Waiting to Copy: On the Dynamics of the Market for Technology*

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Preliminary Version

Abstract

We examine the appropriability problem of an inventor who brings to the market a successful innovation that can be legally copied. We study this problem in a dynamic model in which imitators can "enter" the market either by copying the invention at a cost or by buying knowledge (a license) from the inventor. The first imitator to enter the market can then resell his acquired knowledge to the remaining imitators. This dynamic interaction in the licensing market dramatically affects the conventional wisdom on the need for intellectual property rights. Our main result reveals that, in equilibrium, imitators delay their entry into the market and thus the inventor retains monopoly rents for some time. Second, we show that the innovator strictly prefers to offer non-exclusive rather than exclusive licenses which would forbid reselling by the imitators. Last, we prove that when the innovator faces a large number of imitators, her equilibrium reward converges to monopoly profits.

JEL: L24, O31, O34, D23, C73.

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INTRODUCTION

Cesaroni and Mariani (2001) show that in the chemical industry a large share of technologies are both exploited in-house and simultaneously licensed-out to potential competitors.[1] Cesaroni (2001) describes more specifically the case of Himont, a chemical firm which developed a process of production of polypropylene.[2] Although Himont was using its new technology to sell in the market, it was also active in licensing its process, called Spheripol. The market for licenses was characterized by intense competition. Indeed, other firms producing polypropylene with different

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[1] Their study includes European, North American and Japanese firms.

[2] A plastic material with a wide range of applications.

processes of production were also offering licenses. Given such behavior, entry of new competitors was unavoidable and Himont could at least reap revenues from also licensing aggressively. Furthermore, Arora et. al. (2001) argue that such licensing agreements among established firms and potential competitors are common not only for chemicals but also for electronics, software and business services.

In this paper, we build on these facts and show that such competition to provide licenses has essential implications for the rewards of inventors and for the need for intellectual property rights. The central premise in the economics of innovation is that, without intellectual property rights, the rewards from innovative activities are vulnerable to ex-post expropriation by imitators. Imitators immediately copy the innovation, dissipating the rents of inventors and thus discouraging initial investments in research. This well-known appropriability problem was first pointed out by Arrow (1962).

The purpose of this paper is to reexamine this "conventional wisdom" in the presence of a market for licenses, that we call market for technology.[3] We preserve the essential features of the appropriability problem by examining the following situation. We consider an environment in which imitators can legally copy an innovation at a certain cost.[4] Moreover, incentives to copy are such that, imitators would receive, after paying the imitation cost, a strictly positive equilibrium payoff even if all of them decided to copy. Within the confines of this problem, our point of departure is to consider the dynamics of trading between innovators and imitators in the market for technology. Licenses may be sold in two distinct situations. First, the innovator, under the threat of profitable imitation, may sell knowledge (a license, henceforth) to the imitators. If entry on the market cannot be prevented, the innovator can at least reap some licensing payments. Second, since knowledge is a non-rival good, imitators who initially buy a license may subsequently compete with the innovator by reselling licenses to the remaining imitators.

Before presenting our results, we make a clarifying observation. We implicitly assume that the lack of intellectual property rights is *not* an obstacle for the parties to conclude mutually beneficial licensing agreements to exchange knowledge services, designs, codes, etc. Even if the inventor is not legally protected against imitation or reinvention, she can still choose to sell her knowledge. It has been argued that the existence of asymmetric information can be an obstacle to licensing in the absence of intellectual property rights. Indeed, the innovator needs to convince the potential licensee of the value of her invention but is then exposed to expropriation.[5] We, however, consider a case in which the innovation is already on the market, success is publicly observable, and thus asymmetries of information are minimal.

To capture the elements previously discussed and, in particular, the dynamics of trading in the market for technology, we initially develop a model with one innovator and two imitators. Our model has the following extensive form structure. Time is divided in an infinite sequence

[3] We use the term of Arora et. al. (2001). Technology refers to knowledge rooted in engineering but also drawn from production experience.

[4] The cost of reverse engineering the commercialized product. Technologies, displayed in the market, usually conceal some details. Software is an example. The source code that is necessary for imitation can be, most of the time, kept secret. For an excellent discussion, see Gans and Stern (2002).

[5] See Anton and Yao (1994, 2002).

of periods. The innovation is introduced in the market at period zero. In each period, imitators can enter the market either by copying at a cost or by buying a license.[6] Once an imitator enters the product market, he also becomes a competitor of the inventor in the market for technology. In other words, he will compete with the innovator by offering a license to the remaining entrant as observed in the case of Himont.[7]

As a benchmark result, we establish that when the market for technology is missing both imitators immediately enter the market by copying at period zero.[8] Indeed, there is no benefit from delaying entry since the imitation cost will remain fixed over time. By contrast, the main result of this paper reveals that, in the presence of a market for technology, imitators will *delay* their entry into the market. Thus, the inventor will appropriate monopoly rents for some time even in the absence of intellectual property rights.

This result is due to the dynamics of the price of licenses. When a market for technology exists, entry will take place by licensing. Before any imitator enters the market, the inventor offers two licenses at a price equal to the imitation cost. Once an imitator enters the market by buying what we call the *first* license, the price of the *second* license, the license that will be sold to the remaining imitator, is determined competitively in the market for technology.

Consider, for instance, the outcome of what we call the *no-delay licensing equilibrium*. After the first license is bought, the innovator and the first entrant *immediately* sell the second license to the remaining imitator at a price equal to its marginal cost, zero.[9] These price dynamics ensure an equilibrium payoff to the second entrant strictly higher than the equilibrium payoff to the first entrant. This gives rise to a war of attrition in which each imitator delays his entry on the market with the hope that his rival will enter before him and decrease the price of the license. Note that what drives this result is the existence, in equilibrium, of a *pecuniary externality*: After the first imitator enters the market, the equilibrium price of the second license decreases to zero.

As imitators delay their entry, the innovator retains monopoly profits for some time. The expected duration of the *delay* can be considerable. Consider for instance the extreme case in which the imitation cost is close but smaller than the present value of equilibrium triopoly profits. In such an environment, when the market for technology is missing, imitators immediately copy and the innovator only appropriates the present value of triopoly profits. We show that, when a market for technology exists, the expected duration of the delay converges to infinity and the innovator receives a reward arbitrarily close to the present value of monopoly profits.

In the environment described until now, it is natural to wonder whether the inventor would not prefer to offer *exclusive* licenses forbidding resale by the first entrant. Our analysis uncovers that the inventor strictly prefers to sell *non-exclusive* licenses. If exclusive licenses were offered both imitators would immediately buy a license at period zero. Thus, the innovator would collect,

[6] "Enter" means *use* the innovation in the market. Licenses offers are take it or leave it offers.

[7] If imitators enter simultaneously, they obtain their corresponding payoffs and the game ends. The model also allows an imitator who enters by copying to sell licenses. This assumption does not affect our results for the case of two imitators and simplifies our calculations.

[8] By missing we mean a form of market incompleteness such that knowledge trades cannot be executed.

[9] Since knowledge is a durable good and we assume that transferring it implies no real cost, its marginal cost is zero.

in form of licensing fees, the imitation costs, but imitators would *not* delay their entry into the market. The innovator would like to keep the license prices high for some time and then decrease them to encourage delay. Imitators, however, anticipate that such a promise is not credible and that the price of licenses will be the same over time. Non-exclusive licenses, on the other hand, work as a commitment device for the innovator: by changing the *structure* of the market for technology from monopoly to duopoly, they provide credibility to future reductions in license prices.

Next, we show the existence of a key connection between the delay until the first imitator enters the market (i.e. the delay to buy the first license) and the delay until the second license is sold. Although the no-delay licensing equilibrium is the unique symmetric markov perfect equilibrium, there can also exist other subgame perfect equilibria. All of these are characterized by different delays until the second license is sold.[10] So, the first entrant retains, until the second license is traded, duopoly profits in the market. Thus, if the delay in the market for technology is not too long, imitators continue "playing" a war of attrition. Otherwise, the game becomes a preemption game in which entry takes place quasi instantaneously. In this last case, the appropriability problem emerges once again but for a different rationale: Entry occurs immediately when imitators expect that knowledge will be slowly diffused in the market for technology.

Last, we extend our model to the case of a large number of potential imitators. We find the striking result that the innovator's equilibrium payoff converges to monopoly profits. The intuition, in one of the two cases we consider, is as follows. The unique symmetric markov perfect equilibrium in the market for technology is such that after the first entry, the inventor and the first imitator propose licenses at a zero price, almost instantaneously, to all remaining imitators. If the number of potential imitators is large enough, the profits of the first entrant will thus be significantly reduced and can be insufficient to cover the initial entry cost.[11] All imitators would thus shy away from initially entering the market, effectively granting the innovator monopoly rents forever.

The rest of the paper is organized as follows. Section 1 presents the model. The benchmark result when the market for technology is missing is shown in Section 2. In Section 3, we establish our main result. In Section 4, we explore two themes: the choice of exclusive versus non-exclusive licenses and the multiplicity of perfect equilibria. Section 5 extends our model to consider a large number of imitators. Section 6 examines the related literature. Section 7 concludes by discussing several issues raised by our paper and by proposing a different perspective on the increasing popularity of secrecy as a means to protect innovative rents.

1. THE MODEL

We consider an economy in which an inventor ("she"), denoted by s, has developed an innovation that represents an improvement over the previous state of the art.[12] The innovation

[10] Although in all of them the license is sold at a zero price.
[11] This follows since all of remaining imitators will find at least weakly profitable to enter.
[12] The innovation may be either a product improvement or a cost reducing process.

is not protected by intellectual property rights.[13] Two imitators, denoted by $h \in \{i, j\}$, may "adopt" the innovation by either: (i) Using a costly *imitation* technology (henceforth, copying); or by (ii) Buying technical knowledge from the inventor (henceforth, licensing).

We first describe the product market (hereafter, market). Time is broken into a countable infinite sequence of intervals, each called a period of real time length $\Delta \in \mathbb{R}_{++}$.[14] The innovation is introduced to the market at the beginning of period zero. At that date, the imitators might already be producing with an older technology or selling an older product.[15] To simplify the exposition, without loss of generality, the profits of the imitators who do not use the innovation are normalized to zero.

Imitators can adopt the innovation either by copying, c, or by licensing, ℓ. To clarify the terminology that we will use throughout the paper, when an imitator adopts the innovation at period t we will say that he enters the market. Besides, we will describe him as active in the market from that period on. We assume that the innovator and each active imitator obtain the same equilibrium profit flow regardless of their mode of entry. We denote by π_n the equilibrium profit flow when n firms are active in the market.[16] Moreover, we make the standard assumption that $\pi_1 > 2\pi_2 > 3\pi_3 > 0$.

We assume that all parties are risk neutral and maximize the sum of their discounted expected payoffs (i.e., profits plus potential licensing payments). Let $r > 0$ be the (common) rate of time preference and $\delta := \exp(-r\Delta) \in (0, 1)$ the discount factor between time periods. The profits of a firm during a period in which n firms are active in the market is $\int_0^\Delta \pi_n e^{-rt} dt = (1 - \delta)\, \Pi_n$, where $\Pi_n := \int_0^\infty \pi_n e^{-rt} dt = (\pi_n / r)$ is the present value of market profits when there are n active firms.

Next, we provide an outline of the imitation technology and the market for technology. The imitation technology can be described as follows. An imitator by spending, at any period t, an amount of real resources $\kappa \in \mathbb{R}_{++}$ obtains instantaneously a perfect version of the invention. We interpret κ, the imitation cost, as a one-time sunk cost that must be incurred to reverse engineer the fine details of the innovation. An alternative to copying is to enter the market by licensing. The inventor, being the creator of the innovation, possesses the required (indivisible) knowledge to transfer the innovation. If an imitator buys this piece of knowledge at period t, he will be able to instantaneously obtain a perfect version of the innovation at zero cost.[17]

We make the following simplifying assumptions regarding licensing agreements. First, we suppose that licenses are fixed-fee contracts.[18] At those periods t, such that no imitator has

[13] The lack of intellectual property rights is formalized in Assumption 1 of section 2. This assumption postulates that: (i) Copy is not legally forbidden; and that (ii) The present value of equilibrium profits when all imitators copy the invention is higher than the imitation cost.

[14] In what follows we will refer to period $(t + k)\,\Delta$ directly as period $t + k$ for all $k \in \{0, 1, ..\}$.

[15] Our model is sufficiently general to encompass situations in which the innovation is either drastic or non-drastic.

[16] These profits depend on market conditions, the type of competition and the features of the innovation. To make our argument most general we specify equilibrium profits in reduced form.

[17] The zero cost assumption is a convenient normalization. Further, to simplify, we also assume that transferring knowledge from the inventor to the imitators implies no real cost.

[18] In a previous version we showed that the results were unchanged when we allowed for two part tariffs and linear demand.

entered the market yet, we denote by p^t_{sh} the price at which the inventor offers to sell a license to imitator $h \in \{i, j\}$. If the inventor offers no license to imitator $h \in \{i, j\}$ we denote that by a price $p^t_{sh} = +\infty$. Second, the licenses offered by the innovator can be either *exclusive* or *non-exclusive*. If an imitator enters the market before his rival by signing a non-exclusive licensing contract with the inventor, he can then resell the acquired knowledge to the other imitator in subsequent periods. We also assume that if an imitator enters by copying, he also becomes a competitor of the inventor in the market for technology.[19] Specifically, at each period t, in which imitator i is active in the market and entered by copying or by purchasing a non-exclusive license and imitator j has not entered the market yet, the innovator offers a license at a price p^t_{sj} to imitator j and imitator i offers a license at a price p^t_{ij}.

Formally, relevant economic activity occurs within the framework of the following extensive form game. At the beginning of each period in which no imitator has entered yet:

(i) The innovator announces, on a take-it-or-leave-it basis, a pair of licensing contracts (i.e., a pair of prices p^t_{si} and p^t_{sj} and the exclusivity feature of the contract);

(ii) The imitators simultaneously decide whether to enter the market or not and, conditional on entrance, how to enter: They choose either c or ℓ.

The game continues in this manner as long as no imitator chooses to enter the market. If at period t both imitators enter simultaneously, the game ends and all parties receive their corresponding payoffs. However, if only one of them enters the market, say imitator i, the game continues as follows. From the beginning of period $t + 1$ on:

(i) If imitator i enters by either copying or purchasing a non-exclusive license, he and the innovator simultaneously announce prices for a single license: p^t_{ij} and p^t_{sj} respectively. Otherwise, when imitator i enters the market by buying an exclusive-license, only the innovator offers a license contract to imitator j; and

(ii) Imitator j decides whether to enter or not and, conditional on entrance, how to enter: By either copying or buying a license from one of the sellers if it is feasible for imitator i to sell a license (i.e., if imitator i entered the market by copying or by buying a non-exclusive license).

Until imitator j decides to enter the market, the innovator and imitator i receive their corresponding market profits. The game continues as long as there is still at least one imitator who has not entered the market yet.

Payoffs in this extensive form are calculated as follows. Suppose that the following outcome occurs: Imitator i enters the market at period t and imitator j at period $t + 1$. Besides, both imitators enter by buying a non-exclusive license from the innovator. The innovator's payoff at period 0 is then

$$V_s = (1 - \delta^t)\Pi_1 + \delta^t \left[(1 - \delta)\Pi_2 + p^t_{si}\right] + \delta^{t+1} \left[p^{t+1}_{sj} + \Pi_3\right] \tag{1}$$

[19] The results given here do not actually depend on this assumption. However it simplifies our calculations substantially.

