Uncertain delivery in markets for lemons*

João Correia-da-Silva

CEMPRE and Faculdade de Economia. Universidade do Porto.

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Abstract. The notion of uncertain delivery is extended to study exchange economies in which agents have different abilities to distinguish between goods (for example a car in good condition versus a car in bad condition). In this setting, it is useful to distinguish goods not only by their physical characteristics, but also by the agent that is bringing them to the market. Equilibrium is shown to exist, and characterized by the fact that agents always receive the cheapest bundle among those that they cannot distinguish from truthful delivery. Several examples are presented as an illustration.

Keywords: General equilibrium, Asymmetric information, Adverse selection, Uncertain delivery, Pool, Delivery rates.

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1 Introduction

Economic agents usually trade goods without having perfect knowledge of their characteristics. This applies to firms hiring workers with unknown productivity, to consumers buying used cars with unknown quality, and to financial institutions buying assets with unknown return. Each trader enters the market with specific prior knowledge and observation abilities concerning the characteristics of the goods being traded.

This is a particular kind of asymmetric information (adverse selection), famous since the seminal contribution of Akerlof (1970). This information asymmetry differs from that considered in the models of trade with adverse selection of Prescott and Townsend (1984a, 1984b), Gale (1992, 1996), Bisin and Gottardi (1999, 2006) and Rustichini and Siconolfi (2008). In these models, agents enter the market having private information about their type (endowments and preferences in each of the possible states of nature).

In this paper, each agent's private information is described by a partition of the set of commodities, such that the agent can distinguish goods that belong to different sets of the partition. This formalization follows the works of Minelli and Polemarchakis (2001) and of Meier, Minelli and Polemarchakis (2006).

In these works, there are only markets for classes of goods that everyone can distinguish. If there is an agent in the economy that does not distinguish apples from oranges, then the other agents cannot trade apples (or oranges) among themselves. This seems very restrictive, and here an alternative framework is considered. The agents are allowed to buy any good. Nevertheless, an agent that buys high quality goods and is not able to observe the quality of the goods should probably expect to receive low quality goods instead.

Frequently, the owner of a good has superior information about its characteristics. This is the case that is focused in this paper. Agents are assumed to have perfect information about their endowments, but have differential information about the characteristics of the goods which are being offered by the other agents in the market.

Consider a market in which melons of different quality are being offered. Some agents are able to observe the quality of the melons, while others are not. Now suppose that an agent who cannot distinguish the good from the bad melons, decides, nevertheless, to buy 10 good melons. The seller may deliver 10 good melons (truthful delivery), 2 good and 8 bad melons, or even 10 bad melons. Buying 10 good melons, all that this uninformed buyer guarantees is delivery of 10 melons (good or bad).

Instrumental to the treatment of trade with adverse selection are the concepts of "pool" and "delivery rate", introduced by Dubey, Geanakoplos and Shubik (2005). We can think of a set of commodities, like melons (good and bad), as a pool. An agent that buys 10 good melons but is unable to distinguish the good from the bad melons, may receive 2 good melons and 8 bad melons (the delivery rates are 0,2 and 0,8). An agent that can distinguish the good melons surely receives 10 good melons.

An agent that buys a good may be delivered one of a set of possibilities, and takes as given the probabilities of receiving each of the possible deliveries. This is closely related to what was termed as "uncertain delivery" by Correia-da-Silva and Hervés-Beloso (2008a, 2008b and 2009), in a series of papers that study ex-ante trade of contingent goods, with agents having different abilities to verify the occurrence of the exogenous states of nature.

The workings of the economy are as follows. Goods are distinguish goods not only by their physical characteristics, but also by the agent that is bringing them to the market. There are prices and rates of delivery for each of these generalized goods, that agents take as given. The only restriction imposed on delivery rates is that each agent is unable to distinguish what she receives from what she bought. An equilibrium is composed by prices, delivery rates, orders and deliveries, such that each agent makes an order that maximizes the utility of the delivered bundle, with the (correctly anticipated) deliveries being feasible.

The main result of this paper is the existence of equilibrium (Section 2). A byproduct of the existence proof is the finding of an equilibrium in which agents receive the cheapest bundle among those that they cannot distinguish from truthful delivery. This is what should be expected from optimizing agents that set delivery rates for the goods they offer.

To illustrate the main intuitions offered by the model, the examples presented by Meier, Minelli and Polemarchakis (2006) are explained and solved (Section 3).

It is left for future work the study of the case in which agents set, simultaneously, the prices of the goods they offer and the respective delivery rates. The relevance of this is made evident in the work of Wilson (1980).

2 The model

We consider an economy in which a finite number of agents, $i \in \mathcal{I} = \{1, ..., I\}$, trade a finite number of goods, $l \in \mathcal{L} = \{1, ..., L\}$.

