Tipping Points and Business-as-Usual in a Global Carbon Commons

Rodrigo Harrison* and **Roger Lagunoff†**

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Abstract

This paper formulates a dynamic model of global carbon consumption in the absence of an effective international agreement. Each period, countries extract carbon from the global ecosystem. A country's output depends both on its carbon usage and on the ecosystem ("stored carbon"). The desired mix of extracted versus stored carbon by each country is determined by its stochastically evolving factor elasticities.

We characterize *Business-as-usual (BAU) equilibria* as smooth, Markov Perfect equilibrium profiles of carbon usage across countries. A BAU equilibrium is shown to generate lower aggregate output and higher carbon use each period than the socially efficient path, although some countries might actually use less carbon under BAU. We characterize properties of *tipping points*, threshold levels of stored carbon stocks below which the global commons collapses, spiraling downward toward a steady state of marginal sustainability. We show that if the profile of carbon factor elasticities reaches a high enough threshold, a tipping point will be breached. Even in this case, there remains a time span (a "negotiation window") in which a collapse may be averted if the countries agree to implement the efficient profile of carbon usage.

JEL Codes: C73, D82, F53, Q54, Q58

Key Words and Phrases: carbon consumption, global carbon commons, climate change, tipping points, international carbon agreements.

*Instituto de Economía Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Macúl, Santiago 780436. CHILE

†Department of Economics, Georgetown University, Washington, DC 20057, USA. +202-687-1510, lagunofr@georgetown.edu, www.georgetown.edu/lagunoff/lagunoff.htm.

1 Introduction

Human consumption is based on carbon usage. The resulting increases in anthropogenic GHGs alarm scientists and policy experts, many of whom focus attention on finding an effective international response to limit carbon emissions.[1]

This paper formulates a dynamic model of global carbon consumption in the absence of such a response. Our objective is to understand the strategic incentives of nation states in a "business-as-usual" (or "BAU" from here on) scenario. What are the long run implications of BAU? How does it compare with socially efficient usage? Are some countries hurt more than others? Are outcomes under BAU sustainable or is economic collapse inevitable? What determines the transition, if any, from sustainability to collapse?

To makes sense of the last few questions in particular, our model integrates a strategic model of emissions into a nonlinear dynamic model of carbon. A key feature of this model is that consumption and economic output may collapse and shrink if a key state variable falls below some critical threshold — a *tipping point*.

Tipping points are commonly discussed and modeled in the earth science literature, a sample of which includes Lenton, et al. (2008), Kerr (2008), Rockstrom, et al. (2009), and Anderies, et al. (2013). Most of these posit nonlinear dynamical systems that describe a *safe operating spaces (SOS) for humanity*, i.e., levels of methane and CO_2 concentrations, degrees of biodiversity, and so on,... that sustain levels of activity consistent with innovation, growth and development. Tipping points are typically described as the boundaries of these regions. Rockstrom et al. (2009) employ the term *planetary boundaries* to describe these constraints.

These earth science models typically contain a very detailed accounting of the different forms of carbon mass, but do not model human incentives explicitly. By contrast, the present paper posits a much simpler physical model of carbon. We abstract, for instance, from complicated marine-atmospheric diffusion processes and plant photosynthesis and respiration. The upside is that the model offers a rich and tractable characterization of incentives.

Our particular focus on *strategic* incentives of nations also distinguishes our work from the burgeoning literature that integrates GE economies with tipping models of climate dynamics. These include IAMs (integrated assessment models) of Nordhaus (2006, 2007, 2008) and Lemoine and Traeger (2014), Hope (2006), Stern (2006), and Cai, Judd, and Lontzek (2012), all of whom incorporate the possibility of abrupt changes in the earth's climate and eco-system.[2]

[1]e.g., IPCC Fourth Assessment Report: Climate Change 2007.

[2]See also Krusell and Smith (2009), Acemoglu et al. (2012), and Golosov et al. (2014) for useful

A study of the incentives of state actors is, to us, a sensible addition to the IAM literature since the most critical policy choices are made by large, powerful nations with divergent interests. For this reason, a business-as-usual setting is likely to be more complex than is suggested by an atomistic market with clearly defined property rights and well functioning contract enforcement.[3]

We posit a model in which each country produces a composite consumption good for its citizens. Production depends both on a carbon-based input and on the renewable ecosystem. The ecosystem is an open access source of stored carbon from which countries can freely extract. Hence, while carbon extraction is essential for production, its aggregate extraction depletes an ecosystem that is also essential to the production process. Some preservation of the ecosystem and its repository of stored carbon is, therefore, beneficial for purely economic reasons.[4]

The carbon dynamics in the model distinguish only between emitted and "stored" (non-atmospheric) carbon. The latter is summarized by a carbon resource stock ω_t at each date t representing all usable sources of non-atmospheric carbon in the global ecosystem. The stock ω_t may be thought of as known reserves of "stored" or "preserved" carbon in fossils, soil, or biomass. Once extracted, carbon is used in the production process and emitted into the atmosphere. The simple distinction between stored and emitted carbon is the basis for all dynamic changes in the model.

As both stored and extracted carbon are used in the production processes of all countries, each country's desired mix of the two carbon sources is determined by its relative output elasticities. Countries with relatively high output elasticities of extracted carbon prefer to extract more than countries with low elasticities. The elasticities are assumed to evolve stochastically, and the country-specific shocks to these may be serially correlated. This assumption captures a common feature in studies of climate change: both environmental costs and factor composition vary over time, are difficult to forecast, and often vary widely across countries. Heterogeneity reflects variation in geographic, demographic, and politico-economic influences. Burke, et. al. (2011) find, for example, widely varying estimates of the effect of climate change on US agriculture when climate model uncertainty is taken into account. Desmet and Rossi-Hansberg (2014) document substantial cross country variation in a calibrated model of spatial differences in welfare losses across countries due to global warming.

A *business-as-usual (BAU) equilibrium* is a smooth Markov Perfect equilibrium profile of carbon usage across countries. The two key state variables in the BAU

quantitive assessments of carbon taxation and cap and trade policies.

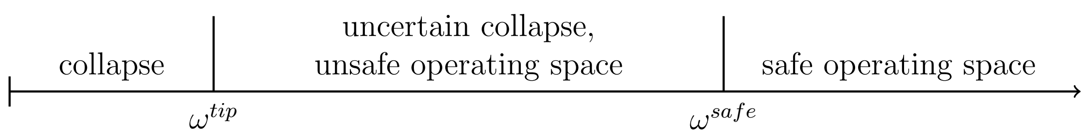
[3]See also Calcutt and Petkov (2012) and Harrison and Lagunoff (2014) for analyses of mechanism design problems in international environmental agreements.

[4]Climate change is one potential side effect of depletion, although ours is not a paper about climate change per se.

model are the carbon stock ω_t and the profile of country-specific carbon elasticities.

Though the dynamics of our model, with the inclusion of tipping points, are quite different, the present framework builds on the classic common pool framework of Levhari and Mirman (LM) (1980). Related work includes Cave (1987), Dutta and Sundaram (1993), Sorger (1996), Finus (2001), Barrett (2003), Dutta and Radner (2004, 2006, 2009), and Battaglini and Harstad (2012), all of whom examine strategic incentives in dynamic games with a commons or with climate externalities. The definition of the BAU equilibrium here largely coincides with one used in this literature (e.g., Dutta and Radner (2009)). The over-depletion problem in the present model echoes the "tragedy of the commons" theme running through these models.

The next section (Section 2) analyzes a benchmark case in which residual carbon stock grows or decays at a fixed rate. Section 3 then introduces a tipping model of carbon cycle dynamics. The results in Section 3 characterize threshold levels of stored carbon stocks in equilibrium that delineate safe operating spaces (SOS — in the sense of Rockstrom, et al. (2009)) from states in which collapse is possible. Regions of state space in which collapse is possible may, in turn, be delineated from those in which collapse is inevitable. In terms of carbon the stock, ω_t, these regions are shown schematically below.



A number of recent versions of IAMs (Cai, et al. (2012) and Lemoine and Traeger (2014)), have adopted dynamic programming methodologies to calculate time consistent mitigation policies in the presence of tipping. We also use dynamic programming techniques, in this case to solve for BAU equilibria. We find parametric configurations to support both SOS and collapse, although the historical pattern suggests a troubling trend toward the latter. Our results show the following:

- The BAU equilibrium generates lower aggregate output and higher carbon use each period than the path chosen by a socially efficient planner, although some countries might actually use *less* carbon under BAU. The difference, moreover, between BAU and the socially efficient path of carbon usage grows through time.
- A global economy in a BAU equilibrium can remain in SOS if elasticities of carbon usage remain low.

- However, if over time the carbon factor elasticities become large enough, the BAU equilibrium will eventually breach a tipping point, precipitating a collapse.

It is worth noting that under BAU, output is not monotone in carbon usage. Countries with either very high or very low carbon factor elasticities have larger output than those with intermediate elasticities. This means that even as the global commons reaches a tipping point, the leading carbon emitters are not the first to suffer consequences of the decline. As the threshold tipping stock is breached, we show that countries will *accelerate* their rates of extraction under BAU rather than slow them down. This is because once a collapse becomes inevitable, the marginal continuation value of preserving the carbon stock eventually vanishes.

The results underscore the idea that while the proximate cause of tipping is the depletion of the carbon stocks (or, equivalently, accretion of atmospheric carbon), the "deeper" parameters that drive the tipping and collapse are technological: the factor elasticities that determine the mix of extracted and stored carbon.

Finally, we show that collapse may in some instances be avoided, but only if the international community moves away from its business-as-usual regime. Specifically:

- Even if the tipping point is breached, there remains a time span (a "negotiation window") in which a collapse may be averted if the countries agree to implement planner's solution.

The upshot is that an effective international agreement provides an additional buffer against large shocks to the carbon stock. This suggests a somewhat more optimistic scenario than the other results. This is clearly contingent, however, on whether countries can coordinate on an international agreement, a mined topic in the literature.[5]

2 The Baseline Model without Tipping

2.1 An Overview

This section lays out a rudimentary baseline model of carbon usage. The model consists of an infinite horizon global economy with n countries. Each country makes

[5]See, for instance, Finus (2001), Barrett (2003), Calcott and Petkov (2012), and Harrison and Lagunoff (2014).

use of an essential resource — carbon — each period. Carbon is extracted from an open access, carbon-based commons which we refer to as the "global ecosystem."[6]

The framework is loosely based the classic common pool model of Levhari and Mirman (LM) (1980). Each period, identical users in the LM model choose how much of a depletable, open access resource to consume. Examples include fisheries or forestry. There are no direct costs or externalities from usage. Conservation is therefore valued in LM only for instrumental reasons: preserving the stock allows one to smooth consumption.

The present study modifies the classic model by adding the renewable ecosystem into each country's production technology and by further introducing heterogeneous shocks that affect each countries desired mix of productive inputs. In Section 3, we add a nonlinear tipping component into the carbon dynamic.

In the baseline model of this section, however, there are no tipping points, and no threat of environmental catastrophe. Later, we modify the model to allow these possibilities.

2.2 Consumption and Dynamics of the Carbon Stock

Countries make inter-temporal strategic decisions regarding how much carbon to extract and use. Country i's $(i = 1, \ldots, n)$ carbon extraction in date t is denoted by c_{it}. Let $C_t = \sum_i c_{it}$ represent the level of global carbon consumption at t. Consumption of C_t units of carbon produces C_t units of emissions, and so the two terms are sometimes used interchangeably. The global consumption C_t is consumed from a stock ω_t that represents all usable sources of non-atmospheric carbon in the global ecosystem. The stock ω_t may be thought of as known reserves of "stored" or "preserved" carbon in fossils, soil, or biomass. Once extracted, carbon is emitted into the atmosphere. The simple distinction between stored and released carbon forms the basis for all dynamic changes in the model.

The ecosystem evolves over time according to dynamic law of motion

$$\omega_{t+1} = A(\omega_t - C_t)^\gamma \tag{1}$$

with the initial stock fixed at some level ω_0. We assume that $\gamma < 1$ allowing for depreciation (e.g., plant respiration) if the stock is large enough, while $A > 1$ allows for accumulation due to the natural process of reabsorption (e.g., photosynethesis).

[6]Strictly speaking fossil fuels extracted from the geosphere are not part of the ecosystem. Their extraction, however, passes through and may upset the ecosystem and, consequently, the term "global ecosystem" is used to represent all forms of "stored" (non-atmospheric) carbon.

When $C_t = 0$, there is no human consumption in which case the law of motion in (1) is globally stable as it balances the dynamic forces of release and recapture of carbon through sequestration. Indeed, generalizing the model to allow for a time-varying ergodic process $\{A_t\}$, the evolution of the stock in (1) then produces a stable carbon cycle.[7]

The law of motion in (1) is not intended as a literal description of earth's carbon processes but rather as a tractable way to model dynamic trade offs of a climate externality. We later generalize (1) by introducing a non convexity in the carbon dynamic in order to discuss tipping points, environmental collapse and other features that focus on the potential instabilities of a business-as-usual equilibrium in the global carbon commons. We refer to the dynamic in (1) as the *baseline model.* Despite its simplicity, the baseline model will be shown to produce a rich set of equilibrium ecosystem dynamics.

Let $\mathbf{c}_t = (c_{1t} \ldots, c_{nt})$ denote the date t profile of resource consumption (and emissions). The entire dynamic path profile of resource consumption is the given by

$$\mathbf{c} \;=\; \{\mathbf{c}_t\}_{t=0}^{\infty}$$

A path $\mathbf{c}$ is *feasible* if it is consistent with the dynamic constraint (1) and $C_t \leq \omega_t$ at each date t.

Each country maximizes the payoff of its representative citizen,

$$\sum_t \delta^t \log y_{it} \tag{2}$$

where y_{it} is a composite output consumed by the representative consumer from country i at date t. All countries discount the future according to δ. A transversality condition entails a joint restriction $A\delta\gamma < 1$.

The production of y_{it} depends on both extracted carbon and the carbon-based global ecosystem according to the production technology

$$y_{it} = c_{it}^{\theta_{it}} (\omega_t - C_t)^{1-\theta_{it}}. \tag{3}$$

In (3), $\theta_{it} \in [0, 1]$ is the output elasticity of extracted carbon, while $1 - \theta_{it}$ is the output elasticity of the global ecosystem net of aggregate consumption. Countries with larger θ_{it} in date t will typically extract and emit more carbon, other things equal.

[7] An even richer model would allow A to depend on the existing stock. For tractability, however, we assume it is fixed and exogenous.

This formulation accounts for the fact that all countries' economies have carbon requirements, but production also requires that country draw upon a viable eco-system. The relative elasticities vary both over time and between countries, the latter reflecting the fact that both benefits and costs of extraction differ across countries. Warmer average temperatures resulting from GHG emissions are viewed differently in Greenland than in Sub-saharan Africa. Time variation comes from the fact that countries may be hit with serially correlated shocks. The shocks capture the unpredictability of technological change and the persistence of climatic change within each country.

A *type profile* in date t is a vector

$$\theta_t = (\theta_{1t}, \theta_{2t}, \ldots, \theta_{it}, \ldots \theta_{nt}),$$

and is publicly observed at the beginning of each period t. Let $\theta^t = \{\theta_0, \theta_1, \ldots, \theta_t\}$ be the history of realized type profiles up to and including date t, and let

$$\theta^\infty \ = \ \{\theta_0, \theta_1, \ldots, \theta_t, \ldots\}$$

the infinite time path of elasticity profiles.

Fixing the initial profile θ_0, the profile θ_t is assumed to evolve according to a stationary, though not necessarily ergodic, Markov process $F(\theta_t|\theta_{t-1})$. In what follows, "almost everywhere" will refer to the paths θ^∞ in the probability space $(\Theta^\infty, \mathcal{F}, P)$ such that F is the Markov kernel associated with a filtration $\{\mathcal{F}_t\}$ on the space $(\Theta^\infty, \mathcal{F}, P)$. We allow for F to exhibit both persistence across time and correlation of carbon elasticities across countries.

2.3 The Business-As-Usual Equilibrium

In any period, the state of the global economy is summarized the pair (ω_t, θ_t) consisting of the ecosystem and the elasticity profile. A *Markov-contingent plan* is a profile

$$c^*(\omega_t, \theta_t) = (c_1^*(\omega_t, \theta_t), \ldots, c_n^*(\omega_t, \theta_t))$$

that specifies each country's usage $c_i^*(\omega_t, \theta_t)$ as a function of the state (ω_t, θ_t).