Similarly, the present value of the payoff for each imitator is

$$V_i = \delta^t \left[(1-\delta)\Pi_2 - p_{si}^t\right] + \delta^{t+1}\Pi_3;\ V_j = \delta^{t+1}(\Pi_3 - p_{sj}^{t+1}) \tag{2}$$

Last, we assume that all parties observe the history up to the beginning of period t and that the buyer(s) observed the price offers and the nature of the contract made by the seller(s), at the beginning of period t in the market for technology. A history at the beginning of period t consists of a sequence of license contracts proposed by the seller(s), a sequence of entry decisions chosen by the imitators and a sequence of decisions of how to enter the market. We use subgame-perfect equilibria (SPE) as our solution concept. That is, we require strategies to form a Nash equilibrium following any feasible history. In certain sections of the paper, we restrict our attention to Markov Perfect Equilibria (MPE). In MPE strategies are functions only of payoff-relevant histories, determined in our model by the number of imitators who are active in the market at each time period.

2. BENCHMARK: APPROPRIATION WITHOUT A MARKET FOR TECHNOLOGY

We analyze, in this section, the SPE when the market for technology *is missing*. Thus, imitators can only enter the market by copying. We establish that both imitators enter the market without delay at the beginning of period zero. This result can be considered as the foundation for the "conventional wisdom" calling for intellectual property rights.

It is important to note that although we consider an economy without intellectual property rights, the imitation cost κ works as an entry barrier determining a natural measure of protection for the inventor.[20] A value of κ such that $\kappa > \Pi_2$ is sufficient to completely protect the inventor from imitation. Indeed, given that the imitation cost is strictly higher than the present value of duopoly profits, no imitator copies in equilibrium. The innovator therefore retains monopoly profits Π_1, even though intellectual property rights are not protected. The goal of this paper is to study the dynamics of entry and appropriation with and without a market for technology. Thus, to make our problem interesting, we impose the following assumption on imitation costs through Sections 2-4.

Assumption 1: $0 < \kappa < \Pi_3$

In an economy in which copying is legal, Assumption 1 ensures that it is profitable for both imitators. Under assumption 1, we obtain the following result.

Proposition 1 *Suppose that the market for technology is missing. Then*
(i) there is a unique SPE in which both imitators copy immediately at period $t = 0$
(ii) the equilibrium payoffs for the innovator and the imitators are Π_3 *and* $\Pi_3 - \kappa$ *respectively.*

[20] Actually, one can interpret intellectual property protection as policy measures that augment the level of κ.

Proof. See the Appendix. ■

When the market for technology is missing both imitators enter the market immediately. Indeed, there is no benefit from delaying entry since the entry cost will remain fixed throughout their planning horizon at the value of the imitation cost, κ. Furthermore, by delaying entry, imitators lose profits during the time periods in which they do not use the innovation. Therefore, if entry occurs, it will take place at period zero for sure. Assumption 1 assures that entry is indeed profitable for both imitators.[21]

Proposition 1 summarizes the "conventional wisdom" justifying the need for intellectual property rights. In the absence of such protection, imitators enter immediately following a successful innovation and compete away the rents of the initial inventor. Foreseeing the risk that their reward, Π_3, might be insufficient to cover their research costs, innovators might thus shy away from initially investing in research. The purpose of this paper is to challenge this line of thought and to show that delay can actually occur in equilibrium when a market for technology exists.

3. APPROPRIATION IN THE PRESENCE OF A MARKET FOR TECHNOLOGY

This section presents our main result when a market for technology exists: We show that, in equilibrium, imitators will delay their entry into the market and thus the innovator will collect monopoly profits for some time. We assume that the inventor is constrained to offer *non-exclusive* licensing contracts. Although this might initially appear to be a strong assumption, we show in section 4 that the inventor will prefer to offer such *non-exclusive* rather than *exclusive* licensing agreements. To present the intuition of this seemingly paradoxical result we must first identify the sources of rents for the inventor.

A. Copying and Licensing

Because we start studying the MPE of our game, in order to ultimately determine the equilibrium entry times of the imitators, we need to analyze two different types of subgames. First, the subgame which starts at the beginning of period $\tau \geq t+1$ after any feasible history in which a single imitator has entered at the beginning of period $\tau \geq t$. Second, the subgame which starts at the beginning of period $\tau \geq t$ after any feasible history in which entry has not occurred yet.[22] For clarity and future reference, we call the first subgame, the competitive subgame and the second one, the monopoly subgame.[23]

We first examine the competitive subgame. Specifically, suppose that imitator i has entered at the beginning of period t. Both the innovator and imitator i can, in subsequent periods, offer licensing contracts to imitator j.[24] The SPE of this subgame will be characterized by prices p_{sj}^{τ}

[21] Both obtain profits of $\Pi_3 - \kappa > 0$.

[22] Notice that after a history in which the imitators simultaneously enter, the game effectively ends. At every period the firms just compete on the product market.

[23] Formally, we consider a partition which maps the set of all feasible histories of the game into a set of *two* disjoint and exhaustive subsets of this set. The partition mapping that defines the payoff relevant history is defined by the number of imitators who are active in market at each feasible history.

[24] Imitator i entered either by copying or by signing a non-exclusive license. In both cases he can compete on the market for technology (i.e transfer knowledge to imitiator j for a fee).

and p^{τ}_{ij} of the licenses and by the choice of imitator j relative to the timing and mode of entry. For presentation purposes, a complete characterization of pure strategy SPE of this subgame will be performed in section 4. In this section, to emphasize our main ideas, we focus our attention on the MPE of this subgame that we call the no-delay licensing Nash equilibrium.[25] For simplicity, we present here the equilibrium outcome that results when the precepts of the no-delay licensing Nash equilibrium are followed.

Definition 1 *In the no-delay licensing Nash equilibrium a license is sold to imitator j immediately at period $t+1$ at a zero price.*

In the no-delay licensing equilibrium, a license is immediately sold to the remaining imitator at a price equal to its marginal cost. In the appendix, we present strategies that give rise to this equilibrium outcome and show that they form a subgame perfect equilibrium of the competitive subgame. Those strategies prescribe that both the innovator and imitator i propose licenses at a zero price in every period. Imitator j then has no incentive to delay his entry since the license is offered to him at its minimal price. Furthermore, for both sellers it is a best response to offer the license at a zero price given that his rival adopts the same strategy. We call the license that is offered competitively after the first entry the *second* license.[26]

It is important to point out that the no-delay licensing equilibrium is *not* necessarily the unique SPE of this subgame. Indeed there could exist other SPE in which both the innovator and imitator i keep prices high for some periods of time and imitator j delays his entry to benefit from a lower price in later periods. In section 4, we examine the implications that this multiplicity of SPE has for our main results. We also prove the existence of a condition that guarantees that the no-delay licensing equilibrium is the unique SPE of this subgame. However these issues are not necessary to understand the intuition of the mechanism that we highlight and so we discuss them in depth in section 4.

We now examine the expected payoffs of the imitators when the no-delay licensing equilibrium is being played. Imitator j, the follower imitator (i.e., the imitator who enters second), will enter at the beginning of period $t+1$ by obtaining a license at a zero price. His expected *equilibrium* payoff in period t units is therefore

$$V_j = \delta \Pi_3 \tag{3}$$

The expected payoff of imitator i depends on his mode of entry. If he entered the market by copying, his expected payoff in period t units would be

$$V_i^c = (1-\delta)\Pi_2 + \delta \Pi_3 - \kappa$$

as: (i) He obtains a flow of duopoly profits during period t; and: (ii) Since the no-delay equilibrium is played, the remaining imitator will immediately enter at period $t+1$ and therefore his profits

[25] The no-delay licensing equilibrium is the unique MPE.

[26] This types of competition was observed in the case of Himont mentionned in the introduction.

decrease to triopoly profits π_3 thereon.[27] If he instead entered the market by licensing, his expected payoff in period t units would be[28]

$$V_i^\ell = (1-\delta)\Pi_2 + \delta\Pi_3 - p_{si}$$

Observe that the only distinction between V_i^c and V_i^ℓ resides in the entry cost. In particular, neither by licensing nor by copying, does imitator i expects to obtain future licensing profits. Price competition in the market for technology will reduce licensing profits to zero.

Therefore, to determine the mode of entry of imitators we need to examine the prices at which the inventor offers to sell the licenses. We thus turn our attention to the monopoly subgame. We establish that the inventor will always offer two licenses at prices smaller or equal to the imitation cost. Thus, the imitators will always enter the market by licensing rather than by copying.

Lemma 1 *In the monopoly subgame, the innovator offers two licenses at prices $p_{si} \leq \kappa$ and $p_{sj} \leq \kappa$. Thus, copying never occurs in a MPE.*

Proof. See the Appendix. ■

The intuition behind this result is as follows. The innovator can always do weakly better by adding a second license at a price equal to κ at every period than by offering only one license. By adopting this licensing strategy, she does not change the entry costs and thus the entry decision of the imitators (the previously excluded imitator could always enter by copying if he paid the imitation cost κ) but collects licensing revenues when the imitators do enter the market.[29] Given these license prices, imitators, if they enter the market, will always do so by purchasing a license from the inventor and not by copying.

We still need to establish the exact license prices that the innovator will set. But first we summarize the payoffs of the imitators. If the leader imitator (i.e., the one who enters first, denoted by superscript 1), enters at period t, then, according to equation (3) and lemma 1, the payoffs for the leader and the follower imitator in period t units are

$$V_h^1 = (1-\delta)\Pi_2 + \delta\Pi_3 - p_{sh};\ V_h^2 = \delta\Pi_3 \tag{4}$$

for $h \in \{i,j\}$. On the other hand, if both imitators enter simultaneously their payoffs in period t units are

$$V_i^b = \Pi_3 - p_{si}; V_j^b = \Pi_3 - p_{sj} \tag{5}$$

[27] Observe that if we assumed that imitator i could not become a seller were he copied the invention instead of buying knowledge from the inventor, he would obtain the same expected payoff in period t units. Indeed, in equilibrium, at the beginning of period $t+1$, the inventor would sell knowledge to imitator j and he would immediately accept.

[28] Recall that in an MPE prices do not depend on calendar time but just on the number of imitators who are active in the market.

[29] The same idea applies to show that it is preferable to offer two licenses at a price of κ rather than no license at all.

A number of properties of these payoffs underlie our main results. First, we observe that as δ goes to one (or equivalently as Δ shrinks to zero), the payoff of the follower imitator is always strictly higher than the payoff of the leader imitator. This, formally, ensures a "war of attrition" between the imitators that yields delay to enter the market. Each imitator delays his entry time into the market with the hope of buying the *second* license and thus to pay a zero price for knowledge in the future. Second, we note that when the inventor raises the price of knowledge p_s she increases the difference between V^1 and V^2 and thus she magnifies the incentives of the imitators to wait longer before entering the market.

B. Appropriation in the absence of legal protection

Here we present our main result. We obtain the equilibrium license prices chosen by the innovator and the equilibrium entry time of the imitators. For tractability, we present our results for the limit of our discrete-timing game as the length of each period becomes arbitrarily small. For that, we initially fix a $\Delta > 0$ and then we inspect the limiting behavior of markov perfect equilibria when Δ shrinks to zero. Characterizing the limiting case has one important advantage: We are able to explicitly compute the innovator's equilibrium expected payoff and to compare it with the payoff that she obtains when the market for technology is missing.

Because the innovator will set license prices below or at most equal to the imitation cost, we directly denote the behavior strategy for imitator $h \in \{i, j\}$ by $\psi_h(p_{si}, p_{sj})$ and we interpreted it as the probability of buying a license at period t conditional on reaching period t. The innovator's strategy must specify at each period at which entry has not happened yet a pair of license prices $\{p_{si}, p_{sj}\}$. These strategies are (part) of a MPE if: (i) For any pair of license prices chosen by the innovator, the pair of behavior strategies selected by the imitators are, for each period, a Nash equilibrium between the imitators; and (ii) Given the equilibrium behavior strategies of the imitators, the innovator chooses a pair of license prices that maximizes her expected payoff. We say that a MPE is symmetric if when $p_{si} = p_s = p_{sj}$, then $\psi_i(p_s) = \psi_j(p_s)$.

As we observed before when the length of a period, Δ, shrinks to zero, the payoff of the follower imitator becomes strictly higher than the payoff of the leader imitator. This gives rise to a "war of attrition" in which each imitator delays his time to enter the market with the hope of buying the *second* license at a zero price. Meanwhile, the innovator, being the sole user of the innovation in the market, appropriates temporal monopoly profits. Proposition 2, the main result of this paper, formally captures this economic idea.

Proposition 2 *As the length of each period, Δ, converges to zero, there exists a unique symmetric MPE such that*
(i) the innovator sets prices $p_{si} = p_{sj} = \kappa$ for the licenses;
(ii) the distribution of entry times of each imitator converges to an exponential distribution with hazard rate equal to

$$\lambda = \frac{r\left(\Pi_3 - \kappa\right)}{\kappa}$$

(iii) the inventor's equilibrium expected payoff is

$$V_s(\kappa) = \frac{\pi_1}{r + 2\lambda} + \frac{2\lambda}{r + 2\lambda}(\Pi_3 + \kappa)$$

Proof. See the Appendix. ■

Result (ii) of proposition 2 indicates that the limiting distribution of entry times is an exponential distribution with *hazard rate* equal to λ. This is a typical result in war of attrition games. We examine next, in subsection C, the parameters that influence the size of this instantaneous entry rate for each imitator and the magnitude of the innovator's equilibrium payoff given in result (iii).

The equilibrium entry times of the imitators are used to derive result (i) which describes the optimal pricing decision of the inventor. The inventor chooses p_{si} and p_{sj} to maximize her expected payoff. If the innovator sets the same price p_s for both imitators, the hazard rate is given by

$$\lambda(p_s) = \frac{r(\Pi_3 - p_s)}{p_s}$$

and the expected payoff for the inventor can be expressed as

$$V_s(p_s) = \frac{\pi_1}{r + 2\lambda} + \frac{2\lambda}{r + 2\lambda}(\Pi_3 + p_s)$$

In this synthetic form, we observe that the license price p_s has several effects. First, and most obvious, a higher price for knowledge raises the licensing revenues that the inventor collects when she sells the *first* license (i.e. the license to the first imitator). Second, a higher price for knowledge decreases the hazard rate $\lambda(p_s)$ and thus delays entry into the market by the imitators. Indeed, as p_s increases, it becomes more attractive for the imitators to delay their entry times with the hope of buying the *second* license at a zero price if the rival enters first. There is nevertheless a countervailing effect: As imitators delay their entry times, the licensing profits are obtained later, potentially decreasing the overall period-0 expected payoff of the inventor. Result (i) demonstrates that this third effect is dominated by the previous ones. The inventor chooses the license price that maximizes the delay in entry times (i.e., the license price that minimizes the hazard rate). The incentive to preserve monopoly rents for a longer period clearly dominate the potential loss in licensing revenues.

The essential message of proposition 2 is that, when a market for technology exists, the inventor retains monopoly profits for a time period, even in complete absence of intellectual property rights. The innovator benefits in two ways from the existence of a market for technology. She collects licensing revenues but, more importantly, the *dynamics of the equilibrium license prices* in this market encourage imitators to delay their entry times and thus the inventor preserves monopoly rents for a time period.

