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AN EXPERIMENTAL GAME

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# THE VALUE OF FIAT MONEY WITH AN OUTSIDE BANK: AN EXPERIMENTAL GAME<sup>#</sup>

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## Abstract

Why people accept intrinsically worthless fiat money in exchange for real goods and services has been a longstanding puzzle in economics. Attempts to explain the broad acceptance of fiat money have relied on either assuming that someone will exchange the fiat money for real consumption at the end of the horizon, or on pushing the puzzle of fiat money into infinite future in overlapping generations settings. We examine an alternative route that can explain the value of fiat money through a debt instrument which allows consumption to be moved backward in time. In this paper, we present empirical evidence that the theoretical predictions about the behavior of such economies work reasonably well in a laboratory experiment. The invention of fiat money and related debt instruments allow society to replace expensive commodities by costless paper and cut the dead weight loss associated with the former.

*JEL-classification:* C73, C91

*Keywords:* Experimental Gaming, Bank, Fiat money

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# 1. INTRODUCTION

A longstanding puzzle in the use of fiat money is: what supports the value of intrinsically worthless paper money? In this paper, we test experimentally the proposition that the presence of an outside or central bank is sufficient to support the value of fiat money. Included in the reasons for the value of fiat money that have been offered, are those of Hahn (1965, 1971), Jevons (1875), and Shubik and Wilson (1977). They constitute a small sample of studies from an extensive literature on the subject most of which are sufficient, but not all are necessary. The reasons given for symbolic money to be valued are that: (1) it serves as a means to overcome the failure of the double coincidence of wants (e.g., when A wants a good owned by B, but has no good that B wants), because money is assumed to be wanted by all; (2) it minimizes transactions costs, providing a valued convenient way to trade; (3) it carries default penalties, i.e., one is penalized for owing money; (4) value is supported by high enough dynamic expectations;<sup>1</sup> and (5) it is supported by an outside bank.<sup>2</sup>

Monetary theory is a complex mixture involving economic optimization, expectations, trust and institutional considerations. Often there are several mechanisms that can be used to achieve the same ends. We consider only two, as experimentation enables us to validate the value of money in the presence of an outside bank. This game has the property that the economy is able to substitute a nearly costless symbol of trade for a valuable commodity such as gold in the financing of transactions. From the observations noted above we know that a bank is not necessary but is sufficient to achieve this result.<sup>3</sup>

In this paper we investigate the behavior of a minimal economy that includes an outside bank and a default penalty on unpaid loans. We chose to address reasons (3) and (5) listed above, as both bank loans and default penalties exist in the field and it is

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<sup>1</sup> For example, they might believe that prices will be stable in a booming economy in the future.

<sup>2</sup> Without going into technical details, for (3) if an individual has the strategic opportunity to default he will do so unless there is a penalty that is typically denominated in some form of disutility or loss related to the money value of the loss, For (4) see Grandmont (1983) for analysis of the role of expectations in supporting the value of money.

<sup>3</sup> Our view is that a modern monetary economy has much of its complexity reflected in the institutions and the laws that have evolved in the development of that society, thus assuming the existence of an outside bank is at least as reasonable as assuming trading based on pair-wise search. The former is better suited to the economies we live in and the latter is better suited to studies in early economic anthropology. They answer different questions.

straightforward to consider them experimentally. The other reasons merit separate investigation, and are outside the scope of this paper. We view financial institutions and the related laws as consequences of social evolution through custom and analysis. A minimal game tends to capture the analysis more than the evolution, as the time span of evolution is generally too long to replicate in the laboratory.

We consider a finitely repeated game in which any money held at the end is worthless. However there is a banking system that allows individuals to borrow in such a way that they can avoid ending the game with worthless paper. The terminal period of the game is known exactly or with some uncertainty in advance and individuals can borrow at a specified interest rate, but must pay a bankruptcy penalty for ending in debt. The individuals can maximize their payoff by ending the game with zero money balances (Shubik, 1980, Dubey and Geanakoplos, 1992). We show experimentally that the economy becomes “cash consuming” as the theory predicts<sup>4</sup> if the holdings of the outside or government bank are not counted. In an exchange economy where fiat money is utilized the price level will be determined by the relationship among the initial amount of money held by the traders, the length of the game, the natural discount rate  $\beta$  for intertemporal consumption, and the bank rate of interest  $\rho$ . We compare the predictions suggested by theory with data observed in laboratory economies populated by profit motivated human agents and minimally intelligent (MI) algorithmic traders (specified later in detail) simulated on a computer.

## 2. BASIC THEORY

Consider an economy with two types of traders, who can trade two goods for money. One type of trader has an endowment of  $(a, 0, m)$ <sup>5</sup> and the other has  $(0, a, m)$ , where  $a, m > 0$ . In this economy, the traders of each type may borrow from a single bank at an announced rate of interest and then bid for the two available goods. The bank stands ready to lend a one-period loan to anyone at a fixed rate of interest  $\rho \geq 0$ .

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<sup>4</sup> In the sense that all of the original endowment of fiat is drained away as interest payments to the outside bank, where they are not necessarily recycled into the economy. They could be retired or destroyed, or held as reserves. The definition of how much money a central bank holds and what are its reserves raises national accounting questions beyond the scope of this paper.

<sup>5</sup> That is,  $a$  units of good A, 0 units of good B, and  $m$  units of money.

The individuals can pay the loan back at the beginning of the following period, or roll a part or all of the unpaid balance and interest over and add it to the next period's loan. One can only go bankrupt at the end where any outstanding debt is charged against the trader's total earnings from the entire game.

Even at this level of simplicity several basic issues arise. Should the bank be a strategic player or a dummy? We have chosen it to be a dummy. Does it fix in advance the quantity of money to be lent, or the interest rate to be charged, or fix both as its *modus operandi*? As the bank is a dummy we have chosen to specify an interest rate as a parameter in the game. We implicitly assume that the bank always has sufficient funds to lend, and thereby avoid having to discuss the details of the meaning of bank reserves. The bank permits the loans to be rolled over. The roll over condition is an important feature in finance that enables borrowers to delay any day of reckoning by replacing a current constraint by a future one.

For simplicity we stipulate that any positive money balances carried from one period to the next do not earn any interest.<sup>6</sup> A more general game would permit traders to deposit in, as well as borrow from, a bank thereby earning returns on any surplus financial capital. In the model economy, we limit the players to borrowing for two reasons. The first is to keep the game as simple as possible by defining a smaller choice set for players, i.e., confining their financial decision to the amount of borrowing instead of the amounts of borrowing and lending/deposit. The second reason is that in illustrating the value of paper money that has no given positive terminal value we need to investigate the behavior of the players at the terminal points and to compare it with the predictions of a finite period dynamic programming model of trade with a specified salvage value condition.

The experiment requires that the individuals make two decisions, a financial decision to borrow an amount  $d$  and a market decision to bid amounts  $b_i$  with  $i = 1, 2$  for each of the two goods. This game is known as the "sell-all" game where all individuals

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<sup>6</sup> Historically in U.S., real deposit rates have been close to zero, while historical interest rates on loans have been positive.

put up for sale their entire endowment of goods.<sup>7</sup> The derivations are given for both games in two appendices in Quint and Shubik (2008). For the sell-all game of  $T$  periods, borrowing, bids and prices in the first period, and the subsequent periods are:

$$b_1 = \frac{(1+\rho)^T m}{2(1+\rho)^T \frac{(1-\beta^T)}{1-\beta} - 2(1+\rho)^{T-1} \frac{(1-\beta^T)}{1-\beta}} = \frac{(1+\rho)(1-\beta)}{2\rho(1-\beta^T)} m$$

And

$$p_1 = \frac{2b_1}{a} = \frac{(1+\rho)(1-\beta)}{2\rho(1-\beta^T)a} m$$

With the inter-period linkage being

$$b_t = (1+\rho)^{t-1} \beta^{t-1} b_1$$

$$p_t = (1+\rho)^{t-1} \beta^{t-1} p_1$$

And finally

$$d_t = \frac{(1+\rho)^{t+1}}{\rho} \frac{1-\beta^t}{1-\beta^T} m - \frac{(1+\rho)^t}{\rho} \frac{1-\beta^{t-1}}{1-\beta^T} m - (1+\rho)^t m$$

In the game and analysis we confine the structure of the utility function to:

$$\sum_{t=1}^T \beta^{t-1} 10\sqrt{x_{it}y_{it}} + \beta^T \mu \min[m_{iT+1}, 0]$$

where  $\beta$  is a natural time discount rate for consumption,  $10\sqrt{\cdot}$  is the utility function for a level of consumption of  $x_{it}$  units of good A and  $y_{it}$  units of good B during period  $t$ , and  $\mu$  is the penalty for bankruptcy (i.e., holding a negative cash balance at the end of the game in period  $T$ ). There are  $T$  time periods, followed by full settlement at the beginning of period  $T+1$ . In the finite games one may set  $\beta$  such that  $0 < \beta \leq 1$ . In order to preserve the boundedness of the payoffs in infinite horizon models,  $0 \leq \beta < 1$ . Zero expected monetary inflation requires the Fisher condition  $1+\rho = 1/\beta$  to hold. If exogenous uncertainty is present in the economy, the noninflationary condition must be replaced by a somewhat more complex condition (see Karatzas et al. 2006).