Using equation (3), the payoff of a Markov-contingent plan $\mathbf{c}^*$ to the representative citizen in country i may be expressed as

$$U_i(\omega_t, \mathbf{c}^*, \theta_{it}) \ \equiv \ E\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t}[\theta_{i\tau} \log\left(c_i^*(\omega_\tau, \theta_\tau)\ \right) + (1-\theta_{i\tau}) \log\left(\omega_\tau - C^*(\omega_\tau, \theta_\tau)\ \right)] \middle| \ \omega_t, \theta_t\right] \tag{4}$$

A *Markov Perfect equilibrium (MPE)* is a Subgame Perfect equilibrium in which each country's strategy is a Markov-contingent plan.[8] The MPE is often considered to be the " business-as-usual" benchmark since it is arguably the most natural scenario that prevails in the absence of any agreement among the participants. The MPE requires no special coordination, no monitoring beyond the initial quota, and no explicit sanctions.

We further restrict attention to *smooth* MPE, that is, Markov-contingent plans that are both Subgame Perfect and smooth functions of the state (almost everywhere). This restriction rules out certain MPE that use discontinuities in the state to create triggers on which participants can tacitly coordinate. From here on, we refer to any smooth MPE as a *Business-as-usual (BAU) equilibrium.*

In any BAU equilibrium, country i's Markov-contingent plan $c_i^*(\omega_t, \theta_t)$ must maximize its long run payoff from date t, given the carbon dynamic (1) and production technology (3), and given any past history of consumption and elasticity profiles.

By incorporating the production technology in (9) directly into the long run payoff for country i, the Markov contingent consumption $c_i^*(\omega_t, \theta_t)$ may be found as a solution to the Bellman equation

$$U_i(\omega_t, \mathbf{c}^*, \theta_{it}) = \max_{c_{it}} \left\{ \theta_{it} \log c_{it} + (1-\theta_{it}) \log(\omega_t - C_t) \; + \; \delta \; E \left[U_i(\omega_{t+1}, \mathbf{c}^*, \theta_{i\,t+1}) \Big| \; \omega_t, \theta_{it} \right] \right\} \tag{5}$$

after for every state (ω_t, θ_t).

To calculate the MPE it is simpler to work with extraction rates rather than levels. The extraction rate e_{it} is therefore defined implicitly by $c_{it} = e_{it}\omega_t$. Denote the global extraction rate by $\mathcal{E}_t = \sum_i e_{it}$. Using extraction rates rather than levels, the corresponding Euler equation for each country i is

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} + \delta[\frac{\partial E[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1}) \big| \; \theta_{it}]}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_{it}}] \;\; = \;\; 0$$

with $\dfrac{\partial \omega_{t+1}}{\partial e_{it}} = -A\gamma\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1}$.

[8]Two clarifications are useful here. First, in the MPE each country's Markov-contingent plan c_i^* maximizes $U_i(\mathbf{c}^*, \omega_t, \theta_{it})$ given c_{-i}^* in any state (ω_t, θ_{it}) over the set of *full history-contingent* consumption plans. For brevity, we omit the specification of full history contingent strategies. Second, payoffs corresponding to infeasible paths must be formally defined as well. For our purposes, the simplest approach is to define the payoff on the extended real line, setting flow payoffs equal to $-\infty$ whenever $C_t > \omega_t$.

A unique stationary solution to the Euler equation for country i is given by:[9]

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} \;-\; (1-\theta_{it}) \;=\; \frac{A\delta\gamma}{1-A\delta\gamma} \quad i=1,2,\ldots,n \tag{6}$$

With minor algebra, we obtain a unique BAU equilibrium in extraction rates:

$$e_i^*(\omega_t,\theta_t) \;=\; \frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}})} \quad \text{and} \quad \mathcal{E}^*(\omega_t,\theta_t) \;=\; \frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}})} \tag{7}$$

where $\phi_{it} = \theta_{it}(1-A\gamma\delta)$. Country i's consumption/emissions in the BAU equilibrium is increasing in its resource elasticity and decreasing in the effective discount factor $A\delta\gamma$. Its emissions also vary with the global elasticity profile, decreasing in extraction rates $\theta_{jt},\ j \neq i.$ of other countries. Appendix 5.1 contains a detailed derivation.

For a point of reference, the BAU equilibrium extraction rates in (7) generalize the "Fish War" equilibria of Levhari and Mirman (1980). In the LM model, production depends only on the extracted resource, whereas here it depends on both extracted and stored resources.[10]

In a straightforward way, the extraction rates in (7) correspond to BAU equilibrium carbon levels in the following result.

Proposition 1 *There is a unique Business-as-usual equilibrium* $\mathbf{c}^*$ *described by country-specific and aggregate carbon consumption*

$$c_i^*(\omega_t,\theta_t) \;=\; \omega_t\frac{\frac{\phi_{it}}{1-\phi_{it}}}{1+\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}}} \quad \textit{and} \quad C^*(\omega_t,\theta_t) \;=\; \omega_t\frac{\sum_{j=1}^n \frac{\phi_{it}}{1-\phi_{it}}}{1+\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}}}, \tag{8}$$

and composite consumption y_i^* *by country* i *is convex in* θ_{it}*, ceteris parabis.*

The Proposition follows directly from (7) which, in turn, is derived in Appendix 5.1. We omit the details here.

[9] See Appendix 5.1 for an explicit derivation.

[10] To see this, observe that if $A=1$ and $\theta_{it}=1$ for all i and all t as in the Levhari-Mirman (LM) model, then the indirect preferences reduce to payoffs in their model. In that case our extraction rates corresponds precisely to those in the LM. Namely, $\phi_{it} = (1-\gamma\delta)$ for all i and t, and

$$e_{it}^*(\theta=\mathbf{1}) = \frac{(\frac{(1-A\gamma\delta)}{1-(1-A\gamma\delta)})}{1+\frac{(1-A\gamma\delta)}{1-(1-A\gamma\delta)}n} = \frac{(1-A\gamma\delta)}{n(1-A\gamma\delta)+A\gamma\delta}$$

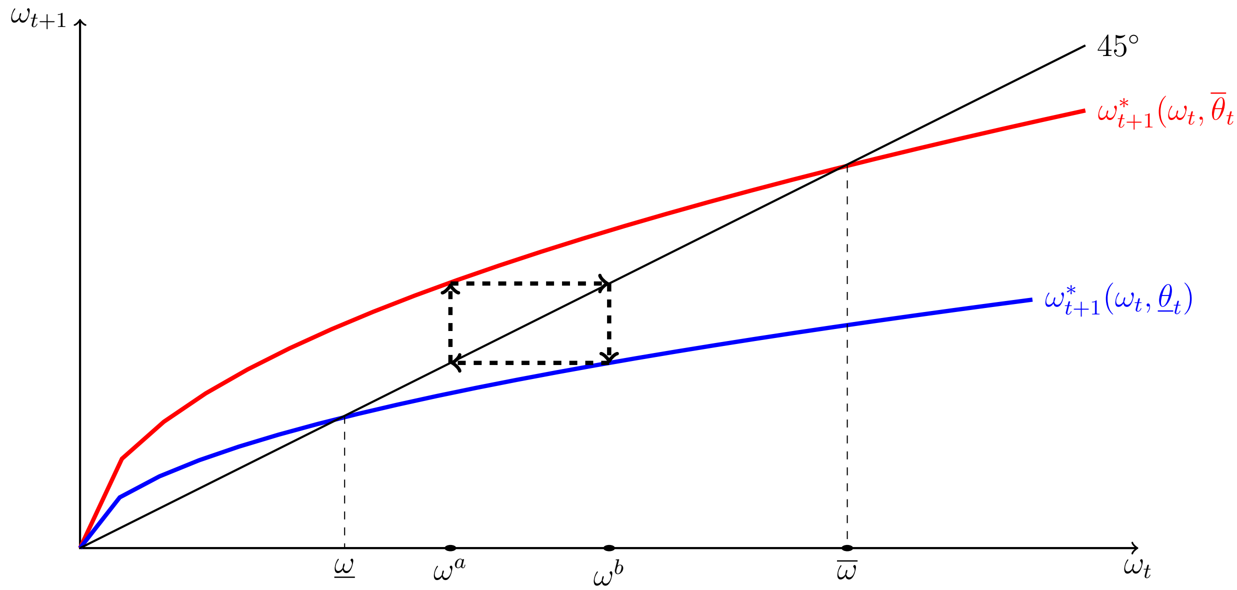


Figure 1: Stable carbon cycle on two profiles, ω^a and ω^b.

Using BAU equilibrium carbon levels in (8) an equilibrium consumption is given by

$$y_i^*(\omega_t, \theta_t) \;=\; (c_i^*(\omega_t, \theta_t))^{\theta_{it}} \, (\omega_t - C^*(\omega_t, \theta_t))^{1-\theta_{it}} \tag{9}$$

and the law of motion for the carbon stock is given by

$$\omega^*_{t+1}(\omega_t, \theta_t) \;=\; A(\omega_t - C_t^*(\omega_t, \theta_t))^{\gamma} \;=\; A\omega_t^{\gamma}(1 - \mathcal{E}^*(\theta_t))^{\gamma} \tag{10}$$

The BAU equilibrium law of motion (10) determines how the global commons adjusts over time to changes in the stock and in the elasticity profile. The law of motion in (10) converges to a stationary, ergodic process on the carbon stock. The resulting carbon cycle is illustrated in a particularly simple case. Consider a two-state stationary, irreducible Markov process on the two bounds $\underline{\theta}$ and $\overline{\theta}$ with p denoting the switching probability between the two. Figure 1 displays a literal cycle when $p = 1$, that is, when the process alternates deterministically between $\underline{\theta}$ and $\overline{\theta}$. The equilibrium dynamic then cycles between carbon stocks, ω^a and ω^b.

For a continuous Markov kernel, the carbon cycle is a limit distribution defined on a subset $[\underline{\omega}, \overline{\omega}]$ of the carbon space. Figure 2 illustrates a range of laws of motion as θ_t varies on a limit set defined by extremal profiles $\underline{\theta} = (\underline{\theta}, \ldots, \underline{\theta})$ and $\overline{\theta} = (\overline{\theta}, \ldots, \overline{\theta})$. Starting from any initial state (ω_0, θ_0), the process converges to a limit distribution on stocks lying between the two fixed points $\underline{\omega} = (A(1 - \mathcal{E}^*(\underline{\theta}))^{\gamma})^{1/(1-\gamma)}$ and $\overline{\omega} = (A(1 - \mathcal{E}^*(\overline{\theta}))^{\gamma})^{1/(1-\gamma)}$.

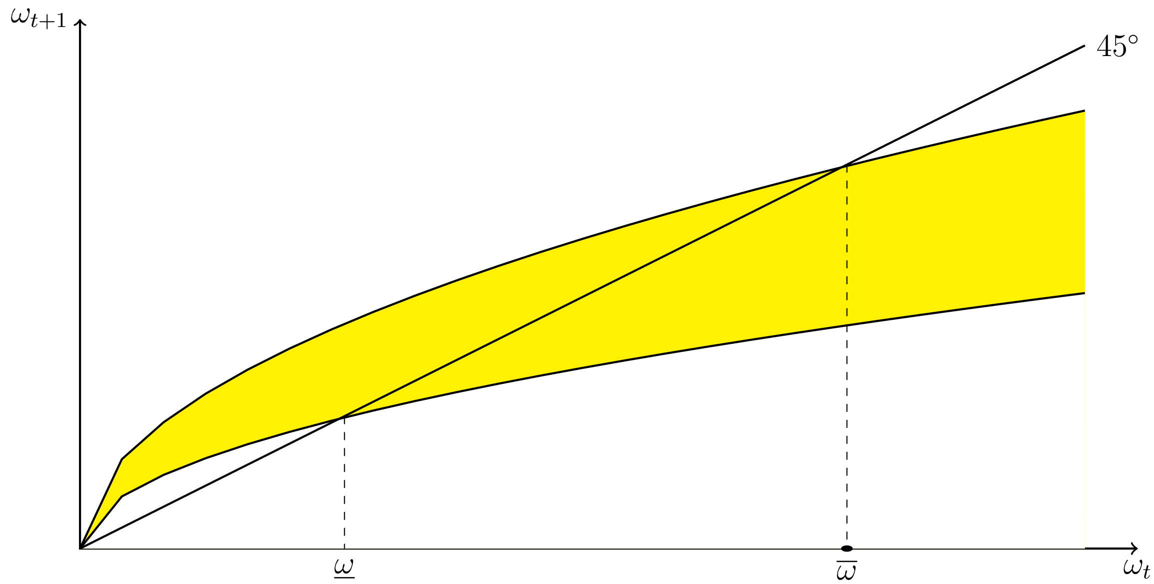


Figure 2: Equilibrium dynamic on carbon stocks as θ varies on its support.

From Equation (10), the BAU equilibrium path of carbon stocks can be computed recursively from ω_0, i.e.,

$$\omega^{*1}(\omega_0, \theta^0) = \omega_1^*(\omega_0, \theta_0), \quad \omega^{*2}(\omega_0, \theta^2) = \omega_2^*(\omega_1^*(\omega_0, \theta_0), \theta_1), \ \ldots, \quad \omega^{*t}(\omega_0, \theta^t) = \cdots \tag{11}$$

Hence, $\omega_t = \omega^{*t}(\omega_0, \theta^t)$ is the equilibrium stock achieved at t starting from stock ω_0 and given the realized history θ^t of elasticity profiles. Similarly, one can compute carbon consumption and composite output paths, $y^{*t}(\omega_0, \theta^t)$ and $c^{*t}(\omega_0, \theta^t)$, respectively.

A Numerical Illustration. To illustrate further, the BAU equilibrium time path may be calibrated using historical data for carbon consumption of the five currently largest CO_2 emitters: China, U.S.A., India, Russian Federation and Japan. These five have arguably the largest strategic influence on global carbon usage. For each of these countries we use data from the World Bank for the period between 1980 and 2009, of its emission of CO_2 per capita and population.[11] Using the data and parameters set to the following numerical values below, we back out the corresponding time path of carbon stocks and elasticities.

Parameter	Value
δ : discount factor	0.9615
ω_0: initial stock of resource	$50 \times C_0$
γ: rate of depreciation in the stock	0.99999
A: stock reabsorbtion factor	1.03

Here, ω_0 is set at the estimated oil reserves in 1980. Parameters γ and A were selected so that the net effect of depreciation/reabsorbtion process is increasing.

Figure 3 presents the calibrated BAU time path of output elasticities for carbon usage for the top two emitters, China and the U.S. The path suggests that China followed a more carbon-dependent technology over the last decade, while the U.S. displays a decreasing dependence. Before that, the U.S. has been the more intensive user of carbon (relative to the ecosystem).

A simple observation (see Equation 44 in Appendix 5.4) shows that in the BAU equilibrium, higher carbon consumption is associated with countries with higher extraction elasticities. Moreover, ceteris paribus, the bigger the carbon stock ω_t the lower the output needed to generate a given consumption level in the BAU equilibrium.

Proposition 1 also establishes the convexity of y_i^* in θ_i. Hence, composite consumption is first decreasing in θ_{it} starting from very low elasticities, reaching some minimum output corresponding to an elasticity $\hat{\theta}$ after which point output increases θ_{it}. This is displayed in Figure 4. Hence, countries with either very high or very low resource elasticities have larger consumption. Convexity also makes clear why reversing course is problematic: starting from a high elasticity θ_{it}, as a country's carbon footprint recedes, output must initially fall before growth is possible again.

[11]Data for Russian Federation before 1992 is assumed equal to per capita emissions of the USSR.