To capture the usual context in which the seller has superior information about the quality of the goods that she brings to the market, it is useful to consider a generalized notion of a good, incorporating in its description the name of the agent that is endowed with the good. This allows us to study markets in which agents may not have the ability to distinguish good cars from bad cars in general, but are able to observe the quality of their own cars.

Such reformulation implies the need to adapt endowments and preferences to this new setup. We refer to good l that is in the initial endowment of agent i as the generalized good (l, i).

Endowments in terms of these generalized commodities, $f_i \in \mathbb{R}^{LI}_{+}$, relate to the usual definition of endowments, $e_i \in \mathbb{R}_+^L$, as follows:

$$f_i \in \mathbb{R}_+^{IL}, \quad \text{with} \quad f_i^{(l,i)} = e_i^l \quad \text{and} \quad f_i^{(l,j)} = 0, \ \forall l, i, j \neq i.$$

Similarly, the utility functions in terms of these generalized commodities, V_i, can be obtained from the usual utility functions, $U_i : \mathbb{R}_+^L \to \mathbb{R}$, as follows:

$$\begin{aligned} & V_i : \mathbb{R}_+^{LI} \to \mathbb{R}; \\ & V_i(x_i) = U(z_i), \quad \text{where} \quad z_i^l = \sum_j x_i^{(l,j)}. \end{aligned}$$

Agents wish to maximize their utility functions, $V_i(x_i)$, which are continuous, concave and strictly increasing[1].

Each agent has specific abilities to distinguish the different goods that are traded in the market. These observation abilities are described by a partition of the set of generalized goods, P_i, such that $(l', j') \in P_i(l, j)$ if and only if agent i cannot distinguish good (l', j') from good (l, j).

The inability to distinguish between two goods, (l, j) and (l', j'), implies that an agent that buys certain quantities of (l, j) and (l', j'), say $y_i^{(l,j)}$ and $y_i^{(l',j')}$, may receive different quantities, $x_i^{(l,j)}$ and $x_i^{(l',j')}$, such that:

$$x_i^{(l,j)} + x_i^{(l',j')} = y_i^{(l,j)} + y_i^{(l',j')}.$$

More generally, when buying $y_i = (y_i^{(1,1)}, ..., y_i^{(1,I)}, y_i^{(2,1)}, ..., y_i^{(L,I)})$, agent i will receive $x_i = (x_i^{(1,1)}, ..., x_i^{(1,I)}, x_i^{(2,1)}, ..., x_i^{(L,I)})$ such that:

$$\begin{bmatrix} x_i^{(1,1)} \\ \vdots \\ x_i^{(1,I)} \\ x_i^{(2,1)} \\ \vdots \\ x_i^{(L,I)} \end{bmatrix} = \begin{bmatrix} k_i^{(1,1),(1,1)} & \cdots & k_i^{(1,1),(1,I)} & k_i^{(1,1),(2,1)} & \cdots & k_i^{(1,1),(L,I)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_i^{(1,I),(1,1)} & \cdots & k_i^{(1,I),(1,I)} & k_i^{(1,I),(2,1)} & \cdots & k_i^{(1,I),(L,I)} \\ k_i^{(2,1),(1,1)} & \cdots & k_i^{(2,1),(1,I)} & k_i^{(2,1),(2,1)} & \cdots & k_i^{(2,1),(L,I)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_i^{(L,I),(1,1)} & \cdots & k_i^{(L,I),(1,I)} & k_i^{(L,I),(2,1)} & \cdots & k_i^{(L,I),(L,I)} \end{bmatrix} \cdot \begin{bmatrix} y_i^{(1,1)} \\ \vdots \\ y_i^{(1,I)} \\ y_i^{(2,1)} \\ \vdots \\ y_i^{(L,I)} \end{bmatrix},$$

[1]By strictly increasing, it is meant that an increase in consumption of any of the goods is strictly desired by the agents: $x_i \geq x_i'$ and $x_i \neq x_i'$ implies that $V_i(x_i) > V_i(x_i')$.

where $k_i^{(l',j'),(l,j)}$ denotes the number of units of good (l', j') that agent i receives for each unit of good (l, j) that she buys.

The delivery matrix, k_i, is endogenous (equilibrating variable). The delivery matrices that are compatible with the abilities of agent i to distinguish commodities are such that, for each (l, j):

$$\sum_{(l',j') \in P_i(l,j)} k_i^{(l',j'),(l,j)} = 1 \quad \text{and} \quad k_i^{(l',j'),(l,j)} = 0, \ \forall (l', j') \notin P_i(l, j).$$

The set of matrices that satisfy these conditions is denoted K_i, and $K = \prod_{i=1}^{I} K_i$.

It should be clear that if agent i is able to distinguish all the commodities, that is, if $P_i(l, j) = \{(l, j)\}$, $\forall (l, j)$, then K_i has a single element (the identity matrix). In this case, agent i is sure of receiving exactly the bundle that she buys.