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<sup>7</sup> There is a closely related and somewhat more complex “buy-sell” game (see Huber, et al. 2007a) in which the individuals make an additional decision  $q$  about the amount of their endowed good they offer for sale. Much of the current paper is limited to results from the simpler sell-all game.

In the above expressions we note a considerable simplification when  $1+\rho = 1/\beta$ ,  $b_t = b_1$ ,  $p_t = p_1$ , and  $d_t = \beta^{T-t}m/(1-\beta^T)$ .

In the experiment it is specified that at the termination of play after settlement of debt, any amount of money retained by the traders is of no value and any unpaid debt is of negative salvage value; it is subtracted from the payout to the players.

In actual economic life the length of the lags in payments and delivery of goods varies considerably. Lags are possible in delivery of both money and goods. For the sake of simplicity we assume that the goods traded arrive in time to be utilized in the same period in which they are traded and payments are delayed until the following period.

When the goods endowments of the individuals are given by  $(a, 0)$  and  $(0, a)$  and the initial amount of money held by each agent is  $m$ , Quint and Shubik (2008) show that the competitive market price of the goods at time  $t$  is given above together with the amount borrowed and the bid.

In our six treatments, we fixed parameters  $a = 200$ ,  $\rho = 0.05$ , and  $m = 1,000$ . For three treatments each we used  $T = 10$  and  $20$ . With  $\rho = 0.05$ , depending on the value of  $\beta$  the theoretically predicted equilibrium price path is inflationary ( $\beta = 1$ ), flat ( $\beta = 1/1.05$ ) or deflationary ( $\beta = 1/1.15$ ). We label these six treatments after this theoretical price path and the number of periods (10 or 20): INFL\_10, INFL\_20, FLAT\_10, FLAT\_20, DEFL\_10, and DEFL\_20 (see overview in Table 1) to get one inflationary, one flat, and one deflationary price path of 10 and 20 periods respectively.<sup>8</sup>

(Insert Table 1 about here)

Each treatment of the economy is cash consuming. The interpretation is that the initial amount of government money  $m$  per capita that the traders are endowed with is not merely used for transactions but is also eaten up in the finite horizon game by the interest payments on extra money borrowed from the bank. As the horizon gets longer, the size of the equilibrium initial borrowing drops, and so do the initial prices (see Tables 2 and 3).

(Insert Tables 2 and 3 about here)

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<sup>8</sup> To check the robustness of our results we conducted one additional run with a higher interest rate of 15 percent. Specifically,  $a = 200$ ,  $\beta = 1/1.15$ ,  $\rho = 0.15$ ,  $m = 1,000$ ,  $T = 20$ . As  $\beta(1+\rho)=1$  the predicted price path is flat and the treatment is thus labeled FLAT\_20\_rho\_15%. Results for this treatment are presented at the end of the paper.

Mathematically the system is no longer homogeneous of order zero in prices; the initial supply of fiat money makes the system non-homogeneous, but the cash consumption removes this feature by the end of the game.

It is also worth noting that when  $\beta = 1$  there is a singularity in the system in the sense that the infinite horizon utility is unbounded. Furthermore with  $\rho = 0$  no cash is removed from the economy and in the limit the payoffs become unbounded. In contrast, as  $\rho$  approaches but does not attain zero, the price approaches but does not quite attain a lower limit.

There are many variants and extensions of this model that merit investigation but are not covered in this experiment. Much of the basic theory has been explored by Bennie (2006) who derives explicit formulae for cyclical endowments. This calls for models with a bank that makes loans as well as accepts deposits. Further results with exogenous uncertainty, i.e., uncertain assets under low and high information conditions have been considered by Bennie (2006).

## **2.1. OTHER JUSTIFICATIONS FOR THE VALUE OF FIAT**

Huber et al. (2007a and 2007b) report experiments in which fiat money and personal credit were given value. In comparing those results with the results of the present experiment it is important to stress a feature of economic dynamics. Because there may be many institutional arrangements that can achieve the same purpose it can be highly misleading to seek a single cause. Here we show that the presence of an outside bank is sufficient to support the value of fiat.

Experimental gaming requires terminal conditions. The contrast between fiat money and gold illustrates the importance of understanding the distinction between a durable assets and its stream of services. If gold is valued at zero at the end, it is a wasting asset; it is more reasonable to attach a positive value representing at least the value of the discounted stream of consumption or production services still present. With fiat a zero salvage value indicates that the economy has ended, and as there are no more transactions, fiat is worthless.



### **2.1.1. Fiat Money and Expectations**

In the experiment with a “sell-all” market without borrowing or lending in Huber et al. (2007a) the support came through expectations specified in the experiment in the terminal conditions. This confirmed the theoretical structure analyzed by Grandmont (1983). As the amount of money was fixed there was no opportunity for inflation as encountered in our INFL-treatments here.

The high level of efficiency we observe in our markets (95.9 to 99.2 percent) mirrors the efficiency of 97.6 percent we saw in sell-all markets without a bank in Huber et al. 2007a. This suggests that (i) the market mechanism itself produces a high level of efficiency and (ii) a bank has no obvious beneficial or negative consequences for overall efficiency in this model.

However, we do find a marked difference in the average money holdings at the end of a period between the sell-all markets in Huber et al. (2007a) and the markets explored in the present experiment. In Huber et al. (2007a) between 30 and 34 percent of the money endowment was kept unspent, and this fraction remained quite stable over the 20 periods of trading. In the markets explored here we see falling unspent money balances and an overall average of roughly 7.5 percent. This difference can be attributed to the salvage values of positive money holdings being zero here and. positive in Huber et al. (2007a).

### **2.1.2. Personal Credit and a Perfect Clearinghouse**

In Huber et al, (2007b) fiat or outside money does not exist. Instead each individual is allowed to personalize credit, and a clearinghouse arrangement prevents worthless credit from being issued.

## **3. THE EXPERIMENTAL SET UP**

We conducted and report on six (plus one control) treatments of this market game. For each treatment, we conducted two experimental runs for a total of twelve runs (plus one, see Table 1). In each run, the participants traded two goods labeled A and B, for money. Each run had ten participants, five of them endowed with ownership claim to 200 units of A and none of B, while the other five had ownership claim to 200 units of B and

none of A. All had the same starting endowment of money  $m = 1,000$ . The interest rate for loans ( $\rho$ ) was fixed at 5 percent per period in all treatments.<sup>9</sup>

All goods are consumed at the end of each period with no balances of goods A and B carried over from one period to the next; endowments of goods are reinitialized at the start of each period. Money holdings (positive and negative) are carried over to the following period.

The trading mechanism is a simple call market: all traders submit two numbers for the amount of money they invest to buy goods A and B. The maximum amount a trader can invest is the sum of his beginning of the period money holdings plus the loan he takes out from the bank. The computer then calculates the total money bid for good A and for good B. We use a sell-all model in which all endowments of goods are sold. To derive the price for A the sum of all bids for A is divided by the total endowment of good A (5 times 200 = 1,000). The same is done for B. Traders endowed with A receive as income 200 times the price of A, and similarly for B.<sup>10</sup>

Each period the ending money balance of each trader is his starting money balance minus the amount of money tendered for the two goods plus the income from selling his 200 endowed units of either A or B minus interest on any loan. Money holdings influence earnings directly only in the last period of the session when negative money holdings are divided by four and deducted from the total points earned. Positive money holdings at the end of the session have no value and are discarded. This may be interpreted as stating that the expected value of leftover fiat money is zero.

In each period the traders can earn points that are converted to dollars at a pre-announced exchange rate at the end of the experiment. Specifically

$$\text{Points earned} = 10 \cdot \sqrt{x_{ij} * y_{ij}} \cdot \beta^{\text{Period}-1}$$

with  $x_{ij}$  and  $y_{ij}$  the number of units of A and B held at the end of a period. The last term with  $\beta$  is the discount rate of points and with  $\beta < 1$ , points earned in later periods are not as valuable (in take home dollars) as points earned in the beginning.  $\beta$  is thus the main variable to distinguish our treatments, as it defines the theoretical price path. By

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<sup>9</sup> As mentioned earlier, we conducted a seventh treatment where the interest rate is set to 15 percent as a robustness check and is labeled FLAT\_20\_rho\_15%.

<sup>10</sup> For more details see Huber et al. 2007a.

varying the value of  $\beta$ , we chose  $(1+\rho)\beta$  to be greater than, less than or equal to 1 so as to expect to encounter inflation, deflation, and a steady price level in the respective treatments. In INFL\_10 and INFL\_20, with 10 and 20 periods respectively,  $\beta = 1$ , and therefore  $(1+\rho)\beta = 1.05$ , theory predicts inflation. Theoretically the rate of inflation should be lower in the longer treatment INFL\_20. In Treatments FLAT\_10 and FLAT\_20  $(1+\rho)\beta$  is 1 which theoretically should yield flat prices. Finally, in Treatments DEFL\_10 and DEFL\_20  $(1+\rho)\beta = 1.05/1.15 = 0.9134$ , which is expected to generate deflationary price paths. In these treatments loans and prices should be highest at the beginning, when many points can be earned.