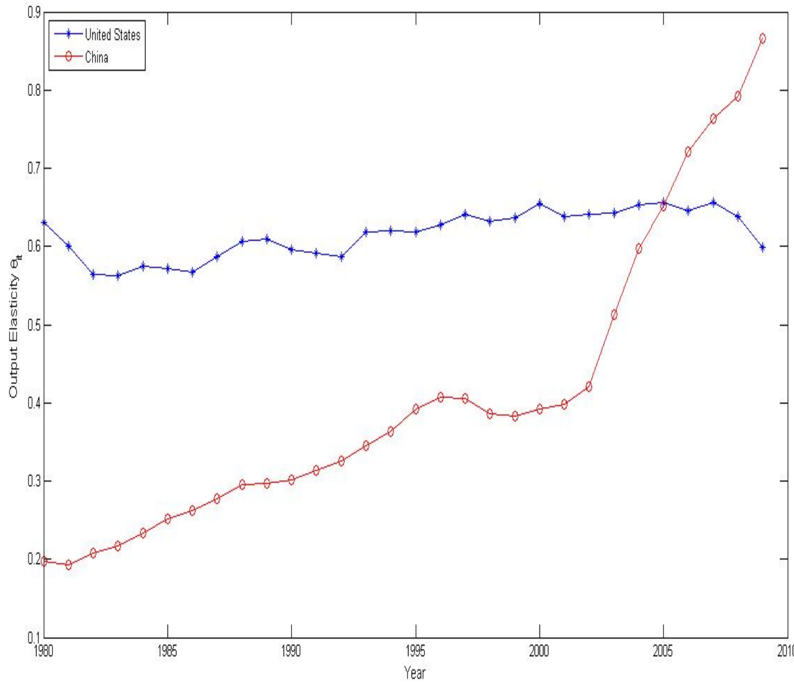


Figure 3: Calibrated equilibrium path of carbon elasticities

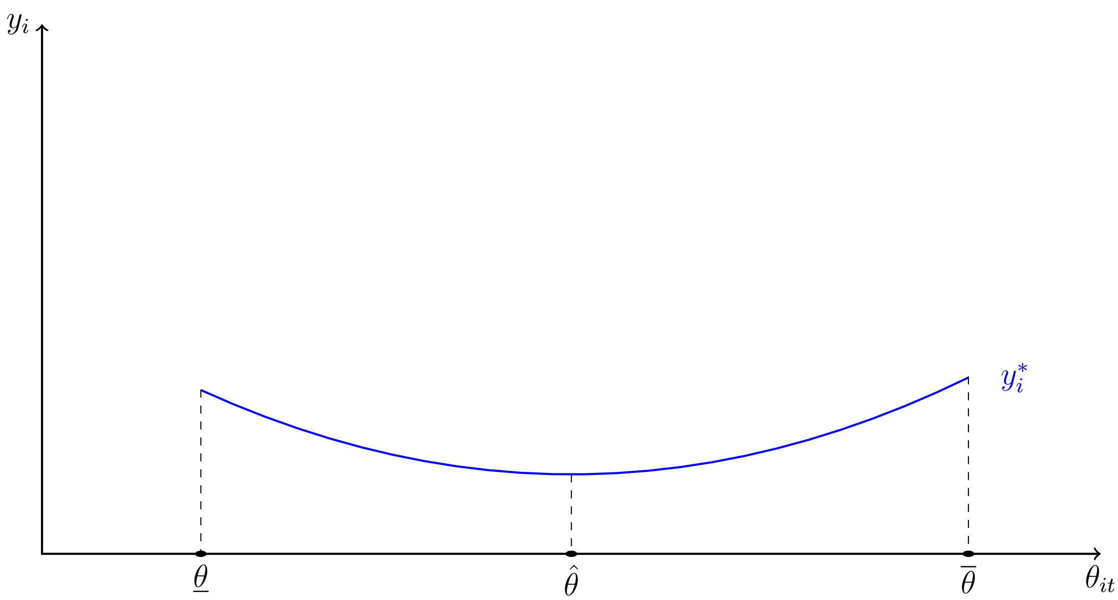


Figure 4: Output distribution across carbon elasticities

From (10), country i's growth rate may be computed as

$$\frac{y^*_{it} - y^*_{i\,t-1}}{y^*_{i\,t-1}} \;=\; \left(\frac{\omega^*_t(\omega_{t-1},\theta_{t-1})}{\omega_{t-1}}\right)\left(\frac{(e^*_i(\theta_t))^{\theta_{it}}\,(1-\mathcal{E}^*(\theta_t))^{(1-\theta_{it})}}{(e^*_i(\theta_{t-1}))^{\theta_{i\,t-1}}\,(1-\mathcal{E}^*(\theta_{t-1}))^{(1-\theta_{i\,t-1})}}\right) - \;1$$

From the right-hand side, country i's growth consists of the product of an aggregate component ω_t/ω_{t-1} determining the general global trend, and an idiosyncratic component relating to changes in a country's carbon intensity. Starting then from a common intermediate elasticity above $\hat{\theta}$ in Fig. 4, countries receiving larger shocks to their θ_i grow more quickly. Consequently, growth in the carbon stock leads to a broad positive trend in global growth, but countries with increasing carbon utilization accelerate faster.

2.4 Comparison with Efficient Carbon Usage

In this section we compare the BAU equilibrium to the optimal plan chosen by a utilitarian social planner. A Markov contingent plan, denoted here by $c^\circ(\omega_t,\theta_t) = (c^\circ_1(\omega_t,\theta_t),\ldots,c^\circ_n(\omega_t,\theta_t))$, is said to be *optimal* if it solves the planner's problem

$$\max_{\mathbf{c}^\circ}\; E\left[\sum_{i=1}^{n}\sum_{t=0}^{\infty}\;\delta^t \log\, y_{it}\bigg|\;\omega_0,\theta_0\right] \quad \text{subject to (1) and (3).} \tag{12}$$

Using the production technology in (1) to calculate an indirect payoff for each country, a solution $\mathbf{c}^\circ$ must solve the Bellman's equation

$$\sum_{i=1}^{n} U_i(\omega_t,\mathbf{c}^\circ,\theta_{it}) \;=\;$$
$$\max_{\mathbf{c}^\circ_\mathbf{t}}\;\sum_{i=1}^{n}\left\{\theta_{it}\log c_{it} + (1-\theta_{it})\log(\omega_t - C_t) \;+\; \delta\; E\left[U_j(\omega_{t+1},\mathbf{c}^\circ,\theta_{i\,t+1})\bigg|\;\omega_t,\theta_t\right]\right\} \tag{13}$$

Working, as before, with extraction rates e_{it} rather than levels, the corresponding Euler equation for country i's optimal contribution to the planner's solution is

$$\frac{\theta_{it}}{e_{it}} - \sum_{j=1}^{n}\frac{(1-\theta_{jt})}{(1-\mathcal{E}_t)} + \sum_{j=1}^{n}\delta[\frac{\partial E[U_j(\omega_{t+1},\mathbf{e},\theta_{j\,t+1})\big|\;\theta_{jt}]}{\partial\omega_{t+1}}\frac{\partial\omega_{t+1}}{\partial e_{it}}] \;=\; 0 \tag{14}$$

with $\dfrac{\partial\omega_{t+1}}{\partial e_{it}} \;=\; -A\gamma\omega_t^\gamma(1-\mathcal{E}_t)^{\gamma-1}$. Notice that the planner internalizes the effect of country i's extraction rate e_{it} on the global ecosystem that, in turn, affects all countries' welfare. Equation (14) has a stationary solution (see Appendix 5.2 for details):

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - \sum_{j=1}^{n}(1-\theta_{jt}) \;=\; \frac{nA\delta\gamma}{(1-A\delta\gamma)}$$

Since the equation is stationary, the optimal extraction rates will be as well. Solving for the optimal extraction rate yields,

$$e_i^{\circ}(\theta_{it}) \;=\; \frac{\theta_{it}(1-A\gamma\delta)}{n}, \;\text{ and }\; \mathcal{E}^{\circ}(\theta_t) \;=\; \sum_j \frac{\theta_{jt}(1-A\gamma\delta)}{n}, \tag{15}$$

As expected, each country's carbon emission is increasing in its resource elasticity, and decreasing in the effective discount factor $A\delta\gamma$. Recalling the notation $\phi_{it} \equiv \theta_{it}(1-A\gamma\delta)$, we have the following result.

Proposition 2 *The optimal contingent carbon profile* $\mathbf{c}^{\circ}$ *is given by*

$$c_i^{\circ}(\omega_t,\theta_{it}) \;=\; \frac{\phi_{it}}{n}\omega_t \tag{16}$$

for each country i*.*

Note that unlike the BAU equilibrium, the optimal plan for country i depends only on its own elasticity θ_{it}, not those of other countries.

Equation (16) determines aggregate carbon usage $C^{\circ}(\omega_t,\theta_{it}) \;=\; \sum_i \frac{\phi_{it}}{n}\omega_t$ and composite consumption

$$y_{it}^{\circ}(\omega_t,\theta_{it}) \;=\; \left(c_i^{\circ}(\omega_t,\theta_{it})\right)^{\theta_{it}}\left(\omega_t - C^{\circ}(\omega_t,\theta_{it})\right)^{(1-\theta_{it})}$$

for each i. The optimal law of motion for carbon is

$$\omega_{t+1}^{\circ}(\omega_t,\theta_t) \;=\; A(\omega_t - C^{\circ}(\omega_t,\theta_t))^{\gamma} \;=\; A\omega_t^{\gamma}(1-\mathcal{E}^{\circ}(\theta_t))^{\gamma}. \tag{17}$$

As with the BAU, the socially optimal profile $\mathbf{c}^{\circ}$ induces an equilibrium path of carbon stock $\omega^{\circ t}(\omega_0,\theta^t)$, an aggregate carbon consumption $C^{\circ t}(\omega_0,\theta^t)$, and composite consumption $y^{\circ t}(\omega_0,\theta^t)$. The comparison between the BAU equilibrium and the socially optimal plan is summarized in the proposition below.

Proposition 3 *Given BAU equilibrium* $\mathbf{c}^{*}$ *and optimal plan* $\mathbf{c}^{\circ}$*, and for any state* (ω_t,θ_t)*,*

1. *$C^*(\omega_t, \theta_t) > C^\circ(\omega_t, \theta_t)$,*

2. *For each country i, and each profile θ_{-i} of others' elasticities, there exists a cutoff carbon elasticity $\tilde{\theta}_i \in [\underline{\theta}, \overline{\theta}]$ such that for any stock ω_t, and in any date t,*

$$\begin{aligned} c_i^*(\omega_t, \theta_{it}, \theta_{-i}) \; &\geq (>) \; c_i^\circ(\omega_t, \theta_{it}) \quad if \quad \theta_{it} \; \geq (>) \; \tilde{\theta}_i, \quad and \\ c_i^*(\omega_t, \theta_{it}, \theta_{-i}) \; &\leq (<) \; c_i^\circ(\omega_t, \theta_{it}) \quad if \quad \theta_{it} \; \leq (<) \; \tilde{\theta}_i, \quad and \end{aligned}$$

3. *along any path of realized carbon elasticity profiles θ^t, the relative differences between efficient and equilibrium output $\frac{y_i^{\circ\, t}(\omega_0, \theta^t)}{y_i^{*t}(\omega_0, \theta^t)}$, carbon consumption $\frac{c_i^{\circ\, t}(\omega_0, \theta^t)}{c_i^{*t}(\omega_0, \theta^t)}$, and carbon stock $\frac{\omega^{\circ\, t}(\omega_0, \theta^t)}{\omega^{*t}(\omega_0, \theta^t)}$ all increase in t.*

The proof is in the Appendix. The Proposition demonstrates that the BAU equilibrium is always characterized by aggregate over-extraction.

Significantly, however, individual countries may over- *or* under extract in the BAU equilibrium depending on their resource elasticity. High intensity carbon users over-extract in the BAU while low intensity users may actually extract less than in the efficient plan. The possibility of under-extraction in a Markov equilibrium is unusual but not unheard of. Dutta and Sundaram (1993) show this possibility in a LM resource model. Heterogeneity plays a key role in the present model since under-extraction by low intensity carbon users occurs as a compensating response to massive over-extraction by the high intensity users. Low intensity users never fully compensate, however, since over-extraction always occurs in the aggregate.

A consequence of the Proposition is that the BAU equilibrium transition $\omega^*_{t+1}(\omega_t, \theta_t)$ on the carbon stock is lower than its efficient counterpart for every realized θ_t. This is illustrated in Figure 5. The first figure, displays two realized transition paths, one corresponding to the efficient extraction while the other corresponding to the BAU equilibrium. The second figure displays the entire supports of the transition paths, showing that efficient support first order stochastically dominates the BAU support. The difference is, in fact, increasing as asserted in part 3 of the Proposition. Consequently, inefficient carbon extraction at the global level has a cumulative effect over time. Individual output, carbon consumption, and carbon stock shrinks in the BAU equilibrium relative to that of the efficient plan.

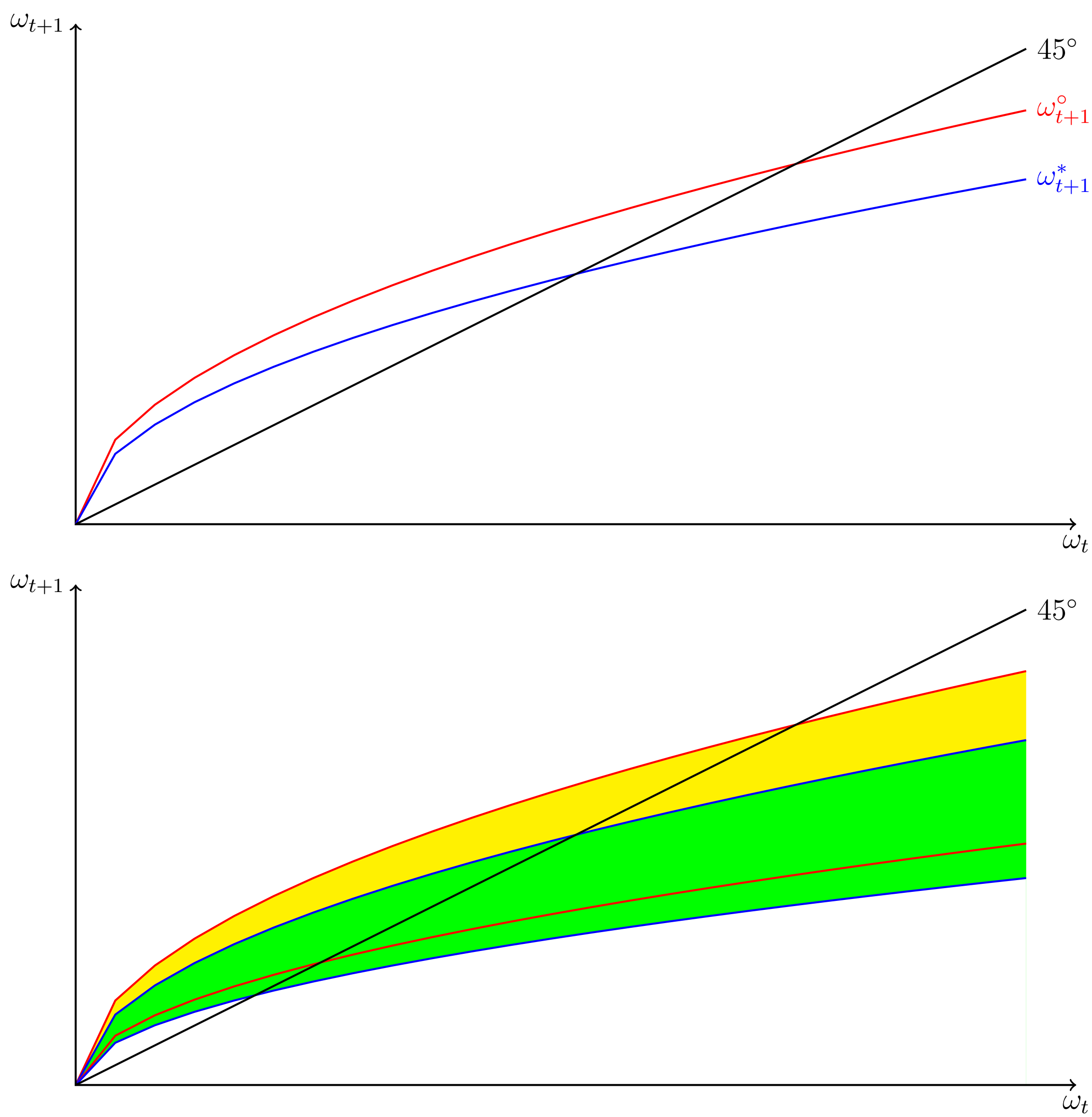


Figure 5: BAU equilibrium paths lie below optimal ones

3 A Tipping Model

In the baseline model thus far the equilibrium dynamics globally converge to a stationary distribution on both output and carbon stock. This section introduces the following non-concavity into the law of motion for carbon. The carbon stock can grow over time only if it exceeds a "recapture threshold" $b \geq 0$, and the natural sequestration process will build up the stock back up to b if the stock falls below the threshold. Expressed formally,

$$\omega_{t+1} = \begin{cases} \max\{b,\ A(\omega_t - C_t - b)^\gamma\} & if \quad \omega_t - C_t \geq b \\ b & \text{otherwise} \end{cases} \tag{18}$$

The original model was a special case where $b = 0$. The specification in (18) is a simple heuristic that exhibits the property that the carbon-based ecosystem may collapse and shrink if the stock falls below some critical carbon threshold — a *tipping point* — that is determined in part by the recapture threshold b. The threshold describes an "environmental poverty trap" since a stock that reaches b remains stuck at b forever.

This is seen clearly in Figure 6. The Figure illustrates the dynamic in (18) for two cases, one with and one without human consumption (as given by the constant C_t in the Figure). In each of these two cases there are three fixed points, one of which is unstable. The unstable fixed points determine the tipping point. In particular, if there are no shocks, i.e., if θ_t remains fixed for all time, then the tipping points are precisely the unstable fixed points in Equation (18).[12] In the general case where the path θ^∞ is stochastic, the tipping points (which we later define formally in Section 3.3) are boundaries of the support of an equilibrium distribution on carbon stocks.