To simplify our problem, we will always assume that agents have perfect information about their endowments.

Assumption 1 (Perfect information about own endowments).

$\forall (i, l) : \ P_i(l, i) = \{(l, i)\}.$

Only goods that exist are priced and traded in the market. Such goods are those for which $\sum_i f_i^{(l,j)} > 0$, a condition which is equivalent to $f_j^{(l,j)} > 0$. This restriction implies straightforward modifications of the spaces in which f_i, k_i, V_i and P_i are defined.

We will denote the set of goods that exist by $\mathcal{M} \subseteq \mathcal{L} \times \mathcal{I}$, and the number of goods that exist by $M \leq L * I$. When the classical interiority assumption holds $(e_i \gg 0, \forall i)$, all the goods are traded in the market and therefore $\mathcal{M} = \mathcal{L} \times \mathcal{I}$ and $M = L * I$.

Taking prices, $p \in \Delta^M$, and delivery rates, $k_i \in K_i$, as given, agent i trades its initial endowments, $f_i \in \mathbb{R}_+^M$, for a bundle, $y_i \in \mathbb{R}_+^M$, that maximizes utility,

$V_i(k_i y_i)$, among those that satisfy the budget restriction, $y_i \in B_i(p)$:

$$B_i(p) = \left\{ y_i \in \mathbb{R}_+^M : \ p \cdot y_i \leq p \cdot f_i \right\}.$$

Notice that Assumption 1 guarantees individual rationality of participating in the market. The agent can always "buy" its own endowments.

Definition 1 (Equilibrium).

An equilibrium of the economy $\mathcal{E} \equiv \{f_i, V_i, P_i\}_{i=1}^I$ *is composed by a price system,* $p^* \in \Delta_+^M$, *individual choices,* $y^* = (y_1^*, ..., y_I^*) \in \mathbb{R}_+^{IM}$, *delivery rates,* $k^* = (k_1^*, ..., k_I^*) \in K$, *and the resulting allocation,* $x^* = (x_1^*, ..., x_I^*) \in \mathbb{R}_+^{IM}$, *which satisfy:*

$y_i^* \in arg \max_{y_i \in B_i(p^*)} V_i(k_i^* y_i), \ \forall i$ *[individual optimality];*

$x_i^* = k_i^* y_i^*, \ \forall i$ *[delivery];*

$\sum_i x_i^* \leq \sum_i f_i$ *[feasibility].*

Theorem 1 (Existence of equilibrium).

There exists an equilibrium of the economy.

Proof:

Consider, for now, bounded choice sets. For choices in the upper bound to imply aggregate excess delivery, let $E = \sum_{(l,j)\in\mathcal{M}} f_j^{(l,j)} + 1$ and define the following convex and bounded choice sets:

$$\overline{Y}_i = \{y_i \in \mathbb{R}_+^M : \ y_i^{(l,j)} \leq E, \forall (l,j)\}.$$

The budget set of agent i, in this bounded economy, is:

$$\overline{B}_i(p) = \left\{ y_i \in \overline{Y}_i : \ p \cdot y_i \leq p \cdot f_i \right\}.$$

Let $\psi_i(y, p, k) = arg \max_{z_i \in \overline{B}_i(p)} \{V_i(k_i z_i)\}$.

The utility function, $V_i(k_i y_i)$, is continuous with respect to both k_i and y_i.

As long as $p \cdot f_i > 0$ (which is always the case for $p \gg 0$), the correspondence $\overline{B}_i(p) = \{y_i \in \overline{Y}_i : \ p \cdot y_i \leq p \cdot f_i\}$ is continuous with non-empty compact values.

When this is the case, we know, from Berge's Maximum Theorem[2], that the demand correspondence, $\psi_i(y, k, p)$, is upper hemicontinuous with nonempty compact values. It is also convex-valued, because V_i is concave and k_i is constant.

Let $\Delta_\epsilon^M = \left\{p \in \Delta^M : \ p \geq \epsilon\right\}$. We will start by finding a fixed point with strictly positive prices, on Δ_ϵ^M, and then let $\epsilon \to 0$ to obtain a sequence of fixed points.

Let $\psi_p^\epsilon(y, p, k) = arg \max\limits_{q \in \Delta_\epsilon^M} \left\{ q \cdot \sum\limits_i (k_i y_i - f_i) \right\}$.

And let $\psi_{k_i}(y, p, k) = arg \min\limits_{d_i \in K_i} \{p \cdot d_i y_i\}$.

All these correspondences (ψ_i, ψ_p^ϵ and ψ_{k_i}) are upper hemicontinuous with nonempty compact and convex values. Therefore, the product correspondence, $\psi^\epsilon = \prod_{i=1}^I \psi_i \times \psi_p^\epsilon \times \prod_{i=1}^I \psi_{k_i}$, also is.