C. Sources of rents for the innovator

Propositions 1 and 2 allow us to discuss the sources and the magnitude of the additional rents obtained by the inventor in the presence of a market for technology. Proposition 1 demonstrates that in the absence of a market for technology, imitators immediately copy and the inventor obtains an equilibrium payoff of Π_3. Proposition 2, shows that the incremental payoff that accrues to the inventor when a market for technology exists is

$$\begin{array}{rcccc} V_s(\kappa) - \Pi_3 & = & \dfrac{[\pi_1 - \pi_3]}{(r + 2\lambda)} & + & \dfrac{2\lambda}{(r + 2\lambda)}\kappa \\ & & \begin{bmatrix} \text{Rewards} \\ \text{from Delayed Entry} \end{bmatrix} & + & \begin{bmatrix} \text{Licensing} \\ \text{Revenues} \end{bmatrix} \end{array} \tag{6}$$

Equation (6) illustrates the fact that the innovator obtains both direct revenues from licensing and indirect benefits from delayed entry.

The length of time during which the innovator retains monopoly rents depends on the equilibrium hazard rate, λ, that, in a symmetric equilibrium, has a compelling economic interpretation. Observe that, if entry has not happened yet, the opportunity cost for each imitator of delaying entry an infinitesimal amount of time equals $r(\Pi_3 - \kappa)$: the flow equilibrium payoff that he would obtain if he were the leader imitator. But, on the other hand, the benefit for each imitator of delaying entry an infinitesimal amount of time equals κ: the difference in the equilibrium payoffs between being the leader and the follower imitator. This benefit is only obtained if the rival imitator enters first: An event that happens with hazard rate equal to λ. Thus, in a behavior symmetric equilibrium, $\lambda\kappa = r(\Pi_3 - \kappa)$, implying that $\lambda = r(\Pi_3 - \kappa)/\kappa$.

So, the expected duration of the time period during which the innovator retains monopoly rents[30]

$$\frac{1}{2\lambda} = \frac{(\kappa/2)}{r(\Pi_3 - \kappa)} \tag{7}$$

depends not only on the benefit of waiting (i.e. the absolute value of κ) but also on the opportunity cost of waiting (i.e. $(\Pi_3 - \kappa)$). Fixing the values for r and Π_3, the key parameter of our model is, κ, the imitation cost. When κ goes to zero, the benefits of waiting are completely eliminated and the imitators enter the market immediately at time 0. When κ increases, the expected duration of "monopoly" time and the overall rents of the innovator increase.[31] Moreover, note that when κ increases not only the benefits of waiting increase but also the opportunity costs of waiting decrease. As κ goes to Π_3 the opportunity cost of waiting goes to zero and, in the limit, entry into the product market never happens. Thus, even in the absence of intellectual property rights, the inventor obtains the present value of monopoly profits. We summarize the preceding discussion in the following corollary.

Corollary 1 *Suppose that a market for technology exists. Then*
(i) the expected duration of monopoly time and the inventor's expected equilibrium payoff are

[30] The formula belows follows immediately from the definition of expectation for an exponential distribution with parameter equal to 2λ (i.e., there are two imitators).

[31] This statement can be easily confirmed by: (i) Totally differentiating $V_s(\kappa)$ with respect to κ; (ii) Considering that $d\lambda/d\kappa = -\frac{1}{\kappa}[r + \lambda] < 0$; and finally: (iii) Using assumption 1.

strictly increasing in the imitation cost, κ
(ii) the inventor's expected equilibrium payoff converges monotonically to the present value of monopoly profits, Π_1*, as* κ *converges to* Π_3*.*

We finish this discussion with an illustrative example

Example

Consider a product that generates monopoly profits of $\pi_1 = \$0.1$ million per year. Suppose $r = 10$ percent. An innovator protected by an infinitely long patent will obtain discounted profits of $\Pi_1 = \$1M$. Suppose that market demand is well approximated by a linear demand and that marginal cost is constant. If firms compete on quantities, we can the derive the value of triopoly profits: $\pi_3 = \$0.025M$ and $\Pi_3 = \$0.25M$. We then vary κ between 0 and Π_3.

We present the results in the following table. In the first column, we report the duration of monopoly time (i.e., the expected delay in entry). In the second, we report the discounted profits of the innovator derived from result (iii) in Proposition 2. In the last three columns, we decompose the percentage contributions of the different revenue streams: (i) Percentage coming from monopoly profits before entry ($\frac{\pi_1}{r+2\lambda}$); (ii) Percentage coming from triopoly profits after entry ($\frac{2\lambda}{r+2\lambda}\Pi_3$); and: (iii) Percentage obtained from licensing revenues ($\frac{2\lambda}{r+2\lambda}\kappa$).

κ ($\$M$)	Dur. Mon. Time (years)	Discounted Profits of innovator ($\$M$)	% Before Entry	% After Entry	% Licensing Revenues
0.01	0.21	0.275	7	89	4
0.02	0.43	0.3	14	80	6
0.04	0.95	0.35	25	65	10
0.07	1.94	0.43	38	49	14
0.1	3.33	0.51	49	37	15
0.2	20	0.82	82	10	8
0.24	120	0.96	96	2	2
0.249	1245	0.99	100	0	0

As κ increases the expected time of *first* entry and the expected equilibrium payoff of the innovator increase. If the cost of reverse engineering the process is \$10000, the innovator expects to retain monopoly profits for more than two months and overall to obtain profits of \$275000 (compared to \$250000 without licensing markets). However, if the cost of reverse engineering is \$100000 entry would be prevented on average for close to 3 years and a half and the innovator would obtain profits of \$510000, a bit more than half the present value of monopoly profits.[32]

[32] Note that, to the best of our knowledge, there is no good estimate of the cost of reverse engineering. Maurer and Scotchmer (2002) argue nevertheless that in certain industries it is reasonable to assume that the cost of an independent inventor is similar to the cost of the initial innovator. In our context this would imply that the innovator could cerainly cover his invention cost.

We observe, in accordance with Corollary 1, that the payoff of the innovator converges to monopoly as κ converges to Π_3. It is interesting to discuss the effects reported in last three columns. The percentage of the overall equilibrium payoff coming from the monopoly position before entry naturally increases with κ. Conversely the percentage of the overall equilibrium payoff coming from triopoly profits after entry decreases with the imitation cost. The more interesting result relates to licensing revenues. We notice that, as κ increases, the percentage of revenue coming from licensing initially increases and then decreases. Indeed, as κ increases, the instantaneous licensing revenues increase. This effect is linear in κ. However, as κ increases, these revenues are obtained at a later date. Given that the effect of κ on delay is non linear, the second effect dominates as κ goes to Π_3. Therefore, the discounted value of expected licensing revenues initially increases with κ and then decreases as the effect of the delay starts dominating.

4. EXCLUSIVITY, DELAY AND APPROPRIATION: FURTHER RESULTS

Section 3 introduced our main result. However, to keep the exposition simple, we concentrated on non-exclusive contracts and focused on one particular equilibrium in the competitive subgame. In this section we examine these issues more thoroughly. First, we study the case of exclusive contracts and show that the innovator will not use them in equilibrium. We then characterize the full set of symmetric perfect equilibrium and identify a condition under which the equilibrium studied in section 3 is the unique symmetric equilibrium.

A. Exclusive contracts

In the previous section we constrained the innovator to offer non-exclusive licenses. This might appear overly restrictive. Certainly, exclusive licenses might be attractive to the innovator as they remove competition in the market for technology. Nevertheless, in this section, we show that the innovator will choose to offer non-exclusive licenses rather than exclusive licensing contracts. We suppose in this sub-section (as opposed to sub-section B) that, if non-exclusive licenses were sold, the no-delay licensing equilibrium would be played in the competitive subgame.

We first examine the subgame that follows after the first imitator, say imitator i, enters the market by signing an exclusive licensing agreement with the inventor. The innovator then has to decide at every period at what price to offer the license to the remaining imitator. We then obtain the following intermediate result.

Lemma 2 *The unique SPE of the subgame starting at $t+1$ after entry at t of imitator i with an exclusive license, is such that the innovator offers a licensing contract at a price κ at every period and imitator j enters immediately at $t+1$.*

Proof. See the Appendix. ■

The innovator would ideally want to promise imitator j to lower the license price in the future to delay his entry into the market. However, this promise is not credible as once that period comes, the innovator has an incentive to keep prices high and it is optimal for the imitator to accept such high offers rather than spend the imitation cost. Therefore, the unique SPE is such that imitator j enters immediately by paying a price of κ to the innovator. We now analyze the SPE of the entire game.

Proposition 3 *The unique SPE when the inventor offers exclusive licenses is such that*
(i) both imitators enter immediately at period $t = 0$ by buying licenses for a price of κ
(ii) the innovator's equilibrium payoff $V_s^e = \Pi_3 + 2\kappa$, is strictly smaller than the equilibrium payoff that she obtains when she offers non-exclusive licenses.

Proof. See the Appendix. ■

It is interesting to compare the results of Propositions 1 and 3. In both cases, imitators correctly perceive that their entry cost will remain fixed in the future and thus decide to enter the market immediately at period zero. However, in the case of Proposition 3, the entry cost remains constant over time due to the exclusivity clauses contained in the licensing contracts. When using exclusive contracts, the innovator cannot commit to lower the price of the second license in the future and, from the point of view of the imitators, she replicates the same economic environment as if the market for technology were missing. The innovator, however, obtains higher rents: She appropriates, in form of licensing revenues, what before were lost imitation costs.

Result (ii) compares the equilibrium payoff for the innovator in the exclusive and non-exclusive cases. The benefits of offering exclusive contracts is that licensing revenues are higher in absolute terms and are obtained earlier (at period 0). However, by offering exclusive contracts, the innovator is unable to commit to lower the price of the second license. Non-exclusive contracts allow the innovator to make this commitment by introducing competition on the market for technology. The imitators therefore delay their entry into the product market. Result (ii) shows that the extra monopoly profits collected due to this delay are larger than the lost licensing revenues.

Some suggestive empirical evidence seems to confirm the importance of non-exclusive contracts in the absence of patents or when patent rights are "weak". Anand and Khanna (2000), report the percentage of non-exclusive licenses signed in their sample of contracts.[33] For chemicals (mostly drugs in the sample), the percentage of non exclusive licenses is 12.36%, for computers 28.48% and for electronics 30.35%. This evidence can be confronted to the data collected in the Carnegie Mellon Survey, reported by Cohen, Nelson and Walsh (2000), that asked managers what are the effective mechanism to appropriate the returns from their firms' innovations. For drugs, 50% of managers reported patents were effective, for computers 41% and for electronics 21%.[34] Thus, the sectors least likely to use patents are also those in which non-exclusive licenses are most prevalent. Our mechanism indeed suggests that in the absence of patents these contracts can become an effective way to protect innovations.

B. Multiplicity of Equilibria

[33] See Table III(i) in thier paper.
[34] See Table I in their paper.

The results of section 3 were derived under the assumption that the no-delay licensing equilibrium would be played. This equilibrium is the unique Markov Perfect equilibrium. In this section, we focus on Subgame Perfect Equilibria. The potential multiplicity of (symmetric) SPE of our timing game stems in part from the fact that there are multiple SPE when the sellers, in the market for technology, compete to sell the second license.[35]

We obtain two important results. First, we determine the existence of a condition under which the no-delay licensing equilibrium is the unique perfect equilibrium of the competitive subgame. This emphasizes the importance of the results of section 3. Second, when this condition is not satisfied, we capture a rich relationship between delay to enter the market (i.e. delay to buy the first license) and delay to trade the second license. More precisely: if the length of time until the second license is traded is *not* too long, imitators will still delay their entry times into the market. Otherwise, they will end up entering the market quasi instantaneously at time zero.

B1. Delay and Multiplicity of Equilibria

To simplify the exposition, for most of this section, we focus on the continuous time formulation of our model.[36] In this context, competition to sell the second license starts instantaneously at time t, with the first entry of, say, imitator i. Let $t_2 \geq t$ be the time at which the second license is sold. The *delay* in trading the second license since the time of the first entry is then $d_2 := (t_2 - t) \geq 0$. We denote by $\widetilde{d}_2 := \widetilde{t}_2 - t > 0$ the maximum amount of time that imitator j is willing to wait to buy the license at a *zero* price rather than copying immediately at time t. From these definitions, it follows that the no-delay licensing equilibrium corresponds to a delay $d_2 = 0$ and that $\widetilde{d}_2$ satisfies $\Pi_3 - \kappa = \Pi_3 e^{-r\widetilde{d}_2}$. Proposition 4 describes the (symmetric) pure strategy SPE of the competitive subgame.

Proposition 4 *Suppose that a market for technology exists. Then*
(i) if $2\Pi_3 > \Pi_2$ *the no-delay licensing equilibrium is the unique SPE of the competitive subgame.*
(ii) if $2\Pi_3 \leq \Pi_2$*, for each* $d_2 \in \left[0, \widetilde{d}_2\right]$ *there exists a SPE in which imitator* j *enters the market at time* $t_2 = t + d_2$ *by buying the license at a zero price.*

Proof. See the Appendix. ■

The first part of proposition 4 establishes the important result that for some market games, the *unique* SPE is the no-delay licensing equilibrium, in which a license is sold immediately. For linear demand and Cournot competition, this condition is always satisfied. The intuition of the result is the following. When $2\Pi_3 > \Pi_2$ the market is such that for any potential candidate equilibrium with delay in the market for technology, a profitable deviation will exist for one of the licensors. Indeed, selling a license to imitator j at the highest acceptable price, Π_3, will allow

[35] We return to the assumption that the inventor offers non-exclusive licensing contracts before the first entry happens.

[36] We interpret the continuous time version of our model as an approximation to our previous discrete time-game for the limiting case in which $\Delta \to 0$.

the deviator to collect profits of $2\Pi_3$ rather than the duopoly profits guaranteed in equilibrium. Thus, if this condition is satisfied, the unique equilibrium of the competitive subgame is the no-delay licensing equilibrium.

It is interesting to note that the degree of competitiveness of the technology and product markets are inversely related. A product market such that $2\Pi_3 > \Pi_2$ can be characterized as not highly competitive: profits erode slowly with an increase in the number of firms. However, such a product market will create a very competitive market for technology in which licensors will be unable to coordinate on keeping prices high.

The second part of the proposition points out, however, that when the condition is not satisfied, the competition game that follows after the first entry accepts a multiplicity of SPE. All of these SPE share a common feature: the second license is always sold at a price equal to zero. The intuition is simple. Because the seller with the lowest price will serve the entire market (i.e., imitator j), each seller has an incentive to undercut his rival. However, all of these SPE are characterized by different delays before the second license is traded. An outcome such that the license is sold with a delay equal to $d_2 > 0$ is a perfect equilibrium for two reasons.[37] First, imitator j, given that $d_2 \leq \widetilde{d}_2$, prefers waiting that amount of time to buy a license at a zero price rather than copying immediately. Second, the sellers, when $2\Pi_3 \leq \Pi_2$, prefer collecting duopoly profits in the market rather than deviating and receiving licensing payments.