The dynamic in (18) is not intended to be a literal description of an earth system. It nevertheless captures what Cai, et al. (2012) argue are two critical features that should be included in any reasonable representation of tipping. Namely, "(i) a fully stochastic formulation of abrupt changes, and (ii) a representation of the irreversibility" of the collapse.[13] The max operator in (18) forces the process to a low but finite steady state b whenever the carbon stock falls below a critical "tipping" point. The fact that the steady state b is independent of human activity is roughly consistent with simulations by Hansen et. al (2013), demonstrating a "soft" or "low-end"

[12] This is roughly consistent with tipping in the planetary boundary models of Rockstrom, et al. (2009) and Anderies, et al. (2013). It differs, however, in the sense that the fixed points in our model are endogenous. Hence, tipping is defined in reference to a particular equilibrium model of human behavior.

[13] Cai, Judd, and Lontzek (2012. p.2). In our case, (i) will come indirectly from dependence of the carbon profile c_t in equilibrium on the stochastically generated factor elasticities.

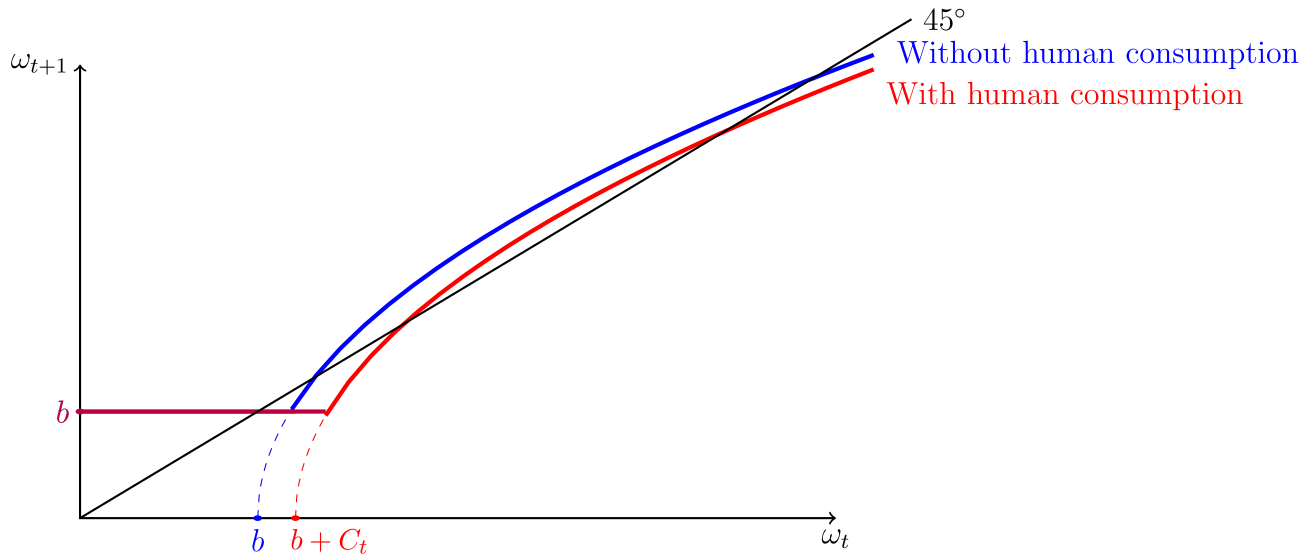


Figure 6: Carbon dynamics with and without human carbon consumption

runaway greenhouse effect. Specifically, their simulations "indicate that no plausible human-made GHG forcing can cause an instability and runaway greenhouse effect" in which extreme, amplified feedbacks fully dissipate the stored carbon stock and evaporate all planetary surface water — as believed to have happened on Venus.[14] Our specification also avoids the dilemma created in the benchmark case of $b = 0$ where full depletion yields a payoff to each country of $-\infty$, a payoff that could associated with human extinction.

3.1 BAU Equilibrium in the Tipping Model

As before equilibria may be defined in terms of extraction rates rather than levels. Consequently, a *BAU equilibrium* in the tipping model is a Markov Perfect equilibrium profile $e_i^*(\omega_t, \theta_t; b),\ i = 1, \ldots, n,\ t = 0, 1, \ldots$. As before, the BAU equilibrium solves a system of Euler equations. The derivation in this case, however, is more complicated.

Define the indicator function $1^*_{\{\omega_t, \mathcal{E}_t\}}$ taking value $1^*_{\{\omega_t, \mathcal{E}_t\}} = 1$ if $A(\omega_t(1-\mathcal{E}_t)-b)^\gamma > b$ and taking value zero otherwise. The indicator registers a value of "1" whenever

[14] Part 7 in Hansen et. al. (2013).

the recapture constraint has slack. The Euler equation is then given by

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it}) \; = $$

$$\delta\gamma\left\{\frac{\omega_t(1-\mathcal{E}_t)}{(\omega_t(1-\mathcal{E}_t)-b)}\left[1+E\left[\left(\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}} - (1-\theta_{i\,t+1})\right)1^*_{\{\omega_{t+1},\mathcal{E}_{t+1}\}}\middle|\,\omega_t,\theta_{i\,t}\right]\right]1^*_{\{\omega_t,\mathcal{E}_t\}}\right\} \tag{19}$$

A complete derivation of (19) is contained in Appendix 5.5. Equation (19) equates country i's marginal flow benefit of extraction with its marginal future cost. Equation (19) coincides with the original BAU Euler equation (14) in the benchmark model whenever $b = 0$. Generally, the equation system in (19) yields no closed form solution. Instead we show:

Proposition 4 *There is a Business-as-usual equilibrium* $\mathbf{c}^*$ *described by country-specific extraction rates* $e_i^*(\theta_t, \omega_t; b)$, $i = 1, \ldots, n$ *that are implicit functions of the forward solutions to (19), and are expressed by*

$$e_i^*(\omega_t, \theta_t; b) \;=\; \frac{\theta_{it}/(G^*(\omega_t,\theta_t,b)+1-\theta_{it})}{1+\sum_j \theta_{jt}/(G^*(\omega_t,\theta_t,b)+1-\theta_{jt})} \tag{20}$$

for each country i*, where*

$$G^*(\omega_t, \theta_t, b) \;\equiv$$

$$E\left[\sum_{\tau=0}^{\infty}(A\delta\gamma)^{\tau+1}\prod_{s=0}^{\tau}\left(\frac{\omega^{*\,t+s}(\omega_t,\theta^{t+s})(1-\mathcal{E}^*(\omega_{t+s},\theta_{t+s};b))}{\omega^{*\,t+s}(\omega_t,\theta^{t+s})(1-\mathcal{E}^*(\omega_{t+s},\theta_{t+s};b))-b}\right)1^*_{\{(\omega_{t+s},\mathcal{E}^*_{t+s})\}}\middle|\theta_t,\omega_t\right] \tag{21}$$

(where we adopt the convention in (21) that $\omega^{*t}(\omega_t, \theta^t) = \omega_t$*).*

See Appendix 5.5 for the full derivation. Here, $G^*(\omega_t, \theta_t, b)$, is the marginal future cost — the loss in expected discounted future payoffs — to country i from extracting more today. Using Eq. (6) from the benchmark model, observe that when $b = 0$, $G^*(\omega_t, \theta_t, 0) = \frac{A\delta\gamma}{1-A\delta\gamma}$.

As in the benchmark model, one may compare the BAU equilibrium to the socially efficient rate extraction rate. The latter may be derived from the planner's Euler equation and expressed as

$$e_i^\circ(\omega_t, \theta_t; b) \;=\; \frac{\theta_{it}}{n(1+G^\circ(\omega_t,\theta_t,b)\;)} \tag{22}$$

where

$$G^\circ(\omega_t, \theta_t, b) \equiv$$

$$E\left[\sum_{\tau=0}^{\infty}(A\delta\gamma)^{\tau+1}\prod_{s=0}^{\tau}\left(\frac{\omega^\circ_{t+s}(\omega_t, \theta^{t+s})(1-\mathcal{E}^\circ(\omega_{t+s}, \theta_{t+s}; b))}{\omega^\circ_{t+s}(\omega_t, \theta^{t+s})(1-\mathcal{E}^\circ(\omega_{t+s}, \theta_{t+s}; b)) - b}\right) 1^\circ_{\{(\omega_{t+s}, \mathcal{E}^\circ_{t+s})\}} \middle| \theta_t, \omega_t\right]. \tag{23}$$

The social marginal cost G° differs from the private marginal cost G^* in that it is evaluated at the socially efficient forward extraction rates $\mathcal{E}^\circ_{t+s},\ s = 1, 2, \ldots$. One can verify that the social marginal cost is higher, and that the comparison between the BAU equilibrium and the socially optimal carbon plan in Proposition 3 remains valid in the tipping model.[15] Namely, aggregate usage is higher in the BAU, though not uniformly so.

3.2 BAU Incentives in the Tipping Model

Two polar cases illustrate how the equilibrium incentives are affected by the recapture threshold b.

Suppose first that the initial stock ω_0 is sufficiently large so that along the BAU path there is no danger of the stock reaching its low steady state of b. Formally, this means $1^*_{\{\omega_t, \mathcal{E}_t\}} = 1$ with probability one in all periods t. The presence of the threshold b in the dynamic $\omega_{t+1} = A(\omega_t - C_t - b)^\gamma$ increases the marginal future cost of current extraction, and so $e^*_i(\omega_t, \theta_t; b) < e^*_i(\omega_t, \theta_t; 0)$.

Next, suppose that the stock converges to the steady state b with certainty. This occurs when, say, $1^*_{\{\omega_t, \mathcal{E}_t\}} = 0$ in the current period t. But this means $\dfrac{\partial \omega_{t+1}}{\partial e_{it}} = 0$, in other words, the country's marginal future cost of extraction is zero. Since current extraction rates do not affect future stocks, each country therefore solves a one period static problem. Each country solves its static first order condition

$$\frac{\theta_{it}}{e_{it}} - \frac{1-\theta_{it}}{1-\mathcal{E}_t} = 0$$

corresponding to the case where discount factors are 0. Country i's BAU equilibrium extraction then coincides with its one-shot optimal extraction rate: $e^*_i(\omega_t, \theta_t; b) =$

[15] Although, the threshold type at which a country over-extracts in BAU equilibrium relative to the planner's problem is generally different.

$\frac{\theta_{it}}{1-\theta_{it}} \left(1 + \sum_j \frac{\theta_{jt}}{1-\theta_{jt}}\right)^{-1}$. It follows that $e_i^*(\omega_t, \theta_t; b) > e_i^*(\omega_t, \theta_t; 0)$. Hence, if recapture constraint holds or will hold with high probability in the near future, countries have little to lose by extracting as much as possible for the present.

Taken together, the two polar cases can be summarized as follows: *the presence of a recapture threshold b reduces the incentives to over-extract, but only when the threat of actually reaching it is low.* Of course, the likelihood of reaching the threshold is endogenous. Even so, there are realized values of the elasticity path profile in which the threshold will be reached regardless of any one country's unilateral conservation efforts.

Ultimately, the question of how likely the threshold is reached is a question about tipping points, which we define in the next section. There, we show that the logic illustrated by these polar cases is broadly representative of the trade offs faced by countries in the tipping model.

3.3 Tipping Points and Collapse

The global commons in a BAU equilibrium will be said to *collapse under BAU at stock* ω_0 , if the equilibrium path of carbon stock, $\{\omega^{*t}\}$ converges to b for almost every path θ^∞ of factor profiles.[16] Formally, the commons *collapses* at ω_0 if

$$\lim_{t\to\infty} \omega^{*t}(\omega_0, \theta^t) \;=\; b \quad a.e.\;\; \theta^\infty$$

More generally, let

$$\mu(\omega_0) \;=\; \text{Prob}\left(\left\{\theta^\infty : \; \lim_{t\to\infty} \omega^{*t}(\omega_0, \theta^t) \;=\; b\right\}\right)$$

denoting the probability of collapse. Since in a collapsing commons the stock spirals downward toward threshold b, a tipping point is the largest stock from which the collapse must occur. Specifically, a *tipping point* is a carbon stock ω^{tip} satisfying

$$\begin{aligned}\omega^{tip} \;&=\; \sup\{\omega_0 : \text{ the commons under BAU collapses at } \omega_0\} \\ &=\; \sup\{\omega_0 : \; \mu(\omega_0) = 1\}.\end{aligned}$$

By these definitions, if the global commons collapses at every initial stock, then the tipping point is infinite.

[16]Recall that the equilibrium path $\{\omega^{*t}\}$ is defined recursively — see Eq. 11.

The tipping point can be distinguished from a carbon threshold above which exists a *safe operating space for humanity*, in the sense of Rockstrom et. al. (2009). In the present model, the global commons under BAU is in a *safe operating space* at ω_0 if

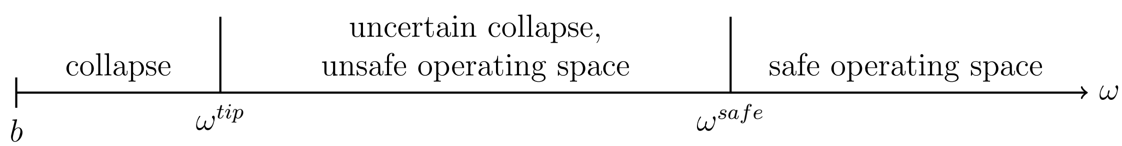
$$\lim_{t\to\infty} \omega^{*t}(\omega_0, \theta^t) \ > \ \omega^{tip} \quad a.e. \ \theta^\infty$$

This leads naturally to a notion of a *safe operating bound*, defined as a carbon stock ω^{safe} satisfying

$$\begin{aligned} \omega^{safe} &= \inf\{\omega_0 : \text{ the commons under BAU is in a safe operating space at } \omega_0\}. \\ &= \inf\{\omega_0 : \ \mu(\omega_0) = 0\}. \end{aligned}$$

Proposition 5 *Suppose that $\omega_0 > b$. Then $\mu(\omega_0)$ is weakly decreasing in ω_0.*

The proof is in the Appendix. By the Proposition, it follows that $\omega^{safe} \geq \omega^{tip}$ with strict inequality if ω^{safe} is finite and F is non-degenerate. In particular, if there is no variation in θ_t, i.e., if θ_t is constant over all t, then $\omega^{safe} = \omega^{tip}$. If the interval $(\omega^{tip}, \omega^{safe})$ is nonempty, then it consists of stocks that are neither safe nor collapsing. Rather, in this interval tipping is stochastically determined by the evolution of factor elasticities. The various regions are delineated below.



Consider, as an example a stationary Markov process on the two profiles $\{\theta', \theta''\}$. Suppose that either profile can be reached from the other every period with positive probability. There are two possibilities. Either the carbon dynamic has a finite tipping point or it does not. The case of a finite tipping point is displayed in Figure 7. Since the equilibrium carbon dynamic for both stocks has fixed points, the tipping point ω^{tip} corresponds to the lowest unstable fix point. From any stock strictly larger than ω^{tip}, the process can avoid collapse with positive probability. In particular, if it reaches stock ω^{safe}, then the commons is guaranteed to avoid collapse, thus defining the *safe operating space (SOS)* described in the planetary boundaries literature of Rockstrom et. al (2009), Anderies et. al. (2013), and others.