Applying the Theorem of Kakutani, we find that there exists a fixed point of ψ^ϵ, that we denote by $(y^\epsilon, p^\epsilon, k^\epsilon)$. Considering a sequence, $\{\epsilon^n\}_{n \in \mathbb{N}}$, that converges to zero, we obtain a sequence of fixed points, $\{(y^n, p^n, k^n)\}_{n \in \mathbb{N}}$. The sequence is contained in a compact set, therefore a subsequence converges to (y^*, p^*, k^*).

We want to consider this subsequence and verify that its limit is an equilibrium.

Suppose that the sequence of prices in the interior of the simplex, $\{p^n\}$, converges to a price on the border of the simplex. There is at least one agent whose income does not tend to zero ($p^* \cdot f_i > 0$), and therefore whose demand (which is u.h.c.) is driven to the bound of the choice set (recall that utility is strictly increasing). This implies that, for sufficiently large n and in the limit, there is aggregate excess delivery of at least some good:

$$\exists (l, j) : \ \sum_i x_i^{(l,j)*} = \sum_i \sum_{(l',j')} k_i^{(l,j),(l',j')*} y_i^{(l',j')*} > \sum_i f_i^{(l,j)}.$$

The budget restrictions imply that $p^n \cdot \sum_i y_i^n \leq p^n \cdot \sum_i f_i$, and the definition

[2] See, for example, Aliprantis and Border (2006).

of ψ_{k_i} implies that $p^n \cdot \sum_i k_i^n y_i^n \leq p^n \cdot \sum_i y_i^n$ (the identity matrix is enough for equality). In the limit:

$$p^* \cdot \sum_i x_i^* = p^* \cdot \sum_i k_i^* y_i^* \leq p^* \cdot \sum_i f_i. \tag{1}$$

From (1), we know that if there is some good (l, j) with strictly positive price ($p^{(l,j)*} > 0$), for which $\sum_i x_i^{(l,j)*} > \sum_i f_i^{(l,j)}$ (excess delivery), there must be another, (l', j'), for which $\sum_i x_i^{(l',j')*} < \sum_i f_i^{(l',j')}$ (excess supply).

To understand the idea of the proof, start by supposing that, in the limit, there is a single good, (l, j), with maximal excess delivery. From the definition of ψ_p, the price of this good tends to 1, and the remaining goods have vanishing prices. Therefore, the aggregate budget restriction becomes:

$$\sum_i y_i^{(l,j)*} \leq \sum_i f_i^{(l,j)}.$$

Given the definition of the ψ_{k_i}:

$$\sum_i x_i^{(l,j)*} = \sum_i \sum_{(l',j')} k_i^{(l,j),(l',j')*} y_i^{(l',j')*} = \sum_i k_i^{(l,j),(l,j)*} y_i^{(l,j)*} \leq \sum_i y_i^{(l,j)*}.$$

Which implies that $\sum_i x_i^{(l,j)*} \leq \sum_i f_i^{(l,j)}$ (no excess delivery). Contradiction.

Now let's move on to the general case. Suppose that, in the limit, there is a set of goods, G, tied for the maximal excess delivery:

$$\forall (l, j) \in G: \sum_i x_i^{(l,j)*} = \sum_i \sum_{(l',j')} k_i^{(l,j),(l',j')*} y_i^{(l',j')*} > \sum_i f_i^{(l,j)}. \tag{2}$$

In the limit, the prices of these goods must add to 1, while the prices of the remaining goods become null. Aggregating the budget restrictions:

$$\sum_i \sum_{(l,j)\in G} p^{(l,j)*} y_i^{(l,j)*} \leq \sum_i \sum_{(l,j)\in G} p^{(l,j)*} f_i^{(l,j)}.$$

Consider the set of goods with highest price, $G_1 \subseteq G$. From the definition of the ψ_{k_i}, we know that buying one unit of a good in G_1 implies delivery of quantities of goods in G_1 that add to 1 unit or less, because buying goods that do not belong to G_1 implies no delivery of goods in G_1. Then:

$$\sum_i \sum_{(l,j)\in G_1} x_i^{(l,j)*} \leq \sum_i \sum_{(l,j)\in G_1} y_i^{(l,j)*}.$$

In fact, considering the set of goods with price higher or equal to some threshold, $t > 0$, denoted $G_t \subseteq G$, we have (from an analogous reasoning):

$$\sum_i \sum_{(l,j)\in G_t} x_i^{(l,j)*} \leq \sum_i \sum_{(l,j)\in G_t} y_i^{(l,j)*}.$$

Observing that $\sum_i y_i^*$ dominates $\sum_i x_i^*$ in the sense of Lorenz (for price inequality among goods instead of income inequality among agents), we obtain:

$$\sum_{(l,j)\in G} \left\{ p^{(l,j)*} \sum_i x_i^{(l,j)*} \right\} \leq \sum_{(l,j)\in G} \left\{ p^{(l,j)*} \sum_i y_i^{(l,j)*} \right\} \Rightarrow$$

$$\Rightarrow \quad \sum_i \sum_{(l,j)\in G} p^{(l,j)*} x_i^{(l,j)*} \leq \sum_i \sum_{(l,j)\in G} p^{(l,j)*} y_i^{(l,j)*} \leq \sum_i \sum_{(l,j)\in G} p^{(l,j)*} f_i^{(l,j)}.$$

Which contradicts (2).