Equilibrium payoffs, in period t units, for the leader and follower imitator may now be written as

$$V_1 = (1 - e^{-rd_2})\Pi_2 + e^{-rd_2}\Pi_3 - p_s; V_2 = e^{-rd_2}\Pi_3 \tag{8}$$

where $d_2 \in \left[0, \widetilde{d}_2\right]$ is a given equilibrium delay.[38] The existence of delay in the market for technology causes one conceptual novelty. The first imitator collects duopoly profits in the market prior to the second entry. It may thus be worthwhile for imitators to become leaders rather than followers. More precisely, as equation (9) highlights, if d_2 is sufficiently high V_1 could be higher than V_2. So, for low values of d_2 our timing-game corresponds to a "war of attrition" whereas for sufficiently large values of d_2 the imitators play a "preemption game".

B2. Delay and Appropriation

We first establish that when the delay on the market for technology is not too long, the imitators still delay their entry into the market.[39] Proposition 5 is indeed a generalization of Proposition 2.

Proposition 5 *Let* $d_2 \in \left[0, \widehat{d}_2\right)$ *for* $\widehat{d}_2 := (1/r)\ln\left(\Pi_2/\left(\Pi_2 - \kappa\right)\right)$ *then*
(i) the innovator sets a price $p_s = \kappa$ *for the licenses;*

[37] The multiplicity of equilibria is *not* due to well-known repeated game theory arguments: None of these equilibria is sustained by a punishment scheme.

[38] To simplify the exposition, we assume that the inventor follows a non-discriminatory pricing policy.

[39] We insist on an important distinction. Note that d_2 is the equilibrium delay in the market for technology. In other words, it is the amount of real time that the second imitator must wait before buying a license at a zero price. On the other hand, when we refer to "delay to enter the market", the delay corresponds to the time elapsed before entry of the first imitator.

(ii) the distribution of entry times of each imitator is exponential with hazard rate equal to

$$\lambda = \frac{r\left[(\Pi_3 - \kappa) + (\Pi_2 - \Pi_3)\left(1 - e^{-rd_2}\right)\right]}{\left[\kappa - (1 - e^{-rd_2})\Pi_2\right]}$$

(iii) the inventor's equilibrium payoff is

$$V_s(\kappa, d_2) = \frac{\pi_1}{(r + 2\lambda)} + \frac{2\lambda\left[(\Pi_3 + \kappa) + (\Pi_2 - \Pi_3)\left(1 - e^{-rd_2}\right)\right]}{(r + 2\lambda)}$$

Proof. See the Appendix. ■

As in Proposition 2, result (i) indicates that the innovator sets a price of κ for the licenses and result (ii) shows that imitators delay their entry into the product market. However, result (iii) establishes that, compared to the expression of Proposition 2, the innovator now collects duopoly profits after the first entry.[40] However when d_2 increases, the equilibrium hazard rate also increases and therefore the expected duration of monopoly time for the innovator decreases.

From this discussion, a natural question emerges: In which SPE does the innovator receive her highest payoff? Unfortunately, the work of several countervailing forces makes difficult to analytically determine her best SPE payoff. On the one hand, the higher the delay in the market for technology, the longer the time period for which the innovator retains duopoly profits after the first entry. Furthermore, as we discussed before, a higher value of d_2 increases the equilibrium hazard rate and therefore the duopoly profits and the licensing payments are obtained earlier. There is nevertheless a powerful offsetting effect: The first entry will occur, on average, earlier and the innovator will thus obtain monopoly profits for a shorter time period.

Last, we discuss the dynamics of entry and appropriation if $d_2 \geq \widehat{d}_2$. In this case, the equilibrium payoff of the leader becomes larger than the equilibrium payoff of the follower. Besides, both of these equilibrium payoffs are greater than the equilibrium payoff of simultaneous entry. This payoff structure gives rise to a preemption game, in which entry into the market takes place immediately at time $t = 0$. Formally.

Proposition 6 *Let* $d_2 \in \left[\widehat{d}_2, \widetilde{d}_2\right]$ *for* $\widehat{d}_2 := (1/r)\ln\left(\Pi_2/\left(\Pi_2 - \kappa\right)\right)$. *Then*
(i) as the length of a period shrinks to zero, $\Delta \to 0$*, the unique perfect equilibrium outcome is that entry will happen for sure at time* $t = 0$*;*
(ii) the innovator sets a price for both licenses equal to κ *and her equilibrium payoff is* $V_s(\kappa, d_2) = \phi V_s^e + (1 - \phi)\, V_s^a$ *where* $\phi\,(\kappa, d_2)$ *is the probability of simultaneous entry at* $t = 0$ *and*

$$V_s^e := 2\kappa + \Pi_3;\ V_s^a := \left[\kappa + (1 - e^{-rd_2})\Pi_2 + e^{-rd_2}\Pi_3\right]$$

Proof. See the Appendix. ■

Observe that the innovator's equilibrium payoff is a convex combination of the payoff that

[40] Note that when $d_2 = 0$, we obtain the same results of Proposition 2.

she receives when imitators enter the market simultaneously at $t = 0$ as in Proposition 3, V_s^e, and when only one of them enters the market at $t = 0$ and the other follows after a delay of at least $\widehat{d}_2$, V_s^a. We know, from Proposition 3, that the innovator's equilibrium payoff for the no-delay licensing equilibrium, $V_s(\kappa)$, is strictly higher than V_s^e.Besides, straightfoward calculations show that $V_s(\kappa)$ is also strictly higher than V_s^a for all values of $d_2 \in \left[\widehat{d}_2, \widetilde{d}_2\right]$.

So, even though ex-post the innovator appropriates with strictly positive probability duopoly profits, sells two licenses instead of one and receives all these payments earlier, she is still strictly worse off than if the no-delay licensing equilibrium were played. Entry the product market happens instantaneously and erodes her monopoly profits substantially.

In our view Propositions 2, 5 and 6 together provide a concise view of appropriation problems in an economy without intellectual property rights. They reveal that appropriation is closely linked to the price and the speed at which the second license will be traded in the market for technology. When imitators expect knowledge to be diffused slowly through the licensing process, entry into the market will be inevitably fast. Thus we show that if an appropriation failure exist, it is not caused by a lack of intellectual property rights per se but rather by a different kind of failure coming from slow diffusion of knowledge in the market for technology.

5. LARGE NUMBER OF IMITATORS

In this section we examine the innovator's appropriability problem in an economy populated by a large number of potential imitators. The set of potential imitators is denoted $\mathcal{P} := \{2, 3...\}$ and the set of *active* imitators by $\mathcal{A} := \{0, 1, ...\}$. The present value of equilibrium profits when there are n active imitators is Π_{n+1} for $n \in \mathcal{A}$.[41] The notion that profits decrease as the number of active rivals increases is formalized by assuming that the sequence of profits $(\Pi_{n+1})_{n\in\mathcal{A}}$ is decreasing. Besides, we also suppose that profits are weakly greater than zero.[42] Thus, the sequence $(\Pi_{n+1})_{n\in\mathcal{A}}$ converges to zero.

In the previous sections, Assumption 1 ($\Pi_3 > \kappa$) guaranteed that copying was a valuable activity for all potential imitators. When the number of potential imitators becomes large, such an assumption becomes difficult to satisfy. We thus consider two cases. The first case, in the continuity of the previous sections, assumes that the imitation cost is smaller than the present value of profits, Π_{n+1}, for all $n \in \mathcal{P}$. To allow for this, the imitation cost must also diminish as the number of active imitators increases. Hence we call this scenario variable imitation cost. The second case is one where, when n is sufficiently large, the profits Π_{n+1} are smaller than the imitation cost.

Independently of the case considered, we show that when the number of *potential* imitators becomes sufficiently large, the number of *active* imitators converges to zero and the innovator's equilibrium payoff converges to the present value of monopoly profits.[43] This result challenges

[41] Note that n imitators and the innovator share the market and thus we obtain profits Π_{n+1}.

[42] This assumption corresponds for instance to a case in which there are no fixed costs.

[43] In this section, in contrast to the previous sections, the assumption that those imitators who enter by copying can also sell knowledge turns out to be important for our main result.

the classical arguments calling for intellectual property rights.[44]

A. Variable Imitation Cost

To maintain the spirit of Assumption 1, we introduce its equivalent for a large economy.

Assumption 2: $\forall n \in \mathcal{P} : (\Pi_{n+1} - \kappa_n) > 0$

Since profits converge to zero as the number of active imitators increases, this assumption requires that the imitation cost depends on n. Moreover, observe that the sequence of imitation costs $(\kappa_n)_{n=2}^{\infty}$ must also converge to zero. It is important to realize that this assumption should not be interpreted literally. Assumption 2 is rather a modelling device which guarantees that even for large economies imitation is still profitable for all potential imitators. Note that it captures well situations where the imitation cost is very small.

We also make an additional assumption that restricts the rate at which the imitation cost varies with n. Assumption 3 turns out to be important for our main result.

Assumption 3: $\lim_{n\to\infty} (\Pi_{n+1}/\kappa_n) = 1$

Under these assumptions, if the market for technology is missing, it is a simple extension of Proposition 1 to show that in a large economy (i.e., as $n \to \infty$) there is a unique SPE in which: (i) All imitators copy the innovation immediately at time zero and (ii) The innovator's equilibrium payoff converges to zero. Thus, as the conventional wisdom suggests, innovation would never take place without intellectual property rights.

We now turn to the question of appropriability in the presence of a market for technology. Similar to the no-delay licensing equilibrium of Section 3, we focus on the equilibrium outcome such that, after the first entry, the innovator and all active imitators immediately offer licenses to the remaining imitators at a zero price.

Proposition 7 *If Assumptions 2 and 3 hold, the unique symmetric MPE is such that:*
(i) the innovator sets a price $p_{sn} = \kappa_n$ for the licenses;
(ii) the distribution of entry times of each imitator is exponential with hazard rate equal to

$$\lambda_n = \frac{r\,(\Pi_{n+1} - \kappa_n)}{(n-1)\,\kappa_n}$$

(iii) as $n \to \infty$ the innovator's equilibrium payoff converges to monopoly profits:

$$\lim_{n\to\infty} V_s^n(\kappa_n) = \Pi_1$$

Proof. See the Appendix. ■

[44] The traditional arguments should be particularly relevant in this case with a large number of imitators who should theoretically reduce the profits of the innovator to zero.

This proposition fundamentally challenges the received conventional wisdom: It states that when the number of potential imitators becomes sufficiently large, the innovator will receive an equilibrium payoff arbitrarily close to monopoly profits.

To give an intuition for this result it is important to first understand the role played by Assumption 3. Note, on the one hand, that when $n \to \infty$ the equilibrium price of the *first* license, κ_n, converges to zero. So, as $n \to \infty$, the benefits of waiting converge to zero.[45] But, on the other hand, the opportunity cost of waiting (i.e. the profitability of copying $(\Pi_{n+1} - \kappa_n)$) also converges to zero, as $n \to \infty$. Assumption 3 guarantees that these two terms decrease at the same rate and for this reason, in the limit, the following ratio

$$\frac{(\Pi_{n+1} - \kappa_n)}{\kappa_n}$$

becomes negligible. Thus as n goes to infinity, the individual hazard rate λ_n converges to zero.

However, the equilibrium payoff of the innovator depends on the time of the first entry and thus on the aggregate hazard rate, $n\lambda_n$. Result (iii) establishes that as $n \to \infty$, $n\lambda_n \to 0$ and the innovator's profits converge to monopoly. Indeed an increase in the number of potential imitators is compensated by a decrease in the hazard rate of each individual imitator thus postponing the time of first entry.

The fact that the individual hazard rate λ_n is divided by $n-1$ summarizes an important message of this paper. Note that the imitator who buys the *first* license provides a *positive* (pecuniary) externality to the rest of the imitators. Indeed the first entrant creates competition in the market for technology and the rest of the imitators benefit by buying licenses at a zero price. So, rational imitators anticipating the possibility of receiving these future positive externalities delay their entry into the market. When the number of imitators becomes large, the incentives of each individual imitator to buy the *first* license are decreased, as the pool of potential first entrants becomes larger.[46]

B. Fixed Imitation Cost

We consider the second case such that the imitation cost remains fixed at a level $\kappa > 0$ independently of n. Therefore for n large enough Assumption 2 is not satisfied and profits become strictly smaller than κ. We denote by K the number of imitators such that $\Pi_{K+2} < \kappa \leq \Pi_{K+1}$. As in the previous case, we focus on the equilibrium such that, after the first entry, the innovator and all active imitators immediately offer licenses to the remaining imitators at a zero price. We obtain the following result.

[45] The benefit of waiting is the difference bewteen the price of the first and the price of the second license.

[46] More formally, consider an imitator, say imitator i, that must decide whether to wait a period of time equal to Δ or buy a license at time t. If he buys it at t, he will be the leader and obtain a payoff equal to V_1. However, if he chooses to buy it at $t+\Delta$, he will buy it at a zero price if at least one of the other $n-1$ imitators bought it before him. In a behavior equilibrium, imitator i must be indifferent between buying at time t and buying at time $t+\Delta$. But when the number of imitators increases, the probability that at least one of them buys a license before imitator i also increases. So, he will remain indifferent between t and $t+\Delta$ if and only if, the other imitators decrease their corresponding probabilities of buying a license in the time interval Δ.

Proposition 8 *As $\delta \to 1$ and for all $n > K$, the unique symmetric MPE is such that no imitator initially enters the market and the innovator's equilibrium payoff equal monopoly profits.*

Proof. See the Appendix. ■

The result of Proposition 8 has a flavor similar to the holdup problem. An imitator is willing to buy the *first* license by making a sunk payment equal to κ only if he anticipates that his profits will cover this cost. He will thus buy the *first* license if and only if he anticipates that at most $K-1$ other imitators will also enter the market. But, after he enters, intense rivalry in the market for technology leads to an entrance of $n-1 > K-1$ imitators as they pay a zero price for the *second* license. Thus entry can never be initially profitable and the innovator will retain monopoly rents forever.[47]

6. RELATED LITERATURE

Our results are related to several branches of the literature. First, our paper is connected with a literature that argues in favor of weakening intellectual property rights. In a sequence of papers, Boldrin and Levine (2002, 2004, 2005, 2007) have proposed a new model of creative activity under perfect competition. These papers share the essential idea that innovative rents equal the discounted value of the revenue stream generated by the first unit(s) of the prototype(s) created by the inventor. We emphasize in our paper a different mechanism based on the dynamics of the market for technology. Our purpose is to depart in a minimal way from the conventional model. Indeed we only introduce the market for licenses. Our work complements theses papers by showing the existence of a different source of rents in the absence of intellectual property rights.[48]

Maurer and Scotchmer (2002) study the consequences of introducing independent invention defence in the patent system.[49] In their paper, the inventor can always find suitable fixed-fee and royalty licensing contracts to deter the duplication of R&D costs by independent inventors.[50] They show that if the cost of independent invention is more than half the initial research cost, the innovator can deter duplication through licensing and still recover her R&D cost. As our work, this paper emphasizes the importance of the market for technology for the innovator's rents but our approach and results differ on several accounts. First, whereas the analysis in Maurer and Scotchmer (2002) is static, our model is built to capture the dynamic trading relationships that exist between innovators and imitators in the market for technology. Second, we emphasize the important of non-exclusive contracts as a commitment device. Third, we find that even for small

[47] These results are closely linked to Boldrin and Levine (2008). The authors argue that if competition on the product market is severe enough and there is a fixed cost of imitation (equivalent to the cost κ in our paper) imitators might refrain from initially entering. The environment they consider does not include a market for technology, but has the inventor moving first and choosing quantities before the imitators. She can therefore choose to produce more than the monopoly quantity, to decrease prices and thus discourage imitation.