### 3.1. A comment on oligopolistic behavior

Huber et al. (2007a, 2007b) noted the influence of the number of players when there are only a few players. This influence is also present in the current experiment (with ten players) and is manifested in the distinction between the competitive and non-cooperative equilibria. However, since that difference is less than 1 percent, and is not the main question investigated in this paper, it is reasonable to use the competitive equilibrium solution as a benchmark for comparing the experimental results.

### 3.2. Overview over the treatments

*Treatment INFL\_10:  $\rho = 0.05$ ,  $\beta=1$ , 10 periods, inflationary prices*

In Treatment 1  $\beta = 1$ , meaning that the value of goods (the points earned for consuming them) is the same in each period. With a positive interest rate this should lead to inflationary prices, as reflected in the levels of equilibrium loans and prices for this and all other treatments are presented in Tables 1 and 2. INFL\_10 ran for 10 periods and we conducted two runs of this treatment with 10 subjects each. The instructions including screenshots are shown in Appendix A.

*Treatment INFL\_20:  $\rho = 0.05$ ,  $\beta=1$ , 20 periods, inflationary prices*

Treatments INFL\_10 and INFL\_20 are distinguished only by the length of the run, with INFL\_20 running 20 periods.

*Treatment FLAT\_10:  $\rho = 0.05$ ,  $\beta=1/1.05$ , 10 periods, flat prices*

This treatment ran for 10 periods and features  $\beta=1/1.05$ . With an interest rate of 5 percent this means that  $(1+\rho)\beta=1$ . Equilibrium prices are therefore steady. We conducted two runs of this treatment. In the first run the software “froze” after period 18, so we have data only for the first 18 periods.

*Treatment FLAT\_20:  $\rho = 0.05$ ,  $\beta=1/1.05$ , 10 periods, flat prices*

This treatment ran for 20 periods and again has  $\beta=1/1.05$ . Equilibrium prices are therefore stable. We conducted two runs of this treatment and the equilibrium prices and loans can be again seen in Tables 1 and 2.

*Treatment DEFL\_10:  $\rho = 0.05$ ,  $\beta=1/1.15$ , 10 periods, deflationary prices*

This treatment ran for 10 periods and here  $\beta=1/1.15$ . With an interest rate of 5 percent this means that  $(1+\rho)\beta = 0.9134$ . This implies that points earned at the beginning are more valuable and thus equilibrium prices and loans decrease over time. We conducted two runs of this treatment.

*Treatment DEFL\_20:  $\rho = 0.05$ ,  $\beta=1/1.15$ , 10 periods, deflationary prices*

This treatment is distinguished from DEFL\_10 only by a higher duration of 20 periods. Again we conducted two runs.

All Treatments were conducted with software written using z-Tree (Fischbacher 2007). Six of the runs were conducted at Yale University in January and February 2007. The other seven runs were carried out at University of Innsbruck in March and October 2007. Average payments were \$22 at Yale and €19 in Innsbruck. While most subjects had participated in experiments in economics before, no student participated in more than one of the runs of the type presented in this paper.

## **4. RESULTS**

Although the rules of the game are simple, the considerations of the terminal conditions, price and borrowing behavior call for a sophisticated strategy. Our concern in this section is to see what aspects of the theoretical predictions of general equilibrium (GE) are confirmed by the observed outcomes of these games. In Section 5 we will also explore how efficiently the markets function when they are populated with appropriately

defined minimally intelligent agents. We focus on the development of prices, loans, money holdings, and efficiency.<sup>11</sup>

#### 4.1. Prices

Equilibrium predictions for price paths are different for the six treatments. GE predicts inflationary prices in treatments INFL\_10 and INFL\_20, stable prices in FLAT\_10 and FLAT\_20, and falling prices in DEFL\_10 and DEFL\_20 (see Table 2 for an overview on GE predictions). Figure 1 shows the realized and equilibrium price paths for all six treatments.

(Insert Figure 1 about here)

We see that in each run the slope of price paths for the two goods conform to the theoretical GE prediction, i.e., we observe inflation in the INFL treatments, relatively stable prices in the FLAT treatments and price decreases in the DEFL treatments. The coefficients in linear regression of prices over time (periods) are presented in Table 4.

(Insert Table 4 about here)

They are significantly positive for each run and each good in the INFL treatments (t-values 6.91 or higher for all price series), not significantly different from zero in seven out of eight FLAT treatments (t-values between -0.58 and 2.03), and significantly negative in the DEFL treatments (t-values of -7.36 or less for all price series). Conformity of the slopes of empirical and theoretical GE price paths implies that these markets were able to assess the implications of different  $\beta$ s in the different treatments.

When looking for differences across treatments with different time horizons we observe that prices in all three shorter treatments (10 periods) are mostly below GE prices, while in the 20-period treatments price levels are close to or above the GE levels. Prices are mostly below GE prices in the 10-period setting because participants did not always spend all their money, and kept a small percentage (usually around 10 percent) unspent.

A comparison of prices in INFL\_10 and INFL\_20 demonstrates the effect of the time horizon on the rate of inflation. Price levels in both treatments started at the same

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<sup>11</sup> The control treatment FLAT\_20\_rho\_15% will be briefly discussed towards the end of the paper and presented in Figure 7.

level, but increased faster in INFL\_10, reaching an average of 19.34 by period 10 vs. only 9.20 in INFL\_20. The data conform to the GE prediction of the effect of time horizon on the rate of inflation in this economy as if the subjects understood what the consequences of their actions in the market were. Similarly, prices in FLAT\_10 are on average 11.34 vs. only 7.65 in FLAT\_20; this difference is also in line with theoretical predictions. For the deflationary treatments the theoretical predictions of price paths and levels hold as well, i.e., both are decreasing and prices are higher in the shorter treatment.

#### **4.2. Borrowing**

The evolution of prices is closely reflected in the evolution of borrowing by traders, as prices strongly depend on the money available to subjects. In GE the size of the average loan increases in the INFL, increases at a slower rate in FLAT treatments, and decreases in DEFL\_10 and DEFL\_20 (see Table 3 for details). Figure 2 shows that the general patterns predicted by GE are present in the experimental data with loans increasing in all runs of the first four treatments, but decreasing in the last two.

(Insert Figure 2 about here)

The total loan balances in INFL\_10 increased over time, and remained mostly below GE levels. In INFL\_20, loan balances increased steadily over time, and remained mostly above GE levels – especially in the last few periods, when subjects tried to get rid of their remaining cash by taking out larger loans.

GE predicts higher loans in the 10-period setting than in the longer setting. This is exactly what we find: loans taken in INFL\_10 were on average 47 percent higher than the respective numbers in the first ten periods of INFL\_20. Similarly, average loans in the first 10 periods of FLAT\_10 are 70 percent higher than those of FLAT\_20, again in line with theoretical predictions. In the deflationary treatments theory suggests higher loan levels in the 10-period market in the first ten periods and the data support this prediction.

Thus the prices and loan data generated in these markets populated with profit-motivated human agents re consistent with the theoretical predictions,

### 4.3. Development of Money Balances over Time

GE prediction for all six treatments is that money holdings are zero after the last period. However, as participants faced some uncertainty about the exact length of the experiments in eight of the runs<sup>12</sup> and as the computational task is quite demanding, even getting close to GE predictions is remarkable. Figure 3 presents the development of average money holdings over time in the six treatments.

(Insert Figure 3 about here)

Comparing the development of average money holdings in the treatments we observe similarities and differences. In all treatments average money holdings decrease, thus the economy is cash consuming. In the shorter treatments INFL\_10, FLAT\_10, and DEFL\_10 money holdings are above GE levels in all six runs, while in the longer treatments money holdings are below GE levels in five of six runs. This is mostly a result of different GE paths, which would have required faster spending in the shorter treatments. However, we consider not the deviations, but the closeness to GE paths as remarkable.

We also see that participants took the length of the experimental economy into account. Table 5 shows the average money holdings after period 10, separated for the 10-period and 20-period economies. Holdings in the longer treatments are more than two times as large as in the shorter treatments, as participants took smaller loans and kept more money at the end of the tenth period in the longer sessions.

(Insert Table 5 about here)

Money holdings yield a proxy for the accuracy of decision making. Traders may decide to keep some of their money unspent for various reasons. However, as loans cost money it is not rational to take a loan and then not spend some of the money (it earns no interest). We compute for each period and each participant the percentage of money left unspent, and calculate the respective averages for traders who did and did not take a loan. Table 6 shows that on average those who took out a loan kept only 2.8 percent of their

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<sup>12</sup> In eight of the runs participants faced some uncertainty about the exact length of the experiment. In the short treatments they were told that the session would last 8 to 12 periods and in the long sessions they knew it would be 18 to 22 periods. The eight runs are both runs of INFL\_10, INFL\_20, DEFL\_10, and FLAT\_20. In all other runs participants knew exactly how many periods they would trade.

overall money balance unspent, while those who did not take a loan kept 11.7 percent unspent. This pattern was visible in every one of the treatments.

(Insert Table 6 about here)

Over time we find that the amount of money left unspent decreases in all markets, but the averages are almost always lower for participants who took a loan than those who did not take a loan. The respective averages are shown in Figure 4.