There is not excess delivery of any good, therefore, x^* is feasible and $p^* \gg 0$. Existence of equilibrium in the bounded economy is established.

To check that this is an equilibrium when the bounds on the choice sets are removed, we must verify that individual choices remain unaltered.

Observe that the bound on the choice sets is large enough for the individual choices, y_i^*, to be in the interior of $\overline{Y}_i$ (otherwise we would not have feasibility). Since preferences are convex, we are sure that the bounds are not binding. If there were a strictly better choice outside $\overline{Y}_i$, then there would also be a strictly better choice in the frontier of $\overline{Y}_i$.

QED

Under general conditions, equilibrium exists. Furthermore, the equilibrium that was found has a nice property: delivered bundles are as cheap as possible (to see this, look at the definition of ψ_{k_i} in the proof of existence). Although delivery rates were taken as given by the agents, they are coincide with those that would be expected from their optimizing behavior. This property induces a natural refinement of the equilibrium concept.

3 Examples

In this section, some examples presented as an illustration.[3] Here the notation is simpler than in the previous section, because it will not be necessary to deal with generalized goods in a formal way.

In the first example, an agent that does not distinguish two goods is not able to consume the high quality good, in spite of being willing to pay any price for a small quantity of this good.

The second example, two partial substitute goods are traded at the same equilibrium price. The agent that does not distinguish the two goods receives both goods, with delivery rates determined by the "leftovers" from the choices of the informed agents.

Finally, an example of a job market with two firms: one that can observe the productivity of workers, and another that cannot. Wages end up reflecting productivity, with the informed firm hiring the more productive workers.

3.1 Cherry picking

Three individuals, $I = \{1, 2, 3\}$, trade three commodities, $L = \{0, r, g\}$, that we can think of as 'money', 'red cherries' and 'green cherries'.

[3]The examples were adapted from those presented by Meier, Minelli and Polemarchakis (2006). The equilibrium concept used here leads to qualitatively different solutions.

Agent 1 is endowed with 'money', agent 2 with 'red cherries' and agent 3 with 'green cherries'.

$$\begin{cases} e_1 = \{12, 0, 0\}; \\ e_2 = \{0, 12, 0\}; \\ e_3 = \{0, 0, 12\}. \end{cases}$$

All agents prefer 'green cherries' to 'red cherries'. Agent 2 does not like 'red cherries' at all. Preferences are described by the following utility functions:

$$\begin{cases} U_1(x_1) = ln(x_1^0) + ln(x_1^r) + 2ln(x_1^g);\,^4 \\ U_2(x_2) = ln(x_2^0) + 2ln(x_2^g); \\ U_3(x_3) = ln(x_3^0) + ln(x_3^r) + 2ln(x_3^g). \end{cases}$$

There is asymmetric information because agent 1 cannot distinguish 'red cherries' from 'green cherries' while agents 2 and 3 are able to distinguish the three commodities.

$$\begin{cases} P_1 = \{\{0\}, \{r, g\}\}; \\ P_2 = \{\{0\}, \{r\}, \{g\}\}; \\ P_3 = \{\{0\}, \{r\}, \{g\}\}. \end{cases}$$

Agents 2 and 3 are perfectly informed, and therefore receive exactly what they buy. On the other hand, agent 1 can buy either 'red cherries' or 'green cherries', but receives whatever kind of cherries is delivered (because agent 1 does not distinguish the two goods).[5]

[5]The group of commodities designated as 'cherries' can be seen as a list containing two alternatives: 'red cherries' and 'green cherries'. In the original context of an economy with uncertain delivery (see Correia-da-Silva and Hervés-Beloso (2008a, 2008b and 2009) for a detailed description), which was of ex-ante contracting for future contingent delivery (agents could not distinguish states of nature, but could distinguish commodities), delivery had to be either of 'red cherries' or 'green cherries'. Here, additional complexity arises from the fact that there is an infinite number of possibilities for the delivery of 10 'cherries' (10 'red' and 0 'green', 5 'red' and 5 'green', 7 'red' and 3 'green', etc.).