[48] As pointed out in section 5, the results for a large number of imitators are also linked to Boldrin and Levine (2008).

[49] Under independent invention defence, independent inventors (imitators who reverse engineer the product) are allowed to share the market with the innovator.

[50] These contracts decrease profits on the market.

reverse engineering costs, the inventor can appropriate large rents due to the dynamics of the market.[51] Note however that a cautious reinterpretation of our model in the spirit of their paper suggests that our findings also argue in favour of an independent invention defence.

The use of licensing to preserve monopoly rents has also been of interest in the literature. Gallini (1984) shows that an incumbent could license her innovation to eliminate the incentives of a rival to develop his own superior technology. Rockett (1990) shows how an innovator can use licensing to crowd out the market by weak competitors and thus to prevent entry of strong rivals. Although our paper relates to this literature there are clear differences. In particular, in our setting imitators are homogeneous and the innovator deters entry through a subtle use of non-exclusive licensing contracts.

Other explanations for the existence of endogenous delay in imitation have also been proposed. Scherer (1980) suggests that technological constraints generates “natural lags” in imitation. This explanation does not depend on the strategic responses of imitators. Benoit (1985) shows how an endogenous delay can emerge if the profitability of a non-patentable innovation is uncertain and gradually revealed over time to a unique imitator. We do not introduce asymmetric information but rather build on the strategic interactions between *several* imitators in the market for technology.

Choi (1998) is closest to our paper in that the effect of imitation on the innovator is obtained through *endogenous* entry that results from *strategic* interaction between imitators. However, in his paper imitation occurs in a world with imperfect patent rights and the interaction between imitators is controlled by information transmission through possible infringement suits. In contrast, our paper focuses on a setting *without* patent rights and in which the strategic interaction is dominated by the dynamics of the price of licenses. For instance, the main results of Choi (1998) are driven by an *informational externality*: If an imitator enters the market and the innovator responds with an infringement suit, the second imitator will receive information concerning the outcome of that suit. In contrast, our results are mainly driven by a *pecuniary externality*: After the first imitator enters the market, the equilibrium price of the *second* license will fall to its marginal cost.[52]

A paper by Arora and Fosfuri (2003) focuses on the interaction between the product market and the market for technology in a static environment. The paper highlights that it can be optimal for a firm to license out its technology to a rival. In a static model, it proves that, under certain conditions, the revenues that the firm expects from licensing to a rival dominates the loss due to the erosion of overall industry profits. This paper shares some similarities with the analysis of what we have called the competitive subgame. More precisely, the trade-off considered in this paper is comparable to the one that guarantees uniqueness of the no-delay licensing equilibrium in the competitive subgame.[53]

[51] The cost of research of the inventor is not explicitely defined in our model. But a similar comparison between the rents of the inventor and the cost κ of reverse engineering is performed in section 3.

[52] We note that also Bernheim (1984) examines the dynamics of entry deterrence. The dynamics are however very different than in our model. In particular, Bernheim (1984) assumes that entrants are ordered in an exogenously given sequence.

[53] See in particular the condition in Proposition 4.

Last, as pointed out in the introduction, licensing without intellectual property rights can be a subtle issue in the presence of asymmetric information. Anton and Yao (1994, 2002) have underlined this concern. For instance, Anton and Yao (1994) study the problem of an inventor who can license her idea to two firms. Buyers are reluctant to buy before learning the quality of the idea but if the idea were revealed, they could potentially steal it. Anton and Yao (1994) propose a subtle solution to this expropriation problem.[54] In the environment that we consider, asymmetries of information should be minimal and the concerns studied by Anton and Yao (1994, 2002) can be left aside. Indeed, the innovation is already commercialized at the start of the game and its success is publicly observable.

7. CONCLUSION

The main contribution of this paper is to provide a concise view of appropriability problems in an economy without intellectual property rights but with markets for technology. The results of this paper reveal that the set of markets in which innovators and imitators interact is crucial to understand how the rents generated by the arrival of new technologies are divided between economic agents.

In the absence of a market for licenses, our model can be seen as a simple theoretical foundation for the received conventional wisdom. However once we allow for simple trading interactions between innovators and imitators, new and surprising insights are obtained. In general, the introduction of markets for technology dissipates the appropriability problem. The innovator collects monopoly profits, at least temporarily, in most of the cases that we have focused on. The counterpart of this result is that each imitator delays his entry on the market with the hope that his rival will enter before him. To generate this delay, the innovator optimally chooses to grant non-exclusive licensing contracts. Indeed, non-exclusivity is crucial to change the structure of the market for technology (from monopoly to duopoly) and thus to create the beneficial competition that results in the reduction of the price of the second license.

In Section 4 we highlighted cases where the appropriability problem might reemerge and these results are suggestive. They show that if an appropriation failure exists, it is not caused by a lack of intellectual property rights per se but rather by a "coordination" problem related to how quickly knowledge is diffused in the market for technology. The message of this paper can be reexpressed as follows: When knowledge is quickly diffused after the first entry appropriability concerns are generally dissipated.

Because we have examined appropriability problems, we formulated a very simple and suitable model for this particular goal. Thus, we have not considered several interesting issues which could be the object of future research. First, our model does not consider the possibility of sequential improvements to the current innovation. In this context, licensing may provide not only knowledge to imitate the current innovation but also to discover a future improvement.

[54] The mechanism that solves this problem starts with full revelation by the inventor. The buyer will still sign a contract under the threat by the inventor that she will reveal the idea to the competitor if he decided to copy without paying. Anton and Yao (2002) also discuss a different mechanism for this problem based on partial disclosure of the idea and the issuance of a bond.

Second, we have assumed that the imitation cost is common knowledge. In an extension of this paper, it would be interesting to presume that imitators have private information about their cost of copying and to examine how this private information affects the innovator's equilibrium payoff. Third, our model considers an stationary environment in which market demand is implicitly the same over time. Examining a framework in which market demand changes over time (i.e. a growing market size) would also be an interesting exercise.

We show that even in the absence of intellectual property rights and in an environment sufficiently close to the traditional one, inventors are able to collect substantial economic rents. An essential future step will be to answer the following question: Without intellectual property rights, do innovators appropriate an equilibrium payoff equal to the social value of their contribution (see Ostroy and Makowski (1995))? Recently, Shapiro (2007) has argued that under the current patent system innovators capture private rewards that exceed their social contributions. Could the dynamics of prices in the market for technology appropriately tailor these rents to their social contributions?

We conclude by offering a different perspective on our results. Two very influential surveys (Yale Survey (1983) and Carnegie Mellon Survey (1994)) have asked managers to rank the most efficient means of protecting their innovations. The Yale Survey, conducted in 1983, reports that for both product and process innovations, secrecy was consistently ranked as one of the worst methods to protect an innovation. The Carnegie Mellon Survey, conducted ten years later, reports, on the contrary, that secrecy was consistently ranked first. As Cohen et al (2000) point out, there is no apparent explanation for the "growth in the importance of secrecy as an appropriability mechanism". This fact is particularly surprising, since the period between 1983 and 1994 was one where patent protection tended to strengthen.

Number of industries ranking secrecy as:

Mechanism	1st	2nd	3rd	4th
Secrecy 1983 (process)	2	10	19	2
Secrecy 1994 (process)	21	10	1	1
Secrecy 1983 (product)	0	0	11	22
Secrecy 1994 (product)	13	11	2	5

We believe that the mechanism highlighted in this paper offers a potential explanation for the previous puzzle. Indeed, a firm choosing secrecy is not protected against imitation by competitors. However, as emphasized in this paper, in the presence of a market for technology, imitators might delay entry. Thus, more active licensing markets might increase the returns that firms can expect from choosing secrecy. Licensing activity did indeed intensify in the period 1983 to 1994. Arora et. al. (2002) using data compiled by the Securities Data Company report that the total number of disclosed licensing deals during the period 1985 to 1989 was 1130 while for the years 1993 and 1994, 2009 and 2426 deals were signed respectively.[55] Our model can thus reconcile

[55] The Securities Data Company contains data on licensing deals and joint venture

the parallel increase in the popularity of both secrecy and licensing, providing therefore a novel explanation for this puzzle. This is yet another perspective on our results.

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APPENDIX

Proof of proposition 1. A pure strategy for imitator $h \in \{i, j\}$ must prescribe at each period t whether to “to copy” or “not to copy”. We proceed in steps.

First, we consider the single decision problem that follows a history in which only one imitator, say imitator j, has copied.

Step1. Suppose that imitator j has chosen “to copy” at period $t-1$. Then, the unique best response of imitator i is “to copy” at period t.

To see that, observe that by copying at any period $t_i \geq t$ he obtains a payoff in period t units of $V_i(t_i) = \delta^{(t_i - t)} (\Pi_3 - \kappa)$ and clearly $t = \arg\max_{t_i \in [t,\infty)} V_i(t_i)$.

Now we turn our attention to those histories that start at period t and in which no imitator has copied yet. Given the symmetry of the game, we study the best response of imitator i to the following two strategies of imitator j.

Step 2. Suppose that the strategy of imitator j dictates “to copy” at period t. Then, the unique best response for imitator i is also “to copy” at period t.

This follows directly because if imitator i chooses “to copy” at period $t_i \geq t$ he receives a payoff in period t units equal to $V_i(t_i) = \delta^{(t_i - t)} (\Pi_3 - \kappa)$ and clearly $t = \arg\max_{t_i \in [t,\infty)} V_i(t_i)$.

Step 3. Suppose that the strategy of imitator j dictates “to copy” at a period $t_j > t$. Then, the unique best response for imitator i is “to copy” at period t.

We know, from step 1, that if imitator i copies at $t_i < t_j$ the strategy of imitator j prescribes “to copy” at period $t_i + 1$ (i.e., Δ units of real time later). Thus, if imitator i chooses “to copy” at period $t \leq t_i < t_j$ he obtains a payoff in period t units of $V_i(t) = \delta^{(t_i - t)} [(1-\delta)\Pi_2 + \delta\Pi_3 - \kappa]$. But if he chooses “to copy” at period $t_i \geq t_j$ he receives a payoff in period t units of $V_i(t) = \delta^{(t_i - t)} [\Pi_3 - \kappa]$. Comparing these payoffs, it is evident that the unique best response for imitator i is “to copy” at period t, without delay.

So, our analysis reveals that there is a unique SPE in which imitators choose “to copy” immediately at all periods in which copy has not occurred yet. Thus the unique equilibrium outcome is such that both imitators choose "to copy" at period zero and such that the equilibrium payoffs for the inventor and the imitators are Π_3 and $\Pi_3 - \kappa$ respectively. ■

Lemma 0: Consider the following strategies for the sellers (imitator i and the inventor) and for imitator j.

$$
\begin{aligned}
p_{yj}^{\tau} &= 0, \text{ for } y = i, s \text{ for all feasible histories if entry has not happened yet, for all } \tau \geq t+1 \\
\sigma_j &= \begin{cases} \text{"enter" by } \ell \text{ buying from } \widehat{y} \text{ if } \widehat{p} \leq \min\{\kappa, (1-\delta)\Pi_3\}, \text{ for all } \tau \geq t+1 \\ \text{"enter" by } c \text{ if } \kappa \leq \min\{\widehat{p}, (1-\delta)\Pi_3\}, \text{ for all } \tau \geq t+1 \\ \text{"wait" or "do not enter" if } \min\{\widehat{p}, \kappa\} > (1-\delta)\Pi_3 \text{ for all } \tau \geq t+1 \end{cases}
\end{aligned}
$$

where $\widehat{y}$ denotes the identity of the seller who offers the minimum license price and $\widehat{p} := \min_{y \in \{i,s\}} p_{yj}$. Furthermore ℓ denotes the strategy of buying a license and c of copying.

These strategies constitute a MPE and they give rise to the no-delay licensing equilibrium outcome.

Proof of Lemma 0. We first note that the strategies described above give rise to the no-delay licensing equilibrium outcome described in the main text. We now prove that these strategies form a subgame perfect nash equilibrium. For the sellers it is clear that, given any history and the strategy of the other players, they do not have strict incentives to deviate from the actions prescribed by their strategies. As to imitator j, we make the following two observations. First, note that if he chooses to enter at the present period, he will do it either by licensing from the seller with lowest license price or by copying the innovation, whatever alternative is more profitable. Second, if he chooses to enter at the current period rather than wait one period and buy a license at the next at a zero price it must be that

$$\max\{\Pi_3 - \widehat{p}, \Pi_3 - \kappa\} \geq \delta \Pi_3$$

where $\Pi_3 - \widehat{p}$ is the payoff in present units of buying by licensing, $\Pi_3 - \kappa$ is the payoff in present units of copying and $\delta\Pi_3$ is the payoff in current units of waiting and buying a license at a zero price next period. It follows then that imitator j will enter by buying a license if and only if (iff, henceforth) $\widehat{p} \leq \min\{\kappa, (1-\delta)\Pi_3\}$. On the contrary, he will enter by copying iff $\kappa \leq \min\{\widehat{p}, (1-\delta)\Pi_3\}$. So, it is easy to see that he will not enter at the current period iff $(1-\delta)\Pi_3 > \min\{\widehat{p}, \kappa\}$. ■

Proof of Lemma 1 As we observed in the main text, the payoff relevant history is determined in our model by the number of imitators who are active in the market at each time period. Thus, the following partition $\mathcal{H} := \{\mathbb{H}_0, \mathbb{H}_1\}$, where $\mathbb{H}_n$ is a set that collects all those histories for which $n = 0, 1$ imitators are active in the market, is the minimal (coarsest) sufficient partition that permit us to define MPE. A strategy for the innovator, σ_s, is Markovian if it is measurable with respect to the corresponding partitions. Formally, if the history $\widehat{h}^{\tau} \in \mathbb{H}_n(\widetilde{h}^{\tau})$ for some $n = 0, 1$, then the continuation strategy $\sigma_s|_{\widehat{h}^{\tau}}$ must equal the continuation strategy $\sigma_s|_{\widetilde{h}^{\tau}}$.

Measurability implies that the inventor's strategy must prescribe the same pair of prices $p_{sh} \in \mathbb{R}_+ \cup \{+\infty\}$ for each period $\tau \geq t$ at which entry has not taken place yet. We claim that a necessary condition for a Markovian licensing strategy, σ_s, to be optimal is that it should dictate to sell two licenses at prices $p_{sh} \leq \kappa$ for $h \in \{i, j\}$. The idea is simple. Any Markovian strategy which does not satisfy this necessary condition is weakly dominated by σ_s and therefore it can not be optimal. There are at most two of these strategies. One, the inventor sells no licences: $p_{sh} = +\infty$ for $h \in \{i, j\}$, denoted by σ_{ij}^{∞}. And the other, the inventor sells just one license, say to imitator i: $p_{sj} = +\infty$. We denote this strategy σ_j^{∞}.