(Insert Figure 4 about here)

This reflects learning effects as well as the desire to get rid of money towards the end of the runs (when it becomes worthless). This is also supported by looking at the number of traders taking a loan (recall that paying interest on loans is the only way to get rid of the endowed money). Figure 5 shows that the number of participants taking a loan increases both for the 10-period and the 20-period treatments.

(Insert Figure 5 about here)

#### **4.4. Efficiency**

Overall efficiency levels (as measured by the points earned as a percentage of the maximum achievable) range from 95.9 to 99.2 percent, reflecting a high degree of “balanced” investments (almost equal investments in the two goods) by most traders beginning in period 1 (see Figure 6).. Efficiency in the last period is higher than in the first period in all runs, suggesting learning effects.

(Insert Figure 6 about here)

Having observed less money saved by those who take a loan we expect them to earn more. This is indeed the case: average earnings for traders who take a loan in a specific period are 99.98 percent of points that can be earned.<sup>13</sup> By comparison, traders who did not take a loan earned on average 92.32 percent of the points they could have achieved in the respective period (see Table 7).

(Insert Table 7 about here)

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<sup>13</sup> Note that in some of the markets (e.g. INFL\_10 and FLAT\_10) earnings of more than 100 percent are achieved by those taking loans. This can happen because these traders buy more goods and thus earn more points at the expense of those without a loan – those end up earning only 85 and 87 percent of the points they could have earned, respectively.



We conclude that traders who took a loan spent almost all of their money, split it more equally than those who did not take a loan and thus they earned significantly more than those without a loan.

#### **4.5. On Uncertainty**

In eight of the runs presented here we permitted a small amount of uncertainty concerning termination, but there was no exogenous uncertainty in each period. In Tables 5 to 8 the treatments with uncertainty are shaded, while the others are not. The main results hold for all treatments. However, the analysis and possibly the comprehension of the game are considerably more complicated when exogenous uncertainty is present. We see higher money holdings at the end (see Table 8) in the treatments with uncertainty than in the other treatments. Treatment INFL\_20, where the length is uncertain, is an exception, as here average money holdings in the end were negative. However, in both runs this was driven by one or two individuals who took exceptionally large loans and ended with up to 27,000 in debt.

(Insert Table 8 about here)

Given that traders were confronted with some uncertainty about the exact number of periods in the sessions, and that the subjects are unlikely to have been able to compute the equilibrium price paths, the results are largely consistent with the GE theory and some deviations from GE price levels should not be surprising.

### **5. MARKET WITH MINIMALLY INTELLIGENT (MI) AGENTS**

We examine the behavior of this economy with an outside bank when it is populated by minimally intelligent (MI) artificial agents who follow simple pre-specified decision rules. The purpose of this examination is to learn the extent to which the properties of the outcomes of this economy may follow from its structure and are robust to behavioral variations of the agents. In previous studies we have contrasted the outcomes of market games against three benchmarks (see Huber et al. 2007a, b). Two of these are competitive and have subgame perfect non-cooperative equilibria derived from optimization by individual economic agents; the third is the outcome from markets populated by minimally intelligent agents who randomly pick their choices from their

opportunity sets (see Gode-Sunder, 1993). Here we face a somewhat more difficult problem that calls for describing the minimal abilities required to operate in multiple markets for goods and credit. We suspect that they are qualitatively different.

### **5.1. Agents with limits**

For an appropriate interpretation of minimally intelligent agents in this economy, several considerations are relevant. First, such agents need external constraints on the domain from which they can choose their actions. Second, these agents do not anticipate the future, and thus their actions are not influenced by consequences that might be foreseeable by more intelligent agents with powers of anticipation. Finally, they choose their actions randomly from the opportunity set available to them. Note that the behavior of an economy that have no value of residual money balances and includes a debt market and is populated with such myopic investors should be expected to differ significantly from the operation of an economy with intelligent agents who can anticipate the future including the possibility of default and its consequences (e.g., bankruptcy) when debt is available. While these considerations are not unique, they appear to be a reasonable start. It is always possible to investigate the consequences of alternative specifications of such agents.

The minimally intelligent agents are here defined as follows: (1) at the beginning of each period, they choose their total spending on goods A and B as a random number drawn from a uniform distribution  $U(0, \max(0, (\text{Beginning cash balance} + \text{credit limit})))$ , where the beginning cash balance is  $m$  in period 1; in the subsequent periods, it is the cash from the sale of the endowed good (A or B) minus any borrowing and interest on that borrowing in the preceding period. The max function ensure that if the beginning cash balance is more negative than the credit limit, spending of the agent during that period must be zero. Unlike intelligent agents, these minimally intelligent agents do not anticipate the penalty associated with outstanding debt at the end of the session. Finally, the total spending chosen is split between goods A and B using a randomly drawn fraction distributed uniformly  $U(0,1)$ . The agents are unintelligent borrowers; if credit is available they may take it much like a subprime borrower with little anticipation or even understanding of the future. Credit limitation rules of good banking prevent them from

overloading on loans and the subsequent disaster. The extra intelligence in a responsible bank makes up for any shortfalls in the individual borrowers.

As far as possible the variables have been set as in the experiment with humans. Thus the cash endowment in period 1 is fixed at 1,000 and identical for all agents. The market is also populated with five traders of each of two types with complementary consumption good endowments of (200,0) and (0,200) respectively. In human subject economies, there is no limit on the amount of borrowing; in MI economies, subjects must borrow enough money to bring any negative cash balance at the beginning of the period to zero, and if their beginning of the period cash balance is positive, they borrow an amount equal to a uniformly drawn random number between zero and the beginning cash balance discounted by interest for one period times a fixed multiplier (we used multiplier 1). The interest rate (5 percent), payoff multiplier (10), payoff exponent (0.5, i.e. the square root), and penalty multiplier for unpaid loans (0.25) are all set as in the human experiment. We report the results of the same three different natural discount rates as in the experiment with humans (0, 5 and 15 percent).

## 5.2. Results

In Huber et al. (2007a) similarly defined agents were able to produce reasonable prices, price paths and a quite high level of efficiency. While the efficiency of markets populated with MI agents, at 79 percent, is similar as in Huber et al. (2007a), these markets fare worse when it comes to prices, price paths, and loans. MI-agents, as set up by us, do not take the natural rate of discount ( $\beta$ ) into account when making their decisions. Therefore price paths, money holdings, and loan levels do not depend on  $\beta$ . Representative price, loan and money holding paths are shown in each panel of Figures 1, 2, and 3. While humans did take  $\beta$  into account and we thus observed inflation, flat prices or deflation, we do not observe different price paths in the MI agent simulation. Here prices decrease slightly over time, as interest on loans slowly depletes the money available to bid for the (fixed number of) goods A and B. Similarly, loans decrease slightly over time, irrespective of  $\beta$ , and as we chose a rather conservative credit limit that precludes the possibility of bankruptcy, MI-agents on average kept most of their initial money endowment until the end of the simulation (see Figure 2). As shown in

Tables 5 and 7 and in Figure 3, MI-agents on average still have 774 of their initial 1,000 cash after period 10, and 612 after period 20. This deviates strongly from GE predictions and from what we observe in our experiments with human subjects who get much closer to GE predictions.<sup>14</sup>

In Figure 6 we compare efficiency (i.e. points earned as percentage of the maximum possible) of markets populated by MI agents with the human experiments. We find that the MI agents have lower efficiency in each period.

In Tables 6 and 8, presenting the percentage of money unspent and the earnings with and without loans respectively, no numbers are given for MI, as we defined our agents to spend all their cash each period and each agent takes a loan each period. The same is true for Figures 4 and 5.

We conclude that a market populated by minimally intelligent agents, as specified here, reaches a satisfyingly high level of efficiency, though still significantly lower than the level reached by humans. However, ignorant of the influence  $\beta$  has, MI-agents are not able to produce the distinct price paths that GE theory predicts and experiments with humans produced.

## 6. CONCLUSIONS

In this paper we report that an outside bank and a default penalty are sufficient to ensure the value of fiat money in a simple laboratory economy. In addition we find that the markets populated by profit-motivated human subjects in our laboratory experiment yield outcomes that take the relationship of natural discount rate and interest rate into account to produce inflationary, flat, or deflationary price paths in line with theory. Minimally intelligent agents, which do not anticipate the future are not able to produce such distinct price paths. While simpler markets populated with such agents function reasonably well (e.g., Huber et al. 2007a), managing a borrowing relationship with a bank is more complex and requires additional abilities such as some elementary way to predict future states. We plan to continue further exploration of the minimal level of trader intelligence necessary to achieve efficiency in various market structures.

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<sup>14</sup> Other specifications of the MI agents, especially higher loans, would bring these markets closer to GE. However, we considered the chosen parameters to be reasonable interpretation of the concept of minimally intelligent agents.

Both commodity as well as fiat money can provide transaction services. However, unlike commodity money, fiat money has no alternative uses, and thus has zero opportunity cost. It is hardly surprising, then, that the use of a fiat or abstract means of payment is a broad systemic property of a modern mass economy. It can be supported institutionally in many ways. In three experiments we have provided evidence for workability of three different arrangements—expectations (Huber et al. 2007a), an efficient clearinghouse (Huber et al. 2007b) and an outside bank in the current paper. We find that markets populated by profit-motivated human subjects and an outside bank work remarkable well and are able to interpret different natural discount rates and produce price paths that reflect general equilibrium predictions.