Since agent 2 will not buy 'red cherries', her budget restriction is (notice that the price of 'money' is normalized to $p^0 = 1$):

$$p^0 x_2^0 + p^r x_2^r + p^g x_2^g = p^r e_2^r \Rightarrow x_2^0 + p^g x_2^g = 12p^r.$$

The optimality condition implies equality between the ratios between marginal utility and price, for each good demanded:

$$x_2^0 = 0.5 p^g x_2^g.$$

From the budget restriction and the optimality condition, we find the demand of agent 2:

$$(x_2^0, x_2^r, x_2^g) = (4p^r, 0, 8\frac{p^r}{p^g}).$$

Similarly, we can obtain the demand function of agent 3:

$$\begin{cases} x_3^0 + p^r x_3^r + p^g x_3^g = 12p^g \\ x_3^0 = p^r x_3^r = 0.5 p^g x_3^g \end{cases} \Rightarrow (x_3^0, x_3^r, x_3^g) = (3p^g, 3\frac{p^g}{p^r}, 6).$$

Looking at the demand of agents 2 and 3 for 'green cherries', we find that $p^r < p^g$, otherwise there would be excess demand. More precisely:

$$8\frac{p^r}{p^g} + 6 \leq 12 \Rightarrow p^r \leq 0.75 p^g.$$

Suppose that agent 1 buys a quantity x_1^{rg} of 'cherries' (guarantees delivery of 'red cherries' and 'green cherries' such that $x_1^r + x_1^g = x_1^{rg}$). If 'red cherries' are cheaper than 'green cherries', then the agent should receive only 'red cherries', and no 'green cherries'.

If the agent received some 'green cherries', then one could wonder why someone is delivering these 'green cherries', instead of trading them in the market for 'red cherries' plus 'money' and delivering the 'red cherries' while keeping the 'money' (there would be an arbitrage opportunity).

Following this reasoning, since $p^r < p^g$, then $x_1^g = 0$. Assuming that agent 1 is aware of this (delivery rates are anticipated and taken as given), we can find her demand function. The quantity x_1^r and the price p^r can also be interpreted as the quantity and price of (undistinguished, or pooled) 'cherries', x_1^{rg} and p^{rg}.[6]

$$\begin{cases} x_1^0 + p^r x_1^r = 12 \\ x_1^0 = p^r x_1^r \end{cases} \Rightarrow \begin{cases} x_1^0 + x_1^0 = 12 \Rightarrow x_1^0 = 6; \\ p^r x_1^r + p^r x_1^r = 12 \Rightarrow x_1^r = \frac{6}{p^r}. \end{cases}$$

For demand to equal supply:

$$\begin{cases} x_1^0 + x_2^0 + x_3^0 = 12 \Rightarrow 4p^r + 3p^g = 6; \\ x_1^r + x_2^r + x_3^r = 12 \Rightarrow \frac{6}{p^r} + 3\frac{p^g}{p^r} = 12; \\ x_1^g + x_2^g + x_3^g = 12 \Rightarrow 8\frac{p^r}{p^g} = 6 \Rightarrow p^r = 0.75p^g. \end{cases}$$

These equations allow the determination of equilibrium prices:

$$p^* = (p^0, p^r, p^g) = (1; 0.75; 1).$$

The allocation is, therefore ($y^* = x^*$ yields truthful deliveries):

$$\begin{cases} x_1^* = (6, \frac{6}{p^r}, 0) = (6, 8, 0); \\ x_2^* = (4p^r, 0, 8\frac{p^r}{p^g}) = (3, 0, 6); \\ x_3^* = (3p^g, 3\frac{p^g}{p^r}, 6) = (3, 4, 6). \end{cases}$$

3.2 Modified cherry picking

Is it always the case that agent 1 does not consume 'green cherries'? In this modified example, we find that agent 1 can consume 'green cherries' if the incentives for agents 2 and 3 to deliver only 'red cherries' disappear.

[6] Agent 1 prefers any interior bundle ($x_1 \gg 0$) to a bundle that is in the frontier of the consumption set. But she cannot get any 'green cherries' and, therefore, her utility is infinitely negative. Recall that we are assuming that, among bundles with $x_1^g = 0$ (in the frontier of the consumption set), the preferences of agent 1 are described by $v_1(x_1) = ln(x_1^0) + ln(x_1^r)$.