Notice that when the innovator uses either σ_{ij}^{∞} or σ_j^{∞} the entry cost of imitator $h \in \{i, j\}$ in the first case and of imitator j in the second one equals κ. The inventor must therefore be weakly better off when she uses σ_s than when she adopts strategy σ_{ij}^{∞}. When she uses σ_s, she can always set $p_{sh} = \kappa$ for $h \in \{i, j\}$ and imitators would face the same entry cost as when she employs σ_{ij}^{∞}. Thus, she will receive at least the same expected market profits by using σ_s as when she uses σ_{ij}^{∞}. But she will obtain higher licensing revenues, if entry happens, because it will take place through licensing. The same argument holds for the case of σ_j^{∞}. ■

Proof of Proposition 2. In this proposition, we determine the unique symmetric MPE when Δ goes to zero. We proceed in a number of steps. Because the proof is long, we succinctly describe each step. In step 1, we show that when the time discount associated with Δ is small enough there exists at least a MPE in behavior strategies. In the second step, we derive sufficient conditions which guarantee uniqueness. In step 3, we obtain the limiting equilibrium distribution when $\Delta \to 0$. In step 4, we determine the expected payoff of the inventor for a given pair of licensing prices p_{si} and p_{sj}. Finally, in step 5, we show that the optimal license prices are $p_{si} = p_{sj} = \kappa$.

Step 1. If $\delta \geq \overline{\delta}_E := [\Pi_2 - \kappa] / \Pi_2$ and if $\min\{p_{si}, p_{sj}\} \in (\overline{p}_E, \kappa]$ (for $\overline{p}_E := \left(1 - \overline{\delta}_E\right) \Pi_2$) then a MPE in behavior strategies exists.

Notice that if $\min\{p_{si}, p_{sj}\} > \overline{p}_E$ then $V_h^2 - V_h^1 > 0$ for $h \in \{i, j\}$ and our game is a "war of attrition". The weak inequality $\delta \geq \overline{\delta}_E$ guarantees that $\overline{p}_E \leq \kappa$. Define $Q(\psi_j) := \psi_j V_i^b + (1 - \psi_j) V_i^1$ as the value for imitator j of buying a license in the current period and $W(\psi_j) := \psi_j V_i^2 + (1 - \psi_j) \delta Q(\psi_j)$ as the value for imitator j of buying a license in the next period. Then, $\mathcal{W}_i\left(\psi_j\right) := W\left(\psi_j\right) - Q\left(\psi_j\right)$ and note that

$$\mathcal{W}_i\left(\psi_j\right) = \psi_j \left[V_i^2 - V_i^b\right] + (1 - \psi_j)\left[\delta Q(\psi_j) - V_i^1\right] = 0 \tag{A1}$$

describes the equilibrium condition for imitator i to be indifferent between buying a license at period t or buying it Δ extra units of time later. We show that $\mathcal{W}_i\left(\psi_j\right) = 0$ has at least one solution $\psi_j^* \in (0, 1)$. To see this, observe that $\mathcal{W}_i(0) = \delta Q(0) - V_i^1 = -V_i^1(1 - \delta) < 0$. By the continuity of $\mathcal{W}_i$ in ψ_j, $\mathcal{W}_i\left(\psi_j\right) = 0$ has at least one solution $\psi_j^* \in (0, 1)$ iff $\mathcal{W}_i(1) = \left[V_i^2 - V_i^b\right] = p_{si} - \Pi_3(1 - \delta) > 0 \iff p_{si} > (1 - \delta)\Pi_3$. But by assumption $p_{si} > \overline{p}_E > \left(1 - \overline{\delta}_E\right)\Pi_3$ and hence $\mathcal{W}_i(1) > 0$. So $\mathcal{W}_i\left(\psi_j\right) = 0$ has at least one solution. By symmetry $\mathcal{W}_j\left(\psi_i\right) = 0$ also has at least one solution.

Step 2. Suppose that $\delta \geq \max\left\{\overline{\delta}_E, \overline{\delta}_U\right\}$ and that $\min\{p_{si}, p_{sj}\} \in (\overline{p}_U, \kappa]$, then there exists a unique MPE in behavior strategies; where

$$\overline{\delta}_U := \frac{-\alpha + \sqrt{\alpha^2 - 8\beta^2}}{4\beta},\ \overline{p}_U := \frac{\beta\,(1-\delta)\,(2\delta-1)}{\delta}$$

and $\beta := \Pi_2 - \Pi_3$; $\alpha := (\kappa - 3\beta)$.

To demonstrate uniqueness, it suffices to show that $\mathcal{W}_i\left(\psi_j\right)$ is strictly increasing in the unit interval. Because $\mathcal{W}_i\left(\psi_j\right)$ and $Q(\psi_j)$ are C^1 functions, we have

$$\mathcal{W}_i^j\left(\psi_j\right) := \frac{d\mathcal{W}_i\left(\psi_j\right)}{d\psi_j} = \left[V_i^2 - V_i^b\right] - \left[\delta Q(\psi_j) - V_i^1\right] + (1-\psi_j)\delta\frac{dQ(\psi_j)}{d\psi_j}$$

Then, using the definition of $Q(\psi_j)$, we have that $\mathcal{W}_i^j\left(\psi_j\right) = (V_i^2 - V_i^b) + V_i^1(1-\delta) + \delta(V_i^1 - V_i^b)(2\psi_j - 1)$. So a sufficient condition for $\mathcal{W}_i\left(\psi_j\right)$ to be strictly increasing in the unit interval is that $\mathcal{W}_i^j\left(0\right) = (V_i^2 - V_i^b) + V_i^1(1-\delta) - \delta(V_i^1 - V_i^b) = \beta\,(1-\delta)\,(1-2\delta) + \delta p_{si} > 0$; where the last equality comes from using our derivations in the main text for V_i^1, V_i^2 and V_i^b. Therefore the condition $p_{si} \geq \overline{p_U}$ is sufficient for uniqueness.[56] So, the only remaining step is to prove that $\overline{p}_U \leq \kappa$. The condition $\overline{p}_U \leq \kappa$ is equivalent to $\mathcal{Z}(\delta) := 2\beta\delta^2 + \alpha\delta + \beta > 0$; where $\alpha := (\kappa - 3\beta)$. Then, recall that $\overline{\delta}_U$ is equal to

$$\overline{\delta}_U := \frac{-\alpha + \sqrt{\alpha^2 - 8\beta^2}}{4\beta}$$

and notice that $\overline{\delta}_U$ is the largest solution to the quadratic equation $\mathcal{Z}(\delta) = 0$. It therefore follows that: (i) $\overline{\delta}_U < 1$, given that when $\delta = 1$, $\mathcal{Z}(\delta) = \kappa > 0$; and (ii) $\forall\delta > \overline{\delta}_U : \mathcal{Z}(\delta) > 0$, given that $\mathcal{Z}(\delta)$ is a strictly convex function of δ. So $\delta > \overline{\delta}_U$ guarantees that $\overline{p}_U \leq \kappa$.[57]

Thus the conditions specified in this step ensure that there is a unique solution to the equation $\mathcal{W}_i\left(\psi_j\right) = 0$. By symmetry, the same arguments apply to imitator j. Finally, we conclude by obtaining an explicit solution for ψ_h^* for $h \in \{i, j\}$. Observe that $\mathcal{W}_i(\psi_j^*) = 0$ can be rearranged as follows

$$\mathcal{W}_i(\psi_j^*) := a\psi_j^2 + b\psi_j + c = 0 \tag{A2}$$

where $a := \delta\left(V_i^1 - V_i^b\right) = \beta(1-\delta) > 0$; $b := \left(V_i^2 - V_i^b\right) + \delta\left(V_i^b - V_i^1\right) + (1-\delta)V_i^1 > 0$ and finally $c := -(1-\delta)V_i^1 < 0$.[58] Recalling that a, b and c are functions of Δ, the unique solution to equation (A2) can be written as

$$\psi_j^*(\Delta) = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{A3}$$

By symmetry we can derive an analogous result for imitator i.

[56] Because if $\overline{p}_U \leq \overline{p}_E$ existence would imply uniqueness and step 1 would be enough.

[57] Note that we consider the case in which $\alpha \leq 0$. The situation in which $\alpha > 0$ is easy to handle. Because $\beta > 0$, $\mathcal{Z}(\delta)$ is an strictly increasing function of δ and $\mathcal{Z}(0) = \beta > 0$. Thus, when $\alpha > 0$, our claim follows directly.

[58] We drop the symbol $*$ just for convenience.

Step 3. When Δ goes to zero the limiting distribution of entry times for each imitator is exponentially distributed with hazard rate $\lambda_i = r(\Pi_3 - p_{sj})/p_{sj}$ and $\lambda_j = r(\Pi_3 - p_{si})/p_{si}$.

We prove this result for imitator j. Note that as Δ goes to zero, δ converges to 1 and therefore the conditions on the parameter δ imposed in steps 1 and 2 guaranteeing existence and uniqueness are indeed satisfied.

As $\Delta \to 0$ our interest resides in obtaining the hazard rate of the distribution of entry times for the imitators. Therefore, at period t, conditional on no imitator having entered into the market yet, we are interested in examining whether the following limit

$$\lim_{\Delta \downarrow 0} \frac{\Pr\{t < t_j \leq t + \Delta | t_j \geq t\}}{\Delta} = \lim_{\Delta \downarrow 0} \frac{\psi_j^*(\Delta)}{\Delta}$$

exists; where t_j is the entry time for imitator j. We can rewrite ψ_j^* as

$$\psi_j^* = \frac{\left(-b + \sqrt{b^2 - 4ac}\right)\left(b + \sqrt{b^2 - 4ac}\right)}{2a} \times \frac{1}{b + \sqrt{b^2 - 4ac}} = \frac{u(\Delta)}{v(\Delta)}$$

where $u(\Delta) := -2c$ and $v(\Delta) := b + \sqrt{b^2 - 4ac}$. As $\Delta \to 0$ we have that $\delta \to 1$, $a \to 0$, $b \to p_{si}$ and $c \to 0$. Using these results, we obtain that $\lim_{\Delta \downarrow 0} u = 0$ and $\lim_{\Delta \downarrow 0} v = 2p_{si}$. Overall $\lim_{\Delta \downarrow 0} \psi_j^* = 0$ and $\lim_{\Delta \downarrow 0} \frac{\psi_j^*(\Delta)}{\Delta} = \lim_{\Delta \downarrow 0} \frac{\partial \psi_j^*}{\partial \Delta} = \lim_{\Delta \downarrow 0} \left(\frac{u'v - uv'}{v^2}\right)$. Because $u'(0) = 2r(\Pi_3 - p_{si})$ and $v'(0) = -2r(\beta + p_{si})$ we have that

$$\lambda_j := \lim_{\Delta \downarrow 0} \frac{\psi_j^*(\Delta)}{\Delta} = \frac{u'(0)}{v(0)} = \frac{r(\Pi_3 - p_{si})}{p_{si}}$$

Recall that a distribution has a constant hazard rate if and only if it is an exponential distribution (See, for example, Proposition B.2, page 297, Marshall and Olkin). Thus, the cumulative distribution function of entry times for imitator j is $G_j(t) = 1 - e^{-\lambda_j t}$, $t \geq 0$. By a symmetric argument it is straightforward to determine the limiting distribution of entry times for imitator i.

Step 4**:** The inventor's discounted expected payoff is: $V_s(p_{si}, p_{sj}) = \frac{\pi_1}{r+\lambda_i+\lambda_j} + \frac{(\lambda_i+\lambda_j)\Pi_3}{r+\lambda_i+\lambda_j} + \frac{\lambda_i p_{si}+\lambda_j p_{sj}}{r+\lambda_i+\lambda_j}$.

The expected payoff of the innovator depends on the time of the first entry, t_1, a random variable that takes values in $[0, \infty)$. By definition $t_1 := \min\{t_i, t_j\}$. Also it is a well-known fact that because t_i and t_j are independent random variables with hazard rates equal to λ_i and λ_j, t_1 has a hazard rate equal to $\lambda_i + \lambda_j$. Now suppose $t_1 = t \in [0, \infty)$. Because the second imitator enters instantaneously at time t, the inventor obtains: (i) A flow of monopoly profits π_1 up to time t; (ii) A flow of triopoly profits π_3 from time t on; and (iii) At time t, she receives either p_{si} or p_{sj} depending on the identity of the first imitator. So, her expected payoff is

$$V_s(p_i, p_j; t) = \frac{\pi_1}{r}\left[1 - e^{-rt}\right] + e^{-rt}\Pi_3 + e^{-rt}\left[\frac{\lambda_i}{\lambda_i + \lambda_j} p_{si} + \frac{\lambda_j}{\lambda_i + \lambda_j} p_{sj}\right]$$

where the last term follows from the definition of hazard rates. Because t_i and t_j are exponentially distributed random variables, t_1 is also exponentially distributed with parameter $\lambda_i+\lambda_j$.[59] Hence, the inventor's discounted expected payoff is

$$V_s(p_{si},p_{sj}) = \frac{\pi_1}{r+\lambda_i+\lambda_j} + \frac{\lambda_i+\lambda_j}{r+\lambda_i+\lambda_j}\Pi_3 + \frac{\lambda_i p_{si}+\lambda_j p_{sj}}{r+\lambda_i+\lambda_j}$$

STEP 5. The payoff maximizing licensing prices are $p_{si}^* = p_{sj}^* = \kappa$.

The goal of the inventor is to choose a pair of prices $\{p_{si}, p_{sj}\}$ in order to maximize $V_s(p_{si}, p_{sj})$. The derivative of $V_s(p_{si}, p_{sj})$ with respect to p_{si} is

$$\frac{\partial V_s}{\partial p_{si}} := V_s^i = \frac{\lambda_i}{D} - \frac{1}{D^2}\frac{\partial \lambda_j}{\partial p_{si}}\left[\pi_1 - \pi_3 - p_{sj}r - p_{sj}\lambda_i + \lambda_i p_{si}\right]$$

where $D := (r+\lambda_i+\lambda_j)$. Using, from step 3, the result for λ_i we have that V_s^i reduces to: $V_s^i = \frac{\lambda_i}{D} - \frac{1}{D^2}\frac{\partial\lambda_j}{\partial p_{si}}\left[\pi_1 - 2\pi_3 + \lambda_i p_{si}\right]$. And because, we know from step 3, that $\partial\lambda_j/\partial p_{si} < 0$, it follows that, for all values of p_{sj}, $V_s(p_{si}, p_{sj})$ is strictly increasing in p_{si} since $\pi_1 > 2\pi_3$ for all market games that we consider. By symmetry, $V_s(p_{si}, p_{sj})$ is strictly increasing in p_{sj} for all values of p_{si}. Thus the optimal pair of prices are $p_{si}^* = p_{sj}^* = \kappa$.

To conclude: Result (i) is a direct consequence of step 5; result (ii) is step 3 for the optimal license prices $p_{si}^* = p_{sj}^* = \kappa$; and finally result (iii) is step 4 for $p_{si}^* = p_{sj}^* = \kappa$. ■

Proof of Lemma 2. Because we concentrate in SPE, the strategy for the inventor dictates, at the beginning of each period $\tau \geq t+1$, the price at which she offers a license to imitator $j : p_{sj}^{\tau} \in \mathbb{R}_+ \cup \{+\infty\}$ for every feasible history. On the other hand, the strategy for imitator j orders for each period $\tau \geq t+1$ and every feasible history whether "to enter" or not, and conditional on entrance, his mode of entry.

Note that any SPE in pure strategies must satisfy the following two properties:

PROPERTY 1 (Copy never occurs): Imitator j enters the market by purchasing a license at a price of κ.

As in lemma 0, the innovator always prefers to offer a license at a price $p_{si} = \kappa$ rather than offer no license: she can at least obtain licensing revenues. Furthermore, if a license is sold, it will always be sold at a price of κ. If not there would be a profitable deviation for the innovator.

PROPERTY 2 (No Delay): Imitator j immediately buys a license.