The case where the tipping "point" is infinite is displayed in Figure 8. In this case the parameters generate certain collapse. Specifically, from any stock ω and any

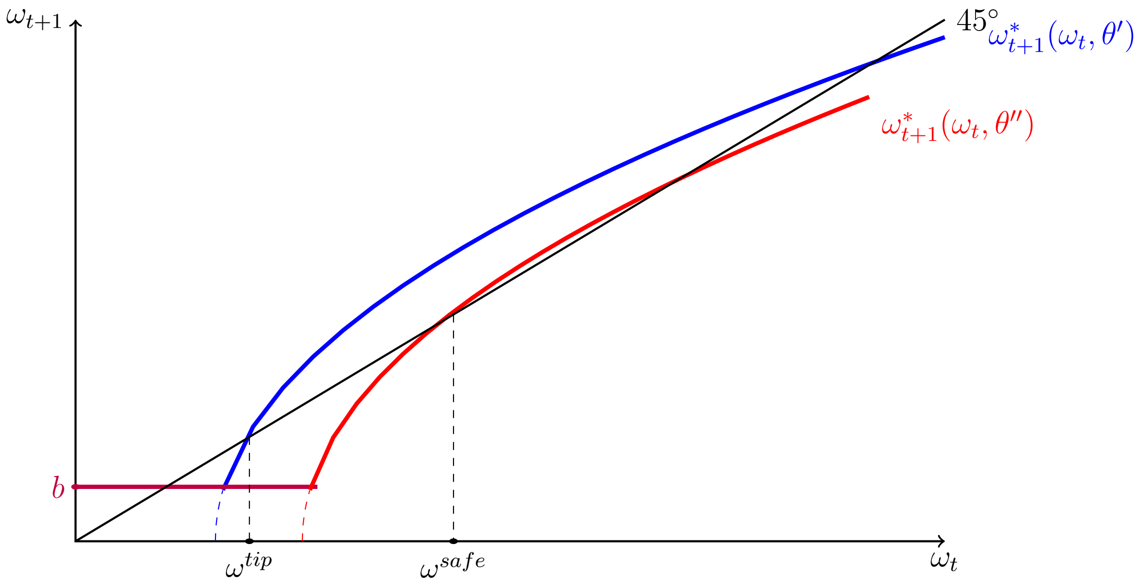


Figure 7: Carbon dynamics with tipping point is ω^{tip}

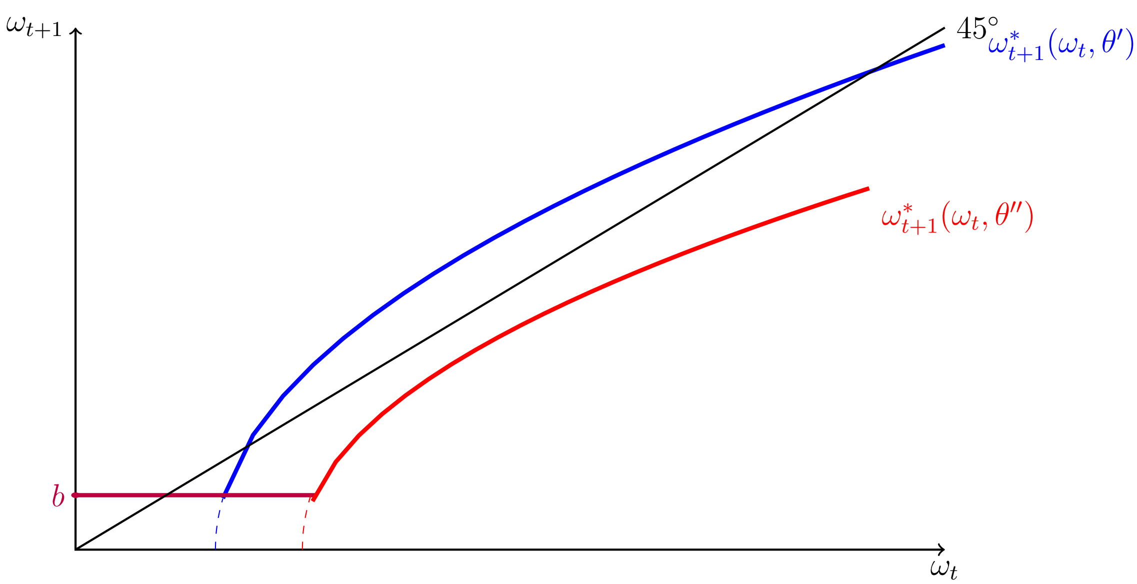
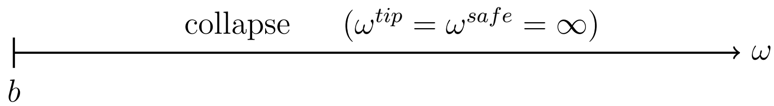


Figure 8: Carbon dynamic with certain collapse.

initial profile, for any time length T, there is a date t at which the process will remain "stuck" at θ'' for T periods starting from t. Since this occurs at infinitely many t, then for T sufficiently long the commons will eventually collapse from ω with probability one. The delineation of space of carbon stocks is, in this case, quite stark as seen below.



Applying the BAU characterization in Proposition 4 and the logic of Section 3.2, we obtain

Proposition 6 *Given the tipping model with threshold b,*

1. *For any stock $\omega > \omega^{safe}$ and for each country i, $e_i^*(\omega, \theta_t, b) < e_i^*(\omega, \theta_t, 0)$.*
2. *There exists a stock $\tilde{\omega} \leq \omega^{tip}$ such that for all $\omega < \tilde{\omega}$ and each i, $e_i^*(\omega, \theta_t, b) > e_i^*(\omega, \theta_t, 0)$.*

Hence, in the safe operating space, the presence of a tipping threshold b induces lower extraction than would be the cased without the threshold. However, there is some region below the tipping point such that the presence of a tipping threshold b induces higher extraction than would be the cased without the threshold. The proof is a straightforward application of the logic of Section 3.2.

3.4 Reaching a Tipping Point

Our main result identifies parameter configurations under which an economy will collapse, and configurations under which tipping points exist.

Theorem 1 *Suppose that $[\underline{\theta}, \overline{\theta}] = [0, 1]$. Then:*

1. *There exists an integer n' and a scalar $\epsilon > 0$ such that if $n \geq n'$ and if, for almost every θ^∞,*

$$\int_{\theta_t \in (1-\epsilon, 1]^n} dF_t(\theta_t|\theta_0) \;\geq\; \epsilon \quad \textit{for all but finitely many } t, \tag{24}$$

   then the global commons under BAU collapses at every ω_0.

2. There exists $\epsilon > 0$ *such that if, for almost every* θ^∞,

$$\theta_t \in [0, \epsilon]^n \quad \forall\, t \tag{25}$$

then the global commons has a finite tipping point ω^{tip}.

Part 1 asserts that if there are sufficiently many countries and if for almost every process the carbon elasticities will become and remain high for T periods for any T, the commons will collapse in the BAU equilibrium. Part 2 asserts that finite tipping points exist if for almost every process the carbon elasticities remain low. In that case, collapse is avoided if the stock starts out large enough. The proof is in the Appendix.

Notice that the sufficient conditions for Part 1 are, in a sense, less restrictive than those of Part 2. To guarantee at least the possibility of reaching a safe operating space, the process cannot stay very long at *any* point in time in profiles with high elasticities of extraction. By contrast, much less is required to guarantee a collapse. The process needs to hit the high range of elasticities at *some* point in time, and remain there for a while. This will be generally true of ergodic processes with full support. Alternatively, this will also hold for super martingales.

The Theorem makes clear that while the proximate cause of tipping is the depletion of the carbon stocks, the "deeper" parameters that drive the tipping and collapse are technological: the factor elasticities that determine the mix of extracted and stored carbon.

The Theorem's logic is straightforward. Let $\mathcal{E}^*(\omega_t, \theta_t, b, n)$ denote the BAU equilibrium aggregate extraction rate, expressed as a function of the relevant parameters. First, as $\theta_t \to \mathbf{0} \equiv (0, \ldots, 0)$, it happens that $\mathcal{E}^*(\omega_t, \theta_t, b, n) \to 0$. This is intuitive since in the limit elasticities are uniformly zero and so countries do not care at all about individual carbon extraction. In that case, the equilibrium law of motion equals the law of motion without human consumption — the latter always has a finite tipping point which will not be approached if the initial stock is large enough.

By contrast, observe that it is *not* true that $\mathcal{E}^*(\omega_t, \theta_t, b, n) \to 1$ as $\theta_t \to \mathbf{1} \equiv (1, \ldots, 1)$. That is, even when the eco-system is not valued at all, countries will not fully extract the stock in equilibrium. This simply because their desire smooth intertemporal consumption leads to some degree of temporal rationing. This was, in fact, first observed by Levhari and Mirman who analyzed precisely the case $\theta_t = \mathbf{1}$ (without the bound b). Nevertheless, it is easy to show that $\mathcal{E}^*(\omega_t, \theta_t, b, n) \to 1$ as both $\theta_t \to \mathbf{1}$ *and* $n \to \infty$. In other words, full extraction does occur in any commons problem when the number of participants is large enough. Thus, when the stochastic process on elasticities moves the global economy to this limiting case for a large enough period of time, a global collapse occurs.

One limitation of Theorem 1 is that it only deals with tail events, i.e., tipping properties are only stated for elasticities close to 1 or close to 0. Whether the tail events occur depends on the likelihood of collapse. Part 1, for instance, can be shown to hold if the Markov process is stationary, ergodic, and has full support. Generally, tipping is more likely in distributions that place increasing weight on higher elasticity profiles.

To see this, start with the original Markov kernel F, then let $\tilde{F}$ be another Markov kernel associated with probability $\tilde{P}$ on the same measurable space $(\Theta^\infty, \mathcal{F})$. The distribution F will be said to *dominate* $\tilde{F}$ (we write $F \succ_D \tilde{F}$) if for all t and all θ_t and all nondecreasing functions $w(\theta)$,

$$\int_{\theta_{1\,t+1}=\underline{\theta}}^{\bar{\theta}} \cdots \int_{\theta_{n\,t+1}=\underline{\theta}}^{\bar{\theta}} w(\theta_{t+1})\ dF(\theta_{t+1}|\theta_t) \ \geq\ \int_{\theta_{1\,t+1}=\underline{\theta}}^{\bar{\theta}} \cdots \int_{\theta_{n\,t+1}=\underline{\theta}}^{\bar{\theta}} w(\theta_{t+1})\ d\tilde{F}(\theta_{t+1}|\theta_t)$$

The definition above is a standard one for multivariate stochastic dominance, although there are others.[17] We use it to show that the likelihood of collapse is stochastically increasing in carbon usage elasticities.

Theorem 2 *Suppose that $\omega_0 > b$ and $F \succ_D \tilde{F}$. Then $\mu(\omega_0) \geq \tilde{\mu}(\omega_0)$ and $\omega^{tip} \geq \tilde{\omega}^{tip}$, and these inequalities are strict if $\tilde{\omega}^{tip} < \infty$.*

3.5 Tipping Points for International Agreements

It may be possible to avoid imminent collapse by having countries agree to implement the globally optimal emissions plan. At this point, another sort of tipping point becomes relevant: the threshold stock above which an optimal emissions plan can forestall collapse. The main result of this section establishes that as long as the initial carbon stock is not too low, it is always possible to construct an agreement to forestall collapse.

The policy tipping point, denoted by ω^{ptip}, is the threshold below which economy collapses under the welfare maximizing planner's solution:

$$\omega^{ptip} = \sup\{\omega_0 :\ \text{the planner's commons collapses at } \omega_0\}.$$

Tipping point ω^{ptip} a point at which it is too late for the countries to avoid collapse even if they sign on to an international agreement that implements the optimal extraction plan. Not surprisingly, one can verify that $\omega^{ptip} \leq \omega^{tip}$. The inequality, moreover, is strict, if the policy tipping point is finite and if $\underline{\theta} > 0$. This follows directly from

[17] See Zoli (2009) or Maasoumi and Yalonetzky (2013).

the fact that $\mathcal{E}^*(\omega, \theta_t; b) > \mathcal{E}^\circ(\omega, \theta_t; b)$ in any state pair (ω_t, θ_t). The policy tipping point thus provides the economy more breathing space to avoid collapse. This is especially important in the case of Part 1 in the Theorem — the parameters under which collapse is certain in the business-as-usual equilibrium. In that case, a coordinated international agreement is necessary.

Below, we prove that policy tipping points are always finite, and so it is always possible to avoid collapse if the initial carbon stock is not too low.

Theorem 3 *In any global commons, the policy tipping point ω^{ptip} is finite.*

As before, the proof is in the Appendix. The policy tipping point thus leads to a more nuanced delineation of the state space as follows

Notice, moreover, that for any stock ω_t with $\omega^{ptip} < \omega_t < \omega^{tip}$, the international community has a stochastic but finite period of time to implement an optimal international agreement in order to avert a collapse.

4 Conclusion

This paper formulates a model of global carbon consumption that integrates strategic incentives of countries into a dynamic model of nonlinear carbon emissions. Our focus is specifically on the strategic interaction among the largest players — the countries themselves. The objective is to understand the strategic incentives to restrict carbon under "business-as-usual" (or "BAU"). In particular, are outcomes under BAU sustainable or is economic collapse inevitable? What determines the transition, if any, from sustainability to collapse?

To evaluate these issues, the papers models a world in which a country's GDP depends on both its carbon usage and on the preservation of the global ecosystem. Each country therefore faces a trade off between, on the one hand, extracting and emitting carbon, and on the other, maintaining a stock of stored or "unextracted" carbon to preserve a healthy ecosystem. Countries naturally differ in how they evaluate this trade off, and even the same country can make different trade offs at different points in time, depending on economic shocks.

The results describe scenarios in which consumption and economic output may collapse and shrink if the carbon stock sustaining the ecosystem falls below some critical threshold — a tipping point. The results delineate between stocks the guarantee a safe operating space for humanity from carbon stocks in which tipping *can* occur. In turn, stocks in which tipping can occur are delineated from those in which tipping *must* occur. These distinctions are roughly consistent with certain *planetary boundaries* as defined by Rockstrom et. al. (2009).

In an unsettling result, we show that if the there are sufficiently many participants in the BAU and if output elasticity of extracted carbon is high enough for a long enough time period, a tipping point will certainly be breached. Unfortunately, it appears that the trend is toward both conditions coming true. A preliminary but suggestive calibration of carbon elasticities shows a noticeable increase over the last twenty-five years and, moreover, the ongoing devolution (i.e., the partitioning large nations into ever smaller ones) has dramatically increased the number of nations since World War II, each making separate carbon decisions. Our future work is directed toward calibrating the tipping process explicitly given the time series on global emissions.

The silver lining is that even in this case, there remains a small window in which tipping may be averted if the countries can depart from BAU and sign on to an effective international treaty to limit emissions.

5 Appendix

5.1 Derivation of the BAU equilibrium for Proposition 1 proof

Starting from the Euler equation,

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} + \delta\Big[\frac{\partial E[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1}) \big| \, \theta_{it}]}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_{it}}\Big] \;\; = \;\; 0$$

with $\dfrac{\partial \omega_{t+1}}{\partial e_{it}} = -A\gamma\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1}$. Then the Euler equation is

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} \; = \; A\gamma\delta[\frac{\partial E[U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})\big|\, \theta_{it}]}{\partial \omega_{t+1}}\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1}]. \qquad (26)$$

Differentiating the value function $U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})$ with respect to ω_{t+1}

$$\frac{\partial E[U_j(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})\big|\, \theta_{i\,t}]}{\partial \omega_{t+1}} \; = \; E\left[\frac{1}{\omega_{t+1}} \; + \; A\delta\gamma\frac{\partial E[U_i(\omega_{t+2},\mathbf{e},\theta_{i\,t+2})\big|\, \theta_{i\,t+1}]}{\partial \omega_{t+2}}\omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma}\Big|\, \theta_{i\,t+1}\right].$$

Substituting this into the first order condition (26), we obtain

$$\begin{aligned}
&\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} \\
= \;& A\delta\gamma\left\{E\left[\frac{1}{\omega_{t+1}} \; + \; \frac{\partial E[U_i(\omega_{t+2},\mathbf{e},\theta_{i\,t+2})\big|\, \theta_{i\,t+1}]}{\partial \omega_{t+2}}\omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma}\Big|\, \theta_{i\,t}\right]\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1}\right\} \\
= \;& A\gamma\delta\left\{\frac{\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma}}{\omega_{t+1}(1-\mathcal{E}_t)} + E\left[\frac{\partial E[U_i(\omega_{t+2},\mathbf{e},\theta_{i\,t+2})\big|\, \theta_{i\,t+1}]}{\partial \omega_{t+2}}\omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma}\Big|\, \theta_{i\,t}\right]\frac{\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma}}{1-\mathcal{E}_t}\right\} \\
= \;& \frac{1}{1-\mathcal{E}_t}A\delta\gamma\left\{(1+E\left[\frac{\partial E[U_i(\omega_{t+2},\mathbf{e},\theta_{i\,t+2})\big|\, \theta_{i\,t+1}]}{\partial \omega_{t+2}}\omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma}\Big|\, \theta_{i\,t}\right]\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma}\right\} \\
= \;& \frac{1}{1-\mathcal{E}_t}A\delta\gamma\left(1+[E\left[(\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}} - (1-\theta_{i\,t+1}))]\Big|\, \theta_t\right]\right).
\end{aligned}$$

Reorganizing terms, we obtain the Euler equation

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it}) \; = \; A\delta\gamma + A\delta\gamma[E\left[(\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}} - (1-\theta_{i\,t+1}))]\Big|\, \theta_t\right]. \quad (27)$$

Define $\hat{s}_{it} \;\equiv\; \frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it})$. Then

$$\hat{s}_{it} = A\delta\gamma + A\delta\gamma E[\hat{s}_{it+1}\big|\, \theta_t].$$

Forward iteration yields a steady state $\hat{s}_{it} = \frac{A\delta\gamma}{(1-A\delta\gamma)}$, and plugging in the last equation we obtain a stationary solution to the Euler equation for country i:

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} \;-\; (1-\theta_{it}) \;=\; \frac{A\delta\gamma}{1-A\delta\gamma} \quad i=1,2,\ldots,n. \tag{28}$$