**Table 1: The Experimental Design**(Interest rate ( $\rho$ ) = 0.05 in all treatments except 0.15 in the last column)

	INFL_10	INFL_20	FLAT_10	FLAT_20	DEFL_10	DEFL_20	FLAT_20, Rho = 0.15
<b>Periods</b>	10	20	10	20	10	20	20
<b>Natural discount rate</b>	$\beta=1$	$\beta=1$	$\beta=1/1.05$	$\beta=1/1.05$	$\beta=1/1.15$	$\beta=1/1.15$	$\beta=1/1.15$
<b>Predicted price change</b>	$(1+\rho)/\beta = 1.05$	$(1+\rho)/\beta = 1.05$	$(1+\rho)/\beta = 1$	$(1+\rho)/\beta = 1$	$(1+\rho)/\beta = 1.05/1.15 = 0.91$	$(1+\rho)/\beta = 1.05/1.15 = 0.91$	$(1+\rho)/\beta = 1$
<b>Predicted prices</b>	inflation	inflation	flat	flat	deflation	deflation	flat

**Table 2: Equilibrium Prices for the Six Treatments**

	INFL_10	INFL_20	FLAT_10	FLAT_20	DEFL_10	DEFL_20
1	10.50	5.25	12.95	8.02	18.19	14.59
2	11.02	5.51	12.95	8.02	16.61	13.32
3	11.58	5.79	12.95	8.02	15.17	12.16
4	12.16	6.08	12.95	8.02	13.85	11.10
5	12.76	6.38	12.95	8.02	12.64	10.14
6	13.40	6.70	12.95	8.02	11.54	9.26
7	14.07	7.04	12.95	8.02	10.54	8.45
8	14.77	7.39	12.95	8.02	9.62	7.72
9	15.51	7.76	12.95	8.02	8.79	7.05
10	16.29	8.14	12.95	8.02	8.02	6.43
11		8.55		8.02		5.87
12		8.98		8.02		5.36
13		9.43		8.02		4.90
14		9.90		8.02		4.47
15		10.39		8.02		4.08
16		10.91		8.02		3.73
17		11.46		8.02		3.40
18		12.03		8.02		3.11
19		12.63		8.02		2.84
20		13.27		8.02		2.59

**Table 3: Equilibrium Loans for the Six Treatments**

	INFL_10	INFL_20	FLAT_10	FLAT_20	DEFL_10	DEFL_20
1	1,100	50	1,590	605	2,639	1,917
2	1,260	105	1,670	635	2,454	1,760
3	1,433	165	1,753	667	2,288	1,616
4	1,621	232	1,841	700	2,139	1,485
5	1,823	304	1,933	735	2,005	1,366
6	2,042	383	2,029	772	1,885	1,258
7	2,278	469	2,131	811	1,778	1,160
8	2,533	563	2,237	851	1,684	1,071
9	2,807	665	2,349	894	1,601	991
10	3,103	776	2,467	938	1,528	918
11		896		985		852
12		1,026		1,035		792
13		1,167		1,086		739
14		1,320		1,141		690
15		1,485		1,198		647
16		1,663		1,257		609
17		1,855		1,320		574
18		2,063		1,386		544
19		2,286		1,456		517
20		2,527		1,528		493

**Table 4: Theoretical and Estimated Slope coefficients (t-statistics) in OLS Regressions of Price on Time**

	Theory	MI Agents	Avg. for Session	Run 1 A	Run 1 B	Run 2 A	Run 2 B
INFL_10	<b>0.64</b>	<b>-0.20</b>	<b>1.36</b>	1.77 (8.69)	1.68 (7.55)	0.98 (9.04)	1.00 (6.98)
INFL_20	<b>0.42</b>	<b>-0.20</b>	<b>0.75</b>	0.74 (6.91)	0.68 (8.61)	0.77 (9.89)	0.82 (8.87)
FLAT_10	<b>0.00</b>	<b>-0.20</b>	<b>0.05</b>	0.06 (0.70)	0.02 (0.46)	0.11 (1.10)	0.01 (0.10)
FLAT_20	<b>0.00</b>	<b>-0.20</b>	<b>0.02</b>	0.00 (0.12)	0.10 (2.03)	-0.01 (-0.58)	-0.01 (-0.20)
DEFL_10	<b>-1.12</b>	<b>-0.20</b>	<b>-0.92</b>	-0.82 (-12.87)	-0.99 (-7.36)	-1.03 (-7.94)	-0.83 (-9.96)
DEFL_20	<b>-0.61</b>	<b>-0.20</b>	<b>-0.51</b>	-0.46 (-25.72)	-0.50 (-19.96)	-0.55 (-14.67)	-0.54 (-13.15)

**Table 5: Money Holdings at the End of Period 10**  
(Treatments with uncertain horizon are shaded)

	<b>Theory</b>	<b>MI agents</b>	<b>Observed Average</b>	Run 1	Run 2
INFL_10	0	774	302	112	491
INFL_20	794	774	580	452	708
FLAT_10	0	774	144	177	111
FLAT_20	614	774	501	324	677
DEFL_10	0	774	185	346	23
DEFL_20	341	774	279	274	284
<b>Average_10</b>	<b>0</b>	<b>774</b>	<b>210</b>	<b>212</b>	<b>275</b>
<b>Average_20</b>	<b>583</b>	<b>774</b>	<b>453</b>	<b>350</b>	<b>556</b>

**Table 6: Percentage of Money Kept Unspent by Borrowers and Non-borrowers**

(Treatments with uncertain horizon are shaded)

(Averages across those periods when at least one trader was in the respective group)

	Percentage of money unspent by borrowers	Percentage of money unspent by Non-borrowers
INFL_10	2.5%	8.9%
INFL_20	7.3%	12.3%
FLAT_10	1.9%	9.4%
FLAT_20	2.8%	14.4%
DEFL_10	1.0%	15.3%
DEFL_20	1.1%	10.0%
<b>Average</b>	<b>2.8%</b>	<b>11.7%</b>

**Table 7: Efficiency: Average Points Earned per Period as Percentage of Maximum Possible for Borrowers and Non-borrowers**

(Treatments with uncertain horizon are shaded)

(Averages across those periods when at least one trader was in the respective group)

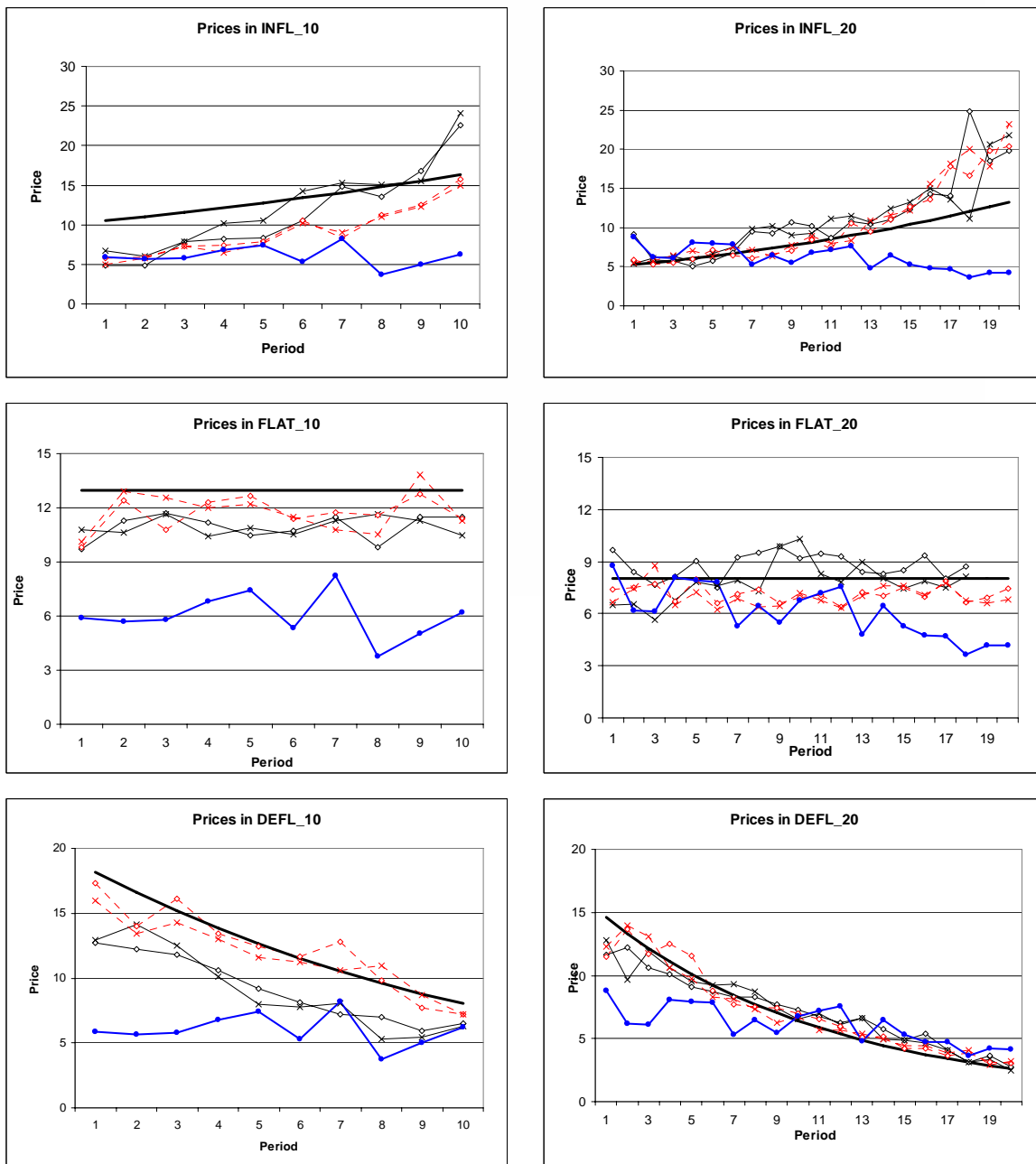
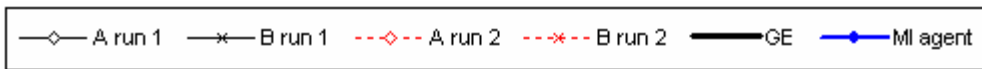
	Percentage of maximum possible points earned	Percentage of maximum possible points earned
INFL_10	104.32%	85.07%
INFL_20	96.95%	87.83%
FLAT_10	102.94%	87.57%
FLAT_20	100.08%	94.95%
DEFL_10	97.89%	99.63%
DEFL_20	97.88%	98.87%
<b>Average</b>	<b>99.98%</b>	<b>92.32%</b>