By property 1 we know that imitator j will never copy in equilibrium. Thus, when a license is offered at period τ he can either accept the current offer or reject it and then accept a future license offer. Rejecting a current offer would be a best response if imitator j obtained a higher payoff by accepting a future offer. However this is clearly impossible according to property 1.

Formally, the following pair of strategies form the unique SPE

$$\begin{aligned} p_{sj} &= \kappa \text{ always} \\ \sigma_j &= \begin{cases} \text{"enter" by } \ell \text{ if } p_{sj} \leq \kappa \\ \text{"enter" by } c \text{ if if } p_{sj} > \kappa \end{cases} \end{aligned}$$

[59] See, for instance, Proposition C.1, page 302, Marshall and Olkin

in which imitator j enters immediately at $t+1$ by buying a licensing at a price equal to κ. ■

Proof of Proposition 3. We consider all feasible subgames that start at the beginning of period t at which neither imitator has entered the market yet. The inventor announces, at the beginning of each period $\tau \geq t$, for each feasible history a pair of license prices $\left\{p_{si}^{\tau}, p_{sj}^{\tau}\right\}$. The imitators, at each period $\tau \geq t$, and for each feasible history, simultaneously decide whether "to enter" or not and, conditional on entrance, how to enter. The proof is similar to that of lemma 2. We proceed in two steps.

Step 1. We observe that any SPE must satisfy the following three properties:

Property 1 (Copy never occurs): The imitators enter the market by purchasing a license.

The argument follows similar lines to those of the previous lemma.

Property 2 (Simultaneous Entry): The imitators enter the market at the same period.

To see this, suppose it were not. Then one of the imitators, say imitator i, must enter the market at period $\tau \geq t$ and imitator j at period $\widehat{\tau} > \tau$. By lemma 2, we know that in equilibrium $\widehat{\tau} = \tau + 1$ and the imitator j's equilibrium payoff in period τ units is $\delta\left(\Pi_3 - \kappa\right)$. However, if imitator j deviates and buys a license at period τ his worst payoff would be $\Pi_3 - \kappa$ which is strictly higher than $\delta\left(\Pi_3 - \kappa\right)$. Thus, the conclusion follows.

Property 3 (No Delay): Whenever the innovator offers a license, her equilibrium offer will be simultaneously accepted by the imitators.

We know that copying will never take place and that entry will occur simultaneously. If the imitators reject the current offer and enter the market in a future period they will be strictly worse off because at any period at which the licenses are traded their price will be equal to κ.

Properties 1, 2 and 3 together imply that, for any history at which the imitators have not entered the market yet, the licenses will be traded immediately. Then, the unique best response for the innovator is to offer at every period and for every feasible history the same pair of equilibrium license prices, $\left\{p_{si}^{\tau}, p_{sj}^{\tau}\right\} = \{\kappa, \kappa\}$ for all $\tau \geq t+1$. Moreover, the following pair of strategies conform the unique SPE

$$
\begin{aligned}
\left\{p_{si}^{\tau}, p_{sj}^{\tau}\right\} &= \{\kappa, \kappa\} \text{ always} \\
\sigma_h &= \begin{cases} \text{"enter" by } \ell \text{ if } p_{sh} \leq \kappa \\ \text{"enter" by } c \text{ if if } p_{sh} > \kappa \end{cases}
\end{aligned}
$$

So, in the unique SPE both imitators will enter at period $t = 0$ and the inventor's discounted equilibrium payoff is $\Pi_3 + 2\kappa$.

Step 2. In the case of exclusive contracts the innovator's discounted equilibrium payoff is $V_s^e = \Pi_3 + 2\kappa$; and in the case of non-exclusive contracts, according to proposition 2 is $V_s = \frac{\pi_1}{r+2\lambda} + \frac{2\lambda}{r+2\lambda}\left(\Pi_3 + \kappa\right)$. To compare them we define $\upsilon := \left[\frac{\pi_1}{r+2\lambda} + \frac{2\lambda}{r+2\lambda}\left(\Pi_3 + \kappa\right)\right] - \left[\Pi_3 + 2\kappa\right]$. Our goal is to show $\upsilon \geq 0$. We have that $\upsilon \geq 0$ if and only if $\pi_1 - \pi_3 \geq 2\kappa(r+\lambda)$. Given that $\lambda = r\left(\Pi_3 - \kappa\right)/\kappa$, it follows that $\upsilon \geq 0 \iff \pi_1 - 3\pi_3 \geq 0$. This last inequality is always strictly satisfied for all market games considered in this paper. Hence, we conclude that it is strictly preferable for the innovator to offer non-exclusive contracts. ■

Proof of Proposition 4. We show the validity of this proposition sequentially.

STEP 1. After the first imitator, say imitator i, enters the market at time t imitator j must enter the market at some finite time $\tau \geq t$.

To see this, suppose it were not. This is impossible because by copying at time t imitator j guarantees himself a payoff in time t units equal to $V_j = \Pi_3 - \kappa > 0$.

STEP 2. In any SPE entry must occur through licensing. Moreover, the price of the license at any $\tau \geq t$ at which entry happens must be equal to zero.

This follows the logic of previous proofs.

STEP 3. For any $d_2 \in \left[0, \widetilde{d}_2\right]$ there exists a SPE whose outcome is for imitator j to enter the market at time $t_2 = t + d_2$ by buying a license at a zero price iff $2\Pi_3 \leq \Pi_2$.

To prove this statement, note that the following strategies support the SPE outcome described above.[60] In particular, for the sellers (the inventor and imitator i)

$$\sigma_s = \sigma_i = \begin{cases} p_j = \kappa \text{ for all } \tau_- \text{ such that } t \leq \tau_- < t_2 \\ p_j = 0 \text{ for all } \tau_+ \text{ such that } \tau_+ \geq t_2 \end{cases}$$

And for the buyer (imitator j)

$$\sigma_j = \begin{cases} \text{“enter” by } \ell \text{ buying from } \widehat{y} \text{ if } \widehat{p} \leq \overline{p}_j = \Pi_3(1 - e^{-r(t_2 - t_j)}), \text{ for } \tau_- \text{ such that } t \leq \tau_- < t_2 \\ \text{“wait”} \quad \text{if } \widehat{p} > \overline{p}_j = \Pi_3(1 - e^{-r(\tau - t_j)}), \text{ for } \tau_- \text{ such that } t \leq \tau_- < t_2 \\ \text{“enter” by } \ell \text{ buying from } \widehat{y} \quad \text{if } \widehat{p} \leq \min\{\kappa, (1-\delta)\Pi_3\}, \text{ for all } \tau_+ \geq t_2 \\ \text{“enter” by } c \quad \text{if } \kappa \leq \min\{\widehat{p}, (1-\delta)\Pi_3\}, \text{ for all } \tau_+ \geq t_2 \\ \text{“wait” or “do not enter”} \quad \text{if } \min\{\widehat{p}, \kappa\} > (1-\delta)\Pi_3 \text{ for all } \tau_+ \geq t_2 \end{cases}$$

where $t_j \in [\tau_-, t_2)$ is the time at which one of the sellers offers a license to imitator j, $\widehat{y}$ denotes the identity of the seller who offers the minimum license price and $\widehat{p} := \min_{y \in \{i,s\}} p_{yj}$

Now we show that for any $d_2 \in \left[0, \widetilde{d}_2\right]$ these strategies form a SPE. First, note that by definition of $\widetilde{d}_2$ imitator j does not have any incentive to deviate and choose “to enter” by copying at any time τ_- such that $t \leq \tau_- < t_2$. Second, suppose that a license is offered at a price $\widehat{p}$ at time $t_j \in [\tau_-, t_2)$. If he accepted the offer, he would obtain a payoff in time t_j units of $\Pi_3 - \widehat{p}$. If he instead rejected it, he would receive a payoff in time t_j units of $e^{-r(t_2 - t_j)}\Pi_3$. Thus, his best response is to accept the offer and enter the market iff $\widehat{p} \in \left[0, \overline{p}_j\right]$ where $\overline{p}_j = \Pi_3(1 - e^{-r(t_2 - t_j)})$.

Consider the strategy of the sellers and a time $t_2 = t + d_2$. For $d_2 = 0$, the proof is done in Lemma 0. Hence, let d_2 be such that $0 < d_2 \leq \widetilde{d}_2$. Then a seller, say imitator i, might deviate from the proposed strategy and offer a license at time t_j for $t_j \in [\tau_-, \tau)$. From the buyer's strategy, the maximum price he can charge is $\overline{p}_j$. Imitator i will not find profitable to deviate at time t_j if $\Pi_3 + \overline{p}_j \leq \Pi_2 \left[1 - e^{-r(t_2 - t_j)}\right] + e^{-r(t_2 - t_j)}\Pi_3$. That is if $2\Pi_3 \leq \Pi_2$. We have therefore shown the first part of the proposition.

To show the uniqueness result of the second part of the proposition, suppose that $2\Pi_3 > \Pi_2$ and that a license is sold at time $t_2 > t$. According to step 2, the equilibrium license price must be zero. Besides we know that for any $t_j \in [\tau_-, t_2)$ imitator j will accept to pay at most $\overline{p}_j$ for

[60] The strategies should be interpreted as prescribing actions at nodes in the limiting case when $\Delta \to 0$

a license. But because $2\Pi_3 > \Pi_2$ offering a license at a price $\overline{p}_j$ is a profitable deviation for the sellers. Because our argument is valid for any value of $t_2 > t$ it follows that the unique SPE, when $2\Pi_3 > \Pi_2$, is the no-delay equilibrium.

Finally, note that the strategies of the sellers and of imitator j for all time periods $\tau_+ \geq t_2$ are identical to the strategy used in the proof of Lemma 0. Hence, the proof is concluded. ■

PROOF OF PROPOSITION 5. We denote by $G(t) : [0, \infty) \to [0, 1]$ the distribution function of entry times for the imitators. We assume momentarily that $G(t)$ has a density denoted by $g(t)$ and, to simplify, we also suppose that the innovator does not price discriminate between the imitators. The proof is in a number of steps.

STEP 1. Imitators may delay their entry into the market only if $d_2 \in \left[0, \widehat{d}_2\right)$ for $\widehat{d}_2 = (1/r)\ln(\Pi_2/\Pi_2 - \kappa) < \widetilde{d}_2$.

To see this, note that $V_2 = V_1$ when $p_s = (1-e^{-rd_2})\Pi_2$. Substituting $\widehat{d}_2$ in this last expression yields $p_s = (1 - e^{-r\widehat{d}_2})\Pi_2 = \kappa$. So, for all $d_2 \in \left[0, \widehat{d}_2\right)$ the inventor might choose a license price, p_s, that satisfies $(1 - e^{-rd_2})\Pi_2 < p_s \leq \kappa$ and hence $V_2 > V_1$. Thus depending on p_s delay to buy the first license may occur.

STEP 2. We obtain the distribution function of entry times assuming that $V_2 > V_1$.

To accomplish this, recall that because we focus on a behavior SPE equilibrium, at time t, if entry has not happened yet, imitators must be indifferent between: (i) Choosing "to enter" at time t; and (ii) Waiting dt extra units of time before entering. This indifference condition requires that the opportunity cost of waiting dt units of extra time be exactly equal to the expected marginal benefit of waiting dt units of extra time. The opportunity cost is the flow of profits that an imitator would obtain if he were the leader imitator at t. That is $MC = rV_1dt$. The marginal benefit is the increase in the payoff that an imitator receives by being the follower rather than the leader imitator. That is $MB(dt) = V_2 - V_1$. However, the marginal benefit is only received if the rival imitator enters first: an event that is determined by the hazard rate $\lambda(t) \equiv (g(t)/1 - G(t))$. So, in a SPE it must be that $\lambda(t)(V_2 - V_1) = rV_1dt$. Using equation (9) of the main text we obtain that for a $d_2 \in \left[0, \widehat{d}_2\right)$ equilibrium entry times are characterized by a constant hazard rate (and thus by an exponential distribution) given by

$$\lambda(p_s) = \frac{r\left[(1 - e^{-rd_2})\Pi_2 + e^{-rd_2}\Pi_3 - p_s\right]}{[p_s - (1 - e^{-rd_2})\Pi_2]}$$

STEP 3. We calculate the equilibrium expected payoff for the inventor and show that $p_s^* = \kappa$.

Like in the proof of proposition 2, assume that the time of the first entry occurred at $t \in [0, \infty)$. Since now the follower imitator enters the market with a delay equal to d_2 (i.e., he enters at time t_2), the inventor obtains: (i) A flow of monopoly profits up to time t; (ii) At time t, she receives the price for the license p_s; (iii) A flow of duopoly profits from time t up to time t_2; and finally: (iv) A flow of triopoly profits from time t_2 on. Hence, her payoff is $V_s(t, p_s, d_2) = \frac{\pi_1}{r}\left[1 - e^{-rt}\right] + e^{-rt}\left[p + \left(1 - e^{-rd_2}\right)\Pi_2 + e^{-rd_2}\Pi_3\right]$. But because the time of the first entry $t_1 := \min\{t_i, t_j\}$ has an exponential distribution with parameter 2λ it follows immediately that her expected payoff is

$$V_s(p, d_2) = \frac{\pi_1}{(r + 2\lambda)} + \frac{2\lambda}{(r + 2\lambda)} \left[p_s + \left(1 - e^{-rd_2}\right) \Pi_2 + e^{-rd_2} \Pi_3\right]$$

To conclude this step it suffices to show that the optimal price chosen by the inventor is equal to κ. If that were the case, note that for all $d_\tau \in [0, d_\tau^u) : V_2 > V_1$ and the assumption made in step 2 would be satisfied. To accomplish this, observe that the innovator chooses a price, p, to maximize $V_s(p, d_\tau)$ subject to the hazard rate found in step 2. Thus

$$\frac{dV_s(p, d_2)}{dp_s} = -\frac{2r}{[r + 2\lambda(p_s)]^2} \frac{d\lambda}{dp_s} (\Pi_1 - \mathcal{M}) + \frac{2\lambda}{(r + 2\lambda)}$$

where $\mathcal{M} := \left[p + \Pi_2 - e^{-rd_2} (\Pi_2 - \Pi_3)\right]$. Since: (i) $d\lambda/dp_s < 0$; (ii) $\arg\min_{p_s \in (0,\kappa]} (\Pi_1 - \mathcal{M}) = \kappa$; and (iii) The least upper bound of κ is Π_3, it is evident that $\inf_\kappa (\Pi_1 - \mathcal{M}) = \Pi_1 - \Pi_3 + \Pi_2 + e^{-rd_2} (\Pi_2 - \Pi_3) > 0$. So, because $dV_s(\kappa; d_2)/dp_s > 0$ the inventor must indeed choose $p_s^* = \kappa$ and her equilibrium payoff is obtained by replacing p_s^* into $V_s(p; d_2)$. ■

Proof of Proposition 6. It is well-known (See, for example, Fudenberg and Tirole [1991]) that, for preemption games, the limiting distribution of discrete-time games (i.e., when Δ goes to zero) may not usually be expressed as an equilibrium in continuous time strategies of the kind we have used in proposition 5. Because of this limitation, we use our discrete time model and then we compute the limiting distribution of the equilibrium outcomes.