Solving for e_{it} directly, if $\mathcal{E}_t \neq 1$ and $\frac{1-\theta_{it}(1-A\delta\gamma)}{\theta_{it}(\mathcal{E}_t-1)(A\delta\gamma-1)} \neq 0$, we get:

$$e_{it} = (1-\mathcal{E}_t)\,\frac{1}{\frac{1}{\theta_{it}}\,(1-\theta_{it}) + \frac{A\delta\gamma}{\theta_{it}(1-A\delta\gamma)}} \text{ for } i=1,2...n.$$

Adding up these equations, the aggregate extraction rate is:

$$\begin{aligned}
\mathcal{E}_t &= \sum_{i=1}^{n}(1-\mathcal{E}_t)\,\frac{1}{\frac{1}{\theta_{it}}\,(1-\theta_{it}) + \frac{A\delta\gamma}{\theta_{it}(1-A\delta\gamma)}} \\
&= \sum_{i=1}^{n}\frac{1}{\frac{1}{\theta_{it}}\,(1-\theta_{it}) + \frac{A\delta\gamma}{\theta_{it}(1-A\delta\gamma)}} - \mathcal{E}_t\sum_{i=1}^{n}\frac{1}{\frac{1}{\theta_{it}}\,(1-\theta_{it}) + \frac{A\delta\gamma}{\theta_{it}(1-A\delta\gamma)}} \\
&= \frac{\sum_{i=1}^{n}\frac{1}{\frac{1}{\theta_{it}}(1-\theta_{it})+\frac{A\delta\gamma}{\theta_{it}(1-A\delta\gamma)}}}{1+\sum_{i=1}^{n}\frac{1}{\frac{1}{\theta_{it}}(1-\theta_{it})+\frac{A\delta\gamma}{\theta_{it}(1-A\delta\gamma)}}}
\end{aligned}$$

Substituting this in the equation for e_{it}, we obtain the MPE extraction rate,

$$\begin{aligned}
e^*_{it}(\theta_t) &= \left(1-\frac{\sum_{j=1}^{n}\frac{1}{\frac{1}{\theta_{jt}}(1-\theta_{jt})+\frac{A\gamma\delta}{\theta_{jt}(1-A\gamma\delta)}}}{1+\sum_{j=1}^{n}\frac{1}{\frac{1}{\theta_{jt}}(1-\theta_{jt})+\frac{A\gamma\delta}{\theta_{jt}(1-A\gamma\delta)}}}\right)\frac{1}{\frac{1}{\theta_{it}}\,(1-\theta_{it}) + \frac{A\gamma\delta}{\theta_{it}(1-A\gamma\delta)}} \\
&= \frac{\left(\frac{\theta_{it}(1-A\gamma\delta)}{1-\theta_{it}(1-A\gamma\delta)}\right)}{1+\left(\sum_{j=1}^{n}\frac{\theta_{jt}(1-A\gamma\delta)}{1-\theta_{jt}(1-A\gamma\delta)}\right)} \;=\; \frac{\left(\frac{\phi_{it}}{1-\phi_{it}}\right)}{1+\left(\sum_{j=1}^{n}\frac{\phi_{jt}}{1-\phi_{jt}}\right)}
\end{aligned} \tag{29}$$

where $\phi_{it} = \theta_{it}(1-A\gamma\delta)$. This we prove that $c^*_{it}(\theta_t)$ is given by Equation (8).

Finally, as for the convexity of y^*_i in θ_{it}, observe that

$$y^*_i(\omega_t,\theta_t) \;=\; \omega_t\left(\frac{\left(\frac{\phi_{it}}{1-\phi_{it}}\right)}{1+\left(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}}\right)}\right)^{\theta_{it}}\left(1-\frac{\sum_j\left(\frac{\phi_{jt}}{1-\phi_{jt}}\right)}{1+\left(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}}\right)}\right)^{(1-\theta_{it})}$$

which can be expressed purely as a function of θ_{it}, as

$$y_i^*(\omega_t, \theta_t) \;=\; \omega_t \,(A\theta_{it})^{\theta_{it}}\,(1 - B\theta_{it})^{(1-\theta_{it})}$$

where A and B are positive constants (implicitly functions of $\theta_{-i\,t}$). Then

$$Dy_i^*(\omega_t, \theta_t) \;=\; y_i^*(\omega_t, \theta_t)\left[\log(\frac{A\theta_{it}}{1 - B\theta_{it}}) + 1 - \frac{B(1-\theta_{it})}{1 - B\theta_{it}}\right]$$

which has a unique critical point and is increasing in θ_{it}.

5.2 Derivation of Planner's Solution for Proposition 2 proof

Starting from the Euler equation

$$\frac{\theta_{it}}{e_{it}} - \sum_{j=1}^{n}\frac{(1-\theta_{jt})}{(1-\mathcal{E}_t)} + \sum_{j=1}^{n}\delta[\frac{\partial E[U_j(\omega_{t+1}, \mathbf{e}, \theta_{j\,t+1})\Big|\,\theta_{jt}]}{\partial \omega_{t+1}}\frac{\partial \omega_{t+1}}{\partial e_{it}}] \;=\; 0$$

with $\dfrac{\partial \omega_{t+1}}{\partial e_{it}} = -A\gamma\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1}$, then the Euler equation becomes

$$\frac{\theta_{it}}{e_{it}} - \sum_{j=1}^{n}\frac{(1-\theta_{jt})}{(1-\mathcal{E}_t)} \;=\; A\gamma\delta\sum_{j=1}^{n}[\frac{\partial E[U_j(\omega_{t+1}, \mathbf{e}, \theta_{j\,t+1})\Big|\,\theta_{jt}]}{\partial \omega_{t+1}}\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1}]. \qquad (30)$$

Differentiating the value function $U_j(\omega_{t+1}, \mathbf{e}, \theta_{j\,t+1})$ with respect to ω_{t+1} yields,

$$\frac{\partial E[U_j(\omega_{t+1}, \mathbf{e}, \theta_{j\,t+1})\Big|\,\theta_{j\,t}]}{\partial \omega_{t+1}} \;=\; E\left[\frac{1}{\omega_{t+1}} \;+\; A\delta\gamma\frac{\partial E[U_j(\omega_{t+2}, \mathbf{e}, \theta_{j\,t+2})\Big|\,\theta_{j\,t+1}]}{\partial \omega_{t+2}}\omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma}\Big|\,\theta_{j\,t+1}\right].$$

Substituting this into the first order condition, we obtain

$$\begin{aligned}
&\frac{\theta_{it}}{e_{it}} - \sum_{j=1}^{n} \frac{(1-\theta_{jt})}{(1-\mathcal{E}_t)} \\
= \;& A\delta\gamma \sum_{j=1}^{n} \left\{ E\left[\frac{1}{\omega_{t+1}} + \frac{\partial E[U_j(\omega_{t+2}, \mathbf{e}, \theta_{j\,t+2}) \big| \, \theta_{j\,t+1}]}{\partial \omega_{t+2}} \omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma} \Big| \, \theta_{j\,t} \right] \omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma-1} \right\} \\
= \;& \frac{1}{1-\mathcal{E}_t} A\gamma\delta \sum_{j=1}^{n} \left\{ \frac{\omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma}}{\omega_{t+1}} + \right. \\
& \left. E\left[\frac{\partial E[U_j(\omega_{t+2}, \mathbf{e}, \theta_{j\,t+2}) \big| \, \theta_{j\,t+1}]}{\partial \omega_{t+2}} \omega_{t+1}^{\gamma-1}(1-\mathcal{E}_{t+1})^{\gamma} \Big| \, \theta_{j\,t} \right] \omega_t^{\gamma}(1-\mathcal{E}_t)^{\gamma} \right\} \\
= \;& \frac{1}{1-\mathcal{E}_t} \left(nA\delta\gamma + A\delta\gamma \sum_{j=1}^{n} E\left[\frac{\partial E[U_j(\omega_{t+2}, \mathbf{e}, \theta_{j\,t+2}) \big| \, \theta_{j\,t+1}]}{\partial \omega_{t+2}} \omega_{t+1}^{\gamma}(1-\mathcal{E}_{t+1})^{\gamma} \Big| \, \theta_{j\,t} \right] \right) \\
= \;& \frac{1}{1-\mathcal{E}_t} \left(nA\delta\gamma + A\delta\gamma \sum_{j=1}^{n} [E\left[(\frac{\theta_{j\,t+1}(1-\mathcal{E}_{t+1})}{e_{j\,t+1}} - \sum_{j=1}^{n}(1-\theta_{j\,t+1}))] \Big| \, \theta_{j\,t} \right] \right).
\end{aligned}$$

Reorganizing terms, we obtain

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - \sum_{j=1}^{n}(1-\theta_{jt}) \; = \; nA\delta\gamma + A\delta\gamma \sum_{j=1}^{n}[E\left[(\frac{\theta_{j\,t+1}(1-\mathcal{E}_{t+1})}{e_{j\,t+1}} - \sum_{j=1}^{n}(1-\theta_{j\,t+1}))] \Big| \, \theta_{j\,t} \right]. \tag{31}$$

Define $s_{it} \;\equiv\; \frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - \sum_{j=1}^{n}(1-\theta_{jt})$. Then

$$s_{it}(\theta_{it}) = nA\delta\gamma + A\delta\gamma E[s_{i\,t+1}(\theta_{i\,t+1}) \big| \, \theta_{it}].$$

Forward iteration yields a steady state $s_{it} = \frac{nA\delta\gamma}{(1-A\delta\gamma)}$ and plugging in the last equation we obtain

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - \sum_{j=1}^{n}(1-\theta_{jt}) \; = \; \frac{nA\delta\gamma}{(1-A\delta\gamma)}.$$

Then:

$$\begin{aligned}
\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - \sum_{j=1}^{n}(1-\theta_{jt}) &= \frac{nA\delta\gamma}{(1-A\delta\gamma)} && (32)\\
\theta_{it}(1-\mathcal{E}_t)(1-A\delta\gamma) - e_{it}\sum_{j=1}^{n}(1-\theta_{jt})(1-A\gamma\delta) &= e_{it}A\gamma\delta n\\
\theta_{it}(1-\mathcal{E}_t)(1-A\gamma\delta) - e_{it}(n-\sum_{j}\theta_{jt})(1-A\gamma\delta) &= e_{it}A\gamma\delta n && (33)\\
\sum_{j}\theta_{jt}(1-\mathcal{E}_t)(1-A\gamma\delta) - (1-A\gamma\delta)\mathcal{E}_t(n-\sum_{j}\theta_{jt}) - \mathcal{E}_tA\gamma\delta n &= 0\\
\sum_{j}\theta_{jt} - n\mathcal{E}_t - \sum_{j}\theta_{jt}A\gamma\delta &= 0\\
\mathcal{E}_t^{\circ} &= \frac{\sum_j \phi_{jt}}{n}
\end{aligned}$$

using Equations (32) and (33) we get

$$e_i^{\circ}(\theta_{it}) = \frac{\phi_{it}}{n} \tag{34}$$

thus verifying Equation (16) in the Propostion.

5.3 Proof of Proposition 3

Part 1. Global Over-extraction. We must show $\mathcal{E}^{o}(\theta) > \mathcal{E}^{\circ}(\theta)$ for all θ.

$$\begin{aligned}
\sum_{i=1}^{n}\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{i=1}^{n}\frac{\phi_{it}}{1-\phi_{it}})} &> \sum_{i=1}^{n}\frac{\phi_{it}}{n}\\
\frac{1}{1+(\sum_{i=1}^{n}\frac{\phi_{it}}{1-\phi_{it}})}\sum_{i=1}^{n}(\frac{\phi_{it}}{1-\phi_{it}}) &> \frac{1}{n}\sum_{i=1}^{n}\frac{\phi_{it}}{n}\\
n\sum_{i=1}^{n}(\frac{\phi_{it}}{1-\phi_{it}}) &> (1+(\sum_{i=1}^{n}\frac{\phi_{it}}{1-\phi_{it}}))\sum_{i=1}^{n}\phi_{it}\\
n\sum_{i=1}^{n}(\frac{\phi_{it}}{1-\phi_{it}}) &> \sum_{i=1}^{n}\phi_{it} + \sum_{i=1}^{n}\phi_{it}\sum_{i=1}^{n}\frac{\phi_{it}}{1-\phi_{it}}\\
\sum_{i=1}^{n}(\frac{\phi_{it}}{1-\phi_{it}})(n-\sum_{i=1}^{n}\phi_{it}) &> \sum_{i=1}^{n}\phi_{it}\\
\sum_{i=1}^{n}(\frac{\phi_{it}}{1-\phi_{it}}) &> \sum_{i=1}^{n}\frac{\phi_{it}}{(n-\sum_{i=1}^{n}\phi_{it})}.
\end{aligned}$$

So it is enough to prove that this last inequality is true for each i for whom $\phi_{it} > 0$. Specifically, we verify

$$
\begin{aligned}
(\frac{\phi_{it}}{1-\phi_{it}}) &> \frac{\phi_{it}}{(n-\sum_{i=1}^{n}\phi_{it})} \\
(\frac{1}{1-\phi_{it}}) &> \frac{1}{(n-\sum_{i=1}^{n}\phi_{it})} \\
n-\sum_{i=1}^{n}\phi_{it} &> 1-\phi_{it} \\
n-1-\sum_{j\neq i}\phi_{jt}+1-\phi_{it} &> 1-\phi_{it} \\
n-1-\sum_{j\neq i}\phi_{jt} &> 0
\end{aligned}
$$

but note that

$$
\begin{aligned}
n-1-\sum_{j\neq i}\phi_{jt} &\geq n-1-\sum_{j\neq i}^{n}\overline{\theta}(1-\gamma\delta) \\
n-1-\sum_{j\neq i}\phi_{jt} &> (n-1)(1-\overline{\theta}(1-\gamma\delta))>0 \\
&\Rightarrow n-1-\sum_{j\neq i}^{n}\phi_{jt}>0.
\end{aligned}
$$

Hence, at the global level there is over-extraction.

Part 2. Over- and Under-extraction by Individual Countries. To evaluate whether a country over or under extracts in the BAU equilibrium, one need only compare e°_{it} to e^{*}_{it}. Country over (under) extracts if $e^{*}_{it} > (<) e^{\circ}_{it}$. We therefore compare:

$$
e^{\circ}_{it}(\theta_t)=\frac{\theta_{it}(1-A\gamma\delta)}{n}=\frac{\phi_{it}}{n} \overset{?}{>} e^{o}_{it}(\theta_t)=\frac{(\frac{\theta_{it}(1-A\gamma\delta)}{1-\theta_{it}(1-A\gamma\delta)})}{1+(\sum_{j=1}^{n}\frac{\theta_{jt}(1-A\gamma\delta)}{1-\theta_{jt}(1-A\gamma\delta)})}=\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}^{n}\frac{\phi_{jt}}{1-\phi_{jt}})}
$$

with, recall, $\phi_{it}=\theta_{it}(1-A\gamma\delta)$. Since $\phi_{it}>0$, country i over-extracts if

$$\frac{(\frac{1}{1-\phi_{it}})}{1+(\sum_{j=1}^{n}\frac{\phi_{jt}}{1-\phi_{jt}})} \ > \ \frac{1}{n}\ ,$$

and solving for ϕ_{it}, country i will over (under) extract if

$$\phi_{it} \ > \ (<)\ 1-\frac{n-1}{\sum_{j\neq i}\frac{\phi_{jt}}{1-\phi_{jt}}} \tag{35}$$

By choosing $\tilde{\theta}$ such that $\tilde{\theta}(1-A\delta\gamma)$ equals the right hand side of (35), we have found out threshold.

Notice, moreover, that the larger the profile of her opponents the (weakly) smaller is the set of types for which is optimal to her over extract.[18]

Part 3. Output paths. The final part must prove that relative output, carbon consumption and carbon stock shrinks in the BAU relative to that of the efficient plan.