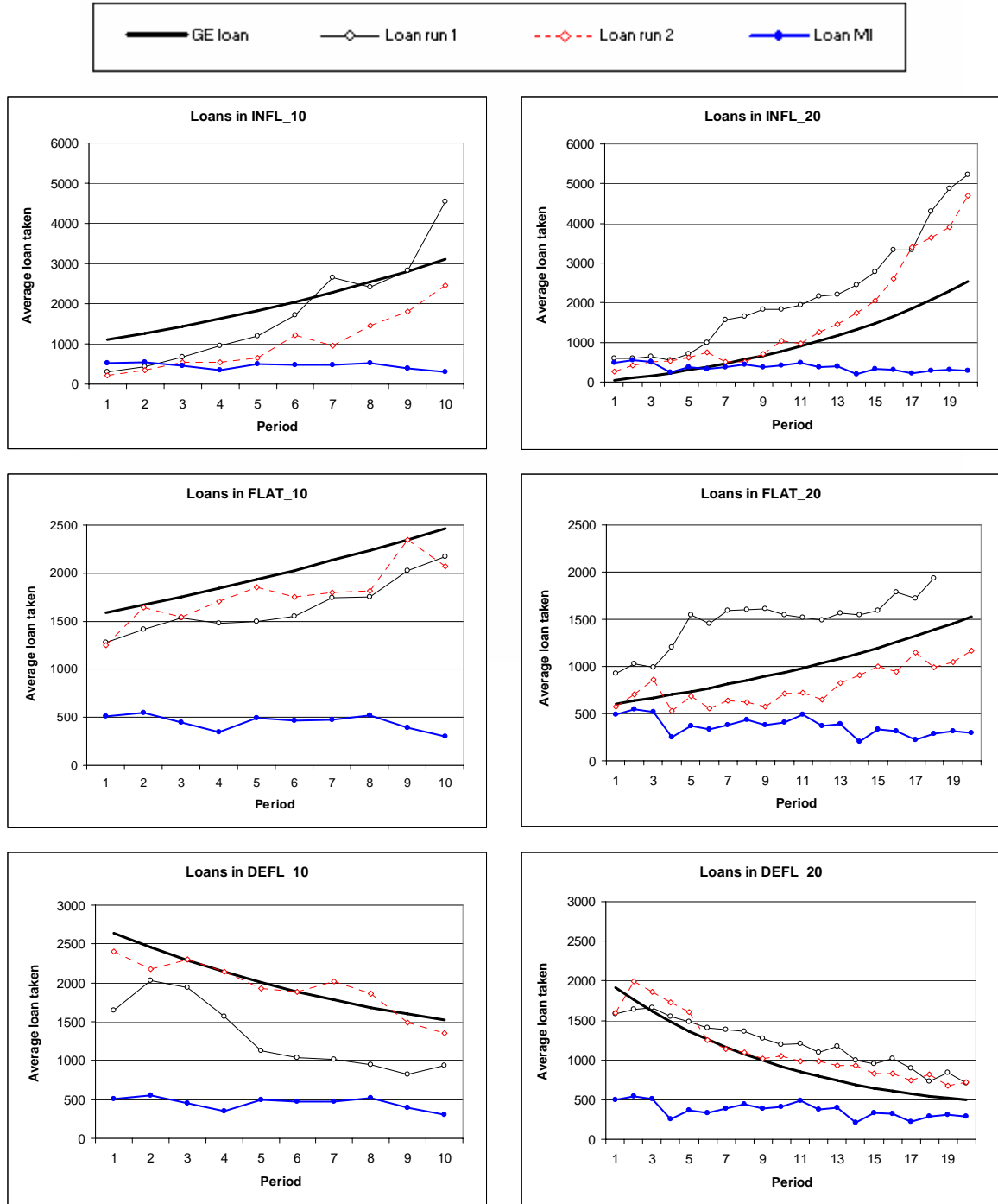
**Table 8: Money Holdings at the End**  
(Treatments with uncertain horizon are shaded)

	<b>Theory</b>	<b>MI agents</b>	<b>Observed Average</b>	Run 1	Run 2
INFL_10	0	774	302	112	491
INFL_20	0	612	-880	-1,179	-580
FLAT_10	0	774	144	177	111
FLAT_20	0	612	501	324	677
DEFL_10	0	774	185	346	23
DEFL_20	0	612	279	274	284

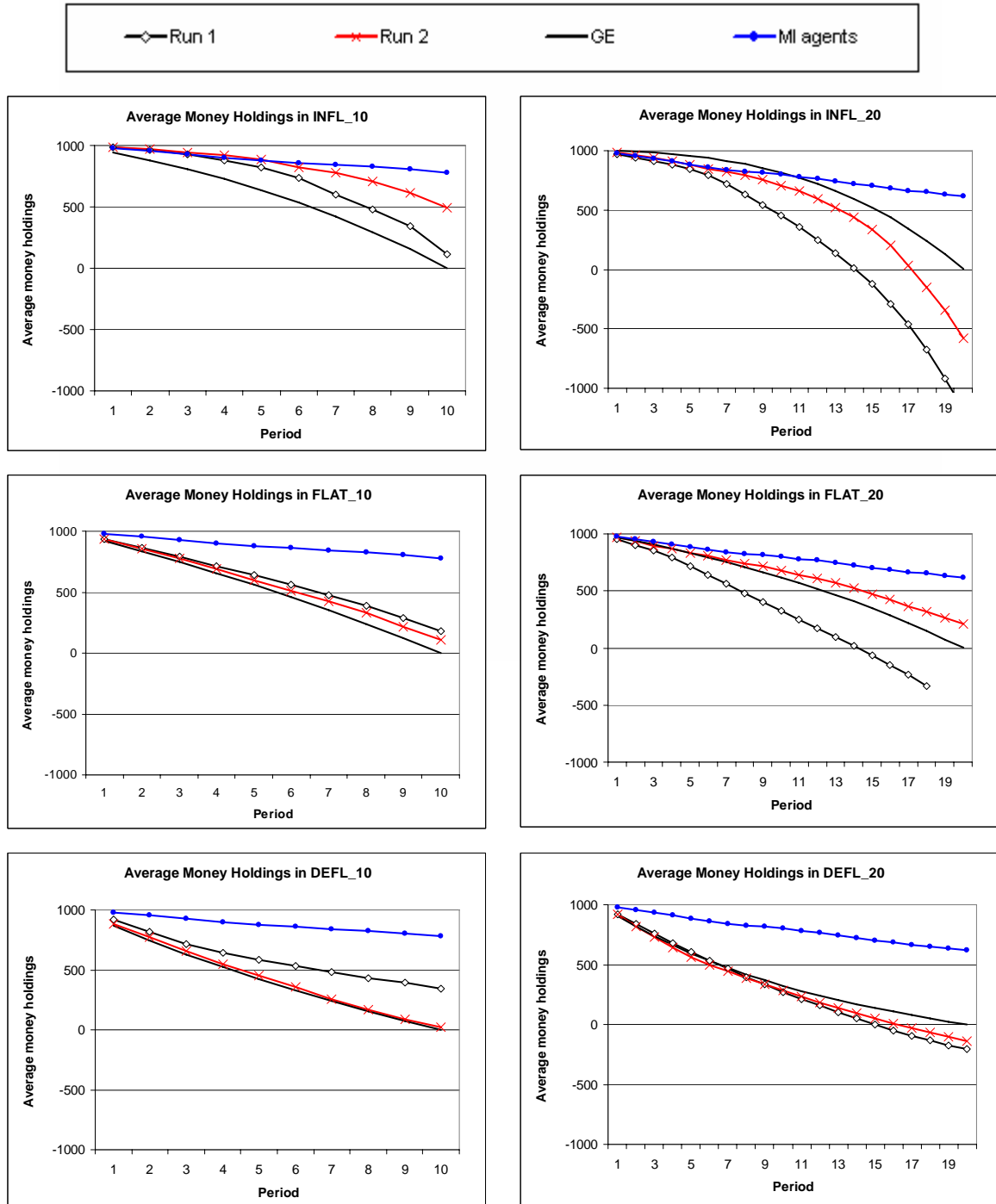
**Figure 1: Time Series of Observed Transaction Prices in the Six Treatments and in Simulations with Minimally Intelligent (MI) Agents Compared to GE Predictions**



