x_j^*(\tau) \\ & \leq \sum_{j \geq n} \frac{\partial}{\partial x_j} V(x_{i < n}^*(\max \{\tau_{n-1}, t\}), x_{i \geq n}^*(\tau); t) \cdot \frac{d}{d\tau} x_j^*(\tau) \\ & \leq \sum_{j \geq n} \frac{\partial}{\partial x_j} V(x_{i < n}^*(\max \{\tau_{n-1}, t\}), x_{i \geq n}^*(\tau); \max \{\tau_{n-1}, t\}) \cdot \frac{d}{d\tau} x_j^*(\tau) < 0. \end{aligned}$$

The first weak inequality in (6) follows from property 1 of the remark:  $(x_{i \in M_-}^*(t), x_{i \in M_+^n}) \leq$



$x_{i < n}^*(\max\{\tau_{n-1}, t\})$  implies

$$\begin{aligned} \frac{\partial}{\partial x_j} V(x_{i \in M_-}^*(t), x_{i \in M_+}^*(\tau), x_{i \geq n}^*(\tau); t) &\leq \\ \frac{\partial}{\partial x_j} V(x_{i < n}^*(\max\{\tau_{n-1}, t\}), x_{i \geq n}^*(\tau); t) &\text{ for all } j \geq n. \end{aligned}$$

The second weak inequality in (6) follows from property 2 of the remark:  $t \leq \max\{\tau_{n-1}, t\}$  implies

$$\begin{aligned} \frac{\partial}{\partial x_j} V(x_{i < n}^*(\max\{\tau_{n-1}, t\}), x_{i \geq n}^*(\tau); t) &\leq \\ \frac{\partial}{\partial x_j} V(x_{i < n}^*(\max\{\tau_{n-1}, t\}), x_{i \geq n}^*(\tau); \max\{\tau_{n-1}, t\}) &\text{ for all } j \geq n. \end{aligned}$$

Finally, the strict inequality in (6) follows from combining all three properties of the remark:  $\tau > \max\{\tau_{n-1}, t\}$  implies  $x_i^*(\tau) > x_i^*(\max\{\tau_{n-1}, t\})$  for all  $i$  and therefore

$$\begin{aligned} \frac{\partial}{\partial x_j} V(x_{i < n}^*(\max\{\tau_{n-1}, t\}), x_{i \geq n}^*(\tau); \max\{\tau_{n-1}, t\}) &\leq \\ \frac{\partial}{\partial x_j} V(x^*(\tau); \max\{\tau_{n-1}, t\}) &< \frac{\partial}{\partial x_j} V(x^*(\tau); \tau) = 0 \text{ for all } j \geq n. \end{aligned}$$

We now show that  $V(x^*(t); t) > V(x; t)$ . Since  $x \neq x^*(t)$  there exists either an  $n \in M_-$  such that the interval  $(\tau_n, \min\{\tau_{n+1}, t\})$  is nonempty, or an  $n \in M_+$  such that the interval  $(\max\{\tau_{n-1}, t\}, \tau_n)$  is nonempty (or both). In the former case, it follows from step 1 above that at least one of the integrals in the first sum of (4) is positive. In the latter case, it follows from step 2 above that at least one of the integrals in the second sum of (4) is negative.

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Figure 1: Directions of Expected Utility Increase