We first compute the optimal carbon path by iterating on the optimal dynamic law of motion in (17), and setting $\phi_{it}=\theta_{it}(1-A\delta\gamma)$ as before, one can display the

[18]Example: suppose a symmetric profile ϕ_{-i}, i.e. $\phi_j=\phi_k$ for all $k,j\neq i$.

$$\phi_i > 1-\frac{n-1}{\sum_{j\neq i}\frac{\phi_j}{1-\phi_j}} = 1-\frac{n-1}{(n-1)\frac{\phi}{1-\phi}} = \frac{2\phi-1}{\phi}. \tag{36}$$

Note that the extreme (highest) profile player i can be facing is a profile of opponents with the highest type, *i.e.* $\theta_j=\overline{\theta}<1$ for all $j\neq i$. Then from equation 36

$$\theta_i > \frac{2\overline{\theta}(1-A\gamma\delta)-1}{\overline{\theta}(1-A\gamma\delta)^2}$$

or

$$\phi_i > \frac{2\overline{\phi}-1}{\overline{\phi}}.$$

So if we require all θ_i over extract, the condition is:

$$\underline{\theta} > \frac{2\overline{\theta}(1-A\gamma\delta)-1}{\overline{\theta}(1-A\gamma\delta)^2}.$$

This implies the following sufficient condition: if $A\delta\gamma \geq \frac{1}{2}$ all types θ_i over extract.

time path of the carbon stock in the planner's optimum as

$$\begin{aligned} \omega^{*\,t}(\omega_0, \theta^t) &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} (1 - \mathcal{E}^{\circ}_{t-\tau}(\theta_{t-\tau}))^{\gamma^\tau} \\ &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} \left(1 - \frac{\sum_j \phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau} . \end{aligned} \tag{37}$$

Using (15) and (37), a country's output path in the social planner's problem is given by

$$y_i^{*t} \;=\; \left(\frac{\phi_{i\,t}}{n}\right)^{\theta_{it}} \left(1 - \frac{\sum_j \phi_{j\,t}}{n}\right)^{(1-\theta_{it})} \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} \left(1 - \frac{\sum_j \phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau} . \tag{38}$$

A particularly useful illustration of (37) is the case without shocks. In that case $\theta_t = \theta_{t'} = \theta$ and so (37) reduces to

$$\omega^{*t}(\omega_0, \theta^t) = \omega_0^{\gamma^t} \left(1 - \frac{\sum_j \phi_j}{n}\right)^{\frac{\gamma(1-\gamma^t)}{1-\gamma}} A^{\frac{1-\gamma^t}{1-\gamma}} \tag{39}$$

in which case the output path simplifies to

$$y_i^{*t} \;=\; \left(\frac{\phi_i}{n}\right)^{\theta_i} \left(1 - \frac{\sum_j \psi_j}{n}\right)^{(1-\theta_i)} \omega_0^{\gamma^t} \left(1 - \frac{\sum_j \phi_j}{n}\right)^{\frac{\gamma(1-\gamma^t)}{1-\gamma}} A^{\frac{1-\gamma^t}{1-\gamma}}. \tag{40}$$

These paths may be compared to the BAU equilibrium. Iterating on the equilibrium law of motion, one derives the time path of the carbon stock as

$$\begin{aligned} \omega^{*t}(\omega_0, \theta^t) &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} (1 - \mathcal{E}^{*}_{t-\tau}(\theta_{t-\tau}))^{\gamma^\tau} \\ &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} \left(1 - \frac{\sum_j (\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1 + (\sum_j \frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau} . \end{aligned} \tag{41}$$

A country's output path in the BAU equilibrium is given by

$$\begin{aligned}
y_i^{*t}(\omega_0,\theta^t) &= \left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{\theta_{it}}\left(1-\frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{(1-\theta_{it})}\omega^{*t}(\omega_0,\theta^t) \\
&= \left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{\theta_{it}}\left(1-\frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{(1-\theta_{it})}\times \\
&\quad \omega_0^{\gamma^t}A^{\frac{1-\gamma^t}{1-\gamma}}\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau}.
\end{aligned} \tag{42}$$

Comparing the BAU in (42) with the optimal output in (38). We see that $y_{it}^{*} < y_{it}^{\circ}$ iff

$$\begin{aligned}
&\left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{\theta_{it}}\left(1-\frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{(1-\theta_{it})}\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau} \\
< \; &\left(\frac{\phi_{it}}{n}\right)^{\theta_{it}}\left(\frac{1-\sum_j\phi_{jt}}{n}\right)^{(1-\theta_{it})}\prod_{\tau=1}^{t}\left(1-\frac{\sum_j\phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}.
\end{aligned}$$

In order to evaluate the relative growth in output paths, we compare:

$$\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau} < \prod_{\tau=1}^{t}\left(1-\frac{\sum_j\phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}$$

which holds due to the fact that the aggregate extraction rate is larger (hence conservation rate is smaller) in the MPE. Moreover the relative difference

$$\prod_{\tau=1}^{t}\left(1-\frac{\sum_j\phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}\Big/\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau}$$

is increasing as time passes. Hence, both the expected ratio $E[\frac{y_{it}^{\circ}}{y_{it}^{*}}]$ and the expected difference $E[y_{it}^{\circ}-y_{it}^{*}]$ are increasing in t.

5.4 Generating Output Elasticities from the BAU Equilibrium

For each country and for a given period of time, output elasticities are calibrated in a simple way. Recall that under BAU equilibrium carbon consumption is characterized by the stationary extraction rates:

$$\begin{aligned} c_{it} &= \omega_t \cdot e_{it} \\ &= \omega_t \cdot \frac{\frac{\phi_{it}}{1-\phi_{it}}}{1+\sum_i \frac{\phi_{it}}{1-\phi_{it}}}. \end{aligned}$$

Which implies

$$\frac{\phi_{it}}{1-\phi_{it}} = \frac{c_{it}}{\omega_t}(1+\sum_i \frac{\phi_{it}}{1-\phi_{it}}) \tag{43}$$

Adding up the last equation over the set of countries we get:

$$\begin{aligned} \sum_i \frac{\phi_{it}}{1-\phi_{it}} &= \sum_i \frac{c_{it}}{\omega_t}(1+\sum_i \frac{\phi_{it}}{1-\phi_{it}}) \\ &= \frac{C_t}{(\omega_t - C_t)} \end{aligned}$$

plugging in to equation 43 and solving for ϕ_{it}, we get

$$\phi_{it} = \frac{c_{it}}{(\omega_t - C_{-it})}$$

where $C_{-it} = \sum\limits_{j \neq i} c_{jt}$.

Finally the output elasticity for country i in period t is given by

$$\theta_{it} = \frac{c_{it}}{(1-\delta\gamma)(\omega_t - C_{-it})}. \tag{44}$$

A simple observation of equation 44, shows that the calibration will assign higher elasticities to countries with higher carbon consumption. Moreover, ceteris paribus, bigger amount of stock of resource available at period t (ω_t) implies lower the output elasticities. ∎

5.5 Derivation of the BAU equilibrium in the Tipping Model

Starting from the Euler equation,

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} + \delta[\frac{\partial E[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1})\Big|\, \theta_{it}]}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_{it}}] 1^*_{\{\omega_t, \mathcal{E}_t\}} \quad = \quad 0$$

with $\frac{\partial \omega_{t+1}}{\partial e_{it}} = -\frac{A\gamma\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)^{1-\gamma}}$. Then the Euler equation is

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} \;=\; A\delta\gamma \frac{\partial E[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1})\Big|\, \theta_{it}]}{\partial \omega_{t+1}} \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)^{1-\gamma}}. \tag{45}$$

Differentiating the value function $U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1})$ with respect to ω_{t+1}

$$\frac{\partial E[U_j(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1})\Big|\, \theta_{i\,t}]}{\partial \omega_{t+1}} \;=$$

$$E\left[\frac{1}{\omega_{t+1}} \;+\; A\delta\gamma \frac{\partial E[U_i(\omega_{t+2}, \mathbf{e}, \theta_{i\,t+2})\Big|\, \omega_{t+1}, \theta_{i\,t+1}]}{\partial \omega_{t+2}} \frac{1-\mathcal{E}_{t+1}}{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}} \; 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \Big|\, \omega_t, \theta_{i\,t}\right].$$

Substituting this into the first order condition (45), we obtain

$$
\begin{aligned}
&\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} \\
= \;& A\delta\gamma \left\{ \frac{\partial E[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1}) \big| \; \theta_{i\,t}]}{\partial \omega_{t+1}} \left(\frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)^{1-\gamma}} \right) \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}} \\
= \;& \delta\gamma \left\{ \frac{\partial E[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1}) \big| \; \theta_{i\,t}]}{\partial \omega_{t+1}} \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)} \omega_{t+1} \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}} \\
= \;& \delta\gamma \left\{ E\left[\frac{1}{\omega_{t+1}} + A\delta\gamma \frac{\partial E[U_i(\omega_{t+2}, \mathbf{e}, \theta_{i\,t+2}) \big| \; \theta_{i\,t+1}]}{\partial \omega_{t+2}} \frac{(1-\mathcal{E}_{t+1})}{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}} \Big| \theta_{i\,t} \right] \times \right. \\
& \left. \qquad \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)} \omega_{t+1} \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}} \\
= \;& \delta\gamma \left\{ \frac{1}{\omega_{t+1}} + E\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}} \right) \frac{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}}{\omega_{t+1}} \times \right. \right. \\
& \left. \left. \frac{(1-\mathcal{E}_{t+1})}{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}} \Big| \omega_t, \theta_{i\,t} \right] 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)} \omega_{t+1} \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}} \\
= \;& \delta\gamma \left\{ \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)} \left[1 + E\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}} \right) (1-\mathcal{E}_{t+1}) 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \Big| \; \omega_t, \theta_{i\,t} \right] \right] 1^*_{\{\omega_t, \mathcal{E}_t\}} \right\}.
\end{aligned}
$$

Reorganizing terms, we obtain the Euler equation

$$
\begin{aligned}
&\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it}) \; = \\
&\delta\gamma \left\{ \frac{\omega_t(1-\mathcal{E}_t)}{(\omega_t(1-\mathcal{E}_t)-b)} \left[1 + E\left[\left(\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}} - (1-\theta_{i\,t+1}) \right) 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \Big| \; \omega_t, \theta_{i\,t} \right] \right] 1^*_{\{\omega_t, \mathcal{E}_t\}} \right\}
\end{aligned}
\tag{46}
$$

which, after iteration, generates the forward solution in the paper.

5.6 Proofs of the Main Results

Proof of Proposition 5 . Let $\omega_0 > \bar{\omega}_0 > b$. It suffices to show

$$
P\left(\left\{ \theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0, \theta^t) \; \leq \; \omega^{tip} \right\} \right) \; < \; P\left(\left\{ \theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\overline{\omega}_0, \theta^t) \; \leq \; \omega^{tip} \right\} \right).
$$

In turn, this holds if

$$
\omega^{*\,t}(\omega_0, \theta^t) \; > \; \omega^{*\,t}(\bar{\omega}_0, \theta^t) \quad \forall \; \theta^t \;\; \forall \; t.
$$

We proceed by induction. Fix $\omega_0 > \bar{\omega}_0 > b$ and observe that $\omega^{*\,t}(\omega_0, \theta^t) = A(\omega^{*\,t-1}(\omega_0, \theta^{t-1}) - C^*(\omega_0, \theta^{t-1}))^\gamma$ and so we proceed by induction. Suppose, by contradiction, that for $t = 1$,

$$\omega_1^*(\omega_0, \theta_0) = A(\omega_0 - C^*(\omega_0, \theta^0))^\gamma \ < \ A(\bar{\omega}_0 - C^*(\bar{\omega}_0, \theta^0))^\gamma = \omega_1^*(\bar{\omega}_0, \theta_0).$$

In particular, this implies

$$\omega_0 - C^*(\omega_0, \theta^0) \ < \ \bar{\omega}_0 - C^*(\bar{\omega}_0, \theta^0).$$

Notice, first, that it is not possible for $\omega_0 - C^*_{-i}(\omega_0, \theta^0) < \bar{\omega}_0 - C^*_{-i}(\bar{\omega}_0, \theta^0)$ for <u>all</u> i. If that were true, then $c_i^*(\omega_0, \theta^0) < c_i^*(\bar{\omega}_0, \theta^0)$ for all i, a contradiction of the fact that $C^*(\omega_0, \theta^0) \ > \ C^*(\bar{\omega}_0, \theta^0)$ when c_i^* is increasing in $\omega_0 - C^*(\omega_0, \theta^0)$ and given $\omega > \bar{\omega}$.

Hence, there is some country j for whom

$$\omega_0 - C^*_{-j}(\omega_0, \theta^0) > \bar{\omega}_0 - C^*_{-j}(\bar{\omega}_0, \theta^0) \quad \text{and} \quad c_j^*(\omega_0, \theta^0) > c_j^*(\bar{\omega}_0, \theta^0).$$

For this country j, payoffs in an arbitrary state ω_0 and for an arbitrary choice c_j can be expressed as

$$\theta_{it} \log c_j + (1 - \theta_{it}) \log(\omega_0 - C^*_{-j}(\omega_0, \theta^0) - c_j) + \delta E[V(\omega_0 - C^*_{-j}(\omega_0, \theta^0) - c_j)].$$

By strict concavity of this objective as a function of c_j, $c_j^*(\omega_0, \theta^0)$ is optimal in state ω_0 only if

$$c_j^*(\omega_0, \theta^0) - c_j^*(\bar{\omega}_0, \theta^0) \ \leq \ (\omega_0 - C^*_{-j}(\omega_0, \theta^0)) - (\bar{\omega}_0 - C^*_{-j}(\bar{\omega}_0, \theta^0)).$$

Proceeding by induction, it can be established that for all t,

$$\omega^{*\,t}(\omega_0, \theta^t) \ > \ \omega^{*\,t}(\bar{\omega}_0, \theta^t) \quad \forall\ \theta^t \ \ \forall\ t.$$

We thus conclude the proof ∎

Proof of Theorem 1. Let $\mathcal{E}^*(\omega_t, \theta_t, b, n)$ denote the equilibrium aggregate extraction rate, expressed as a function of the relevant parameters.

Part 1. We first show that there is a stationary lower bound independent of ω_t, namely, $\underline{\mathcal{E}}(\theta_t, b, n) \leq \mathcal{E}^*(\omega_t, \theta_t, b, n) \ \ \forall\ \omega_+ t$ with the property that for $\theta_t \in (1 - \epsilon, 1]^n$ and ϵ small enough and n large enough, we have

$$\omega_t \ > \ \omega^*_{t+1}(\omega_t, \theta_t; b) \quad \forall\ \omega_t\ .$$

To find a stationary lower bound, observe that $1^*_{\{\omega_t,\mathcal{E}_t\}} = 0$ for all ω_t such that $A(\omega_t(1-\mathcal{E}_t)-b)^\gamma \leq b$ or equivalently, $\omega_t \leq \frac{1}{1-\mathcal{E}_t}\left(b+(\frac{b}{A})^{1/\gamma}\right) \equiv K$. Moreover, K is the upper bound on stocks for which $1^*_{\{\omega_t,\mathcal{E}_t\}} = 0$. Hence, the marginal future cost of extraction, $G^*(\omega_t,\theta_t,b)$ is bounded above by its stationary limit when ω approaches K from the right, so that $1^*_{\{\omega_t,\mathcal{E}_t\}} = 1$. Stated precisely:

$$\forall\ \omega_t \leq K, \quad G^*(\omega_t,\theta_t,b) \leq \lim_{\omega\searrow K} G^*(\omega,\theta_t,b).$$

The Euler equation (19) in this limit is

$$\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it}) \ = \ \delta\gamma\frac{K(1-\mathcal{E}_t)}{K(1-\mathcal{E}_t)-b}\left(1+\frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it})\right). \qquad (47)$$

By construction, (47) is the limiting Euler equation to which $\underline{\mathcal{E}}(\theta_t,b,n)$ is a solution.

Taking $\theta_t \to \mathbf{1}$, it follows that $e_i = \mathcal{E}/n$ and the Euler equation becomes

$$\frac{n(1-\mathcal{E})}{\mathcal{E}} \ = \ \delta\gamma\frac{K(1-\mathcal{E})}{K(1-\mathcal{E})-b}\left(1+\frac{n(1-\mathcal{E})}{\mathcal{E}}\right)$$

which can be expressed as

$$n\frac{1-\mathcal{E}}{\mathcal{E}} \ = \ \frac{\delta\gamma K(1-\mathcal{E})}{(1-\delta\gamma)K(1-\mathcal{E})-b}.$$

Using the fact that $K \equiv \frac{1}{1-\mathcal{E}}\left(b+(\frac{b}{A})^{1/\gamma}\right)$, the Euler equation becomes

$$n\frac{1-\mathcal{E}}{\mathcal{E}} \ = \ \frac{\delta\gamma\left(b+(\frac{b}{A})^{1/\gamma}\right)}{(1-\delta\gamma)\left(b+(\frac{b}{A})^{1/\gamma}\right)-b}. \qquad (48)$$

Since, by construction $\underline{\mathcal{E}}(\mathbf{1},b,n)$ satisfies (48), it is easy to see that

$$\lim_{n\to\infty}\underline{\mathcal{E}}(\mathbf{1},b,n) = 1.$$

Next we show that for ϵ small enough and n large enough,

$$\omega_t \ > \ \omega^*_{t+1}(\omega_t,\theta_t;b) \quad \forall\ \omega_t \quad \forall\ \theta_t \in (1-\epsilon,1]^n. \qquad (49)$$

The inequality (49) may be rewritten as

$$\left(\frac{1}{(1-\mathcal{E}^*(\omega_t,\theta_t,b,n))}\left[(\frac{\omega_t}{A})^{1/\gamma}+b\right]-\omega_t\right) \ > \ 0 \quad \forall\ \omega_t. \qquad (50)$$

To verify that (50) holds, we show that

$$\underline{P} \;\equiv\; \min_{\omega} \left(\frac{1}{(1-\underline{\mathcal{E}}(\theta_t;b,n))} \left[(\frac{\omega}{A})^{1/\gamma} + b \right] - \omega \right) \;\;>\;\; 0 \tag{51}$$

where $\underline{\mathcal{E}}_t(\theta_t;b,n)$ is, recall, a stationary lower bound of $\mathcal{E}^*(\omega_t,\theta_t,b,n)$.