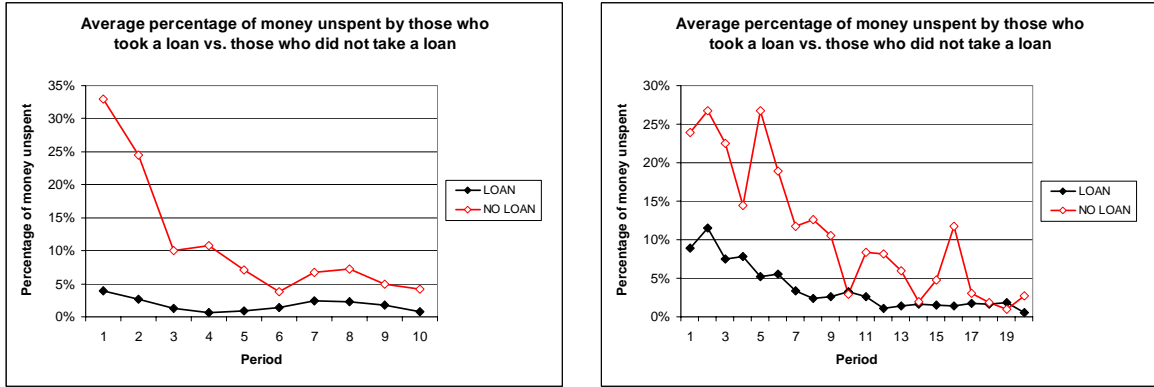
**Figure 2: Time Series of Observed Borrowings in the Six Treatments and in Simulations with Minimally Intelligent (MI) Agents Compared to GE Predictions**



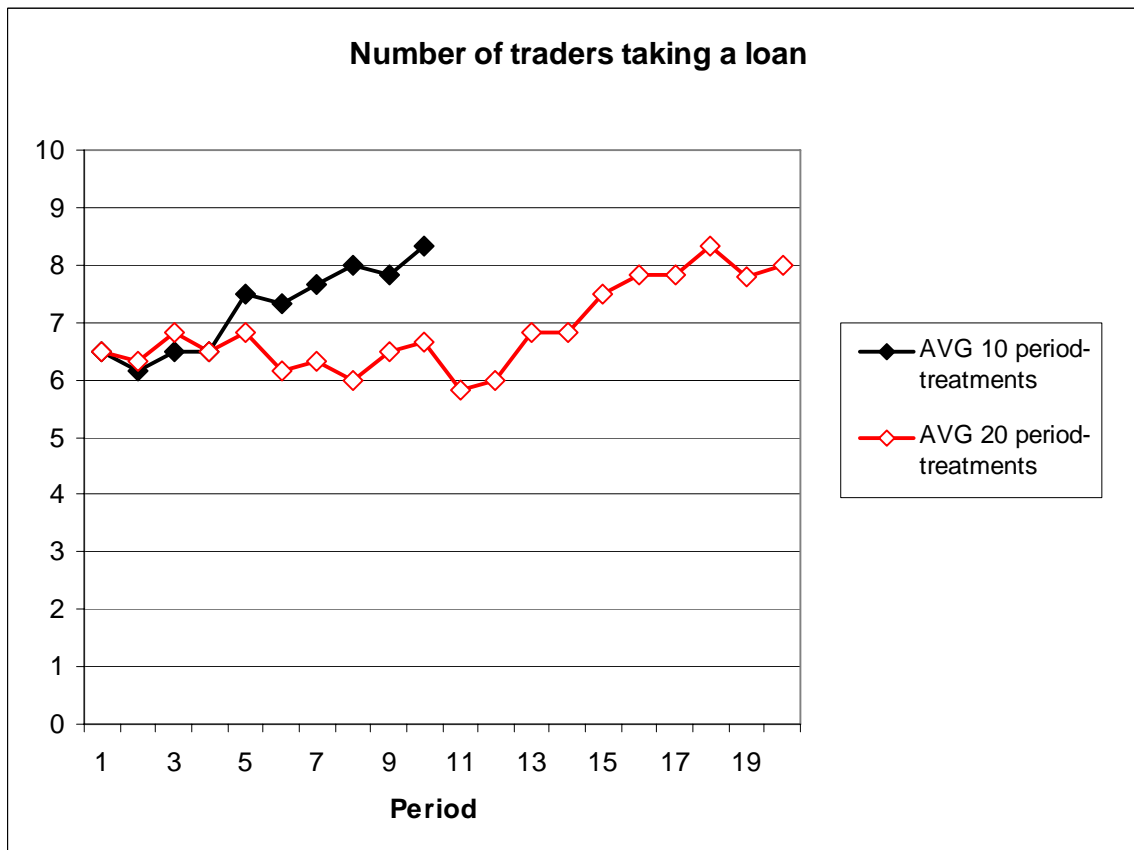
**Figure 3: Time Series of Average Money Balances in the Six Treatments and in Simulations with Minimally Intelligent (MI) Agents Compared to GE Predictions**



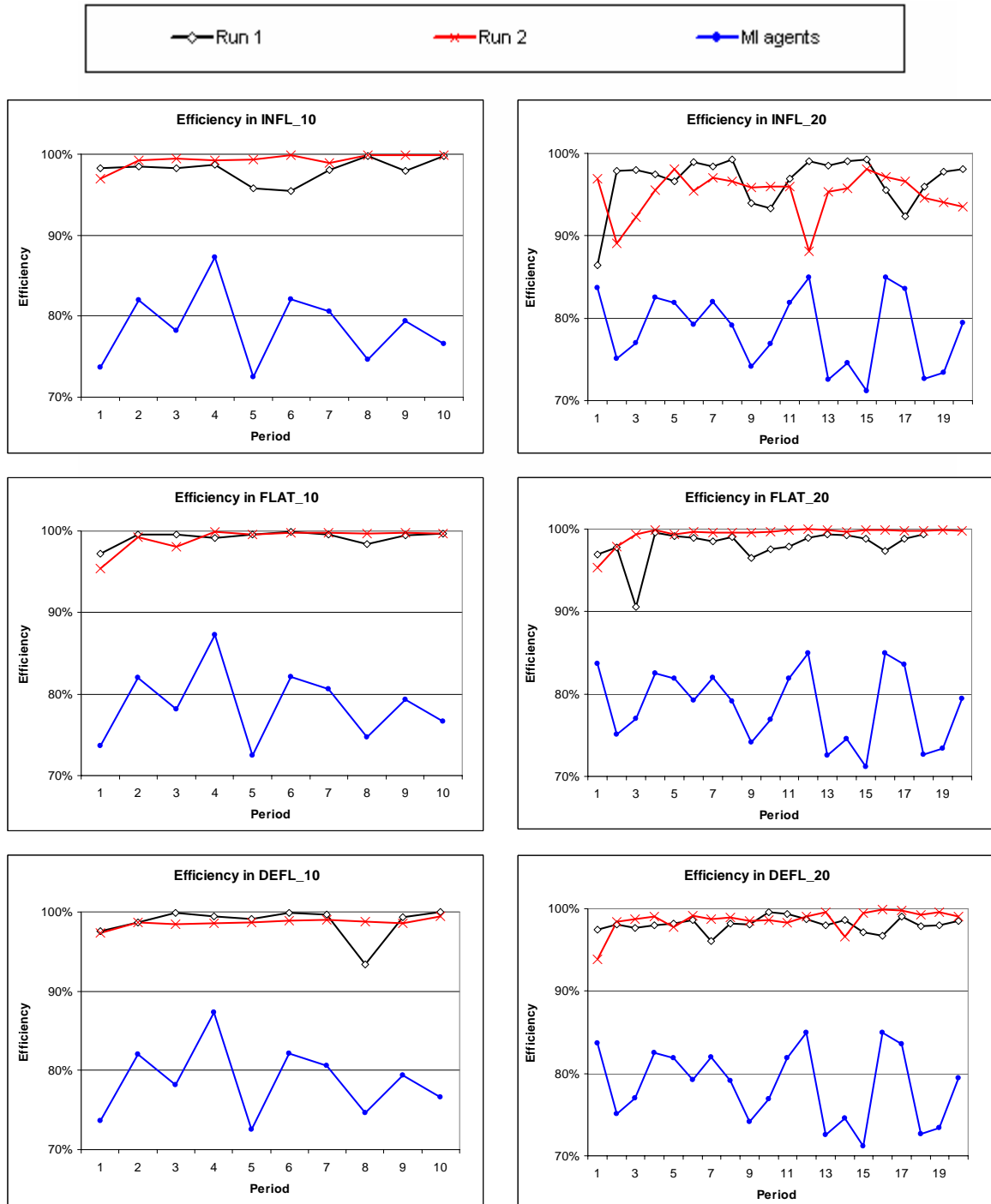
**Figure 4: Average Percentage of Money Kept Unspent by Borrowers vs. Non-borrowers in the 10-period Sessions (left) and the 20-period Sessions (right)**