This occurs if 'green cherries' become much more abundant than 'red cherries'. Let's double the endowments of agent 3 (green cherries):

$$\begin{cases} e_1 = \{12, 0, 0\}; \\ e_2 = \{0, 12, 0\}; \\ e_3 = \{0, 0, 24\}. \end{cases}$$

Demand of agent 2 remains unaltered:

$$\begin{cases} x_2^0 + p^g x_2^g = 12p^r \\ x_2^0 = 0.5p^g x_2^g \end{cases} \Rightarrow (x_2^0, x_2^r, x_2^g) = (4p^r, 0, 8\frac{p^r}{p^g}).$$

While demand of agent 3 doubles:

$$\begin{cases} x_3^0 + p^r x_3^r + p^g x_3^g = 24p^g \\ x_3^0 = x_3^r p^r = 0.5x_3^g p^g \end{cases} \Rightarrow x_3^* = (6p^g, 6\frac{p^g}{p^r}, 12).$$

Now the demand of agents 2 and 3 for 'green cherries' does not exceed supply as long as $p^r \leq 1.5p^g$:

$$x_2^g + x_3^g = 8\frac{p^r}{p^g} + 12 \leq 24 \Rightarrow p^r \leq 1.5p^g.$$

There are three possibilities: (a) the price of 'green cherries' is higher than the price of 'red cherries' and thus agent 1 only consumes 'red cherries' (as in the previous example); (b) the price of 'red cherries' is higher than the price of 'green cherries' and thus agent 1 only consumes 'green cherries'; (c) the prices of 'green cherries' and 'red cherries' coincide.

In case (a), there would be excess supply of 'green cherries', as aggregate consumption is lower than 20:

$$x_1^g + x_2^g + x_3^g = 0 + 8\frac{p^r}{p^g} + 12 < 20.$$

In case (b), there would be excess supply of 'red cherries', as aggregate consumption is lower than 6:

$$x_1^r + x_2^r + x_3^r = 0 + 0 + 6\frac{p^g}{p^r} < 6.$$

Thus, in equilibrium we must have case (c): $p^r = p^g = p^{rg}$. Thus:

$$\begin{cases} x_2 = (4p^{rg}, 0, 8) \\ x_3 = (6p^{rg}, 6, 12) \end{cases} \Rightarrow x_2 + x_3 = (10p^{rg}, 6, 20).$$

The only candidate for an equilibrium allocation gives agent 1 the following consumption bundle: $x_1 = (12 - 10p^{rg}, 6, 4)$.

To check whether this is an equilibrium, we need to find the demand of agent 1. A problem that we face is that agent 1, through her demand, may influence the quality of the cherries (the proportion between red and green cherries).

It is assumed that she takes the proportions of delivered red and green cherries as given. In equilibrium, this proportion must be fulfilled (otherwise it would not be an equilibrium). Since we already have a single candidate for the equilibrium consumption of agent 1, $x_1 = (12 - 10p^{rg}, 6, 4)$, we must assume that agent 1 expects to receive 60% red cherries and 40% green cherries.

The utility and the maximization condition of agent 1 are (with $x_1^{rg} = x_1^r + x_1^g$):

$$u_1(x_1) = lnx_1^0 + ln(0.6x_1^{rg}) + 2ln(0.4x_1^{rg}) \Rightarrow x_1^0 = \frac{1}{3}p^{rg}x_1^{rg}.$$

Putting this together with the budget restriction, demand is obtained:

$$\begin{cases} x_1^0 + p^{rg}x_1^{rg} = 12; \\ x_1^0 = \frac{1}{3}p^{rg}x_1^{rg} \end{cases} \Rightarrow (x_1^0, x_1^r, x_1^g) = (3, 0.6\frac{9}{p^{rg}}, 0.4\frac{9}{p^{rg}}).$$

For demand of 'money' to equal supply:

$$x_1^0 + x_2^0 + x_3^0 = 12 \Rightarrow 3 + 4p^{rg} + 6p^{rg} = 12 \Rightarrow p^{rg} = 0.9.$$

Equilibrium prices are, therefore, $p^* = (1; 0.9; 0.9)$, and the allocation is:

$$x_1^* = (3; 6; 4); x_2^* = (3.6; 0; 8); x_3^* = (5.4; 6; 12).$$

In this case, agent 1 consumes both 'red cherries' and 'green cherries', which are traded at the same price. Agents 2 and 3 optimize by delivering 3.6 units of 'red cherries' and 5.4 units of 'green cherries' to agent 1.

The quantities of red and green cherries that agent 1 buys (y_1^r and y_1^g) are irrelevant, given that they add to 10. If $y^* = x^*$, there is truthful delivery. In any case, the delivery rates adjust to be such that delivery is surely that calculated above: $x_1^r = 6$ and $x_1^g = 4$.

3.3 A job market

Consider an economy with two firms, A and B, that have an initial endowment of 'money' and hire labor. There are two types of labor, 1 and 2 (type 2 is more productive than type 1). Money is designated as good 0. The firms have the same preferences and endowments (8 units of 'money'):

$$\begin{cases} e_A = \{8, 0, 0\} \\ e_B = \{8, 0, 0\} \end{cases} \text{and} \quad \begin{cases} u_A(x_A) = x_{A0} + x_{A1} + 4x_{A2}; \\ u_B(x_B) = x_{B0} + x_{B1} + 4x_{B2}. \end{cases}$$

The price of 'money' is normalized to 1, and the wages of each type of labor are denoted w_1 and w_2.