The first order condition for $\underline{P}$ is

$$(\gamma(1-\underline{\mathcal{E}}(\theta_t;b,n))A^{1/\gamma})^{-1}\omega^{\frac{1-\gamma}{\gamma}} - 1 = 0.$$

Solving for ω, we obtain $\omega^m \equiv (\gamma(1-\underline{\mathcal{E}}(\theta_t;b,n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}$. Substituting ω^m back into the problem we obtain,

$$\begin{aligned}
\underline{P} &= \left(\frac{1}{(1-\underline{\mathcal{E}}(\theta_t;b,n))} \left[(\frac{\omega^m}{A})^{1/\gamma} + b \right] - \omega^m \right) \\
&= \left(\frac{1}{(1-\underline{\mathcal{E}}(\theta_t;b,n))} \left[(\frac{(\gamma(1-\underline{\mathcal{E}}(\theta_t;b,n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}}{A})^{1/\gamma} + b \right] - (\gamma(1-\underline{\mathcal{E}}(\theta_t;b,n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \right) \\
&= \frac{b}{1-\underline{\mathcal{E}}(\theta_t;b,n)} \;-\; (1-\underline{\mathcal{E}}(\theta_t;b,n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right).
\end{aligned}$$

Hence, (50) holds if

$$\underline{P} \;=\; \frac{b}{1-\underline{\mathcal{E}}(\theta_t;b,n)} \;-\; (1-\underline{\mathcal{E}}(\theta_t;b,n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right) \;\;>\;\; 0 \tag{52}$$

holds. But (52) clearly holds in the limit as $\theta_t \to \mathbf{1} \equiv (1,\ldots,1)$ and $n \to \infty$ since $\underline{\mathcal{E}}(\theta_t;b,n) \to 1$ in that case.

Since the argument is strict, it holds for sufficiently large n and θ_t sufficiently close to one. Thus, for any fixed profile if θ_t, there is a finite time length $T(\theta_t)$ such that $\omega^{*t}(\omega_0,\theta^t) \to b$ in at most $T(\theta_t)$ iterations. Let

$$T = \max_{\theta_t \in [1-\epsilon,1]} T(\theta_t).$$

Thus T is a time length (dependent on ω_0) such that if (49) holds for all $\theta_t \in (1-\epsilon,1]^n$ then $\omega^{*t}(\omega_0,\theta^t) \to b$ in at most T iterations.

Observe that (24) implies for any finite $T > 0$, that for a.e. θ_t,

$$\begin{aligned} & \Pr\left(\theta_{t+s} \in (1-\epsilon, 1]^n,\ s = 1, \ldots, T \;\middle|\; \theta_t\right) \\ = & \int_{\theta_{t+1} \in (1-\epsilon,1]^n} \cdots \int_{\theta_{t+T} \in (1-\epsilon,1]^n} \prod_{s=1}^{T} dF(\theta_{t+s}|\theta_{t+s-1}) \qquad (53) \\ \geq & \ \epsilon^T. \end{aligned}$$

It follows that for almost every process $\{\theta_t\}$, there is a date t (infinitely many dates actually) such that (49) holds for realized values $\theta_t, \theta_{t+1}, \ldots, \theta_{t+T}$, in which case $\omega^{*\,t+T}(\omega_t, \theta^{t+T}) = b$. Consequently, the economy collapses at ω_t, concluding the proof of Part 1.

Part 2. The proof here largely reverse engineers some of the logic of part 1. In particular, we now find a stationary *upper* bound $\overline{\mathcal{E}}(\theta_t, b, n) \geq \mathcal{E}^*(\omega_t, \theta_t, b, n) \quad \forall\ \omega_t$ with the property that for $\theta_t \in (0, \epsilon]^n$ and ϵ small enough, we have

$$\omega_t \ < \ \omega^*_{t+1}(\omega_t, \theta_t; b) \quad \text{on a nonnull set of stocks } \omega_t. \qquad (54)$$

Notice that (54) is just the negation of (50).

The simplest upper bound is the extraction rate when each country its static, one shot optimal rate. Namely, we have as our upper bound,

$$\overline{\mathcal{E}}(\theta_t, b, n) \ = \ \sum_i \frac{\theta_{it}}{1-\theta_{it}} \left(1 + \sum_i \frac{\theta_{it}}{1-\theta_{it}}\right)^{-1}$$

Now using an analogous argument to that of the steps from Equations (51) to (52), the Inequality in (54) above holds if

$$\overline{P} \ \equiv \ \min_{\omega} \ \left(\frac{1}{(1-\overline{\mathcal{E}}(\theta_t; b, n))}\left[(\frac{\omega}{A})^{1/\gamma} + b\right] - \omega\right) \ < \ 0 \qquad (55)$$

The first order condition for $\overline{P}$ is

$$(\gamma(1-\overline{\mathcal{E}}(\theta_t; b, n))A^{1/\gamma})^{-1}\omega^{\frac{1-\gamma}{\gamma}} - 1 = 0.$$

Solving for ω, we obtain $\omega^0 \equiv (\gamma(1-\overline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}$. Substituting ω^0 back into the problem we obtain,

$$\overline{P} \ = \ \frac{b}{1-\overline{\mathcal{E}}(\theta_t; b, n)} \ - \ (1-\overline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right).$$

Hence, (54) holds if

$$\overline{P} \;=\; \frac{b}{1-\overline{\mathcal{E}}(\theta_t;b,n)} \;-\; (1-\overline{\mathcal{E}}(\theta_t;b,n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right) \;<\; 0 \tag{56}$$

But (56) is easily observed to hold in the limit as $\theta_t \to \mathbf{0} \equiv (0,\ldots,0)$ since

$$\overline{\mathcal{E}}(\theta_t,b,n) \;\equiv\; \sum_i \frac{\theta_{it}}{1-\theta_{it}} \left(1+\sum_i \frac{\theta_{it}}{1-\theta_{it}}\right)^{-1} \;\to\; 0 \;\text{ as }\; \theta_t \to \mathbf{0}$$

in the limit. Since the inequality is strict, (56) holds for $\theta_t \in [0,\epsilon]^n$ if ϵ is nonzero but sufficiently small. This concludes the proof of Part 2. ∎

Proof of Theorem 2 . Let $\theta_t \geq \tilde{\theta}_t$. Then by the definition of dominance, it suffices to show

$$\omega^{*\,t}(\omega_0,\theta^t) \;<\; \omega^{*\,t}(\omega_0,\tilde{\theta}^t) \quad \forall\, \omega_0 \tag{57}$$

We now proceed to verify (57). We first show $C^*(\omega_t,\theta_t) > C^*(\omega_t,\tilde{\theta}_t)$, in which case (57) holds by an induction argument.

Using the derivation in Appendix 5.5, the Euler equation for c_i^* can be expressed as

$$\frac{\theta_{it}(\omega_0 - C_t)}{c_{it}} - (1-\theta_{it}) \;=\;$$

$$\delta\gamma \left\{ \frac{\omega_t - C_t}{(\omega_t - C_t - b)} \left[1 + E\left[\left(\frac{\theta_{i\,t+1}(\omega_0 - C_{t+1})}{c_{i\,t+1}} - (1-\theta_{i\,t+1})\right) 1^*_{\{\omega_{t+1},C_{t+1}\}} \middle|\, \omega_t, \theta_{i\,t}\right]\right] 1^*_{\{\omega_t,C_t\}} \right\} \tag{58}$$

From (58), it is clear that c_{it}^* is increasing in θ_{it}, and since $\theta_{-i\,t}$ enters c_{it}^* only through its effect on $C_{-i\,t}^*$, it follows from the Envelope Theorem that $C_t^*(\theta_t)$ is increasing in θ_t.

The ordering of tipping points follows from the fact that

$$P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0,\theta^t) \;\to\; b\right\}\right) \;=\; P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0,\theta^t) \;\leq\; \omega^{tip}\right\}\right) \quad \forall\, \omega_0$$

and from Proposition 5.

$$P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0,\theta^t) \;\leq\; \omega^{tip}\right\}\right) \;<\; P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\overline{\omega}_0,\theta^t) \;\leq\; \omega^{tip}\right\}\right)$$

∎

Proof of Theorem 3 . It suffices to show that the planner's solution admits a finite tipping in the worst case: $\theta_t = \mathbf{1}$. In an economy restricted to $\theta_t = \mathbf{1}$, the policy tipping point is defined by the unstable fixed point solution to $\omega_t = \omega^\circ(\omega_t, \mathbf{1}; b)$ or

$$\left(\frac{1}{(1 - \mathcal{E}^\circ(\omega_t, \mathbf{1}; b))} \left[(\frac{\omega_t}{A})^{1/\gamma} + b \right] - \omega_t \right) \;=\; 0 \tag{59}$$

In fact, a finite tipping point exists if we can show that (59) has any solution, stable or unstable. From the planner's solution in (22), it follows that

$$\mathcal{E}^\circ(\omega_t, \mathbf{1}; b) \;=\; \frac{1}{1 + G^\circ(\omega_t, \mathbf{1}; b)} \,.$$

Notice that this expression does not vary in n. Equation (59) then becomes

$$H(\omega_t, b) \;\equiv\; \left(\frac{1 + G^\circ(\omega_t, \mathbf{1}; b)}{G^\circ(\omega_t, \mathbf{1}; b)} \left[(\frac{\omega_t}{A})^{1/\gamma} + b \right] - \omega_t \right) \;=\; 0 \tag{60}$$

To verify that this equation has a solution, observe that $G^\circ(\omega_t, \mathbf{1}; b) \to 0$ as $\omega_t \to 0$ while $G^\circ(\omega_t, \mathbf{1}; b) \to G^\circ(\omega_t, \mathbf{1}; 0) = \frac{A\delta\gamma}{1 - A\delta\gamma}$ as $\omega_t \to \infty$. These limits imply $H(\omega_t, b) \to \infty$ as $\omega \to 0$ and $H(\omega_t, b) \to H(\omega_t, 0) < \omega_t$ as $\omega_t \to \infty$. The Intermediate Value Theorem immediately implies the existence of a solution to (60). Consequently, a finite policy tipping point exists, concluding the proof of the second theorem. ∎

References

[1] Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012), "The Environment and Directed Technical Change," *American Economic Review*, 102: 131-166.

[2] Anderies, J.M., S.R. Carpenter, W. Steffan, and J. Rockstrom (2013), "The Topology of Nonlinear Global Carbon Dynamics: from the tipping points to planetary boundaries." *Environmental Research Letters*, 8: 1-13.

[3] Barrett, S. (2003), *Environment and Statecraft: The Strategy of Environmental Treaty- Making*, Oxford University Press, Oxford.

[4] Battaglini, M. and Bard Harstad (2012), "Participation and Duration of Environmental Agreements," NBER working paper, #18585.

[5] Bodansky, D. (2004), "International Climate Efforts Beyond 2012: A Survey of Approaches," *PEW Center on Global Climate Change.*

[6] Cai, Y., K. Judd, and T. Lontzek (2012), "Tipping Points in a Dynamic Stochastic IAM," mimeo, January.

[7] Calcott, P. and V. Petkov (2012), "International Environmental Agreements: Design of Optimal Transfers Under Heterogeneity," *Environmental Modeling and Assessment*, 17: 209-220.

[8] Cave, J. (1987), "Long-Term Competition in a Dynamic Game: The Cold Fish War," *Rand Journal*, 18: 596-610.

[9] Desmet, K. and E. Rossi-Hansberg (2014), "On the Spatial Economic Impact of Global Warming," mimeo, Princeton University, July.

[10] Dutta, P. and R. Radner (2009), "A Strategic Analysis of Global Warming: theory and some numbers," *Journal of Economic Behavior and Organization*, 71: 187-2009.

[11] Dutta, P. and R. Radner (2006), "A Game-Theoretic Approach to Global Warming," *Advances in Mathematical Economics*, 8: 135-53.

[12] Dutta, P. and R. Radner (2004), "Self-enforcing climate-change treaties," *Proceedings of the National Academy of Sciences.*

[13] Dutta, P. and R. Sundaram (1993), "Tragedy of the Commons?" *Economic Theory*, 3: 413-26.

[14] Finus, M. (2001), *Game Theory and International Environmental Cooperation*, Cheltenham, Edward Elgar.

[15] Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014), "Optimal Taxes on Fossil Fuel in General Equilibrium," *Econometrica*, 82: 41-88.

[16] Hansen, J., M. Sato, G. Russell, and P. Kharecha (2013), "Climate Sensitivity, Sea Level and Atmospheric Carbon Dioxide," *Royal Society Publishing: Philosophical Transactions A*, 371 (2001): 1-42.

[17] Harrison, R. and R. Lagunoff (2014), "Dynamic Mechanism Design for a Global Commons," Georgetown University Working Paper.

[18] Haurie, A. (2008), "A Dynamic Game Model of Strategic R&D Cooperation and GHG Emission Mitigation," mimeo.

[19] Hope C (2006), "The Marginal Impact of CO2 from PAGE2002: An integrated assessment model incorporating the IPCC's five reasons for concern," *Integrated Assessment*, 6: 19-56.

[20] Krusell, P. and A. Smith (2009), "Macroeconomics and Global Climate Change: Transition for a Many-Region Economy," working paper.

[21] Kerr, R. (2008), "Climate Tipping Points Come in from the Cold," *Science* 319: 153.

[22] Lemoine, D. and C. Traeger (2014), "Watch Your Step: Optimal Policy in a Tipping Climate," *American Economic Journal: Economic Policy*, 6: forthcoming.

[23] Lenton, T.M., H. Held, E. Kriegler, J.W. Hall, and W. Lucht (2008), "Tipping Elements in the Earth's Climate System," *Proceedings of the National Academy of Sciences*, 105: 1786.

[24] Levhari, D. and L. Mirman (1980), "The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution," *Bell Journal of Economics*, 14: 322-34.

[25] Maasoumi, E. and G. Yalonetzky (2013), "Introduction to Robustness in Multidimensional Wellbeing Analysis," *Econometric Reviews*, 32: 1-6.

[26] N-V Long (2011), "Dynamic Games in the Economics of Natural Resources: A Survey," *Dynamic Games and Applications* , 1: 115-148.

[27] Nordhaus, W. (2006), "After Kyoto: Alternative Mechanisms to Control Global Warming," *American Economic Review*, 96: 31-34.

[28] Nordhaus, W. (2007), "To Tax or Not to Tax: The Case for a Carbon Tax," *Review of Environmental Economics and Policy.*

[29] Ostrom, E. (1990), *Governing the Commons: The Evolution of Institutions for Collective Action,* Cambridge University Press, New York.

[30] Ostrom, E. (2002), "Common Pool Resource and Institutions: Towards a Revised Theory," *Handbook of Agricultural Economics, Volume 2.*

[31] Rockstrom, J., W. Steffen, K. Noone, A. Persson, F. Chapin, E. Lambin, T. Lenton, M. Scheffer, C. Folke, H. Schellnhuber, B. Nykvist, C. de Wit, T. Hughes, S. van der Leeuw, H. Rodhe, S. Sorlin, P. Snyder, R. Costanza, U. Svedin, M. Falkenmark, L. Karlberg, R. Corell, V. Fabry, J. Hansen, B. Walker, D. Liverman, K. Richardson, P. Crutzen, and J. Foley (2009), " A Safe Operating Space for Humanity," *Nature*, 461: 472-75, DOI: 10.1038/461472a

[32] Rouillon, S. (2010), "Optimal decentralized management of a natural resource," mimeo, Universite Bordeaux.

[33] Sorger, G. (1996), "Markov-Perfect Nash Equilibria in a Class of Resource Games," CIRANO Working paper, April.

[34] Stern, N. (2006), *The Stern Review Report on the Economics of Climate Change.*

[35] United Nations Inter-Governmental Panel on Climate Change (2007), *Fourth Assessment Report: Climate Change.*

[36] Zoli, C. (2009), "From Unidimensional to Multidimensional Inequality, Welfare and Poverty Measurement," mimeo, University of Verona.