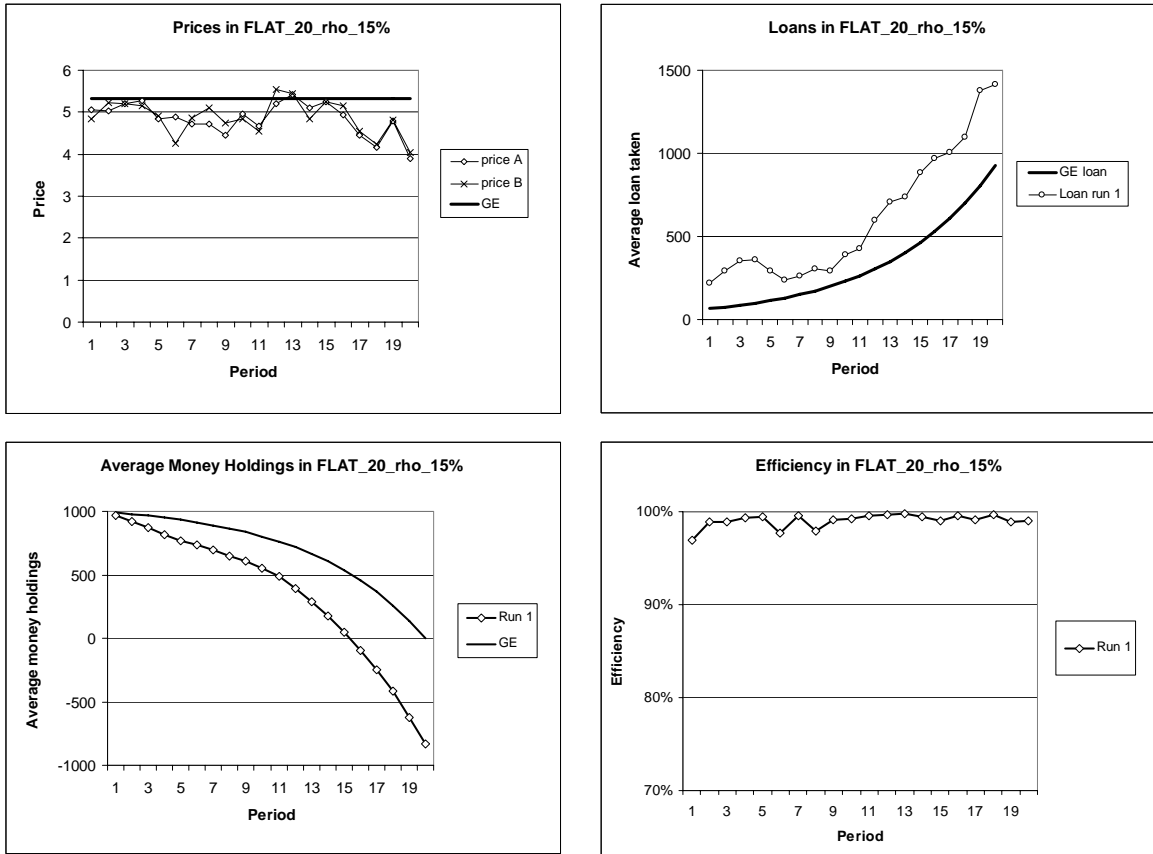
**Figure 5: Average Number of Borrowers by Period**



**Figure 6: Time Series of Allocative Efficiency in the Six Treatments and in Simulations with Minimally Intelligent (MI) Agents**



**Figure 7: Results for Control Treatment FLAT\_20\_rho\_15%**



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## Appendix A: Instructions for INFL\_10

### General

This is an experiment in market decision making. If you follow the instructions carefully and make good decisions, you will earn more money, which will be paid to you at the end of the run.

This run consists of 8 to 12 periods and has 10 participants. At the beginning of **each** period, five of the participants will receive as income the proceeds from selling 200 units of good A, for which they have ownership claim. The other five are entitled to the proceeds from selling 200 units of good B. In addition you will get 1,000 units of money at the start of the experiment. Depending on how many units of goods A and B you buy and on the proceeds from selling your goods and borrowing from a bank, this amount will change from period to period.

During each period we shall conduct a market in which the price per unit of A and B will be determined. All units of A and B will be sold at this price, and you can buy units of A and B at this price. The following paragraph describes how the price per unit of A and B will be determined.

In each period, you are asked to enter the amount of cash you are willing to pay to buy good A, and the amount you are willing to pay to buy good B (see the center of Screen 1). The sum of these two amounts cannot exceed your current holdings of money at the beginning of the period plus the amount you borrow from the bank. The interest rate for money borrowed is 5 percent per period. The computer will calculate the sum of the amounts offered by all participants for good A. ( $= \text{Sum}_A$ ). It will also calculate the total number of units of A available for sale ( $n_A$ , which will be 1,000 if we have five participants each with ownership claim for 200 units of good A). The computer then calculates the price of A,  $P_A = \text{Sum}_A/n_A$ .

If you offered to pay  $b_A$  to buy good A, you will get  $b_A/P_A$  units of good A.

The same procedure is carried out for good B.

Period 2	Remaining time [sec]: 28
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You have:

- Ownership claims for units of good A: 0
- Ownership claims for units of good B: 200
- Units of money: 1180
- Interest rate per period: 0.05

Amount you wish to borrow

Amount you offer to pay to buy A

Amount you offer to pay to buy B

The number of units of A and B you buy (and consume), will determine the amount of points you earn for period t:

$$\text{Points earned}_t = 10 * (b_A/P_A * b_B/P_B)^{0.5} * \text{beta}^{t-1}$$

In this session, beta =1 which means that the last term  $\text{beta}^{t-1}$  is always equal to 1.

*Example: If you buy 100 units of A and 100 units of B in the market you earn*

$$10 * (100 * 100)^{0.5} = 1,000 \text{ points.}$$

Your money at the end of a period (=starting money for the next period) will be :

- your money at the start of the period
- plus the amount you borrow
- plus money from the sale of your initial entitlement to proceeds from A or B
- minus the amount you pay to buy A and B
- minus interest on your loan
- minus repayment of the money borrowed

If you end a period with negative money holdings, you have to take a loan of at least this amount to proceed (roll the loan over). Your final money holdings will be relevant to your score only after the close of the last period. If you have any money left over it is worthless to you. If your money is not enough to pay back any loan then your remaining debt will be divided by 4 and this number will be subtracted from your total points earned. Screen 2 shows an example of calculations for Period 2. There are 10 participants in the market, and half of them have 200 units of A, the other half 200 units of B. Here we see a subject entitled to proceeds from 200 units of good B.

Information on bids and transactions in good A

Information on bids and transactions in good B

Earnings calculation

Cumulative earnings so far. This number/1000 will be the US-\$ you get

Period	2	Remaining time [sec]: 12																																																						
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1	6.5	62	5.9	85	1280	722	722																																																	
2	6.7	104	6.2	104	1166	1044	1766																																																	

The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of run. At the end they will be converted into real Dollars at the rate of 400 points = 1 US-\$ and this amount will be paid out to you.

### How to calculate the points you earn:

The points earned are calculated are calculates with the following formula:

$$\text{Points earned} = 10 * (b_A/P_A * b_B/P_B)^{0.5}$$

To give you an understanding for the formula the following Table might be useful. It shows the resulting points from different combinations of goods A and B. It is obvious, that more goods mean more points, however, to get more goods you usually have to pay more, thereby reducing your money balance, which will limit your ability to buy in later periods.

		Units of good B you buy and consume										
Units		0	25	50	75	100	125	150	175	200	225	250
of A you buy and con- sume	0	0	0	0	0	0	0	0	0	0	0	0
	25	0	250	354	433	500	559	612	661	707	750	791
	50	0	354	500	612	707	791	866	935	1000	1061	1118
	75	0	433	612	750	866	968	1061	1146	1225	1299	1369
	100	0	500	707	866	1000	1118	1225	1323	1414	1500	1581
	125	0	559	791	968	1118	1250	1369	1479	1581	1677	1768
	150	0	612	866	1061	1225	1369	1500	1620	1732	1837	1936
	175	0	661	935	1146	1323	1479	1620	1750	1871	1984	2092
	200	0	707	1000	1225	1414	1581	1732	1871	2000	2121	2236
	225	0	750	1061	1299	1500	1677	1837	1984	2121	2250	2372
	250	0	791	1118	1369	1581	1768	1936	2092	2236	2372	2500

Examples:

- 1) If you buy 50 units of good A and 75 units of good B and both prices are 20, then your points from consuming the goods are 612. Your net change in money is 200 (A or B) \* 20 = 4,000 minus 50 \* 20 minus 75 \* 20 = 1,500, so you have 1,500 more to spend or save in the next period.
- 2) If you buy 150 units of good A and 125 units of good B and both prices are 20, then your points from consuming the goods are 1,369. Your net cash balance is 200 (A or B) \* 20 = 4,000 minus 150 \* 20 minus 125 \* 20 = -1,500, so you have 1,500 less to spend or save in the next period.