Step 1. We obtain for our discrete time model the unique symmetric SPE in behavior strategies.[61]

The equilibrium condition is $\mathcal{W}(\psi) = \psi [V_2 - V_b] + (1 - \psi) [\delta Q(\psi) - V_1] = 0$, for $Q(\psi) = \psi V_b + (1 - \psi) V_1$. This quadratic equation has two roots

$$\psi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a := \delta (V_1 - V_b) > 0$; $b := (V_2 - \delta V_1) + (1 - \delta)) (V_1 - V_b)$; and $c := -(1 - \delta) V_1 < 0$. Because δ is a function of Δ, it follows that as Δ goes to zero, ψ converges to $-(V_2 - V_1) \pm (V_2 - V_1) / [2 (V_1 - V_b)]$. Naturally, we consider the solution corresponding to the positive root $\psi_\tau = (V_1 - V_2) / (V_1 - V_b)$. By definition of V_1, V_2 according to equation (9) and given that $V_b = \Pi_3 - p$ we have that

$$\psi = \frac{\left(1 - e^{-rd_2}\right) \Pi_2 - p_s}{\left(1 - e^{-rd_2}\right) (\Pi_2 - \Pi_3)}$$

Observe that $\psi < 1$ iff $V_2 > V_b$. But because we consider all those values for $d_\tau \in \left[\widehat{d}_2, \widetilde{d}_2\right]$, it follows that $V_2 > V_b$ iff p_s satisfies $\kappa (\Pi_3/\Pi_2) < p_s \leq \kappa$. Thus, at each period, each imitator plans to enter the market independently with probability equal to ψ conditional on no having entered before.

Step 2. Assuming that $V_2 > V_b$, we establish the limiting distribution of entry times when

[61] Recall that the inventor sets the same price for both licenses.

Δ goes to zero.

Fix any real time $t > 0$ and observe that the probability that no imitator will have entered by time t is approximately equal to $(1 - \psi_{\tau})^{n(t,\Delta)}$ where $n(t, \Delta) := (2t) / \Delta$ is the number of decision nodes between time 0 and time t when the real time length of a period is Δ. As Δ goes to zero, $n(t, \Delta)$ increases without bound and hence this probability also converges to zero. The conclusion is, therefore, that at least an imitator will enter the market for sure at time $t = 0$.

Next, we obtain the probability of simultaneous entry at time $t = 0$. For that, consider that we must compute the probability of simultaneous entry conditional on the information that the event in which no imitator enters the market has zero probability. Because both imitators entering simultaneously has probability ψ^2, it is direct to conclude that the probability of simultaneous entry at time $t = 0$ is $\phi(p_s, d_2) = \psi / (2 - \psi)$ and therefore the inventor's expected payoff is

$$V_s(p, d_2) = \phi(p_s, d_2)[2p + \Pi_3] + [1 - \phi(p_s, d_2)]\left[p + \left(1 - e^{-rd_2}\right)\Pi_2 + e^{-rd_2}\Pi_3\right]$$

Step 3. The optimal price is $p_s^* = \kappa$.

The proof follows similar lines to the preceding ones. Observe that $\frac{\partial V_s(p,d_2)}{\partial p_s} = (\partial\phi/\partial p_s)\omega(p_s) + (1 + \phi)$, where $\omega(p_s, d_2) := \left[p_s - \left(1 - e^{-rd_2}\right)(\Pi_2 - \Pi_3)\right]$. Then it is straightforward to show that

$$\frac{\partial\phi}{\partial p_s} = \frac{2}{(2 - \psi)^2}\frac{\partial\psi(p_s, d_2)}{\partial p_s} = \frac{2}{(2 - \psi)^2}\left(-\frac{1}{(\Pi_2 - \Pi_3)[1 - e^{-rd_2}]}\right)$$

and that $(1 + \phi) = 2/(2 - \psi)$. Straightforward mathematical manipulations show that

$$\frac{\partial V_s(p_s, d_2)}{\partial p_s} > 0 \iff -\frac{\omega(p_s, d_2)}{(\Pi_2 - \Pi_3)[1 - e^{-rd_2}]} + (2 - \psi) > 0$$

And after replacing ψ and $\upsilon(p_s, d_\tau)$ by their values, we finally obtain

$$\frac{\partial V_s(p_s, d_2)}{\partial p_s} > 0 \iff 2\left(1 - e^{-rd_2}\right)(\Pi_2 - \Pi_3) - \left(1 - e^{-rd_2}\right)\Pi_3 > 0$$

It is evident that because $2\pi_2 > 3\pi_3 : \partial V_s(p_s, d_2)/\partial p_s > 0$ for all $\kappa(\Pi_3/\Pi_2) < p_s \leq \kappa$ and the optimal price is $p_s^* = \kappa$ validating our assumption that $V_2 > V_b$. The equilibrium payoff for the inventor is directly computed by replacing p_s^* in $V_s(p_s, d_2)$. ■

Proof of Proposition 7. We assume that there are n potential imitators, that time is continuous and, to simplify the exposition, we also suppose that the innovator does not price discriminate between the imitators. The proof follows a number of steps.

Step 1. After the first entry, there is a perfect equilibrium in which $n - 1$ licenses are sold at a zero price to the remaining imitators.

The argument is exactly the same as that of Lemma 0 adapted to the case of n potential imitators. This implies that the payoff, in period t units, for the leader and the follower imitator are respectively $V_1 = \Pi_{n+1} - p_{sn}$ and $V_2 = \Pi_{n+1}$.

Step 2. A necessary condition for a perfect equilibrium in behavior strategy to exist is

$$\mathcal{W}(\psi) := W - V_1 = 0$$

where $W := e^{-r\Delta}\left\{(1-\psi_n)^{n-1}V_1 + \left[1-(1-\psi_n)^{n-1}\right]V_2\right\}$.

To establish this, we first note that each imitator must be indifferent between: (i) Entering by buying a license at real time t; and: (ii) Entering by buying a license at real time $t+\Delta$. Conditional on no imitator having entered up to time t, if one of them enters immediately at time t he will obtain a payoff in time t units equal to

$$V_1 = \Pi_{n+1} - p_{sn}$$

However, if he decides to enter at time $t+\Delta$ he will obtain: (i) A payoff equal to V_2 if at least one of the other $n-1$ imitators have entered in the interval of time Δ; and: (ii) A payoff equal to V_1 if none of the other $n-1$ imitators have entered in the interval of time Δ. So by entering at time $t+\Delta$ he will obtain a payoff in time t units equal to

$$W = e^{-r\Delta}\left\{[1-\psi_n]^{n-1}V_1 + \left[1-[1-\psi_n]^{n-1}\right]V_2\right\}$$

where ψ_n is the probability for each imitator of entering in the interval of time Δ, conditional on not having entered before real time t. That is

$$\psi_n := \Pr\{t < t_h \leq t+\Delta | t_h > t\}$$

In a perfect equilibrium it must be that $W(\psi_n) := W - V_1 = 0$. Or equivalently that

$$\mathcal{W}(\psi_n) = \left\{[1-\psi_n]^{n-1} - e^{r\Delta}V_1 + \left[1-[1-\psi_n]^{n-1}\right]\right\}V_2 = 0 \tag{A4}$$

Step 3. Note that by the Binomial Theorem, $[1-\psi_n]^{n-1}$ can be written as

$$[1-\psi_n]^{n-1} = 1-[n-1]\psi_n + \sum_{\upsilon=2}^{\upsilon=n-1}\binom{n-1}{\upsilon}[-\psi_n]^{\upsilon}$$

Step 4. The equilibrium distribution of entry times is exponential with hazard rate equal to

$$\lambda_n = \frac{r[\Pi_{n+1}-p_{sn}]}{[n-1]p_{sn}}$$

To establish this, we first use the result of step 3 in equation (A4) to obtain

$$\mathcal{W}(\psi_n) = -(V_2-V_1)\sum_{\upsilon=2}^{\upsilon=n-1}\binom{n-1}{\upsilon}[-\psi_n]^{\upsilon} + [n-1][V_2-V_1]\psi_n + \left[1-e^{r\Delta}\right]V_1 = 0$$

Hence, as $\Delta \to 0$, $W(\psi_n)$ goes to

$$-(V_2 - V_1) \sum_{\upsilon=2}^{\upsilon=n-1} \binom{n-1}{\upsilon} [-\psi_n]^{\upsilon} + [n-1][V_2 - V_1]\psi_n = 0$$

and thus the unique solution for $\psi_n \in [0,1]$ is $\psi_n = 0$. By using L'Hopital's rule and the implicit function theorem, it can be shown that the following limit

$$\lambda_n := \lim_{\Delta \downarrow 0} \frac{\psi_n(\Delta)}{\Delta}$$

is finite and well defined. For this reason, we can divided both sides of (A4) by Δ and take the limit of it when Δ goes to zero

$$\lim_{\Delta \downarrow 0} \left\{ -(V_2 - V_1) \sum_{\upsilon=2}^{\upsilon=n-1} \binom{n-1}{\upsilon} \left[-\frac{\psi_n}{\Delta}\right]^{\upsilon} \Delta^{\upsilon-1} + [n-1][V_2 - V_1]\frac{\psi_n}{\Delta} + \frac{(1-e^{r\Delta})}{\Delta} V_1 \right\} = 0$$

It is evident that the limit of the first terms goes to zero as Δ goes to zero and $\lim_{\Delta \downarrow 0} \frac{(1-e^{r\Delta})}{\Delta} = -r$. So

$$\lambda_n = \frac{rV_1}{[n-1][V_2 - V_1]} = \frac{r[\Pi_{n+1} - p_{sn}]}{[n-1]p_{sn}}$$

STEP 5. We show that $p^*_{sn} = \kappa_n$.

We follow the same steps of previous propositions (i.e. Propositions 2 and 5). Assume that the time of the first entry occurred at real time $t \in [0, \infty)$. Since the follower imitators would enter the market with a zero delay, the inventor would obtain: (i) A flow of monopoly profits up to time t; (ii) At time t, she would receive the price for the license p_{sn}; and (iii) A flow of profits equal to π_{n+1} from time t on. Hence, her payoff would be $V^n_s(t, p_{sn}) = \frac{\pi_1}{r}\left[1 - e^{-rt}\right] + e^{-rt}[p_{sn} + \Pi_{n+1}]$. But because now the time of the first entry $t_1 := \min\{t_i, t_j, ...t_n\}$ has an exponential distribution with parameter $n\lambda_n$ it follows that her expected payoff is

$$V^n_s(p_{sn}) = \frac{\pi_1}{[r + n\lambda_n]} + \frac{n\lambda_n}{[r + n\lambda_n]}[p_{sn} + \Pi_{n+1}]$$

To conclude this step it suffices to show that the optimal price chosen by the inventor is equal to κ_n. To accomplish this, observe that the innovator chooses a price, p_{sn}, to maximize $V_s(p_{sn})$ subject to the hazard rate found in the previous step. So

$$\frac{dV^n_s(p_{sn})}{dp_{sn}} = \frac{\partial \lambda_n}{\partial p_{sn}} \frac{rn}{(r + n\lambda_n)^2} \{-\Pi_1 + p_{sn} + \Pi_{n+1}\} + \frac{n\lambda_n}{(r + n\lambda_n)}$$

and since we have assumed that for all $n \in \mathcal{P} : (\Pi_{n+1} - \kappa_n) > 0$ and $p_{sn} \leq \kappa_n$, it follows that $\sup_{p_{sn}}[p_{sn} + \Pi_{n+1} - \Pi_1] < 2\Pi_{n+1} - \Pi_1 < 0$ for all $n \in \mathcal{P}$. Finally because $\partial\lambda_n/\partial p_{sn} < 0$, it ensues that $dV_s(p_{sn})/dp_{sn} > 0$ for all $p_{sn} \in (0, \kappa_n]$ and so $p^*_{sn} = \kappa_n$ as stated.

STEP 6. When $n \to \infty$, $V^n_s(\kappa_n) \to \Pi_1$.

To this end note that the innovator's equilibrium payoff is

$$V_s^n(\kappa_n) = \frac{\pi_1}{[r + n\lambda_n]} + \frac{n\lambda_n}{[r + n\lambda_n]}[\kappa_n + \Pi_{n+1}]\text{; for } \lambda_n = \frac{nr[\Pi_{n+1} - \kappa_n]}{[n-1]\kappa_n}$$

So

$$\lim_{n\to\infty} V_s^n(\kappa_n) = \pi_1 \lim_{n\to\infty} \frac{1}{[r + n\lambda_n]} + \lim_{n\to\infty} \frac{n\lambda_n}{[r + n\lambda_n]} \lim_{n\to\infty}[\kappa_n + \Pi_{n+1}]$$

And

$$\lim_{n\to\infty}(n\lambda_n) = r \lim_{n\to\infty}\left[\frac{n}{n-1}\right] \lim_{n\to\infty}\left[\frac{\Pi_{n+1}}{\kappa_n} - 1\right] = 0$$

Because: (i) $\lim_{n\to\infty}\left[\frac{n}{n-1}\right]$ is bounded; and (ii) By assumption 3: $\lim_{n\to\infty}\left[\frac{\Pi_{n+1}}{\kappa_n} - 1\right] = 0$. Thus, because $\lim_{n\to\infty}[\kappa_n + \Pi_{n+1}] = 0$, it follows directly that

$$\lim_{n\to\infty} V_s^n(\kappa_n) = \Pi_1$$

and the proof is completed. ■

Proof of Proposition 8. Because we focus on MPE, the payoff relevant history is determined by the number of imitators who are active in the market at each time period. Thus, the following partition $\mathbb{H} := \{\mathbb{H}_a\}$, where $\mathbb{H}_a$ is a set that collects all those histories for which $a \in \mathcal{A} = \{0, 1, 2...\}$ imitators are *active* in the market, is the coarsest sufficient partition that allows us to define MPE. A strategy for the innovator, σ_s, is $Markovian$ if it is measurable with respect to the corresponding sets $\mathbb{H}_a$. Formally, if the history $\widehat{h}^\tau \in \mathbb{H}_a(\widetilde{h}^\tau)$ for some $a \in \mathcal{A}$ then the continuation strategy $\sigma_s|_{\widehat{h}^\tau}$ must equal the continuation strategy $\sigma_s|_{\widetilde{h}^\tau}$.

Measurability implies that the inventor's strategy must prescribe the same list of prices $\{p_{sn}\}_{a=0}^{\infty} \in \mathbb{R}_+ \cup \{+\infty\}$ for each period $\tau \geq t$ at which the same number of active imitators are in the market.

Consider first the subgame following entry of at least one imitator. In particular let a be the number of active imitators.

Step1: The unique symmetric markov perfect equilibrium is such that the innovator and all a imitators offer $n - a$ licenses at a zero price to all the remaining imitators.

First note that as in the case of the no-delay licensing equilibrium this is clearly a SPE. We want to show however that it is the unique symmetric MPE.

As in previous proofs, we can show that entry necessarily occurs by licensing. If imitators entered by copying there would be a profitable deviation for the active players.

Furthermore, when the license is sold it must be sold at a price of zero. Therefore, all licenses are sold for a price of zero.

Step2: No imitator initially enters.

Given step 1, imitators know that if they are the first to enter, their profits after entry will immediately be reduced to Π_{n+1}. Furthermore, if $K > n$, $\Pi_{n+1} < \kappa$. Therefore the first entrant expects profits of $V_1 = (1 - \delta)\Pi_2 + \delta\Pi_{n+1} - \kappa$. And as $\delta \to 1$ and $n \to \infty$: $V_1 \to \Pi_{n+1} - \kappa < 0$.

Thus entry cannot be profitable and the innovator keeps monopoly rents forever. ■