Two workers supply labor, at the expense of their time of leisure. Their endowments are 4 units of time.

$$\begin{cases} e_1 = \{0, 4, 0\} \\ e_2 = \{0, 0, 4\} \end{cases} \text{and} \quad \begin{cases} u_1(x_1) = x_{10} - \frac{1}{x_{11}}; \\ u_2(x_2) = x_{20} - \frac{1}{x_{22}}. \end{cases}$$

There is asymmetric information because firm A can distinguish between the two types of labor, but firm B cannot.

$$\begin{cases} P_A = \{\{0\}, \{1\}, \{2\}\}; \\ P_B = \{\{0\}, \{1, 2\}\}. \end{cases}$$

For demand of labor by firm A to be finite, it is necessary that $w_1 \geq 1$ and $w_2 \geq 4$. In fact, it is clear that in equilibrium we will have $w_2 = 4$, otherwise there would be no demand for labor of type 2. Notice that, at this wage, firm A is willing to hire any quantity of labor of type 2.

Optimization by the workers yields demand for leisure. Agent 2 dedicates half of the time to work and the other half to leisure.

$$w_2 x_{22}^2 = 1 \Rightarrow x_{22} = w_2^{-1/2} = 2.$$

If $w_1 = 4$, then the worker of type 1 would also wish to sell 2 units of labor. Only firm B could use this labor (firm A would not find it profitable to hire labor of type 1 at $w_1 = 4$). But, clearly, firm B would not be willing to pay $w_1 = 4$, knowing that the labor would not be 100% of type 2. Thus: $w_1 < 4$ and all the labor hired by firm B is of type 1 (workers of type 2 are not be willing to work for a wage lower than 4, which is what they receive from firm A).

Firm B is aware of this reasoning, and knows, therefore, that the labor hired is 100% of type 1. For firm B to demand a finite amount of labor, it is necessary that $w_1 = 1$. In sum, these considerations imply that equilibrium prices are $w_1^* = 1$ and $w_2^* = 4$.

Again, optimization by the worker yields agent 1's demand for leisure.

$$w_1 x_{11}^2 = 1 \Rightarrow x_{11} = w_1^{-1/2} = 1.$$

From the budget restrictions, we obtain the money income of each agent:

$$\begin{cases} x_{10} + w_1 x_{11} = w_1 e_{11} \Rightarrow x_{10} = w_1(4 - x_{11}) \Rightarrow x_{10} = 3; \\ x_{20} + w_2 x_{22} = w_2 e_{22} \Rightarrow x_{20} = w_2(4 - 2) \Rightarrow x_{20} = 8. \end{cases}$$

The corresponding money income and leisure time of each agent are:

$$\begin{cases} x_1^* = (1, 3, 0); \\ x_2^* = (8, 0, 2). \end{cases}$$

Ignoring the indeterminacy on the allocation of labor of type 1 (firm B is assumed to hire all the supply), we find:

$$\begin{cases} x_A^* = (x_{A0}^*, x_{A1}^*, x_{A2}^*) = (4, 0, 2); \\ x_B^* = (x_{B0}^*, x_{B1}^*, x_{B2}^*) = (7, 1, 0). \end{cases}$$

4 Concluding remarks

In an economy in which agents trade goods with uncertain quality, the ability to observe the quality of the good is very useful. To study markets such as the used car market (Akerlof, 1970), it is natural to assume that agents know the quality of the goods that they bring to the market, but not the quality of the goods brought by the other agents.

To model this kind of information asymmetry, we have considered a generalized notion of a good, incorporating in its description the agent that is endowed with this good. This allowed us to study economies in which agents may not have the ability to distinguish good cars from bad cars, but are able to observe the quality of their own cars.

Equilibrium is shown to exist, and characterized by the fact that agents always receive the cheapest delivery that is consistent with their observation abilities (that is, that they cannot distinguish from truthful delivery)

In this model, the price of the same good may vary across sellers (observe that each seller faces buyers with different information), but price discrimination is not allowed (the price of a good does not depend on the buyer). Notice also that if two agents sell the same good, with equal delivery rates, they must sell it at the same price.

Since the prices of goods depend on the agent that brings them to the market, assuming price-taking behavior is questionable. It still provides a natural benchmark, but its comparison with alternative assumptions would be interesting, as well as the study of a replicated economy.

There could be objections to the assumption of agents taking own delivery rates as given, but it can be verified that the equilibrium delivery rates coincide with those that would be expected from optimizing agents.

Anyway, the message of the work of Wilson (1980) suggests the study of the case in which agents set, simultaneously, the price of the goods they bring to the market, and the corresponding delivery rates. This is left for future work.

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