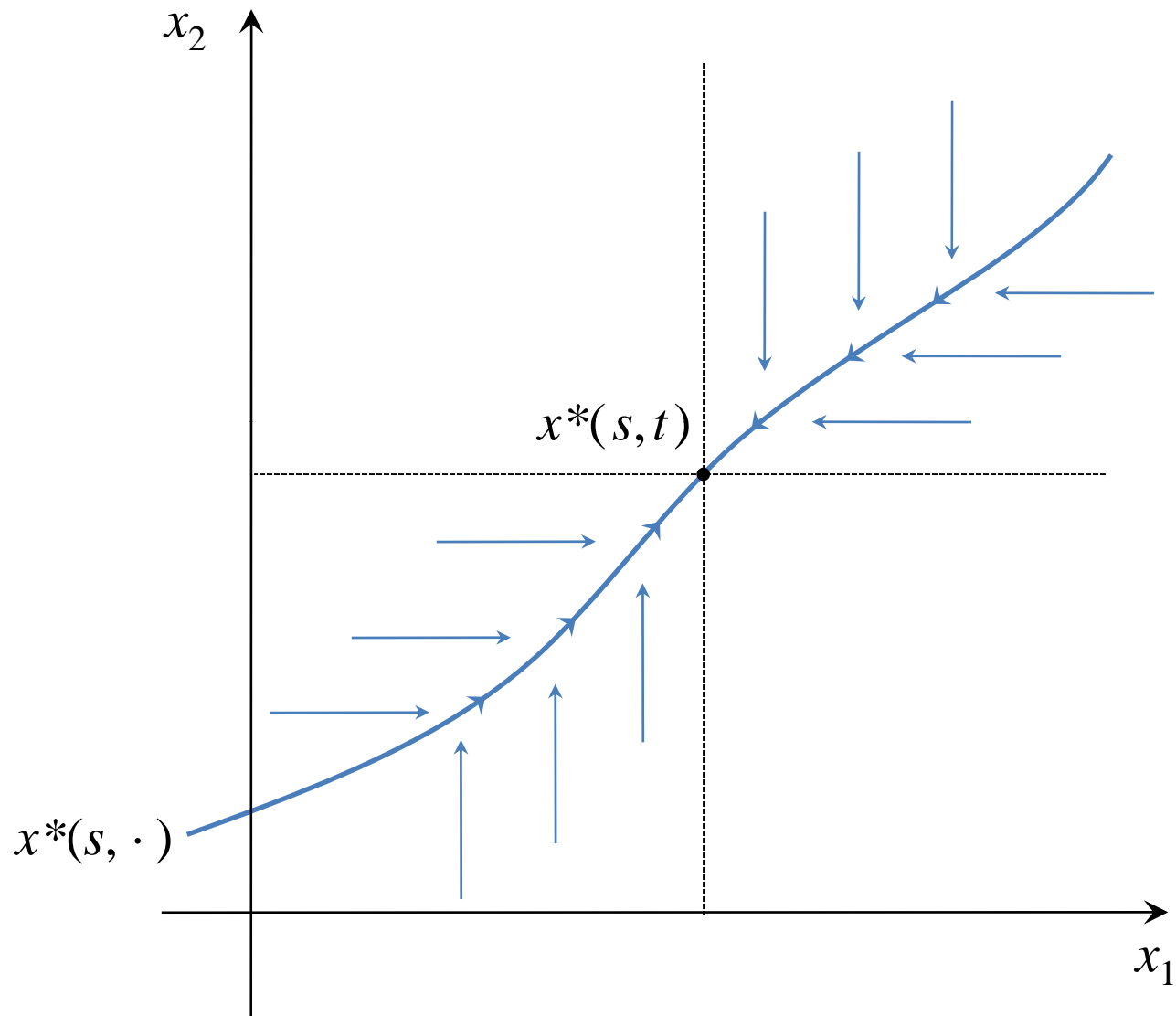


Figure 2: Case i)  $x \geq x^*(s, t)$

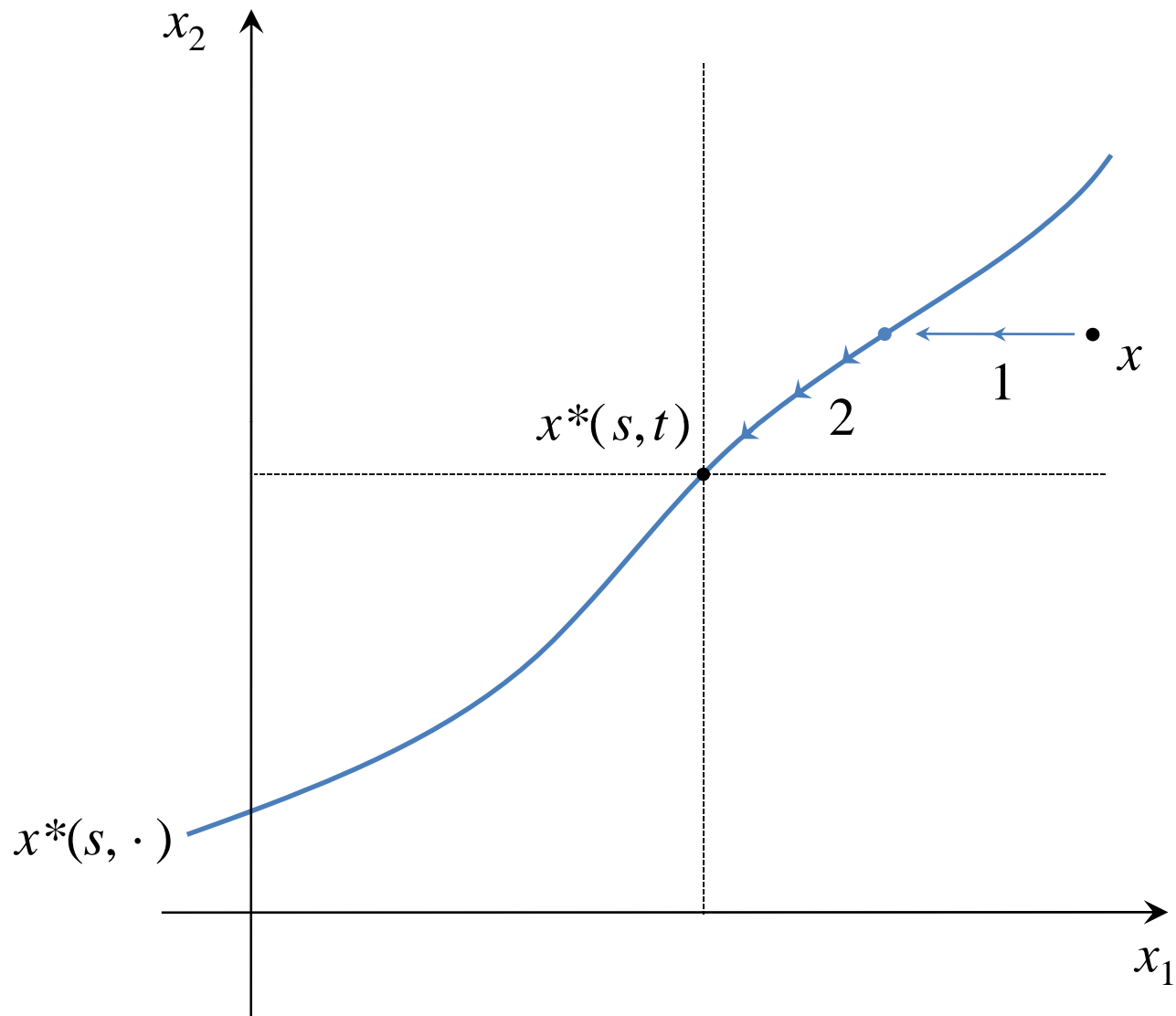


Figure 3: Case ii)  $x_1 \geq x_1^*(s, t)$  and  $x_2 < x_2^*(s, t)$

